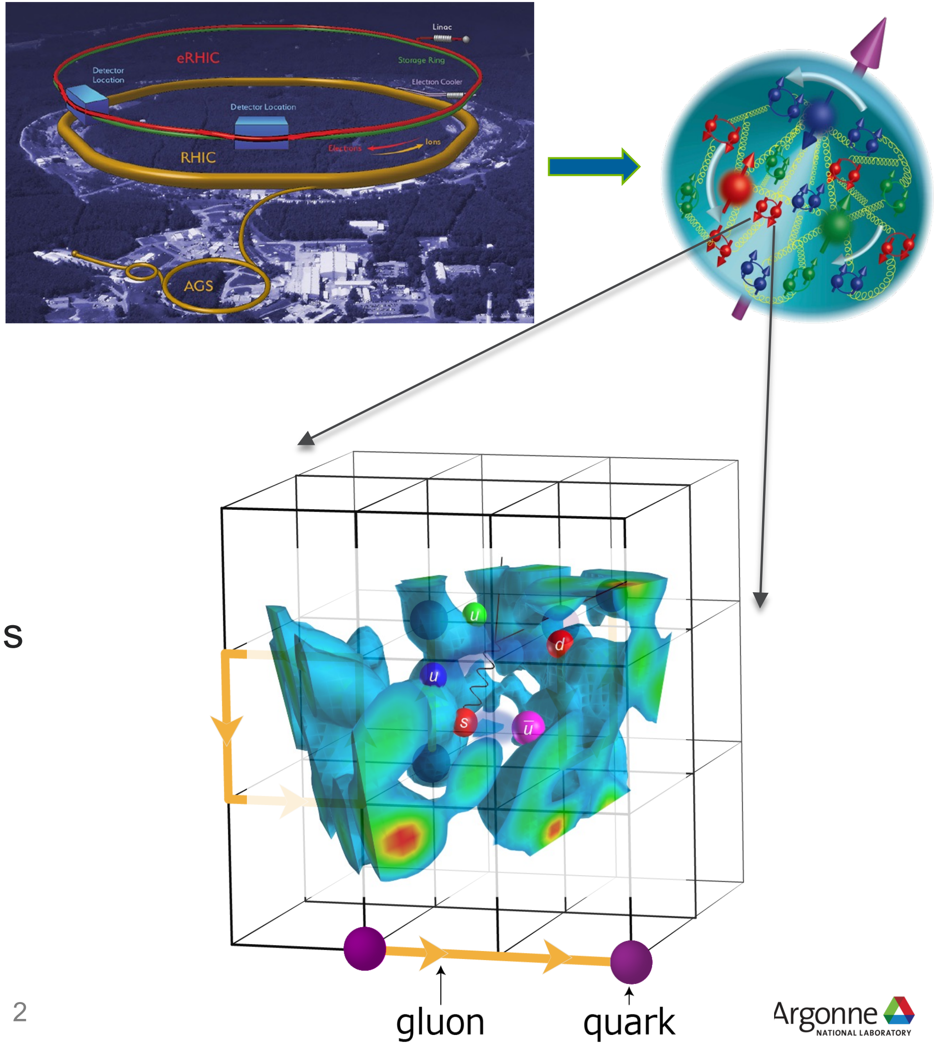
Lattice Calculation of TMD Physics in the EIC Era

Physics Opportunities at an Electron-Ion Collider XI, Florida International University, Miami, FL, USA February 25, 2025

Yong Zhao

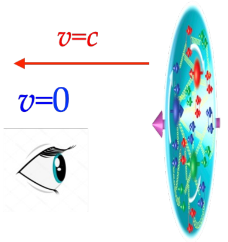
OUTLINE

- **Large-momentum effective theory**
  - Theoretical framework
  - Collins-Soper kernel
  - Soft function and TMDPDFs
- **New approach without Wilson lines**
  - TMDs from Coulomb-gauge correlations
  - Numerical applications
  - Better interpolators for boosted hadron
- **Summary**



LARGE-MOMENTUM EFFECTIVE THEORY (LAMET)

- Revisit Feynman's parton picture in the infinite momentum frame:

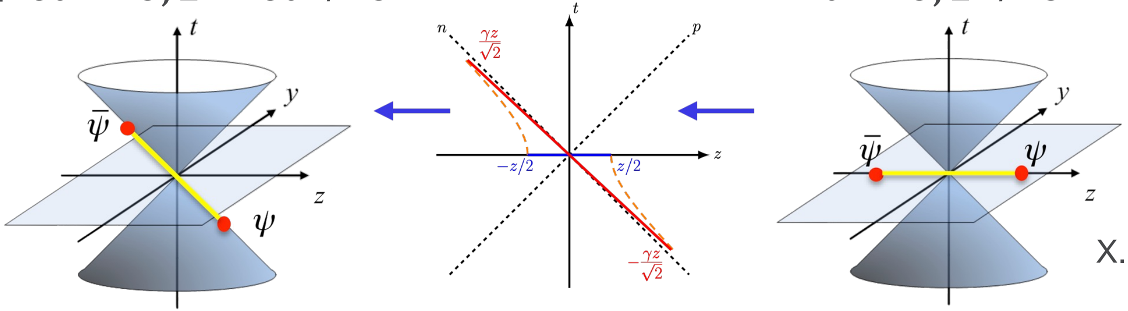


Simulating $\langle P = \infty | O(t=0) | P = \infty \rangle$? ✗

$P \ll 2\pi/a$

- Nevertheless, it is possible to simulate at proton at large momentum:

