The Cubic Casimir and photon $\longrightarrow \chi_c$ diffraction (in the background field of a proton) at the EIC

Adrian Dumitru
Baruch College & CUNY Grad School

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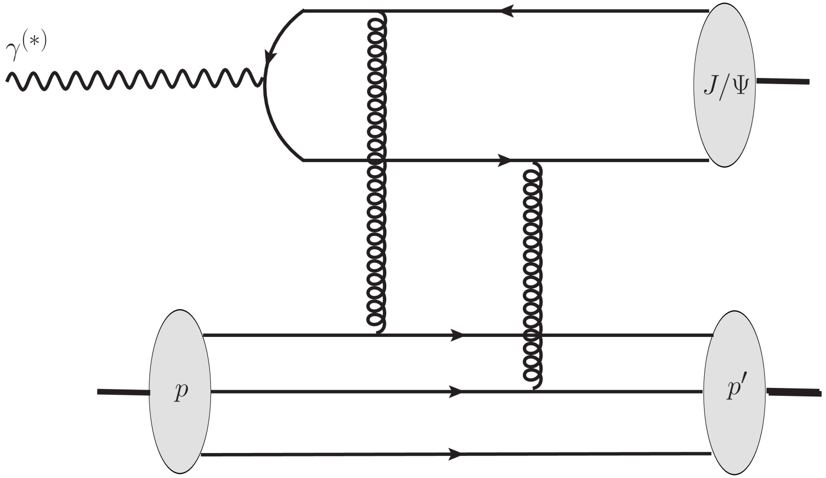
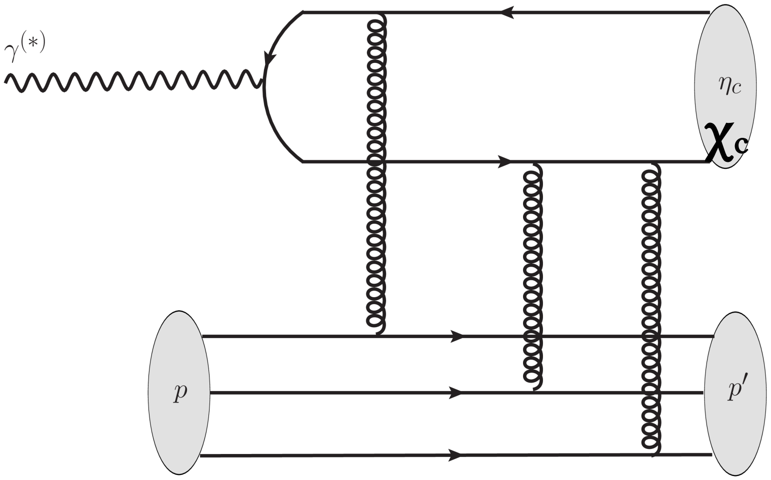
talk based on
S. Benic, A.D., A. Kaushik, L. Motyka, T. Stebel,
arXiv:2404.19134

Exclusive charmonium production processes



$$\delta^{ab}\,\langle A^{+a}A^{+b}\rangle$$

C and P even



$$d^{abc}\,\langle A^{+a}A^{+b}A^{+c}\rangle$$

C and P odd

eikonal fields

proportional to cubic Casimir / anomaly coefficient of fund. rep.

* hard scale(s): m_c, |t|, Q^2

* J/Ψ vector meson production observed & well measured at HERA (differential cross section dσ/dt)

* C-odd exchange related to cubic Casimir has not been seen (yet; except GlueX @ threshold & TOTEM in non-pert. regime)

Motivation, C-odd ggg / Odderon exchange :

* not "generic" for color-SU(Nc): exists only in Nc ≥ 3, not for SU(2); requires non-vanishing anomaly coefficient / cubic Casimir for N-repr

