Helicity and Unpolarized TMDs of Nucleon from Lattice QCD

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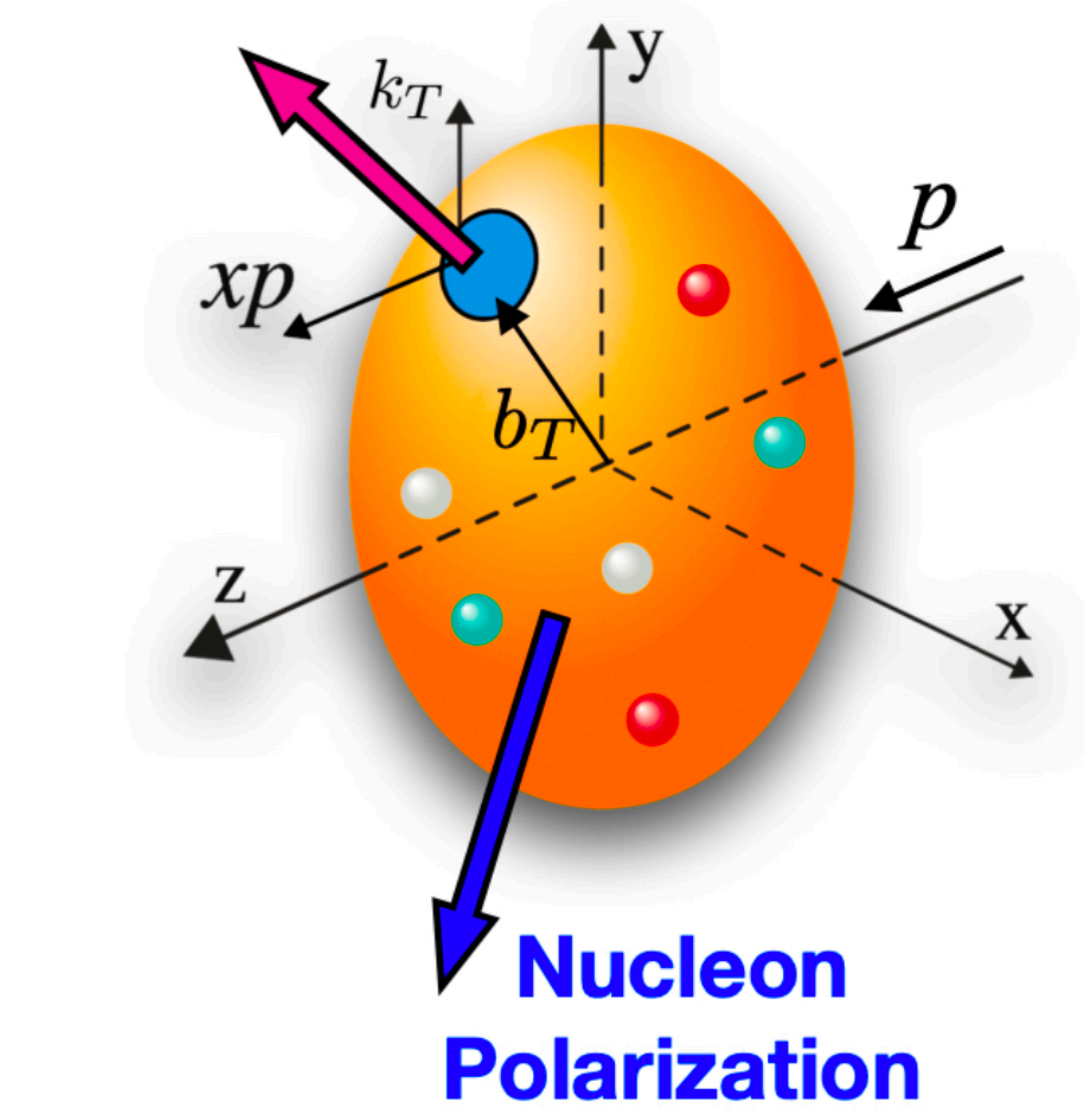
POETIC XI, Florida International University, Feb. 24th, 2025

Transverse-momentum-dependent distributions



$f(x, \vec{k}_T)$

- Leading-power ("twist-2") quark TMDs

Nucleon Polarization	Quark Polarization: Un-Polarized (U)	Quark Polarization: Longitudinally Polarized (L)	Quark Polarization: Transversely Polarized (T)
U	f_1 = Unpolarized		$h_1^\perp$ = Boer-Mulders
L		g_1 = Helicity	$h_{1L}^\perp$ = Worm-gear
T	$f_{1T}^\perp$ = Sivers	$g_{1T}^\perp$ = Worm-gear	h_1 = Transversity; $h_{1T}^\perp$ = Pretzelosity

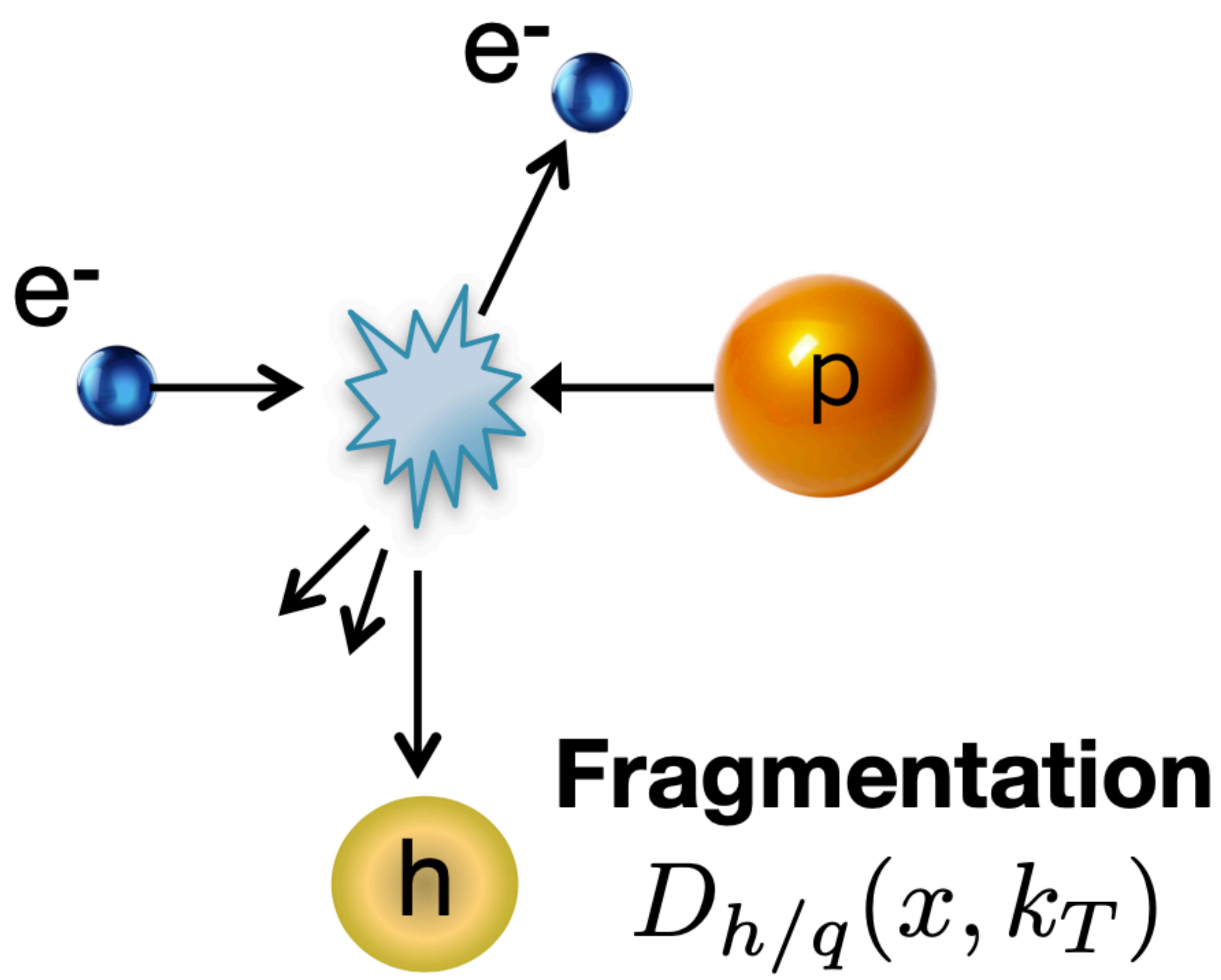
• TMD handbook, arXiv:2304.03302

- 3D image: longitudinal momentum fraction x and confined motion $\vec{k}_T$.
- Nucleon spin structure: orbital motion, Spin-orbit correlations…