$z + ct = 0, z - ct \neq 0$ $\qquad$ $t = 0, z \neq 0$



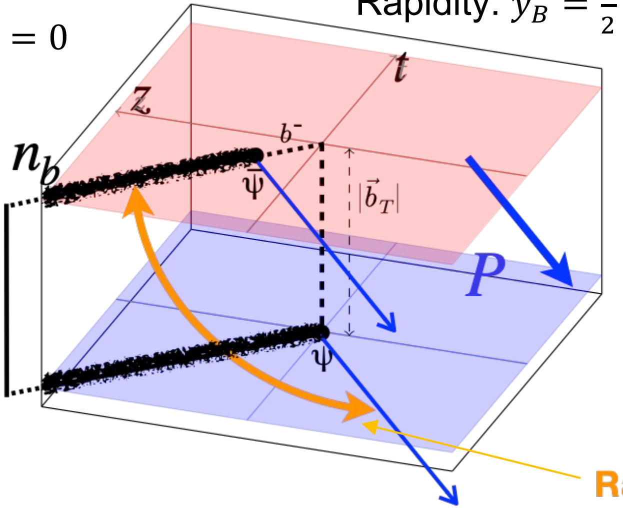
X. Ji, PRL 110 (2013)

TRANSVERSE MOMENTUM DISTRIBUTIONS

- Beam function:

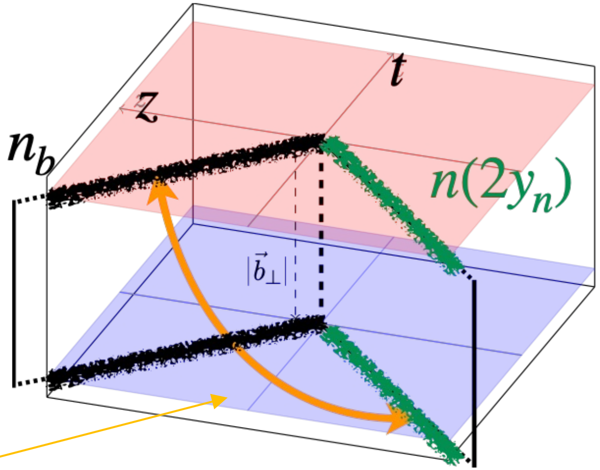
$n_b^2 = 0$

Rapidity: $y_B = \frac{1}{2}\ln\frac{n_b^+}{n_b^-} = -\infty$



Hadronic matrix element

- Soft function:



Vacuum matrix element

Rapidity divergences

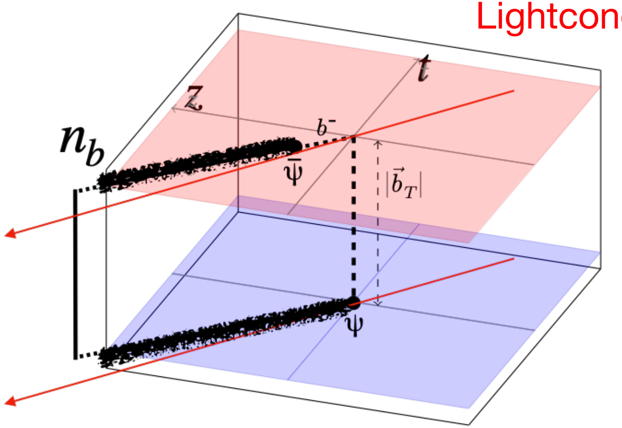
$$f_i(x, b_T, \mu, \zeta) = \lim_{\epsilon\to 0} Z_{UV} \lim_{\tau\to 0} \frac{B_i}{\sqrt{S^q}}$$

Collins-Soper scale: $\zeta = 2(xP^+e^{-y_n})^2$

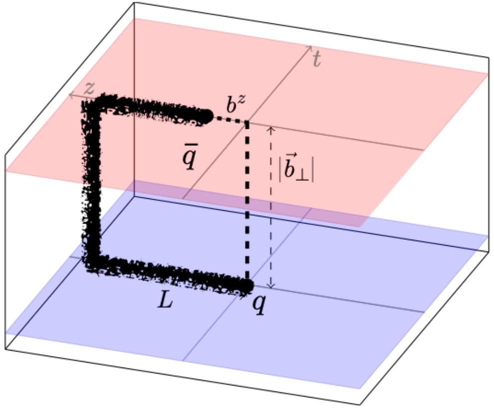
τ: rapidity divergence regulator

TMDS FROM LAMET

- Beam function (in Collins' scheme):
- Quasi-beam function:



Lorentz boost and $L \to \infty$



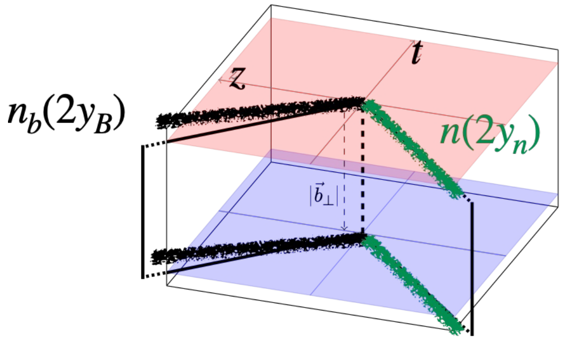
$n_b(y_B) = (n_b^+, n_b^-, 0_\perp) = (-e^{2y_B}, 1, 0_\perp)$

Spacelike but close-to-light-cone ($y_B \to -\infty$) Wilson lines, not directly calculable on the lattice 🙁

Equal-time Wilson lines, directly calculable on the lattice 🙂

SOFT FUNCTION (IN COLLINS' SCHEME)

- Not directly calculable on the lattice, but has the asymptotic behavior:



$$\xrightarrow{y_n - y_B \to \infty} S_r(b_T, \mu)\, e^{-2(y_n - y_B)\gamma_\zeta(b_T,\mu)}$$

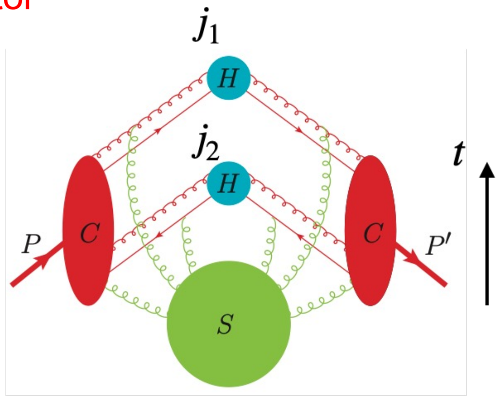
Collins-Soper kernel

Intrinsic soft factor

- Can be extracted from a meson form factor:

$$\lim_{P^z \gg m_\pi} \langle \pi(-P)|j_1(b_\perp)j_2(0)|\pi(P)\rangle = S_r^{-1}(b_\perp,\mu)\int dxdx' H(x,x',\mu)$$
$$\times \phi^\dagger(x,b_\perp,P^z,\mu)\phi(x,b_\perp,P^z,\mu)$$

$\phi(x,b_\perp,P^z,\mu)$: quasi-TMD wave function ✓



- Ji, Liu and Liu, NPB 955 (2020), PLB 811 (2020);
- Ji and Liu, PRD **105** (2022);
- Deng, Wang and Zeng, JHEP **09** (2022).

