Extracting the Pion Off-Shell Electromagnetic Form Factors

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Some references (off-shell)

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Light-Front Motivations

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- **After Integrate in k^-: Bethe-Salpeter Amplitude (Wave Function)**
- **LF Lorentz Invariant Hamiltonian: $P^2 = P^+P^- - P_\perp^2$**

• Light-Front Coordinates

Four-Vector $\Longrightarrow$ $x^\mu = (x^0, x^1, x^2, x^3) \; = \; (x^+, x^-, x_\perp)$

$x^+ = t + z \qquad x^+ = x^0 + x^3 \qquad \Longrightarrow$ **Time**

$x^- = t - z \qquad x^- = x^0 - x^3 \qquad \Longrightarrow$ **Position**

Scalar product

$$x \cdot y = x^\mu y_\mu = x^+ y_+ + x^- y_- + x^1 y_1 + x^2 y_2 = \frac{x^+ y^- + x^- y^+}{2} - \vec{x}_\perp \vec{y}_\perp$$

Metric Tensor:

$$g^{\mu\nu} = \begin{pmatrix} 2 & 2 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \; and \; g_{\mu\nu} = \begin{pmatrix} 0 & 1/2 & 0 & 0 \\ 1/2 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

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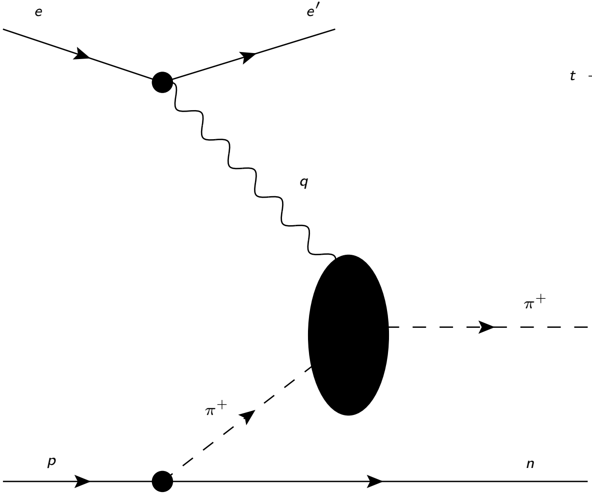
  $\Longrightarrow$ **Extracted from Cross sections**

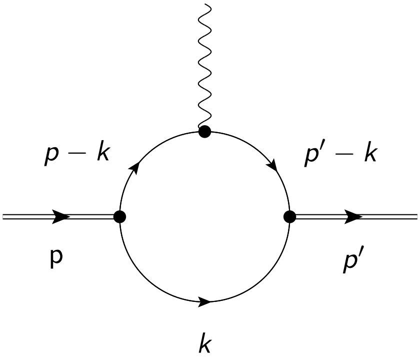
Data from Experiments: Electroproduction



t — channel

Electroproduction: Diagrammatic representation of the pion pole contribution to $p(e, e')\pi^+ n$ process
The black blob represents the half-on-mass shell photo absorption amplitude

Electromagnetic Form Factors



Feynman triangular diagram representing the matrix element of the pion EM current within the Mandelstam framework

Electromagnetic Form Factors

$\Longrightarrow$ **Form factors are essential for our understanding of internal hadron structure and the dynamics**

- **On-shell Case:** $\Longrightarrow$ $F_1(Q^2, t)$
- **Off-shell Case:**

$\Longrightarrow$ **Two Electromagnetic Form Factors:** $F_1(Q^2, t)$ and $F_2(Q^2, t)$

- **Structure:**

$$< p'|\mathcal{O}|P >= (p' + p)^\mu F_1(Q^2, t) + (p' - p)^\mu F_2(Q^2, t)$$

- **Most General Structure**

$$\Gamma_\mu = e[P^\mu \; G_1(q^2, p^2, p') + q^\mu \; G_2(q^2, p^2, p')]$$

- $P^\mu = (p'^\mu + p^\mu)$, and, $q^u = (p'^\mu - p^\mu)$

- $\Longrightarrow \quad G_1 = F_1(p^2, p'^2, q^2)$ **and** $G_2 = F_2(p^2, p'^2, q^2)$

- **Ward-Takahashi Identity-WTI**

$$q^\mu \Gamma_\mu = e\Delta_0^{-1}[\Delta(p) - \Delta(p')]\Delta_0^{-1}(p),$$

where, $\Delta_0(p) = \frac{1}{p^2 - m^2 + \imath\epsilon}$, **and,** $\Delta(p) = \frac{1}{p^2 - m^2 - \Pi(p^2) + \imath\epsilon}$

- **Assuming (Standard renormalization)**

$$\Longrightarrow \quad \{ \quad \Pi(m_\pi^2) = 0$$

- **From the WTI , we have,**

$$(p'^2 - p^2)G_1(q^2, p^2, p'^2) + q^2 G_2(q^2, p^2, p'^2)$$
$$= \Delta^{-1}(p') - \Delta^{-1}(p)$$