TMDs from global analyses of experimental data

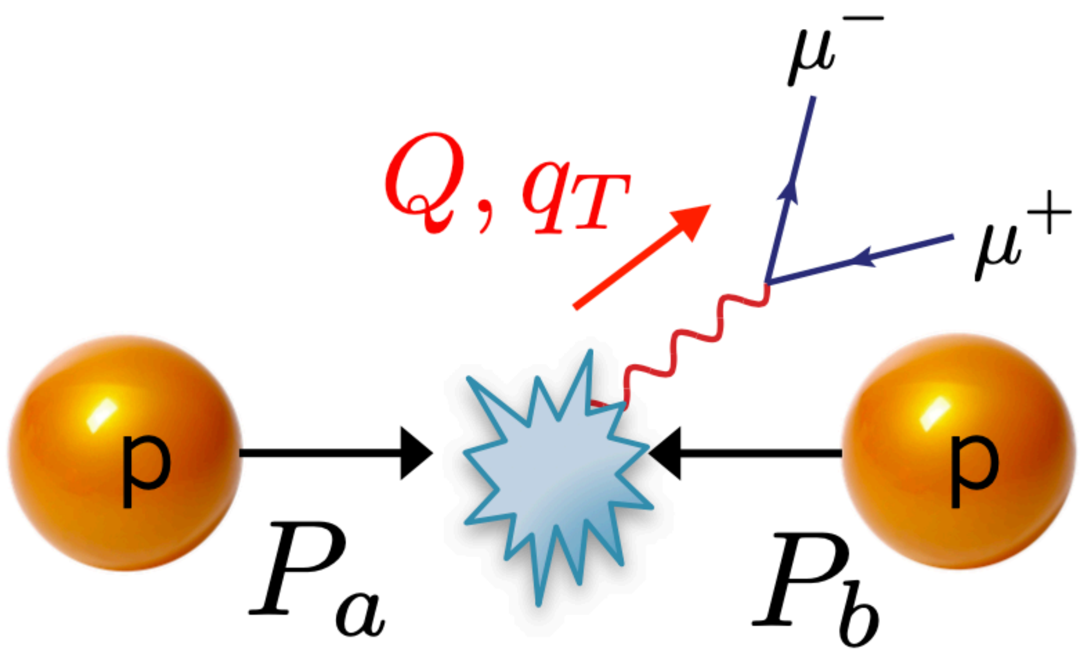
Semi-Inclusive DIS

$$\sigma \sim f_{q/P}(x,k_T)D_{h/q}(x,k_T)$$



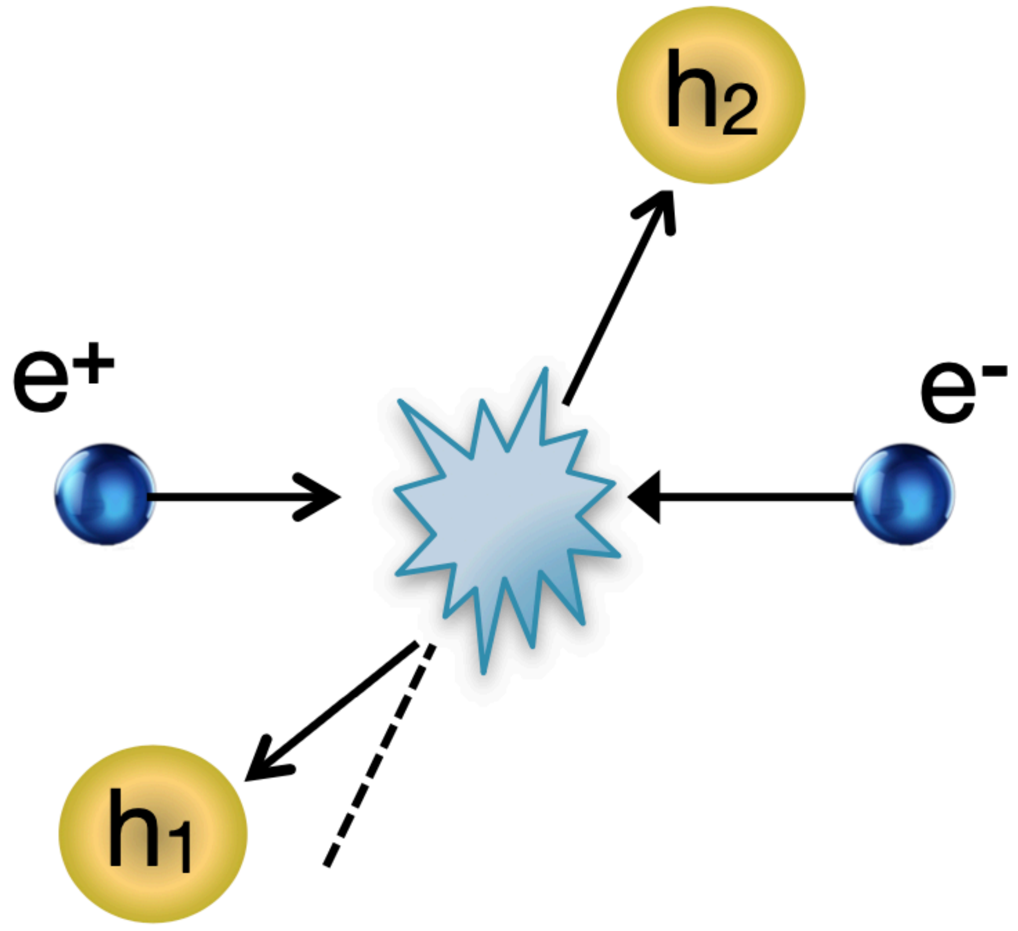
Drell-Yan

$$\sigma \sim f_{q/P}(x,k_T)f_{q/P}(x,k_T)$$



Dihadron in e+e-

$$\sigma \sim D_{h_1/q}(x,k_T)D_{h_2/q}(x,k_T)$$



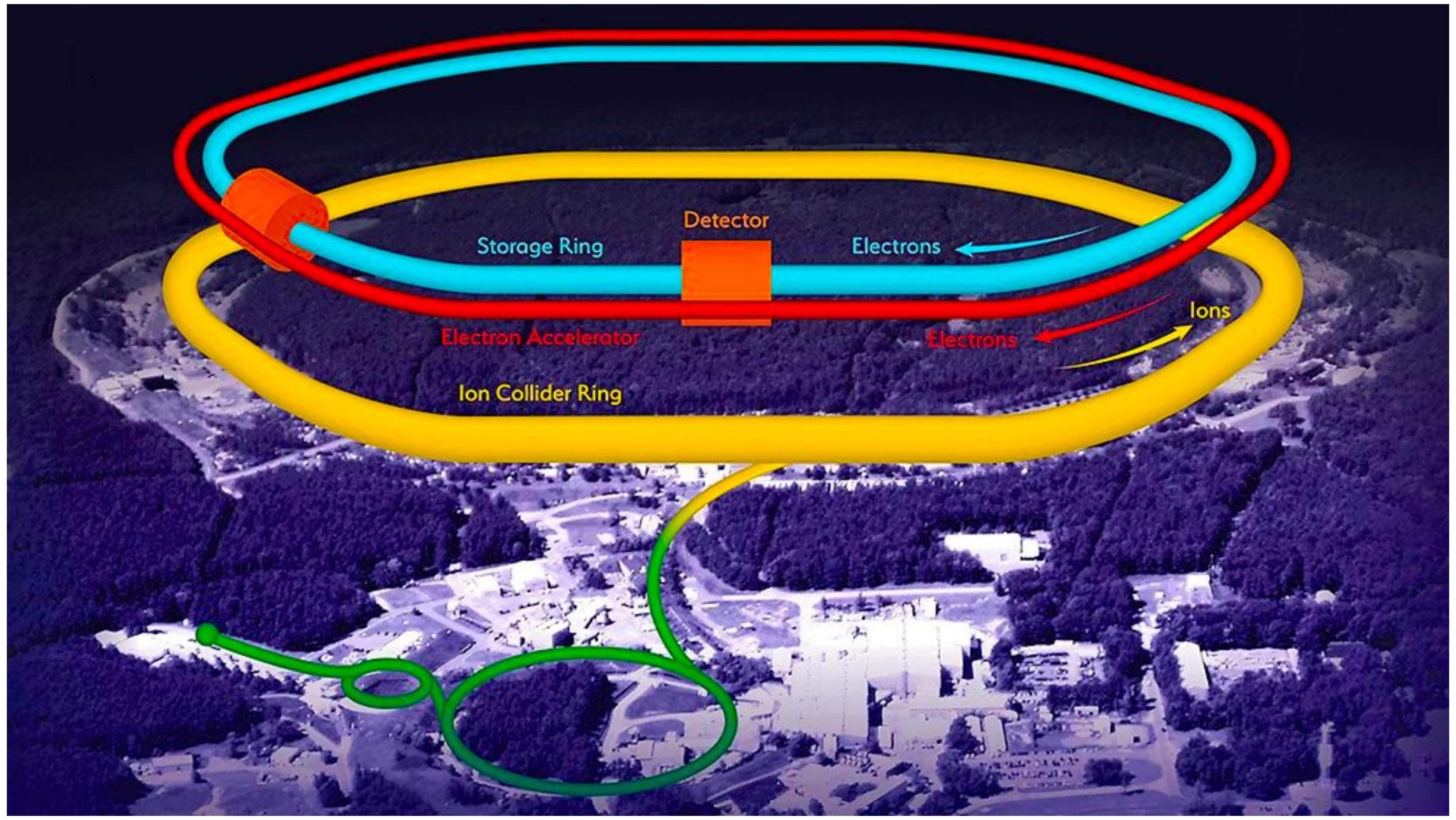
$$\frac{d\sigma_{\mathrm{DY}}}{dQdYdq_T^2} = H(Q,\mu)\int d^2\vec{b}_T e^{i\vec{q}_T\cdot\vec{b}_T} f_q(x_a,\vec{b}_T,\mu,\zeta_a)\, f_q(x_b,\vec{b}_T,\mu,\zeta_b)[1+\mathcal{O}(\frac{q_T^2}{Q^2})]$$

