Unveiling chiral-odd (transversity) dimeson GDAs

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Based on: SB[1], Renaud Boussarie[2], Bernard Pire[2], Lech Szymanowski[3], In Preparation

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Outline

Generalized $\pi\pi$ Distribution Amplitudes = The twin sisters of π GPDs

GDAs: basic properties

The process $e^- e^+ \to (\pi^+\pi^o)(\pi^-\pi^o)$

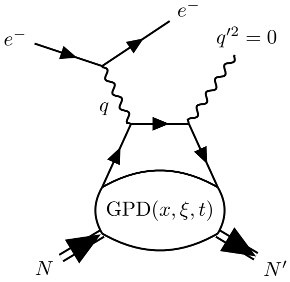
The two interfering amplitudes

Cross-section estimates

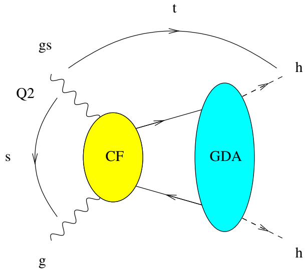
How to separate transversity GDAs contributions?

Conclusions

Crossing DVCS amplitude to $\gamma^*\gamma \to N\bar{N}$ or $\gamma^*\gamma \to \pi\pi$



(a) Factorization of GPDs in DVCS when $Q^2 >> -t$



(b) Factorization of GDAs in $\gamma^*(Q^2)\gamma \to h\,h$ when $Q^2 >> s$

Measured at BELLE
Bright future at BELLE 2 and BESS III
Other channels to be studied : $e^- e^+ \to \gamma\pi\pi$ (seen at BABAR)
$e^- e^+ \to \gamma N\bar{N}$

Nucleon GPDs and their (moderate ?) success story

$\to$ THE CHIRAL-EVEN SECTOR

Nucleon GPDs contribute to DVCS, TCS, DDVCS, DVMP... (and Nucleon EMT)
Meson GPDs very difficult to unravel (but Sullivan processes at EIC; see 2203.16947)

$\to$ THE CHIRAL-ODD SECTOR (also called TRANSVERSITY)

Chiral-odd quark GPDs DO NOT contribute to DVCS, TCS, DVMP at leading twist
Some hope in $2 \to 3$ processes such as $\gamma N \to \gamma \rho_T N'$ at large invariant mass of $\gamma \rho_T$
Various attempts (Goloskokov-Kroll) to modelize DVMP with twist 3 DA and C-O GPDs.

- Gluon transversity GPDs contribute at NLO to DVCS / TCS amplitudes

The transversity sector is the most elusive part of hadron tomography !

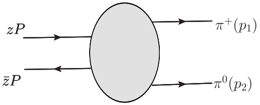
Definition of GDAs : a straightforward generalization of GPDs.
For simplicity consider spin 0 mesons (Wilson lines not explicitly written)

$$\left\langle \pi^0(p_1)\,\pi^+(p_2)\left|\bar{u}(x)\,\gamma^+ d(0)\right|0\right\rangle \longleftrightarrow \left\langle \pi^+(p_2)\left|\bar{u}(x)\,\gamma^+ d(0)\right|\pi^0(p_1)\right\rangle$$

$$\left\langle \pi^0(p_1)\,\pi^+(p_2)\left|\bar{u}(x)\,\sigma^{+i} d(0)\right|0\right\rangle \longleftrightarrow \left\langle \pi^+(p_2)\left|\bar{u}(x)\,\sigma^{+i} d(0)\right|\pi^0(p_1)\right\rangle$$