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- **Real photon with** $q^2 = 0$

$$(p^2 - m_\pi^2)\, G_1(0, p^2, m_\pi^2) = \Delta^{-1}(p)$$

- **And,**

$$G_2(q^2, p^2, m_\pi^2)$$
$$= \frac{(m_\pi^2 - p^2)}{q^2} \left[G_1(0, p^2, m_\pi^2) - G_1(q^2, p^2, m_\pi^2) \right]$$

$F_2(Q^2, t)$ Off-Shell electromagnetic form factor

- **Pion initial state off-mass shell** $(p^2 = t)$
- **Pion Final state on-mass shell** $(p'^2 = m_\pi^2)$

Then,

$$F_2(Q^2, t) = \frac{t - m_\pi^2}{Q^2}\left[F_1(0, t) - F_1(Q^2, t)\right]$$

- **With** $F_i(Q^2, t) \equiv G_i(q^2, t, m_\pi^2)$ $(i = 1, 2)$
- $Q^2 (= -q^2)$ **is the four-momentum transfer in the spacelike region.**
- **Also** $G_2(q^2 = -Q^2, m_\pi^2, m_\pi^2) = F_2(Q^2, m_\pi^2) = 0$
- **For both the initial and final mesons are on-shell**
- **Implicate** $\Longrightarrow$ **Conservation of the electromagnetic current**

Pion-Photon vertex

- **For half on-shell ($p'^2 = m_\pi^2$) and half off-shell ($p^2 = t$):**

$$\Gamma_\mu(p', p)|_{p'^2=m_\pi^2, p^2=t} = e\Big[(p'+p)^\mu F_1(Q^2, t) + (p'-p)^\mu \frac{(t - m_\pi^2)}{Q^2}\left(F_1(0,t) - F_1(Q^2,t)\right)\Big]$$

- **Pion charge normalization:** $F_1(Q^2 = 0, m_\pi^2) = G_1(0, m_\pi^2, m_\pi^2) = 1$

- **In the electroproduction process, directly measuring the form factor $F_2(Q^2, t)$**

$\Longrightarrow$ **Is impractical due to the transversality of the electron current**

- **But** $F_2(Q^2, t)$ **tends to zero as** $t \to m_\pi^2$
- **However:**

- **In the electroproduction process, directly measuring the form factor $F_2(Q^2, t)$**

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- **But** $F_2(Q^2, t)$ **tends to zero as** $t \to m_\pi^2$

- **However:**

$\Longrightarrow$ **The ratio of** $F_2(Q^2, t)$ **to** $t - m_\pi^2$ **remains nonzero when** t **approaches** m_π^2

New electromagnetic form factor

- **New form factor $g(Q^2, t)$, defined**

$$g(Q^2, t) \equiv \frac{F_2(Q^2, t)}{t - m_\pi^2} = \frac{1}{Q^2}[F_1(0, t) - F_1(Q^2, t)]$$

- **Ref. H.-M. Choi, T. Frederico, C.-R. Ji, and J. P. B. C. de Melo, Pion off-shell electromagnetic form factors: Data extraction and model analysis, Phys. Rev. D 100, 116020 (2019).**

- **H.-M. Choi, C.-R. Ji, T. Frederico, and J. P. B. C. de Melo, 3D imaging of the pion off-shell electromagnetic form factors. Proc. Sci. LC2019 (2020) 035.**
- **J. Leão, J. de Melo, T. Frederico, H.M. Choi and C. -R. Ji Off-shell pion properties: Electromagnetic form factors and light-front wave functions, Phys.Rev.D 110 (2024) 7**

- **Master equation off-shell form factor sum rule****

$$F_1\left(Q^2,t\right) - F_1(0,t) + Q^2 g\left(Q^2,t\right) = 0$$

- **Derivative with respect to $Q^2 \Longrightarrow$ evolution equation:**

$$\frac{\partial}{\partial Q^2} F_1\left(Q^2,t\right) + g\left(Q^2,t\right) + Q^2 \frac{\partial g\left(Q^2,t\right)}{\partial Q^2} = 0.$$

** **Master Equation: Consequence of Ward-Takahashi identity**

In on-mass shell limit, $g(Q^2 = 0, m_\pi^2)$

$\Longrightarrow \quad t = m_\pi^2$ **and at** $Q^2 = 0$

- $g(Q, t)$ **is connected with charge radius for on-shell pion EFF**

$$g(Q^2 = 0, m_\pi^2) = -\frac{\partial}{\partial Q^2} F_1(Q^2 = 0, m_\pi^2) = \frac{1}{6}\langle r_\pi^2 \rangle$$

- **On-mass shell solution for** $g(Q^2, m_\pi^2)$ **is given by**

$$g(Q^2, m_\pi^2) = \frac{1}{6}\langle r_\pi^2 \rangle + \alpha Q^2 + \cdots$$

- **The master equation allows extract both $F_1(Q^2, t)$ and $F_2(Q^2, t)$**

$\Longrightarrow$ **Electroproduction process cannot measure directly $F_2(Q, t)$!!!**