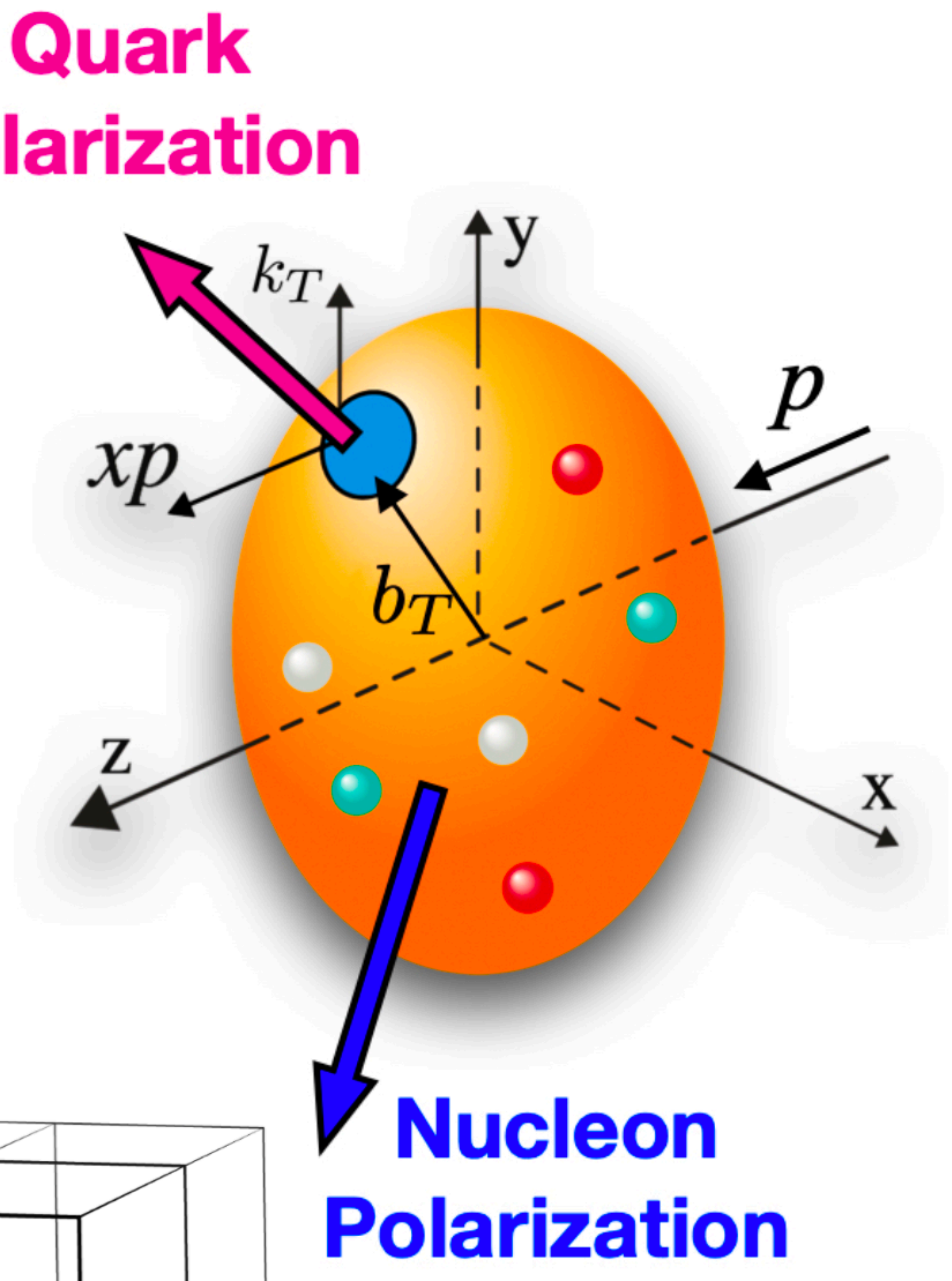
Perturbative hard kernels

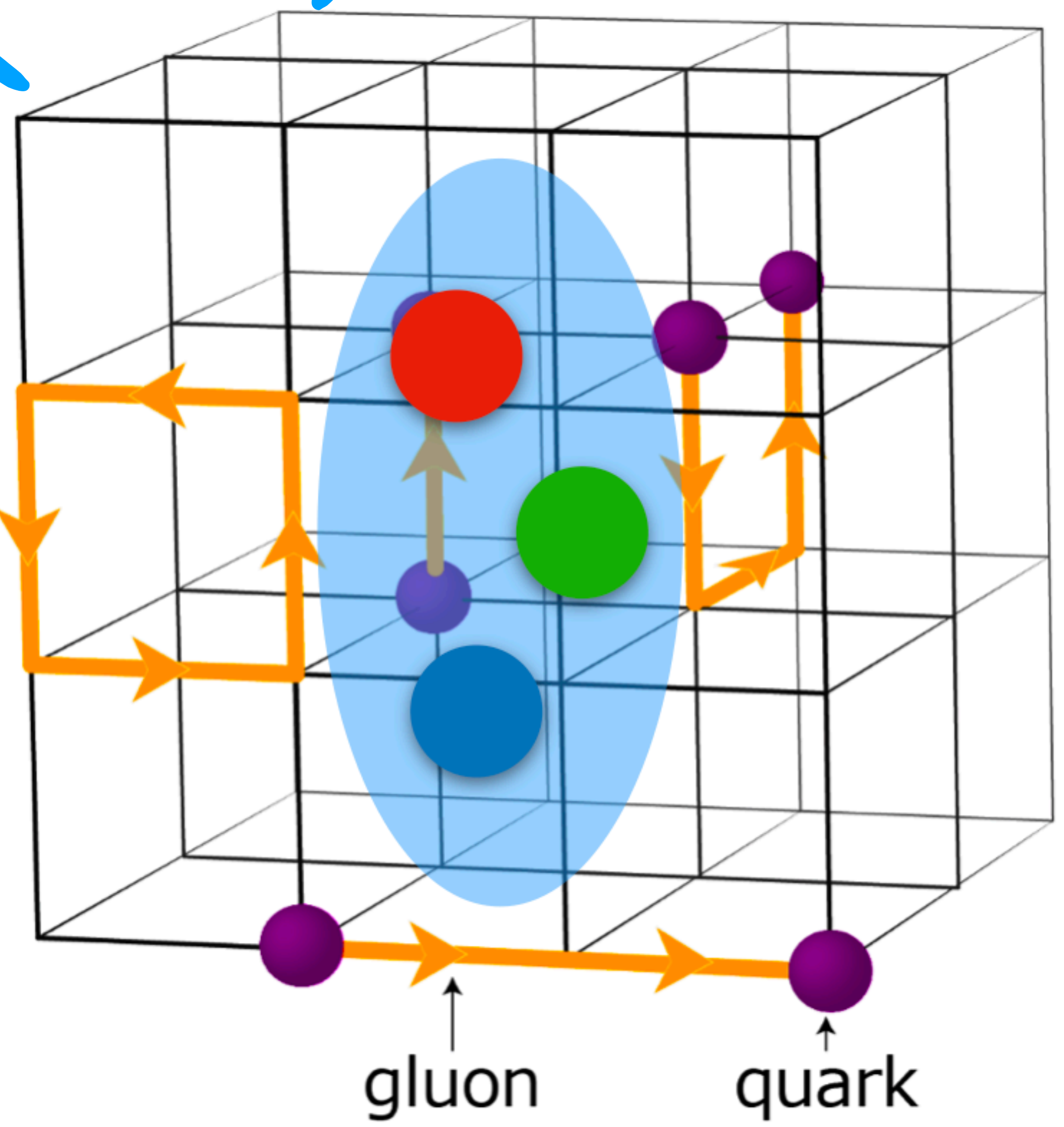
Nonperturbative TMDs

$$q_T^2 \ll Q^2$$

Synergy between Lattice QCD and Global Analyses







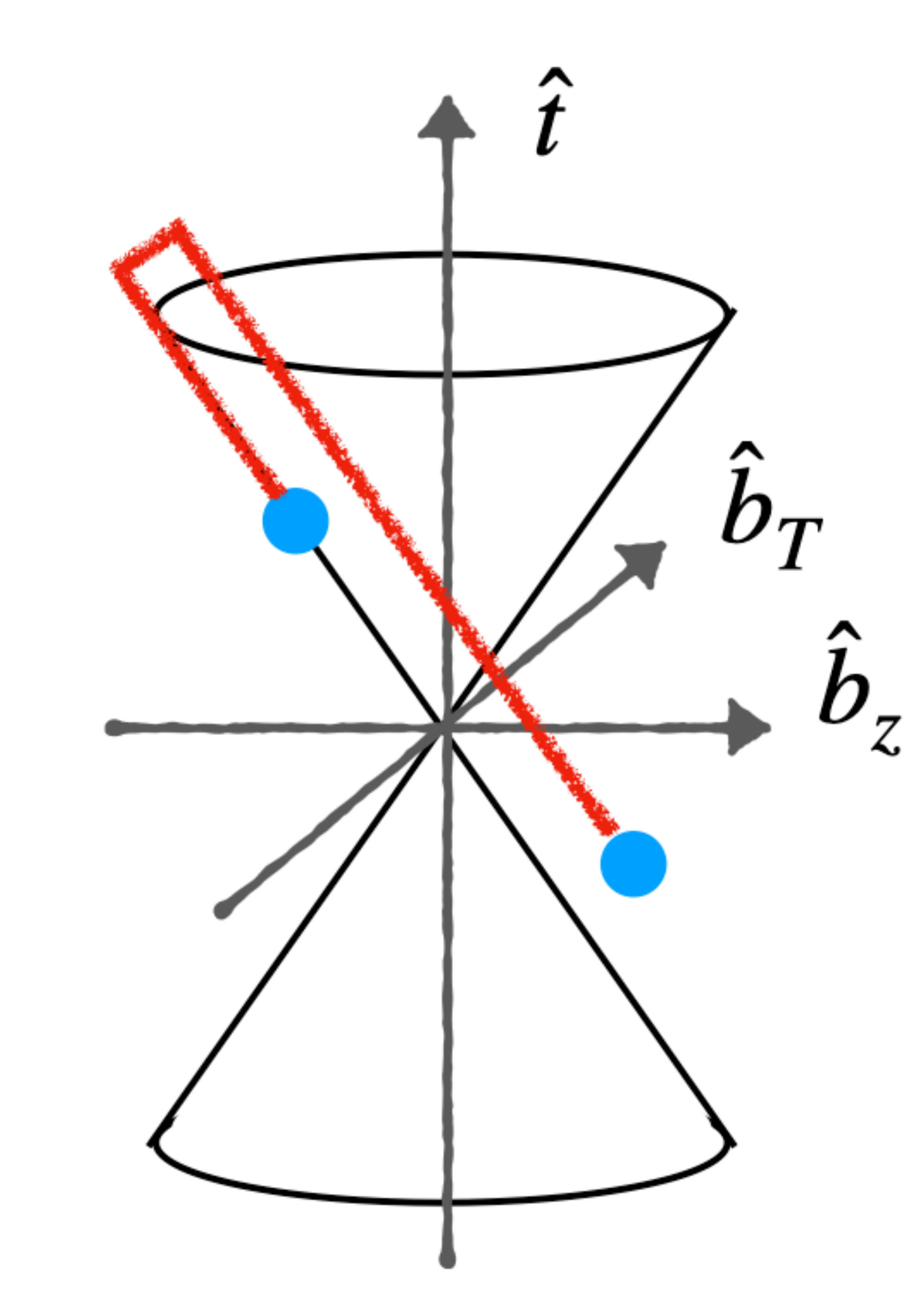
- Global analysis of experimental data.
- Complementary knowledge from lattice QCD is essential.

The definition of TMDs

Beam function

$$f_q(x,\vec{b}_T,\mu,\zeta) = \lim_{\epsilon\to0} Z_{\rm UV}(\epsilon,\mu,\zeta) \lim_{\tau\to0} \frac{B_q(x,\vec{b}_T,\epsilon,\tau,\zeta)}{\sqrt{S_q(\vec{b}_T,\epsilon,\tau)}}$$

UV regulator **Rapidity regulator** **Soft function**



$$B_q(x,\vec{b}_T,\epsilon,\tau,\zeta) = \langle P,\lambda\,|\,\overline{\psi}(\frac{b^+}{2},b_\perp)\Gamma W_{\sqsupset^+}\psi(-\frac{b^+}{2},0)\,|\,P,\lambda\rangle$$

- λ: nucleon polarization
- Γ: quark polarization
- $W_{\sqsupset^+}$: light-cone staple-shaped Wilson link

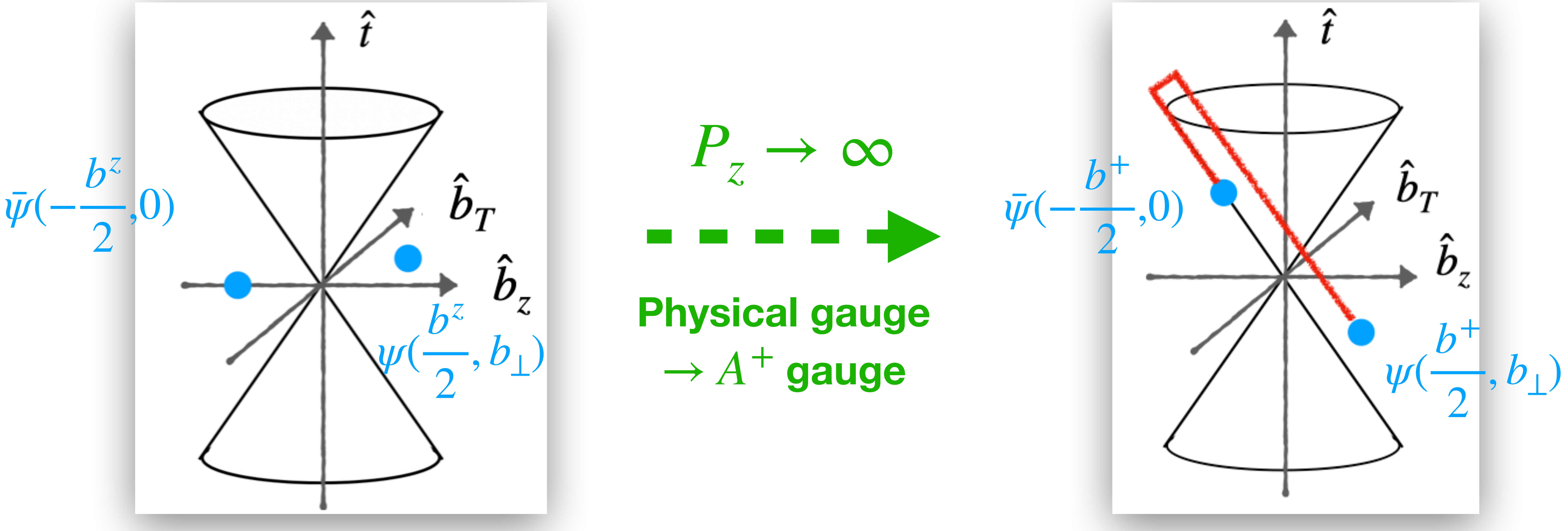
✘ **Light-cone correlations: forbidden on Euclidean lattice**

TMDs from lattice: quasi TMDs

- **XG**, W.-Y. Liu, Y. Zhao, PRD 109 (2024) 9, 094506
- Y. Zhao, PRL 133 (2024) 24, 241904

- Equal-time correlators in physical gauges: $\vec{\nabla}\cdot\vec{A}=0,\ A^z=0,\ A^t=0,\ \ldots$
- Approaching light-cone TMDs in the large momentum limit.
- Computable from lattice QCD.

- Ji, Liu and Liu, NPB 955 (2020), PLB 811 (2020);
- A. Vladimirov, A. Schäfer Phys.Rev.D 101 (2020), 074517
- I. Stewart, Y. Zhao et al., JHEP 09 (2020) 099
- X. Ji et al., Phys.Rev.D 103 (2021) 7, 074005
- I. Stewart, Y. Zhao et al., JHEP 08 (2022) 084



Quasi TMD

$$\langle P|\bar{\psi}(\frac{b_z}{2},b_\perp)\Gamma\psi(-\frac{b_z}{2},0)\big|_{\vec{\nabla}\cdot\vec{A}=0}|P\rangle$$
$$\big|_{A^z=0}$$
$$\big|_{A^t=0}\ \cdots$$

Light-cone TMD

$$\langle P|\bar{\psi}(\frac{b^+}{2},b_\perp)\Gamma W_{\sqsupset^+}\psi(-\frac{b^+}{2},0)|P\rangle$$
$$=\langle P|\bar{\psi}(\frac{b^+}{2},b_\perp)\Gamma\psi(-\frac{b^+}{2},0)\big|_{A^+=0}|P\rangle$$

TMDs from lattice: quasi TMDs

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Yong Zhao' talk tomorrow

- Equal-time correlators in physical gauges: $\vec{\nabla}\cdot\vec{A}=0,\ A^z=0,\ A^t=0,\ \ldots$
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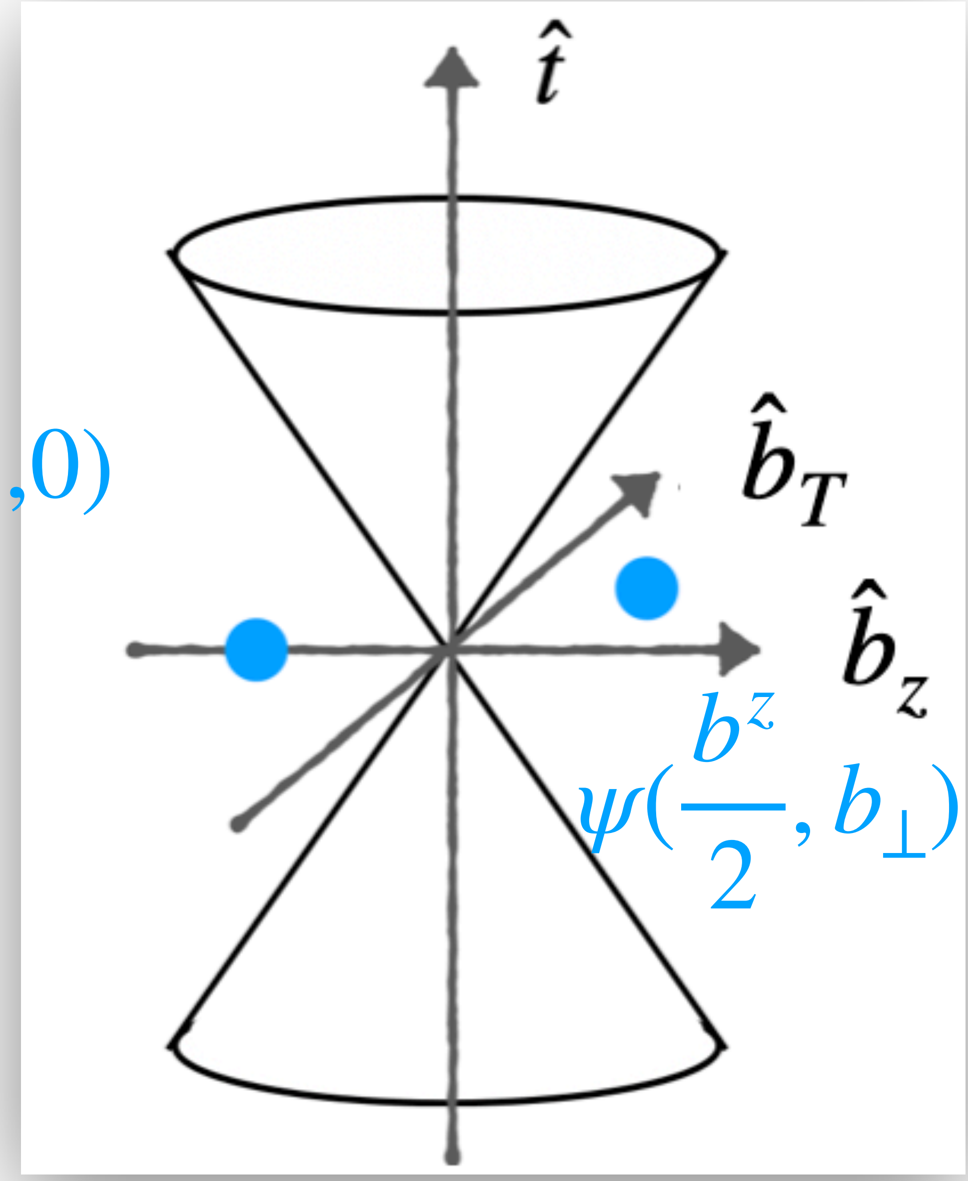
$\bar\psi(-\frac{b^z}{2},0)$

