Correspondence between Color Glass Condensate and High-Twist Expansion

Yu Fu

Duke University

February 28, 2025

In collaboration with: Zhongbo Kang, Farid Salazar, Xin-Nian Wang, and Hongxi Xing

References: arXiv:2310.12847 [hep-ph], arXiv: 2406.01684 [hep-ph]

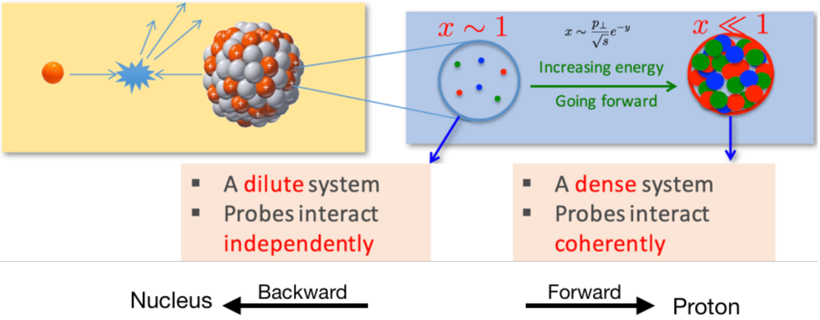
Physics Opportunities at an Electron-Ion Collider XI, Miami, Feb 24-28, 2025

Outline

- Multiple scattering in QCD matter
  - Dilute v.s. Dense medium
  - High-Twist Expansion v.s. Color Glass Condensate(CGC)
- Matching between CGC and High-Twist Expansion (direct photon production as an example)
- Summary and outlook

I.Multiple scattering in QCD matter

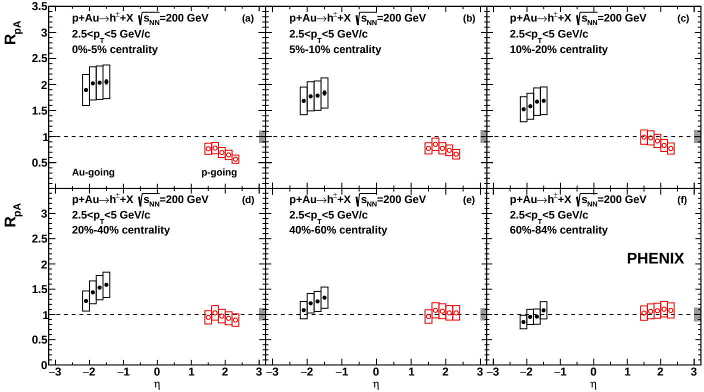
Forward v.s. Backward



- Two important kinematics variables
  - longitudinal momentum fraction: $x \sim \frac{Q}{\sqrt{s}} e^{-y}$
  - transverse momentum transfer: Q
- Forward rapidity($y > 0$): proton-going; sensitive to small-x
- Backward rapidity($y < 0$): nucleus-going; sensitive to large-x

Experimental phenomena in dilute and dense medium

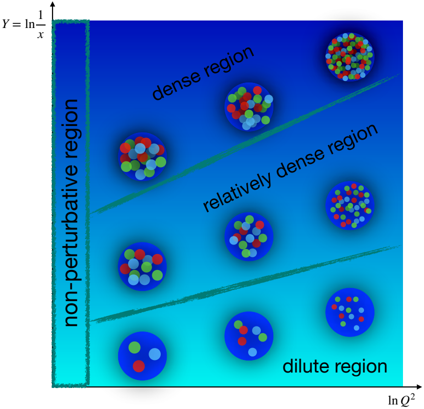
- Nuclear modification factor: $R_{pA} = \frac{1}{A}\frac{\sigma_{pA}}{\sigma_{pp}}$



  - Forward region(dense): Suppression
  - Backward region([relatively] dilute): Enhancement

How do we theoretically explain these phenomena?

