Calculating parton distribution functions with NJL model

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Outline

Overview of the Nambu-Jona-Lasinio model[1]

- The NJL model is a low-energy, effective theory of the strong interaction that mimics many key features of QCD, such as dynamical chiral symmetry breaking (DCSB). Thus, it is a useful tool to help understand non-perturbative phenomena in low energy QCD.
- Unlike QCD, the NJL model considers only the quarks as the explicit degrees of freedom, neglecting the gluons
- Dynamics due to gluon-quark interaction and gluon self-couplings are absorbed into the four-fermion contact interaction.
- In order to preserve the chiral symmetry, we include the pseudoscalar and vector diquarks in addition to the scalar and axial-vector ones already included in the previous work.

[1] Nambu and Jona-Lasinio, Phys. Rev. 122, 345; Phys. Rev. 124, 246.

Confining NJL[2]

- Confinement is simulated by the introduction of an infrared cutoff in the proper-time regularization scheme.
- Doing this eliminates free quark propagation (it gets rid of the imaginary part of hadron decaying into quarks). (In a way that maintains covariance.)
- The nucleon is modeled by a relativistic quark-diquark bound state satisfying the Faddeev equation.
- We calculate PDF, FF, TMD, GPD, etc. with this model, as well as study nucleon in-medium modification, and the binding of atomic nuclei.

[2] H. Mineo et. al, Nucl. Phys. A 735, 482 (2004)

Proper-time regularization scheme

As an effective theory, NJL model is non-renormalizable, thus it needs a regularization prescription in order to be well-defined. We use the proper-time regularization scheme

$$\begin{aligned}\frac{1}{X} &= \frac{1}{(n-1)!}\int_0^\infty d\tau\ \tau^{n-1}e^{-\tau X}\\ &\longrightarrow \frac{1}{(n-1)!}\int_{1/\Lambda_{UV}^2}^{1/\Lambda_{IR}^2} d\tau\ \tau^{n-1}e^{-\tau X},\end{aligned}$$

where X represents a product of propagators that have been combined using Feynman parametrization. Only the ultraviolet cutoff Λ_{UV} is needed to render the theory finite, while Λ_{IR} is introduced to mimic confinement.

NJL Lagrangian

In the qq channel, the NJL Lagrangian for $SU(2)$ flavor is given by:

$$\begin{aligned}\mathcal{L} = \bar{q}(i\partial\!\!\!/ - \hat{m})q &+ G_s \left(\bar{q}\gamma_5 C\tau_2\beta_A\bar{q}^T\right)\left(q^T C^{-1}\gamma_5\tau_2\beta_A q\right) \\ &+ G_a \left(\bar{q}\gamma_\mu C\vec{\tau}\tau_2\beta_A\bar{q}^T\right)\left(q^T C^{-1}\gamma^\mu\tau_2\vec{\tau}\beta_A q\right) \\ &- G_p \left(\bar{q}C\tau_2\beta_A\bar{q}^T\right)\left(q^T C^{-1}\tau_2\beta_A q\right) \\ &+ G_a \left(\bar{q}\gamma_\mu\gamma_5 C\tau_2\beta_A\bar{q}^T\right)\left(q^T C^{-1}\gamma^\mu\gamma_5\tau_2\beta_A q\right), \qquad (1)\end{aligned}$$

where $C = i\gamma_2\gamma_0$ is the charge conjugation matrix and $\beta_A = \sqrt{\frac{3}{2}}\lambda_A$ $(A = 2, 5, 7)$. q is the quark field, $\hat{m} \equiv \mathrm{diag}[m_u, m_d]$ is the current quark mass matrix, which we take as $m_u = m_d$. $\vec{\tau}$ are the Pauli matrices for the $SU(2)$ flavor, and G_s, G_p, and G_a are the coupling constant of the four-fermi interaction in each diquark interaction channel. We will respect the three-flavor chiral symmetry and thus take $G_p = G_s$.

Mass gap equation [3]

The NJL gap equation in the Hartree-Fock approximation is shown below, where the thin line represents the elementary quark propagator, $S_0^{-1}(k) = \not{k} - m + i\varepsilon$, and the shaded circle represents the $\bar{q}q$ interaction kernel. Higher-order terms, attributed to meson loops, for example, are not included in the gap equation kernel.

$$i\,S^{-1}(k) = i\,S_0^{-1}(k) - \sum_{\Omega} K_\Omega\,\Omega \int \frac{d^4\ell}{(2\pi)^4}\,\mathrm{Tr}[\bar{\Omega}\,i\,S(\ell)],$$

The dressed quark propagator thus has the solution
$S(k)^{-1} = \not{k} - M + i\varepsilon$, where

$$M = m + 12\,i\,G_\pi \int \frac{d^4\ell}{(2\pi)^4}\,\mathrm{Tr}_D[S(\ell)]$$

[3] I. C. Cloët, W. Bentz, and A. W. Thomas, Phys. Rev. C 90, 045202 (2014)

Dressed mass as a function of the coupling [4]

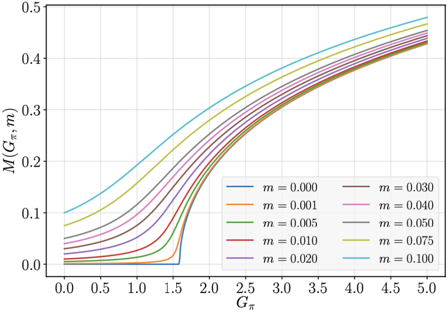


Figure 2.2. Dressed mass for the $1+1$ NJL model as a function of G_π and the bare mass.

[4] P. R. Ramírez, PhD dissertation, Illinois Institute of Technology, 2021

Bethe-Salpeter equation

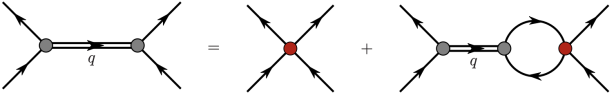


FIG. 2. (Color online) NJL Bethe-Salpeter equation for the quark-antiquark t matrix, represented as the double line with the vertices. The single line corresponds to the dressed quark propagator and the BSE $\bar{q}q$ interaction kernel, consistent with the gap equation kernel used in Eq. (5), is given by Eq. (2).