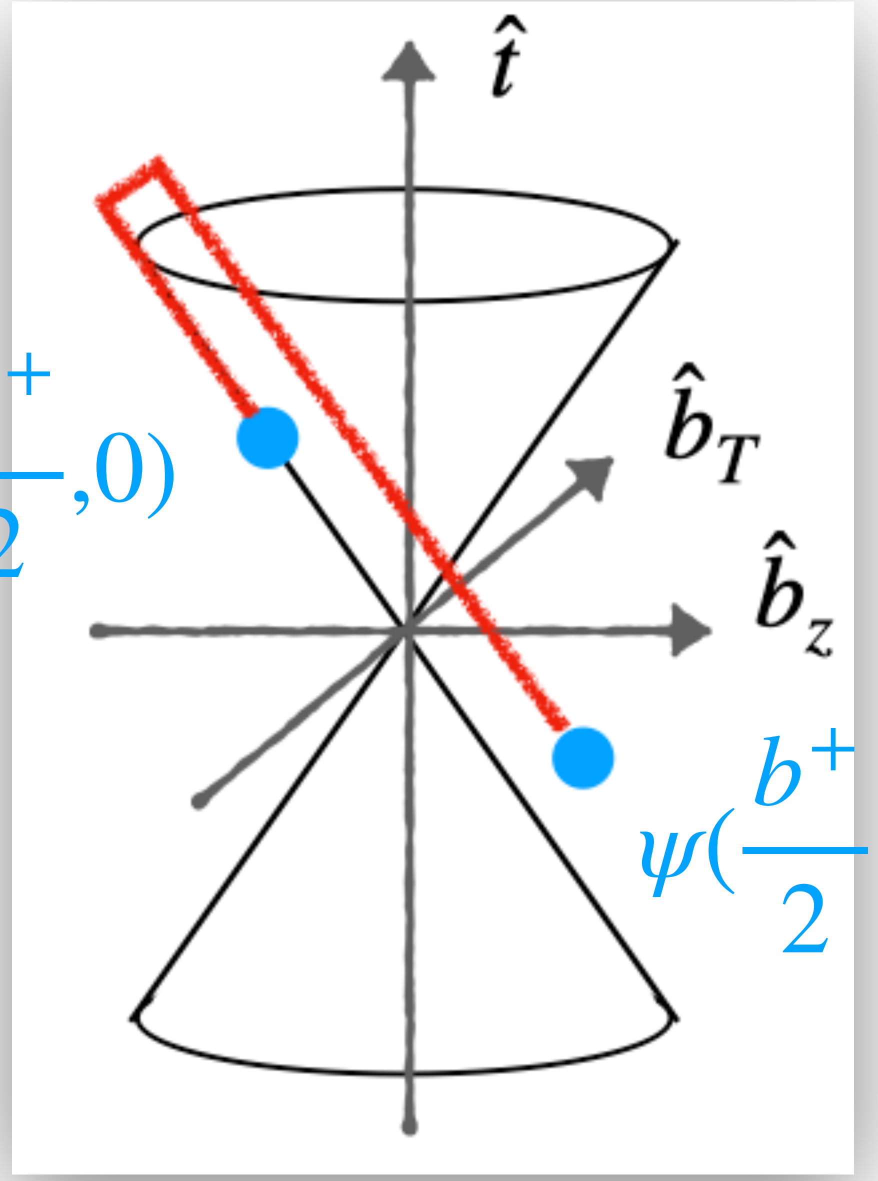


$\psi(\frac{b^z}{2},b_\perp)$

$P_z \to \infty$

Physical gauge
$\to A^+$ gauge

$\bar\psi(-\frac{b^+}{2},0)$



$\psi(\frac{b^+}{2},b_\perp)$

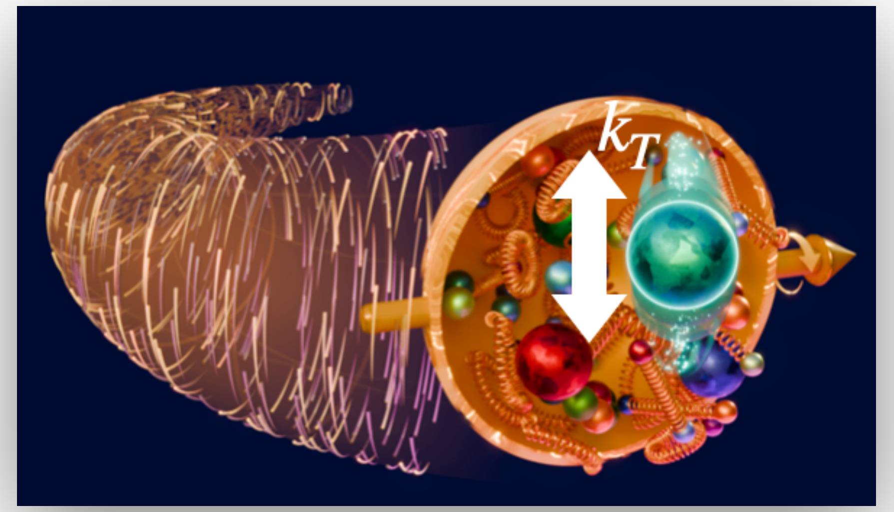
Quasi TMD

$$\langle P|\bar\psi(\frac{b_z}{2},b_\perp)\Gamma\psi(-\frac{b_z}{2},0)\,|_{\vec\nabla\cdot\vec A=0}\,|_{A^z=0}\,|_{A^t=0}\cdots|P\rangle$$

Light-cone TMD

$$\langle P|\bar\psi(\frac{b^+}{2},b_\perp)\Gamma W_{\sqsupset^+}\psi(-\frac{b^+}{2},0)|P\rangle$$
$$=\langle P|\bar\psi(\frac{b^+}{2},b_\perp)\Gamma\psi(-\frac{b^+}{2},0)\,|_{A^+=0}|P\rangle$$

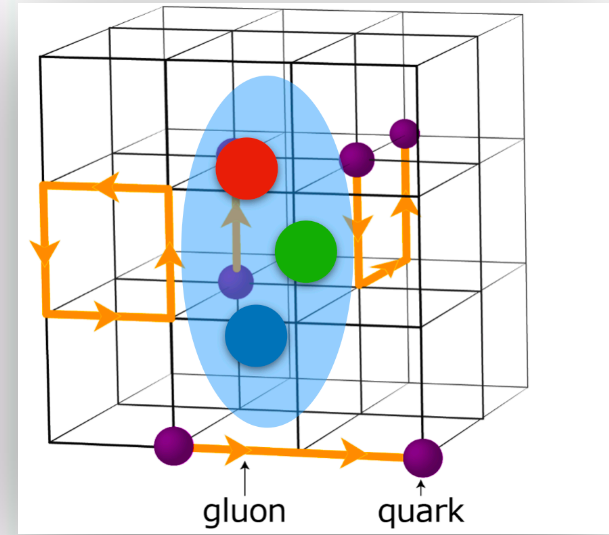
Large P_z expansion and perturbative matching



Quasi TMDs

Collins-Soper kernel

$$\frac{\tilde{f}(x, \vec{b}_T, \mu, P_z)}{\sqrt{S_r(\vec{b}_T, \mu)}} = C(\mu, xP_z) e^{\frac{1}{2}\gamma_\zeta(\mu, b_T)\ln\frac{(2xP_z)^2}{\zeta}} f(x, \vec{b}_T, \mu, \zeta)\{1 + \mathcal{O}[\frac{1}{(xP_z b_T)^2}, \frac{\Lambda^2_{\rm QCD}}{(xP_z)^2}]\}$$



Soft-factor

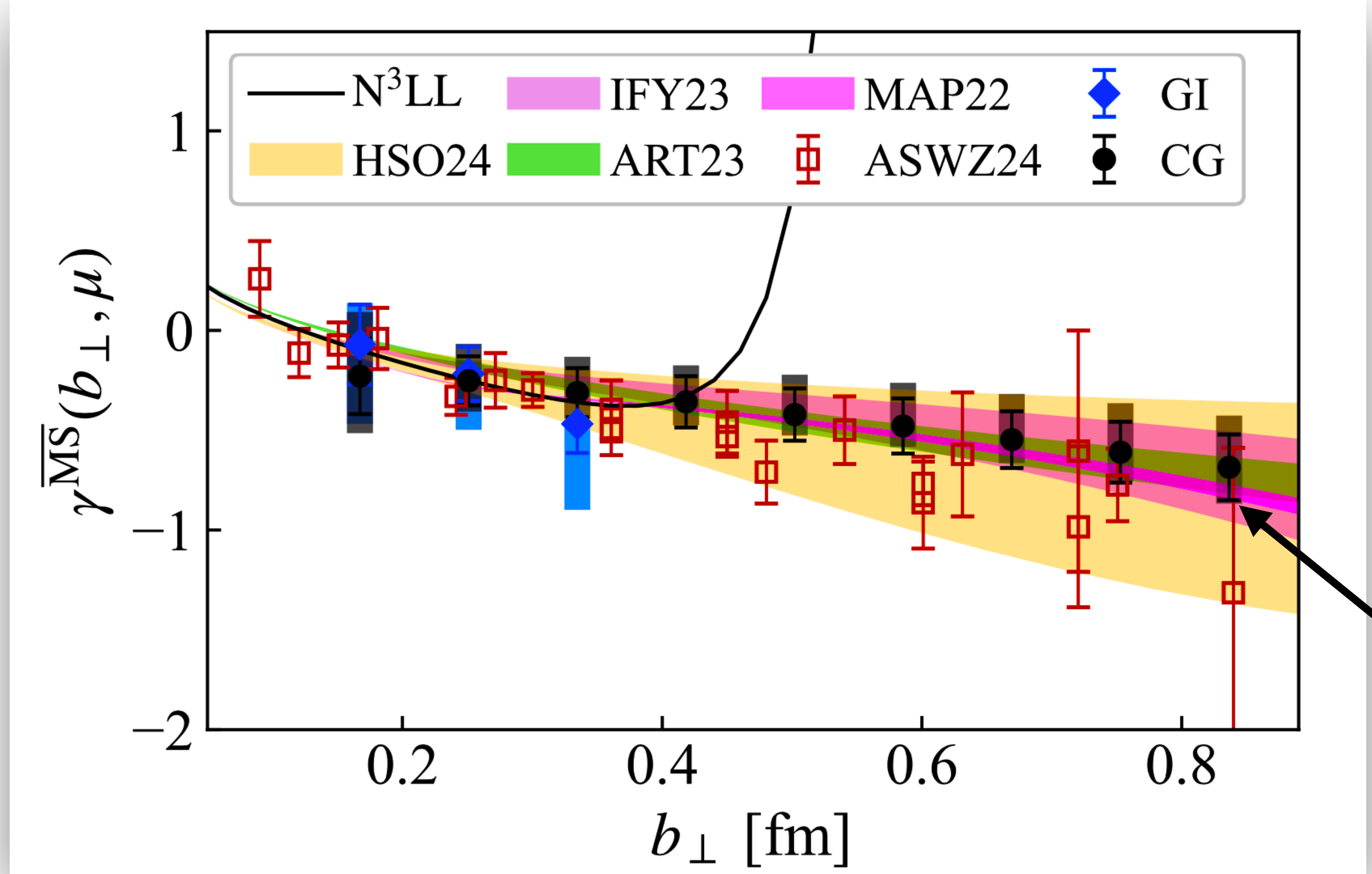
Physical TMD

$P_z < a^{-1}$

- Have same IR physics as light-cone TMDs: **large momentum expansion & power corrections**.
- Quasi TMDs differ from the light-cone TMDs (Collins scheme) by order of P_z (or rapidity $\mathrm{y_B}$) $\to \infty$ and a (or ϵ) $\to 0$ limit, inducing a **perturbative matching** $C(\mu, xP_z)$.

The Collins-Soper kernel from quasi-TMDs

- CS (rapidity evolution) kernel of TMDs can be derived from evolution of quasi-TMDs.



$$\gamma^{\overline{\rm MS}}(b_\perp,\mu) = \frac{1}{\ln(P_2/P_1)} \ln\left[\frac{\tilde f(x,b_\perp,P_2,\mu)}{\tilde f(x,b_\perp,P_1,\mu)}\right] + \delta\gamma^{\overline{\rm MS}}(x,\mu,P_1,P_2)$$

$P_1 \to P_2$

Perturbative correction

- First-principle determination with no model dependence.
- Consistent with most recent global fits.

