A New Look to Azimuthal Modulations and Extraction of GPDs in DVCS

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papers in preparation

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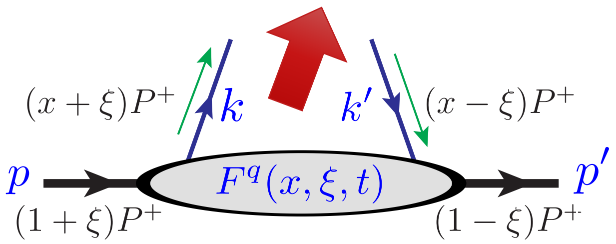
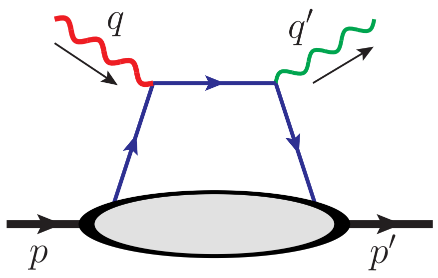
DVCS and GPDs

❑ Generalized parton distribution (GPD)



$$F^q(x,\xi,t) = \int \frac{\mathrm{d}z^-}{4\pi} e^{-ixP^+z^-} \langle p' | \bar{q}(z^-/2)\gamma^+ q(-z^-/2) | p \rangle$$
$$= \frac{1}{2P^+}\left[H^q(x,\xi,t)\,\bar{u}(p')\gamma^+ u(p) - E^q(x,\xi,t)\,\bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m} u(p)\right]$$

❑ Deeply virtual Compton scattering (DVCS)



$$T^{\mu\nu}(p,p',q) = i\int d^4z\, e^{i(q+q')\cdot z}\langle p'|\mathcal{T}\{J^\nu(z/2)J^\mu(-z/2)\}|p\rangle$$

(depends on the choice of n)

$$\simeq \frac{\bar{u}(p',s')}{2P\cdot n}\left\{\left[\mathcal{H}(\xi,t)\gamma\cdot n - \mathcal{E}(\xi,t)\frac{i\sigma^{n\Delta}}{2m}\right](g_\perp^{\mu\nu}) + \left[\widetilde{\mathcal{H}}(\xi,t)\gamma\cdot n\gamma_5 - \widetilde{\mathcal{E}}(\xi,t)\frac{\gamma_5\Delta\cdot n}{2m}\right](-i\,\epsilon_\perp^{\mu\nu})\right\} u(p,s)$$

Factorize into GPDs

$\Delta = p - p'$

$P = (p+p')/2$

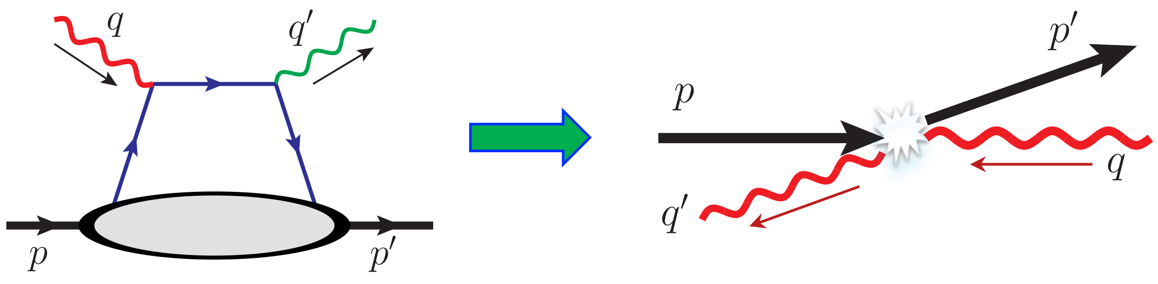
$$\{\mathcal{H},\mathcal{E}\}(\xi,t) \equiv \sum_q e_q^2 \int_{-1}^{1} dx\,\{H^q,E^q\}(x,\xi,t)\left[\frac{1}{x-\xi+i\epsilon} + \frac{1}{x+\xi-i\epsilon}\right]$$
$$\{\widetilde{\mathcal{H}},\widetilde{\mathcal{E}}\}(\xi,t) \equiv \sum_q e_q^2 \int_{-1}^{1} dx\,\{\widetilde{H}^q,\widetilde{E}^q\}(x,\xi,t)\left[\frac{1}{x-\xi+i\epsilon} - \frac{1}{x+\xi-i\epsilon}\right]$$

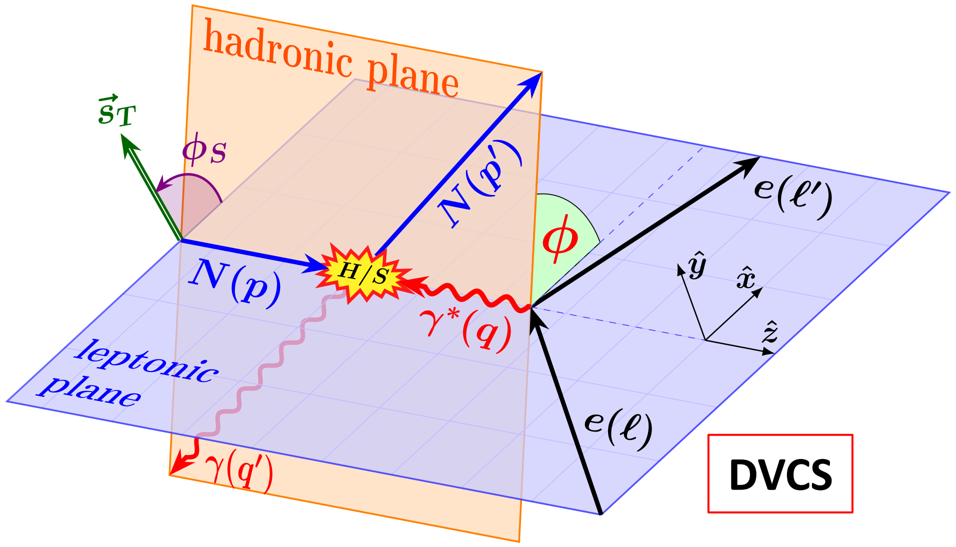
← GPD moments

- Both Re and Im parts
- 8 real dof's in total!

How to separate the GPD moments in DVCS?

❑ Azimuthal dependence in the Breit frame?





Breit frame observables: $(x_B,\, Q^2,\, t,\, \phi_S,\, \phi)$

$$\Rightarrow \frac{d\sigma}{dx_B\, dQ^2\, dt\, d\phi_S\, d\phi}$$

[A.V. Belitsky et al., 2002]
[B. Kriesten et al., 2020, 2022]
[Y. Guo et al., 2021, 2022]
…

ϕ dependence in the DVCS amplitude: $e^{i(\lambda_N - \lambda_{\gamma^*})\phi}$ + polarization ⇒ separate GPDs (?)

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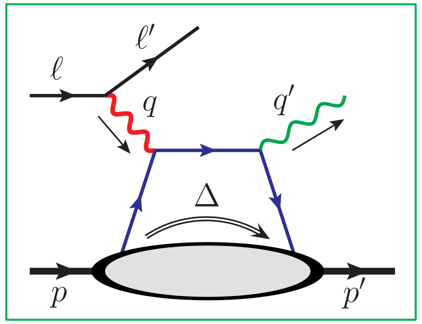
Bethe-Heitler subprocess!

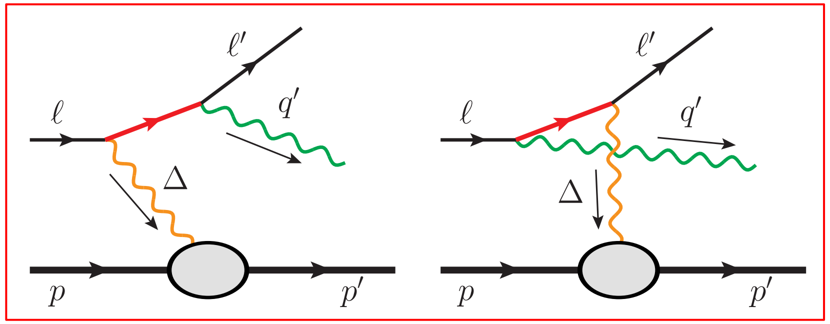
DVCS is not a physical process

$$N(p) + e(\ell) \to N(p') + e(\ell') + \gamma(q')$$

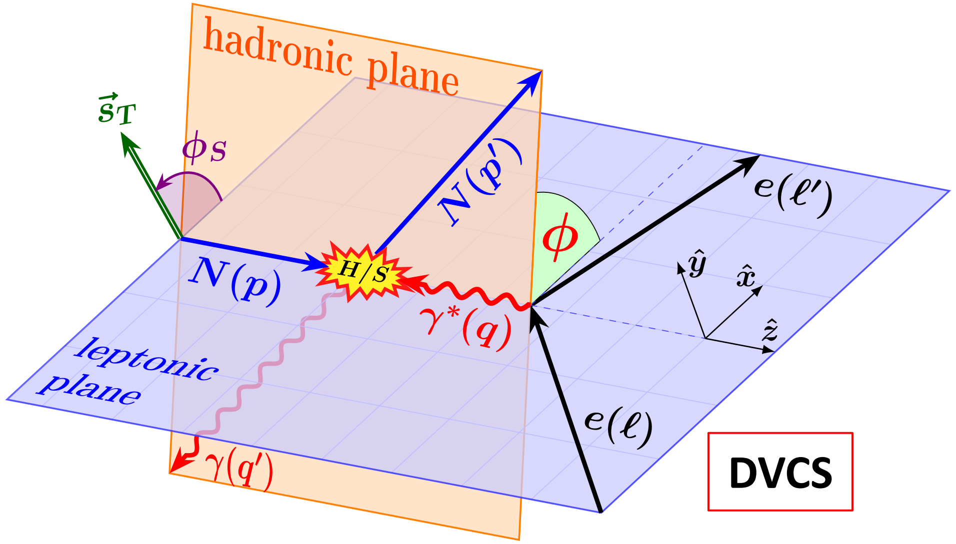
$$\Delta = p' - p$$

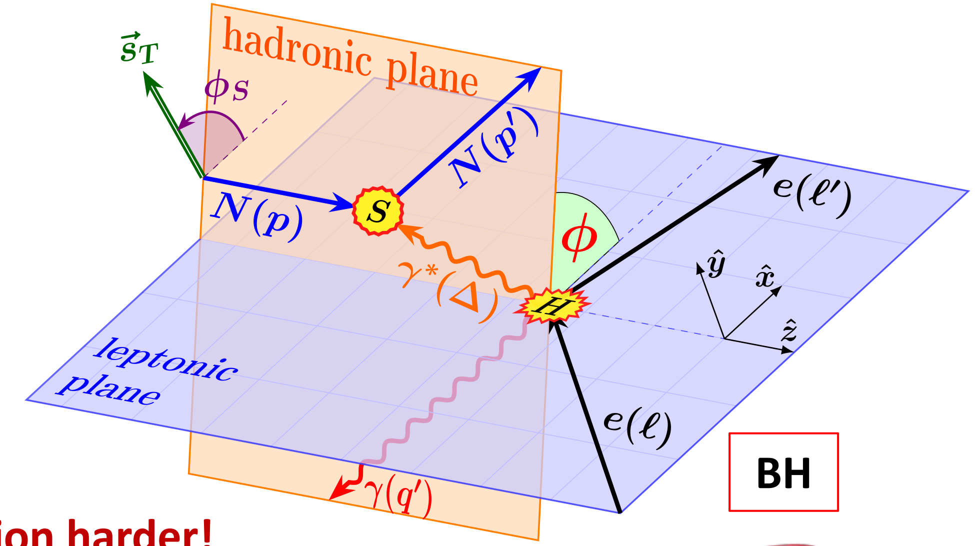


DVCS



Bethe-Heitler (BH) process





Breit frame is *NOT* convenient for describing BH

- The virtual photon $\gamma^*(q)$ does not exist
- The z axis is purely kinematical
- ϕ dependence in the denominators

$$\mathcal{P}_1 = (\ell - \Delta)^2 = -2\ell \cdot \Delta + t \quad \supset 2\ell_x \, \Delta_x \propto \cos\phi$$

$$\mathcal{P}_2 = (\ell - q')^2 = -2\ell \cdot q' \quad \supset 2\ell_x \, q'_x \propto \cos\phi$$

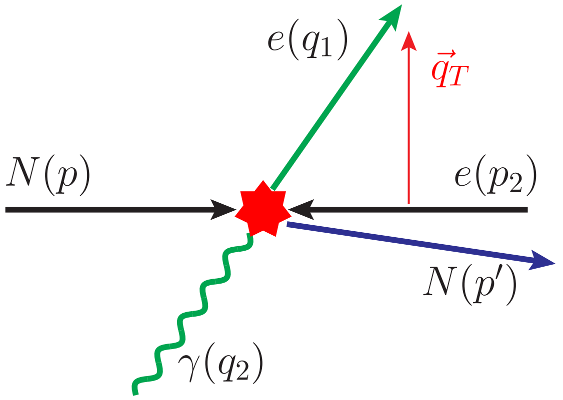
BH contaminates the azimuthal distribution $\Longrightarrow$ makes GPD extraction harder!