$$\frac{1}{2}\mathrm{tr}\,\{t^a, t^b\}t^c = A_F d^{abc}$$

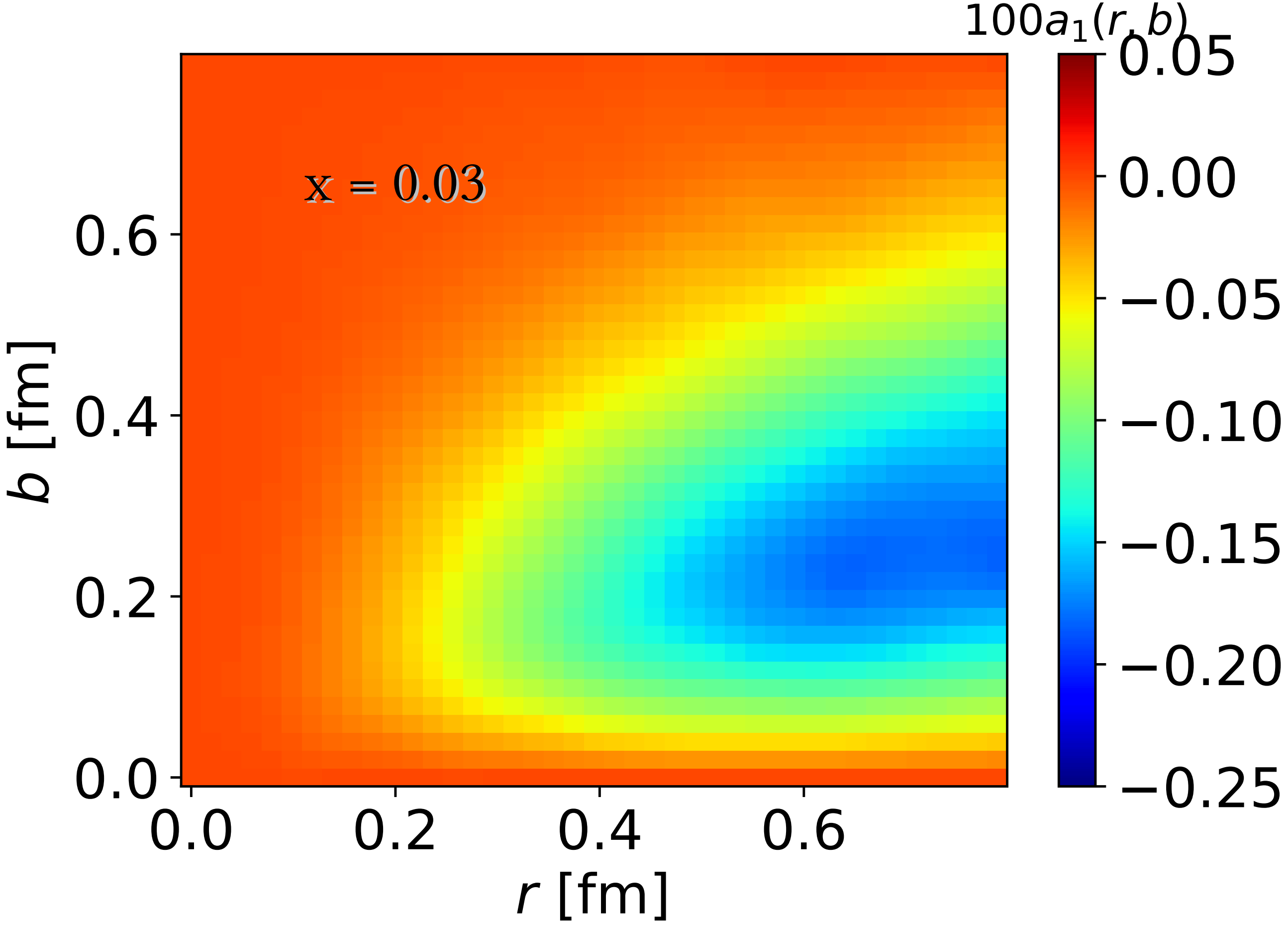
* coupling to proton depends critically on its structure!

- non-Gaussian color field fluctuations, $\langle A^{+a}(\vec{q}_1)\, A^{+b}(\vec{q}_2)\, A^{+c}(\vec{q}_3)\rangle \neq 0$
- magnitude ~ 100-1000 smaller than gg exchange (prediction of light-front |qqq (g)› models)
- t-dependence much weaker than gg exchange, proton much more likely to "survive" high $t \gtrsim 1\ \mathrm{GeV}^2$
- if EIC does not see this –> major puzzle for partonic proton structure

* high-energy evolution very different from gg / Pomeron exchange

Odderon amplitude O(**r**,**b**) / cos ∡(**r**,**b**):

