GPDs through Universal Moment Parametrization

Quark and Gluon GPD Global Analysis with DVCS and DVMP at NLO

Fatma Aslan

Behalf of the GUMP Collaboration
(Yuxun Guo, Gabriel Santiago, Xiangdong Ji)

CENTER for NUCLEAR FEMTOGRAPHY

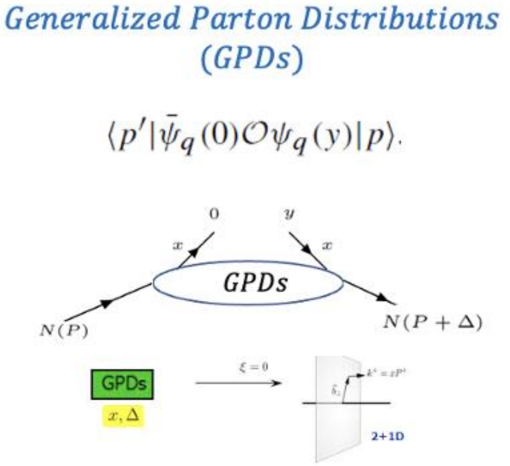
Jefferson Lab

Outline

- *GPDs and Challenges of the GPD Pheno*
  - *Parametrization*
  - *Lack of experimental data*
- *The GUMP Project*
- *Conformal wave expansion of GPDs*
- *DVMP and DVCS*
- ρ *meson production fits using H1 data*

GPDs and the Challenges of GPD pheno

Parametrizing the GPDs

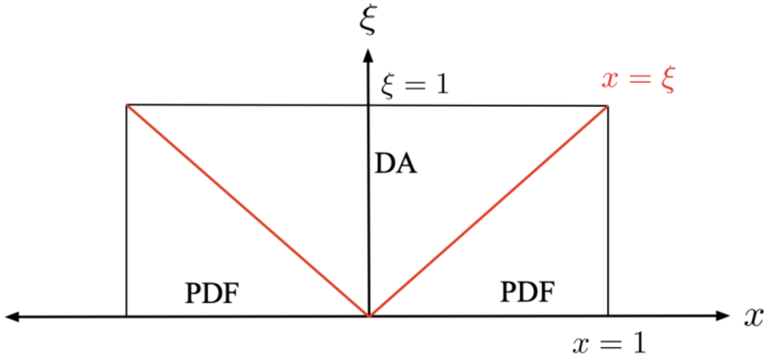


GPDs give information about

- *Longitudinal momentum distribution*
- *Transverse spatial structure*
- *Spin and orbital angular momentum*
- *Nucleon mass*
- *Mechanical properties*
- ……

That much information comes at a cost!

Kinematic regions of $GPDs(x, \xi, t)$



Constraints on $GPDs(x, \xi, t)$

- **Kinematical limits of the variables:** $x \in [-1, 1],\ \xi \in [-1, 1],\ t \leq 0,\ |t| \geq |t|_{min} = \dfrac{4\xi^2 M^2}{1-\xi^2}$
- **The forward limit:** $\lim_{\xi,t\to 0} GPD(x, \xi, t) = PDF(x)$
- **Continuity at $x = \xi$:** Continuity between the two kinematic regions
- **Polynomiality:** $F_n(\xi, t) = \sum_{k=0,even}^{n} \xi^k F_{n,k}(t)$