A better way to think about DVCS

❑ Single diffractive hard exclusive process (SDHEP)

[Qiu & Yu, PRD 107 (2023) 014007]
[arxiv: 2409.06882 and paper in preparation]

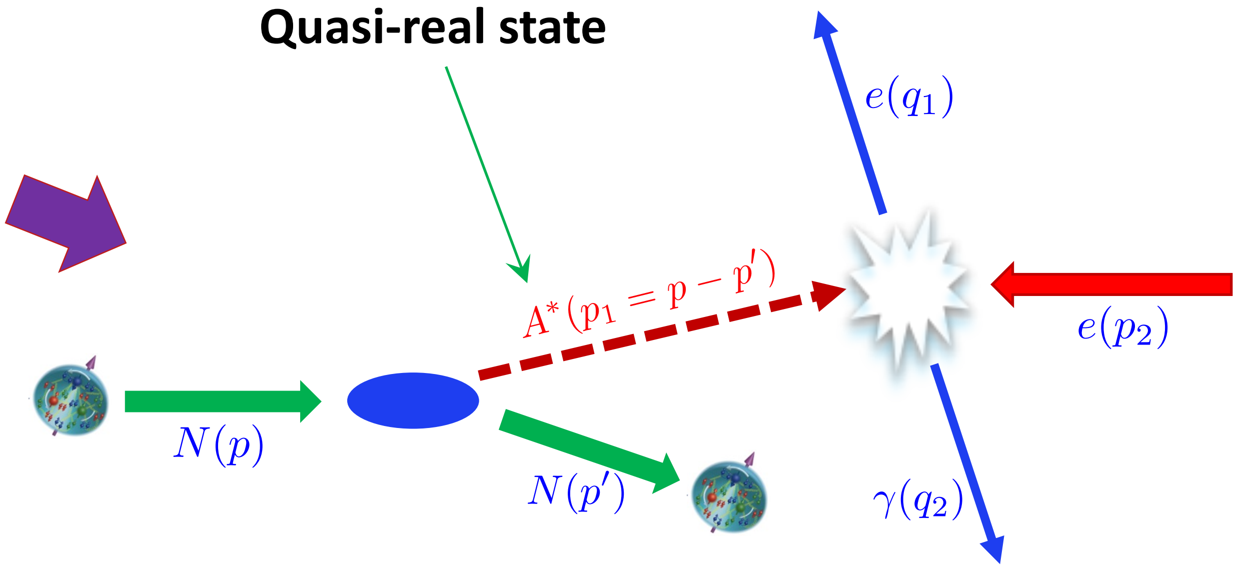
DVCS in lab frame $N(p) + e(p_2) \to N(p') + e(q_1) + \gamma(q_2)$



Two scales:

- **Hard q_T**
- **Soft t**

$t = (p - p')^2$



❑ Two-stage process paradigm

Single diffractive: $N(p) \to N(p') + A^*(p_1 = p - p')$

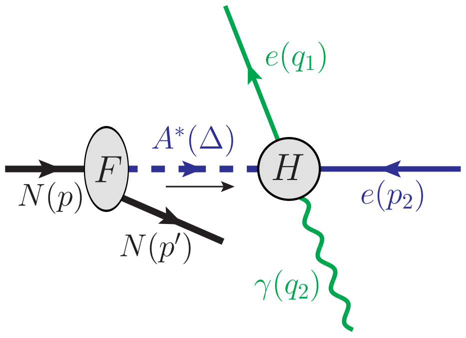
↓ **factorize**

Hard exclusive: $A^*(p_1) + e(p_2) \to e(q_1) + \gamma(q_2)$

Necessary **condition for factorization:**

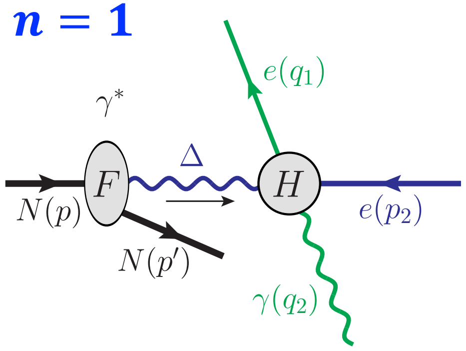
$$q_T \gg \sqrt{-t} \simeq \Lambda_{\text{QCD}}$$

Channel expansion and power counting



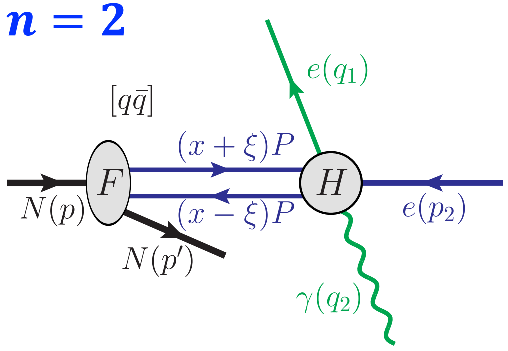
$=$

$\boldsymbol{n = 1}$



$+$

$\boldsymbol{n = 2}$



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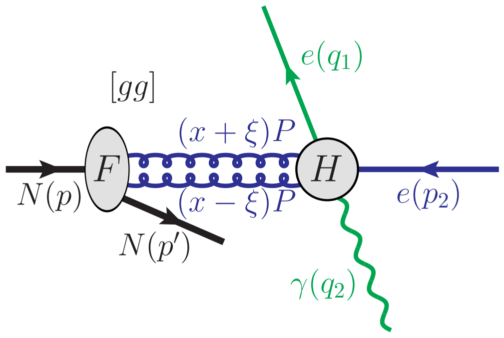
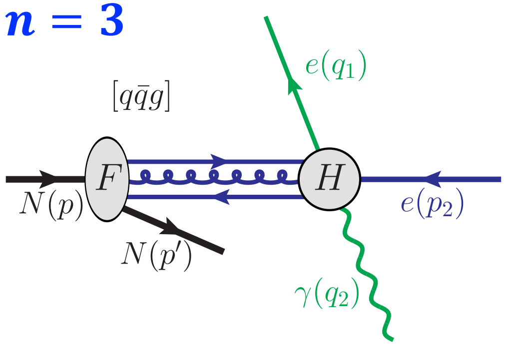


$\Delta = p_1 = p - p'$

$\boldsymbol{n = 3}$

$+$



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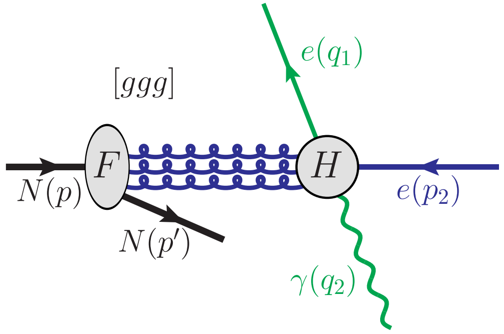

$+\cdots \quad (\boldsymbol{n > 3})$

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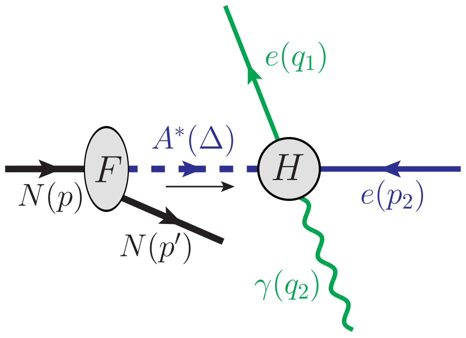
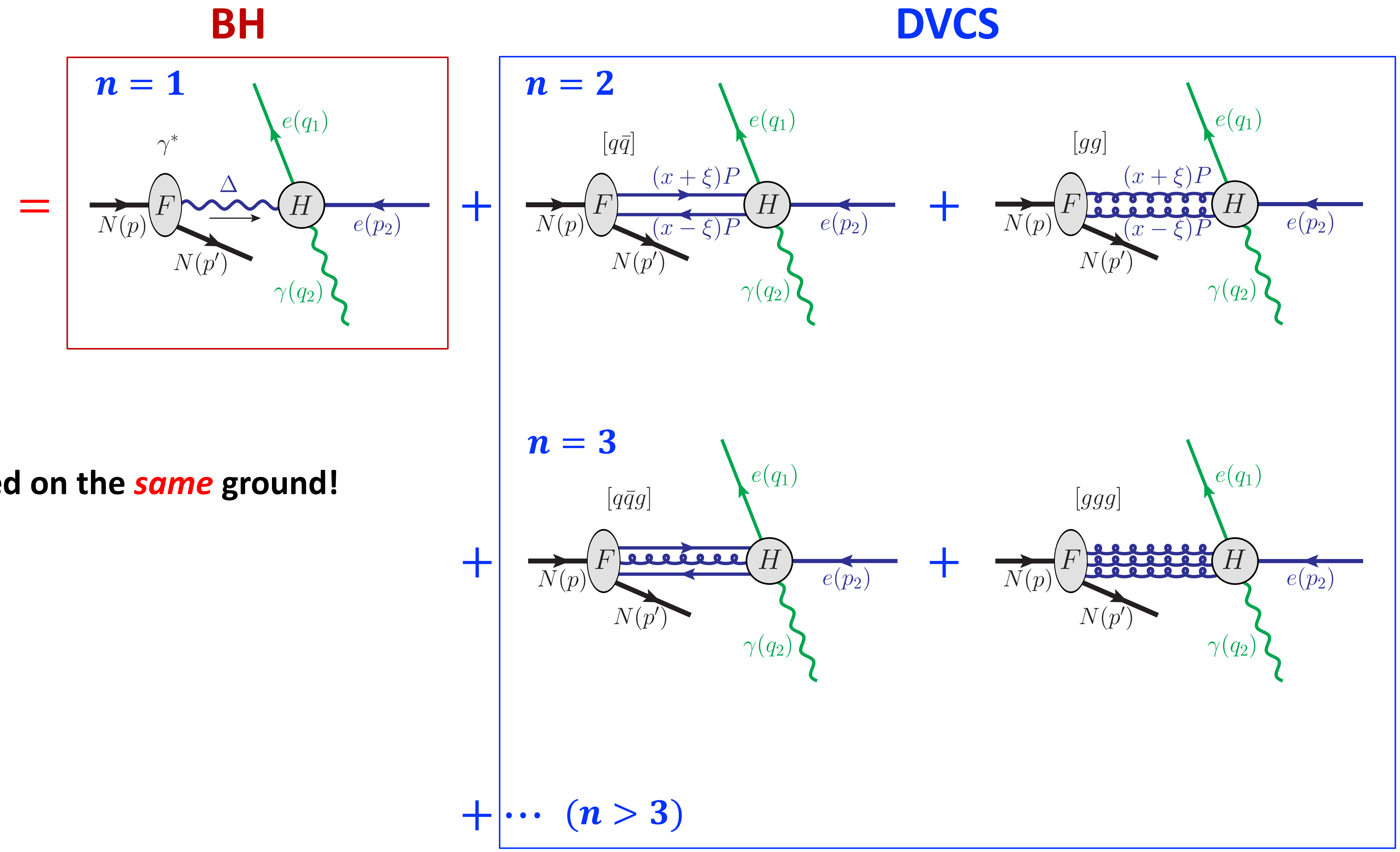
Channel expansion and power counting

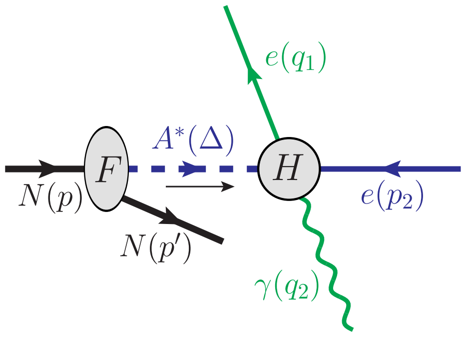




$\Delta = p_1 = p - p'$

BH and DVCS are treated on the *same* ground!