- **Same operators, same decomposition**, *(Mueller-94,Diehl-98,Polyakov-99)*
  Spin 0 : 2 GPDS, 2 GDAs . Spin 1/2 : 8 GPDs, 8 GDAs.
- **Same Collinear factorization : $\mathcal{A}$= coefficient function x GDA**
- **Same ERBL evolution equations for GDAs and meson DAs**
  (Efremov, Radyushkin, Brodsky, Lepage)
  Known asymptotic solutions $\phi^V(z,\zeta,s) \sim 6z\bar{z}(\zeta-\bar{\zeta})B(s)e^{i\delta(s)}$ Note: phase $e^{i\delta(s)}$
  But who believes in asymptopia ? why not holographic shape $\sim \sqrt{z\bar{z}}$ or anything else ?
- **Feasibility of measuring it in spacelike and timelike regions** (BELLE, BABAR).
  First extraction by Kumano-Song-Teryaev 2018

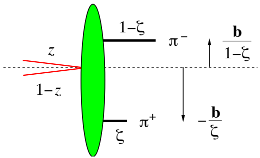
- **Graphical representation of GDA correlator and kinematical variables:**



GPD$(x, \xi, t) \to$ GDA(z, ζ, s)

z =light cone momentum fraction of quark
$\zeta, \bar{\zeta}$ = light cone momentum fraction of final mesons
(related to angle of meson $\vec{p}_i$ in dimeson CMS)

- **Impact parameter picture**: similar to GPDs (Fourier transform $p_T \to b_T$)



Double Distributions link GPDs and GDAs

- Spectral Representation of GPDs and GDAs : Various angles for the integration line
- BUT One universal Double distribution

$$H^q(x,\xi,t) = \int d\beta\, d\alpha\, \delta(x-\beta-\xi\alpha) f^q(\beta,\alpha,t) + \mathrm{sgn}(\xi) D^q\left(\frac{x}{\xi},t\right) ,$$

$$-\frac{1}{2}\Phi^q(z,\zeta,s) = (1-2\zeta)\int d\beta\, d\alpha\, \delta((1-2z)-\beta(1-2\zeta)-\alpha) f^q(\beta,\alpha,s) + D^q(1-2z,s) .$$

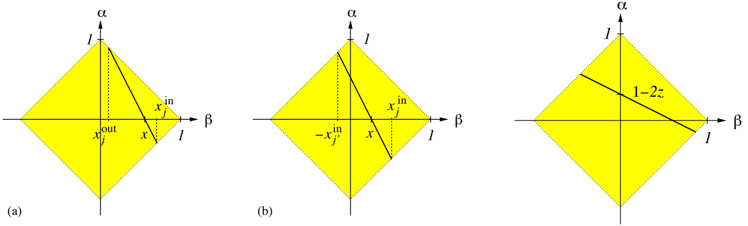
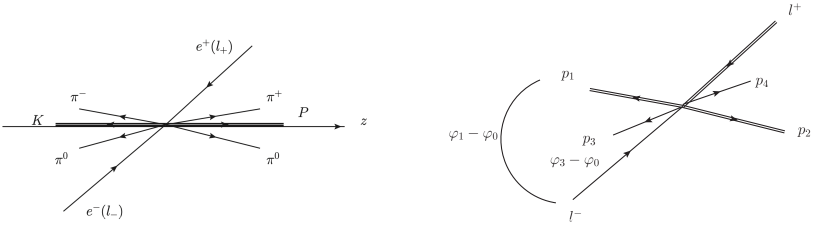


Figure: From the same DD, different lines of integration lead to GPDs in the DGLAP (l) or ERBL (m) regions and to GDAs (right).

$e^-(l^-)e^+(l^+) \to (\pi^+(p_1)\pi^0(p_2)) + (\pi^-(p_3)\pi^0(p_4))$



Longitudinal (left) and transverse (right) view of our process

- Production of two dipions with invariant masses $s_{12}, s_{34} << Q^2 = s$

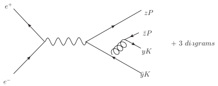
$$e^-(l^-)e^+(l^+) \to (\pi^+(p_1)\pi^0(p_2)) + (\pi^-(p_3)\pi^0(p_4))$$
$$(l^+ + l^-)^2 = Q^2 = s \;,\; (p_1+p_2)^2 = s_{12}, \;(p_3+p_4)^2 = s_{34}$$