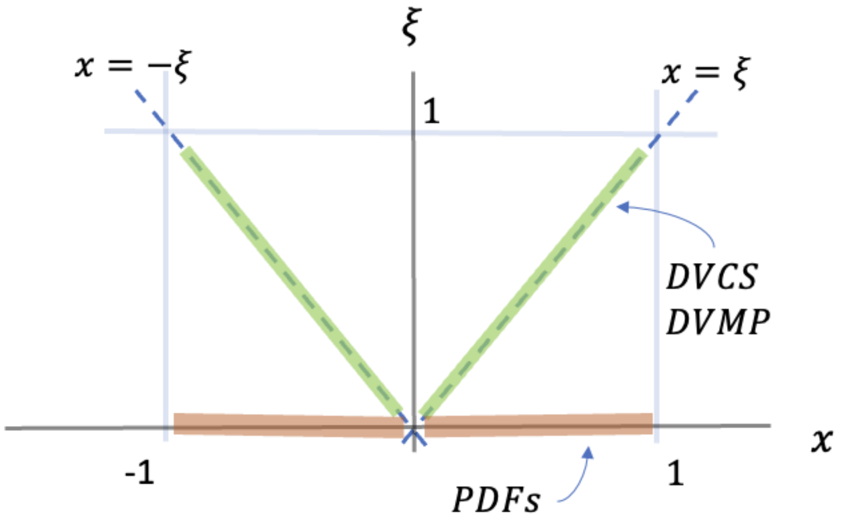
We need to parametrize a high dimensional function of three variables, (x, ξ, t), that is defined in two kinematical regions and must obey certain theoretical constraints

GPDs and the Challenges of GPD pheno

Lack of experimental data

Experimental data

$GPD\ (x, \xi)$

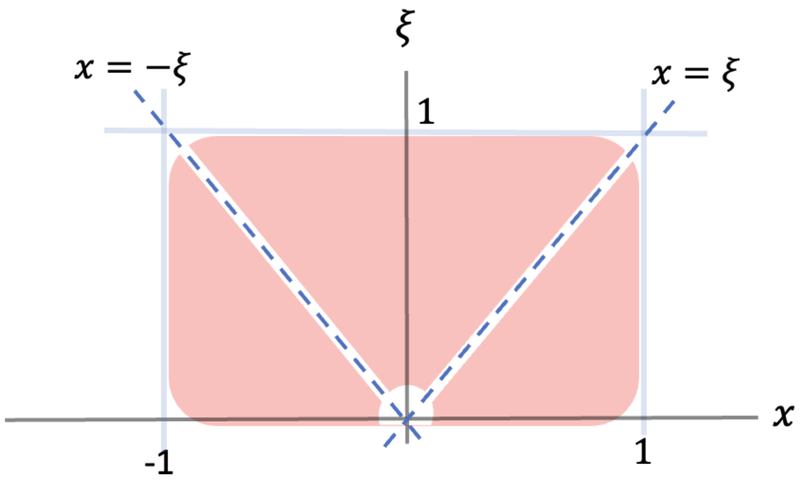


Experimental data give information only on $x = \pm\xi\ *$ *and* $\xi = 0$

$$\mathcal{F}^q(x_B, t, Q^2) \overset{\text{LO}}{\propto} i\pi \underbrace{F^q(x, x, t, \mu_F^2)}_{GPD\ at\ x=\xi} + \underbrace{\text{PV}\int_{-1}^{1} dx \, \frac{F^q(x, \xi, t, \mu_F^2)}{\xi - x}}_{\text{negligible at high energies}}$$