Anatomy of QCD matter



⇒ Color Glass Condensate
Strong field, Wilson line
BK/JIMWLK evolution

See review: Gelis, Iancu, Venugopalan, 2003

⇒ High-Twist formalism
Multiparton correlations
DGLAP type evolution

Qiu, Sterman (1991);Kang, Wang, Wang, Xing (2013)

⇒ Leading twist
Collinear factorization
DGLAP evolution

Collins, Soper (1981)

Theoretical framework for incoherent multiple scattering

High-Twist Expansion: for QCD scattering in non-dense medium

- Power suppression   Perturbative Expansion

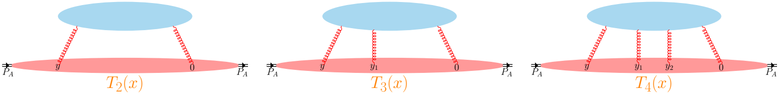
$$\sigma \sim \quad [\alpha_s^0 C_2^{(0)} + \alpha_s^1 C_2^{(1)} + \alpha_s^2 C_2^{(2)} + \cdots] \otimes T_2(x)$$

Twist Expansion

$$+Q^{-1}[\alpha_s^0 C_3^{(0)} + \alpha_s^1 C_3^{(1)} + \alpha_s^2 C_3^{(2)} + \cdots] \otimes T_3(x)$$

$$+Q^{-2}[\alpha_s^0 C_4^{(0)} + \alpha_s^1 C_4^{(1)} + \alpha_s^2 C_4^{(2)} + \cdots] \otimes T_4(x)$$

$$+\cdots$$



- Nuclear enhancement

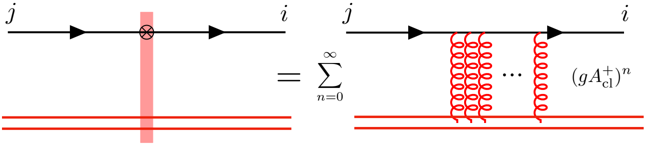
$$T_4(x) \propto \int dy^- dy_1^- dy_2^- \langle F(0^-)F(y_2^-)F(y_1^-)F(y^-)\rangle \propto A^{1/3}$$

$$\Rightarrow \frac{1}{Q^2} \xrightarrow{\text{nuclear size}} \frac{A^{1/3}}{Q^2}$$

Theoretical framework for coherent multiple scattering

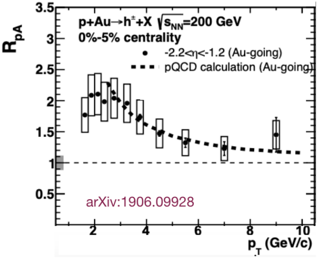
Color Glass Condensate: for QCD scattering in dense medium

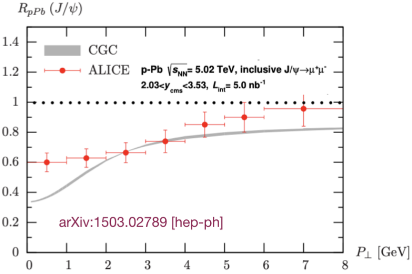
- Probe can not resolve different small-x gluons.
- All small-x gluons are treated equivalently, and be resumed.
- Coherent multiple scattering is encoded in the “shock wave”.



- Quark propagation: $\mathcal{T}^{q}_{ij} = 2\pi\delta(l^-)\gamma^- \int dy_\perp e^{-l_\perp \cdot y_\perp} V_{ij}(y_\perp)$
  Light-like Wilson line: $V_{ij}(y_\perp) = \mathcal{P}exp(i \int dy^- g A^+_{cl}(y^-, y_\perp) t^c_{ij})$

HT vs CGC





- High Twist Expansion:

  Enhancement in backward region

- Color Glass Condensate:

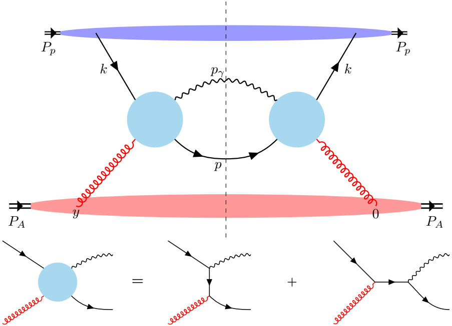
  Suppression in forward region

How to build a unified picture to describe the dilute and dense limits?