FACTORIZATION FORMULA FOR THE QUASI-TMDS

$$\frac{\tilde{f}_{i/p}^{\mathrm{naive}[s]}(x, \mathbf{b}_T, \mu, \tilde{P}^z)}{\sqrt{S_r(b_T, \mu)}} = C(\mu, x\tilde{P}^z) \exp\left[\frac{1}{2}\gamma_\zeta(\mu, b_T)\ln\frac{(2x\tilde{P}^z)^2}{\zeta}\right] \times f_{i/p}^{[s]}(x, \mathbf{b}_T, \mu, \zeta) + \mathcal{O}\left[\frac{1}{(x\tilde{P}^z b_T)^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{(x\tilde{P}^z)^2}\right]$$

- Collins-Soper kernel $\gamma_\zeta(\mu, b_T)$;
- No flavor mixing, easy flavor separation;
- Spin-dependence, e.g., Sivers function;
- Full (x, b_T) dependence.
- Twist-3 PDFs from small b_T expansion.
- Higher-twist TMDs.

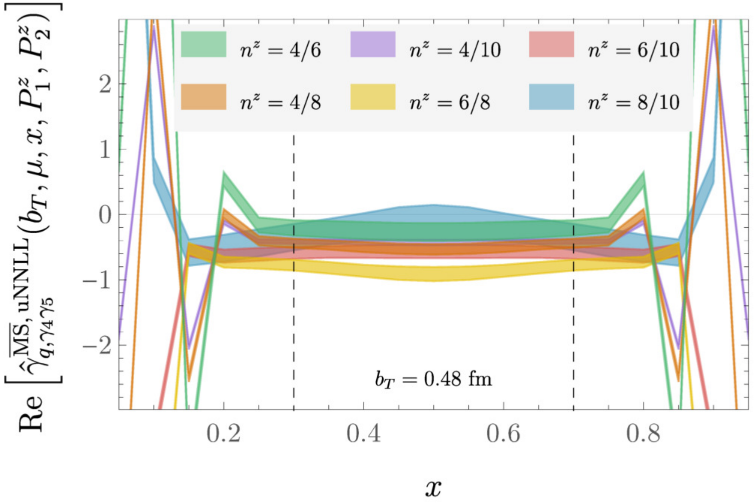
- Ji, Sun, Xiong and Yuan, PRD91 (2015);
- Ji, Jin, Yuan, Zhang and YZ, PRD99 (2019);
- Ebert, Stewart, YZ, PRD99 (2019), JHEP09 (2019);
- Ji, Liu and Liu, NPB 955 (2020), PLB 811 (2020);
- Ebert, Schindler, Stewart and YZ, JHEP 09 (2020);
- Vladimirov and Schäfer, PRD 101 (2020);
- Ji, Liu, Schäfer and Yuan, PRD 103 (2021);
- Ebert, Schindler, Stewart and YZ, JHEP 04 (2022).
- Rodini and Vladimirov, JHEP 08 (2022).

STATE-OF-THE-ART COLLINS-SOPER KERNEL

$$\gamma_\zeta(\mu, b_\perp) = \frac{d}{d\ln P^z} \ln \frac{\tilde{f}(x, b_\perp, \mu, P^z)}{C(\mu, xP^z)}$$

- Physical quark masses, large Lorentz boosts
- Continuum extrapolation with a=0.15, 0.12, 0.09 fm
- Controlled Fourier transform
- Lattice renormalization and operator mixing subtraction
- Next-to-next-to-leading logarithmic (NNLL) order matching

