

Leading & Next-to-Leading Power TMD Factorization in the EIC Era

Physics Opportunities at an Electron-Ion Collider XI



Leonard Gamberg

- w/ Paul Anderson, Nobuo Sato, Richard Whitehill (JAM)
- Zhongbo Kang, Ding-Yu Shao, John Terry, Fany Zhao



Comments NLP unpolarized observables

- Discussion of NLP effects in SIDIS

- NLP factorization **TMD and or Collinear P_T**

- **The observable** $R_{SIDIS} = \frac{\sigma_L}{\sigma_T} \sim \frac{F_{UU,L}}{F_{UU,T}}$ Feynman “Photon-Hadron Phys.” 1972, Ravndal, PLB 1973

- **The observable** $\langle \cos \phi \rangle$ Georgi & Cahn, PRL 1978, PLB 1978

Critique of the perturbative QCD calculation of azimuthal dependence in lepton production
emphasize importance intrinsic k_T the early days/birth of TMD physics

Leads to..

1. The challenge of matches of “low” to “high” transverse momentum spectrum q_T or P_{hT}
2. Factorization BUT @ NLP order α_s ... issues ... necessary (but not sufficient) consistency checks
3. Ongoing work

Importance of NLP TMDs & Factorization

- Importance of NLP “TMD-like” observables underscored while suppressed by $(M/Q)^n$ wrt LP observables
 - ◉ NLP/SLP TMDs as sizable as leading-power TMDs in situations where Q is not that large...e.g. the kinematics of fixed-target experiments
- Their understanding is required for a complete description of “benchmark processes” SIDIS, DY & e^+e^- ...
- Are of interest offer a mechanism to probe physics of quark-gluon-quark correlations, provide novel information about the partonic structure of hadrons; only recently unexplored beyond α_s
 - ◉ Such correlations may be considered quantum interference effects, related to average transverse forces acting on partons inside (polarized) hadrons as well as other phenomena.
- Experimental information SIDIS available DESY/Zeus, Fermi-LAB, HERMES, COMPASS, JLab **extra slides**
- **Opportunity for EIC with its large kinematical coverage TMD program ideal for making further groundbreaking progress in this area**
 - ◉ **NB: Iff factorization can be established beyond “tree level” @ next to leading order**
-Global analysis in terms of NLP TMDs

TMD fact at NLP w & w/o polarization (Literature)



TMD Handbook

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10 - Subleading TMDs

L. Gamberg, A. Metz, I. Stewart

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From a historical perspective it is very interesting that the subleading-power $\cos \phi_h$ azimuthal modulation of the unpolarized SIDIS cross section was important for the development of the TMD field, since one of the earliest discussions of transverse parton momenta in DIS is related to this observable [290, 291, 1237]; see also Sec. 5.1 for more details. Generally, although suppressed by Λ/Q with respect to leading-power observables, subleading TMD observables are typically not small, especially in the kinematics of fixed-target experiments. In fact, the first-ever observed SSA in SIDIS was a sizeable power-suppressed longitudinal target SSA for pion production from the HERMES Collaboration [480]. Those measurements, which triggered many theoretical studies and preceded the first measurements of the (leading-power) Sivers and Collins SSAs, were critical for the growth of TMD-related research.

e-Print: 2304.03302 [hep-ph]

[290] R. N. Cahn, *Azimuthal Dependence in Leptoproduction: A Simple Parton Model Calculation*, *Phys. Lett. B* **78** (1978) 269.

[1237] F. Ravndal, *On the azimuthal dependence of semiinclusive, deep inelastic electroproduction cross-sections*, *Phys. Lett. B* **43** (1973) 301.

[480] HERMES collaboration, A. Airapetian et al., *Observation of a single spin azimuthal asymmetry in semiinclusive pion electro production*, *Phys. Rev. Lett.* **84** (2000) 4047

TMD factorization @ NLP w & w/o polarization (vast subject & incomplete ...Literature)

F. Rivindal PLB 1972

Georgi Politzer PRL 1978

Cahn PLB 1978 (response to Georgi Politzer PRL 1978)

A.Kotzinian NPB (1994)

J. Levelt, P.Mulders Phys. Rev. D(1994)

R. Tangerman, P. Mulders hep-ph/9408305 [hep-ph] (1994)

P.Mulders, R. Tangerman, NPB 461(1996)

D. Boer, P. Mulders, Phys.Rev.D 57 (1998)

L. Gamberg, D. Hwang, A Metz, M. Schlegel, PLB 639 (2006), uncanceled rapidity div. @tw3-factorization

Boer Vogelsang DY PRD 2006

Koike Nagashima Vogelsang SIDIS NPB 2006 Large P_T

A.Bacchetta, D. Boer, M. Diehl, P. Mulders JHEP (2008) factorization at NLP consistency checks on matching

A.P. Chen, J.P. Ma, Phys. Lett. B 768 (2017)

I. Feige, D.W. Kolodrubetz, I. Moul, I.W. Stewart, J. High Energy Phys. 11 (2017)

I. Balitsky, A. Tarasov, J. High Energy Phys. 07 (2017)

I. Balitsky, A. Tarasov, J. High Energy Phys. 05 (2018)

M.A. Ebert, I. Moul, I.W. Stewart, F.J. Tackmann, G. Vita, H.X. Zhu, J. High Energy Phys. 12 (2018)

M.A. Ebert, I. Moul, I.W. Stewart, F.J. Tackmann, G. Vita, H.X. Zhu, J. High Energy Phys. 04 (2019)

Moul, I.W. Stewart, G. Vita, arXiv:1905.07411, 201

A. Bacchetta Bozzi, Echevarria, Pisano, Prokudin, Radici, Physics Letters B 797 (2019)

A.Vladimirov Moos, Scimemi, & S.Rodini JHEP 2022

M. Ebert A. Gao I. Stewart JHEP 06 (2022)

S. Rodini, A. Vladimirov JHEP 08 (2022)

L.Gamberg, Z.Kang, D.Shao, J.Terry, F.Zhao arXiv: e-Print:221.13209 (2022)

I.Balitsky, JHEP 03 (2023) and 2024

L.Gamberg, Z.Kang, D.Shao, J.Terry, F.Zhao in prep 2024

Also Spin transverse spin-dependence Qui Serman collinear higher twist 1991 NLB

X. Ji, J.W. Qiu, W. Vogelsang, and F. Yuan, Phys.Rev.Lett. 97 (2006), Phys.Lett.B 638 (2006), Phys.Rev.D 73 (2006)

Challenges of *SLP/NLP* TMDs

NLP TMD observables challenging in comparison to the current state-of-the-art of leading power observables
Treatments in the literature are mostly limited to a tree-level formalism until recently

****First studies beyond tree level : "Matches & Mis-matches" Bacchetta et al. JHEP 2008, Chen et al. PLB 2017**

More recently

A.P. Chen, J.P. Ma, PLB (2017)

Bacchetta et al. PLB 2019

MIT group, Gao, Ebert, Stewart JHEP 2022

Gamberg, Kang, Shao, Terry, Zhao arXiv: e-Print:221.13209

Vladimirov, Rodini, Scimemi, Moos, JHEP 2021, 2022, JHEP 2023, PRD 2024

Balitsky 2023 rapidity only TMD evolution

- ***In arXiv: e-Print:221.13209 present a systematic procedure for stress testing TMD factorization for DY & SIDIS at NLP using CSS formalism which addresses disagreements in the literature***

R_{SIDIS} NLP TMDs?

- NLP TMD observables challenging in comparison to the current “state-of-the-art” of leading power observables
Treatments in the literature are mostly limited to a tree-level formalism (until recently ...)
- The best (and least) known of these is the ratio of the longitudinal to transverse cross section (SIDIS) $(m/Q)^2$
role of **longitudinal** and **transverse** photon contributions in SIDIS

$$R = \frac{\sigma_L}{\sigma_T} = \frac{4 (m^2 + \langle p_{\perp}^2 \rangle)}{Q^2}$$

where $\langle p_{\perp}^2 \rangle$ is the intrinsic parton transverse momentum.

Feynman 1972 Photon Hadron Interactions, Ravndal PLB 1973 ... for details see Cahn 1989 PRD
often assumed that $F_{UU,L}$ is negligible at low transverse momentum

R_{SIDIS} NLP TMDs?

In 21st century interpretation as TMD physics

$$R = \frac{\sigma_L}{\sigma_T} \approx \frac{F_{UU,L}}{F_{UU,T}}$$

$$\frac{d\sigma}{dx dy dz d\phi_S d\phi_h dP_{h\perp}^2} = \frac{\alpha^2}{xQ^2} \frac{y}{2(1-\varepsilon)} \left\{ F_{UU,T} + \varepsilon F_{UU,L} + \dots \right.$$

Bacchetta et al. JHEP 2007

$$F_{UU,T} = \mathcal{C}[f_1 D_1]$$

$$F_{UU,L} = \mathcal{C}[\dots]$$

$$\mathcal{C}[w f D] = x \sum_a e_a^2 \int d^2\mathbf{p}_T d^2\mathbf{k}_T \delta^{(2)}(\mathbf{p}_T - \mathbf{k}_T - \mathbf{P}_{h\perp}/z) w(\mathbf{p}_T, \mathbf{k}_T) f^a(x, p_T^2) D^a(z, k_T^2)$$

R_{SIDIS} NLP TMDs?

- Nonetheless a sizable contribution up to 20%
Estimate Bacchetta et al. (MAP) JHEP **10**, 127 (2022)

$$\frac{d\sigma}{dx dy dz d\phi_S d\phi_h dP_{h\perp}^2} = \frac{\alpha^2}{xQ^2} \frac{y}{2(1-\varepsilon)} \left\{ F_{UU,T} + \varepsilon F_{UU,L} + \right.$$

- ε ratio of longitudinal and transverse photon flux ...

$$\varepsilon = \frac{1-y}{1-y+y^2/2}$$

- Findings demonstrate that $F_{UU,L}$ cant be ignored, can be substantial & essential for an accurate interpretation of $F_{UU,T}$ which is associated with leading twist TMDs.

• ***What is the physics here?***

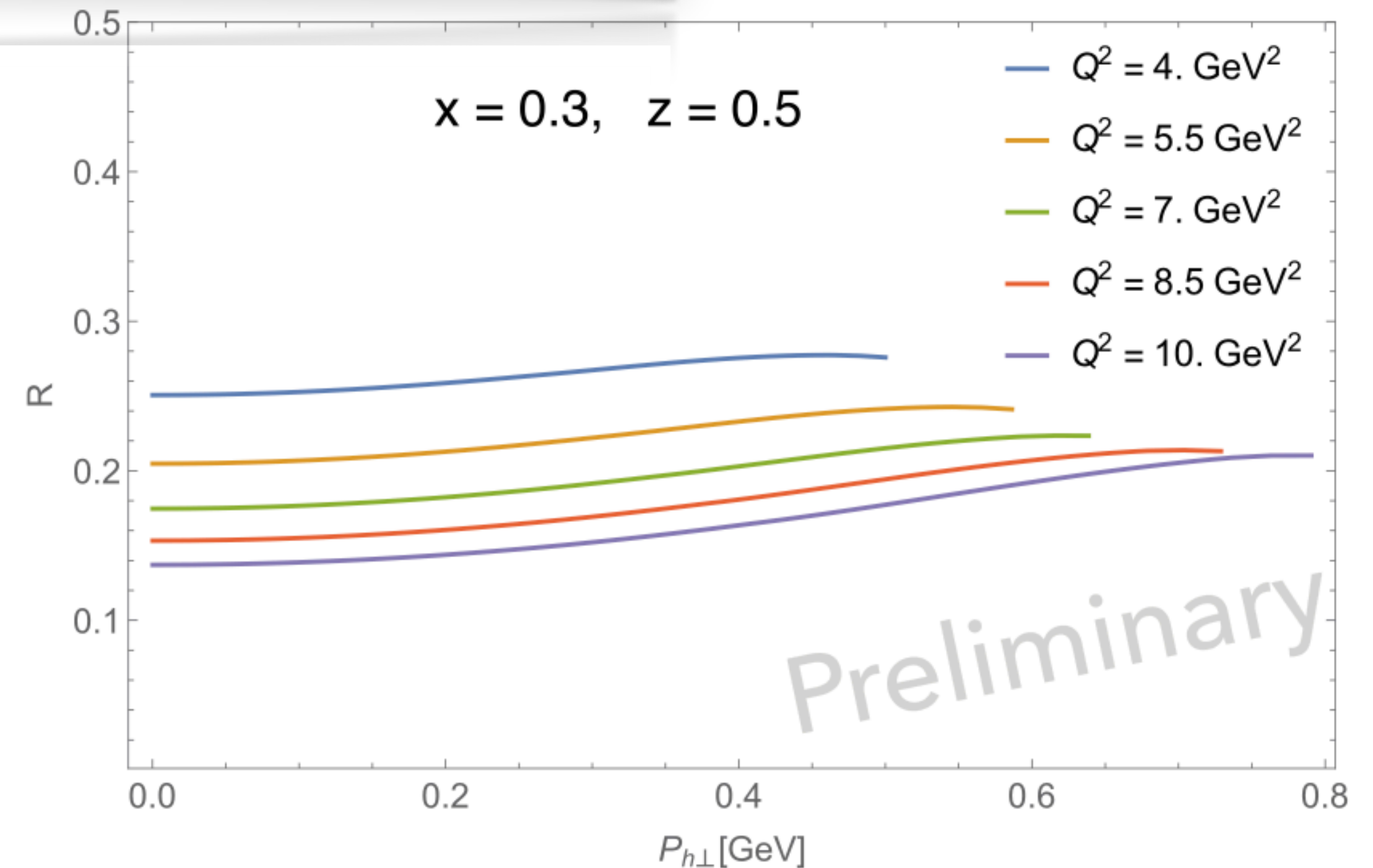




Fig. 18 Estimate of $R_{SIDIS} = F_{UU,L}/F_{UU,T}$ versus the hadron transverse momentum $P_T (P_{hT})$ at fixed values of x and z and for different values of Q^2 , compatible with JLab22 kinematics, using MAP22 TMD analysis [134]



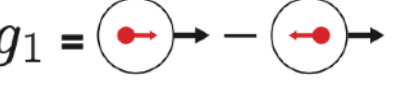



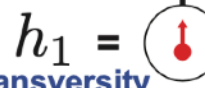
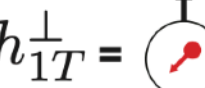
Context TMD Correlator at tree level “twist 4”

K. Goeke, A. Metz, M. Schlegel PLB 2005

$$\Phi^{[\gamma^-]} = \frac{M^2}{(P^+)^2} \left[f_3(x, \vec{k}_T^2) - \frac{\varepsilon_T^{ij} k_{Ti} S_{Tj}}{M} f_{3T}^\perp(x, \vec{k}_T^2) \right]$$

Correlator at tree level @ “twist” 4
previously of academic interest

Leading Quark TMDPDFs  Nucleon Spin  Quark Spin

		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 = \text{Unpolarized}$ 		$h_1^\perp = \text{Boer-Mulders}$ 
	L		$g_1 = \text{Helicity}$ 	$h_{1L}^\perp = \text{Worm-gear}$ 
	T	$f_{1T}^\perp = \text{Sivers}$ 	$g_{1T}^\perp = \text{Worm-gear}$ 	$h_1 = \text{Transversity}$  $h_{1T}^\perp = \text{Pretzelosity}$ 

Subleading Quark TMDPDFs

		Quark Chirality	
		Chiral Even	Chiral Odd
Nucleon Polarization	U	f^\perp, g^\perp	e, h
	L	f_L^\perp, g_L^\perp	e_L, h_L
	T	$f_T, f_T^\perp, g_T, g_T^\perp$	$e_T, e_T^\perp, h_T, h_T^\perp$

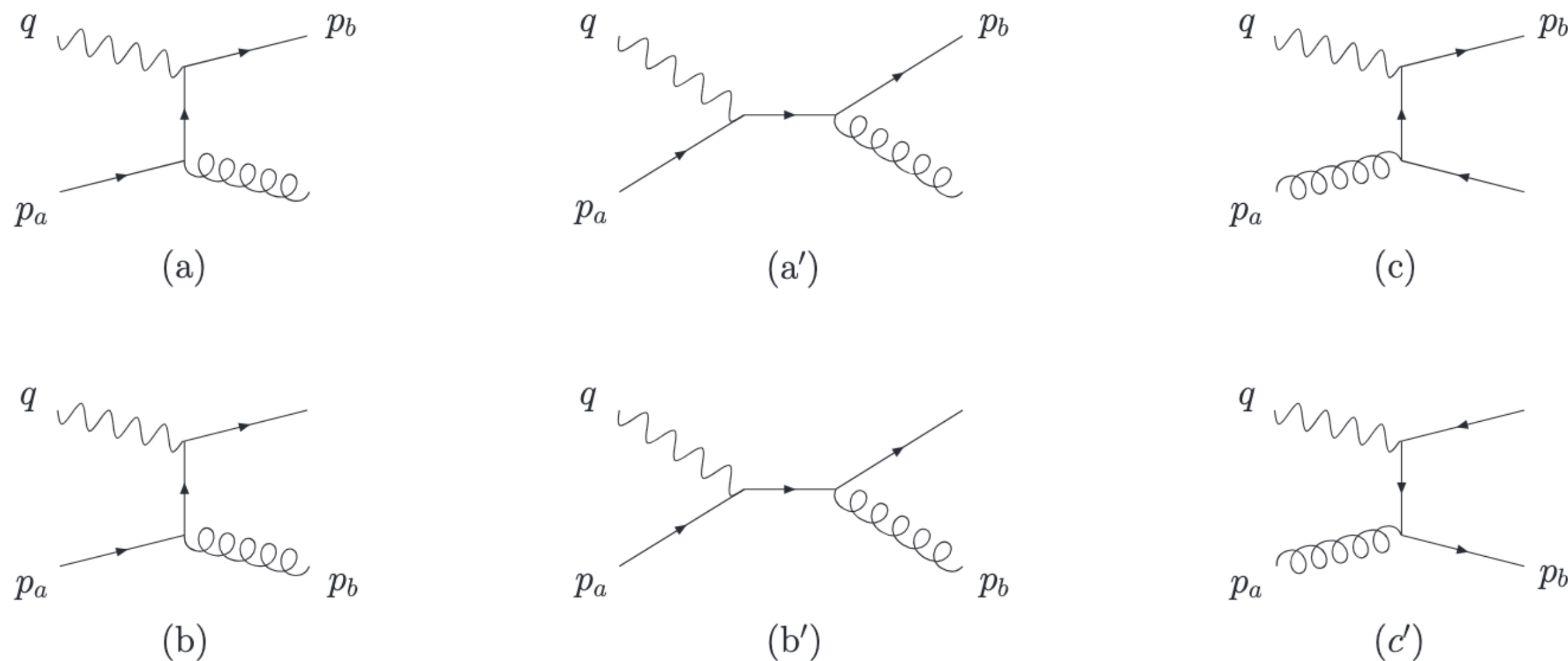
$$\Phi^{[\gamma^+]} = f_1(x, \vec{k}_T^2) - \frac{\varepsilon_T^{ij} k_{Ti} S_{Tj}}{M} f_{1T}^\perp(x, \vec{k}_T^2),$$

$$\Phi^{[\gamma^i]} = \frac{M}{P^+} \left[\frac{k_T^i}{M} \left(f^\perp(x, \vec{k}_T^2) - \frac{\varepsilon_T^{jk} k_{Tj} S_{Tk}}{M} f_T^{\perp'}(x, \vec{k}_T^2) \right) + \dots \right]$$