II.Correspondence between CGC and High-Twist Expansion (Example: $p + A \to \gamma + X$)

Direct photon production in pA within HT Expansion

- Leading twist(LT): single scattering
  - Consider quark-gluon initiated channel



  - Leading twist collinear factorization

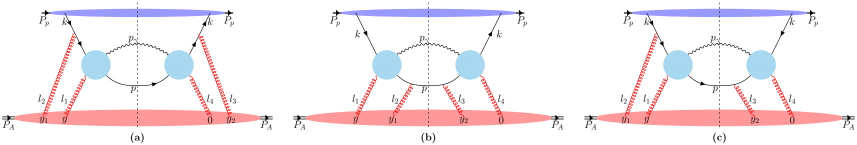
$$\frac{\mathrm{d}\sigma^{HT}}{E_\gamma^{-1}\mathrm{d}^3\boldsymbol{p_\gamma}}\Big|_{\mathrm{LT}} = f_{q/p}(x_q) \otimes f_{g/A}(x) \otimes H^{(2)}_{q+g\to\gamma+q}$$

Direct photon production in pA within HT Expansion

- Next-to-leading twist(NLT):

  Incoherent: Hard scattering + Soft gluon scattering

  ▶ Initial/Final state scattering and initial-final interference



(24 diagrams in total)

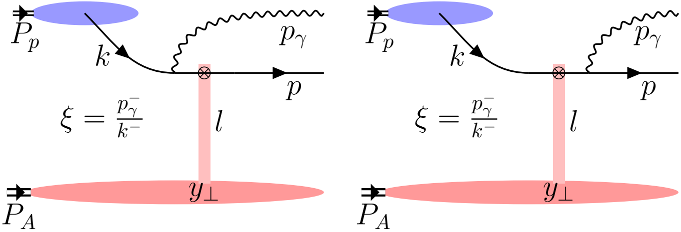
▶ NLT contribution to the differential cross-section

$$\frac{\mathrm{d}\sigma^{HT}}{E_\gamma^{-1}\mathrm{d}^3\boldsymbol{p_\gamma}}\Big|_{\mathrm{NLT}} = f_{q/p} \otimes \left\{ T_{gg}, x\frac{\partial T_{gg}}{\partial x}, x^2\frac{\partial^2 T_{gg}}{\partial x^2} \right\} \otimes H^{(4)}_{q+gg\to\gamma+q}$$

T_{gg}: twist-4 gluon correlation

Direct photon production in pA within CGC formalism

- Coherent multiple scattering from CGC
  - Amplitudes: Initial radiation + Final radiation



  - Differential cross-section within CGC

$$\frac{\mathrm{d}\sigma^{CGC}}{E_\gamma^{-1}\mathrm{d}^3 p_\gamma} = f_{q/p}(x_p) \otimes \int \mathrm{d}^2 l_\perp \frac{l_\perp^2}{p_{\gamma\perp}^2 \left(\xi l_\perp - p_{\gamma\perp}\right)^2} F(x, l_\perp)$$

  - Dipole correlator

$$F(x, l_\perp) = \int \mathrm{d}^2 y_\perp \mathrm{d}^2 y'_\perp e^{-i l_\perp \cdot (y_\perp - y'_\perp)} \frac{1}{N_c} \left\langle \mathrm{Tr}\left[V^\dagger(y'_\perp) V(y_\perp)\right]\right\rangle_x$$