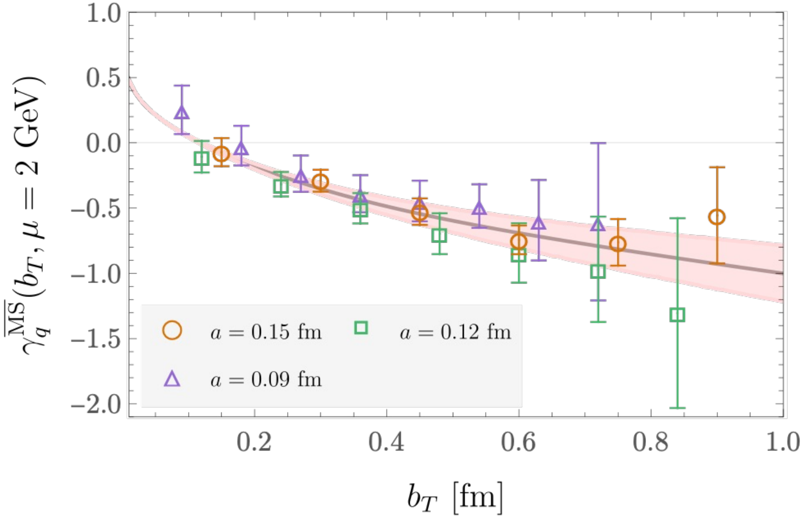
CS kernel extracted in the *x*-space



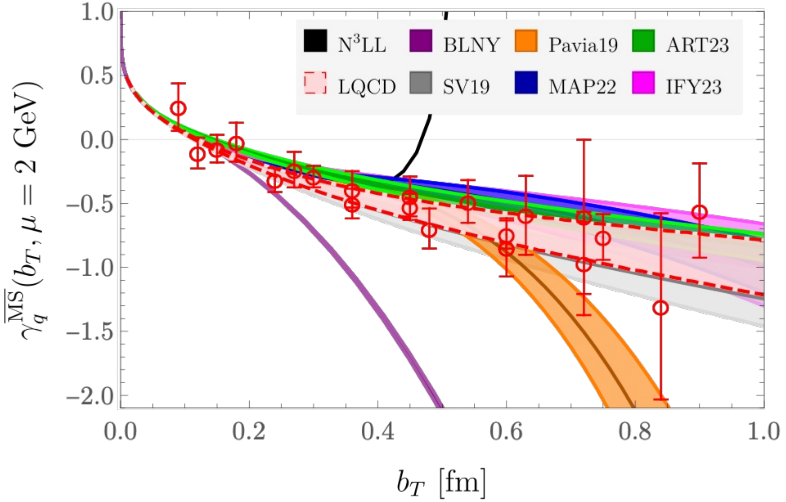
Almost flat at moderate *x*, an important indictor of the validity of factorization

- A. Avkhadiev, P. Shanahan, M. Wagman and YZ, PRD 108 (2023);
- A. Avkhadiev, P. Shanahan, M. Wagman and YZ, PRL 132 (2024).

STATE-OF-THE-ART COLLINS-SOPER KERNEL



Nice agreement with phenomenology ☺



- A. Avkhadiev, P. Shanahan, M. Wagman and YZ, PRD 108 (2023);
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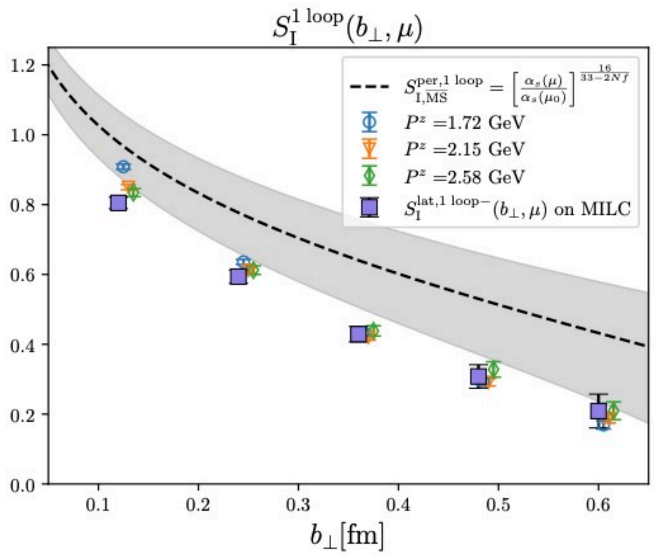
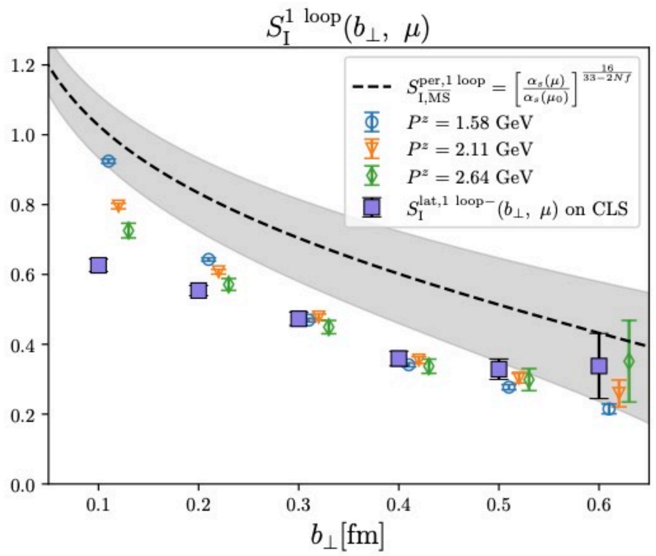
SV19: Scimemi and Vladimirov, JHEP 06 (2020)
Pavia19: Bacchetta et al., JHEP 07 (2020).
MAP22: Bacchetta et al., JHEP 10 (2022).
ART23: Moos et al., JHEP 05 (2024).
IFY23: Isaacson et al., PRD 110 (2024).

SOFT FUNCTION

M.-H. Chu, et al. (LPC), JHEP 08 (2023).