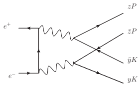
- Pion momenta parametrized on Sudakov basis P, K

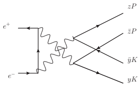
$$p_1 = zP + p_1^- K + \delta^T_{12}/2 \;,\; p_2 = \bar{z}P + p_2^- K - \delta^T_{12}/2$$
$$p_3 = yK + p_3^+ P + \delta^T_{34}/2 \;,\; p_4 = \bar{y}K + p_4^- P - \delta^T_{34}/2$$

- Each dipion contributes to the factorized amplitude through its GDAs $\Phi^V(z, \zeta_{ij}, s_{ij})$ and $\Phi^T(z, \zeta_{ij}, s_{ij})$

The scattering amplitudes







The 1 photon exchange amplitude (C−; see left) is the usual e-m form factor one (Φ^T does NOT contribute):

$$\mathcal{T}^- = \frac{e^2 g^2 C_F}{N_c} \bar{v}(l') \hat{P} u(l) \frac{2}{s^2} \int_0^1 \frac{dzdy}{yz} \Phi^V(z, \zeta_{12}, s_{12}) \Phi^V(y, \zeta_{34}, s_{34})$$

The 2 photon exchange amplitude (C+; see center + right) has two components ($\Phi^V \Phi^V$ and $\Phi^T \Phi^T$ DO contribute):

$$\mathcal{T}^V = C \bar{v}(\ell') \hat{\ell}_\perp u(\ell) \frac{8}{s^2} \int dzdy \frac{\Phi^V(y, \zeta_{12}, s_{12})}{y\bar{y}} \frac{\Phi^V(z, \zeta_{34}, s_{34})}{z\bar{z}} \frac{y\bar{y} + z\bar{z}}{(\bar{z}y - \bar{z}u - \bar{u}y)(\bar{y}z - \bar{y}u - uz)} ,$$

$$\mathcal{T}^T = -C \frac{8}{s^2 f_{2\pi}^{\perp 2}} [\delta_{12}^{\mu} \delta_{34}^{\nu} + \delta_{12}^{\nu} \delta_{34}^{\mu} - g_{\perp \mu\nu} \delta_{34} \cdot \delta_{12}] \, \bar{v}(l') \gamma_T^{\mu} u(l) l_\perp^{\nu} \cdot$$

$$\int dzdy \frac{\Phi^T(z, \zeta_{12}, s_{12})}{z\bar{z}(2\zeta_{12} - 1)} \frac{\Phi^T(y, \zeta_{34}, s_{34})}{y\bar{y}(2\zeta_{34} - 1)} \frac{-1 + \bar{y}z + \bar{z}y}{(\bar{z}y - \bar{z}u - \bar{u}y)(\bar{y}z - \bar{y}u - uz)} ,$$

Cross-section

$$d\bar{\sigma} = \frac{1}{S}\frac{1}{2s}\frac{1}{(2)^9}\frac{1}{(2\pi)^7}\bar{\Sigma}\,|\mathcal{T}|^2\, dcos\theta\, ds_{12}\, ds_{34}\, d\Omega_{12}\, d\Omega_{34}\,\frac{\sqrt{\lambda(s, s_{12}, s_{34})}}{s}\,\frac{\sqrt{\lambda(s_{12}, m_\pi^2, m_\pi^2)}}{s_{12}}\,\frac{\sqrt{\lambda(s_{34}, m_\pi^2, m_\pi^2)}}{s_{34}},$$

$$\bar{\Sigma}|\mathcal{T}|^2 = \bar{\Sigma}|\mathcal{T}^+|^2 + \bar{\Sigma}|\mathcal{T}^-|^2 + 2\bar{\Sigma}\mathcal{R}e\mathcal{T}^+\mathcal{T}^{-*}$$