Lattice data

$GPD\ (x, \xi)$



Lattice data has limitations on $x = \pm\xi$

$*$ *Neglecting the real parts of the CFFs and TFFs*

The GUMP Project

Experimental data

$GPD\ (x,\xi)$

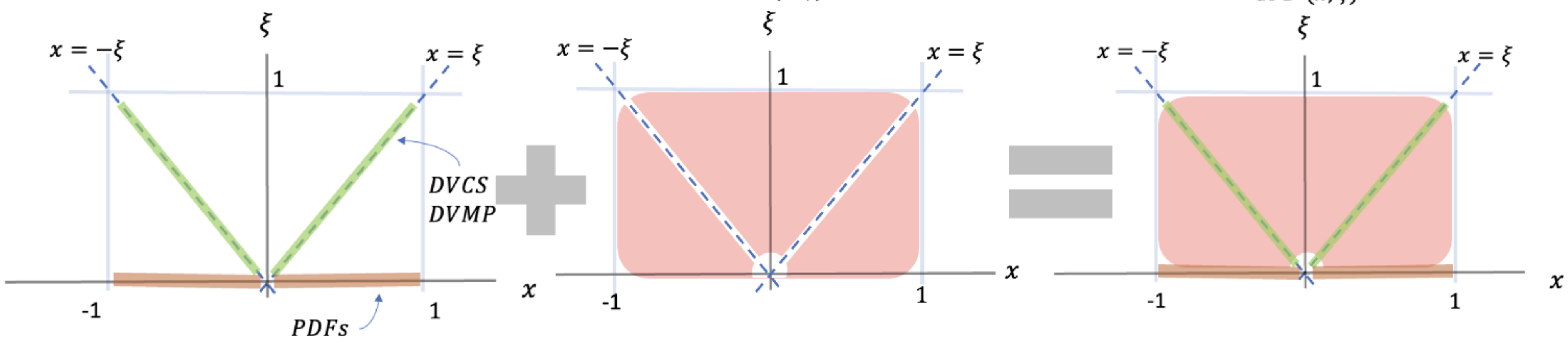
Lattice data

$GPD\ (x,\xi)$

THE GUMP PROJECT

Experimental + Lattice data

$GPD\ (x,\xi)$



Experimental data give information only on $x = \pm\xi *$ *and* $\xi = 0$

Lattice data has limitations on $x = \pm\xi$

Experimental and Lattice data are complementary !

*Combining all the possible constraints on GPDs including both experimental and lattice calculations and parametrizing the t dependence of the **conformal moments** of valence, sea distributions*

Conformal wave expansion of GPDs

- *Consider we expand the GPD in terms of a complete set of polynomials*: $F(x,\xi,t)=\sum \rho_C(x)C_i(x)F_i^C(\xi,t)$,

- *Then the* $F_i^C(\xi,t)$ *is simply the moment of* $F(x,\xi,t)$: $F_i^C(\xi,t)=\int_{-1}^{1}\mathrm{d}xC_i(x)F(x,\xi,t)$.

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- ❑ *There are infinite possible options for* $C_i(x)$; *which choice is the most suitable one for GPDs?*

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❑ *There are infinite possible choices for* $C_i(x)$; *which choice is the most suitable one for GPDs?*

- *Diagonal evolution at LO* $\int_{-1}^{1} \frac{\mathrm{d}x'}{|\xi|} \left[V^{(0)} \left(\frac{x}{\xi}, \frac{x'}{\xi} \right) \right]_+ C_j^{\frac{3}{2}} \left(\frac{x}{\xi} \right) = \gamma_j C_j^{\frac{3}{2}} \left(\frac{x'}{\xi} \right)$;

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LO Evolution:
The evolution is ***diagonal*** *in conformal space,*
and there is ***no mixing*** *of conformal moments.*

NLO and Beyond:
The evolution is ***non − diagonal*** *in conformal space,*
there is ***mixing*** *of conformal moments*
but it is still easier than the coordinate space evolution
because it is matrix multiplicative

$$\frac{d}{d\ln Q^2}\begin{pmatrix} \mathcal{F}_1 \\ \mathcal{F}_2 \\ \vdots \\ \mathcal{F}_n \end{pmatrix} = \frac{\alpha_s(Q^2)}{2\pi}\begin{pmatrix} \gamma_1^{(0)} & 0 & 0 & \dots & 0 \\ 0 & \gamma_2^{(0)} & 0 & \dots & 0 \\ 0 & 0 & \gamma_3^{(0)} & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & \gamma_n^{(0)} \end{pmatrix}\begin{pmatrix} \mathcal{F}_1 \\ \mathcal{F}_2 \\ \vdots \\ \mathcal{F}_n \end{pmatrix} + \left(\frac{\alpha_s(Q^2)}{2\pi}\right)^2 \begin{pmatrix} \gamma_{11}^{(1)} & \gamma_{12}^{(1)} & \dots & \gamma_{1n}^{(1)} \\ 0 & \gamma_{22}^{(1)} & \dots & \gamma_{2n}^{(1)} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \gamma_{nn}^{(1)} \end{pmatrix}\begin{pmatrix} \mathcal{F}_1 \\ \mathcal{F}_2 \\ \vdots \\ \mathcal{F}_n \end{pmatrix} + \cdots$$