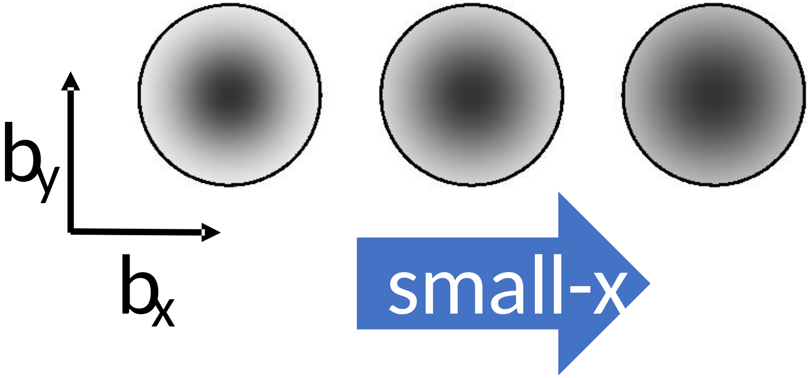
from Erratum to
A.D., H. Mantysaari, R. Paatelainen,
PRD 107 (2023) L011501



* |p> ≈ |qqq⟩ + |qqqg⟩

* note: for say r = 0.25 fm, max amplitude at rather <u>small b ~ 0.1 – 0.2 fm</u>

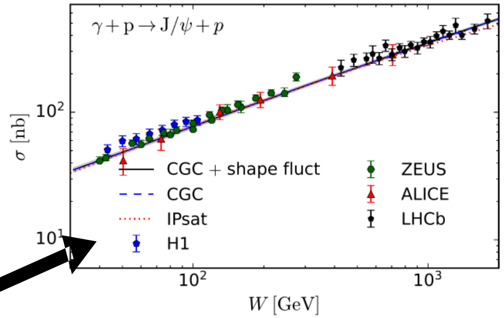
conventional picture of the proton:
snapshots of **Pomeron** at different x



Gluon saturation: proton resembles a **black-disc** as x -> 0

cross sections **grow** with energy!

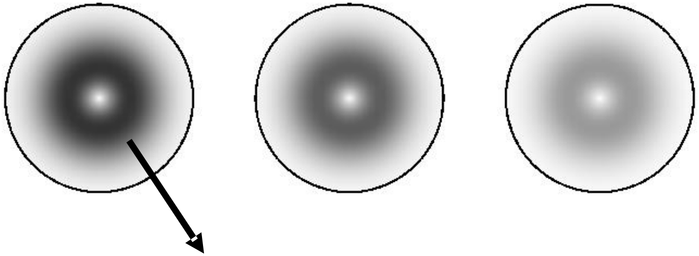
$\gamma p \to J/\psi\ p$



Mantysaari, Salazar, Schenke (2022)

proton through the **Odderon** “lens”:

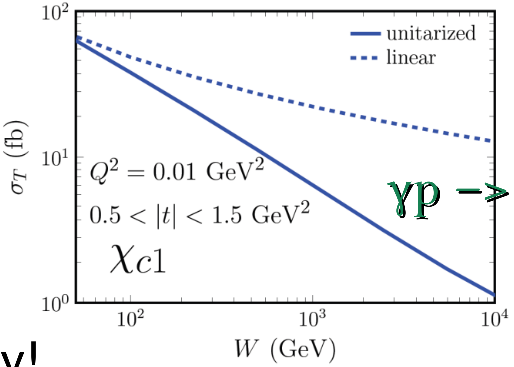
Kovchegov, Szymanowski, Wallon, 2004
Hatta, Iancu, Itakura, McLerran (2005)



Proton getting “whiter” as x -> 0

Odderon occupies a smaller region inside the proton (slope parameter << R^2)

cross sections **drop** with energy!