• D. Bollweg, **XG**, S. Mukherjee, Y. Zhao, PLB 852 (2024) 138617

Domain wall fermion, physical quark masses
$64^3 \times 128$, a = 0.084 fm

Ratio of TMDs from quasi-TMDs

$$\frac{\tilde{f}(x, \vec{b}_T, \mu, P_z)}{\sqrt{S_r(\vec{b}_T, \mu)}} = C(\mu, xP_z)e^{\frac{1}{2}\gamma_\zeta(\mu, b_T)\ln\frac{(2xP_z)^2}{\zeta}} f(x, \vec{b}_T, \mu, \zeta)\{1 + \mathcal{O}[\frac{1}{(xP_z b_T)^2}, \frac{\Lambda^2_{QCD}}{(xP_z)^2}]\}$$

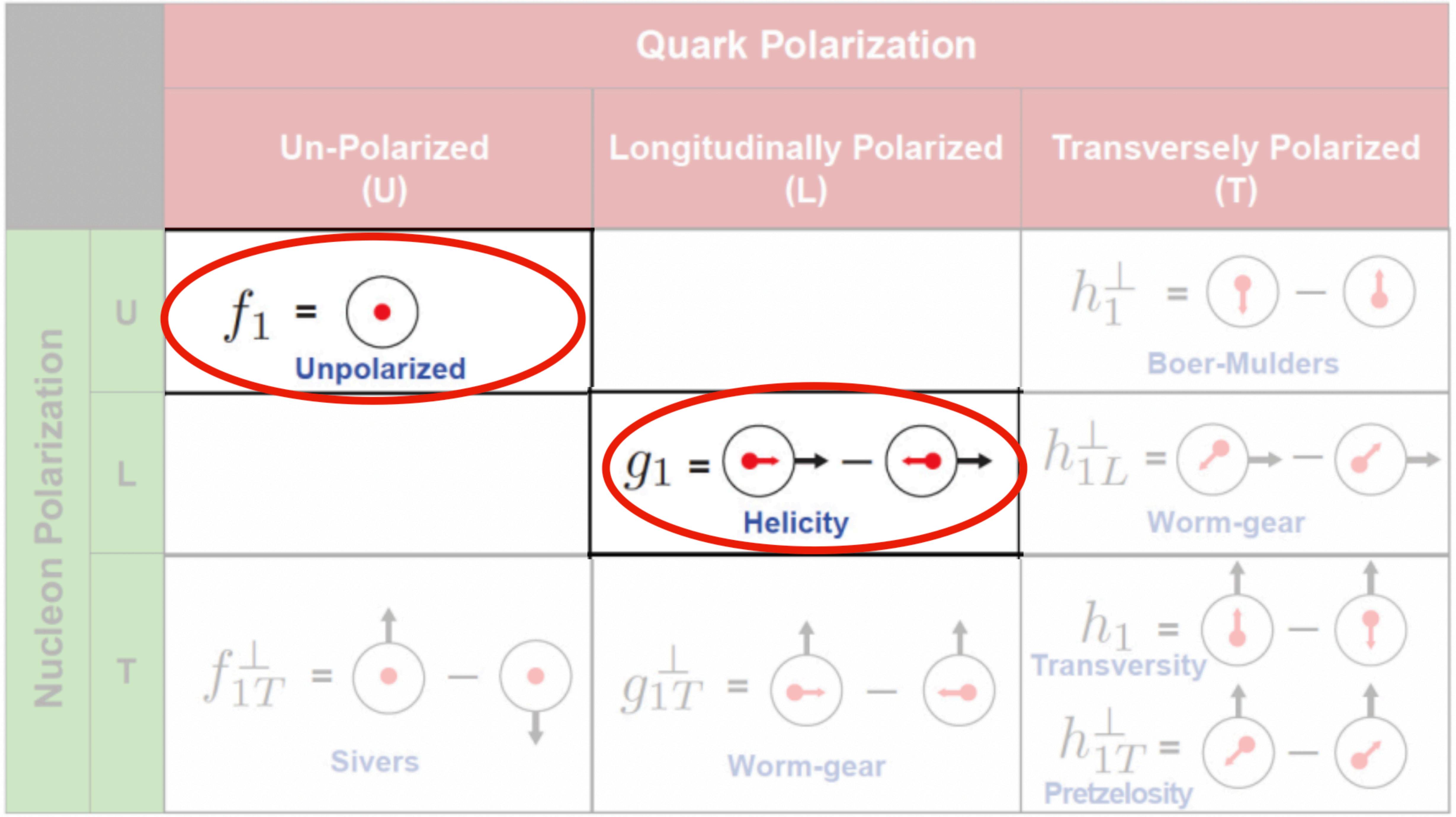
Constructing ratios cancels soft factor.

$$\frac{\tilde{f}_1(x, b_T, P_z, \mu)}{\tilde{f}_2(x, b_T, P_z, \mu)} = \frac{f_1(x, b_T, \zeta, \mu)}{f_2(x, b_T, \zeta, \mu)} + \mathrm{p.c.}$$

Quasi-TMDs TMDs

- Ratios of quasi-TMDs = ratios of light-cone TMDs up to power corrections (p.c.)
- Perturbative corrections and scale evolution also cancels:
  - renormalization-group-invariant (RGI) ratios.
  - valid to all orders in perturbation theory.

Unpolarized and helicity TMDs from lattice



$$\tilde{h}(b_T, b_z, P_z, \mu)$$

$$= \langle \lambda; P_z | \bar{\psi}(b_T, \frac{b_z}{2}) \Gamma \psi(0, -\frac{b_z}{2}) |_{\vec{\nabla}\cdot\vec{A}=0} | \lambda; P_z \rangle$$

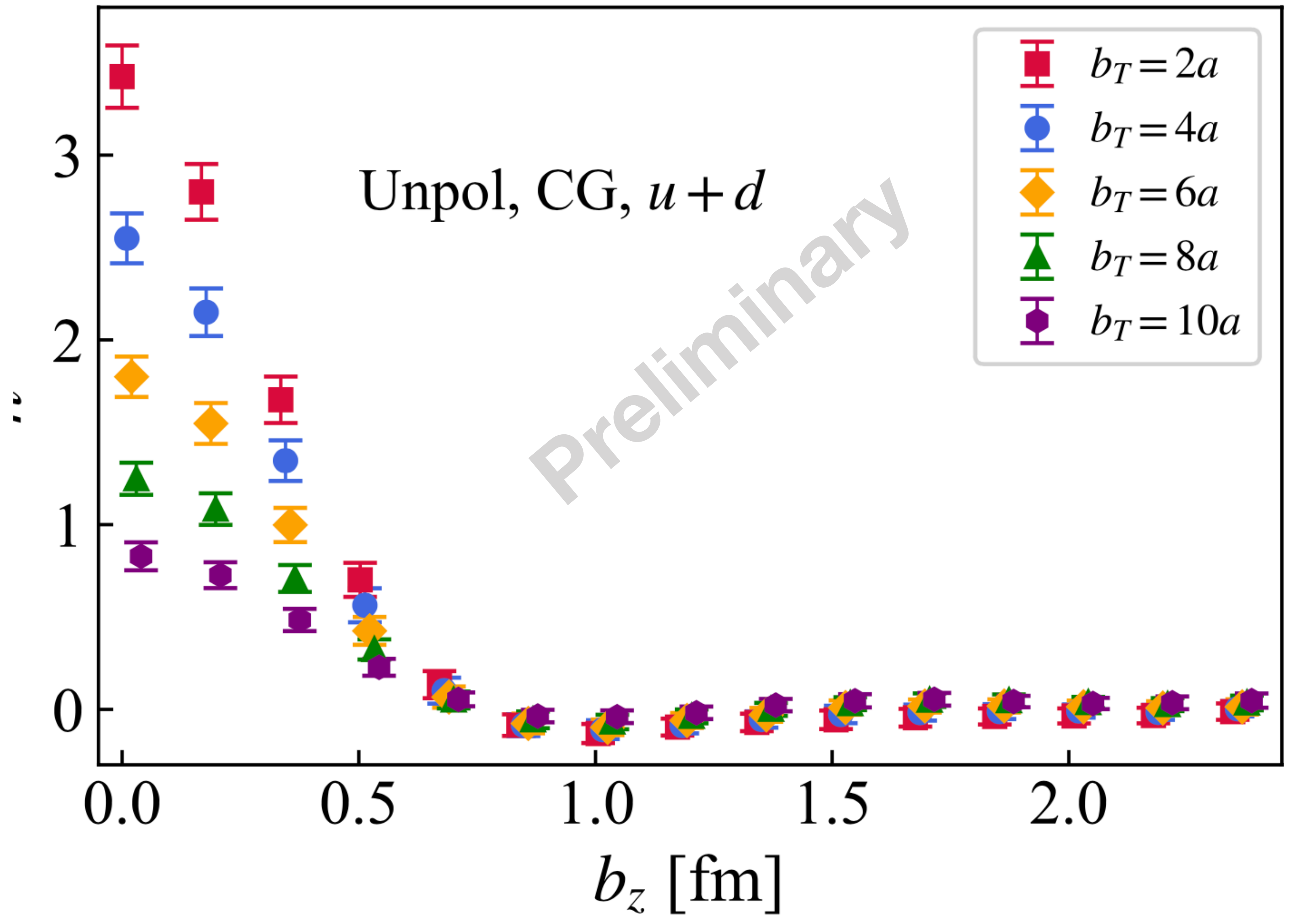
- Unpolarized TMDs: $\Gamma = \gamma^t$, $\psi = u - d$ and $u + d$ (disconnected diagrams ignored)
- Helicity TMDs: $\Gamma = \gamma^z \gamma^5$, $\psi = u - d$

Lattice setup:

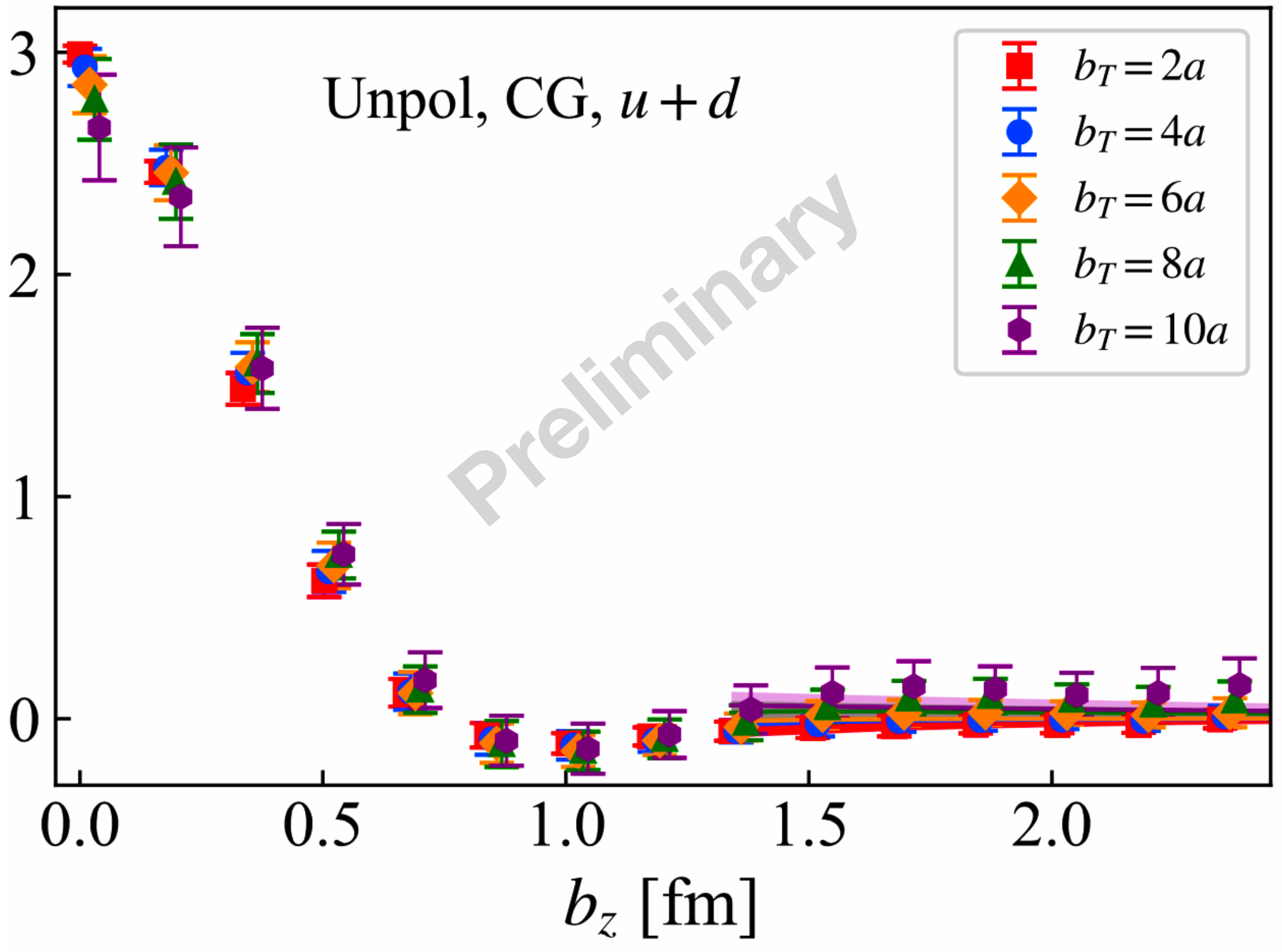
- 2+1 flavor Domain-wall (chiral) fermion discretization.
- Physical quark masses, $64^3 \times 128$ lattice with spacing $a = 0.084$ fm.
- Nucleon momentum up to $P_z = 1.62$ GeV, b_T up to 1 fm.