Conformal wave expansion of GPDs

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$\rho_j(x,\xi)$: Partial wave function

Conformal moment

$$F(x,\xi,t) = \sum_{j=0}^{\infty} \xi^{-j-1} \frac{2^j \Gamma\left(\frac{5}{2}+j\right)}{\Gamma(\frac{3}{2})\Gamma(j+3)} \left[1 - \left(\frac{x}{\xi}\right)^2\right] C_j^{\frac{3}{2}}\left(\frac{x}{\xi}\right) \mathcal{F}_j(\xi,t) \quad \text{for } |x| < \xi \, ;$$

Normalization constant to match the Mellin moment in the forward limit

Gegenbauer polynomials

Weight function of the Gegenbauer polynomials

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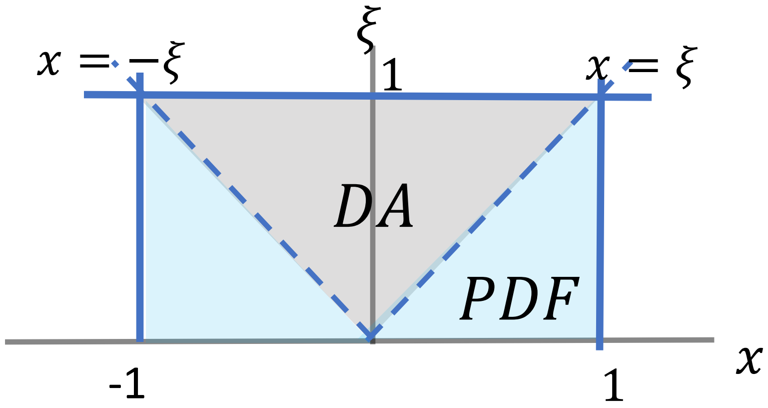
This series diverges!
Cannot represent GPDs



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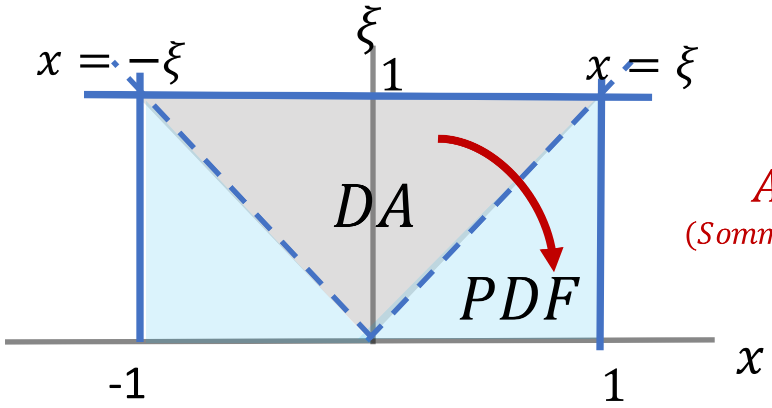

Analytic continuation
(Sommerfeld − Watson Transformation)

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Analytic continuation
(Sommerfeld − Watson Transformation)

Mellin Barnes Integral: $F(x,\xi,t)=\frac{1}{2i}\int_{c-i\infty}^{c+i\infty}dj\,\frac{p_j(x,\xi)}{\sin(\pi[j+1])}\mathcal{F}_j(\xi,t)$

Parametrization of GPDs

Inverse conformal transform

$$F(x,\xi,t) = \frac{1}{2i}\int_{c-i\infty}^{c+i\infty} dj\, \frac{p_j(x,\xi)}{\sin(\pi[j+1])}\mathcal{F}_j(\xi,t)$$

↓ *polynomiality*

$$\mathcal{F}_j(\xi,t) = \sum_{k=0,\text{even}}^{j+1} \xi^k \mathcal{F}_{j,k}(t)$$