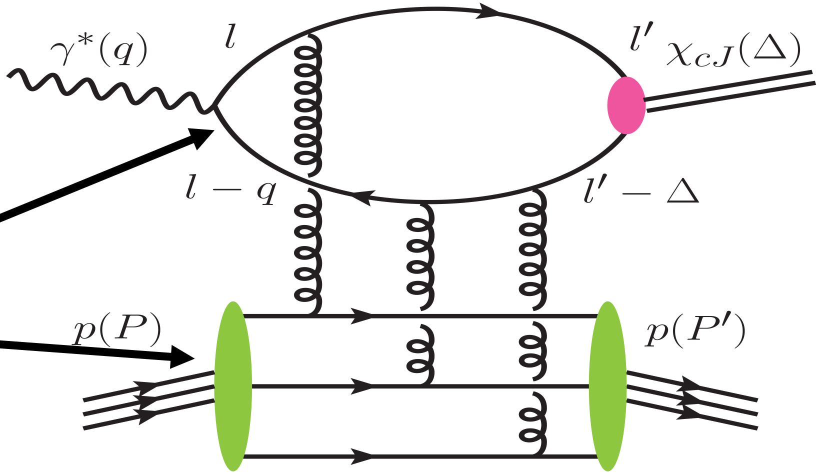
$\gamma p \to \chi_c\ p$

SB, Dumitru, Kaushik, Motyka, Stebel (2024)

Eikonal amplitude for $\gamma^{(*)} \to \chi_{cJ}$

$$\gamma^*(q)p(P) \to \mathcal{H}(\Delta)p(P')$$

$$\langle \mathcal{M}_{\lambda\bar{\lambda}} \rangle = 2q^- N_c \int_{\boldsymbol{r}_\perp \boldsymbol{b}_\perp} \mathrm{e}^{-\mathrm{i}\boldsymbol{\Delta}_\perp \cdot \boldsymbol{b}_\perp} \boxed{\mathrm{i}\mathcal{O}(\boldsymbol{r}_\perp, \boldsymbol{b}_\perp)} \mathcal{A}_{\lambda\bar{\lambda}}(\boldsymbol{r}_\perp, \boldsymbol{\Delta}_\perp)$$



reduced
amplitude

$$\mathcal{A}_{\lambda\bar{\lambda}}(\boldsymbol{r}_\perp, \boldsymbol{\Delta}_\perp) = \int_z \int_{\boldsymbol{l}_\perp \boldsymbol{l}'_\perp} \sum_{h\bar{h}} \Psi^{\gamma}_{\lambda,h\bar{h}}(\boldsymbol{l}_\perp, z) \Psi^{\mathcal{H}*}_{\bar{\lambda},h\bar{h}}(\boldsymbol{l}'_\perp - z\boldsymbol{\Delta}_\perp, z) \mathrm{e}^{\mathrm{i}(\boldsymbol{l}_\perp - \boldsymbol{l}'_\perp + \frac{1}{2}\boldsymbol{\Delta}_\perp)\cdot \boldsymbol{r}_\perp}$$

These amplitudes for J = 0, 1, 2 have been computed in arXiv:2402.19134

C-even charmonium wave functions

scalar part: “boosted Gaussian” model

$$\Psi^{\mathcal{H}}_{\bar{\lambda},h\bar{h}}(\boldsymbol{k}_\perp, z) \equiv \frac{1}{\sqrt{z\bar{z}}} \underbrace{\bar{u}_h(k)\Gamma^{\mathcal{H}}_{\bar{\lambda}}(k,k')v_{\bar{h}}(k')}_{\text{spin structure}} \boxed{\phi_{\mathcal{H}}(\boldsymbol{k}_\perp, z)}$$

spin 1 polarization vector

$$\Gamma^{\mathcal{H}}_{\bar{\lambda}}(k,k') = \begin{cases} \mathrm{i}\gamma_5 \;, & \mathcal{H} = \mathcal{P} \\ 1 \;, & \mathcal{H} = \mathcal{S} \\ \mathrm{i}\gamma_5 \not{E}(\bar{\lambda}, \Delta_0) \;, & \mathcal{H} = \mathcal{A} \\ \frac{1}{4}\left(\gamma_\mu(k_\nu - k'_\nu) + \gamma_\nu(k_\mu - k'_\mu)\right) E^{\mu\nu}(\bar{\lambda}, \Delta_0) \;, & \mathcal{H} = \mathcal{T} \end{cases}$$