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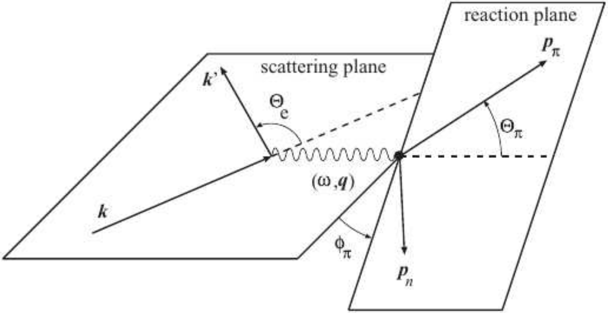
- $g(Q^2, t)$ **Is the new observable form factor**

Cross-section



- **Figure from ref.** H. P. Blok et al.; Phys.Rev.C 78 (2008) 045202

- **Exclusive reaction:** $^{1}\mathrm{H}\,(e, e'\pi^{+})\,n$

$\Longrightarrow$ **Longitudinal (L), Transverse (T), and also interference terms (LT and TT)**

$$(2\pi)\frac{d^2\sigma}{dtd\phi} = \frac{d\sigma_{\mathrm{T}}}{dt} + \epsilon\frac{d\sigma_{\mathrm{L}}}{dt} + \sqrt{2\epsilon(\epsilon+1)}\frac{d\sigma_{\mathrm{LT}}}{dt}\cos\phi + \epsilon\frac{d\sigma_{\mathrm{TT}}}{dt}\cos 2\phi$$

$$\epsilon = \left(1 + \frac{2|\mathrm{q}|^2}{Q^2}\tan^2\frac{\theta_e}{2}\right)^{-1}$$

- ϵ **Polarization of the virtual photon**
- q **is its three momentum**
- θ_e **is the angle between initial and final electron momentum**

- **Sullivan process: Chew-Low formulation**

For small t for the pion pole contribuition

$$N\frac{d\sigma_{\rm L}}{dt} = 4\hbar c\left(eG_{\pi NN}\right)^2 \frac{-tQ^2}{\left(t-m_\pi^2\right)^2} F_\pi^2\left(Q^2\right),$$

- **Flux fator for σ_L:**

$$N = 32\pi\left(W^2-m_p^2\right)\sqrt{\left(W^2-m_p^2\right)^2+Q^4+2Q^2\left(W^2+m_p^2\right)}.$$

- **Invariant mass (virtual photon-nucleon system):**

$$W = \sqrt{M_p^2+2M_p\omega-Q^2}$$

- M_p **proton mass**

- **Pion Nucleon form factor (monopole):**

$$G_{\pi NN}(t) = G_{\pi NN}\left(m_\pi^2\right)\left(\frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2 - t}\right),$$

$G_{\pi NN}\left(m_\pi^2\right) = 13.4$ **and** $\Lambda_\pi = 0.80\ \mathrm{GeV}$

- **For extraction of F_π from the Jefferson Lab***

$\star$ **Ref. G. M. Huber et al. (Jefferson Lab F_π Collaboration) Phys. Rev. C 78, 045203 (2008)**

Obs.: The value of Λ_π is agree with the deuteron proprieties
See T. E. O. Ericson, B. Loiseau, and A. W. Thomas
Phys. Rev. C 66, 014005 (2002)

Models

- **Previus work: Covariant Model***

$$\Gamma_{\pi}^{\mathrm{CON}}(k,p) = g_{\pi\bar{q}\bar{q}},$$

$\Longrightarrow$ $g_{\pi q\bar{q}}$ **pion-quark coupling constant**

- **pointlike vertices**
- **fermion-loop was regulated by dimensional regularization**
- **UV divergence eliminated redefining the renormalized form factor**

$$F_1^{\mathrm{ren}}\left(Q^2,t\right) = 1 + \left(F_1\left(Q^2,t\right) - F_1\left(0,m_\pi^2\right)\right)$$

⋆ **Ref.; Ho-Meoyng Choi, T. Frederico, Chueng-Ryong Ji and J.P.B.C. de Melo, Phys.Rev.D 100 (2019) 11, 116020**

- **Pion microscospic current:**

Mandelstam amplitude for the photoabsorption

- **Here two constraints**

i) Covariance

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ii) Current conservation (on-mass shell, initial and final pion)

Effective Lagrangian

Effective Lagrangian with pion and quark degrees of freedom

$$\mathcal{L} = -\imath \frac{m}{f_\pi} \vec{\pi} \cdot \bar{q} \gamma_5 \vec{\tau} q,$$

- **Coupling of the constituent quark / pseudoscalar isovector pion field / $\mathrm{SU}(2)$ flavor symmetry**
- **f_π is the pion decay constant and m is the constituent quark mass**

- **Mandelstam formula** $\Longrightarrow$ **Pion-photon absorption amplitude**

$$\Gamma^{\mu}\left(p',p\right) = -2ie\frac{m^2}{f_\pi^2}N_c\int\frac{d^4k}{(2\pi)^4}\,\mathrm{Tr}\left[S(k)\gamma^5 S\left(k-p'\right)\right.$$
$$\left.\times\gamma^{\mu}S(k-p)\gamma^5\right]\Gamma_\pi\left(k,p'\right)\Gamma_\pi(k,p),$$

$$\begin{cases} S(p) \textit{ is } \textbf{constituent quark propagator} \\ N_c = 3 \textbf{ is the number of colors} \\ q = (p'-p) \textbf{ the momentum transfer} \\ k \;\; \textbf{the spectator quark momentum} \end{cases}$$