Channel expansion and power counting

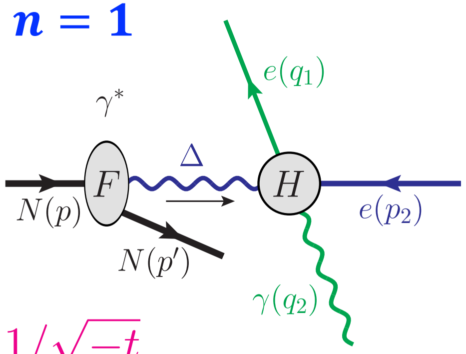


$\Delta = p_1 = p - p'$

$=$

Leading power (LP)

$n = 1$

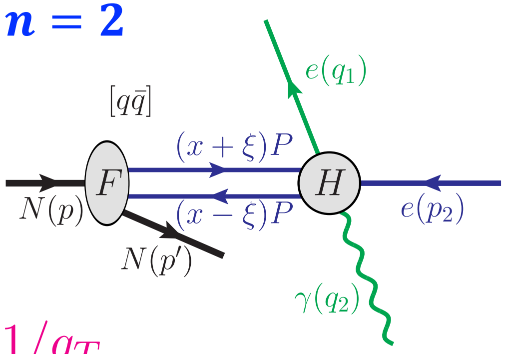


$1/\sqrt{-t}$

$+$

Next-to-leading power (NLP)

$n = 2$



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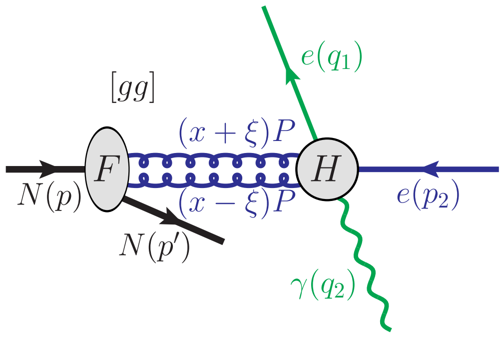
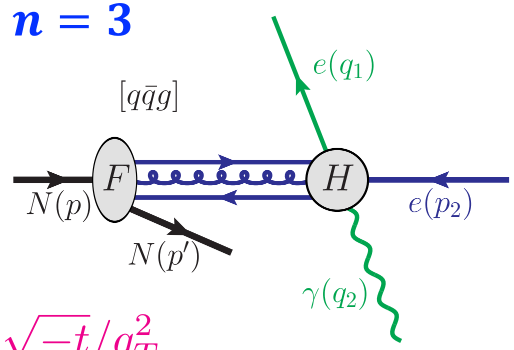


$1/q_T$

NNLP

$+$

$n = 3$



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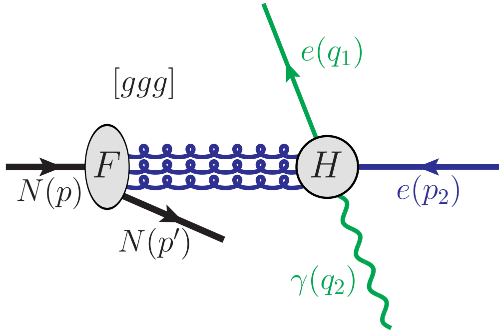
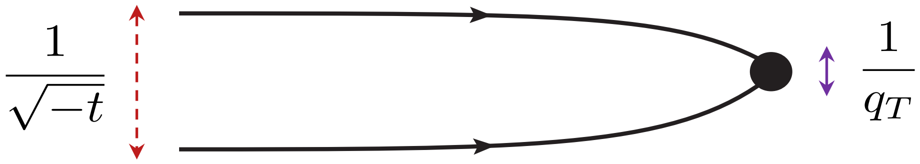


$\sqrt{-t}/q_T^2$

One more physically polarized parton in A^*

one more suppression of $\sqrt{-t}/q_T$



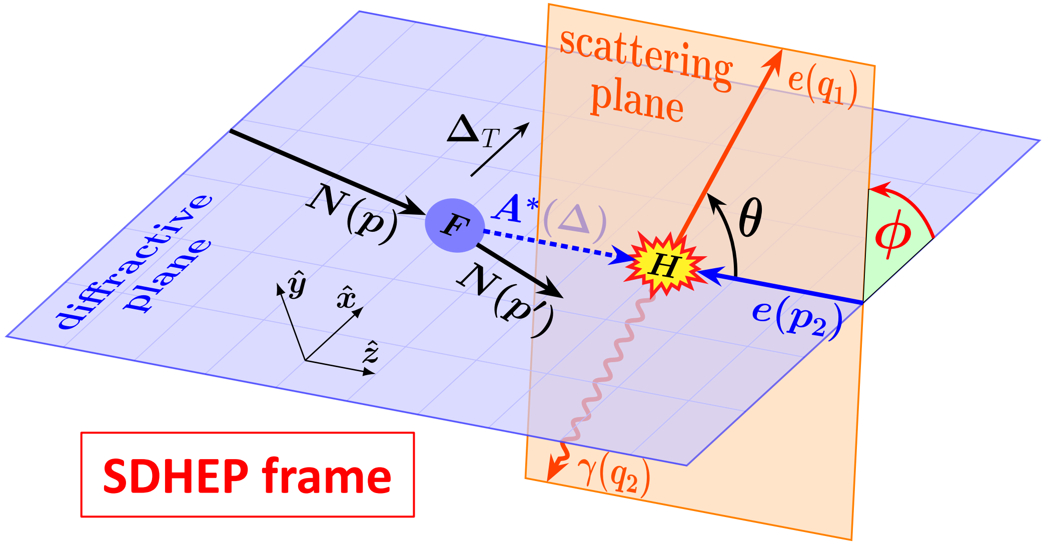
- **Consistent power counting**
- **Channel expansion = power expansion**

NNNLP…

$+\cdots\ (n > 3)\quad \mathcal{O}\left(-t/q_T^3\right)$

SDHEP frame and ϕ distribution

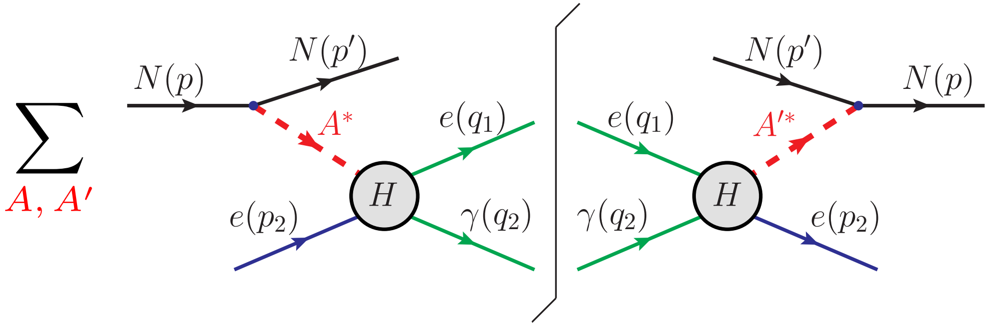
SDHEP frame observables: $(t,\ \xi,\ \phi_S,\ \theta,\ \phi)$



SDHEP frame

$$\mathcal{M}(t,\xi,\phi_S,\theta,\phi) = \sum_{A^*} e^{i(\lambda_A - \lambda_e)\phi} F_{N\to NA^*} \otimes G_{A^* e \to e\gamma}$$

$|\mathcal{M}|^2$



Interference of $(\lambda_A,\ \lambda'_A)$ channels

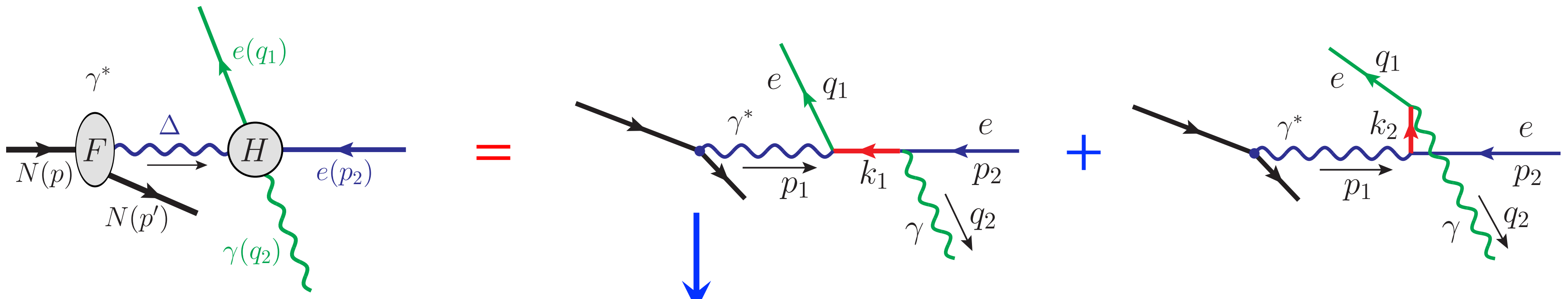
$$\Delta\lambda_A = \lambda_A - \lambda'_A$$

$\cos[(\Delta\lambda_A)\phi]$
$\sin[(\Delta\lambda_A)\phi]$

ϕ distribution is determined by A^* spin states!

$n = 1$: γ^* channel --- BH subprocess

- **Advantage: the quasi-real state A^* has well-defined helicity for all $n = 1, 2, 3, \ldots$**



$$F_N^\mu(p,p') = \langle N(p')|J^\mu(0)|N(p)\rangle = \bar{u}(p',s')\left[F_1(t)\,\gamma^\mu - F_2(t)\,\frac{i\sigma^{\mu\nu}\Delta_\nu}{2m}\right]u(p,s)$$

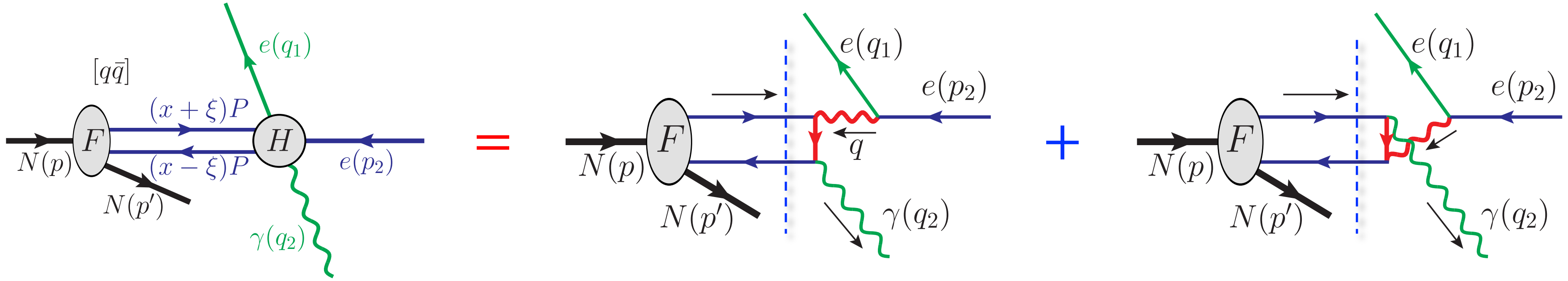
$$\mathcal{M}^{[1]} = \frac{-e}{t}\,F_N^\mu(p,p')\,G_\mu^\gamma(\Delta,p_2,q_1,q_2) = \frac{e}{t}\left[\sum_{\lambda=\pm1}(F_N\cdot\epsilon_\lambda^*)(\epsilon_\lambda\cdot G^\gamma) - 2(F_N\cdot n)(\bar{n}\cdot G^\gamma)\right]$$

$\lambda_A^\gamma = \pm 1$ $\qquad$ $\lambda_A^\gamma = 0$

- **Only the transverse polarization γ_T^* is at LP** $\qquad \mathcal{O}(1/\sqrt{-t})$
- **The longitudinal polarization γ_L^* is at NLP** $\qquad \mathcal{O}(1/q_T)$ $\longleftrightarrow$ **Combine with $n = 2$ (DVCS)**

$n = 2$: $[q\bar{q}]$ channel --- DVCS (twist-2)