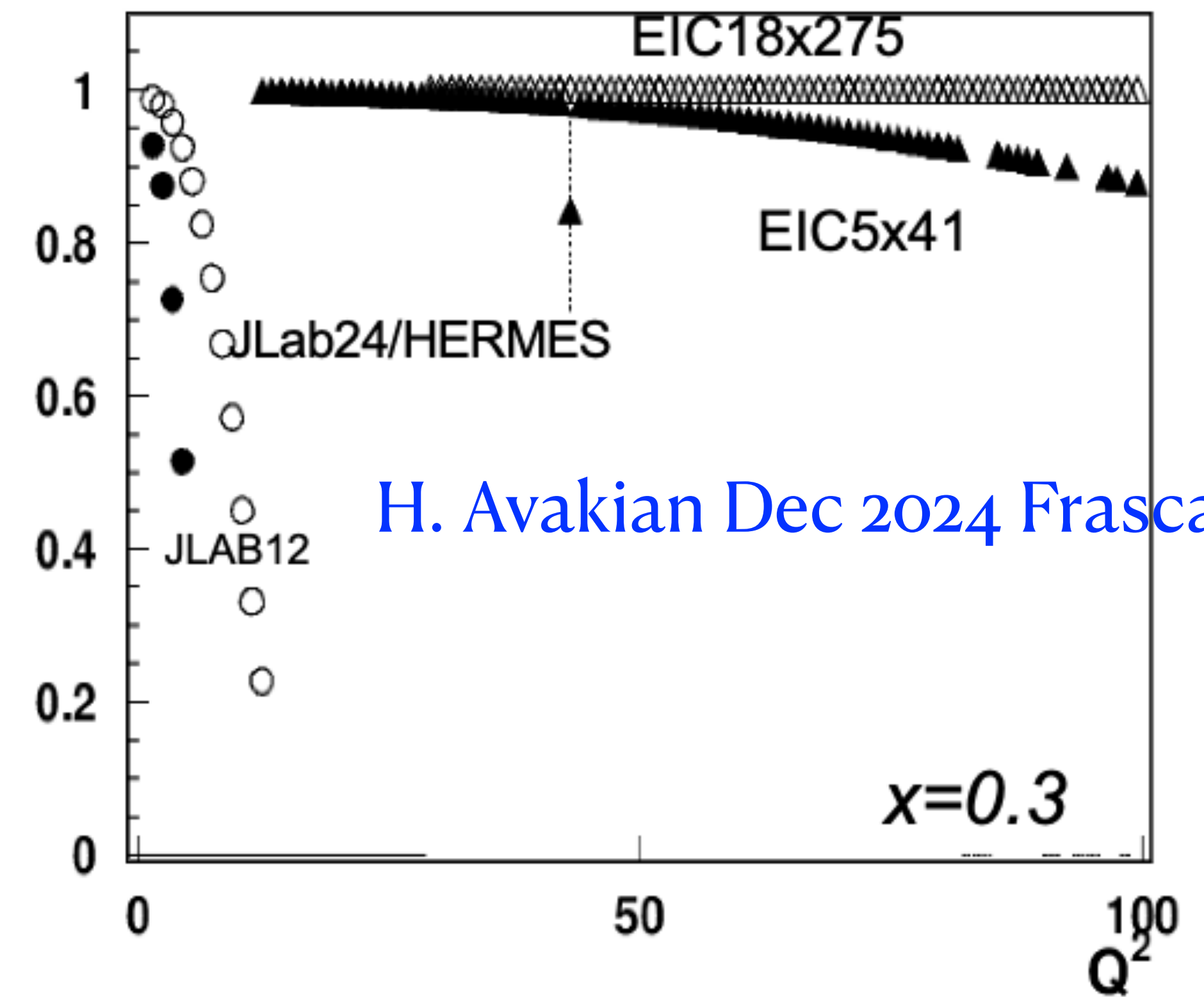
$R_{SIDIS} \text{ \& EIC}$

- However @ large P_T , $F_{UU,L} \sim F_{UU}^{\cos 2\phi_h}$!! see Bacchetta et al. JHEP 2008 "Matches & Mis-matches":
 in principle hard gluon radiation – "collinear P_T factorization applies CSS 1985 in principle Catani et al. 1997-2015, Vogelsang Koike ... many others

$$\frac{d\sigma}{dx dy dz d\phi_S d\phi_h dP_{h\perp}^2} = \frac{\alpha^2}{xQ^2} \frac{y}{2(1-\varepsilon)} \left\{ F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)} \cos\phi_h F_{UU}^{\cos\phi_h} + \varepsilon \cos(2\phi_h) F_{UU}^{\cos 2\phi_h} \right.$$



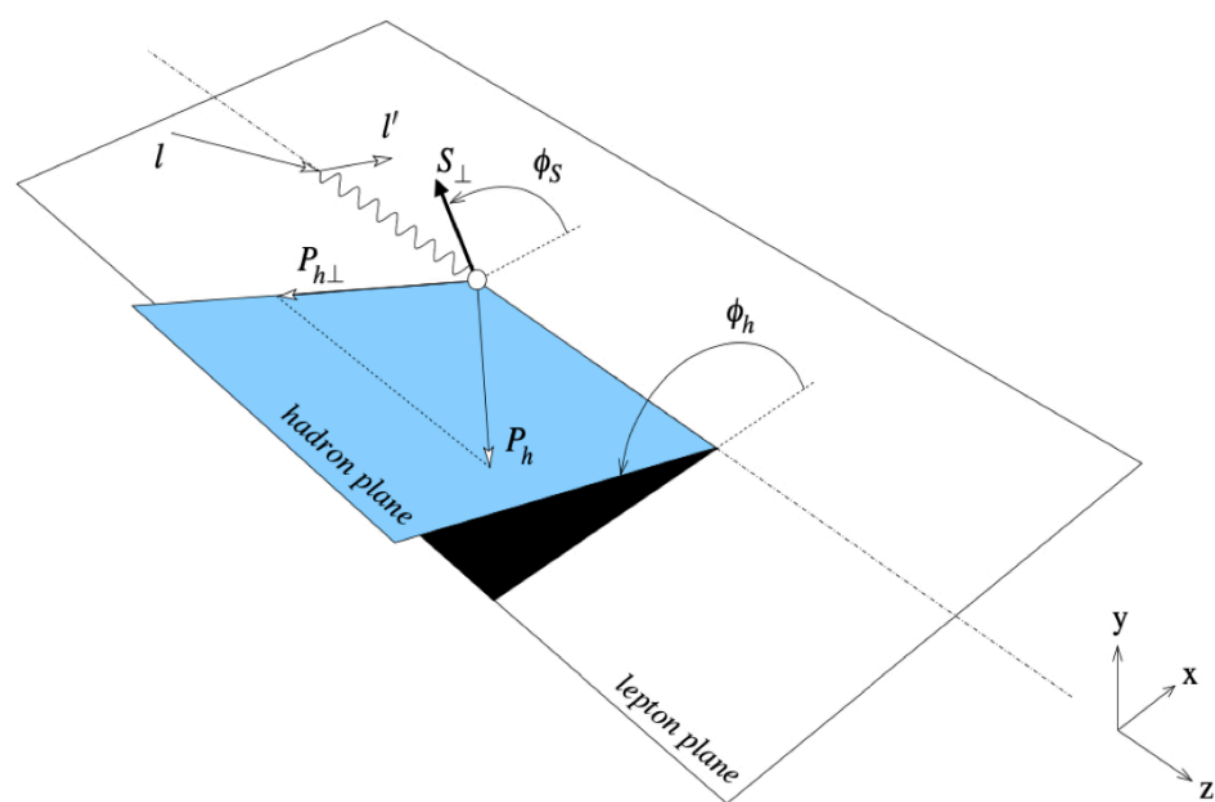
\mathcal{E}



H. Avakian Dec 2024 Frascati

TMD Factorization & Collinear Factorization

- **TMD** applicable $\Lambda_{QCD} \sim P_{h\perp} \ll Q$ **Collinear** applicable $P_{h\perp} \sim Q \gg \Lambda_{QCD}$
- $P_{h\perp} \sim \mathbf{k}_T$ or \mathbf{p}_T intrinsic transverse momentum partons CS described via TMDs
- $P_{h\perp} \gg \mathbf{k}_T$ or \mathbf{p}_T generated transverse momentum in the final state as perturbative radiation & non-perturbative structure is given by collinear pdfs & FFs



$$E' E_h \frac{d\sigma_{ep \rightarrow e' h X}}{d^3 l' d^3 P_h} \approx \hat{\sigma}_{eq \rightarrow e' q'} \otimes f_1 \otimes \tilde{D}_{h/q'}$$

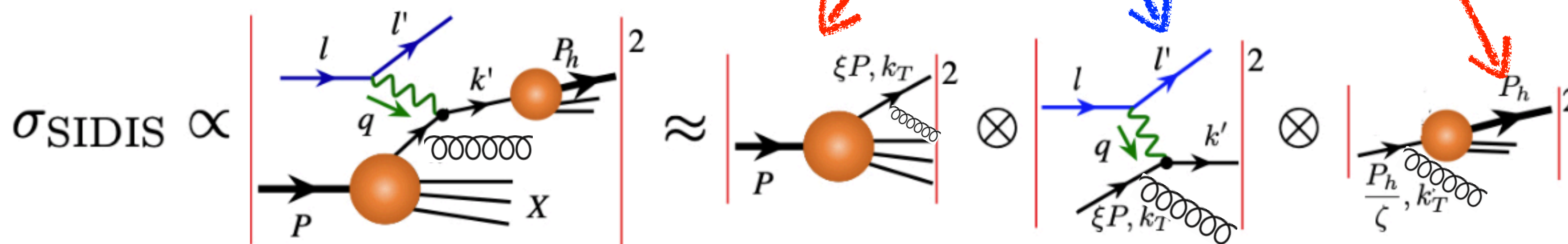
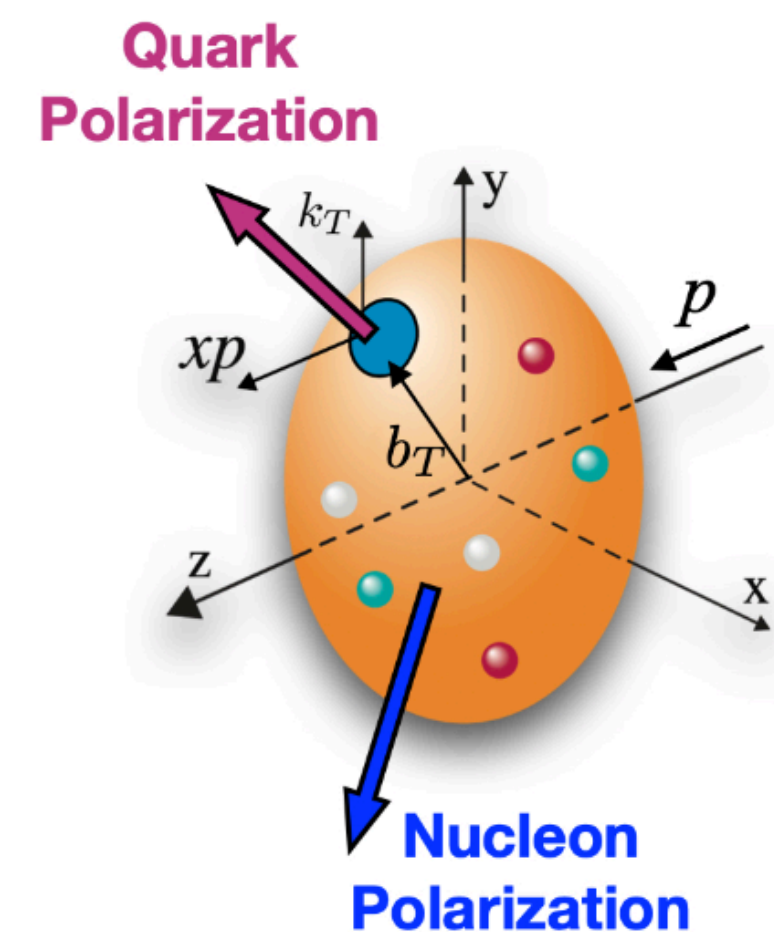


Figure 1.1: Illustration of the momentum and spin variables probed $\overline{\text{MD}}$ parton distributions.

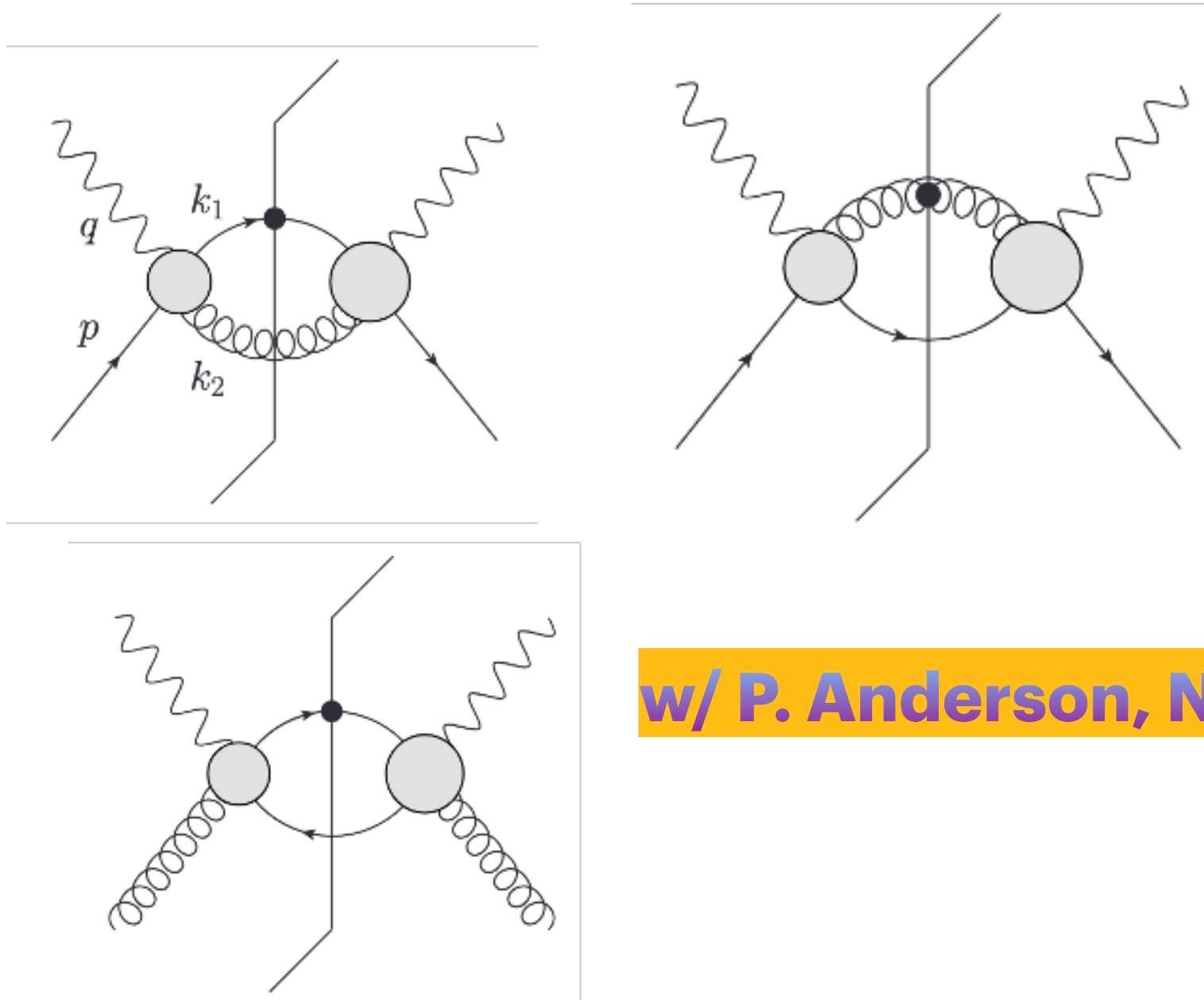
R_{SIDIS} & σ_L at large p_T

$$\frac{d\sigma}{dx dy dz dq_T^2 d\varphi} = \frac{\pi\alpha^2 yz}{4Q^2} \left[\underbrace{\sinh^2 \vartheta F_{UU,L}}_{\sigma_0} - \frac{1}{2} \underbrace{(2 + \sinh^2 \vartheta) F_{UU,T}}_{\sigma_1} - \underbrace{\sinh 2\vartheta F_{UU}^{\cos \varphi}}_{\sigma_1} \cos \varphi + \frac{1}{2} \underbrace{\sinh^2 \vartheta F_{UU}^{\cos 2\varphi}}_{\sigma_2} \cos 2\varphi \right]$$

e.g.

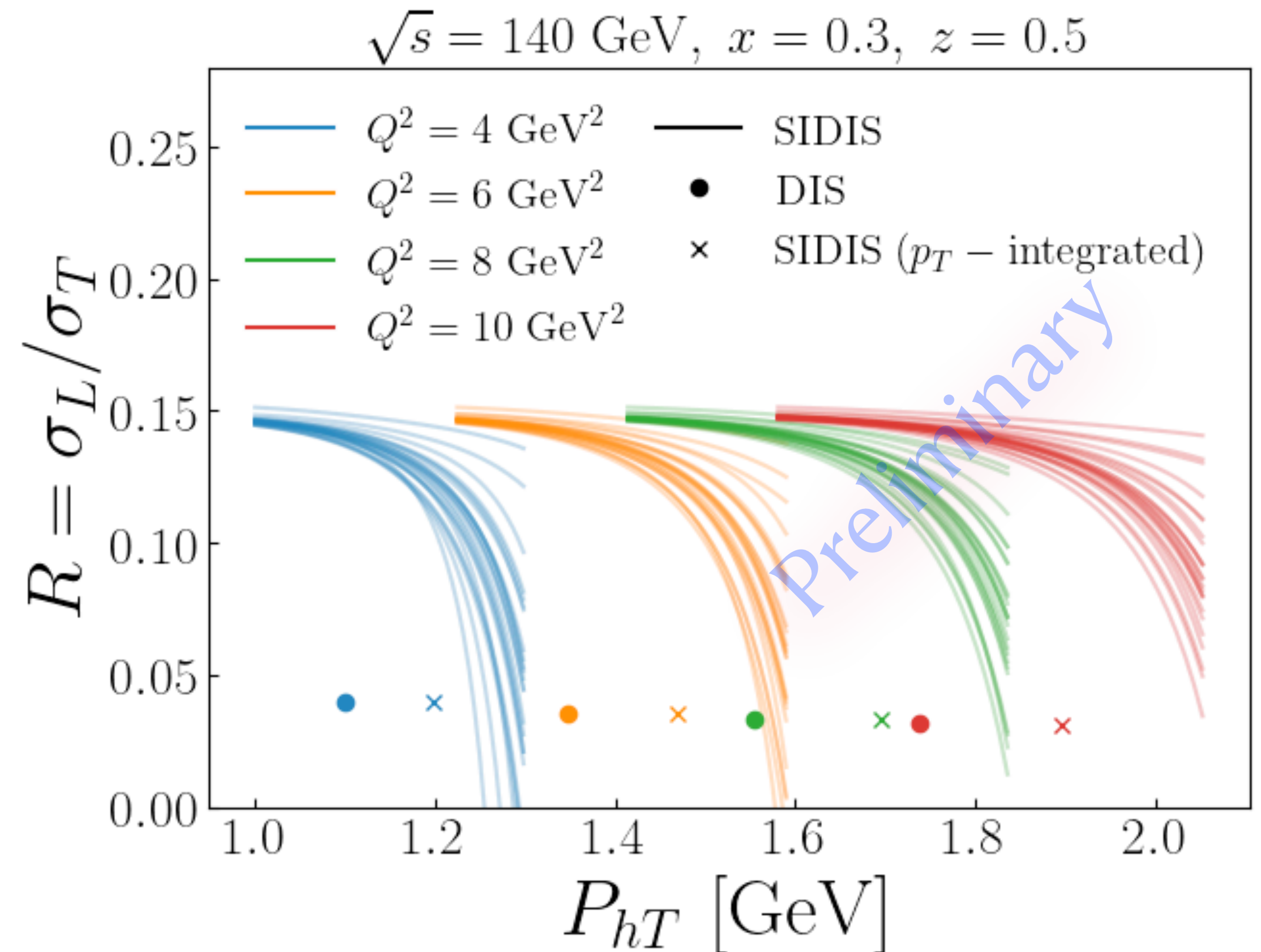
$$\sinh \vartheta \equiv \frac{2}{y} \sqrt{1-y}$$

$$F_{UU,T} = \frac{1}{Q^2} \frac{\alpha_s}{(2\pi z)^2} \sum_a x e_a^2 \int_x^1 \frac{d\hat{x}}{\hat{x}} \int_z^1 \frac{d\hat{z}}{\hat{z}} \delta\left(\frac{q_T^2}{Q^2} - \frac{(1-\hat{x})(1-\hat{z})}{\hat{x}\hat{z}}\right) \times \left[f_1^a\left(\frac{x}{\hat{x}}\right) D_1^a\left(\frac{z}{\hat{z}}\right) C_{UU,T}^{(\gamma^*q \rightarrow qg)} + f_1^a\left(\frac{x}{\hat{x}}\right) D_1^g\left(\frac{z}{\hat{z}}\right) C_{UU,T}^{(\gamma^*q \rightarrow gq)} + f_1^g\left(\frac{x}{\hat{x}}\right) D_1^a\left(\frac{z}{\hat{z}}\right) C_{UU,T}^{(\gamma^*g \rightarrow q\bar{q})} \right]$$