Naive power expansion of CGC

- Differential cross-section within CGC

$$\frac{\mathrm{d}\sigma^{CGC}}{E_\gamma^{-1}\mathrm{d}^3 p_\gamma} = f_{q/p}(x_p) \otimes \int \mathrm{d}^2 l_\perp \frac{1}{p_{\gamma\perp}^2} \frac{l_\perp^2 F(x,l_\perp)}{\left(\xi l_\perp - \boldsymbol{p}_{\gamma\perp}\right)^2}$$

- Twist or power expansion

$$\frac{l_\perp^2 F(x,l_\perp)}{(\xi l_\perp - p)^2} = \boxed{\frac{l_\perp^2 F(x, l_\perp)}{p_{\gamma\perp}^2}}_{\mathrm{LT}} + \boxed{\frac{\xi^2 l_\perp^4 F(x, l_\perp)}{p_{\gamma\perp}^4}}_{\mathrm{NLT}} + \dots$$

  - Twist-2 gluon PDF = 2nd moment dipole correlator:

$$\lim_{x\to 0} x f_{g/A}(x) \simeq \frac{N_c}{2\pi^2\alpha_s} \int \frac{\mathrm{d}^2 l_\perp}{(2\pi)^2} l_\perp^2 F(x, l_\perp)$$ R. Baier, et al; arXiv:hep-ph/0403201

  - Leading twist cross section:

$$\left.\frac{\mathrm{d}^3\sigma^{\mathrm{CGC}}}{E_\gamma^{-1}\mathrm{d}^3 p_\gamma}\right|_{\mathrm{LT}} = \lim_{x\to 0} \left.\frac{\mathrm{d}^3\sigma^{\mathrm{HT}}}{E_\gamma^{-1}\mathrm{d}^3 p_\gamma}\right|_{\mathrm{LT}}$$

CGC and leading twist expansion match at small-x!

Naive power expansion of CGC

- Differential cross-section within CGC

$$\frac{\mathrm{d}\sigma^{CGC}}{E_\gamma^{-1}\mathrm{d}^3 p_\gamma} = f_{q/p}(x_p) \otimes \int \mathrm{d}^2 l_\perp \frac{1}{p_{\gamma\perp}^2} \frac{l_\perp^2 F(x, \boldsymbol{l}_\perp)}{\left(\xi l_\perp - \boldsymbol{p}_{\gamma\perp}\right)^2}$$

- Twist or power expansion

$$\frac{l_\perp^2 F(x, l_\perp)}{(\xi l_\perp - p)^2} = \boxed{\frac{l_\perp^2 F(x, l_\perp)}{p_{\gamma\perp}^2}}_{\text{LT}} + \boxed{\frac{\xi^2 l_\perp^4 F(x, l_\perp)}{p_{\gamma\perp}^4}}_{\text{NLT}} + \cdots$$

  - Twist-4 gluon correlation = 4th moment of dipole correlator:

$$\lim_{x\to 0} T_{gg}(x,0,0) \simeq \frac{N_c^2}{2(2\pi)^4 \alpha_s^2} \int \frac{\mathrm{d}^2 l_\perp}{(2\pi)^2} l_\perp^4 F(x, l_\perp)\Big|_{\text{Twist-4}}$$

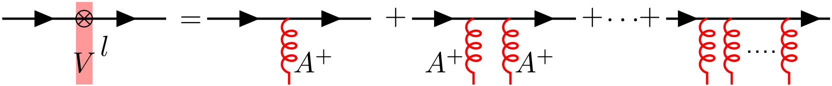
  - Next-to-Leading twist cross section:

$$\frac{\mathrm{d}^3\sigma^{\mathrm{CGC}}}{E_\gamma^{-1}\mathrm{d}^3 p_\gamma}\Bigg|_{\mathrm{NLT}} = f_{q/p} \otimes \left\{ T_{gg}, \cancel{x\frac{\partial T_{gg}}{\partial x}}, \cancel{x^2 \frac{\partial^2 T_{gg}}{\partial x^2}} \right\} \otimes H_{q+gg}^{(4)} \neq \lim_{x\to 0} \frac{\mathrm{d}^3\sigma^{\mathrm{HT}}}{E_\gamma^{-1}\mathrm{d}^3 p_\gamma}\Bigg|_{\mathrm{NLT}}$$

Can NOT recover the **derivative terms** in HT at twist-4!