- **Advantage: the quasi-real state A^* has well-defined helicity for all $n = 1, 2, 3, \ldots$**



$$\mathcal{M}^{[2]} \simeq \sum_q \int_{-1}^{1} dx \left[F^q(x,\xi,t)\, G^q(x,\xi;\hat{s},\theta,\phi) + \widetilde{F}^q(x,\xi,t)\, \widetilde{G}^q(x,\xi;\hat{s},\theta,\phi) \right] + \mathcal{O}\left(\sqrt{-t}/q_T^2\right)$$

GPDs (H, E): defined with γ^+

$(\widetilde{H}, \widetilde{E})$: defined with $\gamma^+\gamma_5$

$$\lambda_A^{q\bar{q}} = 0$$

Combine $n = 1$ and $n = 2$ channels

❑ Amplitude level

LP $\mathcal{M}_{\rm I}:\quad A^* = \gamma_T^* \; (\lambda_A^\gamma = \pm 1)$

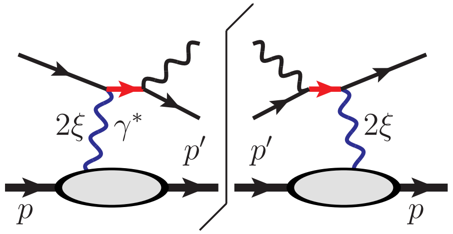
NLP $\mathcal{M}_{\rm II}:\quad (1)\; A^* = \gamma_L^* \; (\lambda_A^\gamma = 0);\quad (2)\; A^* = [q\bar{q}] \; (\lambda_A^q = 0)$ $+ [gg]$ **(high order)**

NNLP: …

❑ Cross section level

$$|\mathcal{M}_{\rm I} + \mathcal{M}_{\rm II} + \cdots|^2 = \underbrace{|\mathcal{M}_{\rm I}|^2}_{\rm LP} + \underbrace{2\,{\rm Re}\,(\mathcal{M}_{\rm I}\mathcal{M}_{\rm II}^*)}_{\rm NLP} + \cdots$$

LP $|\mathcal{M}|^2_{\rm LP} = |\mathcal{M}_{\rm I}|^2$ **No ϕ modulation. $\lambda_A^\gamma = +1$ and $\lambda_A^\gamma = -1$ do NOT interfere until NNLP.**



"twist-2"

Combine $n = 1$ and $n = 2$ channels

❑ Amplitude level

LP $\mathcal{M}_{\rm I}$: $A^* = \gamma_T^*$ $(\lambda_A^\gamma = \pm 1)$

NLP $\mathcal{M}_{\rm II}$: (1) $A^* = \gamma_L^*$ $(\lambda_A^\gamma = 0)$; (2) $A^* = [q\bar{q}]$ $(\lambda_A^q = 0)$ $+ [gg]$ **(high order)**

NNLP: …

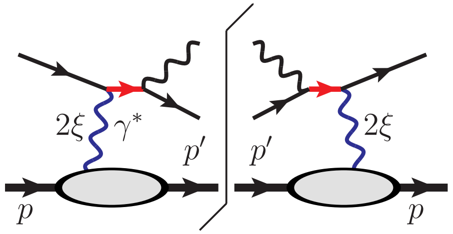
❑ Cross section level

$$|\mathcal{M}_{\rm I} + \mathcal{M}_{\rm II} + \cdots|^2 = \underbrace{|\mathcal{M}_{\rm I}|^2}_{\rm LP} + \underbrace{2\,{\rm Re}\,(\mathcal{M}_{\rm I}\mathcal{M}_{\rm II}^*)}_{\rm NLP} + \cdots$$

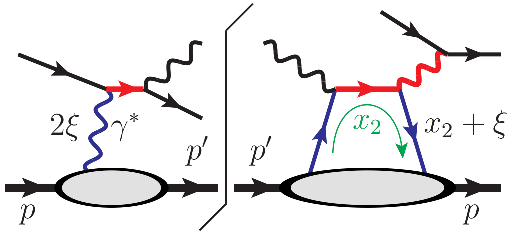
LP $|\mathcal{M}|^2_{\rm LP} = |\mathcal{M}_{\rm I}|^2$ **No ϕ modulation. $\lambda_A^\gamma = +1$ and $\lambda_A^\gamma = -1$ do NOT interfere until NNLP.**

NLP $|\mathcal{M}|^2_{\rm NLP} = 2\,{\rm Re}\,(\mathcal{M}_{\rm I}\mathcal{M}_{\rm II}^*)$ ➡ **$\cos\phi$ or $\sin\phi$ modulation.**

➡ **Signal of GPDs.**



"twist-2"



"twist-3"

Cross section within NLP

$$\frac{d\sigma}{dt\, d\xi\, d\phi_S\, d\cos\theta\, d\phi} = \frac{1}{(2\pi)^2} \frac{d\sigma^{\text{unpol}}}{dt\, d\xi\, d\cos\theta} \cdot \Big[1 + \lambda_e \lambda_N A_{LL}^{\text{LP}} + \lambda_e s_T A_{TL}^{\text{LP}} \cos\phi_S$$
$$+ \left(A_{UU}^{\text{NLP}} + \lambda_e \lambda_N A_{LL}^{\text{NLP}}\right) \cos\phi + \left(\lambda_e A_{UL}^{\text{NLP}} + \lambda_N A_{LU}^{\text{NLP}}\right) \sin\phi$$
$$+ s_T \left(A_{TU,1}^{\text{NLP}} \cos\phi_S \sin\phi + A_{TU,2}^{\text{NLP}} \sin\phi_S \cos\phi\right)$$
$$+ \lambda_e s_T \left(A_{TL,1}^{\text{NLP}} \cos\phi_S \cos\phi + A_{TL,2}^{\text{NLP}} \sin\phi_S \sin\phi\right) \Big]$$

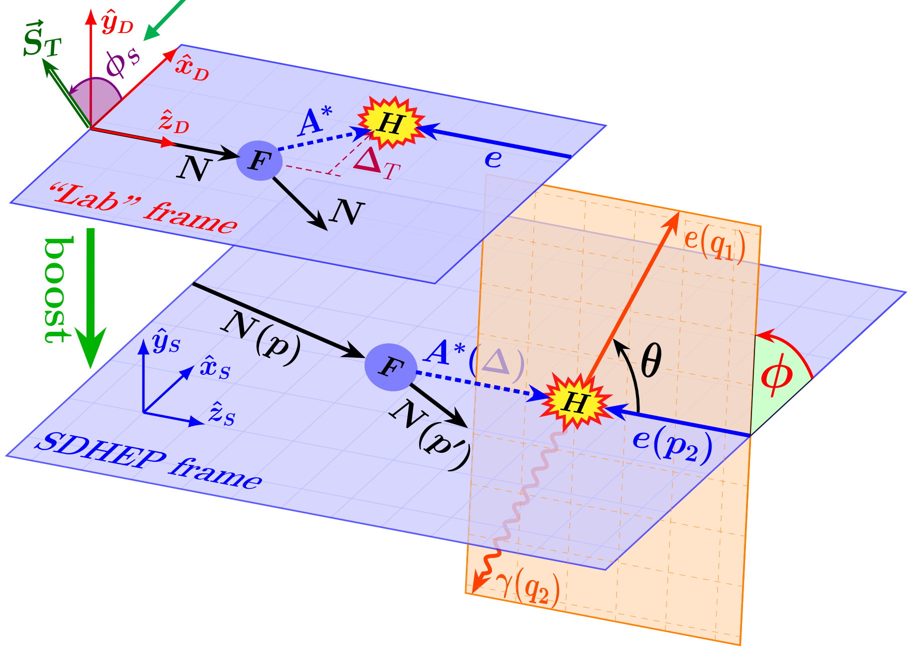
Cross section within NLP

$$\frac{d\sigma}{dt\, d\xi\, d\phi_S\, d\cos\theta\, d\phi} = \frac{1}{(2\pi)^2} \frac{d\sigma^{\rm unpol}}{dt\, d\xi\, d\cos\theta} \cdot \Big[1 + \lambda_e \lambda_N A_{LL}^{\rm LP} + \lambda_e s_T A_{TL}^{\rm LP} \cos\phi_S$$
$$+ \left(A_{UU}^{\rm NLP} + \lambda_e\lambda_N A_{LL}^{\rm NLP}\right)\cos\phi + \left(\lambda_e A_{UL}^{\rm NLP} + \lambda_N A_{LU}^{\rm NLP}\right)\sin\phi$$
$$+ s_T \left(A_{TU,1}^{\rm NLP}\cos\phi_S \sin\phi + A_{TU,2}^{\rm NLP}\sin\phi_S\cos\phi\right)$$
$$+ \lambda_e s_T \left(A_{TL,1}^{\rm NLP}\cos\phi_S\cos\phi + A_{TL,2}^{\rm NLP}\sin\phi_S\sin\phi\right)\Big]$$

In the experimental setting (fixed lab frame),

- **Nucleon spin vector** $\vec{s}_N = (s_T, 0, \lambda_N)$
- **Electron spin vector** $\vec{s}_e = (0, 0, \lambda_e)$

ϕ_S: of nucleon transverse spin in the "Lab" frame



Subscripts: (nucleon, electron)

U = Unpolarized
L = Longitudinally polarized
T = Transversely polarized

Cross section within NLP

$$\frac{d\sigma}{dt\,d\xi\,d\phi_S\,d\cos\theta\,d\phi} = \frac{1}{(2\pi)^2}\frac{d\sigma^{\rm unpol}}{dt\,d\xi\,d\cos\theta}\cdot\Big[1+\lambda_e\lambda_N A_{LL}^{\rm LP}+\lambda_e s_T A_{TL}^{\rm LP}\cos\phi_S$$
$$+\left(A_{UU}^{\rm NLP}+\lambda_e\lambda_N A_{LL}^{\rm NLP}\right)\cos\phi+\left(\lambda_e A_{UL}^{\rm NLP}+\lambda_N A_{LU}^{\rm NLP}\right)\sin\phi$$
$$+s_T\left(A_{TU,1}^{\rm NLP}\cos\phi_S\sin\phi+A_{TU,2}^{\rm NLP}\sin\phi_S\cos\phi\right)$$
$$+\lambda_e s_T\left(A_{TL,1}^{\rm NLP}\cos\phi_S\cos\phi+A_{TL,2}^{\rm NLP}\sin\phi_S\sin\phi\right)\Big]$$

LP: from γ_T^* squared

$$\frac{d\sigma^{\rm unpol}}{dt\,d\xi\,d\cos\theta} = \frac{\alpha_e^3}{(1+\xi)^2}\frac{m^2}{s\,t^2}\Sigma_{UU}^{\rm LP}$$

$$\Sigma_{UU}^{\rm LP} = \left[\frac{1}{\sin^2(\theta/2)}+\sin^2(\theta/2)\right]\left[\left(\frac{1-\xi^2}{2\xi^2}\frac{-t}{m^2}-2\right)\left(F_1^2-\frac{t}{4m^2}F_2^2\right)-\frac{t}{m^2}(F_1+F_2)^2\right]$$

$$A_{LL}^{\rm LP} = \frac{1}{\Sigma_{UU}^{\rm LP}}\cdot\left[\frac{1}{\sin^2(\theta/2)}-\sin^2(\theta/2)\right](F_1+F_2)\left[F_1\left(\frac{-t}{\xi m^2}-\frac{4\xi}{1+\xi}\right)-\frac{t}{m^2}F_2\right]$$

$$A_{TL}^{\rm LP} = \frac{1}{\Sigma_{UU}^{\rm LP}}\cdot\frac{\Delta_T}{2m}\left[\frac{1}{\sin^2(\theta/2)}-\sin^2(\theta/2)\right](F_1+F_2)\left[-4F_1+\frac{1+\xi}{\xi}\frac{-t}{m^2}F_2\right]$$

Quadratic in (F_1, F_2)

Control the rate (unpolarized cross section). No ϕ modulation.