↓ *parametrization*

$$\mathcal{F}_{j,k}(t) = \underbrace{\sum_{i=1}^{i_{\max}} N_{i,k} B(j+1-\alpha_{i,k}, 1+\beta_{i,k})}_{\text{Common ansatz for PDFs}} \underbrace{\frac{j+1-k-\alpha_{i,k}}{j+1-k-\alpha_{i,k}(t)}}_{\text{Regge Trajectory}} \underbrace{\beta(t)}_{\text{Observed exp. fall off}}$$

Common ansatz for PDFs

$$f(x) = \sum_{i=1}^{i_{\max}} N_i x^{-\alpha_i}(1-x)^{\beta_i}$$

Regge Trajectory

$$x^{-\alpha_i(t)}$$

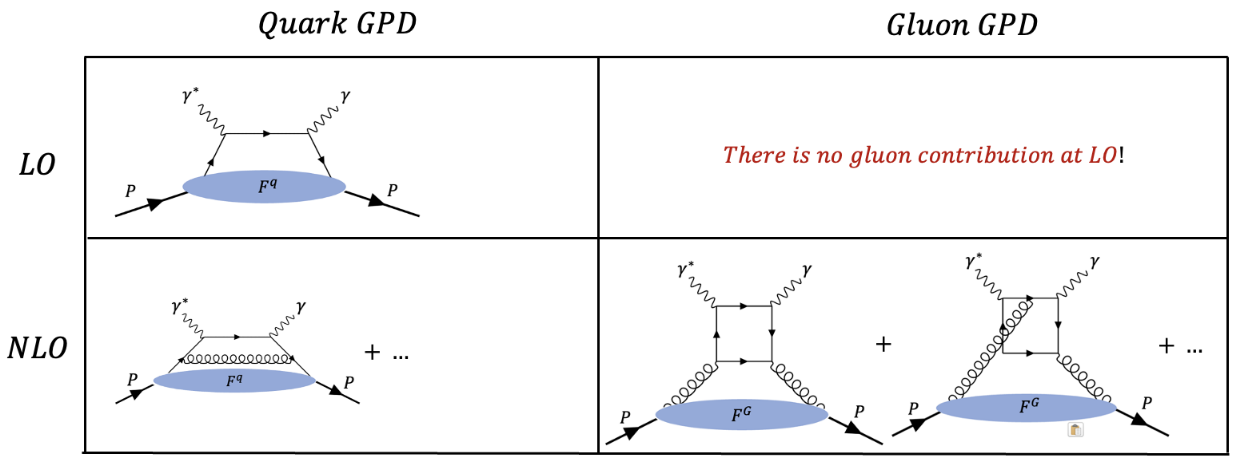
Observed exp. *fall off*

$$e^{-b|t|}$$

DVCS and DVMP; complementary rather than redundant

DVCS

- *The gluons start contributing the hard part at NLO. Their impact at LO arises only through evolution.*

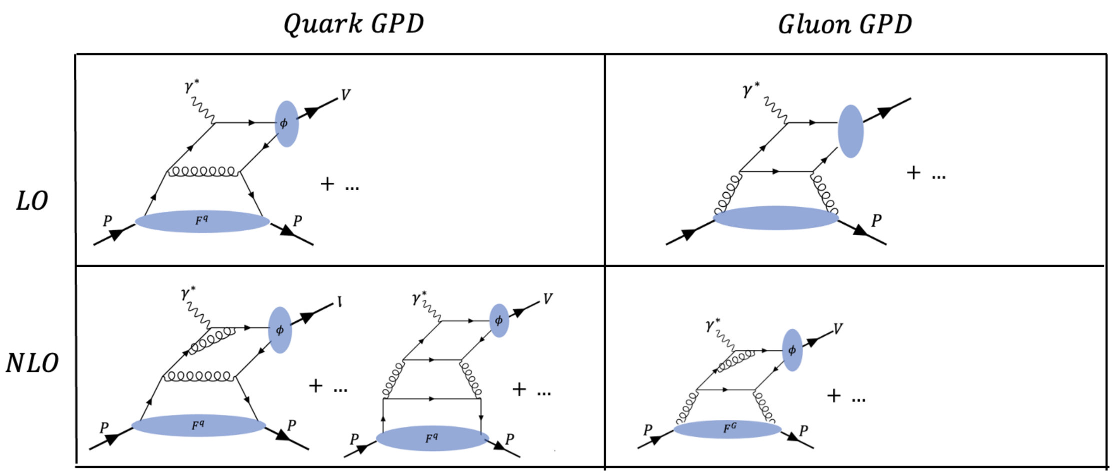


- *Contributions from vector GPDs H and E, and axial vector GPDs $\tilde{H}$ and $\tilde{E}$*
- *At leading twist for small x_B*