From CGC to twist-2 collinear factorization

- Expand CGC vertex
  Quark propagation: $\mathcal{T}^q = 2\pi\delta(l^-)\gamma^- \int \mathrm{d}^2 y_\perp e^{-l_\perp \cdot y_\perp} V(y_\perp)$



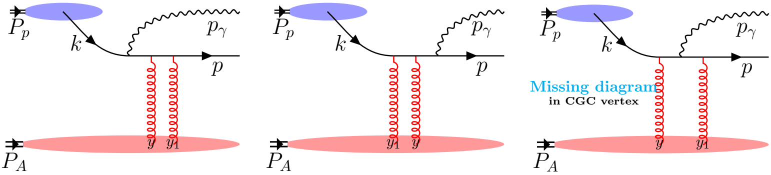
- To 1st order and bring back "sub-eikonal phase"
  LO vertex: $\Gamma(l) \sim \gamma^- \int \mathrm{d}^2 y_\perp \mathrm{d}y^- e^{-il_\perp \cdot y_\perp} \mathbf{e^{il^+y^-}} igA^+(y^-, y_\perp)$
  - Leading twist cross section:

$$\left.\frac{\mathrm{d}^3\sigma^{\mathrm{CGC_{sub}}}}{E_\gamma^{-1}\mathrm{d}^3 p_\gamma}\right|_{\mathrm{LT}} = f_{q/p} \otimes \int_{y,y'} \mathcal{H}_S \, \langle \mathrm{Tr}[A^+(y)A^+(y')] \rangle_x$$

  - Hard part: $\mathcal{H}_S = \frac{\xi^2\left[1+(1-\xi)^2\right]}{p^4_{\gamma\perp}} e^{ixP_A^+(y^- - y'^-)} \delta^{(2)}(y_\perp - y'_\perp)(\partial_{y_\perp} \cdot \partial_{y'_\perp})$
- Matches exactly to High-Twist Expansion at twist-2 beyond small-x

From CGC to twist-4 collinear factorization

- Expand CGC vertex to 2nd order and bring back "sub-eikonal phase"



- Phase in "Missing diagram": $\left[1 - e^{-i\frac{(y^- - y_1^-)}{\tau_{\gamma,\mathrm{form}}}}\right]$

  (photon formation time: $\tau_{\gamma,\mathrm{form}} \sim \frac{p_{\gamma\perp}^2}{xP_A^+}$)

  - Landau-Pomeranchuk-Migdal (LPM) effect:
    - $\tau_{\gamma,\mathrm{form}} \gg y^- - y_1^-$ (coherent) $\to$ contribution vanishes
    - $\tau_{\gamma,\mathrm{form}} \ll y^- - y_1^-$ (incoherent) $\to$ contribution survives

- $\left.\frac{\mathrm{d}^3\sigma^{\mathrm{CGC}_{\mathrm{sub}}}}{E_\gamma^{-1}\mathrm{d}^3 p_\gamma}\right|_{\mathrm{NLT}} = f_{q/p} \otimes \int_{z,y,y'} \mathcal{H}_D \langle \mathrm{Tr}[A^+(z)A^+(y)A^+(y')A^+(0)]\rangle_x$
  - Derivative terms are in $\mathcal{H}_D$

- Matches exactly to High-Twist Expansion at twist-4 beyond small-x

Summary and Outlook

Summary:

- Naive power expansion of CGC only recovers part of the complete HT Expansion result at twist-4.
- **4th moment of the dipole correlation** corresponds to twist-4 gluon-gluon correlation function at small-x.
- **Important missing ingredients in CGC**: sub-eikonal phases and diagrams related to LPM effect.
- **Consistency between CGC and HT Expansion** to twist-4 level after bringing back sub-eikonal phase.