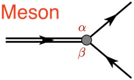
The NJL BSE, consistent with the gap equation of Fig. 1, is illustrated in Fig. 2 and reads

$$\mathcal{T}(q) = \mathcal{K} + \int \frac{d^4k}{(2\pi)^4}\, \mathcal{K}\, S(k+q)\, S(k)\, \mathcal{T}(q), \qquad (7)$$

This works for both mesons and diquarks very similarly.

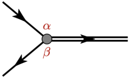
Bethe-Salpeter vertices

Meson



$$\Omega^{\sigma}_{\alpha\beta} = \sqrt{-Z_\sigma}\,(\mathbb{1})_{\alpha\beta}\,,\quad \Omega^{\eta}_{\alpha\beta} = \sqrt{Z_\eta}\,(\gamma_5)_{\alpha\beta}\,,\quad \Omega^{\omega}_{\alpha\beta} = \sqrt{-Z_\omega}\,(\gamma^\mu)_{\alpha\beta}\,,\quad \Omega^{f_1}_{\alpha\beta} = \sqrt{-Z_5}\,(\gamma^\mu\gamma_5)_{\alpha\beta}$$

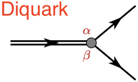
$$\Omega^{a_0}_{\alpha\beta} = \sqrt{-Z_{a_0}}\,(\tau_i)_{\alpha\beta}\,,\quad \Omega^{\pi}_{\alpha\beta} = \sqrt{Z_\pi}\,(\gamma_5\,\tau_i)_{\alpha\beta}\,,\quad \Omega^{\rho}_{\alpha\beta} = \sqrt{-Z_\rho}\,(\gamma^\mu\,\tau_i)_{\alpha\beta}\,,\quad \Omega^{a_1}_{\alpha\beta} = \sqrt{-Z_{a_1}}\,(\gamma^\mu\gamma_5\tau_i)_{\alpha\beta}$$



$$\overline{\Omega}^{\sigma}_{\beta\alpha} = \sqrt{-Z_\sigma}\,(\mathbb{1})_{\beta\alpha}\,,\quad \overline{\Omega}^{\eta}_{\beta\alpha} = \sqrt{Z_\eta}\,(\gamma_5)_{\beta\alpha}\,,\quad \overline{\Omega}^{\omega}_{\beta\alpha} = \sqrt{-Z_\omega}\,(\gamma^\nu)_{\beta\alpha}\,,\quad \overline{\Omega}^{f_1}_{\alpha\beta} = \sqrt{-Z_5}\,(\gamma^\mu\gamma_5)_{\alpha\beta}$$

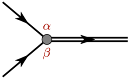
$$\overline{\Omega}^{a_0}_{\beta\alpha} = \sqrt{-Z_{a_0}}\,(\tau_i^\dagger)_{\beta\alpha},\quad \overline{\Omega}^{\pi}_{\beta\alpha} = \sqrt{Z_\pi}\,(\gamma_5\,\tau_i^\dagger)_{\beta\alpha},\quad \overline{\Omega}^{\rho}_{\beta\alpha} = \sqrt{-Z_\rho}\,(\gamma^\nu\,\tau_i^\dagger)_{\beta\alpha},\quad \overline{\Omega}^{a_1}_{\alpha\beta} = \sqrt{-Z_{a_1}}\,(\gamma^\mu\gamma_5\tau_i^\dagger)_{\alpha\beta}$$

Diquark



$$\Omega^{s}_{\alpha\beta} = \sqrt{Z_s}\,(\gamma_5\,C\,\tau_2\,\beta_A)_{\alpha\beta}\,,\qquad \Omega^{p}_{\alpha\beta} = \sqrt{-Z_p}\,(C\,\tau_2\,\beta_A)_{\alpha\beta}\,,$$

$$\Omega^{a}_{\alpha\beta} = \sqrt{-Z_a}\,(\gamma^\mu\,C\,\tau_i\tau_2\,\beta_A)_{\alpha\beta}\,,\qquad \Omega^{v}_{\alpha\beta} = \sqrt{-Z_v}\,(\gamma^\mu\gamma_5\,C\,\tau_2\,\beta_A)_{\alpha\beta}$$



$$\overline{\Omega}^{s}_{\alpha\beta} = \sqrt{Z_s}\left(C^{-1}\,\gamma_5\,\tau_2\,\beta_A\right)_{\alpha\beta},\qquad \overline{\Omega}^{p}_{\alpha\beta} = \sqrt{-Z_p}\left(C^{-1}\,\tau_2\,\beta_A\right)_{\alpha\beta},$$

$$\overline{\Omega}^{a}_{\alpha\beta} = \sqrt{-Z_a}\left(C^{-1}\,\gamma^\nu\,\tau_2\tau_i^\dagger\,\beta_A\right)_{\alpha\beta},\qquad \overline{\Omega}^{v}_{\alpha\beta} = \sqrt{-Z_v}\left(C^{-1}\,\gamma^\nu\gamma_5\,\tau_2\,\beta_A\right)_{\alpha\beta}$$

Faddeev equation

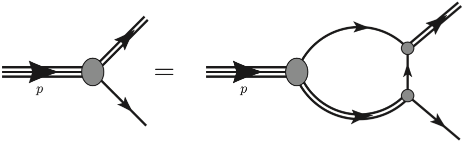


FIG. 3. Homogeneous Faddeev equation for the nucleon in the NJL model. The single lines represent the quark propagator and the double lines the diquark propagators.