- **Frame**
- $\begin{cases} \textit{Offshell}: & \textbf{initial pion:} \quad p^2 = t < 0 \\ \textit{On-shell}: & \textbf{final pion} \quad p'^2 = m_\pi^2 \end{cases}$

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$\Longrightarrow$ **The equation above:** **Satisfy current conservation:**

$$q_\mu \Gamma^\mu \left(p', p\right)\Big|_{p^2=p'^2=m_\pi^2} = 0$$

If both pion in on-shell

- $\Longrightarrow \quad q^+ = q^0 + q^3 = 0$ **(with LF energy** $p^- = p^0 - p^3$ **)**
- $\Longrightarrow$ **Momentum** $(p^+, p_\perp)$ **satisfying**

$$p^- = \frac{p_\perp^2 + t}{p^+}, \quad \textbf{Off-shell}$$

- $\vec{p}'_\perp = \vec{q}_\perp/2$, **pion initial off-shell pion**

$$p'^- = \frac{p'^2_\perp + m_\pi^2}{p^+}, \quad \textbf{On-shell}$$

- $\vec{p}'_\perp = \vec{q}_\perp/2$, **final state on-mass shell pion**
- **Final state on-shell pion momentum:** $p'^+ = p'^-$
- **Defines the kinematics, for** $t < 0$ **and** $t = m_\pi^2$

Bethe-Salpeter amplitude

- **Symmetric (SYM) vertex to smear the $q\bar{q}$ bound-state vertex**

$$\Gamma_\pi^{(\mathrm{SYM})}(k,p) = N\left(\frac{1}{k^2 - m_R^2 + i\epsilon} + \frac{1}{(p-k)^2 - m_R^2 + i\epsilon}\right).$$

$\Longrightarrow$ **Pauli-Villars regularization mass m_R plays the role of a momentum cutoff**

$\Longrightarrow$ **Fixed by fitting the pion decay constant (Exp.) see PDG)**

- **Model for the BS amplitude**

$$\Psi_i(k,p) = \frac{m}{f_\pi} S(k)\gamma^5 \Gamma(k,p)\tau_i S(k-p).$$

- **de Melo, Naus, and Frederico, Phys. Rev. C 59, 2278 (1999)**
- **de Melo, Frederico, Pace, and Salmé,Nucl. Phys. A707, 399 (2002); ibid., Braz. J. Phys. 33. 301 (2003)**

Wave function

- **Wave function // LF projection SYM vertex**

$$\Psi(x, k_\perp, t = p^+ p^- - p_\perp^2)$$
$$= \frac{\mathcal{N}}{t - M_0^2\left(m^2, m^2\right)} \left[\frac{1}{x\left(t - M_0^2\left(m_R^2, m^2\right)\right)} + \frac{1}{(1-x)\left(t - M_0^2\left(m^2, m_R^2\right)\right)} \right]$$

$$M_0^2\left(m_a^2, m_b^2\right) = \frac{k_\perp^2 + m_a^2}{x} + \frac{(p-k)_\perp^2 + m_b^2}{1-x}$$

- $\begin{cases} m_a \text{ and } m_b \text{ // } \textbf{quark masses} \\ m_R \text{ //} \textbf{regulator mass} \\ x = \frac{k^+}{p^+} \end{cases}$

Wave function Normalização

$\Longrightarrow$ **Normalization constant** $\mathcal{N} = N\sqrt{N_c}\frac{m}{f_\pi}$

- **Number of colors** $N_c = 3$**,** f_π **Weak pion decay constant**
- **Wave Function:** $\begin{cases} \textbf{On-shell for } t = p^+p^- - p_\perp^2 = m_\pi^2 \\ \textbf{Off-shell otherwise} \end{cases}$

$$\int_0^1 dx \int \frac{d^2k_\perp}{16\pi^3} \left|\Psi\left(x, k_\perp\right)\right|^2 = 1.$$

- **TMD - Transverse Momentun Distribution**

$$f\left(x, k_{\perp}\right) = \frac{1}{16\pi^3}\left|\Psi\left(x, k_{\perp}\right)\right|^2$$

- **PDF - Parton Distribution Function**

$$f(x) = \int d^2 k_{\perp} f\left(x, k_{\perp}\right)$$

- **Sum Rule**

$$\int dx \int d^2 k_{\perp} f\left(x, k_{\perp}\right) = \int dx f(x) = 1$$

Pion Electromagentic Form Factors

- **Off-mass shell EM form factor** $F_1\left(Q^2, t\right)$

$\Longrightarrow$ **With the final state on-shell, and** $\Gamma^+\left(p', p\right)$ **for the EM-current**

$\Longrightarrow$ **Also** $q^+ = 0$

$$F_1\left(Q^2, t\right) = \frac{\Gamma^+\left(p', p\right)}{2ep^+}.$$

- **Electromagnetic form-factor** $F_2\left(Q^2, t\right)$

$$F_2\left(Q^2, t\right) = \frac{t - m_\pi^2}{Q^2}\left[F_1(0, t) - F_1\left(Q^2, t\right)\right],$$