$$\bar{\Sigma}\left|\mathcal{T}^+\right|^2 = \frac{e_\ell^4 e_{f_1}^2 e_{f_2}^2}{32 N_c^2 s^3} \times \Big\{ 2s^2 u\bar{u}\left(u-\bar{u})^2\right)\Psi_V\Psi_V^* - 2s\left(u-\bar{u})^2\right)\left[2\left(\boldsymbol{\ell}\cdot\boldsymbol{\delta}_{12}\right)\left(\boldsymbol{\ell}\cdot\boldsymbol{\delta}_{34}\right) - \boldsymbol{\ell}^2(\boldsymbol{\delta_{12}}\cdot\boldsymbol{\delta_{34}}\right]\frac{\Psi_V\Psi_T^* + \Psi_T^q\Psi_V^{q*}}{s}$$

$$+4\left[-8(\boldsymbol{\ell}\cdot\boldsymbol{\delta}_{34})^2(\boldsymbol{\ell}\cdot\boldsymbol{\delta}_{12})^2 + 2u\bar{u}(u-\bar{u})^2\boldsymbol{\delta_{12}}\cdot\boldsymbol{\delta}_{34}\frac{(\boldsymbol{\ell}\cdot\boldsymbol{\delta}_{34}\,\boldsymbol{\ell}\cdot\boldsymbol{\delta}_{12}) - \boldsymbol{\delta_{12}}\cdot\boldsymbol{\delta}_{34}\boldsymbol{\ell}^2/4}{\boldsymbol{\ell}^2} + \frac{1}{2s}(\ldots\ldots)\right]\Psi_T\Psi_T^*\Big\}$$

$$\bar{\Sigma}|\mathcal{T}^-|^2 = (e^2 g^2\frac{C_F}{N_c})^2 2s^2 u\bar{u}\,|\Psi_\gamma|^2$$

$$2\mathcal{R}e\mathcal{T}^+\mathcal{T}^{-*} = 4s(u-\bar{u})\frac{e^2 g^2 C_F}{N_c}\frac{-2}{9}\frac{e^2}{16 N_c}\mathcal{R}e[\Psi_\gamma^*(\boldsymbol{l_T}^2\,\Psi^V - (\boldsymbol{l_T}^2\,\boldsymbol{\delta}_{12}\cdot\boldsymbol{\delta}_{34} - 2\boldsymbol{l_T}\cdot\boldsymbol{\delta_{12}}\,\boldsymbol{l_T}\cdot\boldsymbol{\delta_{34}})\Psi^T/4]\,,$$

$$\Psi_V \equiv \frac{8}{s^2}\int\frac{dy}{y\bar{y}}\int\frac{dz}{z\bar{z}}\left(\frac{y\bar{y}+z\bar{z}}{(y\bar{z}-u\bar{z}-y\bar{u}+i0)(\bar{y}z-uz-\bar{y}\bar{u}+i0)}\right)\Phi_V(y,\zeta_{12},s_{12})\,\Phi_V(z,\zeta_{34},s_{34})$$

$$\Psi_T \equiv \frac{-32}{s^2 f_{2\pi}^{\perp 2}}\int\frac{dy}{y\bar{y}}\int\frac{dz}{z\bar{z}}\left(\frac{1}{y\bar{z}-u\bar{z}-y\bar{u}+i0} + \frac{1}{\bar{y}z-uz-\bar{y}\bar{u}+i0}\right)\frac{\Phi_T(y,\zeta_{12},s_{12})\,\Phi_T(z,\zeta_{34},s_{34})}{(2\zeta_{12}-1)(2\zeta_{34}-1)}$$

$$\Psi_\gamma = \frac{2}{s^2}\int dz\int dy\,\frac{\Phi_V(z,\zeta_{12},s_{12})\Phi_V(y,\zeta_{34},s_{34})}{y\bar{z}}\,.$$

Cross-section estimates (Preliminary)