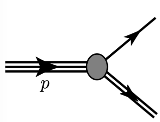
$$X^a_{\alpha,i}(P,p) = \int \frac{d^4\ell}{(2\pi)^4} Z^{ab,ij}_{\alpha\beta}(p,\ell)\, S_j(\tfrac{1}{2}P+\ell)_{\beta\gamma}\, \tau^{ki}_{bc}(\tfrac{1}{2}P-\ell)\, X^c_{\gamma,j}(P,p).$$

where

$$Z^{ab,ij}_{\alpha\beta}(p,\ell) = \left[\Omega^{b,ik}\, S^T_k(-\ell-p)\, \overline{\Omega}^{a,kj}\right]_{\alpha\beta}$$

The nucleon-quark-diquark vertex

The nucleon vertex function is parametrized by



$$\Gamma_s = \sqrt{-Z_N}\,\alpha_1\,\chi_t\,u(p,s), \qquad \Gamma_a = \sqrt{-Z_N}\left[\alpha_2\,\frac{p^\mu}{M_N}\,\gamma_5 + \alpha_3\,\gamma^\mu\gamma_5\right]\frac{\tau_i}{\sqrt{3}}\,\chi_t\,u(p,s),$$

$$\Gamma_p = \sqrt{-Z_N}\,\alpha_4\,\gamma_5\,\chi_t\,u(p,s), \qquad \Gamma_v = \sqrt{-Z_N}\left[\alpha_5\,\frac{p^\mu}{M_N} + \alpha_6\,\gamma^\mu\right]\chi_t\,u(p,s),$$

Therefore the Faddeev equation reads

$$\begin{bmatrix} \alpha_1 \\ \alpha_2\,\frac{p^\mu}{M_N}\gamma_5 + \alpha_3\,\gamma^\mu\gamma_5 \\ \alpha_4\,\gamma_5 \\ \alpha_5\,\frac{p^\mu}{M_N} + \alpha_6\,\gamma^\mu \end{bmatrix} u(p,\lambda)$$

$$= \frac{3}{M}\begin{pmatrix} 1 & \sqrt{3}\,\gamma^\sigma\,\gamma_5 & \gamma_5 & \gamma^\sigma \\ \sqrt{3}\,\gamma_5\,\gamma^\mu & -\gamma^\sigma\,\gamma^\mu & \sqrt{3}\,\gamma^\mu & -\sqrt{3}\,\gamma^\sigma\gamma^\mu\gamma_5 \\ \gamma_5 & \sqrt{3}\,\gamma^\sigma & 1 & \gamma^\sigma\gamma_5 \\ -\gamma^\mu & \sqrt{3}\,\gamma^\sigma\,\gamma^\mu\gamma_5 & \gamma^\mu\gamma_5 & -\gamma^\sigma\gamma^\mu \end{pmatrix}\begin{pmatrix} \Pi_{Ns} & 0 & 0 & 0 \\ 0 & \Pi^{Na}_{\sigma\nu} & 0 & 0 \\ 0 & 0 & \Pi_{Np} & 0 \\ 0 & 0 & 0 & \Pi^{Nv}_{\sigma\nu} \end{pmatrix}\begin{bmatrix} \alpha_1 \\ \alpha_2\,\frac{p^\nu}{M_N}\gamma_5 + \alpha_3\,\gamma^\nu\gamma_5 \\ \alpha_4\,\gamma_5 \\ \alpha_5\,\frac{p^\nu}{M_N} + \alpha_6\,\gamma^\nu \end{bmatrix} u(p,\lambda)$$

Model parameters

The two-flavor NJL has the following parameters:

$\bar{q}q$ couplings:	G_π, G_ρ, G_ω
qq couplings:	$G_s(= G_p)$, $G_a(= G_v)$
masses:	$m_u = m_d$
regularization:	Λ_{IR}, Λ_{UV}

We assign values *a priori* to the following parameters:

$$\Lambda_{IR} = 240\ MeV \text{ and } M = 400\ MeV$$

The remaining parameters can then be fixed by

$$\Lambda_{UV} \leftrightarrow f_\pi$$
$$G_{\pi,\rho,\omega} \leftrightarrow m_{\pi,\rho,\omega}$$
$$G_s,\, G_a \leftrightarrow M_N,\, M_\Delta$$

Quark light-cone momentum distributions

The leading twist spin-independent and spin-dependent quark light-cone momentum distributions in the nucleon are defined by the following equations:

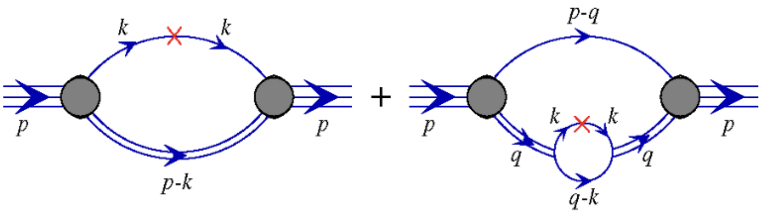
$$f_q(x) = \int \frac{d\xi^-}{2\pi} e^{ixp^+\xi^-} < p,s|\bar{\psi}_q(0)\frac{1}{2}\gamma^+\psi_q(\xi^-)|p,s>_c,$$

$$\Delta f_q(x) = \int \frac{d\xi^-}{2\pi} e^{ixp^+\xi^-} < p,s|\bar{\psi}_q(0)\frac{1}{2}\gamma^+\gamma_5\psi_q(\xi^-)|p,s>_c,$$

where ψ_q is the quark field of flavor q, x is the Bjorken scaling variable and the subscript c means that only connected matrix elements are included.

Feynman diagrams for the PDF

At our model scale, there is no sea quark and no gluons. The Feynman diagram for calculating the valence quark PDFs are shown below.



The single line represents the quark propagator and the double line the diquark propagator. The shaded oval denotes the quark-diquark vertex function and the red cross represents the operator insertion which has the form of $\gamma^+\delta(x - k^+/p^+)\frac{1}{2}(1 \pm \tau_z)$ for the spin-independent distribution and $\gamma^+ \to \gamma^+\gamma_5$ for the spin-dependent one.