- **As a reminder, the FFactor** $g(Q^2, t)$ **is obtained by combining** F_1 **and and** F_2
- **VIP: Use of** γ^+ **eliminates the instantaneous terms**

- **Kinematics (on-shell):**

$$p^\mu = (p_0, \frac{-q\cos\alpha}{2}, 0, \frac{-q\sin\alpha}{2})$$
$$p'^\mu = (p_0, \frac{q\cos\alpha}{2}, 0, \frac{q\sin\alpha}{2}),$$

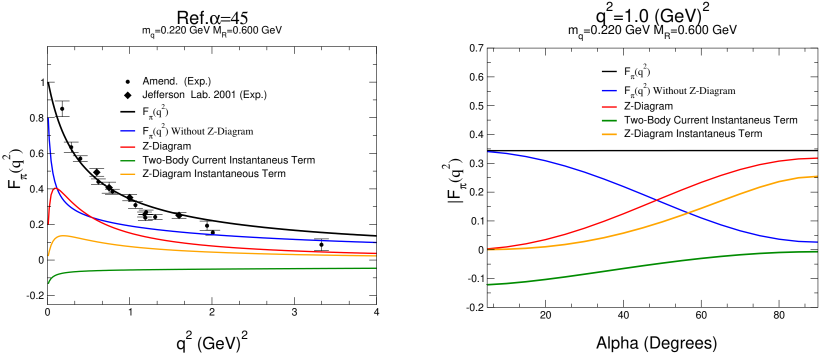
where $p^0 = \sqrt{m_\pi^2 + \frac{q^2}{4}}$**.**

- **Transfer momemtum:** $q^\mu = p'^\mu - p^\mu = (0, q\cos\alpha, 0, 0, q\sin\alpha)$.
- **Light-front coordinates:** $(t+z, t-z, x, y) = (+, -, \perp)$**,**

$$p^\mu = (p_0 - \frac{q\sin\alpha}{2}, p_0 + \frac{q\sin\alpha}{2}, -\frac{q\cos\alpha}{2})$$
$$p'^\mu = (p_0 + \frac{q\sin\alpha}{2}, p_0 - \frac{q\sin\alpha}{2}, \frac{q\cos\alpha}{2})$$

Momentum transfer: $q^\mu = (q\sin\alpha, -q\sin\alpha, q\cos\alpha)$.

Results



Pion electromagnetic form factor (on-shell) with $J^\mu = J^+$

- **Ref. de Melo, Frederico, Pace, Salme, Nucl.Phys. A 707 (2002) 399.**
- **Yabusaki, Ahmed, Paracha, de Melo and El-Bennich, Phys.Rev.D 92 (2015) 3, 034017.**

Model	$\sqrt{\langle r_\pi^2 \rangle}[\mathrm{fm}]$	$f_\pi[\mathrm{MeV}]$	$g\left(0, m_\pi^2\right)\left[\mathrm{GeV}^2\right]$
$\Gamma_\pi^{(\mathrm{SYM})}$	**0.736**	**92.40**	**2.32**
$\Gamma_\pi^{(\mathrm{CON})}$	0.713 ± 0.013	...	2.18 ± 0.08
Exp. [PDG]	0.672(8)	92.28(7)	1.93(5)

- **Quark masses:** $m_u = m_{\bar{d}} = \ 0.220\ \ GeV$
- **Regulator mass:** $m_R = \ 0.600\ \ GeV$

$\Longrightarrow$ **(fit f_π Exp. value)!!**

- f_π, and charge radius r_π, for $m_q = 0.22.\mathrm{GeV}$

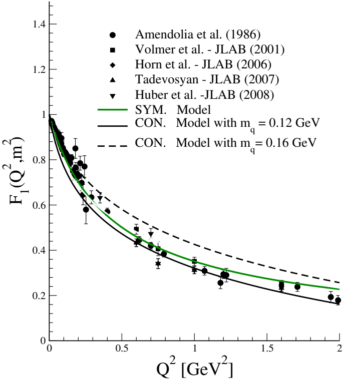
Different values of the regulator mass m_R

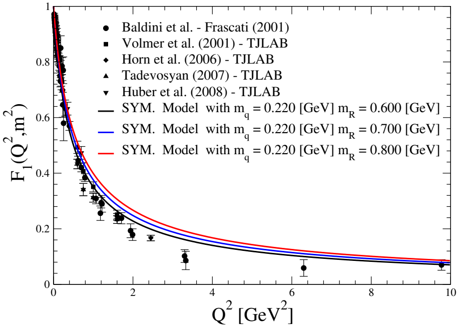
The fractional percent deviations are

$$\Delta f_\pi = \left| \frac{f_\pi^{f_T^{\mathrm{SM}}} - f_\pi^{\mathrm{Exp}}}{f_\pi^{\mathrm{SYM}}} \right| \times 100 \text{ and } \Delta r_\pi = \left| \frac{r_\pi^{S_T{}^{\mathrm{YM}}} - r_\pi^{\mathrm{Exp}}}{r_\pi^{\mathrm{STM}}} \right| \times 100$$

m_R[GeV]	f_π[MeV]	Δf_π(%)	r_π[fm]	Δr_π(%)
0.6	92.4	0.1	0.736	8.7
0.7	97.0	4.9	0.695	3.4
0.8	100.9	8.5	0.675	0.4