To estimate the cross-sections, we use asymptotic DAs and normalize the C-odd GDA with $f_{2\pi}^{\perp} = 1$ GeV. We take the same phases $\delta(s_{ij})$ for the C-odd and C-even GDAs

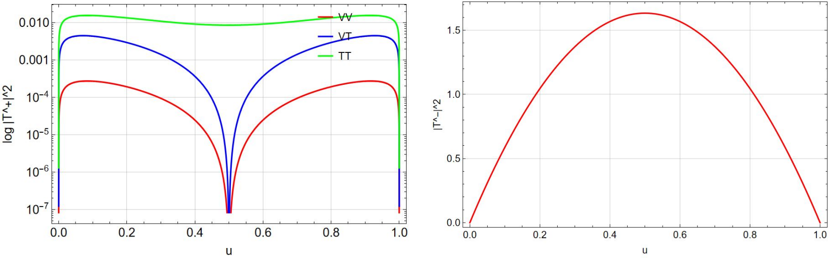
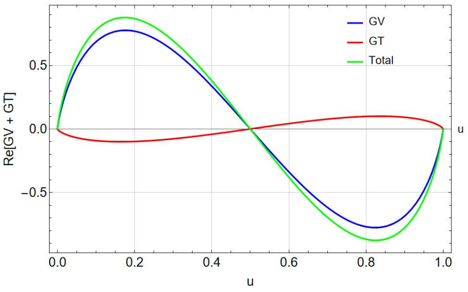


Figure: Charge-even (left) and charge-odd (right) squared amplitudes as a function of u

Chiral-odd GDAs dominate the charge-even cross-section, but the charge-odd cross section is much larger.

Separating the various components

- Define charge conjugation odd -asymmetries to separate $\mathcal{T}^+\mathcal{T}^{-*}$ $u \leftrightarrow \bar{u}$
  $\int du(u-\bar{u})\bar{\Sigma}|\mathcal{T}|^2 = \int du(u-\bar{u})2\mathcal{R}e\mathcal{T}^+\mathcal{T}^{-*}$
  Recall $u-\bar{u} = \cos\theta$; (θ = angle between $\vec{l}$ and $\vec{p}_1 + \vec{p}_2$) ; this is a forward-backward asymmetry.



- $\int d\varphi_1 d\varphi_3 \cos(2\varphi_0 - \varphi_1 - \varphi_3)2\mathcal{R}e\mathcal{T}^+\mathcal{T}^{-*}$ singles out the chiral odd contribution $\Psi^*_\gamma \Psi_T$
  $\varphi_1 - \varphi_0$ = angle between $\vec{l}$ and $\vec{\delta}_{12}$; $\varphi_2 - \varphi_0$ = angle between $\vec{l}$ and $\vec{\delta}_{34}$ in the transverse plane.

Conclusions & Future Work

- Chiral-odd GDAs contribute to the process $e^-e^+ \to (\pi^+\pi^0)(\pi^-\pi^0)$ in our kineamtics
- cross sections are small; distinctive interference effects exist but may be difficult to separate.
- the processes $e^-e^+ \to (\pi^+\pi^-)(\pi^-\pi^+)$ and $e^-e^+ \to (\pi^0\pi^0)(\pi^0\pi^0)$ have no leading twist Charge-odd contributions. Smaller but better suited cross-sections.
- feasibility study for Bes III and BELLE 2 needed
- lattice studies needed for both chiral-even and chiral-odd GDAs
- improve inverse Radon transform techniques to link π GPDs and $\pi\pi$ GDAs
- replace $\pi^+\pi^0$ by $p\,\bar{n} \to$ access the chiral-odd proton-antineutron GDA, related to nucleon transversity GPD

Last remark : $\pi\pi$ chiral-even GDAs contribute to and can be extracted from electroproduction of pion pairs at JLab and EIC (cf Warkentin-Diehl-Ivanov-Schaefer)

Thank you!