Helicity distributions

For the calculation of the spin-dependent PDFs, we use the result

$$u(p,s)\bar{u}(p,s) = \left(\not{p} + M_N\right)\frac{1+\gamma_5 \not{s}}{2},$$

where s^μ is the spin vector of the particle satisfying $s^2 = -1$ and $s \cdot p = 0$. In general, s^μ can be written as

$$s^\mu = \left(\frac{\vec{p}\cdot\vec{n}}{M_N},\ \vec{n} + \frac{(\vec{p}\cdot\vec{n})\vec{p}}{M_N\left(M_N + p^0\right)}\right)$$

where $\vec{n} = \frac{\vec{p}}{|\vec{p}|}$ if the particle is longitudinally polarised, $\vec{n}\cdot\vec{p} = 0$ if transversely polarized.
For the helicity distribution, the proton is longitudinally polarized, and the helicity distribution is defined as

$$\Delta f(x) = f_+(x) - f_-(x),$$

i.e., the difference in the distributions of the quark's spin aligned with the proton's versus the quark's spin anti-aligned with the proton's.

Results for the spin-independent PDFs

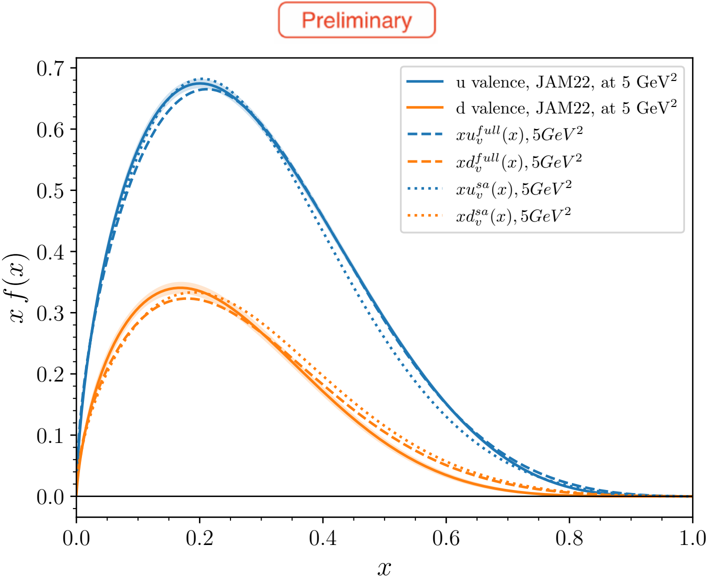
- The model scale is found to be 0.165 GeV^2, which is comparable but slightly higher than the previous 0.16 GeV^2. This is understandable because we've added more complexity to the model.
- The number and momentum sum rules are satisfied

$$\int_0^1 dx f_{q/P}(x) = N_{q/P},$$

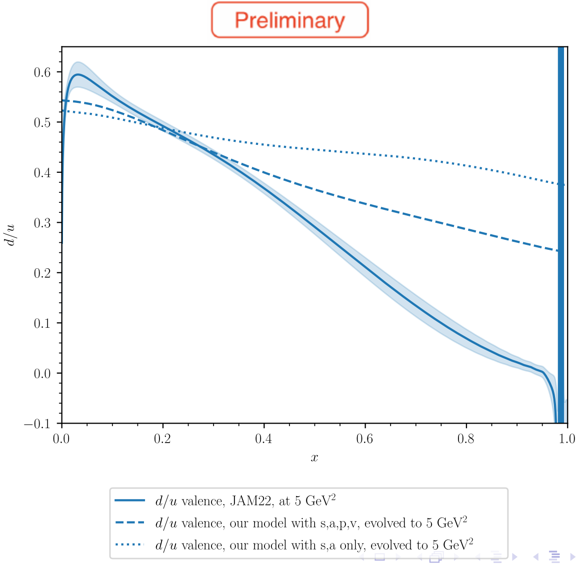
$$\int_0^1 dx x \left[f_{u/P}(x) + f_{d/P}(x) \right] = 1.$$

- To compare our results to the experimental data, need to evolve to a higher energy scale where empirical PDFs are available.