Quasi-TMDs from lattice

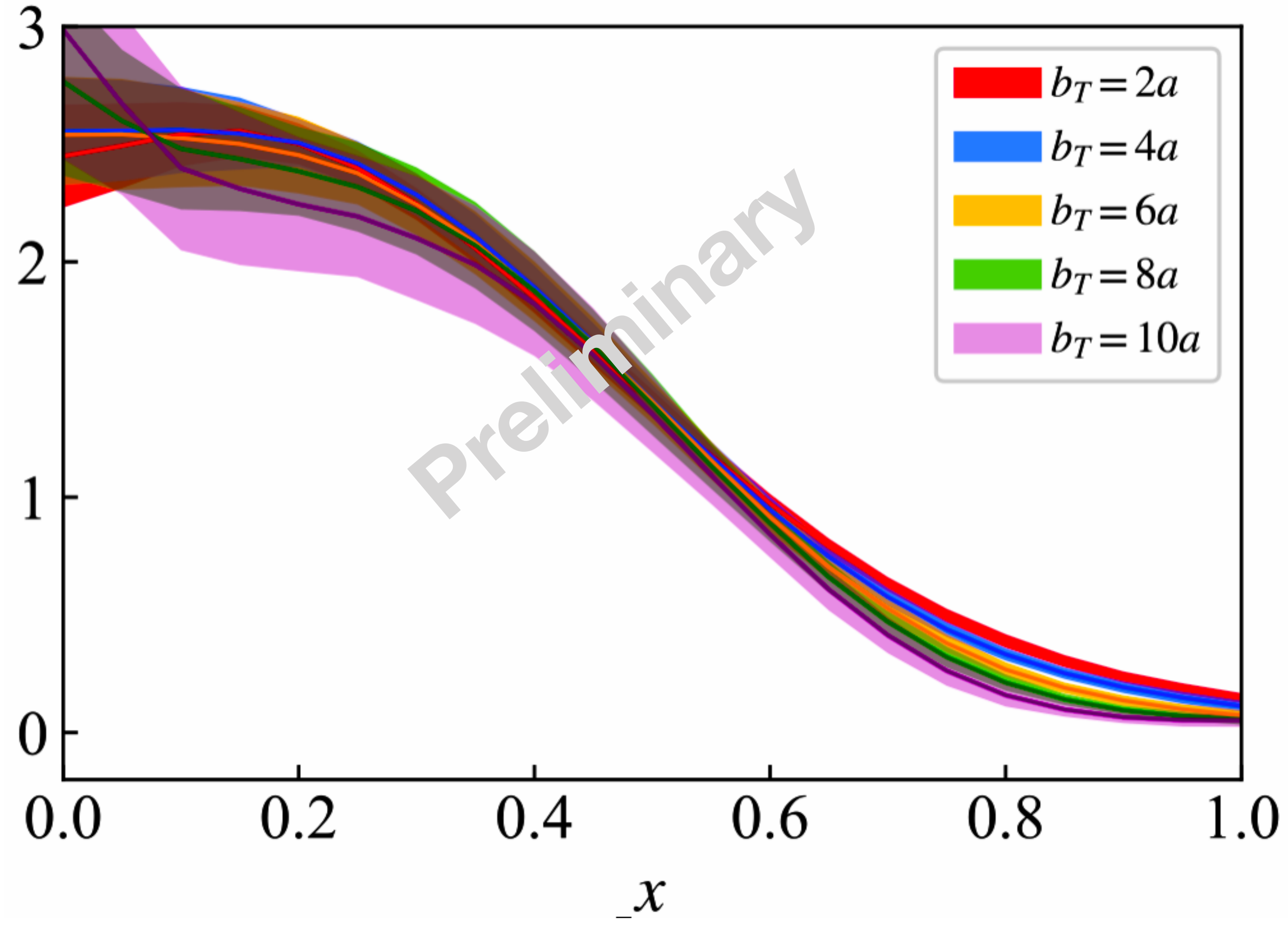
- Bare matrix elements



- Renormalized matrix elements



- Quasi-TMD $\tilde{f}_1(x, b_T, P_z)$



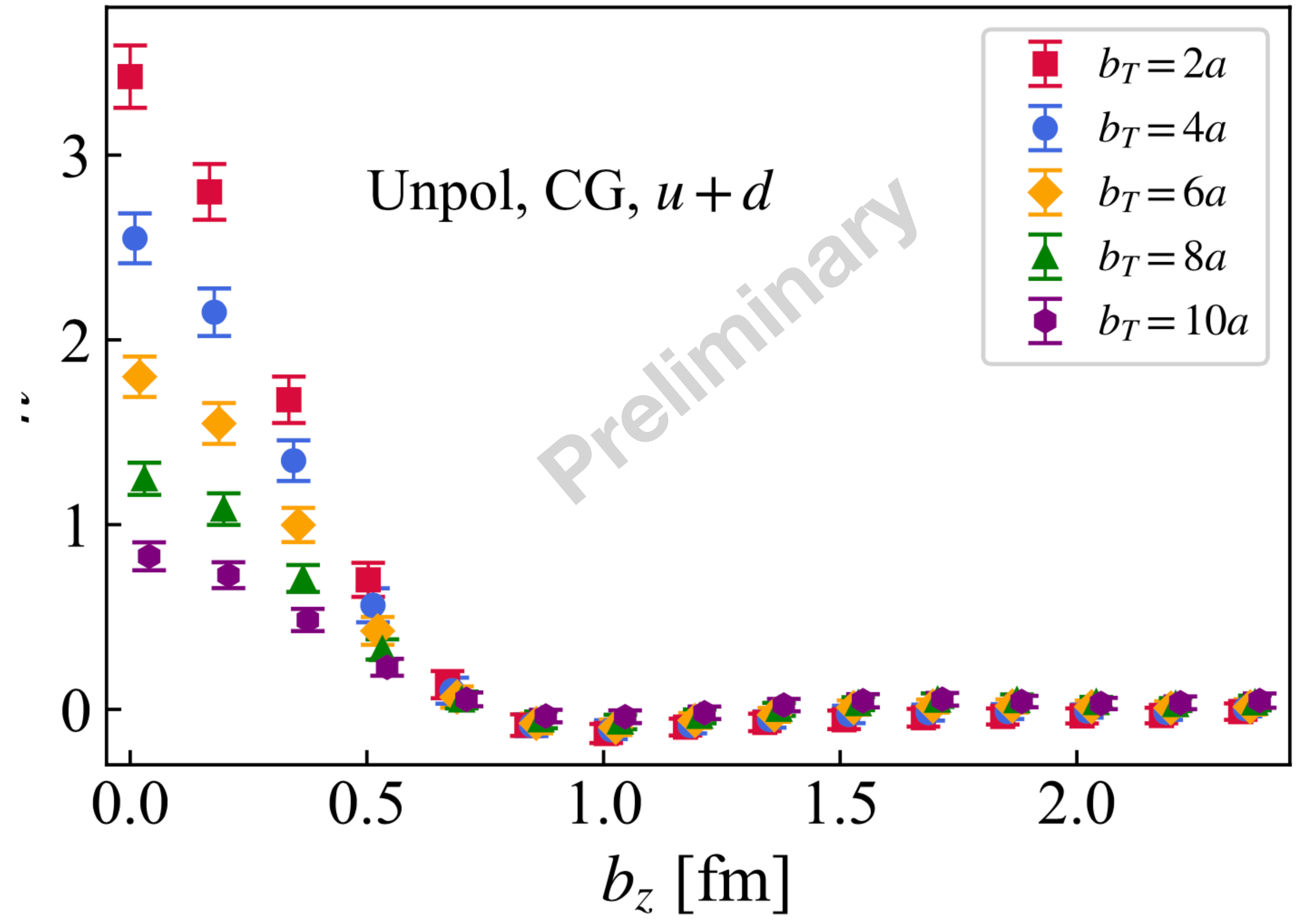
$$[\bar{\psi}(-\frac{\vec{b}}{2})\Gamma\psi(\frac{\vec{b}}{2})\,|_{\vec{\nabla}\cdot\vec{A}=0}]_B = Z_\psi(a)[\bar{\psi}(-\frac{\vec{b}}{2})\Gamma\psi(\frac{\vec{b}}{2})_{\vec{\nabla}\cdot\vec{A}=0}]_R$$

$$\tilde{h}^R(b_T, b_z, P_z) = \frac{\tilde{h}^B(b_T, b_z, P_z, a)}{\tilde{h}^B(b_T, 0, 0, a)}$$

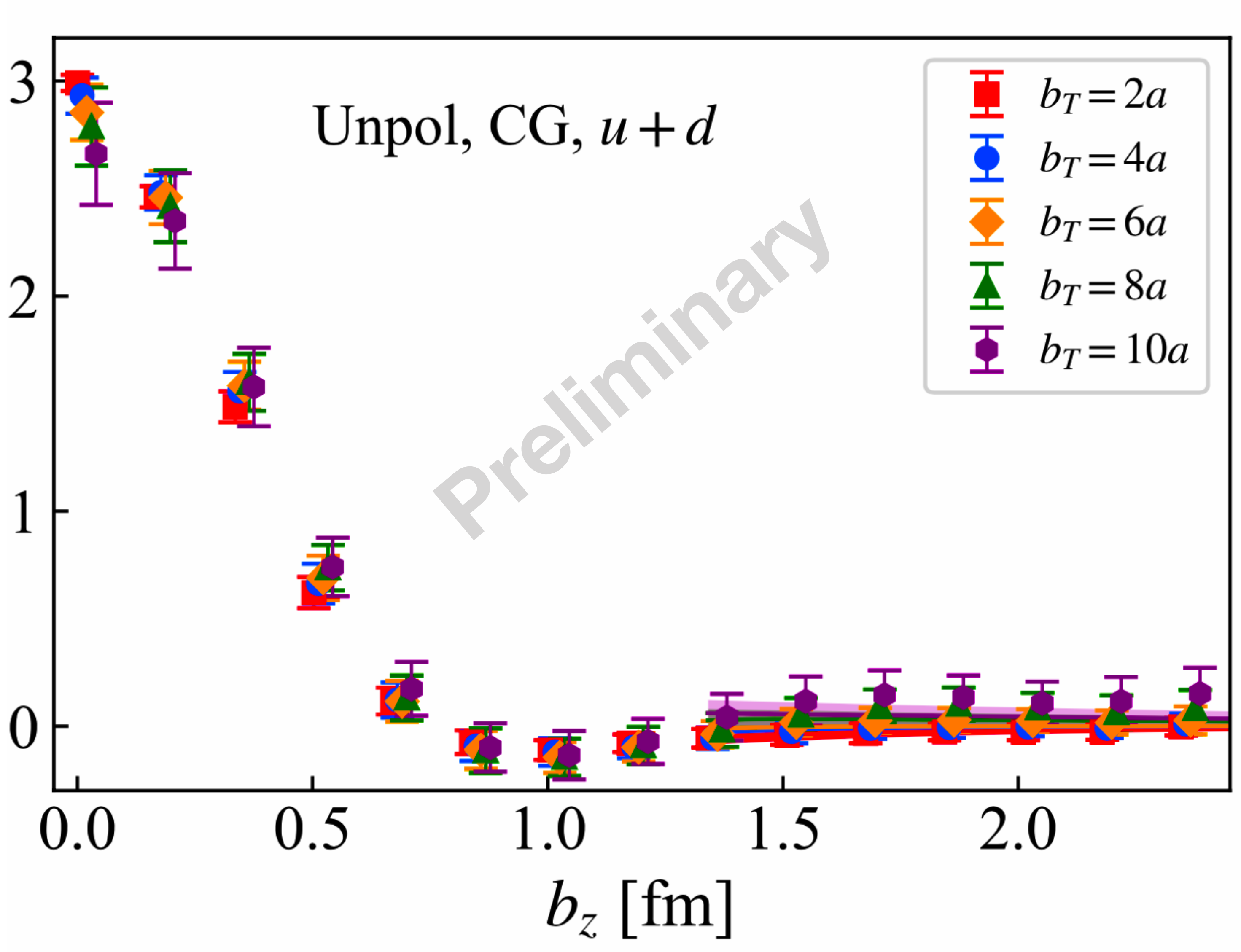
- CG quasi-TMDs: multiplicative renormalization with an overall constant, independent of P_z.
- Ratio scheme renormalization to cancel the $Z_\psi(a)$.