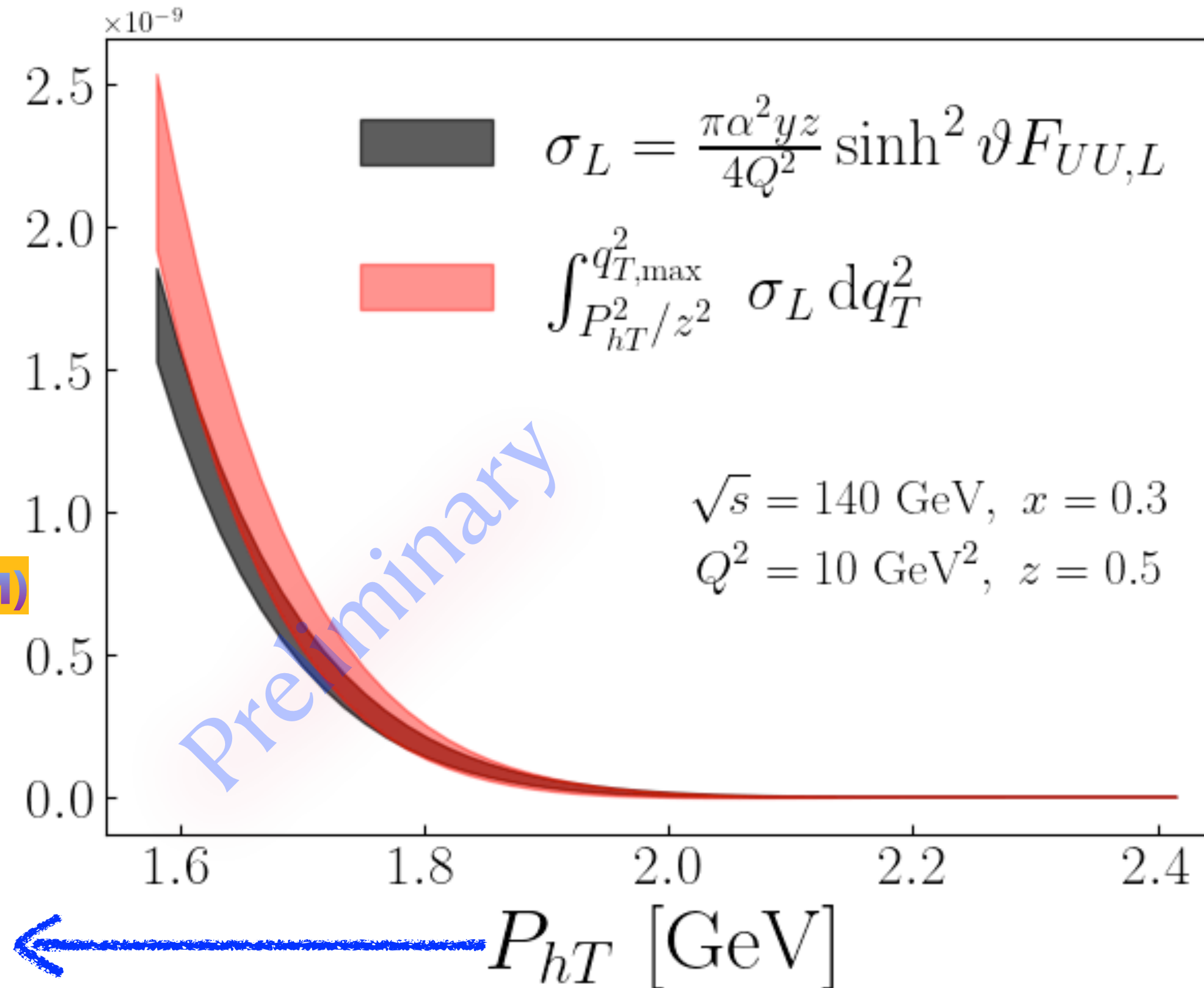
w/ P. Anderson, N. Sato, R. Whitehill (JAM)

- $\gamma^* q \rightarrow qg$ $C_{UU}^{\cos 2\phi_h} = 4C_F \hat{x}\hat{z}$,
- $\gamma^* q \rightarrow gq$ $C_{UU}^{\cos 2\phi_h} = 4C_F \hat{x}(1-\hat{z})$,
- $\gamma^* g \rightarrow q\bar{q}$ $C_{UU}^{\cos 2\phi_h} = 8T_R \hat{x}(1-\hat{x})$,



Longitudinal cross section $\sigma_L(P_{hT}; x, z, Q^2)$

- Dark gray is the P_{hT} differential longitudinal cross section
- Red is the truncated moment of that cross section
- Can this inform us of the low $P_T \approx zq_T \sim k_T$
- Can we extend to low P_{hT} *resum* and learn something about transitions to TMD region?



$$\sinh \vartheta \equiv \frac{2}{y} \sqrt{1-y}$$

w/ P. Anderson, N. Sato, R. Whitehill (JAM)

The observable $\langle \cos \phi \rangle$

No assumption of mechanism

$$\frac{d\sigma}{dx dy dz d\phi_S d\phi_h dP_{h\perp}^2} = \frac{\alpha^2}{xQ^2} \frac{y}{2(1-\varepsilon)} \left\{ F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)} \cos \phi_h F_{UU}^{\cos \phi_h} + \varepsilon \cos(2\phi_h) F_{UU}^{\cos 2\phi_h} \right\}$$

$$\int d\sigma^{(1)} \cos \phi = \int d^2 P_T \cos \phi \frac{d\sigma}{dx_H dy dz_H d^2 P_T}$$

SIDIS Kinematics dictionary

$$Q^2 = -q^2, \quad \mathbf{P}_T = \mathbf{P}_{2T}, \quad \phi,$$

$$x_H = \frac{Q^2}{2P_1 \cdot q}, \quad y = \frac{P_1 \cdot q}{P_1 \cdot k_1}, \quad z_H = \frac{P_1 \cdot P_2}{P_1 \cdot q},$$

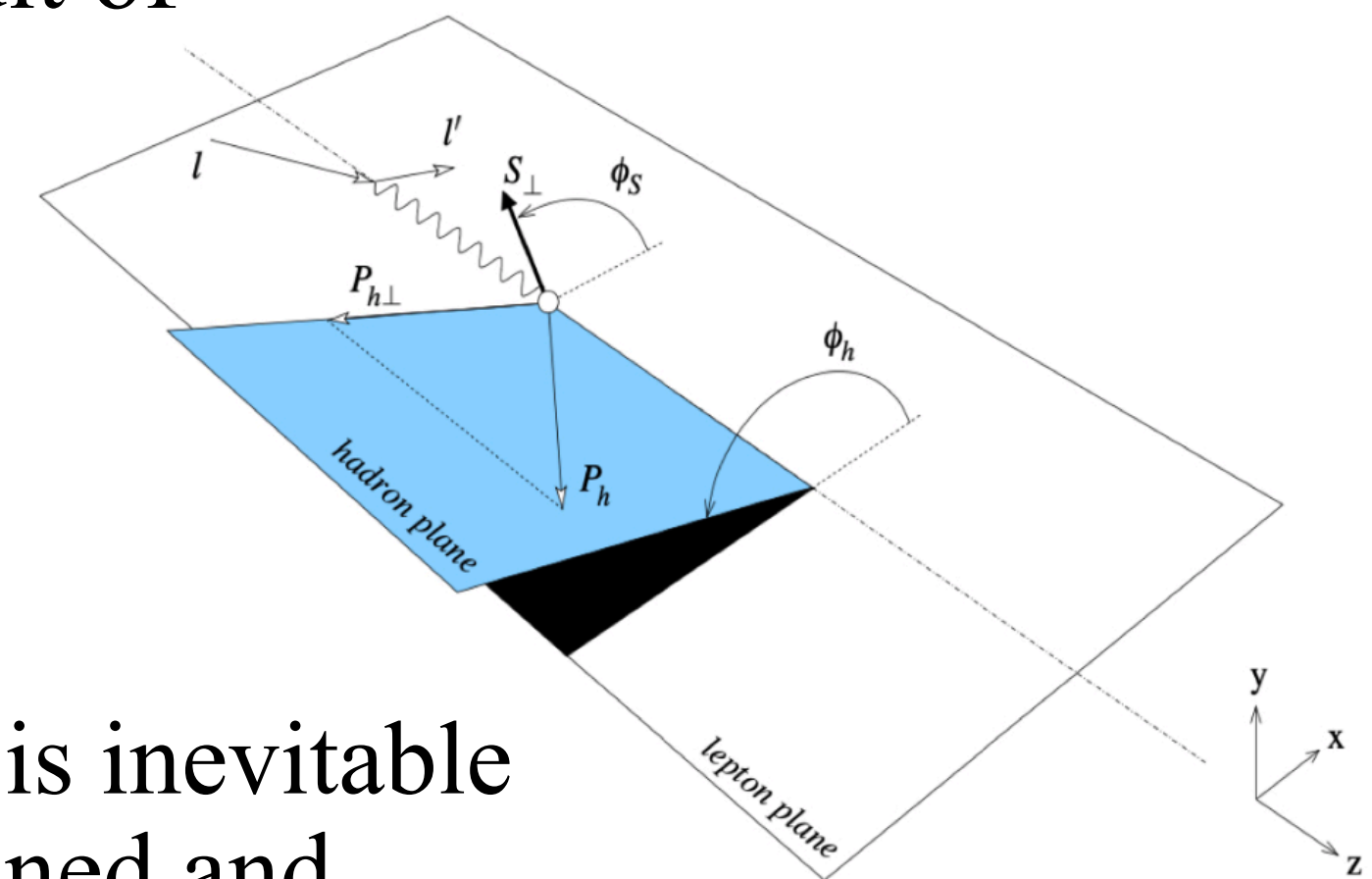
and the parton variables

$$x = \frac{x_H}{\xi} = \frac{Q^2}{2p_1 \cdot q}, \quad z = \frac{z_H}{\xi'} = \frac{p_1 \cdot p_2}{p_1 \cdot q}.$$

TMDs @ “twist-3” NLP

The beginning of TMD physics? $\langle \cos \phi \rangle$

- **Georgi Politzer, PRL 1978** “Measurement $\langle \cos \phi \rangle$ provides clean test of predictions of PQCD
Performed QCD analysis of hard gluon radiation in SIDIS: predict absolute value of final state hadron’s P_T or $P_{h\perp}$, and the angular distribution relative to lepton scattering plane
- ~12-15% ...clean test of QCD “...since such effects would not arise as a result of limited transverse momentum associated with confined quarks...”
- **Cahn, PLB 1978, (& earlier paper by Ravndal, PLB 1972)**
Critique QCD calculation of azimuthal dependence
emphasize importance intrinsic k_T ...
- “We conclude that the azimuthal dependence in vector exchange interactions is inevitable since the partons have transverse momentum as a consequence of being confined and such dependence certainly does not require a special mechanism like gluon bremsstrahlung”
- “...Results cast doubt on the utility of such azimuthal asymmetry as a clean test of quantum chromodynamics ” (i.e. of G&P 78)



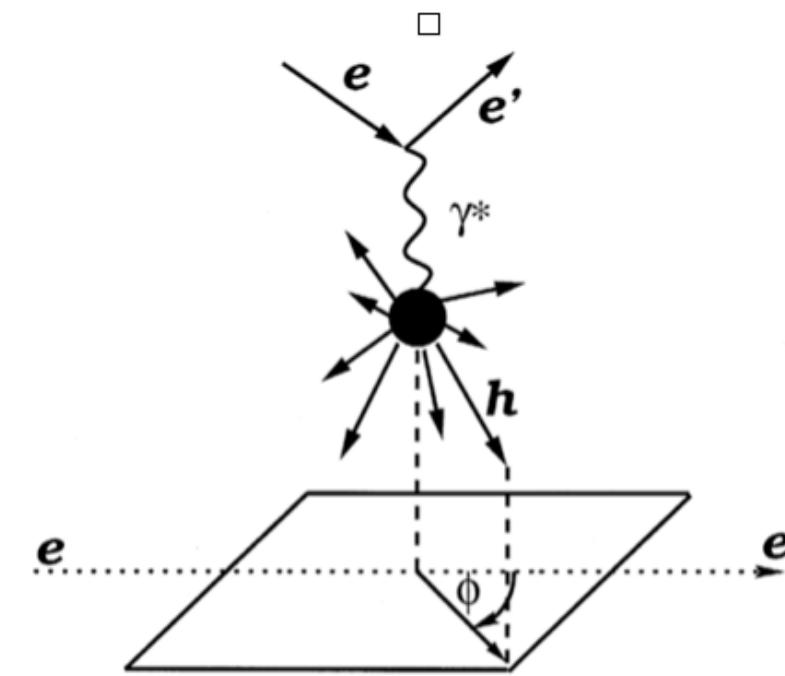
Clean tests of QCD?

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VOLUME 40

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NUMBER 1



Clean Tests of Quantum Chromodynamics in μp Scattering

Howard Georgi

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and

H. David Politzer

California Institute of Technology, Pasadena, California 91125

(Received 25 October 1977)

Hard gluon bremsstrahlung in μp scattering produces final-state hadrons with a large component of momentum transverse to the virtual-photon direction. Quantum chromodynamics can be used to predict not only the absolute value of the transverse momentum, but also its angular distribution relative to the muon scattering plane. **The angular correlations should be insensitive to nonperturbative effects.**

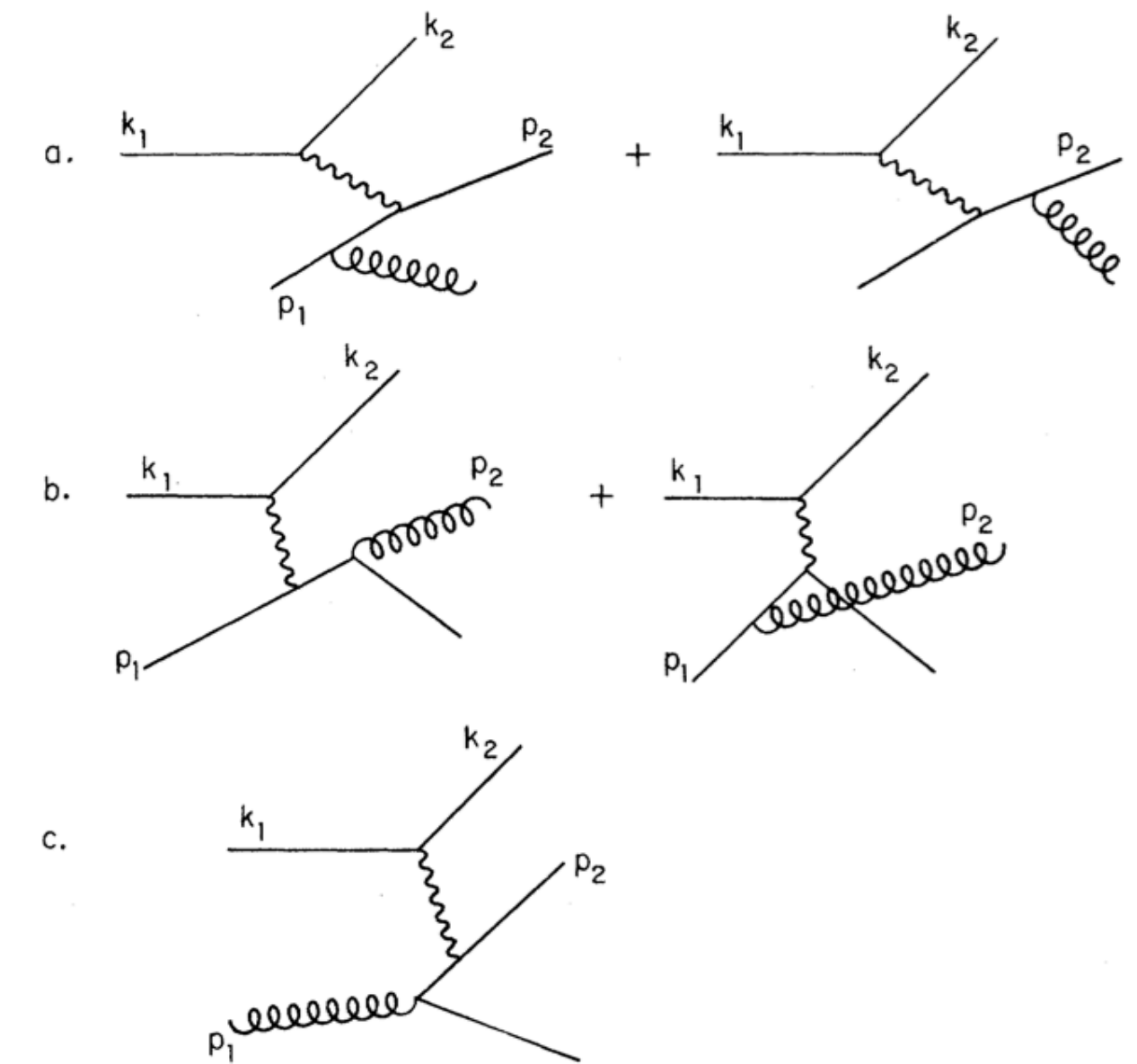


FIG. 1. Diagrams contributing to semi-inclusive μ -parton scattering to first order in α_s . k (p) denotes muon (parton) momentum. The wavy line is a virtual photon. The curly line is a gluon.