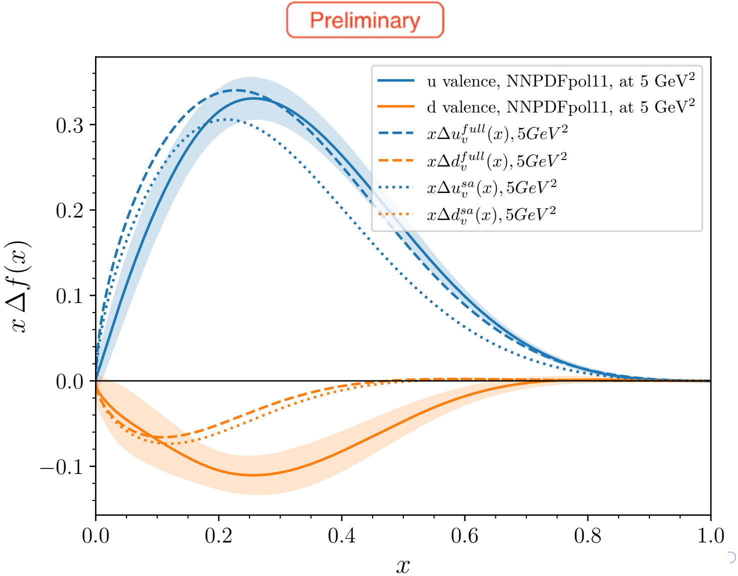
Results for the spin-independent PDFs



Results for the spin-independent PDFs



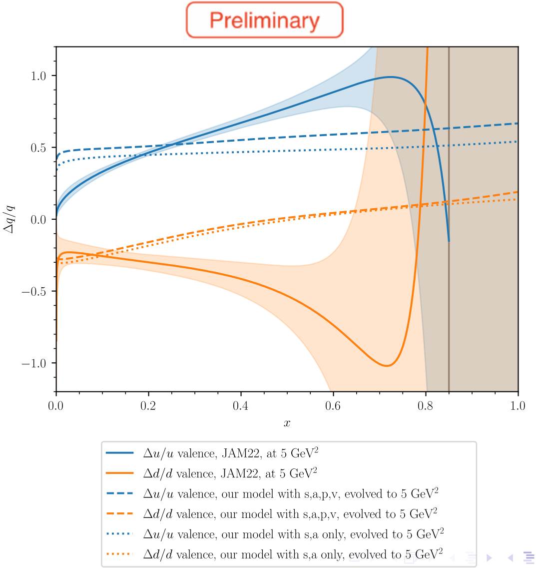
Results for the spin-dependent PDFs



Results for the spin-dependent PDFs

- For the full model we obtain a g_A value of 1.207, which is in excellent agreement to the known value of $g_A = 1.267$. In comparison, the model with only scalar and axial vector diquarks obtains a g_A value of 1.092.
- The positivity constraints for the PDFs are satisfied in our model
$$f_q(x) \geq 0, \;\; f_q(x) \geq |\Delta f_q(x)|.$$
- As $x \to 1$, our result calculated from the full model gives $\Delta u/u \approx 0.67$ while $\Delta d/d \approx 0.19$. In comparison, the model with scalar and axial vector gives a $\Delta u/u \approx 0.54$ while the $\Delta d/d$ ratio approaches 0.14.

Results for the spin-dependent PDFs



Transversity PDFs

The leading twist transversity quark light-cone momentum distributions in the nucleon are defined equivalently by Eq. (1) or Eq. (2) below.

$$p_- \int \frac{d\xi^-}{2\pi} e^{ixp^+\xi^-} \langle p, s|\bar{\psi}_q(0) i\sigma^{+i} \psi_q(\xi^-)|p, s\rangle_c = \bar{u}(p) i\sigma^{+i} u(p) \Delta_T f_q(x), \quad (1)$$

$$p_- \int \frac{d\xi^-}{2\pi} e^{ixp^+\xi^-} \langle p, s|\bar{\psi}_q(0) \sigma^{+j} \gamma_5 \psi_q(\xi^-)|p, s\rangle_c = \bar{u}(p) \sigma^{+j} \gamma_5 u(p) \Delta_T f_q(x). \quad (2)$$

Transversity PDFs

- Defining the two operators for the transversity PDF calculation as $\mathcal{O}_1 = \frac{1}{2} i\sigma^{+i} = \frac{1}{2}\gamma^i\gamma^+$, where $i = 1, 2$, and $\mathcal{O}_2 = \frac{1}{2}\sigma^{+j}\gamma_5 = \frac{1}{2} i\gamma^+\gamma^j\gamma_5$, where $j = 1, 2$, $j \neq i$, we can prove that $\mathcal{O}_1 = \epsilon^{+-ij}\mathcal{O}_2$.
- To get the PDF itself, we need to divide the results of the diagrammatic evaluation by $\frac{1}{2p^+}\bar{u} i\sigma^{+i} u = \frac{1}{2p^+} 2i\epsilon^{+i-j} p_- s_j = i\epsilon^{+i-j} s_j = i\epsilon^{+-ij} s^j$ for $\mathcal{O}_1$, and $\frac{1}{2p^+}\bar{u}\sigma^{+j}\gamma_5 u = \frac{1}{2p^+} i2p^+ s^j = is^j$ for $\mathcal{O}_2$.
- Thus, the PDFs obtained in our calculation using $\mathcal{O}_1$ and dividing by $i\epsilon^{+-ij}s^j$, versus using $\mathcal{O}_2$ and dividing by is^j are formally completely equivalent, and we do get that in our results.

Treatment of γ_5

As is well-known in QCD calculations, a regularization scheme such as dimensional regularization introduces complication for the four-dimensional objects such as the Dirac matrix γ_5. In order to alter the dimension of the space-time, a prescription must be in place to define γ_5 in $d \neq 4$ dimensions. As shown in Ref. [5], one can use the prescription referred to as the Larin scheme, where one replaces γ_5 with

$$\gamma_5 \longrightarrow \frac{1}{24} i\varepsilon_{\mu\nu\rho\sigma}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma, \tag{3}$$

and replace

$$\gamma^\alpha\gamma_5 \longrightarrow \frac{1}{2}\left(\gamma^\alpha\gamma_5 - \gamma_5\gamma^\alpha\right) \tag{4}$$

or

$$\gamma_5\gamma^\alpha \longrightarrow -\frac{1}{2}\left(\gamma^\alpha\gamma_5 - \gamma_5\gamma^\alpha\right). \tag{5}$$

[5] S. Moch, J. A. M. Vermaseren, and A. Vogt, Phys. Lett. B 748, 432 (2015)

Treatment of γ_5

This procedure is equivalent to using

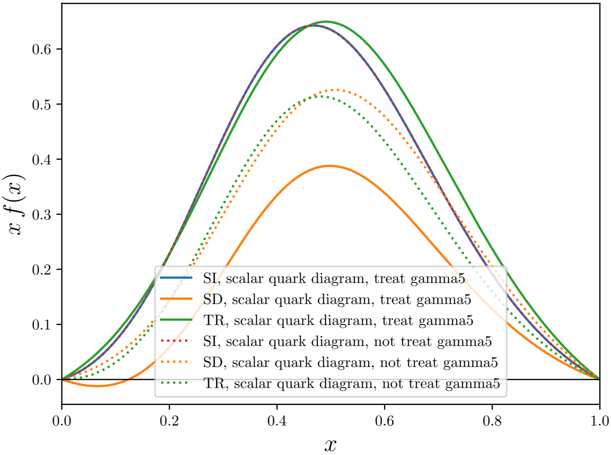
$$\gamma^\alpha\gamma_5 \longrightarrow \frac{1}{6} i\varepsilon^{\alpha\mu\nu\rho}\gamma_\mu\gamma_\nu\gamma_\rho, \tag{6}$$

or

$$\gamma_5\gamma^\alpha \longrightarrow -\frac{1}{6} i\varepsilon^{\alpha\mu\nu\rho}\gamma_\mu\gamma_\nu\gamma_\rho. \tag{7}$$

We will mostly use Eqs.(6)-(7) in our calculation, since it introduces less γ matrices than the first prescription and thus can speed up the computation. However, the two methods are completely equivalent and can be used interchangeably. After the replacements, we perform the trace in d dimensions, and at the end set $d = 4$.