$a = 0.121$ fm, $m_\pi = 670$ MeV

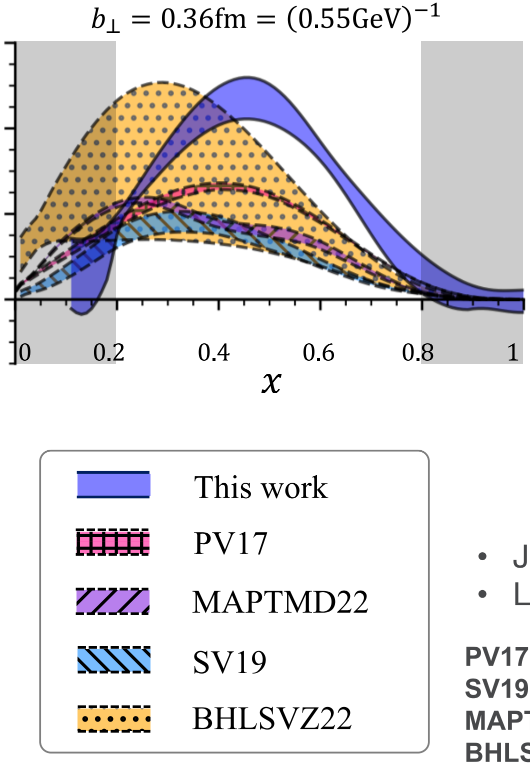


$a = 0.098$ fm, $m_\pi = 662$ MeV

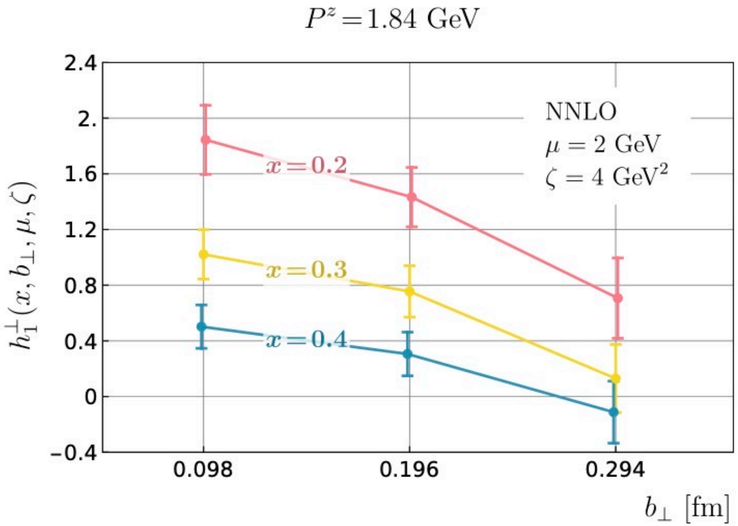
- Current state-of-the-art is at next-to-leading order (NLO)
- Continuum and physical quark mass limits not available so far
- The only calibration so far is perturbative prediction at $a \ll b_\perp \ll \Lambda_{QCD}^{-1}$

NUCLEON VALENCE TMDPDFS

Unpolarized distribution



Boer-Mulders function

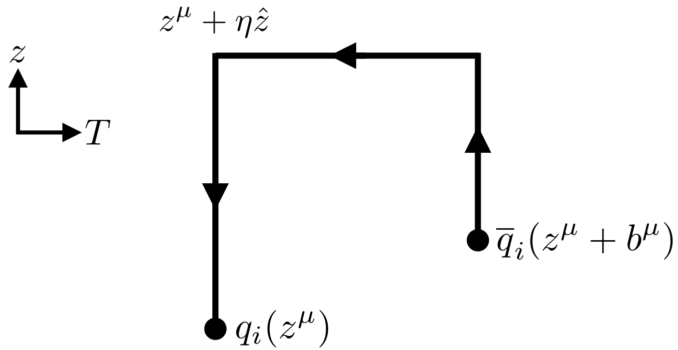


- J.-C. He, M.-H. Chu, J. Hua et al., (LPC), PRD 109 (2024).
- L. Ma et al., (LPC), arXiv: 2502.11807.

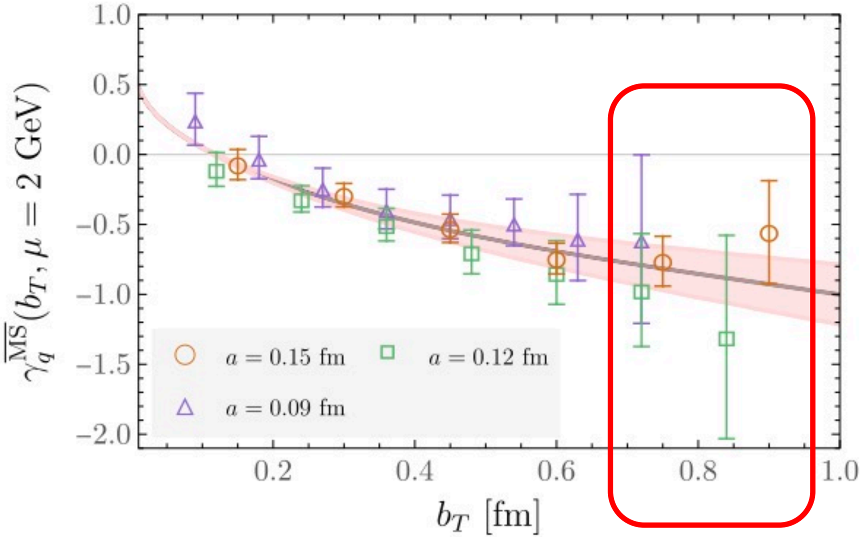
PV17: Bacchetta et al., JHEP 06 (2017).
SV19: Scimemi and Vladimirov, JHEP 06 (2020)
MAPTMD22: Bacchetta et al., JHEP 10 (2022).
BHLSVZ22: Bury et al., JHEP 10 (2022).

SYSTEMATICS IN LATTICE CALCULATIONS

Staple-shaped Wilson line



$$\eta \gg \{b^z, b_T\},\ xP^z \gg b_T^{-1}.$$



- Gauge link induces statistical noise, while signal is exponentially suppressed at large b_T;
- Complex operator mixings due to the breaking of symmetries by the staple;
- Additional systematics due to multiple scales $\{b^z, b_T, \eta\}$ involved.

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  - Numerical applications
  - Better interpolators for boosted hadron
- **Summary**

"Parton distributions from boosted fields in the Coulomb gauge"
Xiang Gao, Wei-Yang Liu and YZ, PRD 109 (2024), 094506

"Transverse momentum distributions from lattice QCD without Wilson lines"
YZ, PRL 133 (2024), 241904

QUASI DISTRIBUTIONS IN THE COULOMB GAUGE

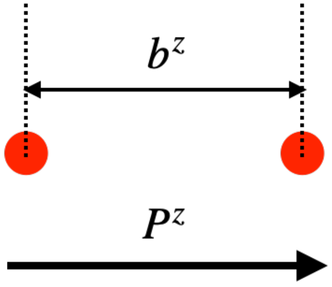
Parton distributions probe the correlation of energetic quarks and gluons dressed in the gauge background, which can be formulated by fixing a physical gauge condition.

Universality in LaMET: $G(A) = 0,\ G(A) = A^0, A^z, \nabla \cdot A$

- Y. Hatta, X. Ji, and YZ, PRD 89 (2014);
- X. Ji, Y.-S. Liu, Y. Liu, J.-H. Zhang and YZ, RMP 93 (2021).