Pert. QCD $\alpha_s = g^2/4\pi$

$$\langle \cos \varphi \rangle_{ep} = -\frac{\alpha_s}{2} \kappa \sqrt{1-z} \frac{(2-y)\sqrt{1-y}}{1+(1-y)^2}$$

Cahn intrinsic k_T

Volume 78B, number 2,3

PHYSICS LETTERS

25 September 1978

AZIMUTHAL DEPENDENCE IN LEPTOPRODUCTION: A SIMPLE PARTON MODEL CALCULATION[☆]

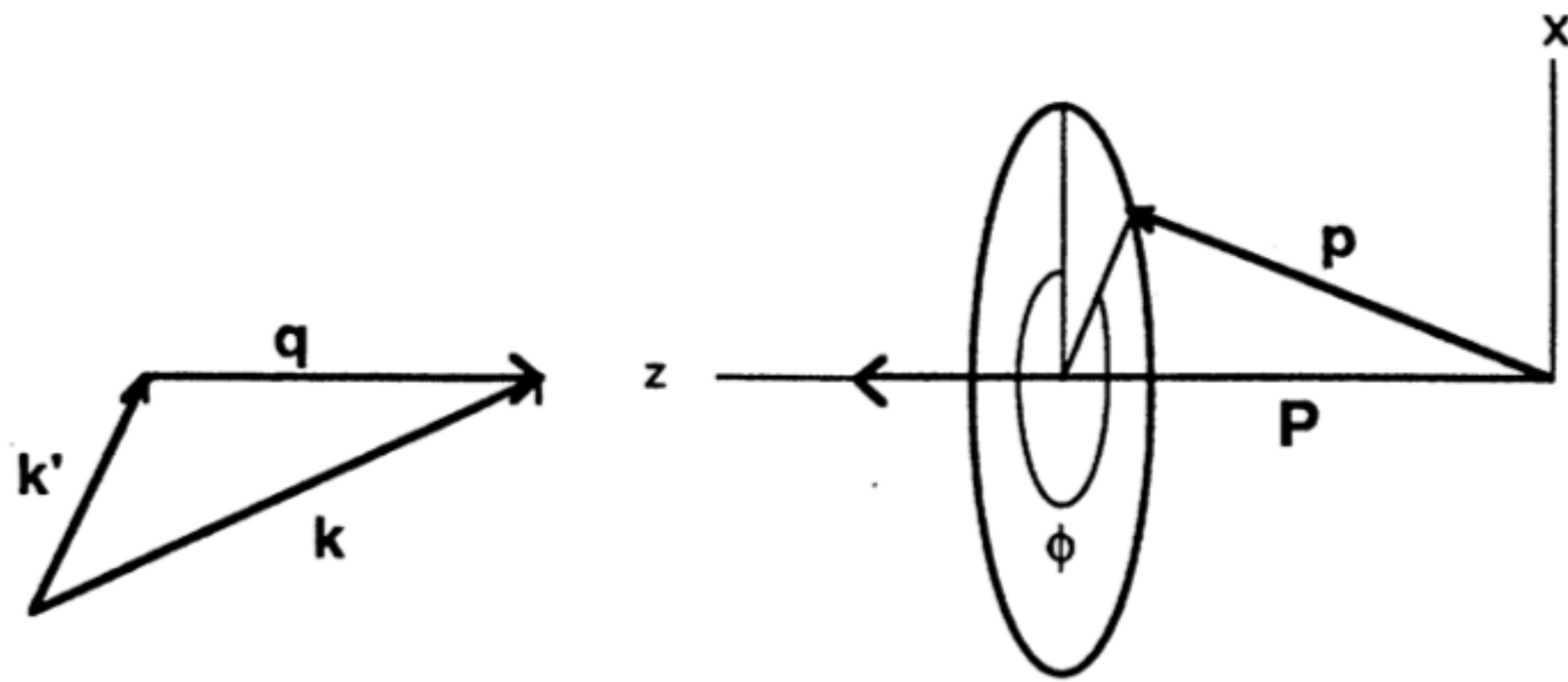
Robert N. CAHN

Department of Physics, University of Michigan, Ann Arbor, MI 48109, USA

Received 5 June 1978

Simple parton model argument allowing for transverse momentum in Mandelstam variables...

Semi-inclusive lepton production, $\ell + p \rightarrow \ell' + h + X$, is considered in the naive parton model. The scattered parton shows an azimuthal asymmetry about the momentum transfer direction. Simple derivations for the effects in ep , νp and $\bar{\nu} p$ scattering are given. Reduction of the asymmetry due to fragmentation of partons into hadrons is estimated. **The results cast doubt on the utility of such azimuthal asymmetry as a clean test of quantum chromodynamics.**



$$\sigma_{ep} \propto \hat{s}^2 + \hat{u}^2 \propto \left[1 - \frac{2p_{\perp}}{Q} \sqrt{1-y} \cos\phi \right]^2 + (1-y)^2 \left[1 - \frac{2p_{\perp}}{Q\sqrt{1-y}} \cos\phi \right]^2$$

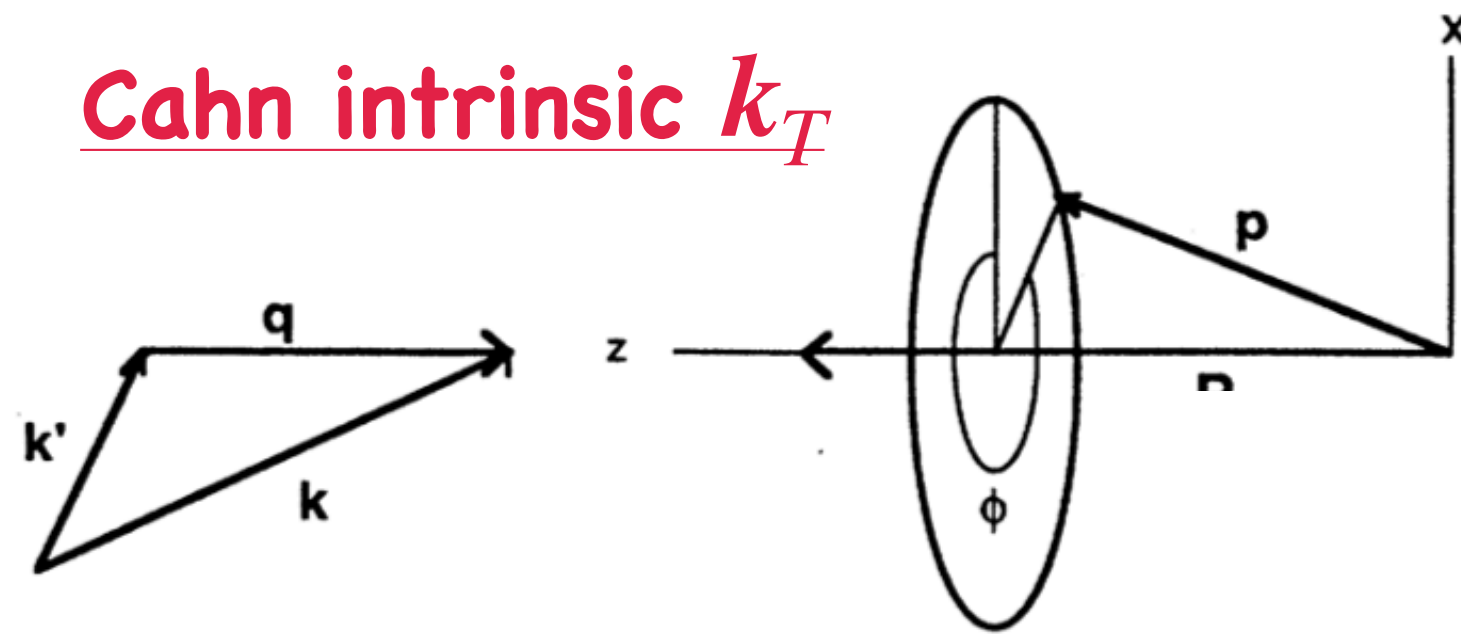
NLP! $\frac{p_{\perp}}{Q}$

$$\langle \cos\phi \rangle_{ep} = - \left[\frac{2p_{\perp}}{Q} \right] \frac{(2-y)\sqrt{1-y}}{1+(1-y)^2}$$

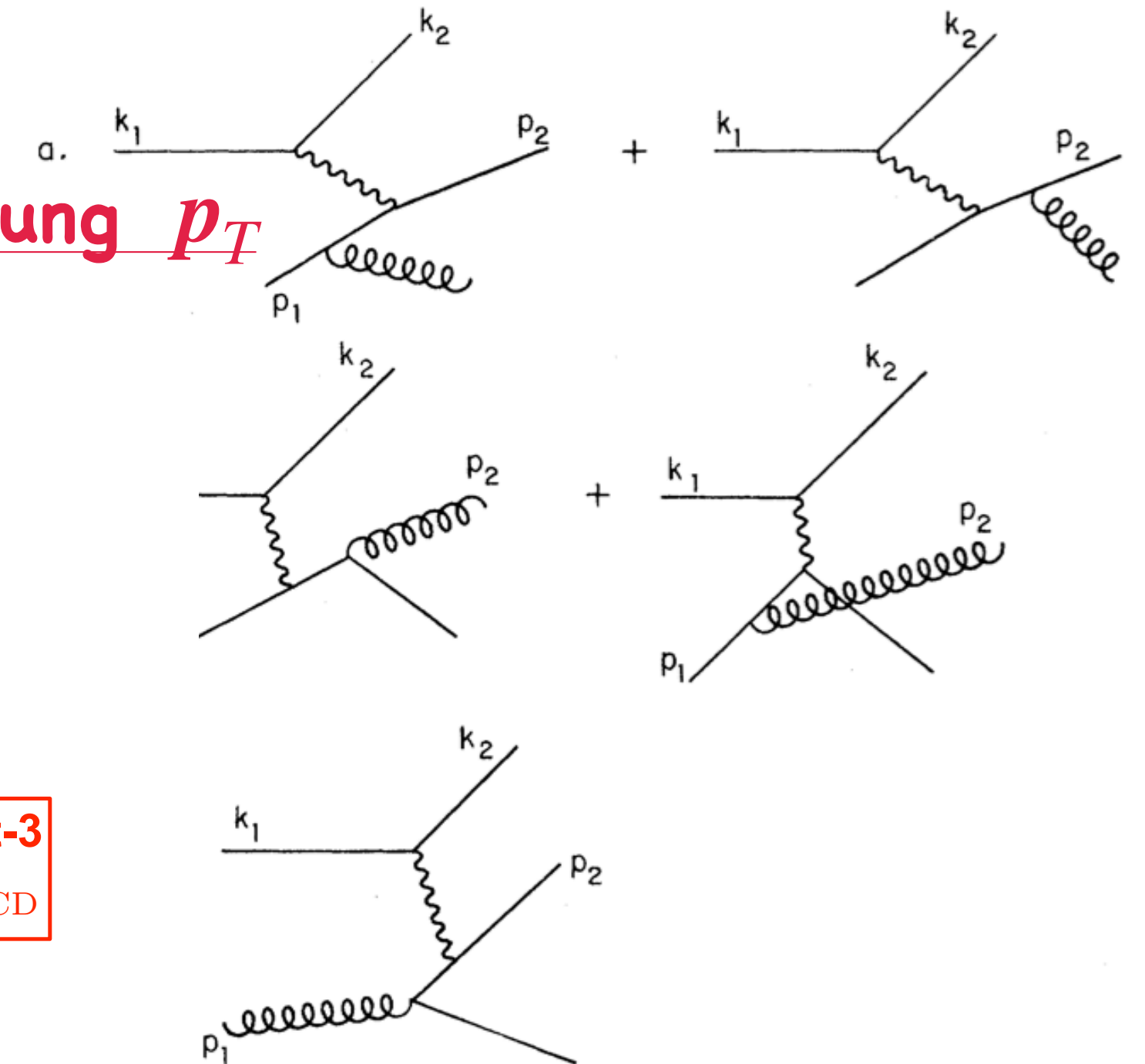
Two mechanisms? Matching...

Factorization & Matching collinear to TMD unpolarized/angle independent Collins Soper Serman NPB 1985

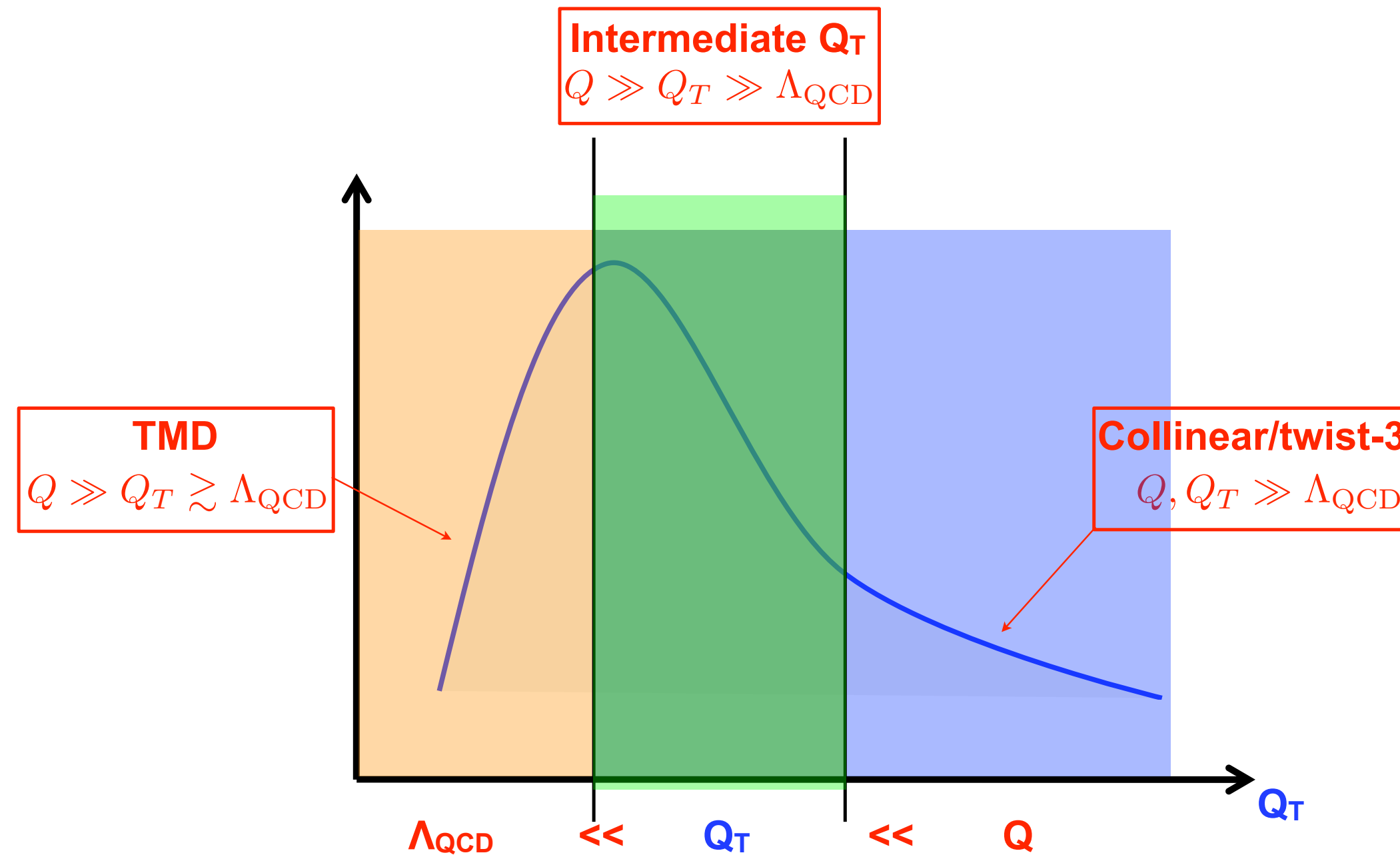
Cahn intrinsic k_T



Georgi & Politzer
hard gluon bremsstrahlung p_T



- “TMD” region
 $(p_T \sim k_T) \sim q_T \ll Q$



- “Collinear” region

$$\Lambda_{qcd} \ll q_T \sim Q$$

A comprehensive study of matching the hi & low Q_T in the overlap region in SIDIS was carried out by JHEP (2008) Bacchetta et al. where attention was given to azimuthal and polarization dependence

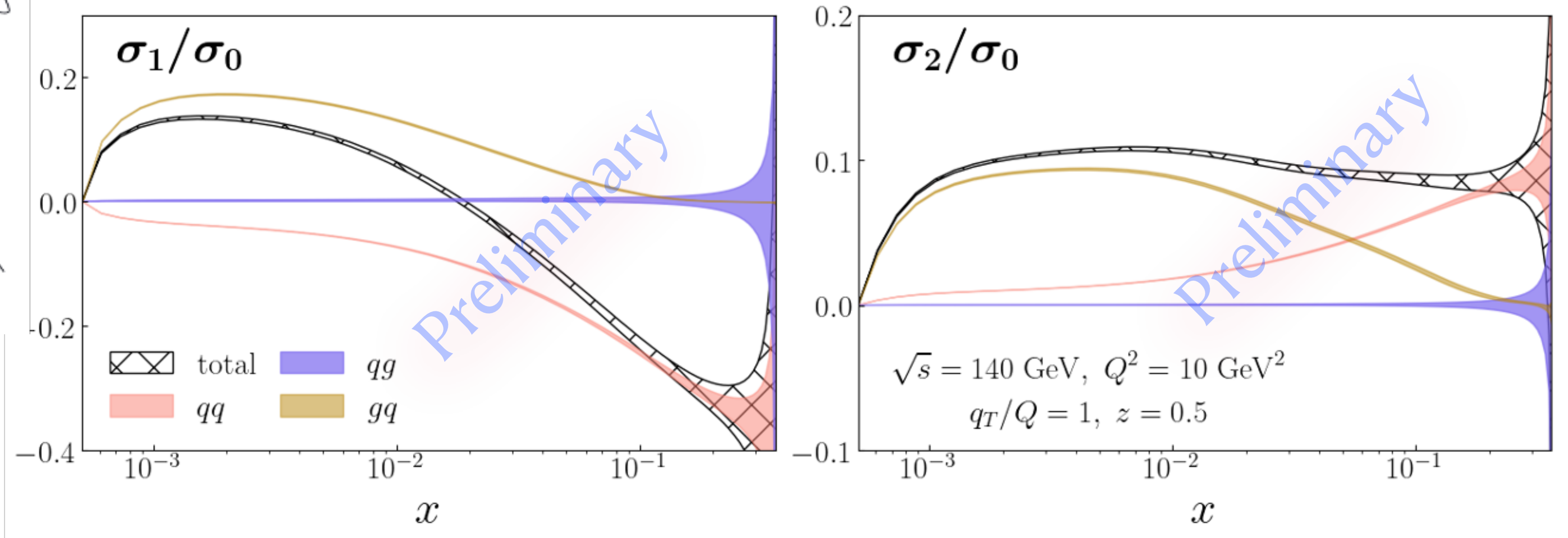
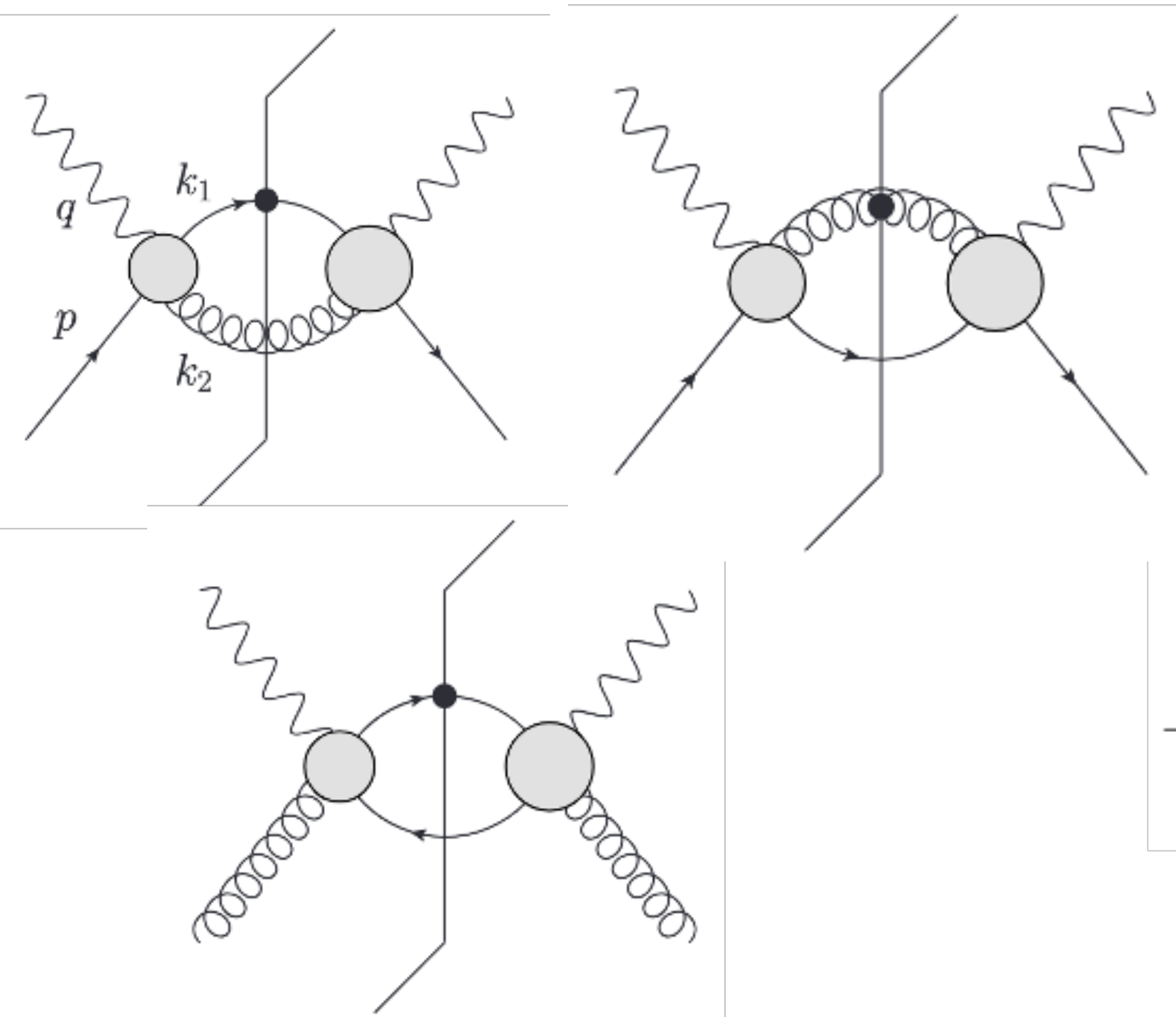
We have explored $\lg p_T$ or q_T angular modulations Cahn effect & $\cos 2\phi_h$

$$\frac{d\sigma}{dx dy dz dq_T^2 d\varphi} = \frac{\pi\alpha^2 yz}{4Q^2} \left[\underbrace{\sinh^2 \vartheta F_{UU,L}}_{\sigma_0} - \frac{1}{2}(2 + \sinh^2 \vartheta) F_{UU,T} - \underbrace{\sinh 2\vartheta F_{UU}^{\cos \varphi}}_{\sigma_1} \cos \varphi + \underbrace{\frac{1}{2} \sinh^2 \vartheta F_{UU}^{\cos 2\varphi}}_{\sigma_2} \cos 2\varphi \right]$$

e.g.