Outlook:

- Consistency between CGC and HT expansion persist at NLO?
- Establish a framework that allows to resum all twists? (modify Wilson lines to keep track of phases?)

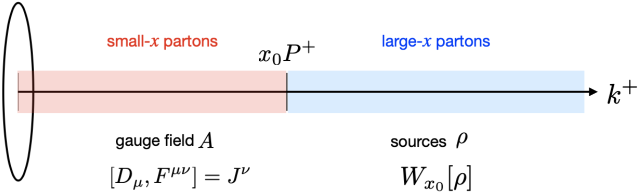
Thank you!

Backup

Theoretical framework for coherent multiple scattering

Color Glass Condensate: for QCD scattering dense medium

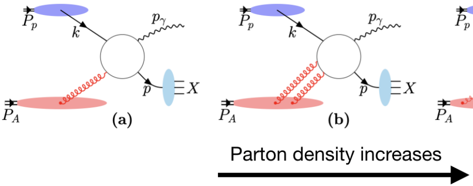
- Separates the partonic content of hadrons according to x



- Large-x partons are treated as static and localized color sources ρ; it generates a current $J^\mu(z) = \delta^{\mu+}\rho(z^-, z_\perp)$
- Sources color charge distribution is dictated by a gauge invariant weight functional $W_{x0}[\rho]$.
- Small-x gluon are treated as classical filed; $\langle A_{cl} A_{cl} \rangle \sim 1/\alpha_s$.
- Expectation value of any observable: $\langle \mathcal{O} \rangle = \int [D\rho] W_{x_0}[\rho] \mathcal{O}[\rho]$

Relation between CGC and high-twist expansion

Take direct photon production as an example



- Higher-twist becomes important at moderate $p_{\gamma\perp}^2$ and small-x:

$$d\sigma \sim \frac{1}{p_{\gamma\perp}^4}[\underbrace{A}_{LT} + \underbrace{B\frac{\langle l_\perp^2\rangle}{p_{\gamma\perp}^2} + C\frac{\langle l_\perp^2\rangle^2}{p_{\gamma\perp}^4} + \cdots]}_{HigherTwist}$$

Hard scale: $p_{\gamma\perp}$

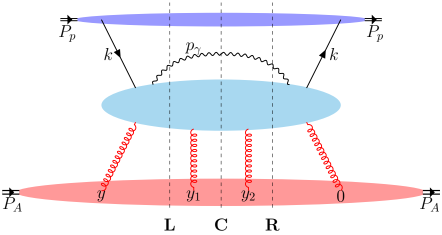
Momentum exchange from medium:$\langle l_\perp^2\rangle \propto Q_s^2 \propto A^{1/3}x^{-0.3}$

Saturation scale grows with energy and nuclear size.

Direct photon production in pA within HT formalism

- Next-to-leading twist(NLT):

  Incoherent: Hard scattering + Soft gluon scattering insertion



  - Category of the diagrams
    - Central cut: contribution from double scattering
    - Left and Right cuts: single-triple interference

Efforts towards a unified picture of dilute and dense limits

Gluon TMD in particle production from low to moderate x

I. Balitsky[a,b] and A. Tarasov[b]
[a] Department of Physics, Old Dominion University, 4600 Elkhorn Ave, Norfolk, VA 23529, U.S.A.
[b] Theory Group, Jefferson Lab, 12000 Jefferson Ave, Newport News, VA 23606, U.S.A.
E-mail: balitsky@jlab.org, atarasov@jlab.org

Abstract: We study the rapidity evolution of gluon transverse momentum dependent distributions appearing in processes of particle production and show how this evolution changes from small to moderate Bjorken x.