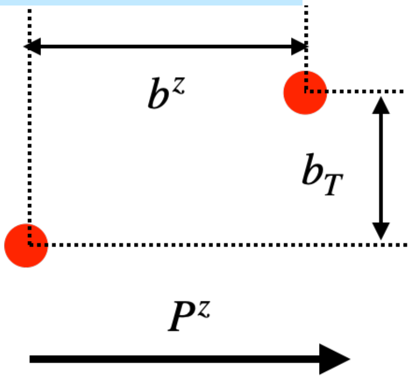
$$\tilde{f}(x, b_\perp, P^z) = \frac{P^z}{P^t} \int \frac{db^z}{2\pi} e^{ixP^z b^z} \langle P | \bar{\psi}(b^z, b_\perp) \frac{\gamma^t}{2} \psi(0) |_{\nabla \cdot A = 0} | P \rangle$$

Quasi-PDF:



X. Gao, W.-Y. Liu and YZ, PRD 109 (2024)

Quasi-TMD:



YZ, PRL 133 (2024)

FACTORIZATION FORMULA

YZ, PRL 133 (2024)

$$\frac{\tilde{B}(x,b_\perp,\mu,P^z)}{\tilde{S}_C(b_\perp,\mu,0)} = \left|C\left(\frac{xP^+}{\mu}\right)\right|^2 \exp\left[\frac{1}{2}\gamma_\zeta(b_\perp,\mu)\ln\frac{2(xP^+)^2}{\zeta}\right] f(x,b_\perp,\mu,\zeta)$$

NLO

Soft function can be extracted from the same meson form factor:

$$\lim_{P^z \gg m_\pi} \langle \pi(-P)|j_1(b_\perp)j_2(0)|\pi(P)\rangle = \frac{1}{[\tilde{S}_C(b_\perp,\mu,0)]^2}\int dxdx' H(x,x',\mu)$$

NLO

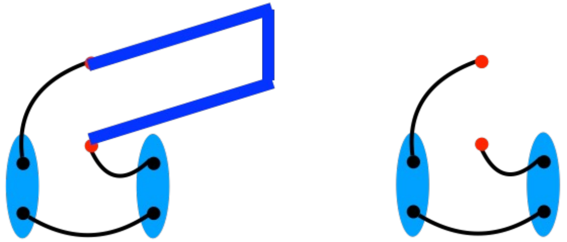
$$\times \phi_C(x,b_\perp,P^z,\mu)\phi_C(x',b_\perp,P^z,\mu)$$

ϕ_C: Coulomb-gauge quasi-TMD wave function ✔

$$\phi_C^* = \phi_C$$

ADVANTAGES

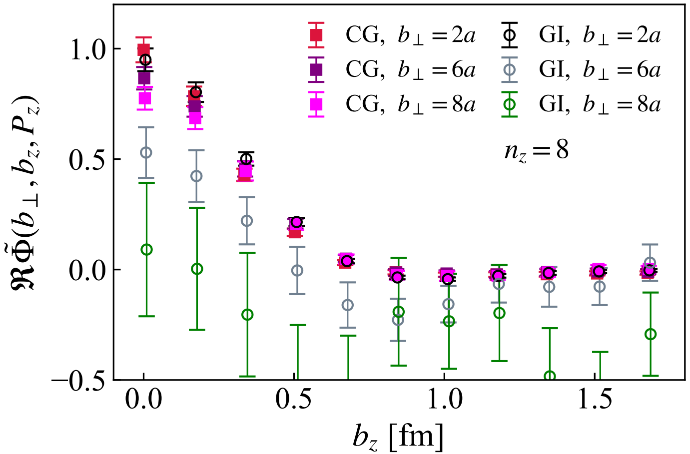
- Significantly improved statistical precision, access to larger b_T;



- Absence of linear power divergence;

$$\bar{\psi}_0(b)\Gamma\psi_0(0) = Z_\psi(a)[\bar{\psi}(b)\Gamma\psi(0)]_r$$

Coulomb gauge (CG) approach vs gauge-invariant (GI) approach



D. Bollweg, X. Gao, S. Mukherjee and YZ, PLB 852 (2024)

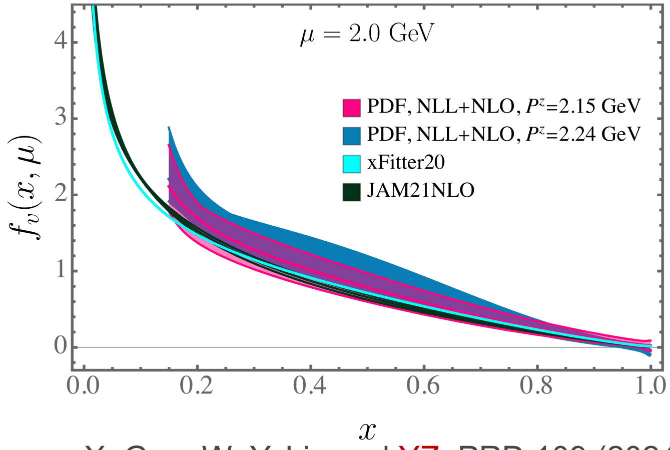
X. Gao, W.-Y. Liu and YZ, PRD 109 (2024)

- Access to larger off-axis momenta thanks to 3D rotational symmetry.

PARTON DISTRIBUTION FUNCTIONS

- $N_f = 2 + 1$ HISQ configurations with Wilson-Clover valence fermions
- $a = 0.06$ fm, $m_\pi = 300$ MeV, $P_z^{max} = 3.04$ GeV