Cross section within NLP

$$\frac{d\sigma}{dt\, d\xi\, d\phi_S\, d\cos\theta\, d\phi} = \frac{1}{(2\pi)^2} \frac{d\sigma^{\text{unpol}}}{dt\, d\xi\, d\cos\theta} \cdot \Big[1 + \lambda_e \lambda_N A_{LL}^{\text{LP}} + \lambda_e s_T A_{TL}^{\text{LP}} \cos\phi_S$$

$$+ \left(A_{UU}^{\text{NLP}} + \lambda_e \lambda_N A_{LL}^{\text{NLP}}\right) \cos\phi + \left(\lambda_e A_{UL}^{\text{NLP}} + \lambda_N A_{LU}^{\text{NLP}}\right) \sin\phi$$

$$+ s_T \left(A_{TU,1}^{\text{NLP}} \cos\phi_S \sin\phi + A_{TU,2}^{\text{NLP}} \sin\phi_S \cos\phi\right)$$

$$+ \lambda_e s_T \left(A_{TL,1}^{\text{NLP}} \cos\phi_S \cos\phi + A_{TL,2}^{\text{NLP}} \sin\phi_S \sin\phi\right)\Big]$$

NLP: from γ_T^*-γ_L^* and γ_T^*-$[q\bar{q}]$ interference

No contribution to the rate,

$\Longrightarrow$ **only to azimuthal modulations** $(\cos\phi, \sin\phi)$

Cross section within NLP

$$\frac{d\sigma}{dt\, d\xi\, d\phi_S\, d\cos\theta\, d\phi} = \frac{1}{(2\pi)^2}\frac{d\sigma^{\rm unpol}}{dt\, d\xi\, d\cos\theta}\cdot\Big[1+\lambda_e\lambda_N A_{LL}^{\rm LP}+\lambda_e s_T A_{TL}^{\rm LP}\cos\phi_S$$

$$+\left(A_{UU}^{\rm NLP}+\lambda_e\lambda_N A_{LL}^{\rm NLP}\right)\cos\phi+\left(\lambda_e A_{UL}^{\rm NLP}+\lambda_N A_{LU}^{\rm NLP}\right)\sin\phi$$
$$+s_T\left(A_{TU,1}^{\rm NLP}\cos\phi_S\sin\phi+A_{TU,2}^{\rm NLP}\sin\phi_S\cos\phi\right)$$
$$+\lambda_e s_T\left(A_{TL,1}^{\rm NLP}\cos\phi_S\cos\phi+A_{TL,2}^{\rm NLP}\sin\phi_S\sin\phi\right)\Big]$$

NLP: from γ_T^*-γ_L^* and γ_T^*-$[q\bar{q}]$ interference

$$A_{XX}^{\rm NLP}=\frac{1}{\Sigma_{UU}^{\rm LP}}\cdot\left(\frac{-t}{m\sqrt{\hat{s}}}\right)\Sigma_{XX}^{\rm NLP}$$

$$\Sigma_{UU}^{\rm NLP}=\frac{\Delta_T}{2m}\frac{1+\xi}{\xi}\left[\frac{2\sin\theta}{\xi}\left(F_1^2-\frac{t}{4m^2}F_2^2\right)-\frac{4+(1-\cos\theta)^2}{\sin\theta\cos^2(\theta/2)}\left(M_1\cdot{\rm Re}\,V_{\mathcal{F}}\right)\right],$$

$$\Sigma_{LL}^{\rm NLP}=-\frac{\Delta_T}{m}\left[\sin\theta(F_1+F_2)\left(\frac{1+\xi}{\xi}F_1+F_2\right)+\frac{3-\cos\theta}{\sin\theta}\left(M_2\cdot{\rm Re}\,V_{\mathcal{F}}\right)\right],$$

$$\Sigma_{TL,1}^{\rm NLP}=2\sin\theta\,(F_1+F_2)\left[F_1+\left(\frac{\xi}{1+\xi}+\frac{t}{4\xi m^2}\right)F_2\right]+\frac{2(3-\cos\theta)}{\sin\theta}\left(M_3\cdot{\rm Re}\,V_{\mathcal{F}}\right),$$

$$\Sigma_{TL,2}^{\rm NLP}=2\sin\theta\,(F_1+F_2)\left(F_1+\frac{t}{4m^2}F_2\right)-\frac{2(3-\cos\theta)}{\sin\theta}\left(M_4\cdot{\rm Re}\,V_{\mathcal{F}}\right),$$

$$\Sigma_{UL}^{\rm NLP}=-\frac{\Delta_T}{m}\frac{1+\xi}{\xi}\frac{3-\cos\theta}{\sin\theta}\left(M_1\cdot{\rm Im}\,V_{\mathcal{F}}\right),$$

$$\Sigma_{LU}^{\rm NLP}=-\frac{\Delta_T}{2m}\frac{4+(1-\cos\theta)^2}{\sin\theta\cos^2(\theta/2)}\left(M_2\cdot{\rm Im}\,V_{\mathcal{F}}\right),$$

$$\Sigma_{TU,1}^{\rm NLP}=\frac{4+(1-\cos\theta)^2}{\sin\theta\cos^2(\theta/2)}\left(M_3\cdot{\rm Im}\,V_{\mathcal{F}}\right),$$

$$\Sigma_{TU,2}^{\rm NLP}=\frac{4+(1-\cos\theta)^2}{\sin\theta\cos^2(\theta/2)}\left(M_4\cdot{\rm Im}\,V_{\mathcal{F}}\right).$$

- **Linear in GPD moments** $V_{\mathcal{F}}=(\mathcal{H},\mathcal{E},\widetilde{\mathcal{H}},\widetilde{\mathcal{E}})^T$
- **Controlled by the real matrix *M*, same for real and imaginary parts of GPD moments**

  $M_i=(M_{i1},M_{i2},M_{i3},M_{i4})$ **(see next slide)**
- **8 asymmetries ⇔ 8 (real) GPD moments**

Cross section within NLP

$$\frac{d\sigma}{dt\,d\xi\,d\phi_S\,d\cos\theta\,d\phi} = \frac{1}{(2\pi)^2}\frac{d\sigma^{\text{unpol}}}{dt\,d\xi\,d\cos\theta}\cdot\Big[1+\lambda_e\lambda_N A_{LL}^{\text{LP}}+\lambda_e s_T A_{TL}^{\text{LP}}\cos\phi_S$$
$$+\left(A_{UU}^{\text{NLP}}+\lambda_e\lambda_N A_{LL}^{\text{NLP}}\right)\cos\phi+\left(\lambda_e A_{UL}^{\text{NLP}}+\lambda_N A_{LU}^{\text{NLP}}\right)\sin\phi$$
$$+s_T\left(A_{TU,1}^{\text{NLP}}\cos\phi_S\sin\phi+A_{TU,2}^{\text{NLP}}\sin\phi_S\cos\phi\right)$$
$$+\lambda_e s_T\left(A_{TL,1}^{\text{NLP}}\cos\phi_S\cos\phi+A_{TL,2}^{\text{NLP}}\sin\phi_S\sin\phi\right)\Big]$$

NLP: from γ_T^*-γ_L^* and γ_T^*-$[q\bar{q}]$ interference

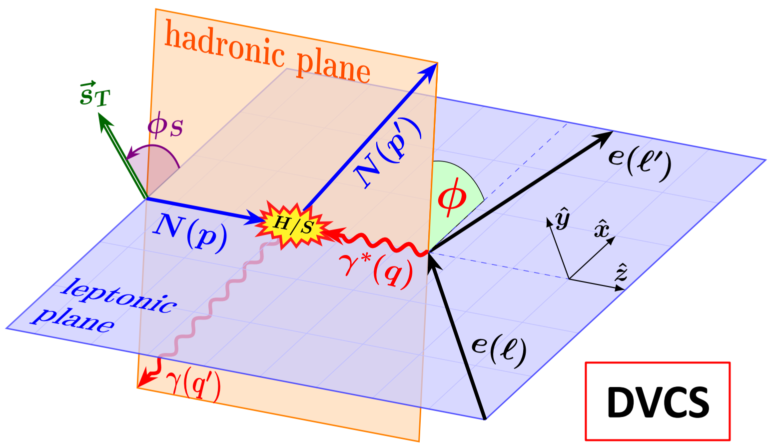
$$M = \begin{bmatrix} F_1 & -\frac{t}{4m^2}F_2 & \xi(F_1+F_2) & 0 \\ (1+\xi)(F_1+F_2) & \xi(F_1+F_2) & \frac{1+\xi}{\xi}F_1 & -\xi F_1-(1+\xi)\frac{t}{4m^2}F_2 \\ \xi(F_1+F_2) & \left(\frac{\xi^2}{1+\xi}+\frac{t}{4m^2}\right)(F_1+F_2) & -\xi F_1+\frac{t}{4m^2}\frac{1-\xi^2}{\xi}F_2 & -\left(\frac{\xi^2}{1+\xi}+\frac{t}{4m^2}\right)F_1-\frac{\xi t}{4m^2}F_2 \\ \xi(F_1+F_2) & \frac{\xi t}{4m^2}(F_1+F_2) & -\xi F_1+\frac{t}{4m^2}\frac{1-\xi^2}{\xi}F_2 & -\left(\xi+\frac{t}{4\xi m^2}\right)F_1-\frac{\xi t}{4m^2}F_2 \end{bmatrix} \begin{matrix} \leftarrow M_1 \\ \leftarrow M_2 \\ \leftarrow M_3 \\ \leftarrow M_4 \end{matrix}$$

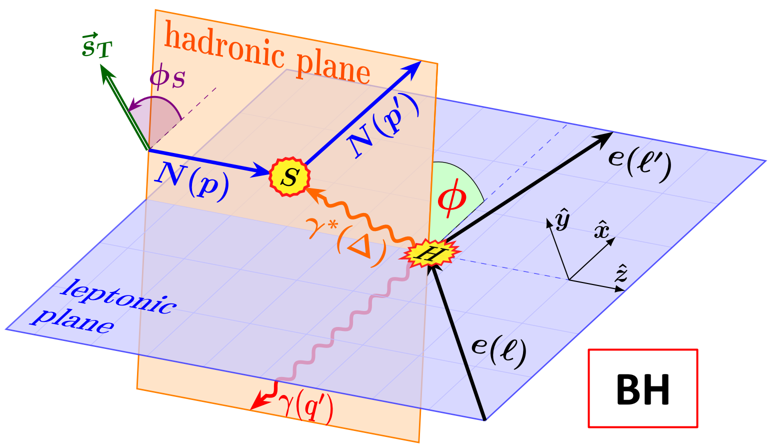
$$\Rightarrow M\cdot\begin{bmatrix}\mathcal{H}\\ \mathcal{E}\\ \widetilde{\mathcal{H}}\\ \widetilde{\mathcal{E}}\end{bmatrix} = \begin{bmatrix}\hat{V}_1\\ \hat{V}_2\\ \hat{V}_3\\ \hat{V}_4\end{bmatrix}$$ ← **Reconstructed from experiments (complex valued)**

$\det M \neq 0$ ⇒ **Unique solution for GPD moments!**

Summary --- SDHEP frame vs. Breit frame

- **Breit frame: centered around $\gamma^*(q)$**

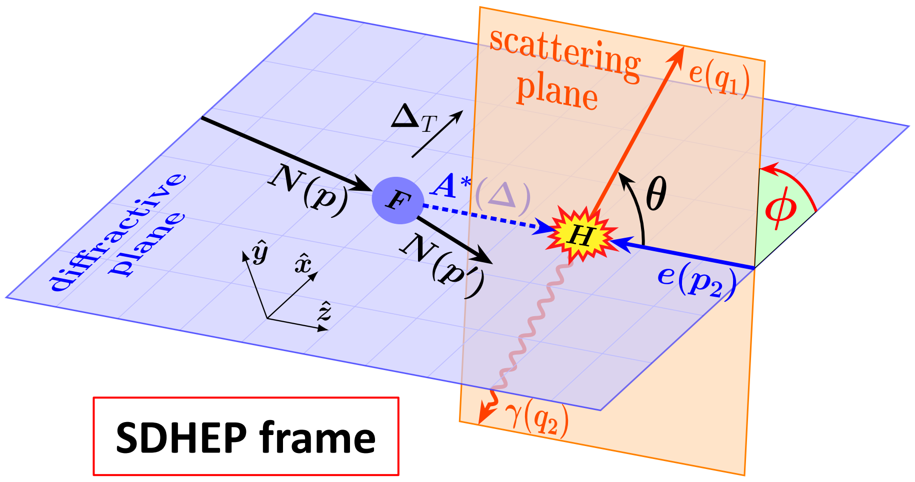




Inconsistent treatments for DVCS and BH

Makes their interference calculation difficult

- **SDHEP frame: centered around $A^*(\Delta)$**



- **Clear physical picture: scale separation**
- $A^* = \gamma^*, [q\bar{q}], [gg], [q\bar{q}g], [ggg], \ldots$
- **Generalizable to high orders and twists**
- **Clear azimuthal distribution**

Thank you!