Keywords: Deep Inelastic Scattering (Phenomenology), QCD Phenomenology

ArXiv ePrint: 1603.06548

Next-to-eikonal corrections in the CGC: gluon production and spin asymmetries in pA collisions

Tolga Altinoluk,[a] Néstor Armesto,[a] Guillaume Beuf,[a] Mauricio Martínez[b] and Carlos A. Salgado[a]
[a] Departamento de Física de Partículas and IGFAE, Universidade de Santiago de Compostela, E-15706 Santiago de Compostela, Galicia-Spain
[b] Department of Physics, The Ohio State University, Columbus, OH 43210, U.S.A.
E-mail: tolga.altinoluk@usc.es, nestor.armesto@usc.es, guillaume.beuf@usc.es, martinezguerrero.1@osu.edu, carlos.salgado@usc.es

Abstract: We present a new method to systematically include corrections to the eikonal approximation in the background field formalism. Specifically, we calculate the subleading, power-suppressed corrections due to the finite width of the target or the finite energy of the projectile. Such power-suppressed corrections involve Wilson lines decorated by gradients of the background field — thus related to the density - of the target. The method is of generic applicability. As a first example, we study single inclusive gluon production in pA collisions, and various related spin asymmetries, beyond the eikonal accuracy.

Keywords: QCD Phenomenology, Hadronic Colliders

ArXiv ePrint: 1404.2219

Gluon-mediated inclusive Deep Inelastic Scattering from Regge to Bjorken kinematics

Renaud Boussarie[a] and Yacine Mehtar-Tani[b]
[a] Centre de Physique Théorique, École polytechnique, CNRS, I.P. Paris, F-91128 Palaiseau, France
[b] Physics Department and RIKEN BNL Research Center, Brookhaven National Laboratory, Upton, NY 11973, U.S.A.
E-mail: renaud.boussarie@polytechnique.edu, mehtartani@bnl.gov

Abstract: We revisit high energy factorization for gluon mediated inclusive Deep Inelastic Scattering (DIS) for which we propose a new semi-classical approach that accounts systematically for the longitudinal extent of the target in contrast with the shockwave limit. In this framework, based on a partial twist expansion, we derive a factorization formula that involves a new gauge invariant unintegrated gluon distribution which depends explicitly on the Feynman x variable. It is shown that both the Regge and Bjorken limits are recovered in this approach. We reproduce in particular the full one loop inclusive DIS cross-section in the leading twist approximation and the all-twist dipole factorization formula in the strict $x = 0$ limit. Although quantum evolution is not discussed explicitly in this work, we argue that the proper treatment of the x dependence of the gluon distribution encompasses the kinematic constraint that must be imposed on the phase-space of gluon fluctuations in the target to ensure stability of small-x evolution.

Keywords: Deep Inelastic Scattering or Small-X Physics, Parton Distributions

ArXiv ePrint: 2112.01412

Quark jets scattering from a gluon field: From saturation to high p_t

Jamal Jalilian-Marian
Department of Natural Sciences, Baruch College, CUNY, 17 Lexington Avenue, New York, New York 10010, USA
and CUNY Graduate Center, 365 Fifth Avenue, New York, New York 10016, USA

(Received 18 September 2018; published 30 January 2019)

We continue our studies of possible generalization of the color glass condensate effective theory of high energy QCD to include the high p_t (or equivalently large x) QCD dynamics as proposed in [Phys. Rev. D 96, 074020 (2017)]. Here, we consider scattering of a quark from both the small and large x gluon degrees of freedom in a proton or nucleus target and derive the full scattering amplitude by including the interactions between the small and large x gluons of the target. We thus generalize the standard eikonal approximation for parton scattering, which can now be deflected by a large angle (and therefore have large p_t) and also lose a significant fraction of its longitudinal momentum (unlike the eikonal approximation). The corresponding production cross section can thus serve as the starting point toward the derivation of a general evolution equation that would contain the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi evolution equation at large Q^2 and the Jalilian-Marian-Iancu-McLerran-Weigert-Leonidov-Kovner evolution equation at small x. This amplitude can also be used to construct the quark Feynman propagator, which is the first ingredient needed to generalize the color glass condensate effective theory of high energy QCD to include the high p_t dynamics. We outline how it can be used to compute observables in the large x (high p_t) kinematic region where the standard color glass condensate formalism breaks down.