Treatment of γ_5 impacting the results for the operators containing γ_5



Results for the transversity PDFs

- The u quark transversity distribution slightly violates its Soffer bound in the small x ($0 < x \lesssim 0.2$) and large x ($0.8 \lesssim x < 1$) regions, while the d quark distribution completely satisfies its Soffer bound. The violation of the Soffer bound is likely due to the approximations made in our model, such as the static approximation, and the on-shell approximation for the diquark states.
- Our calculated results satisfy the positivity constraints for the transversity PDFs

$$f_q(x) \geq |\Delta_T f_q(x)|. \tag{8}$$

Results for the u quark transversity PDFs

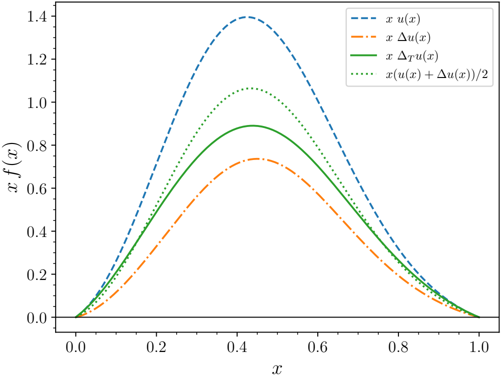


Figure: All 3 leading twist PDFs of u quark in a proton at the model scale, with the Soffer bound for the transversity PDF plotted in dotted line.

Results for the d quark transversity PDFs

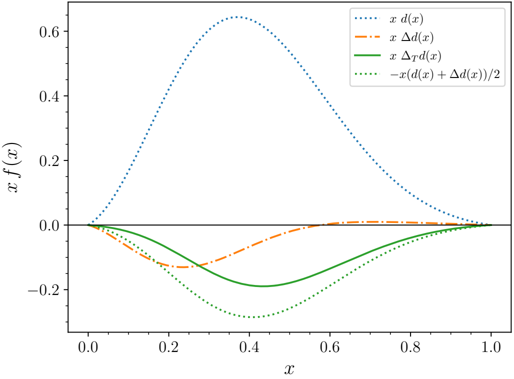
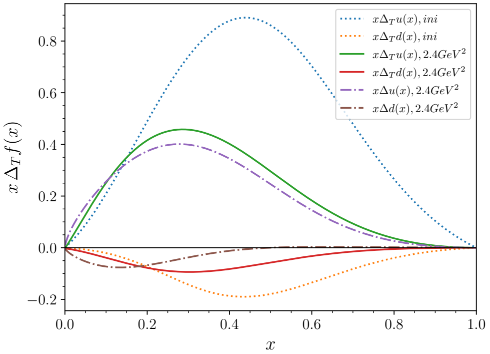


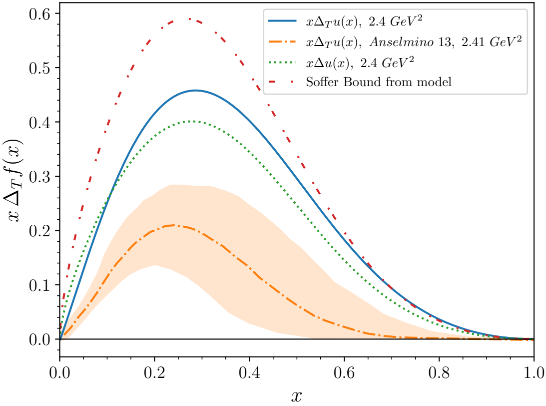
Figure: All 3 leading twist PDFs of d quark in a proton at the model scale, with the Soffer bound for the transversity PDF plotted in dotted line.

Results for the transversity PDFs

The transversity u and d quark distributions in a proton at the model scale, as well as at 2.4 GeV^2, where we compare to their helicity counterparts calculated from our model, evolved to 2.4 GeV^2.

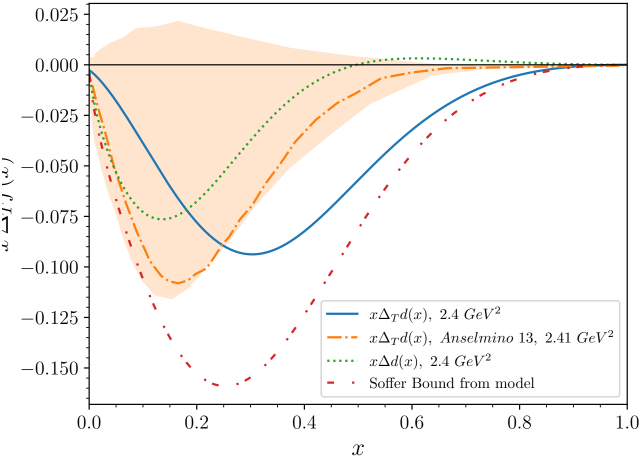


Transversity PDFs of u quark in a proton comparing with experiments [6]



[6] M. Anselmino, M. Boglione, U. D'Alesio, S. Melis, F. Murgia, and A. Prokudin, Phys. Rev. D 87, 094019 (2013)

Transversity PDFs of d quark in a proton comparing with experiments



Results for the transversity PDFs

- For the first moments of the transversity valence quark distributions, we obtain $\int_0^1 dx \Delta_T u_v(x, Q_0^2) = 1.40$ and $\int_0^1 dx \Delta_T d_v(x, Q_0^2) = -0.233$ at the model scale $Q_0^2 = 0.165$ GeV^2.
- This gives a nucleon isovector tensor charge of $g_T = 1.63$ and an isosclar charge of $g_T^0 = 1.17$ at $Q_0^2 = 0.165$ GeV^2.
- This gives a ratio of $\delta d / \delta u = -0.166$.