Jefferson Lab

Thank you!

Two-stage kinematic description

- **Diffractive subprocess** $N(p) \to N(p') + A^*(\Delta = p - p')$

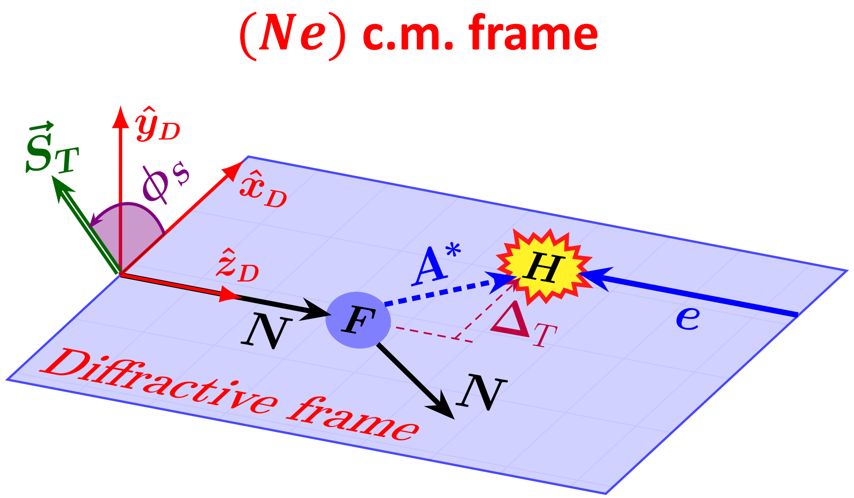
  Describe in diffractive frame: $\hat{x}_D \parallel \vec{\Delta}_T$ **(varying event by event)**

  Trade azimuthal angle of the diffraction for ϕ_S (Jacobian = 1)

  Kinematic variables: $t = \Delta^2,\ \xi = \dfrac{(p - p') \cdot n}{(p + p') \cdot n},\ \phi_S$

  $n = (1, 0, 0, -1)/\sqrt{2}$

  ➡ **determine** $\hat{s} \simeq 2\xi s/(1+\xi)$ **of the hard scattering**



$\hat{x}_D$–$\hat{y}_D$–$\hat{z}_D$: **varying coordinate system**

- **Hard scattering** $A^*(\Delta) + e(p_2) \to e(q_1) + \gamma(q_2)$

Two-stage kinematic description

- **Diffractive subprocess** $N(p) \to N(p') + A^*(\Delta = p - p')$

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Kinematic variables: $t = \Delta^2,\ \xi = \frac{(p-p')\cdot n}{(p+p')\cdot n},\ \phi_S$

$n = (1,0,0,-1)/\sqrt{2}$

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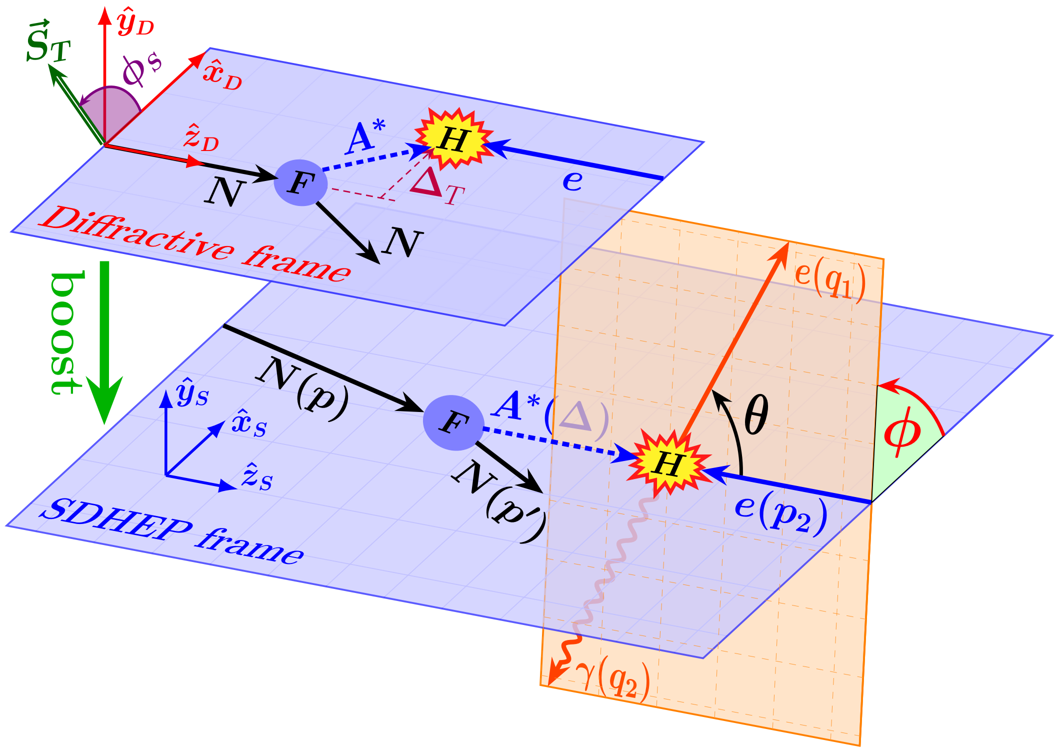


- **Hard scattering** $A^*(\Delta) + e(p_2) \to e(q_1) + \gamma(q_2)$

Describe in SDHEP frame --- $(A^* e)$ c.m. frame: $\hat{z}_S \parallel \vec{\Delta}$

Kinematic variables: $\theta,\ \phi$ $\left[q_T = (\sqrt{\hat{s}}/2)\sin\theta \right]$

➡ $$\frac{d\sigma}{dt\, d\xi\, d\phi_S\, d\cos\theta\, d\phi}$$

$\hat{x}_S$–$\hat{y}_S$–$\hat{z}_S$ **: SDHEP frame coordinate system**

Azimuthal distribution

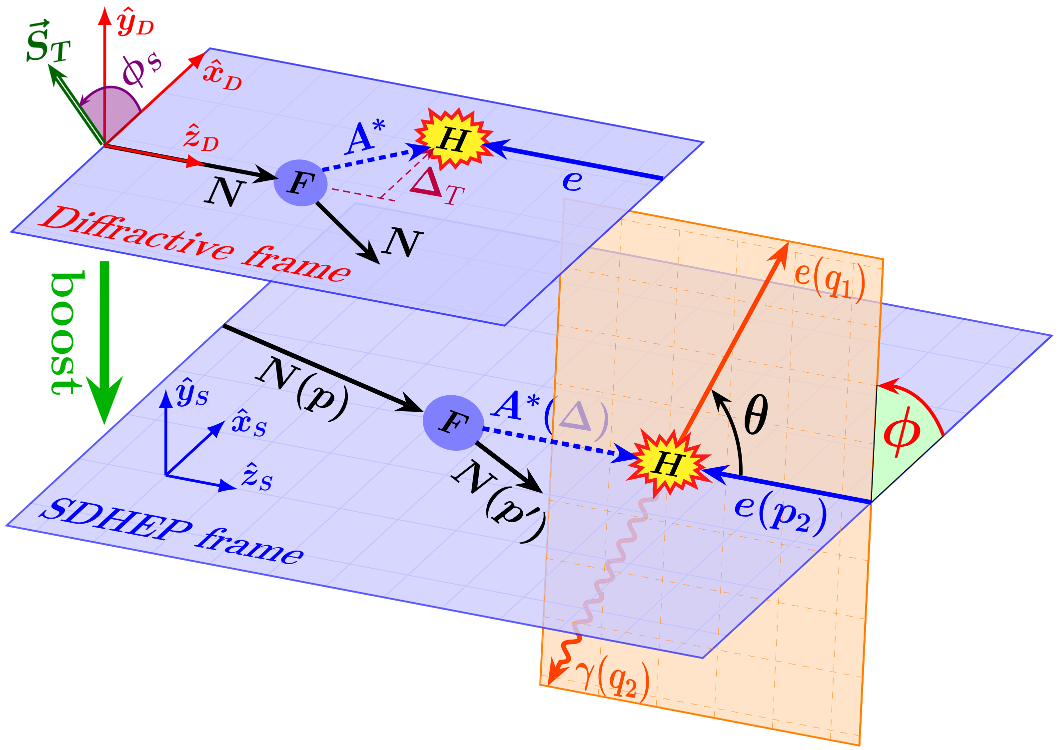
- ϕ_S **in diffraction** $N(p) \to N(p') + A^*(\Delta = p - p')$

$$F_{N\to NA^*}(t, \xi, \phi_S) \propto e^{-i\lambda_N \phi_S}$$

$\lambda_N = \pm 1/2$ **can interfere to give** $\cos\phi_S, \; \sin\phi_S$

transverse spin $s_T \neq 0$

- ϕ **in hard scattering** $A^*(\Delta) + e(p_2) \to e(q_1) + \gamma(q_2)$



Azimuthal distribution

- **ϕ_S in diffraction** $N(p) \to N(p') + A^*(\Delta = p - p')$

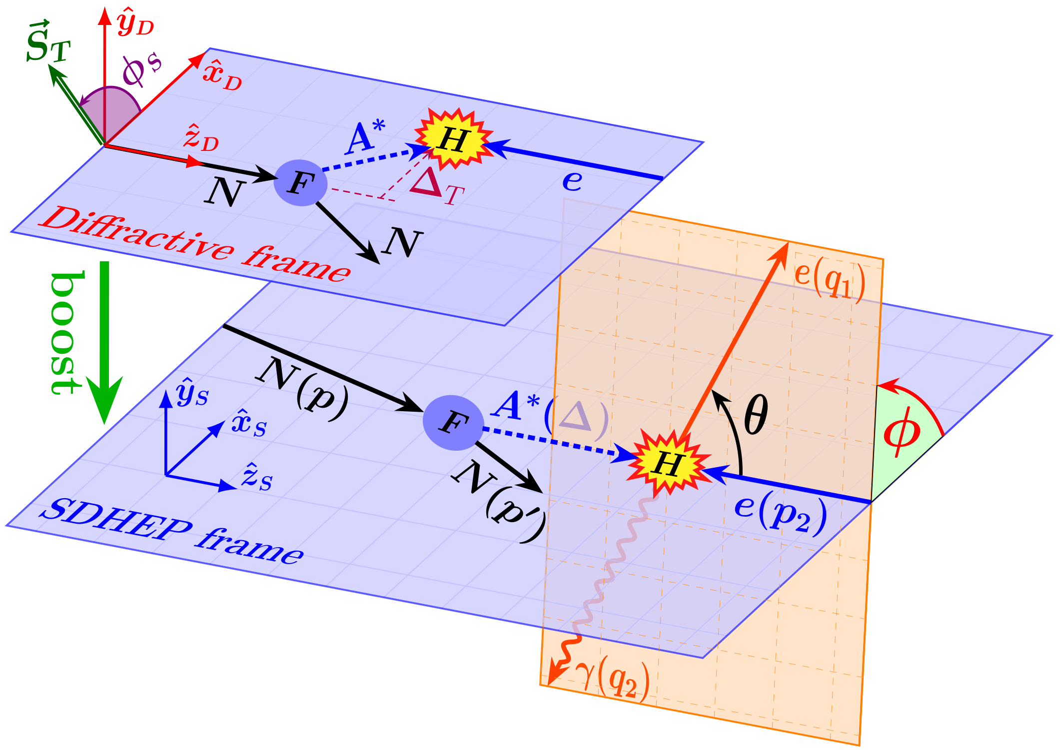
$$F_{N\to NA^*}(t, \xi, \phi_S) \propto e^{-i\lambda_N \phi_S}$$