$$F_{UU,T} = \frac{1}{Q^2} \frac{\alpha_s}{(2\pi z)^2} \sum_a x e_a^2 \int_x^1 \frac{d\hat{x}}{\hat{x}} \int_z^1 \frac{d\hat{z}}{\hat{z}} \delta\left(\frac{q_T^2}{Q^2} - \frac{(1-\hat{x})(1-\hat{z})}{\hat{x}\hat{z}}\right) \times \left[f_1^a\left(\frac{x}{\hat{x}}\right) D_1^a\left(\frac{z}{\hat{z}}\right) C_{UU,T}^{(\gamma^* q \rightarrow qq)} + f_1^g\left(\frac{x}{\hat{x}}\right) D_1^g\left(\frac{z}{\hat{z}}\right) C_{UU,T}^{(\gamma^* q \rightarrow gq)} + f_1^g\left(\frac{x}{\hat{x}}\right) D_1^g\left(\frac{z}{\hat{z}}\right) C_{UU,T}^{(\gamma^* g \rightarrow q\bar{q})} \right]$$

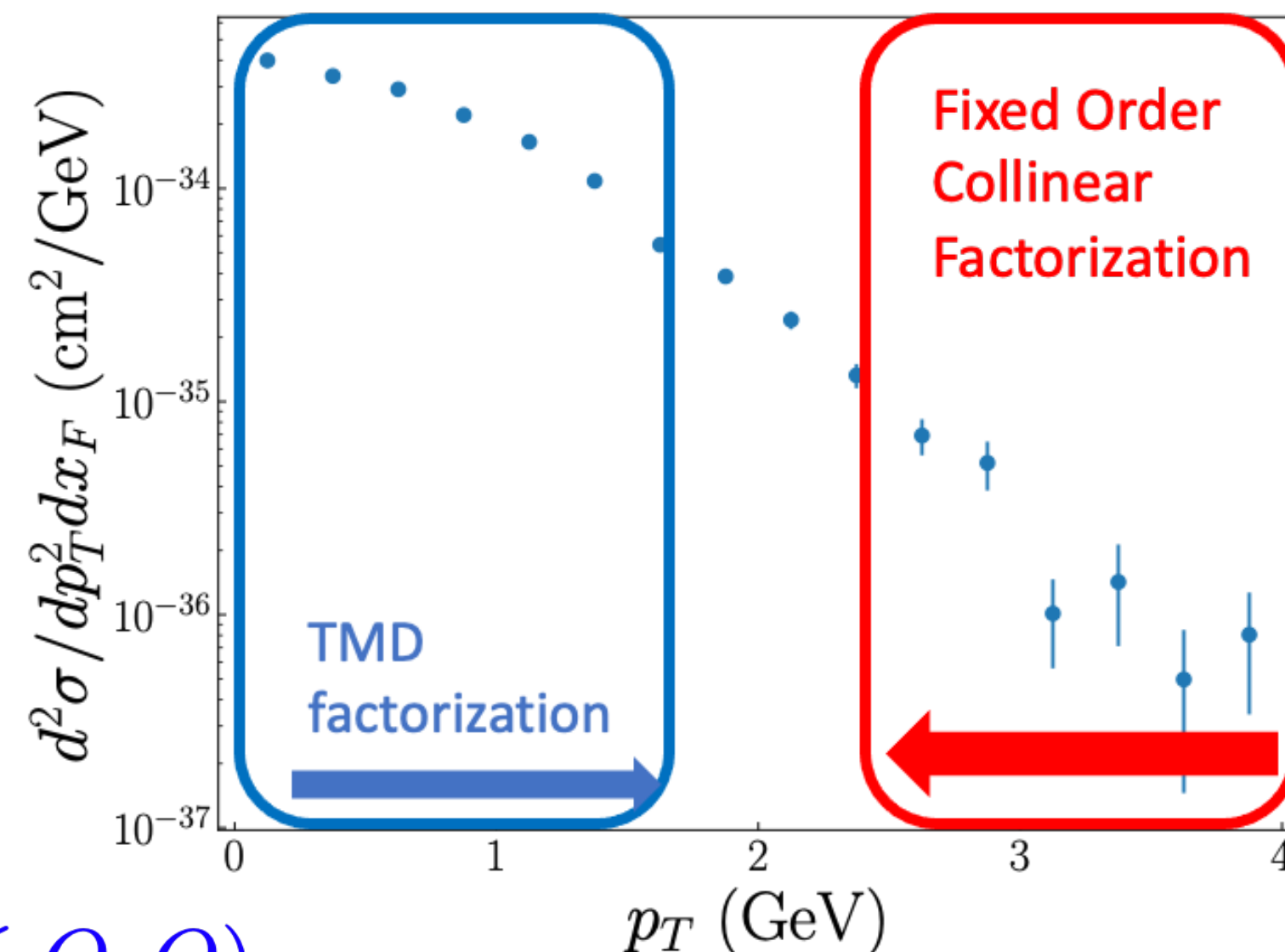
$$\sinh \vartheta \equiv \frac{2}{y} \sqrt{1-y}$$



w/ P. Anderson, N. Sato, R. Whitehill (JAM)

"Mis"-Matches Factorization @ sub-leading power

- Factorization & Matching collinear to TMD unpolarized/angle independent Collins Soper Serman NPB 1985



- Cross section in terms of different "regions"**
- W valid for $q_T \sim k_T \ll Q$ TMD factorization
- FO valid for $k_T \ll p_T \sim Q$ Collinear factorization
- AY subtracts d.c. & in principle,
 $AY \rightarrow W, p_T \rightarrow \infty$ and $AY \rightarrow FO, p_T \rightarrow 0$

$$\frac{d\sigma(m \lesssim q_T \lesssim Q, Q)}{dydq^2 dp_T^2} = W(p_T, Q) + FO(p_T, Q) - AY(p_T, Q) + O\left(\frac{m}{Q}\right)^c$$

$$\langle \cos \phi \rangle = \frac{\int d\sigma^{(0)} \cos \phi + \int d\sigma^{(1)} \cos \phi}{\int d\sigma^{(0)} + \int d\sigma^{(1)}}$$

- Bacchetta, Boer, Diehl, Mulders JHEP (2008) Cahn Effect:
Mis-match/inconsistency breakdown of TMD factorization at NLP?

Attempt to match the high- q_T result for $F_{UU}^{\cos \phi_h}$ to low- q_T result at intermediate q_T as a consistency check for factorization framework that extends Collins-Soper factorization to NLP

“Mis”-matches Factorization @ sub-leading power

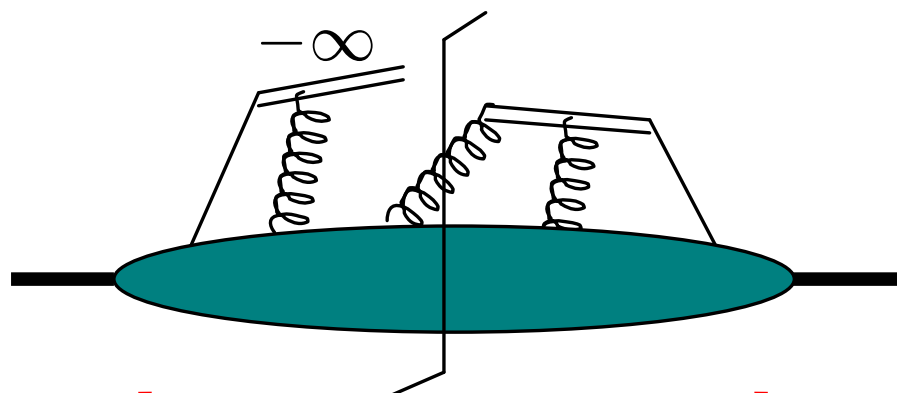
$$\langle \cos \phi \rangle = \frac{\int d\sigma^{(0)} \cos \phi + \int d\sigma^{(1)} \cos \phi}{\int d\sigma^{(0)} + \int d\sigma^{(1)}}$$

To cure mismatch, **Bacchetta Bozzi, Echevarria, Pisano, Prokudin, Radici, PLB (2019)** speculated that soft factor subtraction from leading power (LP) TMD same as NLP TMDs: **PLB (2019)**

What’s the soft factor ???

Advertisement TMD Handbook 2023 e-Print:2304.03302 [hep-ph]

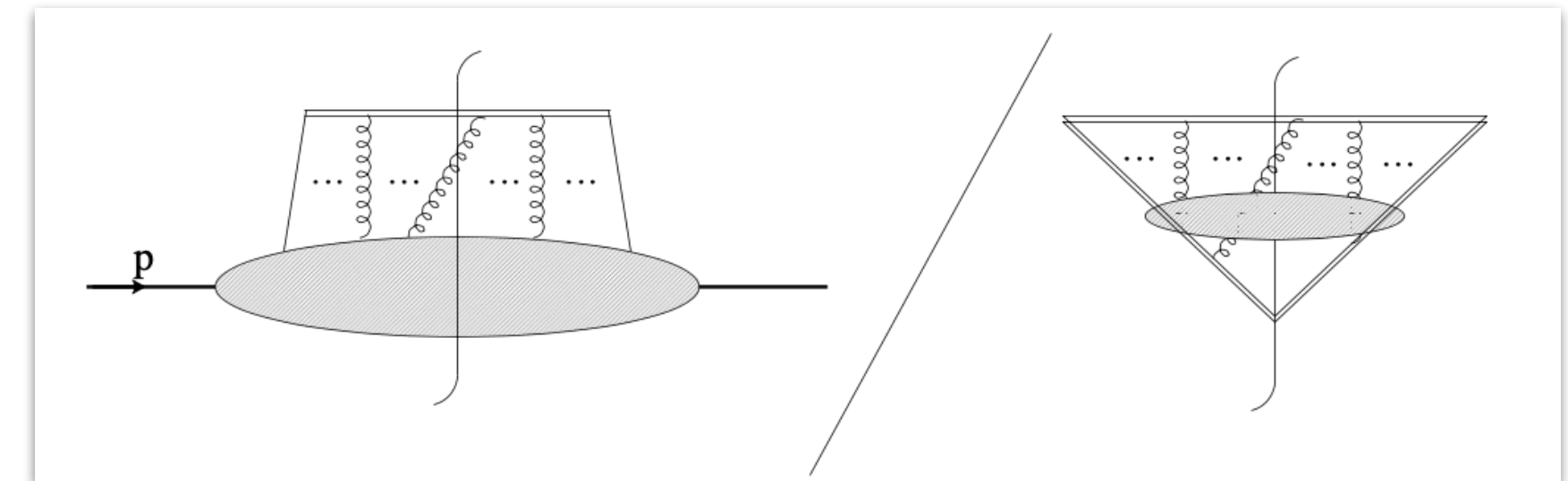
Collins QCD book 2011, Aybat Rogers 2011 PRD, Echevarria et al. 2012 JHEP



$$\tilde{f}_{j/H}^{\text{sub}}(x, b_T; \mu, y_n) = \lim_{\substack{y_A \rightarrow +\infty \\ y_B \rightarrow -\infty}} \underbrace{\tilde{f}_{j/H}^{\text{unsub}}(x, b_T; \mu, y_P - y_B)} \sqrt{\frac{\tilde{S}(b_T; y_A, y_n)}{\tilde{S}(b_T; y_A, y_B) \tilde{S}(b_T; y_n, y_B)}} \times UV_{\text{renorm}}$$

Soft factor subtractor

- 1) **cancel rapidity divergences in “unsubtracted” TMDs**
- 2) separate “right & left” movers i.e. full factorization
- 3) remove double counting of momentum regions



Conjecture on matching $\langle \cos \phi_h \rangle$ (see Bacchetta et al 2019 PLB)

??? $W \rightarrow AY \leftarrow FO$???

$$C[wfD] = \sum_a x e_a^2 \int d^2 \mathbf{p}_T d^2 \mathbf{k}_T d^2 \mathbf{l}_T \delta^{(2)}(\mathbf{p}_T - \mathbf{k}_T + \mathbf{l}_T + \mathbf{q}_T) \times w(\mathbf{p}_T, \mathbf{k}_T) f^a(x, p_T^2) D^a(z, k_T^2) U(l_T^2)$$

schematically

$$F_{UU}^{\cos\phi} = C \left[\frac{\hat{h} \cdot p_{\perp}}{Q} f_1(x, p_{\perp}) \frac{\tilde{D}^{\perp}(z, k_{\perp})}{z} S(l_{\perp}) + \frac{\hat{h} \cdot k_{\perp}}{Q} x f^{\perp}(x, p_{\perp}) D_1(z, k_{\perp}) S(l_{\perp}) \right]$$

Method: get AY term from TMD and match to AY from FO

let one of $p_{\perp}, k_{\perp}, l_{\perp} \rightarrow q_{\perp}$ large, others small

To address these subtleties fresh look redo
TMD @ LP and NLP factorization

Exploit the utility of using
Fierz decamp & "good and bad" LC quark fields

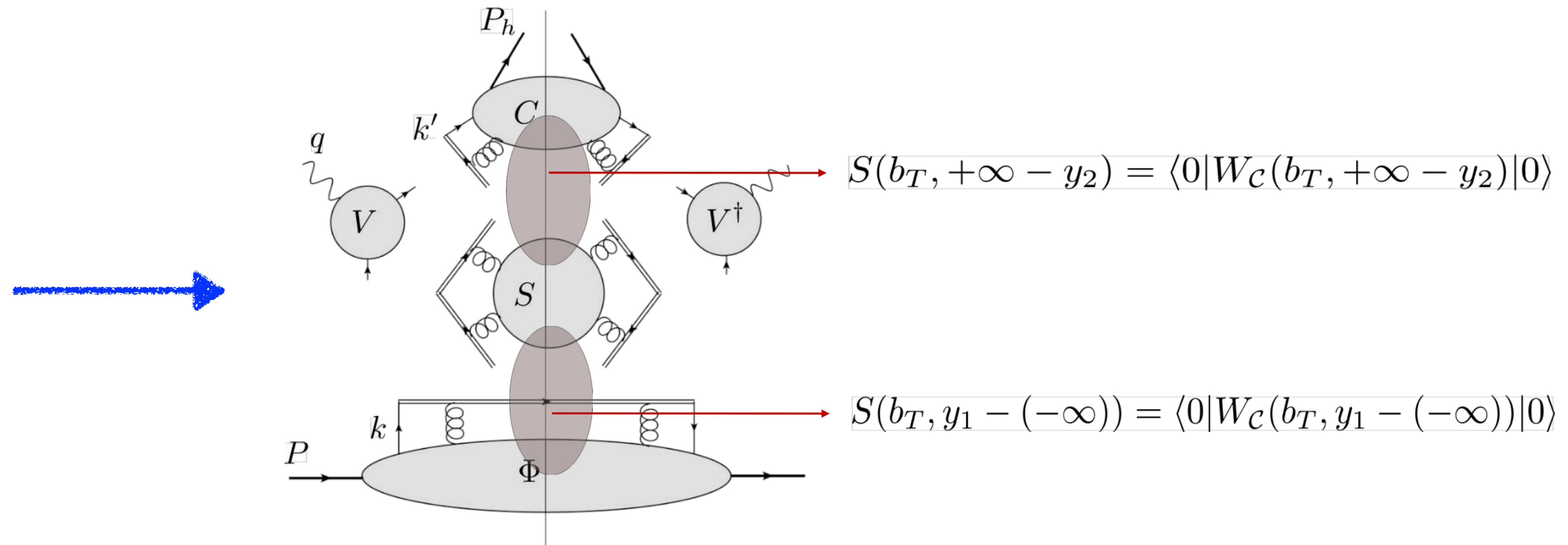
Focus on NLO soft factor calculation

L.Gamberg, Z.Kang, D.Shao, J.Terry, F.Zhao arXiv: e-Print:221.13209 (2022)

L.Gamberg, Z.Kang, D.Shao, J.Terry, F.Zhao in prep 2025

TMD factorization at NLO @ LP

$$\frac{d\sigma^W}{dQ^2 dx_F dp_T^2} = H_{\text{SIDIS}} \int \frac{d^2\vec{b}_T}{(2\pi)^2} e^{i\vec{q}_T \cdot \vec{b}_T} \frac{\Phi^{[\Gamma]}(x, b_T)}{S(b_T, y_1 - (-\infty))} S(b_T, y_1 - y_2) \frac{C^{[\Gamma']}(z, b_T)}{S(b_T, +\infty - y_2)}$$



Courtesy of Andrea Simonelli

Challenges of *SLP/NLP* TMDs

Various sources for power suppressed terms identified and discussed in the literature from Tree level Studies, Mulders, Tangerman (1996), Bacchetta et al. JHEP (2007)

- This includes corrections associated to kinematic prefactors involving contractions between the leptonic and hadronic tensors, referred to as ***kinematic power corrections***
- Another involve subleading terms in quark-quark correlators involving Dirac structures that differ from LP ones referred to as ***intrinsic power corrections***— e.g. Cahn function $f^\perp(x, k_T)$, $e(x, k_T)$...

$$\Gamma_a \in \left\{ \underbrace{\frac{\not{n}}{4}, \frac{\not{n}\gamma^5}{4}, \frac{i}{4}\sigma^{i+}\gamma^5}_{\text{LP}}, \underbrace{\frac{1}{2}, \frac{\gamma^5}{2}, \frac{\gamma^i}{2}, \frac{\gamma^i\gamma^5}{2}, \frac{i}{2}\sigma^{ij}\gamma^5, \frac{i}{4}\sigma^{+-}\gamma^5}_{\text{NLP}} + \dots \right\}$$

- Another from hadronic matrix elements of (interaction dependent) quark-gluon-quark operators, referred to ***dynamic power corrections*** multi-parton qgq correlators