Summary and future work

- We used the framework of the relativistic Faddeev equation in the NJL model to calculate the quark LC momentum distributions in the nucleon based on a straightforward Feynman diagram evaluation.
- We treated the Dirac matrix γ_5 with the Larin scheme, and used consistent regularization prescription for all 3 leading twist PDFs.
- The work can be extended to calculate GPDs and TMDs, or to a finite baryon density calculation.

Thank you for your attention!

Model parameters

The Λ_{UV} and G_π together is determined by the pion decay constant and pion mass.

$$< 0|\bar{\psi}\gamma_\mu\gamma_5\psi|\pi(q) >= 2if_\pi q_\mu$$

From this we can obtain

$$f_\pi = -12i\sqrt{Z_\pi}M\int\frac{d^4k}{(2\pi)^4}\frac{1}{(k^2-M^2)\left((k+q)^2-M^2\right)}|_{q^2=m_\pi^2}$$

By solving the Bethe-Salpeter equation for the mesons, m_π and Z_π can be related to the parameter G_π. Thus, this equation, together with the value of m_π, determines the parameters of Λ_{UV} and G_π.

Model parameters

- Similarly, m_ρ and m_ω determine the parameters G_ρ and G_ω, respectively.
- G_s and G_a are determined by solving the two Faddeev equations for the nucleon and the delta baryon.
- We obtain $G_s = 7.65\ GeV^{-2}$ and $G_a = 4.91\ GeV^{-2}$.
- The corresponding diquark masses are $M_s = 0.679\ GeV$, $M_p = 0.945\ GeV$, $M_a = 0.929\ GeV$, and $M_v = 1.099\ GeV$.
- Compared to the previous values obtained without the pseudoscalar and vector diquark channels, $M_s = 0.768\ GeV$ and $M_a = 0.929\ GeV$, the scalar diquark mass got smaller, while the axial vector diquark is exactly the same. The axial vector diquark mass does not change because the delta baryon Faddeev equation only concerns the axial vector diquark, and is thus unchanged from the previous work.

Isospin factors

By separating the isospin factors, the spin-independent u and d distributions in the proton can be expressed as

$$
\begin{aligned}
u_v(x) = {} & f^{s}_{q/N}(x) + f^{p}_{q/N}(x) + \frac{1}{3} f^{a}_{q/N}(x) + f^{v}_{q/N}(x) \\
& + f^{ss}_{q(D)/N}(x) + f^{pp}_{q(D)/N}(x) + \frac{5}{3} f^{aa}_{q(D)/N}(x) + f^{vv}_{q(D)/N}(x) \\
& + f^{sp}_{q(D)/N}(x) + f^{ps}_{q(D)/N}(x) + \frac{1}{\sqrt{3}} f^{sa}_{q(D)/N}(x) + \frac{1}{\sqrt{3}} f^{as}_{q(D)/N}(x) \\
& + f^{sv}_{q(D)/N}(x) + f^{vs}_{q(D)/N}(x) + \frac{1}{\sqrt{3}} f^{pa}_{q(D)/N}(x) + \frac{1}{\sqrt{3}} f^{ap}_{q(D)/N}(x) \\
& + f^{pv}_{q(D)/N}(x) + f^{vp}_{q(D)/N}(x) + \frac{1}{\sqrt{3}} f^{av}_{q(D)/N}(x) + \frac{1}{\sqrt{3}} f^{va}_{q(D)/N}(x).
\end{aligned}
$$

Isospin factors

And

$$
\begin{aligned}
d_v(x) = {} & \frac{2}{3} f^{a}_{q/N}(x) \\
& + f^{ss}_{q(D)/N}(x) + f^{pp}_{q(D)/N}(x) + \frac{1}{3} f^{aa}_{q(D)/N}(x) + f^{vv}_{q(D)/N}(x) \\
& + f^{sp}_{q(D)/N}(x) + f^{ps}_{q(D)/N}(x) - \frac{1}{\sqrt{3}} f^{sa}_{q(D)/N}(x) - \frac{1}{\sqrt{3}} f^{as}_{q(D)/N}(x) \\
& + f^{sv}_{q(D)/N}(x) + f^{vs}_{q(D)/N}(x) - \frac{1}{\sqrt{3}} f^{pa}_{q(D)/N}(x) - \frac{1}{\sqrt{3}} f^{ap}_{q(D)/N}(x) \\
& + f^{pv}_{q(D)/N}(x) + f^{vp}_{q(D)/N}(x) - \frac{1}{\sqrt{3}} f^{av}_{q(D)/N}(x) - \frac{1}{\sqrt{3}} f^{va}_{q(D)/N}(x).
\end{aligned}
$$

Treatment of γ_5

Within our proper-time regularization prescription, certain operations do not commute, e.g., Lorentz contraction and regularization procedure. For example, for the integral of the type

$$i\int \frac{d^d l}{(2\pi)^d}\frac{l^2}{(l^2-\Delta+i\epsilon)^n}, \tag{9}$$

where $(l^2-\Delta+i\epsilon)^n$ is the combined denominator using Feynman parameterization, we apply the mathematical identity

$$\begin{aligned} & i\int \frac{d^d l}{(2\pi)^d}\frac{l^2}{(l^2-\Delta+i\epsilon)^n} \\ =& i\int \frac{d^d l}{(2\pi)^d}\frac{\Delta}{(l^2-\Delta+i\epsilon)^n} + i\int \frac{d^d l}{(2\pi)^d}\frac{1}{(l^2-\Delta+i\epsilon)^{n-1}}. \end{aligned} \tag{10}$$