$\lambda_N = \pm 1/2$ **can interfere to give** $\cos\phi_S,\ \sin\phi_S$

transverse spin $s_T \neq 0$

- **ϕ in hard scattering** $A^*(\Delta) + e(p_2) \to e(q_1) + \gamma(q_2)$

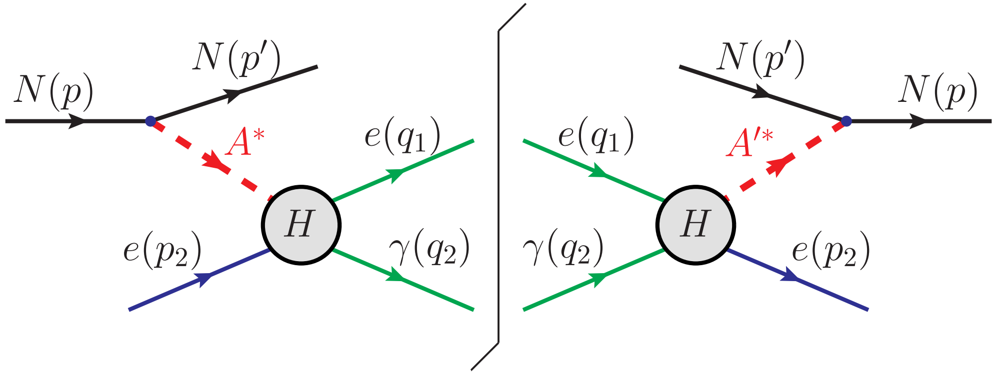
$$\mathcal{M}(t, \xi, \phi_S, \theta, \phi) = \sum_{A^*} F_{N\to NA^*}(t, \xi, \phi_S) \otimes G_{A^* e\to e\gamma}(\hat{s}, \theta, \phi)$$
$$= \sum_{A^*} \left[e^{-i\lambda_N \phi_S} F_{N\to NA^*}(t, \xi)\right] \otimes \left[e^{i(\lambda_A - \lambda_e)\phi} G_{A^* e\to e\gamma}(\hat{s}, \theta)\right]$$



Interference of (λ_A, λ'_A) channels ➡

$\cos[(\Delta\lambda_A)\phi]$
$\sin[(\Delta\lambda_A)\phi]$

$$\Delta\lambda_A = \lambda_A - \lambda'_A$$

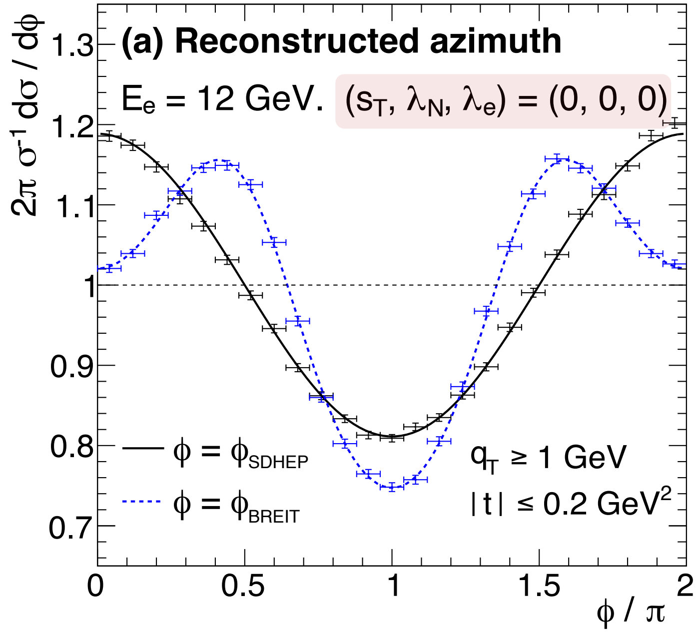


Simple numerical examples

$$\frac{d\sigma}{dt\, d\xi\, d\phi_S\, d\cos\theta\, d\phi} = \frac{1}{(2\pi)^2} \frac{d\sigma^{\rm unpol}}{dt\, d\xi\, d\cos\theta} \cdot \Big[1 + \lambda_e \lambda_N A_{LL}^{\rm LP} + \lambda_e s_T A_{TL}^{\rm LP} \cos\phi_S$$
$$+ \left(A_{UU}^{\rm NLP} + \lambda_e\lambda_N A_{LL}^{\rm NLP}\right)\cos\phi + \left(\lambda_e A_{UL}^{\rm NLP} + \lambda_N A_{LU}^{\rm NLP}\right)\sin\phi$$
$$+ s_T\left(A_{TU,1}^{\rm NLP}\cos\phi_S \sin\phi + A_{TU,2}^{\rm NLP}\sin\phi_S\cos\phi\right)$$
$$+ \lambda_e s_T\left(A_{TL,1}^{\rm NLP}\cos\phi_S\cos\phi + A_{TL,2}^{\rm NLP}\sin\phi_S\sin\phi\right)\Big]$$

Generate 10^6 events and reconstruct

unpolarized



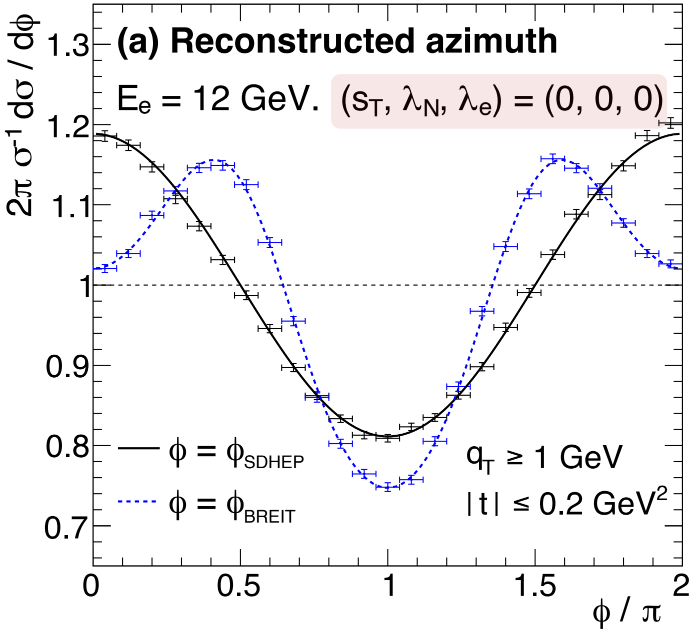
(normalized)

Simple numerical examples

$$\frac{d\sigma}{dt\, d\xi\, d\phi_S\, d\cos\theta\, d\phi} = \frac{1}{(2\pi)^2} \frac{d\sigma^{\text{unpol}}}{dt\, d\xi\, d\cos\theta} \cdot \Big[1 + \lambda_e \lambda_N A_{LL}^{\text{LP}} + \lambda_e s_T A_{TL}^{\text{LP}} \cos\phi_S$$
$$+ \left(A_{UU}^{\text{NLP}} + \lambda_e \lambda_N A_{LL}^{\text{NLP}}\right) \cos\phi + \left(\lambda_e A_{UL}^{\text{NLP}} + \lambda_N A_{LU}^{\text{NLP}}\right) \sin\phi$$
$$+ s_T \left(A_{TU,1}^{\text{NLP}} \cos\phi_S \sin\phi + A_{TU,2}^{\text{NLP}} \sin\phi_S \cos\phi\right)$$
$$+ \lambda_e s_T \left(A_{TL,1}^{\text{NLP}} \cos\phi_S \cos\phi + A_{TL,2}^{\text{NLP}} \sin\phi_S \sin\phi\right) \Big]$$

Generate 10^6 events and reconstruct

unpolarized



(normalized)

SDHEP fit to $1.00 + 0.190\cos\phi$ → $\langle A_{UU}^{\text{NLP}} \rangle = 0.190$

Breit fit to $1.00 + 0.15\cos\phi - 0.12\cos 2\phi - 0.01\cos 3\phi + 0.01\cos 4\phi$

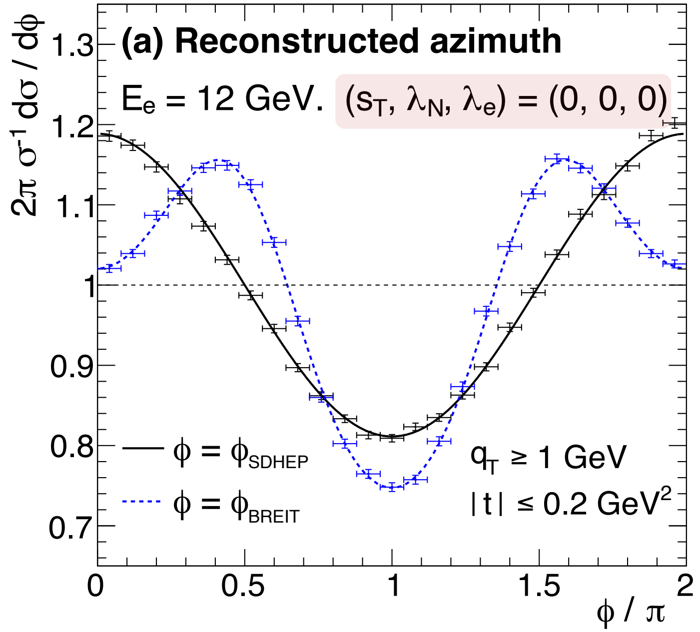
- **No straightforward interpretation of the coefficients.**
- **Need to introduce more gears in GPD extraction.**

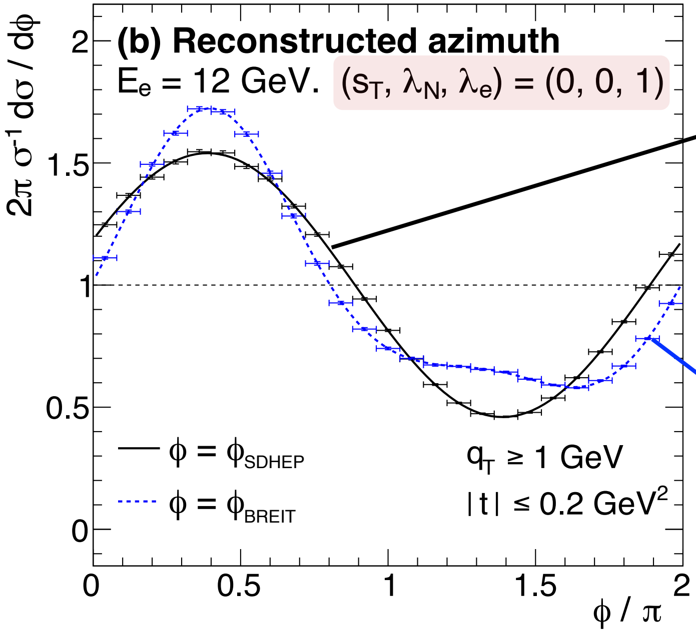
[A.V. Belitsky et al., 2002]
[B. Kriesten et al., 2020, 2022]
[Y. Guo et al., 2021, 2022]
…

Simple numerical examples

$$\frac{d\sigma}{dt\,d\xi\,d\phi_S\,d\cos\theta\,d\phi} = \frac{1}{(2\pi)^2}\frac{d\sigma^{\text{unpol}}}{dt\,d\xi\,d\cos\theta}\cdot\Big[1+\lambda_e\lambda_N A_{LL}^{\text{LP}} + \lambda_e s_T A_{TL}^{\text{LP}}\cos\phi_S + \left(A_{UU}^{\text{NLP}} + \lambda_e\lambda_N A_{LL}^{\text{NLP}}\right)\cos\phi + \left(\lambda_e A_{UL}^{\text{NLP}} + \lambda_N A_{LU}^{\text{NLP}}\right)\sin\phi + s_T\left(A_{TU,1}^{\text{NLP}}\cos\phi_S\sin\phi + A_{TU,2}^{\text{NLP}}\sin\phi_S\cos\phi\right) + \lambda_e s_T\left(A_{TL,1}^{\text{NLP}}\cos\phi_S\cos\phi + A_{TL,2}^{\text{NLP}}\sin\phi_S\sin\phi\right)\Big]$$

Generate 10^6 events and reconstruct





$$1.00 + 0.19\cos\phi + 0.51\sin\phi$$

$$\langle A_{UU}^{\text{NLP}}\rangle = 0.19$$

$$\langle A_{UL}^{\text{NLP}}\rangle = 0.51$$

?

single electron polarization

Jefferson Lab

NLP and LO

❑ Amplitude level

LP $\mathcal{M}_{\rm I}: \quad A^* = \gamma_T^* \ (\lambda_A^\gamma = \pm 1)$

NLP $\mathcal{M}_{\rm II}: \quad (1)\ A^* = \gamma_L^* \ (\lambda_A^\gamma = 0); \quad (2)\ A^* = [q\bar{q}] \ (\lambda_A^q = 0)$ + $[gg]$ **(high order)**

NNLP: …

❑ Cross section level

$$|\mathcal{M}_{\rm I} + \mathcal{M}_{\rm II} + \cdots|^2 = \underbrace{|\mathcal{M}_{\rm I}|^2}_{\rm LP} + \underbrace{2\,{\rm Re}\,(\mathcal{M}_{\rm I}\mathcal{M}_{\rm II}^*)}_{\rm NLP} + \cdots$$

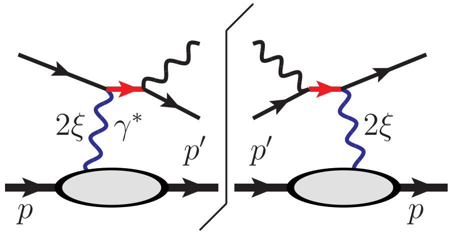
LP $|\mathcal{M}|^2_{\rm LP} = |\mathcal{M}_{\rm I}|^2$ **No ϕ modulation. $\lambda_A^\gamma = +1$ and $\lambda_A^\gamma = -1$ do NOT interfere until NNLP.**

NLP $|\mathcal{M}|^2_{\rm NLP} = 2\,{\rm Re}\,(\mathcal{M}_{\rm I}\mathcal{M}_{\rm II}^*)$ ➡ $\cos\phi$ **or** $\sin\phi$ **modulation.**

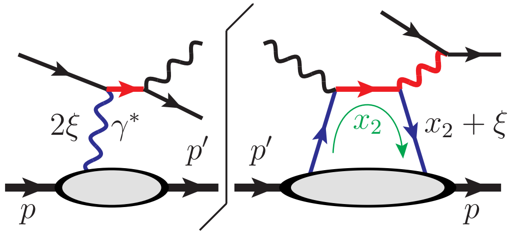
➡ **Interference of different numbers of particles.**

⬇

Unique feature to QFT, beyond non-rel. QM!