All three distributions are not required to span the NLP cross section due to EOM

$$\Phi_{q/P jj'}^{\text{int}}(x, \mathbf{k}_\perp, \mathbf{S}) = \Phi_{q/P jj'}^{\text{kin}}(x, \mathbf{k}_\perp, \mathbf{S}) + \Phi_{q/P jj'}^{\text{dyn}}(x, \mathbf{k}_\perp, \mathbf{S})$$

⁸Gamberg, Kang, Shao, Terry, Zhao 2022

Factorization at sub-leading power ... revisit Tree level

$$\frac{d\sigma}{dx dy d\Psi dz d^2 P_{h\perp}} = \kappa \frac{\alpha_{em}^2 y}{4Q^4 z} L_{\mu\nu} W^{\mu\nu}$$

- “TMD” region $(p_T \sim k_T) \sim q_T \ll Q$

$$W_{\mu\nu} = \frac{1}{(2\pi)^4} \sum_X \int d^4x e^{-iqx} \langle P | J_\mu^\dagger(0) | h, X \rangle \langle h, X | J_\nu(x) | P \rangle,$$

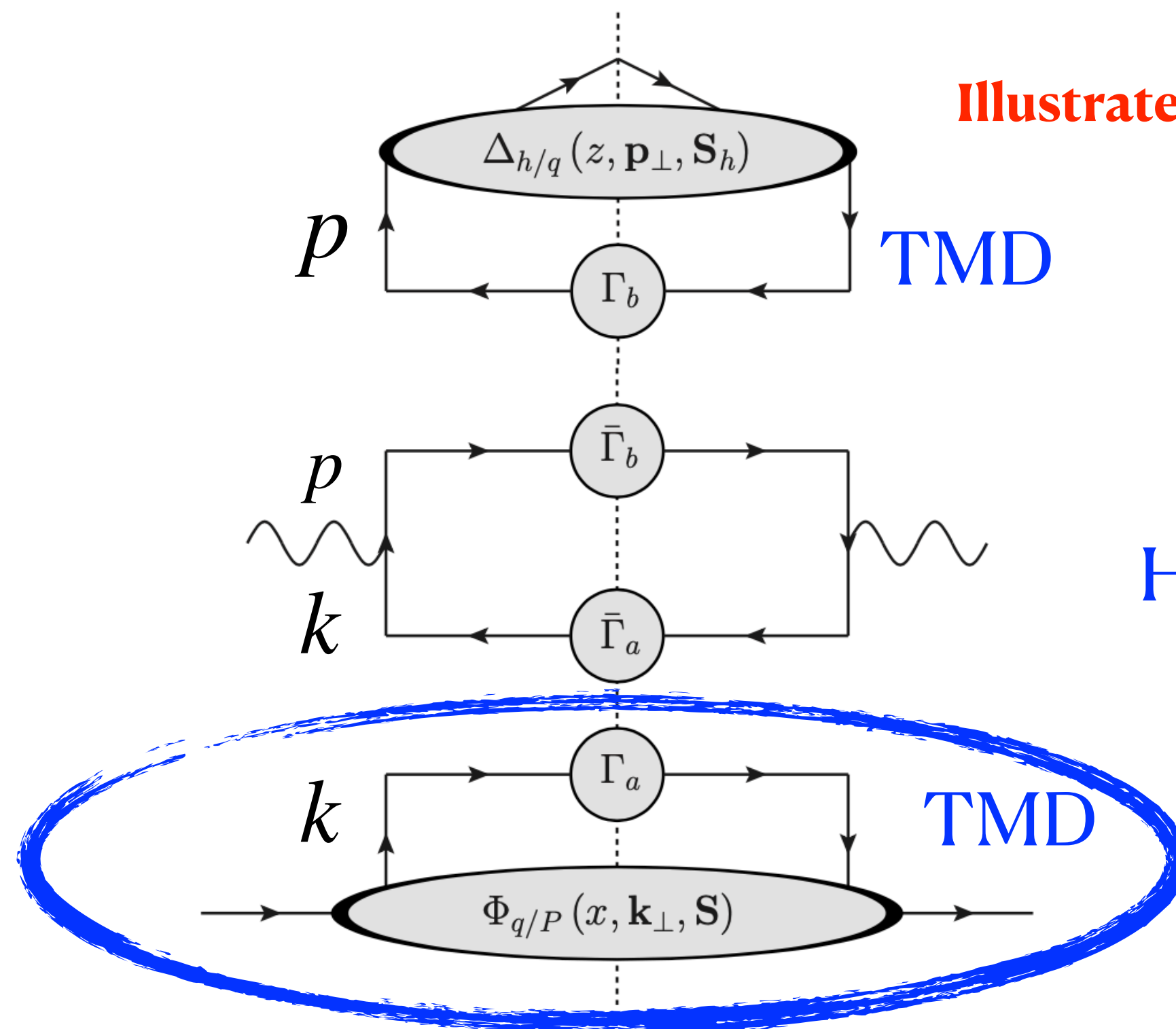
$$J_\mu(x) = J_\mu^{(2)}(x) + J_\mu^{(3)}(x)$$



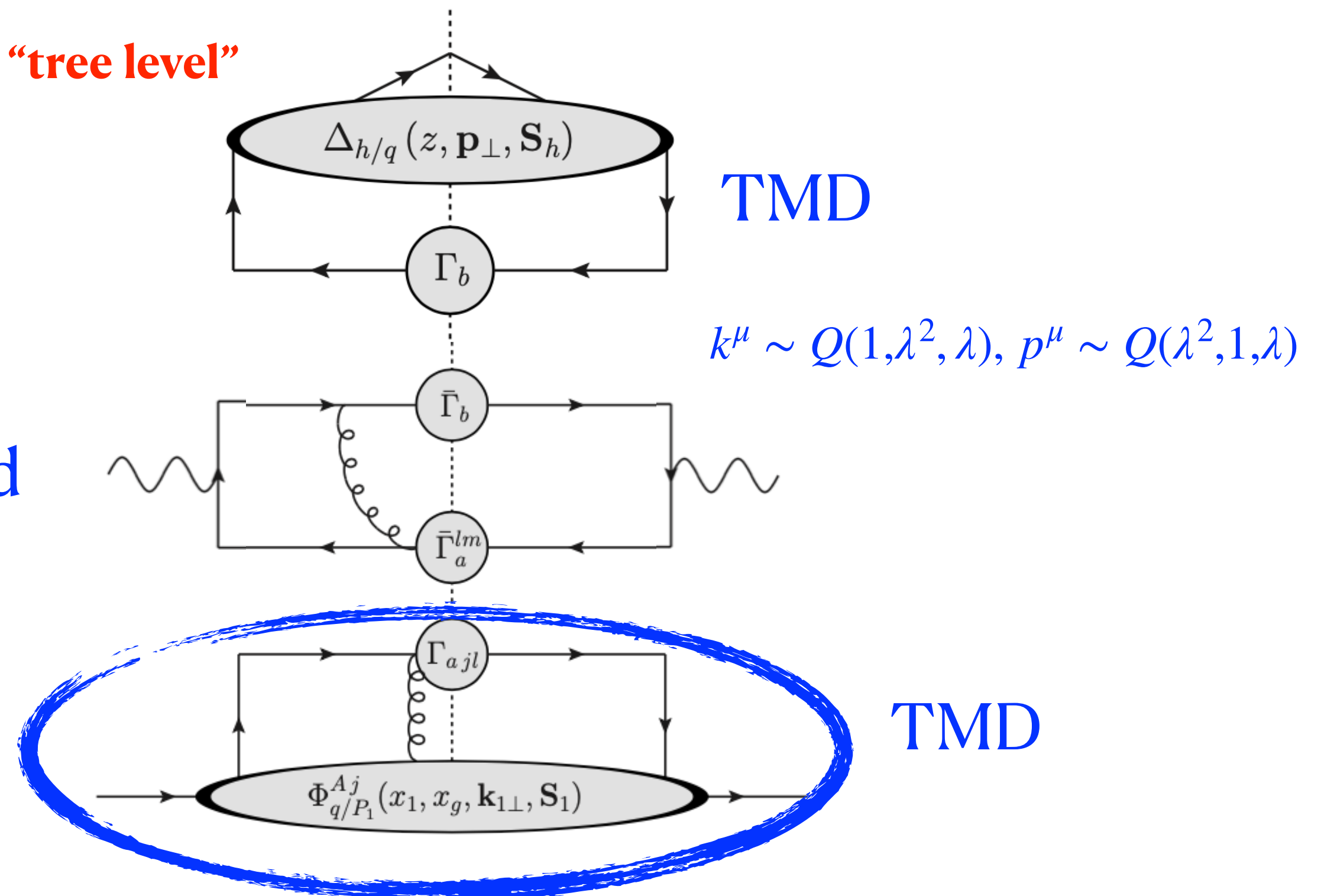
Working @ NLP, the current contains 3(!) contributions:

- One with 2 partons entering from each correlation function
- Another with 3 partons entering from one correlation function
- & partonic kinematic power corrections-momentum scaling

$$k^\mu \sim Q(1, \lambda^2, \lambda), p^\mu \sim Q(\lambda^2, 1, \lambda)$$



Hard



TMD

Tree level factorization sub-leading power

Combining these contributions and multiplying by leptonic tensor get factorized

Cahn and more ... Includes dynamical “tilde” contributions

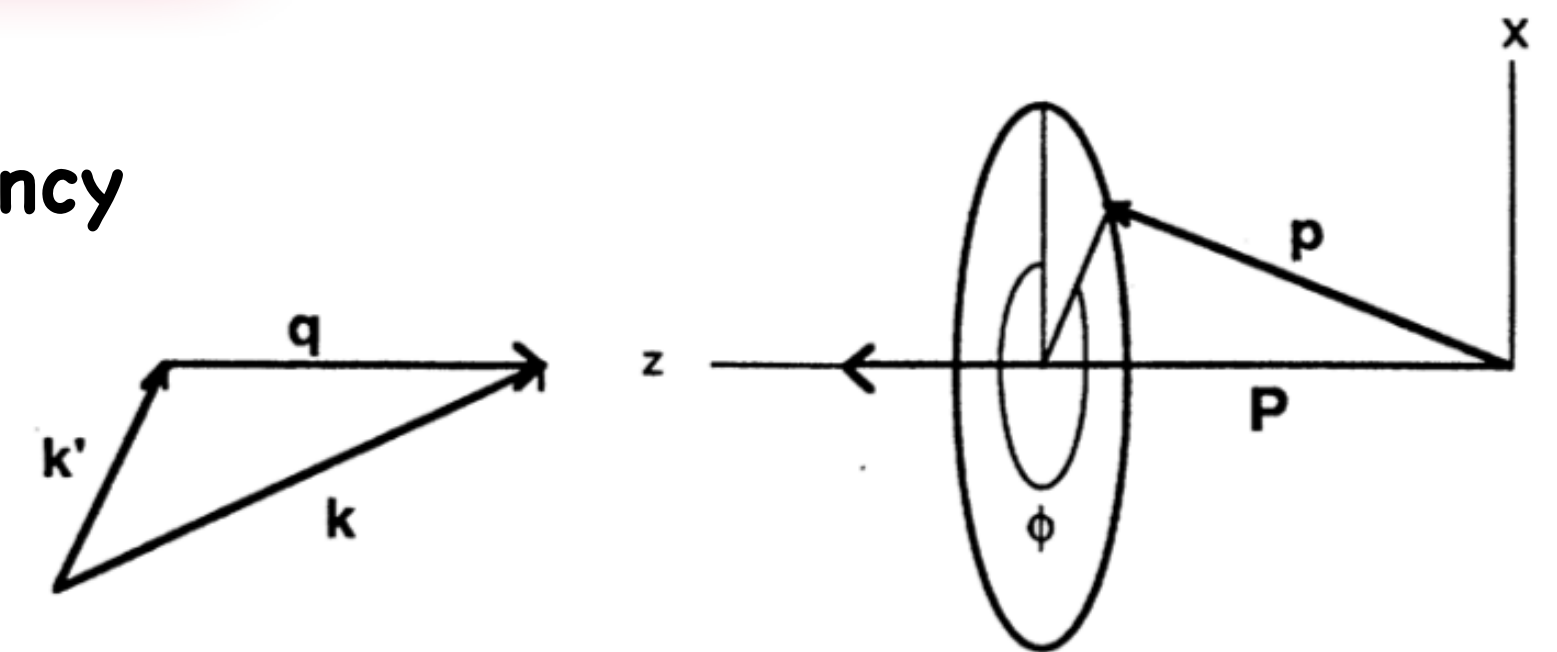
Using “intrinsic & dynamical” basis

$$F_{\text{DIS}}^3(x, z, \mathbf{P}_{h\perp}) = c^{\text{DIS}} \left[\frac{q_{\perp}}{Q} f_1 D_1 \right] - c^{\text{DIS}} \left[\left(x \frac{\mathbf{k}_{\perp} \cdot \hat{x}}{Q} f^{\perp} \right) D_1 - f_1 \left(\frac{\mathbf{p}_{\perp} \cdot \hat{x}}{zQ} D^{\perp} \right) \right] \\ - \int \frac{dx_g}{x_g} c_{\text{dyn } x_g}^{\text{DIS}} \left[\left(x \frac{\mathbf{k}_{\perp} \cdot \hat{x}}{Q} \tilde{f}^{\perp} \right) D_1 \right] + \int \frac{dz_g}{z_g} c_{\text{dyn } z_g}^{\text{DIS}} \left[f_1 \left(\frac{\mathbf{p}_{\perp} \cdot \hat{x}}{zQ} \tilde{D}^{\perp} \right) \right],$$

Cahn and more intrinsic k_T

Different setup then Bacchetta et al 2007 allows us to check RG consistency

Gamberg, Kang, Shao, Terry, Zhao arXiv: e-Print:221.13209



Factorization & resummation at NLO and NLP

Beyond tree level

Note first attempt Bacchetta Boer Diehl Mulders JHEP 2008

- We perform one loop calculation & attempt to establish renormalization group consistency: Regions hard, soft, collinear

$$\begin{aligned}
 F_{\text{DIS}}^3(x, z, \mathbf{P}_{h\perp}) = & H_{\text{DIS}}^{\text{LP}}(Q; \mu) C^{\text{DIS}} \left[\frac{q_{\perp}}{Q} f_1 D_1 \mathcal{S}^{\text{LP}} \right] \\
 & - H_{\text{DIS}}^{\text{int}}(Q; \mu) C^{\text{DIS}} \left[\left(x \frac{\mathbf{k}_{\perp} \cdot \hat{x}}{Q} f_{\perp} D_1 - \frac{\mathbf{p}_{\perp} \cdot \hat{x}}{zQ} f_1 D^{\perp} \right) \mathcal{S}^{\text{int}} \right] \\
 & - \int \frac{dx_g}{x_g} H_{\text{DIS}}^{\text{dyn}}(x_g, Q; \mu) C^{\text{DIS}} \left[x \frac{\mathbf{k}_{\perp} \cdot \hat{x}}{Q} \tilde{f}_{\perp} D_1 \mathcal{S}^{\text{dyn}} \right] \\
 & + \int \frac{dz_g}{z_g} H_{\text{DIS}}^{\text{dyn}}(z_g, Q; \mu) C^{\text{DIS}} \left[\frac{\mathbf{p}_{\perp} \cdot \hat{x}}{zQ} f_1 \tilde{D}^{\perp} \mathcal{S}^{\text{dyn}} \right].
 \end{aligned}$$

- H^{LP} , H^{int} and H^{dynam} represent LP, intrinsic NLP, and dynamic NLP hard functions.
- Additionally, \mathcal{S}^{LP} , \mathcal{S}^{int} and \mathcal{S}^{dyn} denote the LP, intrinsic sub-leading power, and dynamic sub-leading power soft function
- **NB if soft factors are different universality, of TMDs breaks down. Global analysis w/ NLP observables problematic**

NLO-calculation-factorization

Necessary but not sufficient condition to establish factorization

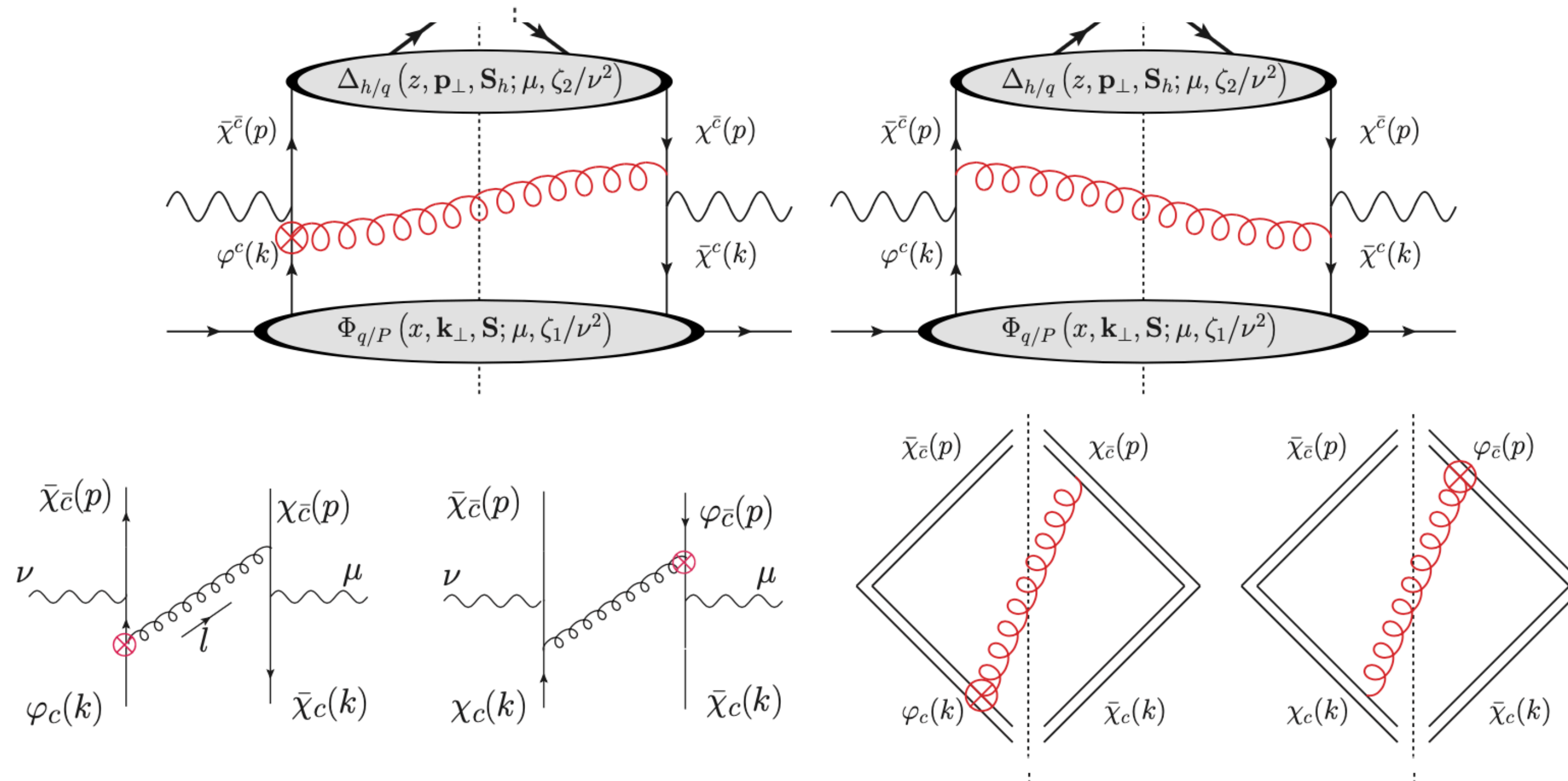
Recipe

- **Calculate: soft, collinear (and anti), & hard**
- **Renormalize**
 - Exploit properties of good and bad fields & power counting
- **Check renormalization group consistency**