$$\frac{d\sigma^{\gamma}}{dt} = \pi\alpha_{\rm em}^2 \frac{x_{\rm B}^2}{Q^4}\left\{ \left|\mathcal{H}(x_{\rm B},t,Q^2)\right|^2 - \frac{t}{4M_{\rm p}^2}\left|\mathcal{E}(x_{\rm B},t,Q^2)\right|^2 + \left|\widetilde{\mathcal{H}}(x_{\rm B},t,Q^2)\right|^2 \right\}$$

DVV_LP

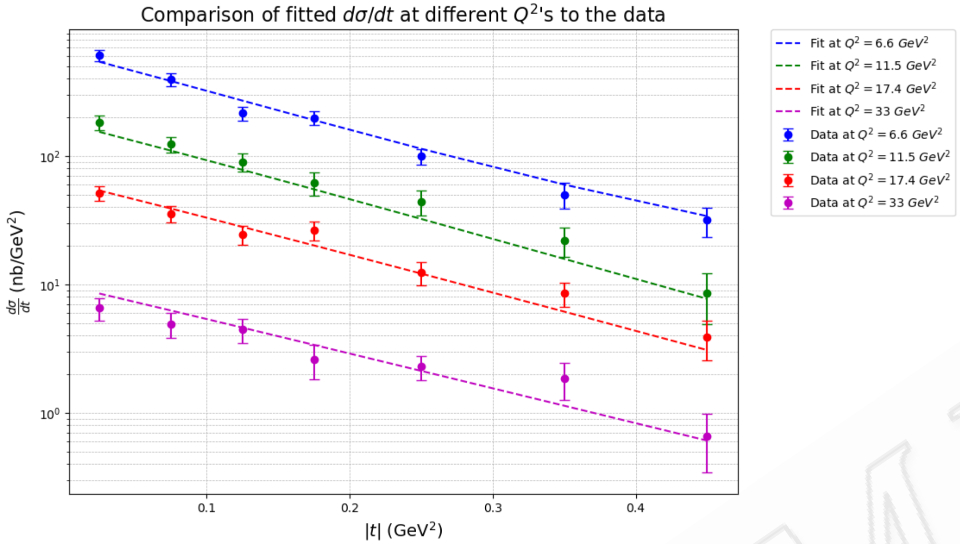
- *The gluons start contributing to the hard part at LO.*