Quasi-TMDs from lattice

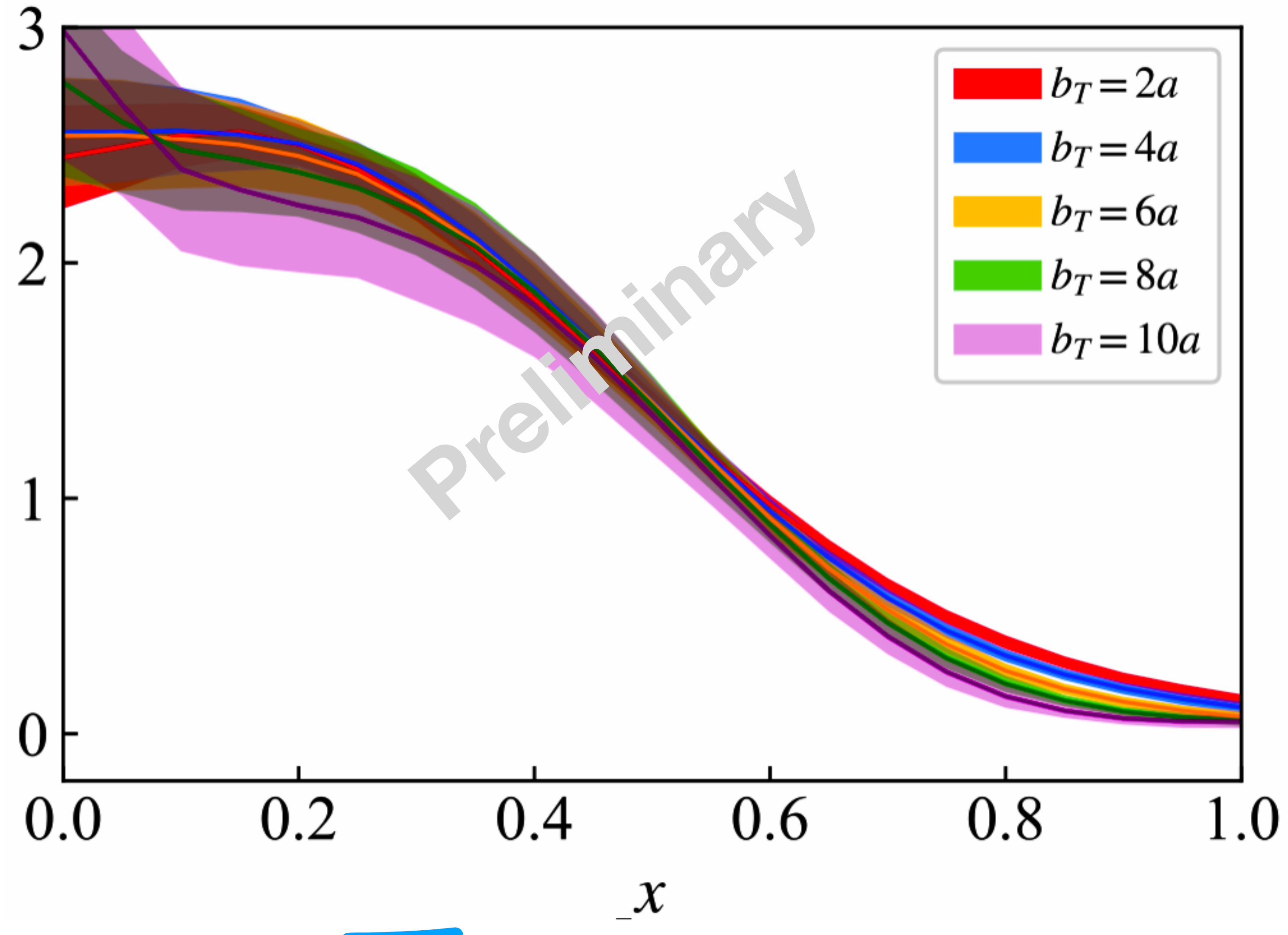
- Bare matrix elements



- Renormalized matrix elements



- Quasi-TMD $\tilde{f}_1(x, b_T, P_z)$



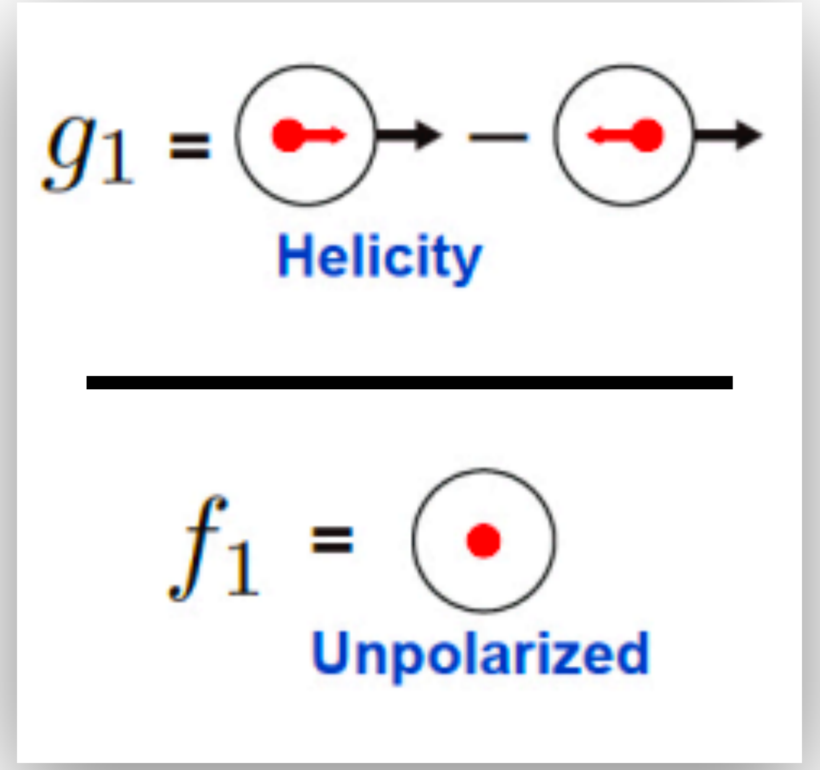
$$[\bar{\psi}(-\frac{\vec{b}}{2})\Gamma\psi(\frac{\vec{b}}{2})\,|_{\vec{\nabla}\cdot\vec{A}=0}]_B = Z_\psi(a)[\bar{\psi}(-\frac{\vec{b}}{2})\Gamma\psi(\frac{\vec{b}}{2})_{\vec{\nabla}\cdot\vec{A}=0}]_R$$

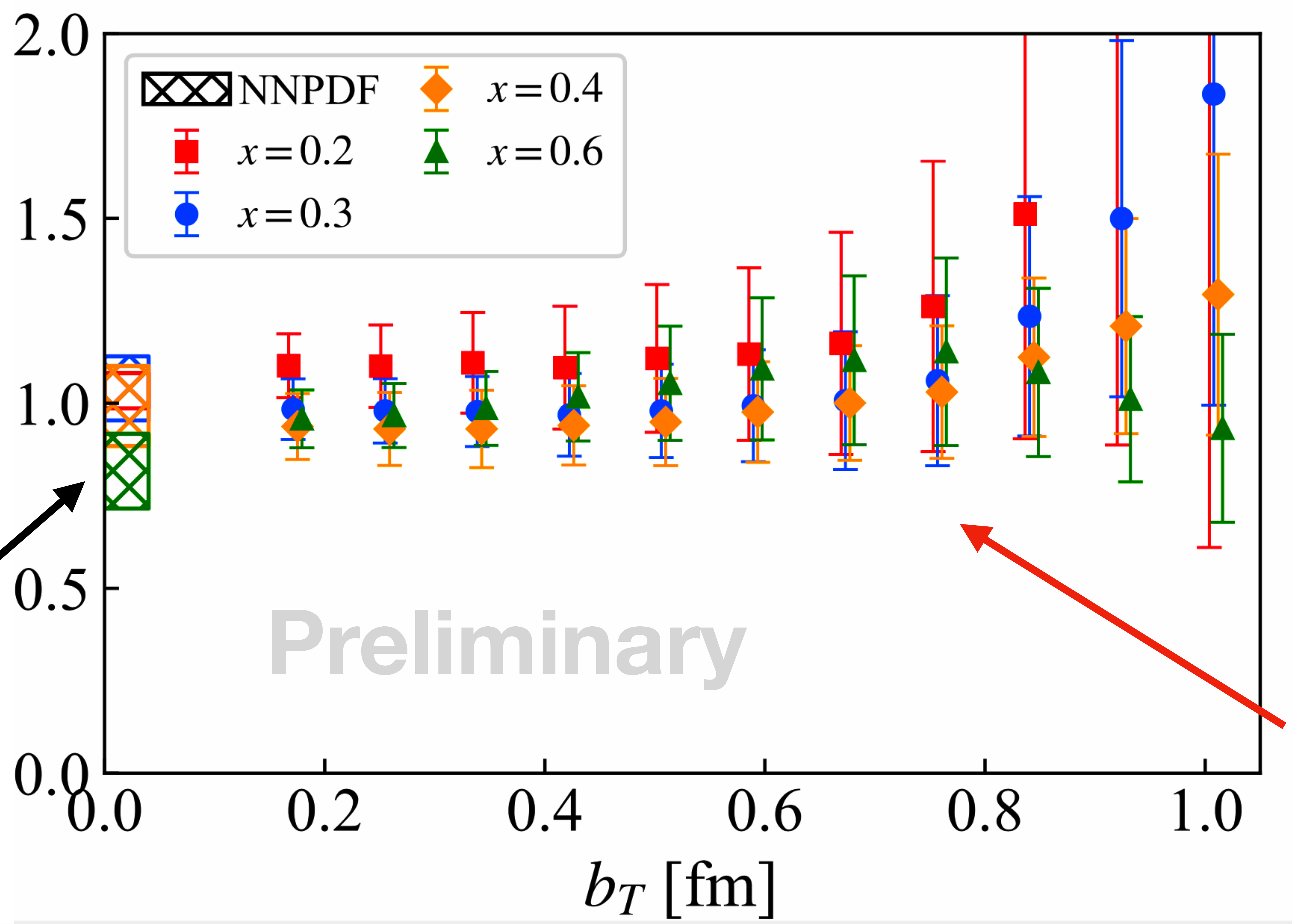
$$b_z \xrightarrow{\text{F.T.}} x$$

$$\tilde{h}^R(b_T, b_z, P_z) = \frac{\tilde{h}^B(b_T, b_z, P_z, a)}{\tilde{h}^B(b_T, 0, 0, a)}$$

Ratios between $u-d$ heli. and unpol. TMDs

$$\frac{g_{1L}^{\Delta u-\Delta d}(x,b_T)}{f_1^{u-d}(x,b_T)}\frac{1}{g_A}=\frac{\tilde{g}_{1L}^{\Delta u-\Delta d}(x,b_T)}{\tilde{f}_1^{u-d}(x,b_T)}\frac{1}{g_A}$$





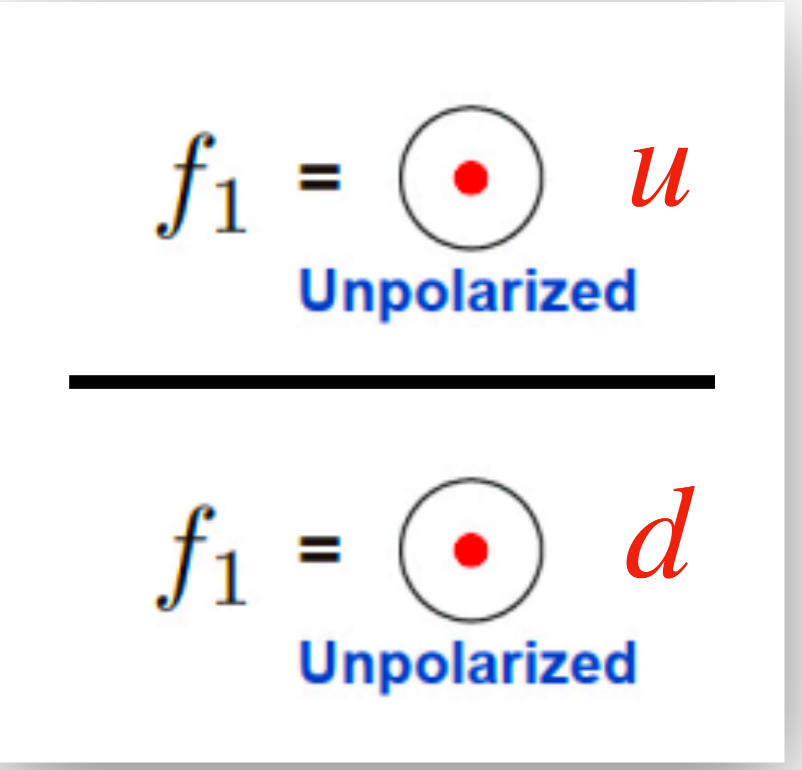
Ratio of collinear PDFs from NNPDF

Our lattice results

- Our preliminary results indicate that the helicity/unpolarized TMD ratios show no significant dependence on finite b_T.
- The TMD ratios are close to the collinear PDF ratios.