$\mathcal{H} = \mathcal{P} \rightarrow \eta_c$

$\mathcal{H} = \mathcal{S}, \mathcal{A}, \mathcal{T}$ } χ_c

spin 2 coupling to EM tensor

spin 2 polarization tensor: in terms of $E^\mu(\bar{\lambda}, \Delta_0)$ via Clebsch-Gordans

transversality condition

$$\Delta_0 \cdot E(\bar{\lambda}, \Delta_0) = 0 \qquad \Delta_0 = k + k'$$

Odderon in the dipole/GTMD framework

Boussarie, Hatta, Szymanowski, Wallon (2019)

Zhou (2013)

Boer, Echevarria, Mulders, Zhou (2016)

Odderon as the imaginary part

$$\mathcal{O}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp) = -\frac{1}{2\mathrm{i}N_c}\mathrm{tr}\left\langle V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp) - V(\boldsymbol{y}_\perp)V^\dagger(\boldsymbol{x}_\perp)\right\rangle$$

decomposition into GTMDs at small-x

$$\mathcal{D}_{SS'}(\boldsymbol{k}_\perp,\boldsymbol{\Delta}_\perp) \approx \frac{(2\pi)^3 g^2}{4M_pN_c}\frac{1}{\boldsymbol{k}_\perp^2-\frac{\boldsymbol{\Delta}_\perp^2}{4}}\bar{u}(P',S')\left[\boxed{F_{1,1}} + \mathrm{i}\frac{\sigma^{i+}}{P^+}k_\perp^i\boxed{F_{1,2}} + \mathrm{i}\frac{\sigma^{i+}}{P^+}\Delta_\perp^i F_{1,3}\right]u(P,S)$$

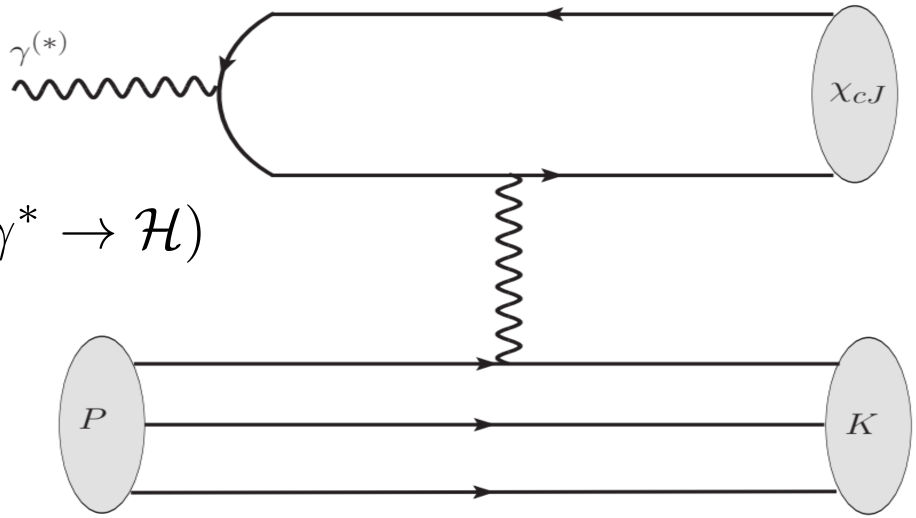
$$f_{1,1}(\boldsymbol{k}_\perp,\boldsymbol{\Delta}_\perp) + \underbrace{\boxed{\mathrm{i}\frac{\boldsymbol{k}_\perp\cdot\boldsymbol{\Delta}_\perp}{M_p^2}g_{1,1}(\boldsymbol{k}_\perp,\boldsymbol{\Delta}_\perp)}}_{\text{spin-independent Odderon}}$$

$$\frac{\boldsymbol{k}_\perp\cdot\boldsymbol{\Delta}_\perp}{M_p^2}f_{1,2}(\boldsymbol{k}_\perp,\boldsymbol{\Delta}_\perp) + \underbrace{\boxed{\mathrm{i}g_{1,2}(\boldsymbol{k}_\perp,\boldsymbol{\Delta}_\perp)}}$$

spin gluon Sivers at |t|->0

$$g_{1,2}(\boldsymbol{k}_\perp,0) = -\frac{1}{2}xf_{1T}^{\perp g}(x,\boldsymbol{k}_\perp)$$