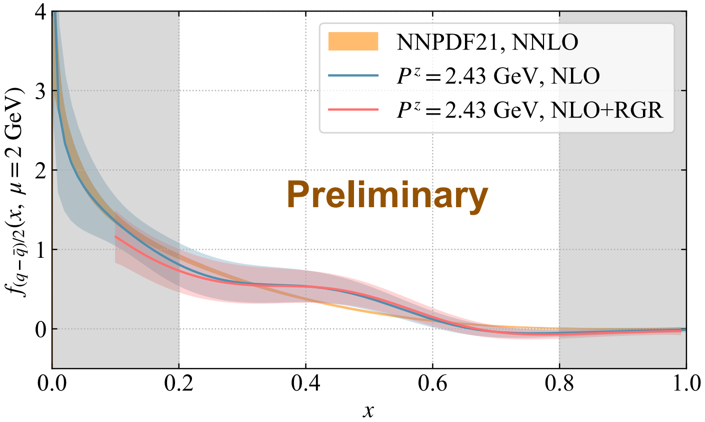
Pion valence-quark PDF



X. Gao, W.-Y. Liu and YZ, PRD 109 (2024)

Proton unpolarized valence PDF

$$f_u - f_d - (f_{\bar{u}} - f_{\bar{d}})$$



X. Gao, J. He, YZ et al, in preparation.

See Jinchen He's poster presentation

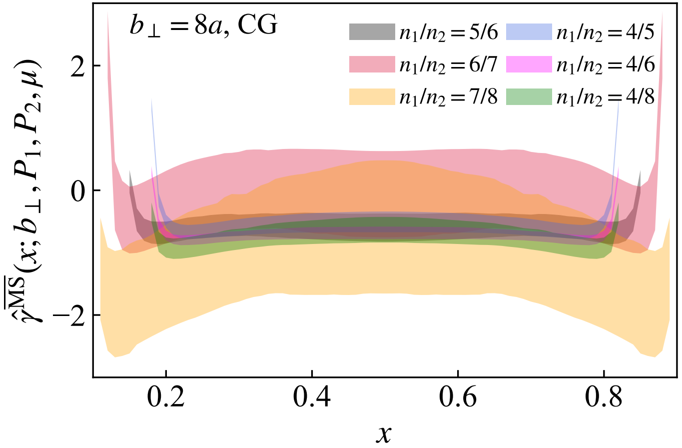
- Off-axis directions used to reach large momentum
- Agreement with the gauge-invariant approach and phenomenology within statistical errors

COLLINS-SOPER KERNEL

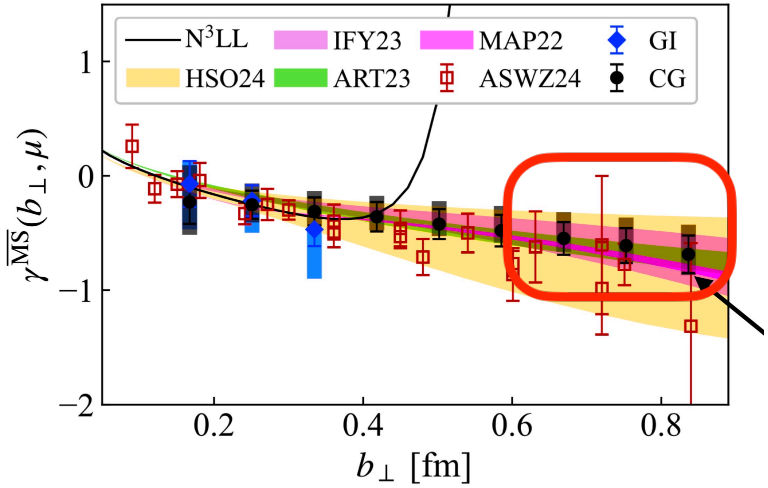
D. Bollweg, X. Gao, S. Mukherjee and YZ, PLB 852 (2024)

- $N_f = 2 + 1$ (chiral) domain-wall fermion configurations
- $a = 0.0836$ fm, $m_\pi = 140$ MeV, $P_z^{max} = 1.85$ GeV.

Nice plateau in *x* and convergence in P_z



Agreement with old method and recent global fits



Accessibility to deeper non-perturbative region!

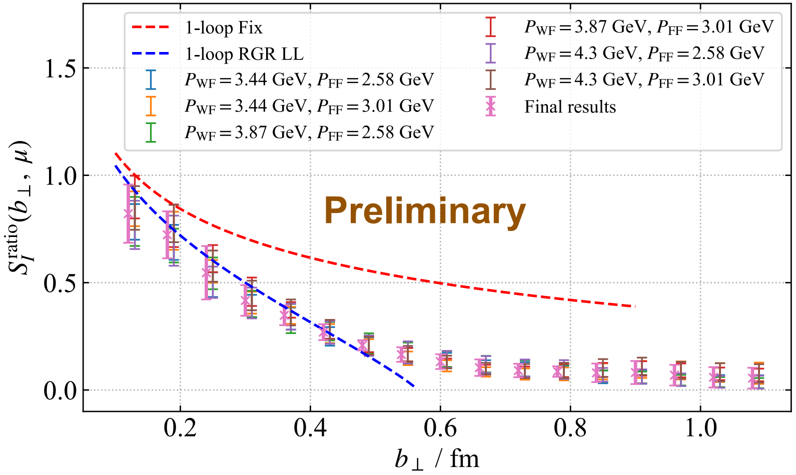
MAP22: Bacchetta et al., JHEP 10 (2022).
ART23: Moos et al., JHEP 05 (2024).
IFY23: Isaacson et al., PRD 110 (2024).
HSO24: Aslan, Rainaldi et al., PRD 110(2024).
ASWZ24: Avkhadiev et al., PRL 132 (2024).

SOFT FUNCTION AND UNPOLARIZED PION TMDPDF

- $N_f = 2 + 1$ HISQ configurations with Wilson-Clover valence fermions
- $a = 0.06$ fm, $m_\pi = 300$ MeV, $P_z^{max} = 3.04$ GeV (off-axis).

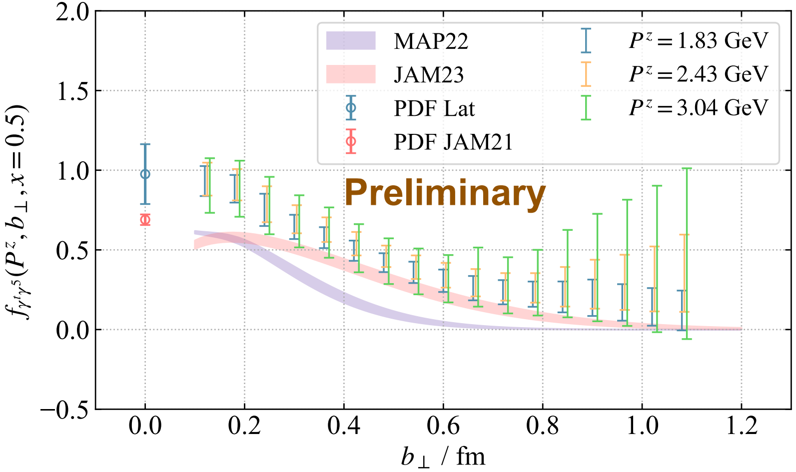
X. Gao, J. He, YZ et al, in preparation.