DOI: 10.1103/PhysRevD.99.014043

Helicity evolution at small x

Yuri V. Kovchegov,[a] Daniel Pitonyak[b] and Matthew D. Sievert[c]
[a] Department of Physics, The Ohio State University, Columbus, OH 43210, U.S.A.
[b] RIKEN BNL Research Center, Brookhaven National Laboratory, Upton, New York 11973, U.S.A.
[c] Physics Department, Brookhaven National Laboratory, Upton, NY 11973, U.S.A.
E-mail: kovchegov.1@osu.edu, dpitonyak@quark.phy.bnl.gov, msievert@bnl.gov

Abstract: We construct small-x evolution equations which can be used to calculate quark and anti-quark helicity TMDs and PDFs, along with the g_1 structure function. These evolution equations resum powers of $\alpha_s \ln^2(1/x)$ in the polarization-dependent evolution along with the powers of $\alpha_s \ln(1/x)$ in the unpolarized evolution which includes saturation effects. The equations are written in an operator form in terms of polarization-dependent Wilson line-like operators. While the equations do not close in general, they become closed and self-contained systems of non-linear equations in the large-N_c and large-N_c & N_f limits. As a cross-check, in the ladder approximation, our equations map onto the same ladder limit of the infrared evolution equations for the g_1 structure function derived previously by Bartels, Ermolaev and Ryskin [1].

Keywords: Resummation, Perturbative QCD

ArXiv ePrint: 1511.06737

Quark branching in QCD matter to any order in opacity beyond the soft gluon emission limit

Matthew D. Sievert[*] and Ivan Vitev[†]
'heoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA

(Received 18 July 2018; published 12 November 2018)

ıld nuclear matter effects in reactions with nuclei at a future electron-ion collider (EIC) lead to a fication of semi-inclusive hadron production, jet cross sections, and jet substructure when compared to acuum. At leading order in the strong coupling, a jet produced at an EIC is initiated as an energetic :, and the process of this quark splitting into a quark-gluon system underlies experimental observables. ;pectrum of gluons associated with the branching of this quark jet is heavily modified by multiple :ring in a medium, allowing jet cross sections and jet substructure to be used as a probe of the um's properties. We present a formalism that allows us to compute the gluon spectrum of a quark jet to bitrary order in opacity, the average number of scatterings in the medium. This calculation goes ıd the simplifying limit in which the gluon radiation is soft and can be interpreted as energy loss of the :, and it significantly extends previous work which computes the full gluon spectrum only to first order acity. The theoretical framework demonstrated here applies equally well to light parton and heavy : branching, and is easily generalizable to all in-medium splitting processes.

DOI: 10.1103/PhysRevD.98.094010

+ many more!

Efforts towards a unified picture of dilute and dense limits

- Aiming to extend the applicability of CGC from small-x (dense) to large-x (dilute) region
  - Emphasis on the sub-eikonal corrections to the parton propagators

    [arXiv:1404.2219;arXiv:1505.01400; arXiv:1512.00279;arXiv:1902.04483;arXiv:1907.03668;arXiv:2012.03886 et.al.]
  - Rapidity evolution of unintegrated gluon distributions

    [arXiv:1505.02151;arXiv:1603.06548;arXiv:1706.01415;arXiv:1712.09389;arXiv:1905.09144;]
  - New semi-classical approaches

    [arXiv:2006.14569;arXiv:2112.01412;arXiv:2309.16576;arXiv:1708.07533;arXiv:1809.04625;arXiv:2308.15545]
- However, no consensus has yet been reached on the relations between HT Expansion and CGC.