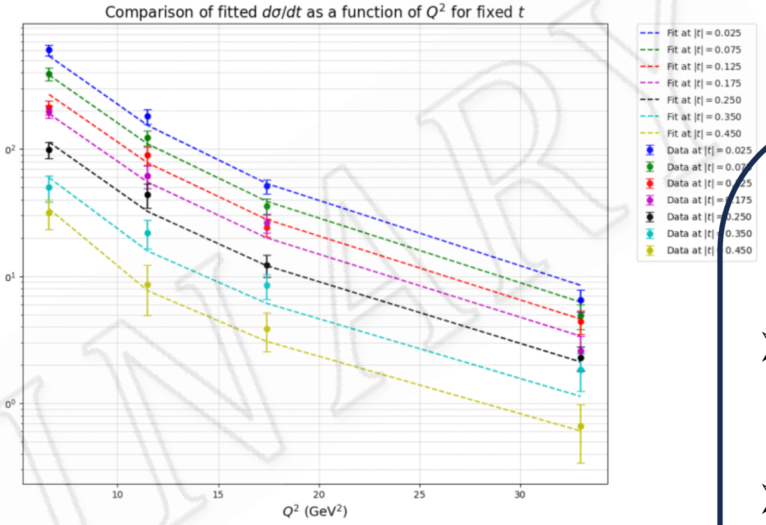


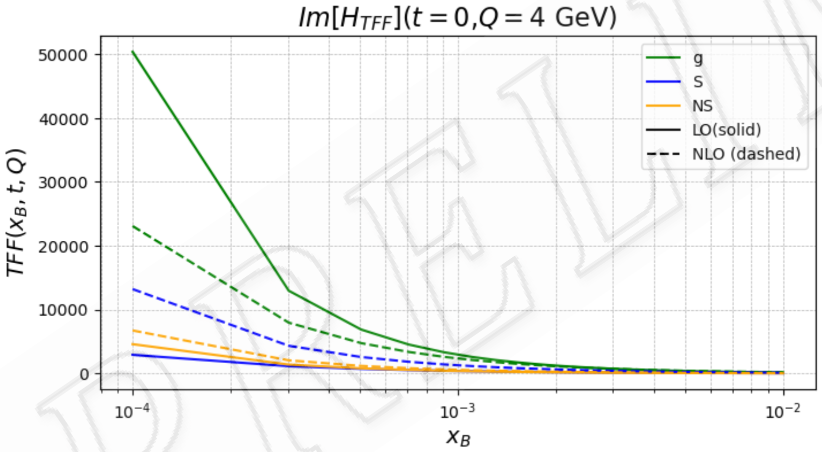
- *Vector GPDs H and E contribute to vector meson production.*
- *At leading twist for small x_B*

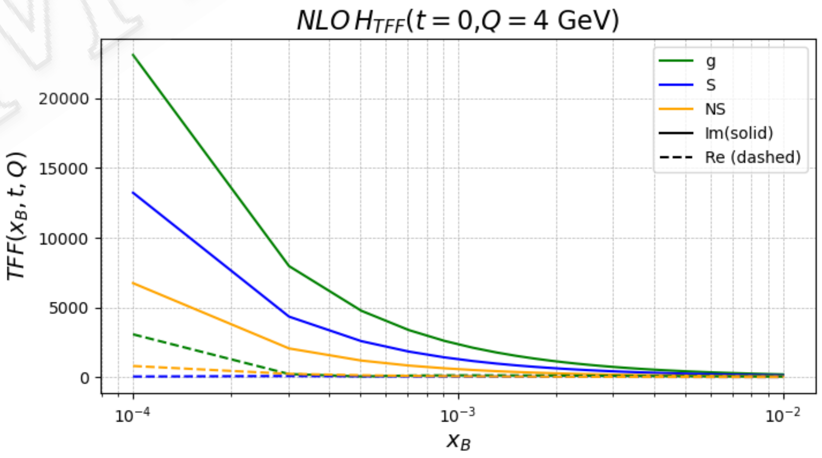
$$\frac{d\sigma^{\rm V_L}}{dt} = 4\pi^2\alpha_{\rm em} \frac{x_{\rm B}^2}{Q^4}\left\{ \left|\mathcal{H}_{\rm V_L}(x_{\rm B},t,Q^2)\right|^2 - \frac{t}{4M_{\rm p}^2}\left|\mathcal{E}_{\rm V_L}(x_{\rm B},t,Q^2)\right|^2 \right\}$$

ρ meson production fits using H1 data









$$\mathcal{F}^q(x_B, t, Q^2) \overset{\text{LO}}{\propto} \underbrace{i\pi\, F^q(x, x, t, \mu_F^2)}_{GPD\ at\ x=\xi} + \underbrace{\text{PV}\int_{-1}^{1} dx\, \frac{F^q(x, \xi, t, \mu_F^2)}{\xi - x}}_{\text{negligible at high energies}}$$

Conclusions and outlook

- *NLO corrections are important and significant*
- *DVCS and DVMP are complementary*
  - *Including DVCS and possibly lattice data*
  - *Incorporating the ZEUS data with H1 data for DVMP*

Thanks for your attention