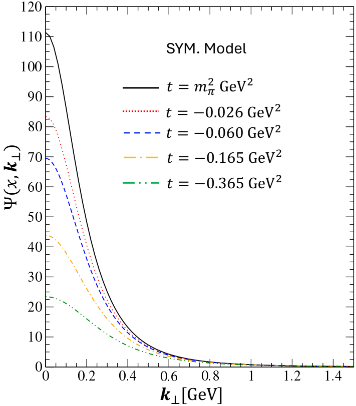
Q^2 (GeV^2)	W (GeV)	$-t$ (GeV^2)	$F_1(Q^2, t)$ Exp.	CON	SYM	$F_1(0, t)$ CON	SYM
0.526	1.983	0.026	0.502±0.013	0.487	0.4471	0.891	0.8926
0.576	1.956	0.038	0.440±0.010	0.462	0.4200	0.869	0.8685
0.612	1.942	0.050	0.413±0.011	0.443	0.3995	0.849	0.8458
0.631	1.934	0.062	0.371±0.014	0.430	0.3860	0.831	0.8244
0.646	1.929	0.074	0.340±0.022	0.419	0.3744	0.814	0.8041
0.660	1.992	0.037	0.397±0.019	0.435	0.3937	0.870	0.8705
0.707	1.961	0.051	0.360±0.017	0.414	0.3717	0.848	0.8440
0.753	1.943	0.065	0.358±0.015	0.394	0.3526	0.827	0.8192
0.781	1.930	0.079	0.324±0.018	0.381	0.3382	0.807	0.7960
0.794	1.926	0.093	0.325±0.022	0.371	0.3289	0.789	0.7742
0.877	1.999	0.060	0.342±0.014	0.366	0.3283	0.834	0.8279
0.945	1.970	0.080	0.327±0.012	0.343	0.3058	0.806	0.7944
1.010	1.943	0.100	0.311±0.012	0.322	0.2868	0.781	0.7638
1.050	1.926	0.120	0.282±0.016	0.307	0.2731	0.758	0.7357
1.067	1.921	0.140	0.233±0.028	0.297	0.2637	0.737	0.7097
1.455	2.001	0.135	0.258±0.010	0.237	0.2227	0.742	0.7160
1.532	1.975	0.165	0.245±0.010	0.219	0.2078	0.714	0.6799
1.610	1.944	0.195	0.222±0.012	0.201	0.1955	0.688	0.6475
1.664	1.924	0.225	0.203±0.013	0.188	0.1860	0.665	0.6182
1.702	1.911	0.255	0.227±0.016	0.177	0.1783	0.644	0.5896
1.416	2.274	0.079	0.270±0.010	0.259	0.2430	0.807	0.7945
1.513	2.242	0.112	0.258±0.010	0.235	0.2238	0.767	0.7450
1.593	2.213	0.139	0.251±0.010	0.217	0.2097	0.738	0.7092
1.667	2.187	0.166	0.241±0.012	0.201	0.1976	0.713	0.6769
1.763	2.153	0.215	0.200±0.018	0.179	0.1816	0.672	0.6257

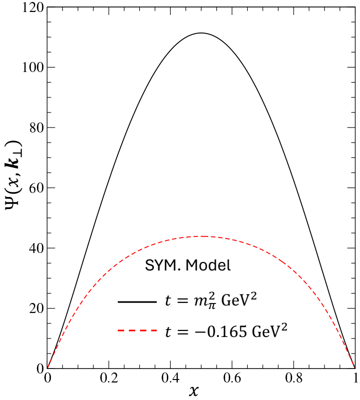




Pion electromagnetic form factor (on-shell)

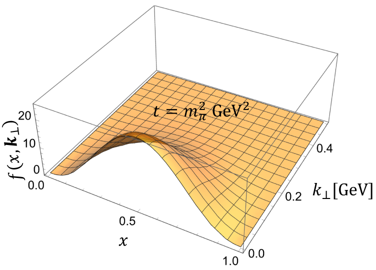
- Ref. Leão, de Melo, Frederico, Choi and Ji, Phys.Rev.D 110 (2024) 7
- Ref. Ho-Meoyng Choi, T. Frederico, Chueng-Ryong Ji and J. de Melo. Phys.Rev.D 100 (2019) 11, 116020.



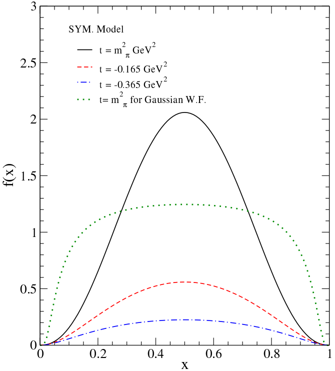


Pion wave function $\Psi(x, k_\perp)$ from the SYM model

- **Left panel: Fixed values $x = 0.5$ for various values of t times $k_\perp$.**
- **Right panel: Fixed values of $k_\perp = 0$ for two value of t times x.**