NLO Ingredients soft factor

The soft region

The soft function is generated through the emissions of soft gluons in the partonic cross section



$$\hat{\mathcal{S}}^{\text{LP}}(b; \mu, \nu) = Z_{\mathcal{S}^{\text{LP}}}(b; \mu, \nu) \mathcal{S}^{\text{LP}}(b; \mu, \nu)$$

$$\hat{\mathcal{S}}^{\text{NLP}}(b; \mu, \nu) = Z_{\mathcal{S}^{\text{NLP}}}(b; \mu, \nu) \mathcal{S}^{\text{NLP}}(b; \mu, \nu)$$

$$\frac{\partial}{\partial \ln \mu} \mathcal{S}^{\text{NLP}}(b; \mu, \nu) = \Gamma_{\mathcal{S}^{\text{NLP}}}^\mu \mathcal{S}^{\text{NLP}}(b; \mu, \nu)$$

$$\frac{\partial}{\partial \ln \nu} \mathcal{S}^{\text{NLP}}(b; \mu, \nu) = \Gamma_{\mathcal{S}^{\text{NLP}}}^\nu \mathcal{S}^{\text{NLP}}(b; \mu, \nu)$$

$$\Gamma_{\mathcal{S}^{\text{int}}}^\nu = \frac{\partial}{\partial \ln \nu} Z_{\mathcal{S}^{\text{NLP}}}(b; \mu, \nu)$$

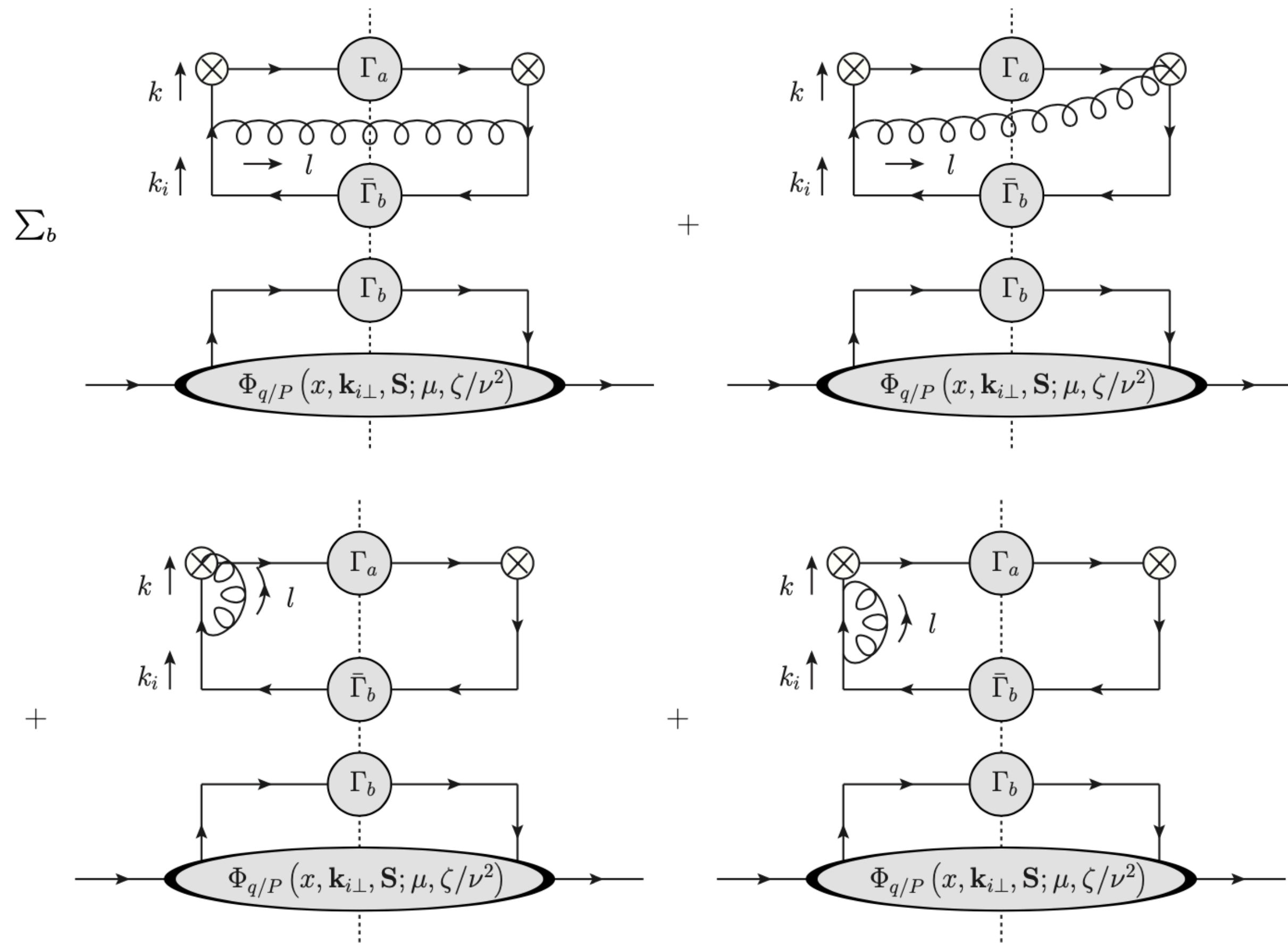
Gamberg, Kang, Shao, Terry, Zhao
arXiv: e-Print:221.13209

Soft emission from sub-leading fields vanish \rightarrow NLO + NLP soft function is half the LP one

$$\Gamma_{\mathcal{S}^{\text{int}}}^\mu = \frac{1}{2} \Gamma_{\mathcal{S}^{\text{LP}}}^\mu, \quad \Gamma_{\mathcal{S}^{\text{int}}}^\nu = \frac{1}{2} \Gamma_{\mathcal{S}^{\text{LP}}}^\nu$$

NLO Ingredients collinear factor

Diagrams associated with the evolution of the collinear region



Renormalize TMDs: soft & UV subtraction

$$\hat{\Phi}^{[\Gamma^a]}(x, \mathbf{b}, \mathbf{S}; \mu, \zeta/\nu^2) = Z_{\Gamma^a \Gamma^b}(b, \mu, \zeta/\nu^2) \Phi^{[\Gamma^b]^0}(x, \mathbf{b}, \mathbf{S}; xP^+)$$

$$\Gamma_3^\nu = \frac{\partial}{\partial \ln \nu} Z_{NLP}(b; \mu, \nu)$$

Necessary condition rapidity RG Consistency

Review Leading power

$$f_1(x, b; \mu, \zeta_1) = f_1(x, b; \mu, \zeta_1/\nu^2) \sqrt{\mathcal{S}^{\text{LP}}(b; \mu, \nu)}$$

$$D_1(z, b; \mu, \zeta_2) = D_1(z, b; \mu, \zeta_2/\nu^2) \sqrt{\mathcal{S}^{\text{LP}}(b; \mu, \nu)}$$

$$\Gamma_2^\nu + \frac{1}{2}\Gamma_S^\nu = 0, \quad \Gamma_2^\nu + \frac{1}{2}\Gamma_S^\nu = 0$$

Next to leading power

$$\frac{d\sigma}{dP_T^2} \sim -H_{\text{DIS}}^{\text{int}}(Q; \mu) \mathcal{C}^{\text{DIS}} \left[\left(x \frac{\mathbf{k}_\perp \cdot \hat{x}}{Q} f^\perp D_1 - \frac{\mathbf{p}_\perp \cdot \hat{x}}{zQ} f_1 D^\perp \right) \mathcal{S}^{\text{int}} \right]$$

$$ib^\mu M^2 f^{\perp(1)}(x, b; \mu, \zeta_1) = ib^\mu M^2 f^{\perp(1)}(x, b; \mu, \zeta_1/\nu^2) \sqrt{\mathcal{S}^{\text{int}}(b; \mu, \nu)} \longleftrightarrow \Gamma_{3\text{int}}^\nu + \frac{1}{2}\Gamma_{S\text{int}}^\nu = 0$$

Non-trivial result

However for cross section

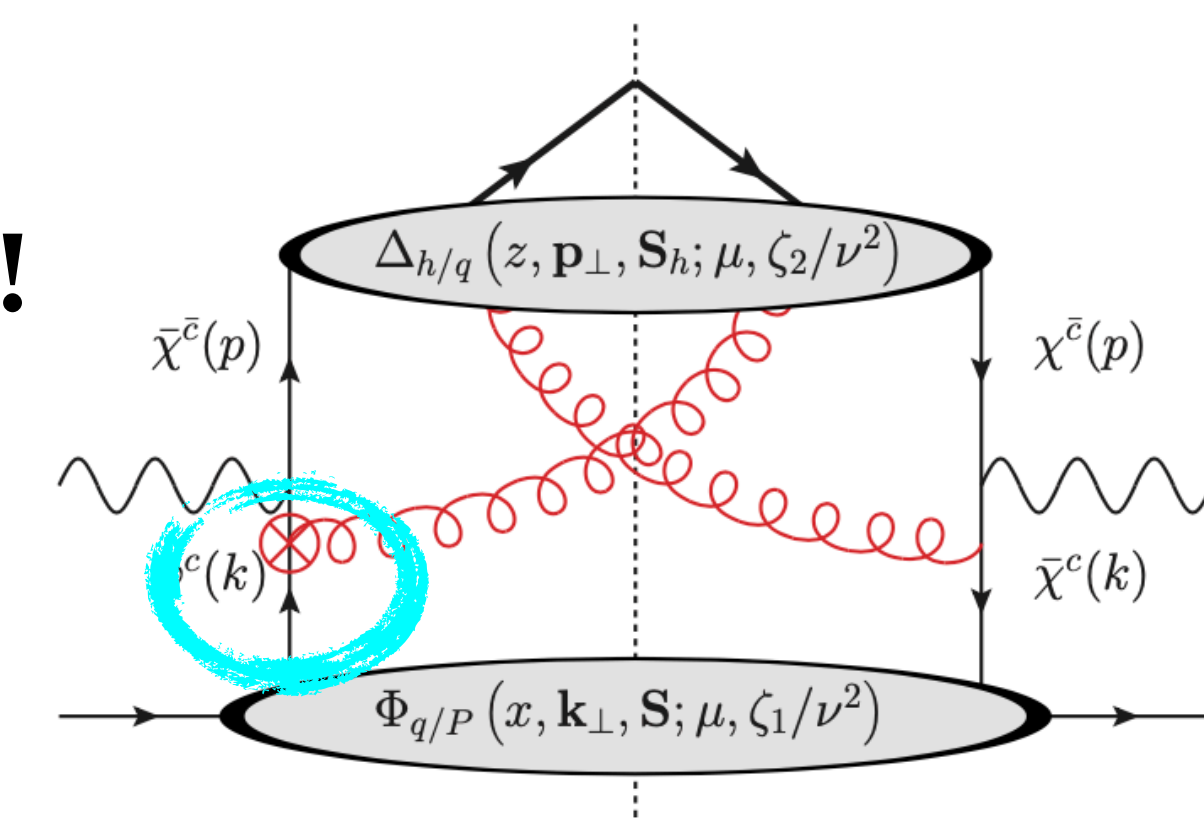
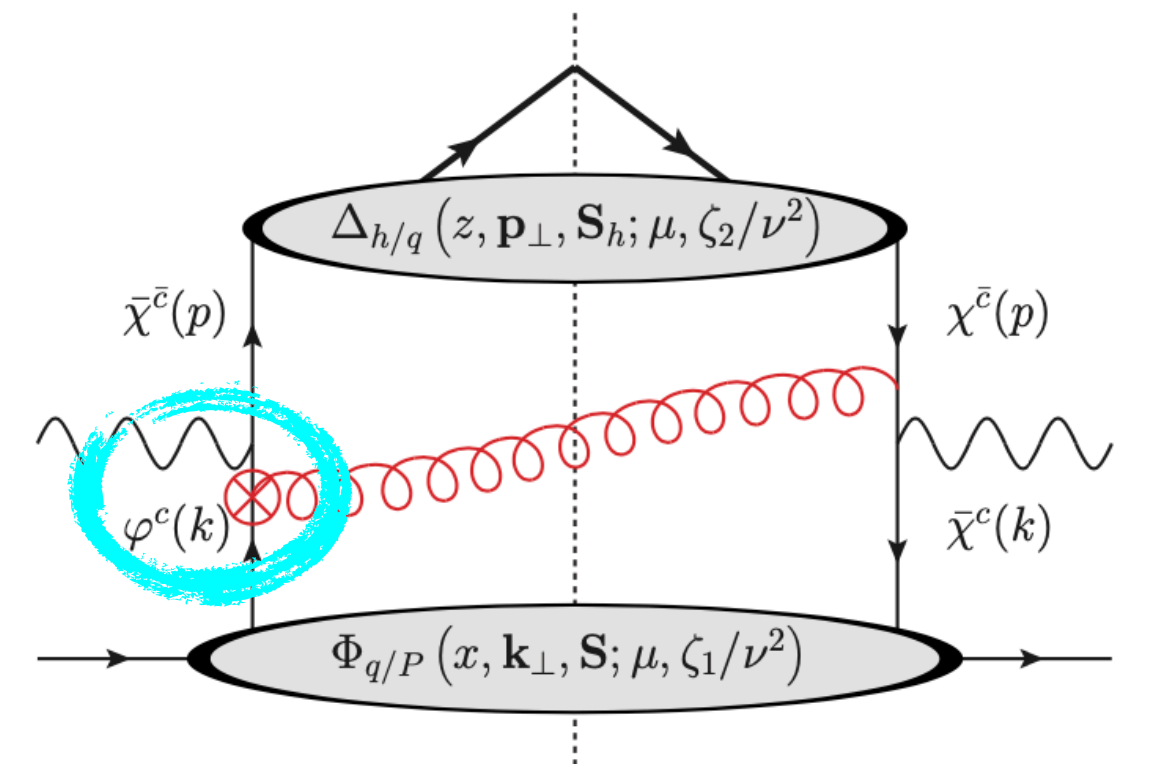
$$D_1(z, b; \mu, \zeta_2) = D_1(z, b; \mu, \zeta_2/\nu^2) \sqrt{\mathcal{S}^{\text{LP}}(b; \mu, \nu)}$$

!!

$$\Gamma_2^\nu + \frac{1}{2}\Gamma_{S\text{int}}^\nu \neq 0$$

$$D_1(z, b; \mu, \zeta_2) = D_1(z, b; \mu, \zeta_2/\nu) \sqrt{\mathcal{S}^{\text{int}}??}$$

$$\Gamma_{2\text{mod}}^\nu + \frac{1}{2}\Gamma_{S\text{int}}^\nu = 0$$



Necessary condition rapidity RG Consistency

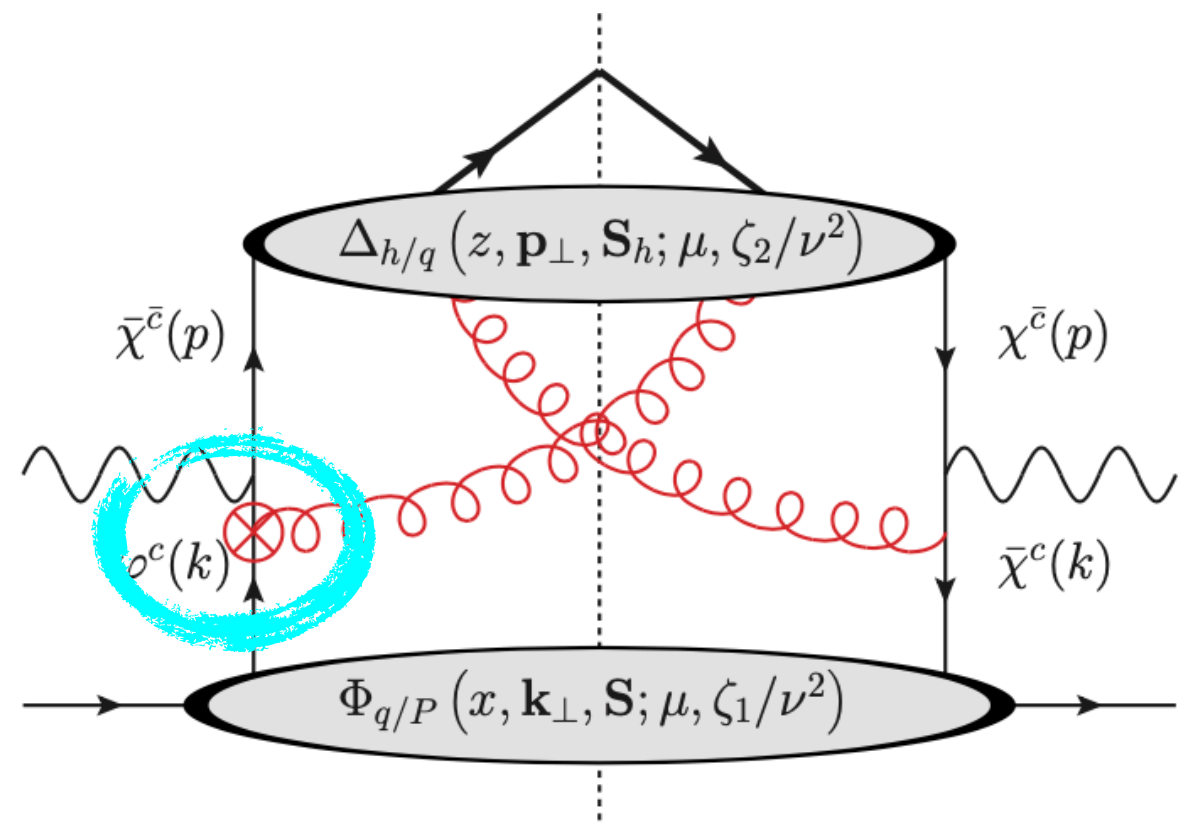
Next to leading power

$$\frac{d\sigma}{d\ln\nu} = 0 \quad \& \quad \frac{d\sigma}{d\ln\mu} = 0$$

$$-H_{\text{DIS}}^{\text{int}}(Q; \mu) C^{\text{DIS}} \left[\left(x \frac{\mathbf{k}_{\perp} \cdot \hat{x}}{Q} f^{\perp} D_1 - \frac{\mathbf{p}_{\perp} \cdot \hat{x}}{zQ} f_1 D^{\perp} \right) S^{\text{int}} \right]$$

$$\Gamma_{S^{\text{int}}}^{\nu} + \Gamma_{3^{\text{int}}}^{\nu} + \Gamma_{2^{\text{mod}}}^{\nu} = 0$$

Have shown



Problem: Breakdown of universality different soft function for D_1 ?!

$$D_1(z, b; \mu, \zeta_2) = D_1(z, b; \mu, \zeta_2/\nu) \sqrt{S^{\text{int}}}$$

NLP

!!