Primakoff photon exchange amplitude



$$\langle \mathcal{M}_{\lambda\lambda'}(\gamma^* p \to \mathcal{H} p)\rangle = -\frac{eF_1(\ell_\perp)}{\ell_\perp^2} n_\mu \mathcal{M}^\mu_{\lambda\lambda'}(\gamma^*\gamma^* \to \mathcal{H})$$

main uncertainty: quarkonium wave function

Important constraint from γγ decay width:

$$\lim_{|t|\to 0} |t| \frac{\mathrm{d}\sigma(\gamma p \to \mathcal{H} p)}{\mathrm{d}|t|} = \frac{8\pi\alpha(2J+1)\ \Gamma(\mathcal{H} \to \gamma\gamma)}{M_{\mathcal{H}}^3}$$

* model independent

* all orders in QCD coupling

* proved via “Collins-Ellis trick” (backup slides)

* for axial vector, Γγγ = 0 (Landau-Yang theorem), so t dσ/dt –> 0

Primakoff in the NRQCD limit at Q^2, t --> 0

$$\frac{\mathrm{d}\sigma(\gamma p \to \mathcal{S} p)}{\mathrm{d}|t|} \to \frac{9\pi q_c^4 \alpha^3 N_c |R'(0)|^2 F_1^2(0)}{m_c^7 |t|}$$

t –> 0, m_c –> ∞

$$\frac{\mathrm{d}\sigma(\gamma p \to \mathcal{A} p)}{\mathrm{d}|t|} \to \frac{3\pi q_c^4 \alpha^3 N_c |R'(0)|^2 F_1^2(0)}{m_c^9}$$

$$\frac{\mathrm{d}\sigma(\gamma p \to \mathcal{T} p)}{\mathrm{d}|t|} \to \frac{12\pi q_c^4 \alpha^3 N_c |R'(0)|^2 F_1^2(0)}{m_c^7 |t|}$$

* NR limits agree with literature Jia, Mo, Pan, Zhang, 2207.14171

* Note the absence of a Coulomb tail for axial quarkonia

* For χ_{c0}, χ_{c2} parameter free (thanks to constraint for t dσ/dt;
note $m_c = M_H/2$ in NR limit)

Primakoff for χ_{cJ} at Q^2=0, high $|t| \gg m_c^2$

Cross sections scale

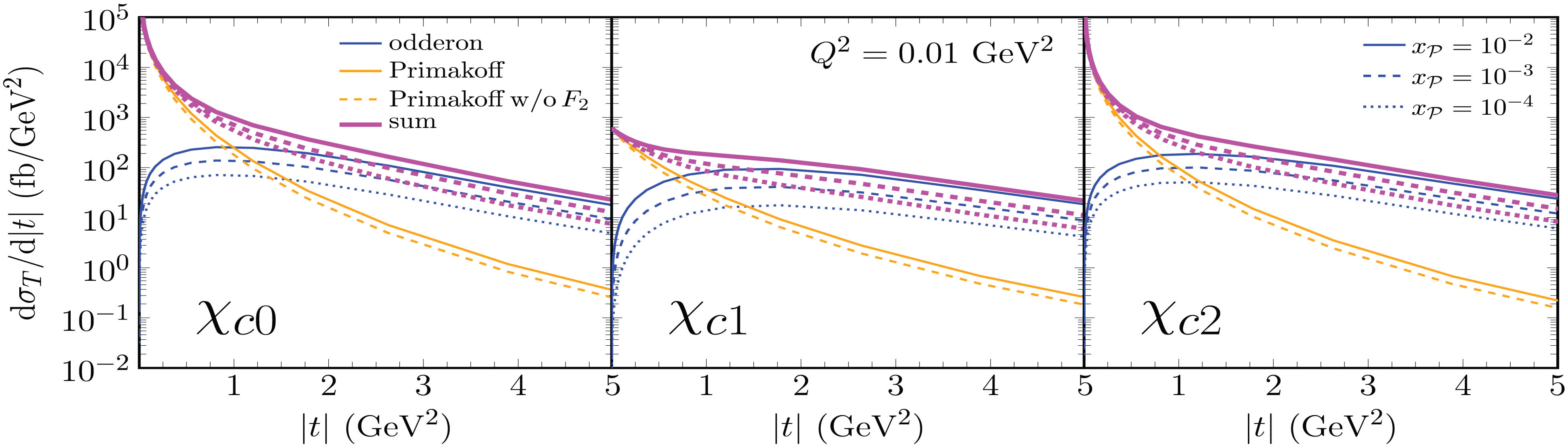
$$\frac{\mathrm{d}\sigma}{\mathrm{d}t} \sim \frac{F_1^2(t) + \frac{|t|}{4M_p^2} F_2^2(t)}{|t|^3}$$

F1(t), F2(t): Dirac and Pauli form factors of the proton

The coefficient involves the meson LC-DA (and depends on J) but the above scaling with |t| is universal !