Intrinsic soft factor



Caveat: the pion form factor comes from LPC with a different lattice ensemble and pion mass

Pion unpolarized TMDPDF



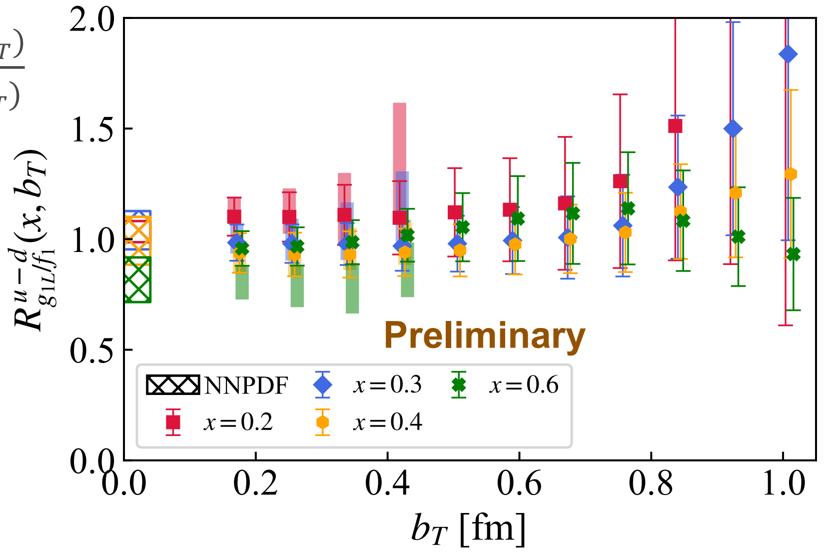
MAP22: Cerutti et al., PRD 107 (2023).
JAM23: Barry et al., PRD 108 (2023).
PDFLat: Gao, Liu and YZ, PRD 109 (2024).
PDFJAM21: Barry et al., PRL 127 (2021).

SPIN-DEPENDENT PROTON TMDPDF

- $N_f = 2 + 1$ (chiral) domain-wall fermion configurations
- $a = 0.0836$ fm, $m_\pi = 140$ MeV, $P_z^{max} = 1.62$ GeV.

X. Gao, YZ et al, in preparation.

$$\frac{g_{1L}^{u_+}(x, b_T) - g_{1L}^{d_+}(x, b_T)}{f_1^{u_v}(x, b_T) - f_1^{d_v}(x, b_T)}$$



The gauge-invariant method (colored vertical bands) can predict $b_T \leq 0.4$ fm.

The Coulomb gauge method is more precise and reliable at very large b_T

Meaningful comparison with phenomenology.

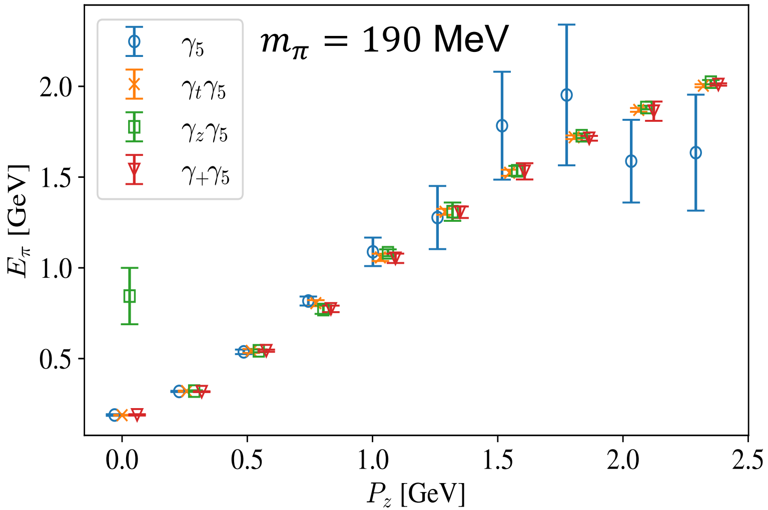
See Xiang Gao's talk on Monday

MAP24: Bacchetta et al., JHEP 08(2024).
ART23: Moos et al., JHEP 05 (2024).

BETTER INTERPOLATORS FOR BOOSTED HADRONS

- Old interpolator for pion: $\langle \pi(p) | \bar{u}\gamma_5 d \rangle \propto m_\pi$
- New interpolators for pion: $\langle \pi(p) | \bar{u}\gamma_5\gamma_\mu d \rangle \propto P_\mu$
- Signal in $\pi\pi$ correlation $\propto \left(\frac{P_\mu}{m_\pi}\right)^2$,while noise stays at the same level regardless of P_μ
- Observed signal-to-noise enhancement factor
  - 50 for pion at ~ 2 GeV, or $O(2500)$ in statistics
  - 10 for nucleon ~ 3 GeV, or $O(100)$ in statistics
- **Extremely valuable for precision nucleon 3D imaging!**

Pion energy at different momenta



R. Zhang, A. Grebe, D. Hackett, M. Wagman and YZ, arXiv: 2501.00729.

SUMMARY

- Much progress has been made in the LaMET calculation of TMD physics;
- The Collins-Soper kernel is under better systematic control now, while more work needs to be done to reliably calculate the soft function and TMDPDFs;
- The Coulomb-gauge method has the potential to significantly improve the **precision** in the non-perturbative region, thus becoming a standard approach for TMD physics in the future.
- The kinematically enhanced interpolators can have a profound impact on **precise** lattice calculation of parton physics.
- There are a lot of exciting new results to look forward to in the future!