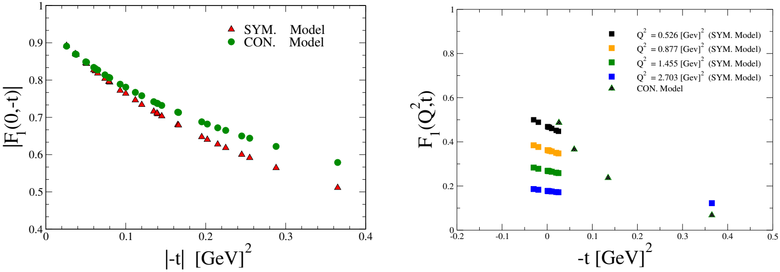
Twist-2 pion TMD $f(x, k_\perp)$ for SYM model; fixed value of $Q^2 = 0$, with two values of $t = m_\pi^2$, and $-0.165\mathrm{GeV}^2$



Twist-2 PDF $f(x)$ with SYM model, at fixed $Q^2 = 0$ and some values of t.

Light-front quark model on-shell pion, from the the Gaussian wave function**

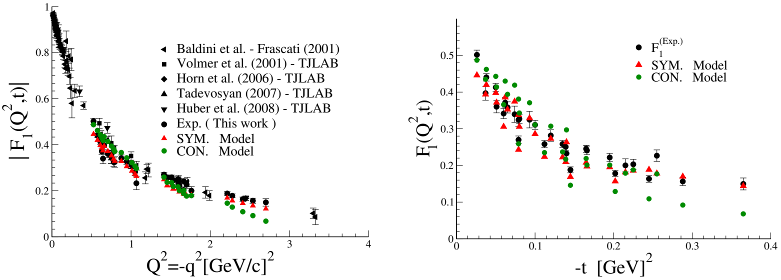
$\star\star$ **H.-M. Choi and C.-R. Ji, Phys. Rev. D 110, 014006 (2024)**



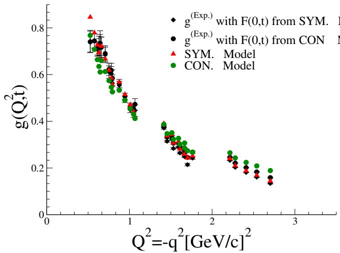
Pion off-shell EM form factor $F_1(Q^2, t)$ **from the CON and SYM models versus** $-t$

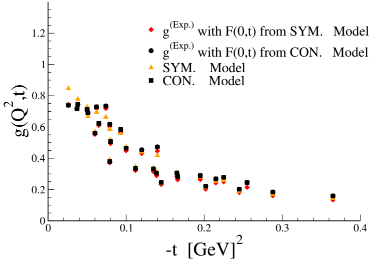
Right panel for $F_1(0, t)$

Left panel: $F_1(Q^2, t)$ **for** $Q^2 = 0.526\,, 0.877\,, 1.455$ **and** $\mathbf{2.703}\,[\mathbf{GeV}]^2$



Pion off-shell EM form factor $F_1\left(Q^2,t\right)$
Left: $F_1\left(Q^2,t\right)$ extracted from the experimental cross sections, CON, and SYM models
Right: $F_1\left(Q^2,t\right)\times -t$ for the same models.
The values of $\left(Q^2,t\right)$ are from the table, along with the experimental and theoretical results.

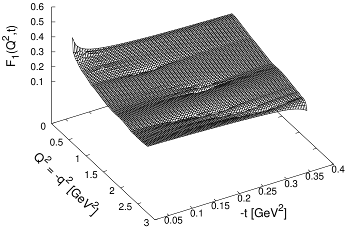


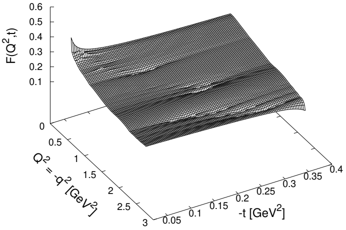


Pion off-shell EM form factor $g\left(Q^2,t\right)$.
Right: $g\left(Q^2,t\right) \times Q^2$, compared with the extracted result from the experimental data* with $F_1(0,t)$ from CON and SYM models

Left: $g\left(Q^2,t\right) \times -t$ with same models

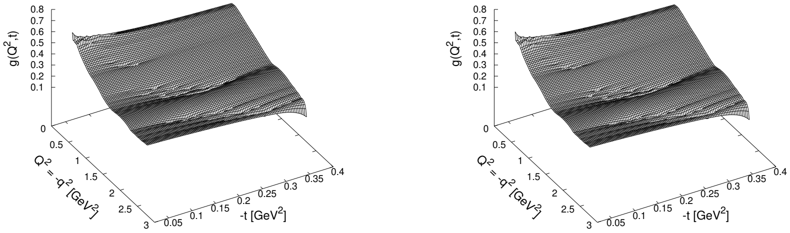
$\star$ H. P. Blok et al. (Jefferson Lab F_π Collaboration) Phys. Rev. C 78, 045202 (2008)