Ratios between valence $u-$ and $d-$unpolarized TMDs

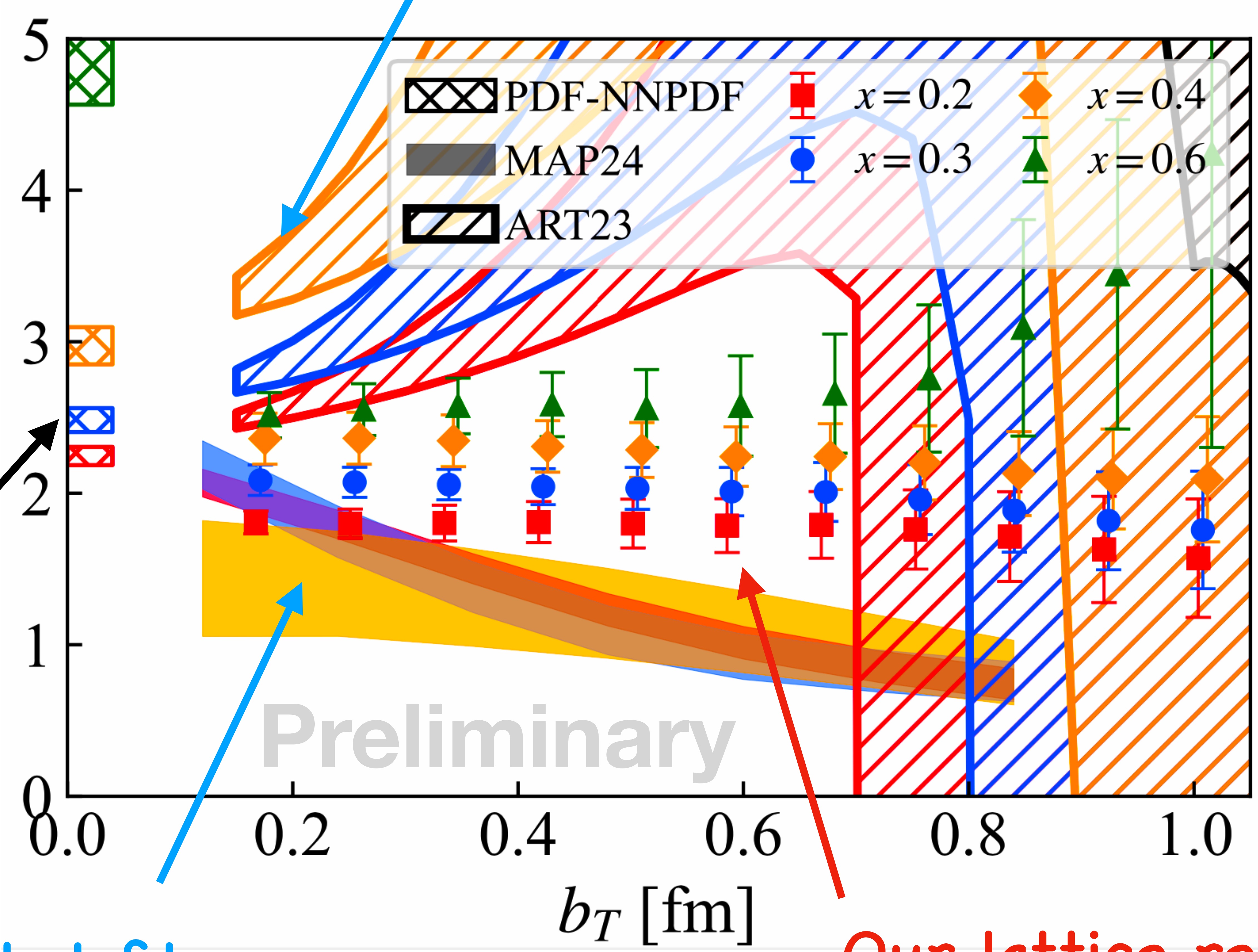
$$\frac{f_1^{u_v}(x,b_T)}{f_1^{d_v}(x,b_T)} = \frac{\tilde{f}_1^{u_v}(x,b_T)}{\tilde{f}_1^{d_v}(x,b_T)}$$



Ratio of collinear PDFs from NNPDF

Global fit from ART23

• *JHEP* 05 (2024) 036



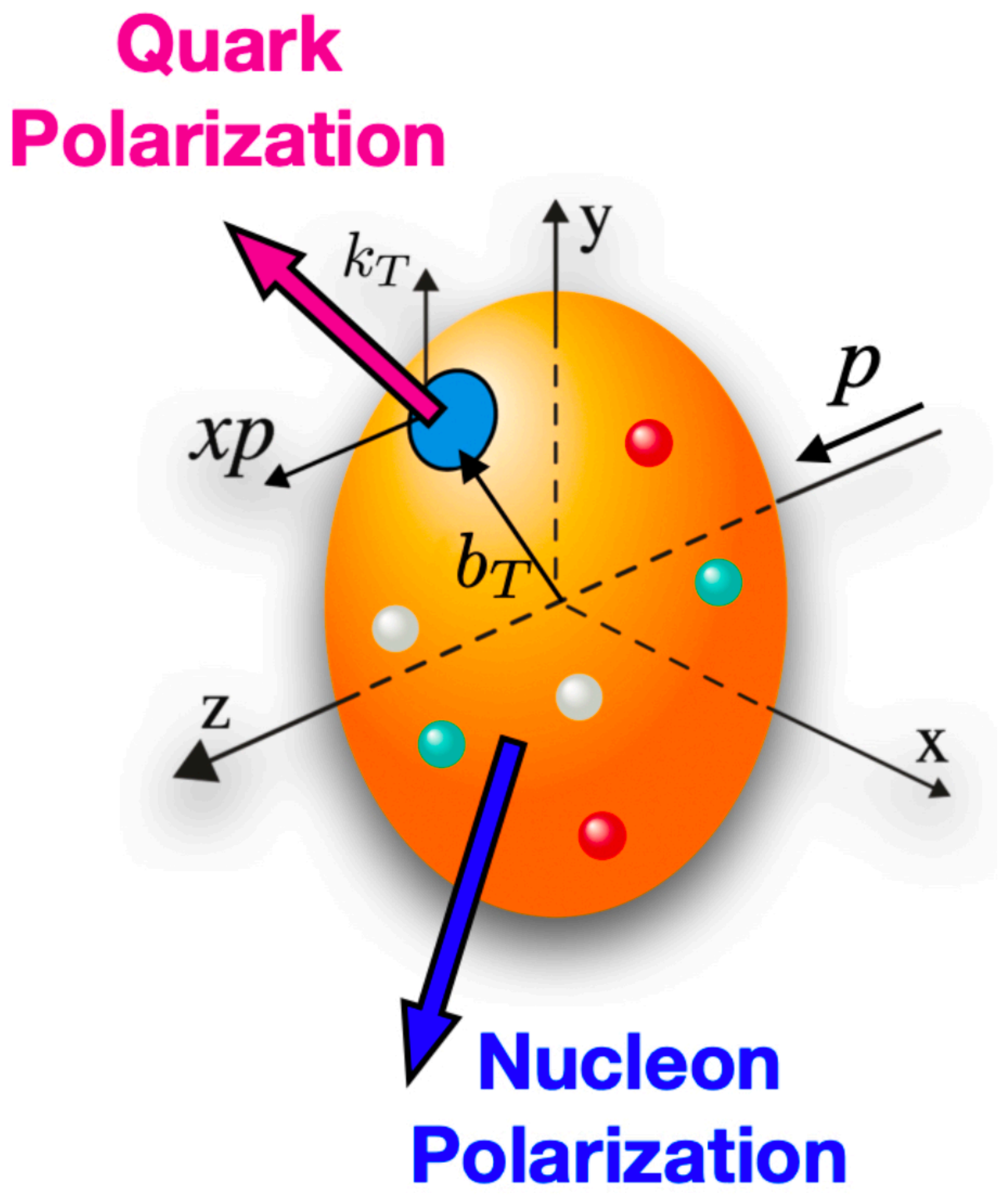
Global fit from MAP24

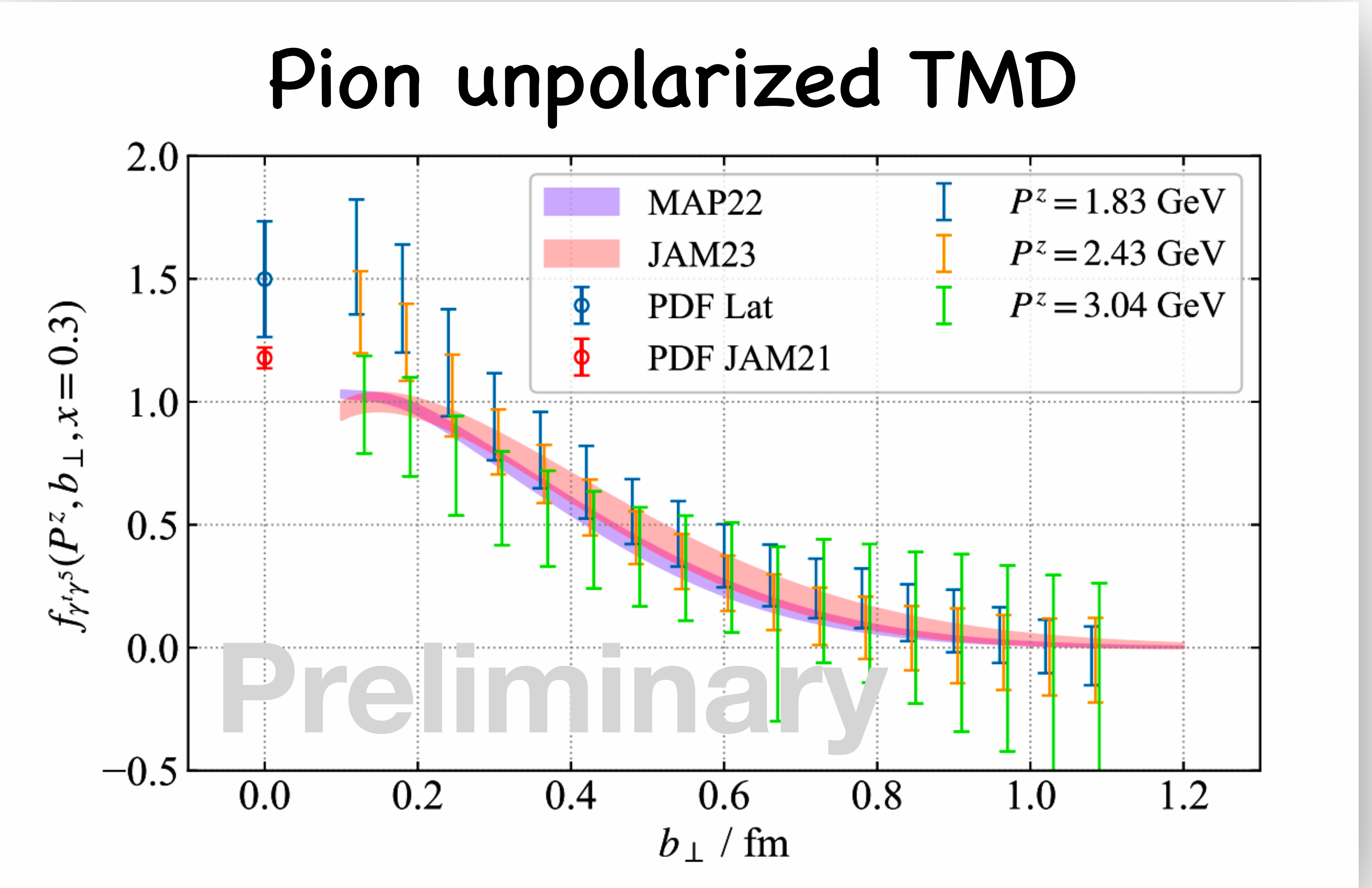
• *JHEP* 08 (2024) 232

Our lattice results

Summary & outlook

- TMDs can be extracted from quasi-TMD correlators.
- We have extracted the CS kernels from the evolution of quasi-TMDs, achieving precision comparable to global analyses.
- We calculated the ratio of nucleon TMDs and observed a weak b_T dependence in the iso-vector helicity and unpolarized TMDs, as well as between the $u-$ and $d-$ quark unpolarized TMDs.
- When combined with the soft factor, the full TMDs can be determined.





Thanks for your attention!