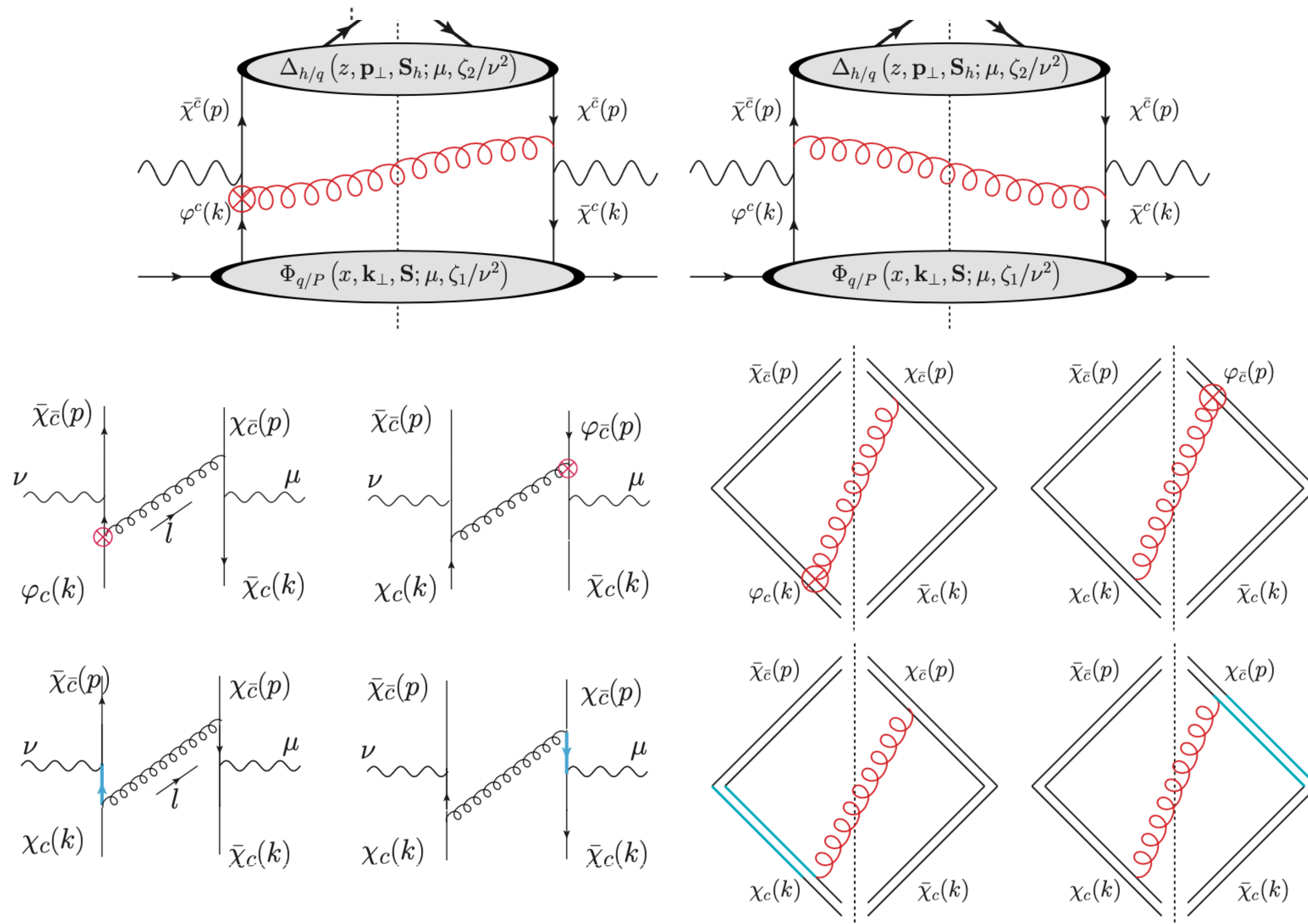
$$D_1(z, b; \mu, \zeta_2) = D_1(z, b; \mu, \zeta_2/\nu^2) \sqrt{S^{\text{LP}}(b; \mu, \nu)}$$

LP

Other contributions? Ingredients soft factor

The soft region

The soft function is generated through the emissions of soft gluons in the partonic cross section



Progress Report
Stay tuned ...

Contributions to the soft factor after applying the eikonal approximation and including the effect from the transverse momentum contributions from the quark propagators.

Summary

We explore NLP $(M/Q)^n$ contributions in large P_T and TMD regions via factorization theorem

- NLP factorization based on “*TMD formalism*”
 - extend the tree level Amsterdam formalism and beyond leading order
 - CSS, Ji Ma Yuan, Akyat Rogers, framework vs. SCET and Background Field Methods
- Consider R_{SIDIS} & revisit “Cahn effect” & matching related to early importance intrinsic k_T
 - “*Intrinsic*” NLP TMDs related thru EOM in terms “*kinematic*” & “*dynamical*”
- Consider RG consistency of matching to collinear factorization
 - Bacchetta, Boer, Diehl, Mulders JHEP 2008, Bacchetta et al. PLB 2019
- Report progress in this necessary condition NLP factorization (not yet sufficient)
- In doing so, we provide the basis for performing global analysis & phenomenology of one the earliest observables used to study intrinsic 3-D momentum structure of the nucleon—Opportunity for EIC study of transverse momentum nucleon structure

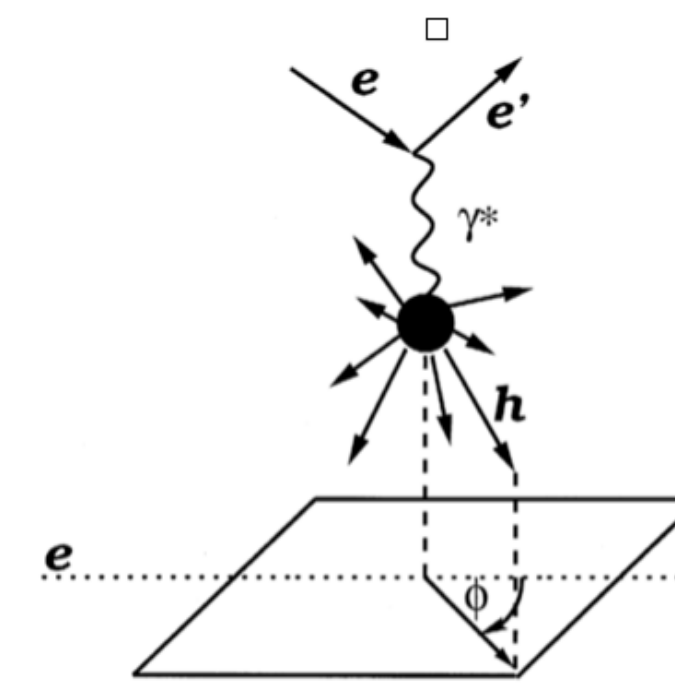
Thank You

$$(p_T \sim k_T) \sim q_T \ll Q$$

“TMD region??”

$$\frac{d\sigma^{ep \rightarrow ehX}}{d\phi} = \mathcal{A} + \mathcal{B} \cos \phi + \mathcal{C} \cos 2\phi + \mathcal{D} \sin \phi + \mathcal{E} \sin 2\phi$$

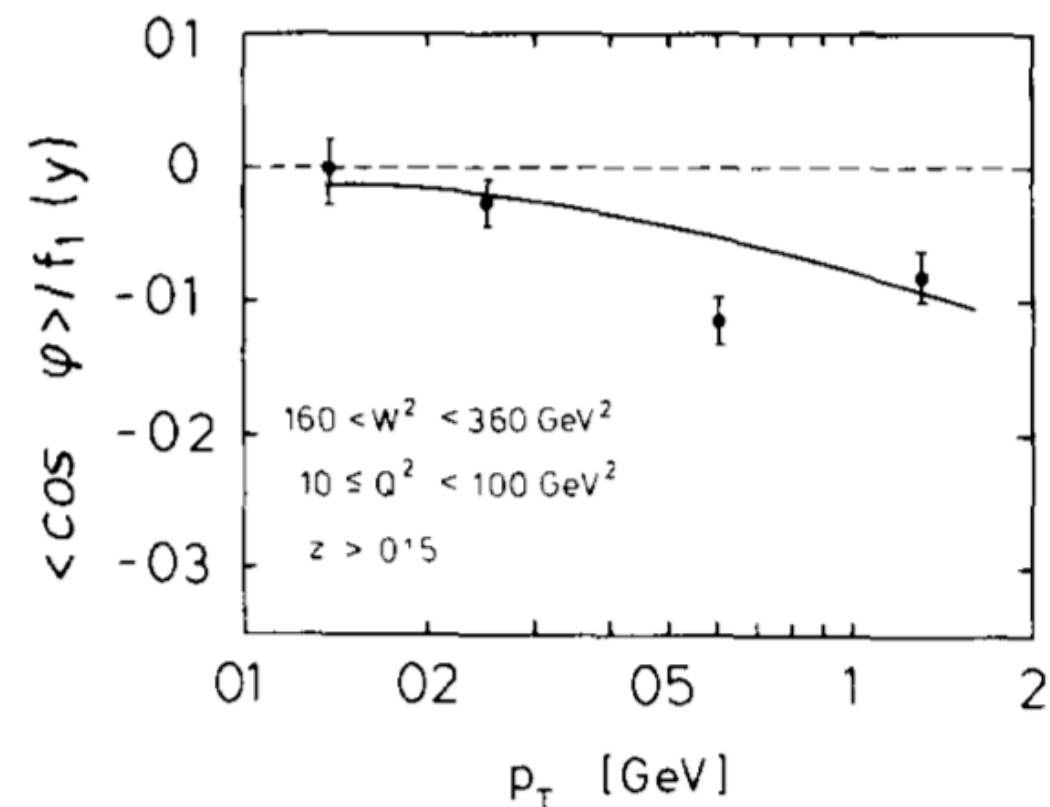
DATA



$$\Lambda_{qcd} \ll q_T \sim Q$$

“Collinear region??”

EMC collaboration Phys. Lett. B 130 (1983) 118, & Z. Phys. C 34 (1987) 277

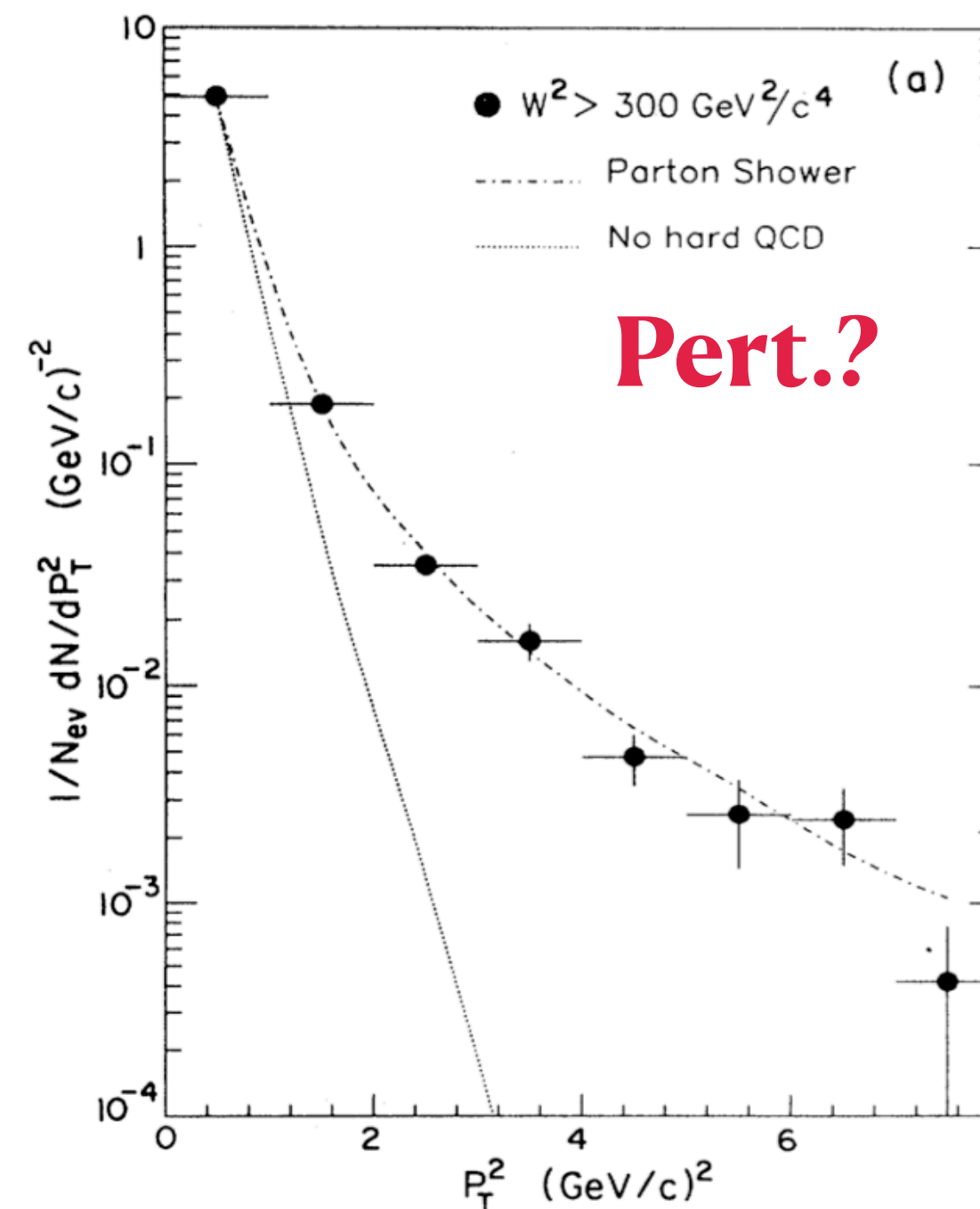


Non-pert.

Fig 4 p_T dependence ($p_T > 50$ MeV) of $\cos \phi$ moment for $160 \leq W^2 < 360 \text{ GeV}^2$, $Q^2 > 10 \text{ GeV}^2$ and $z > 0.15$ compared with model calculations described in ref [8] (statistical errors on model curve from Monte Carlo ± 0.03 not shown)

In conclusion a finite $\langle \cos \phi \rangle$ has been observed in deep inelastic muon scattering. The sign of the effect is negative and shows little Q^2 or W^2 dependence. There is a significant increase of the asymmetry as a function of z and p_T . The general trend of the data is reproduced by a model containing a large effective intrinsic momentum. A contribution from leading order QCD cannot be excluded but is at present not required by the data.

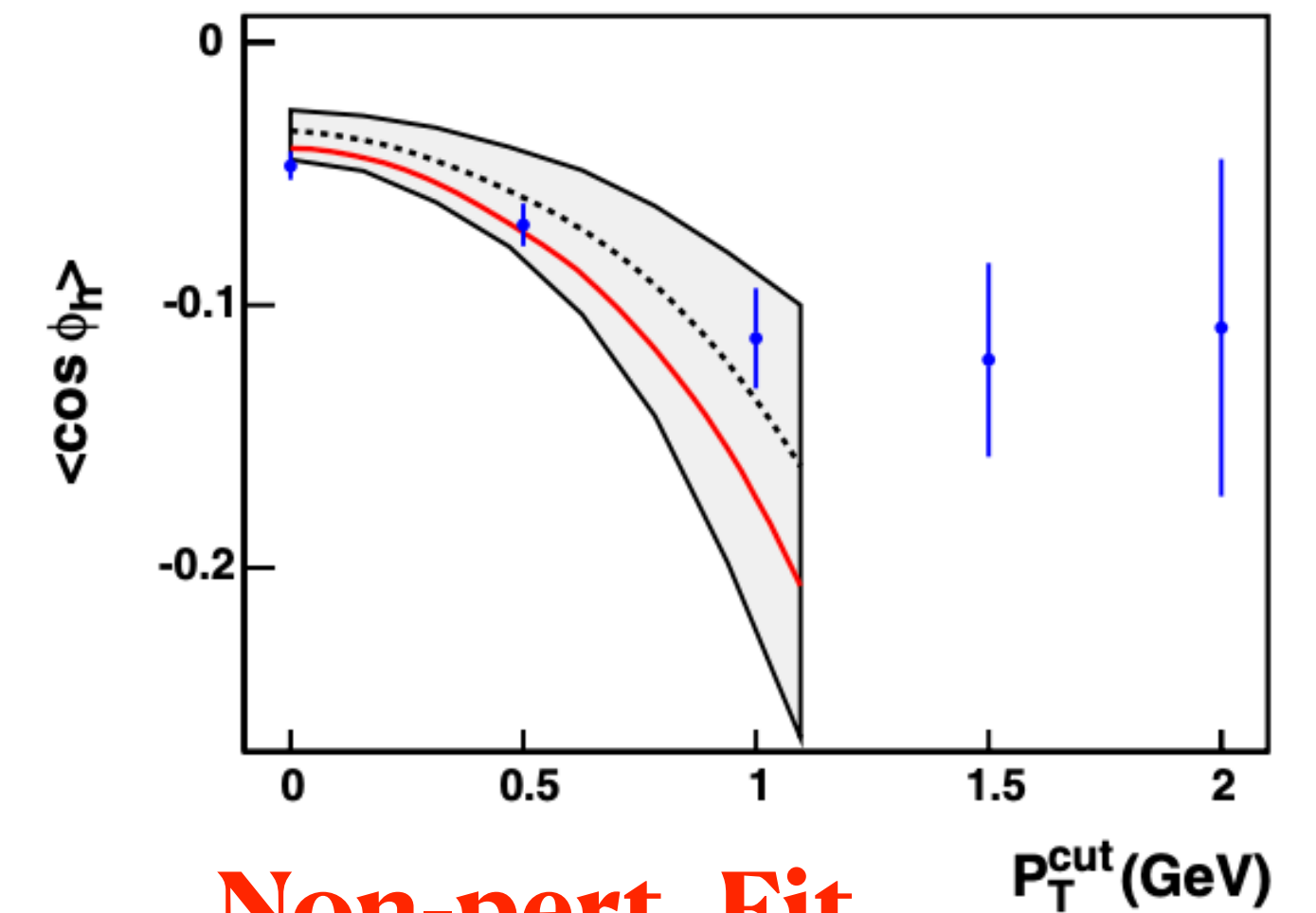
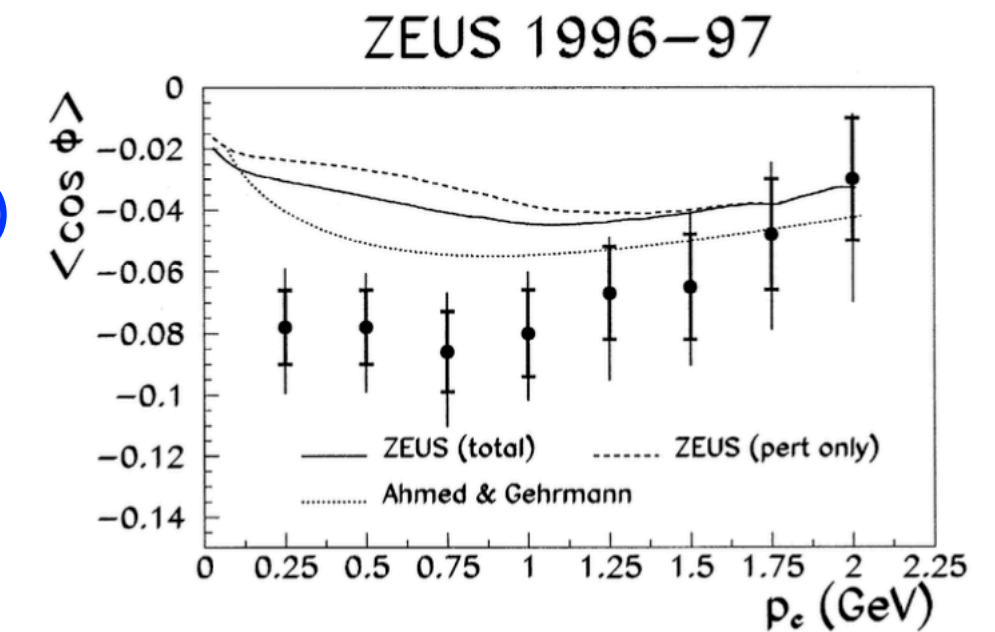
E665 Phys. Rev. D 48 (1993) 5057



Pert.?

ZEUS PLB 481 (1990)

Pert.



Non-pert. Fit

Anselmino, Boglione, D'Alesio, Kotzinian, Murgia, Prokudin

PRD 71, 074006 (2005)

Bands are generated by computing the observable on a subset of JAM replicas (from the recent $W+c$ analysis) and taking the mean \pm standard deviation

- R_{SIDIS} Exp:— JLab Hall C since 2006 PR12-06-104- see Kinney
<https://indico.jlab.org/event/758/contributions/13813/attachments/10635/16065/Kinney-RSIDIS.pdf>
A. Airapetian et al., Phys. Rev. D 87(1), 012010 (2013), arXiv:1204.4161
C. Adolph et al., Nucl. Phys. B 886, (2014), arXiv:1401.6284,
A. Moretti, at COMPASS. SciPost Phys. Proc. 8, 144 (2022), arXiv:2107.10740