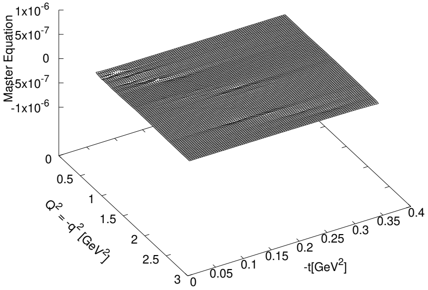
The 3D plots of the form factor $F_1(Q^2, t)$ extracted from the experimental data*. Inputs for "$F_1(0, t)$" from the CON model (left), and SYM model (right panel)

$\star$ **H. P. Blok et al. (Jefferson Lab F_π Collaboration) Phys. Rev. C 78, 045202 (2008).**



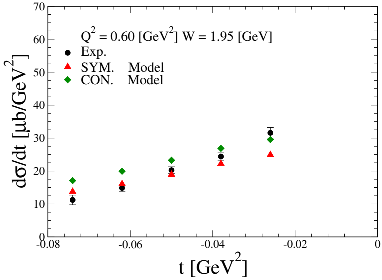
The 3D plots of the form factor $g(Q^2,t)$ extracted from the experimental data*. Inputs for "$F_1(0,t)$" from the CON model (left), and SYM model (right panel)

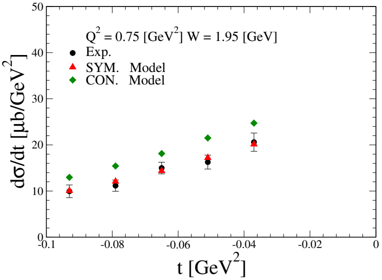
$\star$ H. P. Blok et al. (Jefferson Lab F_π Collaboration) Phys. Rev. C 78, 045202 (2008).



- **Off-shell form factor sum rule****

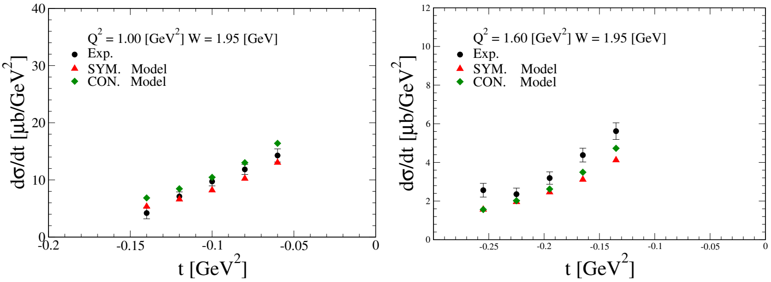
$$F_1\left(Q^2,t\right)-F_1(0,t)+Q^2 g\left(Q^2,t\right)=0$$



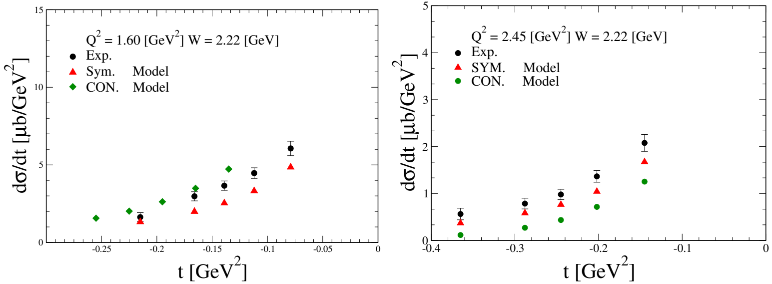


Cross-sections calculated with CON and SYM models, compared with the experimental data, for fixed Q^2 and W, and varying t

$\star$ **H. P. Blok et al. (Jefferson Lab F_π Collaboration) Phys. Rev. C 78, 045202 (2008).**



⋆ **H. P. Blok et al. (Jefferson Lab F_π Collaboration) Phys. Rev. C 78, 045202 (2008).**



⋆ **H. P. Blok et al. (Jefferson Lab F_π Collaboration) Phys. Rev. C 78, 045202 (2008).**

Summary

- Light-front approach correctly describes hadronic bound states
- Take New Informations about Bound States
- Breaking of the rotational invariance has to be evaluated
- Choose $G_{\pi NN}(t)$ with two differents parametrizations
- Use WTI to related $F_1(Q^2,t)$ and $F_2(Q^2,t)$
- Use $F_1(Q^2,t)$ and $F_2(Q^2,t)$ relationship to extract the EMFF $g(Q^2,t)$
- The Light-front pion wave function off-shell is very sensitive for

the t and x

- Show $F_1(Q^2,t) \times |-t|$ from experiments
- Show $g(Q^2,t) \neq 0$ no matter what $G_{\pi NN}(t)$ is used

• The electromagnetic form factors from the models, both on-shell and off-shell F_1 are compared with the experimental data

• Also for $g(Q,t)$ electromagnetic form factor (on and off -shell)

• The cross-section from the models are compared with the experimental data, giving good results

Next:

• Kaon Electromagetic Form Factors / on-shell and off-shell

• Parton Distribution Function off-shell regime (on-shell)

• Gravitational Form factor (Pion, Kaon)

• Frame off-shell dependence in the Light-Front

Thanks to the Organizers

Physics Opportunities at an Electron-Ion Collider

POETIC XI - 2025

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Support LFTC and Brazilian Agencies
FAPESP , CNPq and CAPES
Thanks (Obrigado)!!