"twist-2"



"twist-3": LO

Expectation beyond LO

Amplitude level

F^g and $\widetilde{F}^g$

LP $\mathcal{M}_{\rm I}: \quad A^* = \gamma_T^* \; (\lambda_A^\gamma = \pm 1)$

NLP $\mathcal{M}_{\rm II}: \quad (1)\; A^* = \gamma_L^* \; (\lambda_A^\gamma = 0); \quad (2)\; A^* = [q\bar{q}] \; (\lambda_A^q = 0); \quad (3)\; A^* = [gg] \; (\lambda_A^g = 0, \pm 2)$

NNLP: …

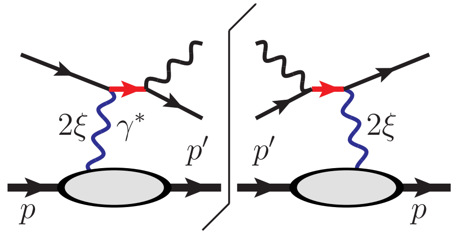
gluon transversity F_T^g

Cross section level

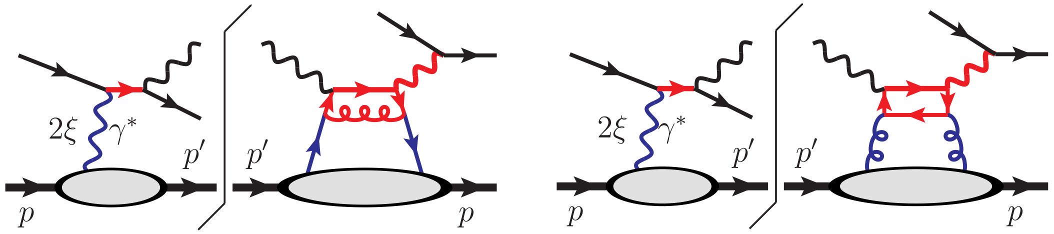
$$|\mathcal{M}_{\rm I} + \mathcal{M}_{\rm II} + \cdots|^2 = \underbrace{|\mathcal{M}_{\rm I}|^2}_{\rm LP} + \underbrace{2\,{\rm Re}\,(\mathcal{M}_{\rm I}\mathcal{M}_{\rm II}^*)}_{\rm NLP} + \cdots$$

LP $|\mathcal{M}|^2_{\rm LP} = |\mathcal{M}_{\rm I}|^2$ **No ϕ modulation. $\lambda_A^\gamma = +1$ and $\lambda_A^\gamma = -1$ do NOT interfere until NNLP.**

NLP $|\mathcal{M}|^2_{\rm NLP} = 2\,{\rm Re}\,(\mathcal{M}_{\rm I}\mathcal{M}_{\rm II}^*)$ ➡ **$\cos\phi$ or $\sin\phi$ modulation, and $\cos 3\phi$ or $\sin 3\phi$ modulation**



"twist-2"



"twist-3": NLO

Expectation beyond NLP

❑ Amplitude level

LP $\mathcal{M}_{\rm I}: \quad A^* = \gamma_T^* \; (\lambda_A^\gamma = \pm 1)$

NLP $\mathcal{M}_{\rm II}: \quad (1)\; A^* = \gamma_L^* \; (\lambda_A^\gamma = 0); \quad (2)\; A^* = [q\bar{q}] \; (\lambda_A^q = 0); \quad (3)\; A^* = [gg] \; (\lambda_A^g = 0, \pm 2)$

NNLP $\mathcal{M}_{\rm III}: \quad (1)\; A^* = [q\bar{q}g] \; (\lambda_A^{qqg} = \pm 1); \quad (2)\; A^* = [ggg] \; (\lambda_A^{ggg} = \pm 1, \pm 3)$

❑ Cross section level

$$|\mathcal{M}_{\rm I} + \mathcal{M}_{\rm II} + \mathcal{M}_{\rm III} + \cdots|^2 = \underbrace{|\mathcal{M}_{\rm I}|^2}_{\rm LP} + \underbrace{2\,{\rm Re}\,(\mathcal{M}_{\rm I}\mathcal{M}_{\rm II}^*)}_{\rm NLP} + \underbrace{2\,{\rm Re}\,(\mathcal{M}_{\rm I}\mathcal{M}_{\rm III}^*) + |\mathcal{M}_{\rm II}|^2}_{\rm NNLP} + \cdots$$

LP $|\mathcal{M}|^2_{\rm LP} = |\mathcal{M}_{\rm I}|^2$ **No ϕ modulation.**

NLP $|\mathcal{M}|^2_{\rm NLP} = 2\,{\rm Re}\,(\mathcal{M}_{\rm I}\mathcal{M}_{\rm II}^*)$ ➡ $\cos\phi$ **or** $\sin\phi$ **modulation, and** $\cos 3\phi$ **or** $\sin 3\phi$ **modulation**

NNLP $|\mathcal{M}|^2_{\rm NNLP} \begin{cases} 2\,{\rm Re}\,(\mathcal{M}_{\rm I}\mathcal{M}_{\rm III}^*) \\ |\mathcal{M}_{\rm II}|^2 \end{cases}$ ➡ **constant,** $\cos 2\phi$ **or** $\sin 2\phi$**, and** $\cos 4\phi$ **or** $\sin 4\phi$ **modulations**

↑ **NLO due to F_T^g and $F_T^{3,g}$**

Expectation beyond NLP

❑ Amplitude level

LP $\mathcal{M}_{\rm I}: \quad A^* = \gamma_T^* \; (\lambda_A^\gamma = \pm 1)$

NLP $\mathcal{M}_{\rm II}: \quad (1)\; A^* = \gamma_L^* \; (\lambda_A^\gamma = 0); \quad (2)\; A^* = [q\bar{q}] \; (\lambda_A^q = 0); \quad (3)\; A^* = [gg] \; (\lambda_A^g = 0, \pm 2)$

NNLP $\mathcal{M}_{\rm III}: \quad (1)\; A^* = [q\bar{q}g] \; (\lambda_A^{qqg} = \pm 1); \quad (2)\; A^* = [ggg] \; (\lambda_A^{ggg} = \pm 1, \pm 3)$

❑ Cross section level

$$|\mathcal{M}_{\rm I} + \mathcal{M}_{\rm II} + \mathcal{M}_{\rm III} + \cdots|^2 = \underbrace{|\mathcal{M}_{\rm I}|^2}_{\rm LP} + \underbrace{2\,{\rm Re}\,(\mathcal{M}_{\rm I}\mathcal{M}_{\rm II}^*)}_{\rm NLP} + \underbrace{2\,{\rm Re}\,(\mathcal{M}_{\rm I}\mathcal{M}_{\rm III}^*) + |\mathcal{M}_{\rm II}|^2}_{\rm NNLP} + \cdots$$

LP $|\mathcal{M}|^2_{\rm LP} = |\mathcal{M}_{\rm I}|^2$ **No ϕ modulation.**

NLP $|\mathcal{M}|^2_{\rm NLP} = 2\,{\rm Re}\,(\mathcal{M}_{\rm I}\mathcal{M}_{\rm II}^*)$ ➡ **$\cos\phi$ or $\sin\phi$ modulation, and $\cos 3\phi$ or $\sin 3\phi$ modulation**

NNLP $|\mathcal{M}|^2_{\rm NNLP} \begin{cases} 2\,{\rm Re}\,(\mathcal{M}_{\rm I}\mathcal{M}_{\rm III}^*) \\ |\mathcal{M}_{\rm II}|^2 \end{cases}$ ➡ **constant, $\cos 2\phi$ or $\sin 2\phi$, and $\cos 4\phi$ or $\sin 4\phi$ modulations**

Clear signature of twist-3 GPDs. ⬅ ***NEW* to this power!** (↑ $\cos 2\phi$ or $\sin 2\phi$)

NLO due to F_T^g and $F_T^{3,g}$ (↑ $\cos 4\phi$ or $\sin 4\phi$)

Expectation beyond NLP

❑ Amplitude level

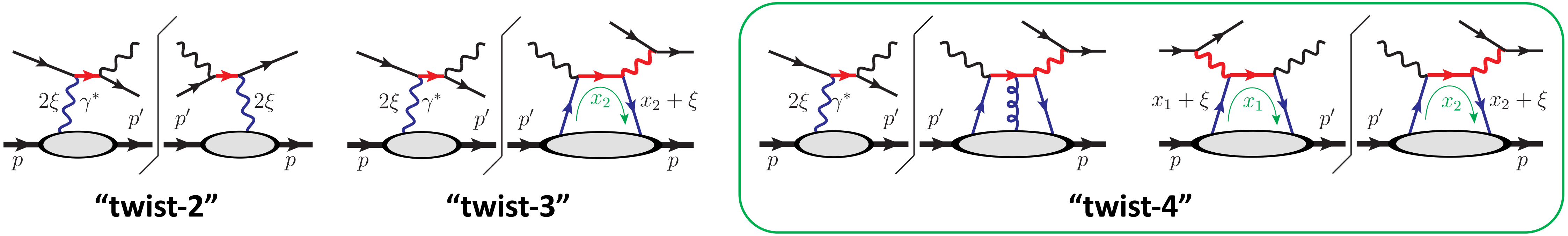
$$\textbf{LP} \quad \mathcal{M}_{\rm I}: \quad A^* = \gamma_T^* \ (\lambda_A^\gamma = \pm 1)$$

$$\textbf{NLP} \quad \mathcal{M}_{\rm II}: \quad (1)\ A^* = \gamma_L^* \ (\lambda_A^\gamma = 0); \quad (2)\ A^* = [q\bar{q}] \ (\lambda_A^q = 0); \quad (3)\ A^* = [gg] \ (\lambda_A^g = 0, \pm 2)$$

$$\textbf{NNLP} \quad \mathcal{M}_{\rm III}: \quad (1)\ A^* = [q\bar{q}g] \ (\lambda_A^{qqg} = \pm 1); \quad (2)\ A^* = [ggg] \ (\lambda_A^{ggg} = \pm 1, \pm 3)$$

❑ Cross section level

$$|\mathcal{M}_{\rm I} + \mathcal{M}_{\rm II} + \mathcal{M}_{\rm III} + \cdots|^2 = \underbrace{|\mathcal{M}_{\rm I}|^2}_{\rm LP} + \underbrace{2\,{\rm Re}\,(\mathcal{M}_{\rm I}\mathcal{M}_{\rm II}^*)}_{\rm NLP} + \underbrace{2\,{\rm Re}\,(\mathcal{M}_{\rm I}\mathcal{M}_{\rm III}^*) + |\mathcal{M}_{\rm II}|^2}_{\rm NNLP} + \cdots$$



➡ **DVCS-square term belongs to NNLP and shall be considered together with twist-3 GPD!**