Treatment of γ_5

However, for the tensor type of integral

$$i\int \frac{d^d l}{(2\pi)^d}\frac{l^\mu l^\nu}{(l^2-\Delta+i\epsilon)^n} = i\int \frac{d^d l}{(2\pi)^d}\frac{\frac{1}{d}g^{\mu\nu}l^2}{(l^2-\Delta+i\epsilon)^n}, \tag{11}$$

we apply the integration by parts relation

$$\begin{aligned} & i\int \frac{d^d l}{(2\pi)^d}\frac{l^2}{(l^2-\Delta+i\epsilon)^n} \\ =& \frac{-1}{2(n-1)} i\int \frac{d^d l}{(2\pi)^d} l_\mu \frac{\partial}{\partial l_\mu}\frac{1}{(l^2-\Delta+i\epsilon)^{n-1}} \\ =& \frac{d}{2(n-1)} i\int \frac{d^d l}{(2\pi)^d}\frac{1}{(l^2-\Delta+i\epsilon)^{n-1}}. \end{aligned} \tag{12}$$

Treatment of γ_5

Thus, performing the Lorentz contractions before or after the regularization procedure will make a difference in the final results. Therefore, we have developed a computational prescription that is guided by preserving symmetries and reproducing exact results. The computational procedure is the following:

1. The epsilon tensors associated with the Bethe-Salpeter/Faddeev vertices are contracted with the result of the d-dimensional trace immediately after the trace is taken.
2. The regularization scheme is then applied to the result as the loop momentum integration is performed.
3. After the integration, the epsilon tensor from the operator is contracted.
4. Finally, the epsilon tensor from the spin projection operator is contracted with the result from 3.

Treatment of γ_5

As an example, the scalar quark diagram can be written down in our model as

$$\Delta_T f^s_{q/N}(x) = \bar{\Gamma}^s_N \int \frac{d^4k}{(2\pi)^4} \delta(xp\cdot n - k\cdot n)\ (iS(k))\ \mathcal{O}_{1,2}\ (iS(k))\ (-\tau_s(p-k))\ \Gamma^s_N$$
$$= \bar{\Gamma}^s_N \int \frac{d^4k}{(2\pi)^4} \delta(xp\cdot n - k\cdot n)\ S(k)\ \mathcal{O}_{1,2}\ S(k)\ \tau_s(p-k)\ \Gamma^s_N . \quad (13)$$

Plugging in the vertex function,

$$\Delta_T f^s_{q/N}(x) = -Z_N\alpha_1^2\ \bar{u}(p,s) \int \frac{d^4k}{(2\pi)^4} \delta(xp\cdot n - k\cdot n)\ S(k)\ \mathcal{O}_{1,2} S(k)\ \tau_s(p-k)\ u(p,s)$$
$$= -Z_N\alpha_1^2 \int \frac{d^4k}{(2\pi)^4} \delta(xp\cdot n - k\cdot n)\ Tr[\ S(k)\ \mathcal{O}_{1,2}\ S(k)\ \tau_s(p-k)\ (\not{p} + M_N)\ \frac{1+\gamma_5\not{s}}{2}]. \quad (14)$$

Plugging in the propagators,

$$\Delta_T f^s_{q/N}(x) = \frac{1}{2} Z_N Z_s \alpha_1^2\ i \int \frac{d^4k}{(2\pi)^4} \delta(xp\cdot n - k\cdot n)\ \frac{Tr[\ (\not{k}+M)\ \mathcal{O}_{1,2}\ (\not{k}+M)\ \ (\not{p}+M_N)\ (1+\gamma_5\not{s})]}{(k^2-M^2+i\ \varepsilon)^2[(p-k)^2 - M_s^2 + i\ \varepsilon]} . \quad (15)$$

Treatment of γ_5

Depending on the type of diquarks involved and which operator is used, there can be occurrence of γ_5 in the Faddeev and conjugate Faddeev vertices Γ_N and $\bar{\Gamma}_N$, or the operator $\mathcal{O}_1$ but not the $\mathcal{O}_2$. Including the explicit γ_5 in Eq. (15), there can be up to 4 γ_5 matrices in one Feynman diagram evaluation. For the scalar quark diagram, however, there are only up to two γ_5 matrices if we use the operator $\mathcal{O}_1$. When operator $\mathcal{O}_1$ is used, the treatment of γ_5 makes a huge impact on our result, as can be seen from Fig. **??**. While if we use the operator $\mathcal{O}_2$, there is no difference in the results whether we treat γ_5 or not.

Treatment of γ_5

As shown in Fig. **??**, although the spin-independent result remains the same since there is no γ_5 in the equation, both the helicity result and the transversity result using the operator $\mathcal{O}_1$ change a lot depending on whether we treat γ_5 with the Larin scheme, or not treat it and simply performing the trace in $d = 4$ dimensions, because of the γ_5 matrix contained in both the helicity and transversity $\mathcal{O}_1$ operator. As a result, treating γ_5 with the Larin scheme is necessary to obtain the agreement of the transversity PDF result using the two operators. Since the two operators are formally equivalent, they must give the same transversity PDF, and we achieve that by taking γ_5 as the Larin scheme.

Treatment of γ_5

As can be seen from Fig. **??**, where $\mathcal{O}_1$ is used for the TR results, for not treating the γ_5, $|\Delta_T f^s_{q/N}(x)|$ is mostly less than $|\frac{f^s_{q/N}(x)+\Delta f^s_{q/N}(x)}{2}|$ (3 dotted lines), while for treating the γ_5, $|\Delta_T f^s_{q/N}(x)|$ is modified to be significantly larger than $|\frac{f^s_{q/N}(x)+\Delta f^s_{q/N}(x)}{2}|$ (3 solid lines). Being the dominant diagram, this leads to the Soffer bound not to be satisfied in our new model, likely due to the approximations of our model. However, as we discussed before, it is essential to treat γ_5 correctly in order to have a consistent result for the transversity PDF between the two equivalent operators. In addition, if we do not treat γ_5, the helicity and transversity distributions turn out to be very similar, for this scalar quark diagram example, which is the dominant diagram in both the helicity and transversity PDFs. But treating the γ_5 sets them apart quite a bit.