If this scaling at high |t| is broken then there's another process at work !

Numerical results, differential X-sections



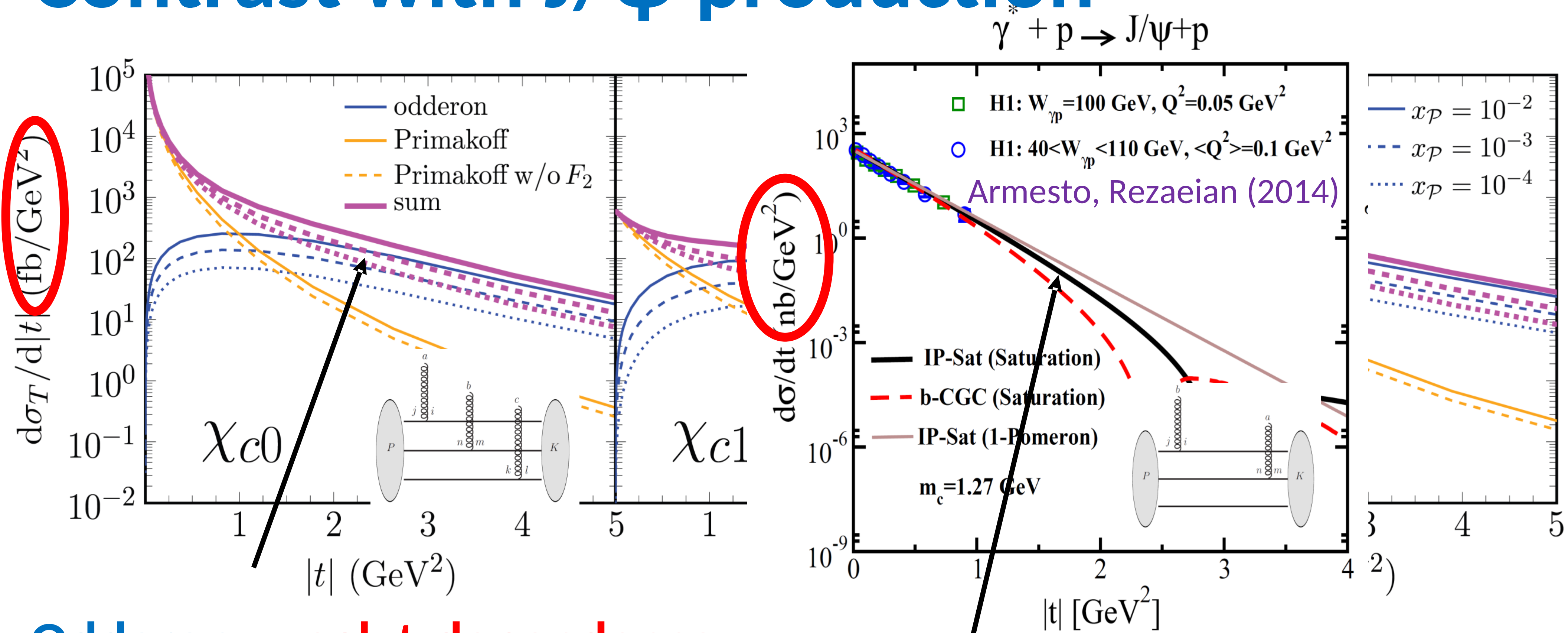
* low |t| dominated by Primakoff, ggg exchange important from |t| ~ 1 GeV^2
slow fall-off with increasing |t| !

* cross-section at |t| ~ 1 GeV^2 is enhanced by factor ~4 due to constructive interference

* X-section for χc1 is finite as t –> 0 (due to Landau-Yang)

* small X-section at |t| ~ 1 GeV^2 and beyond (due to P-odd ggg), high lumin. critical for discovery

Contrast with J/ψ production



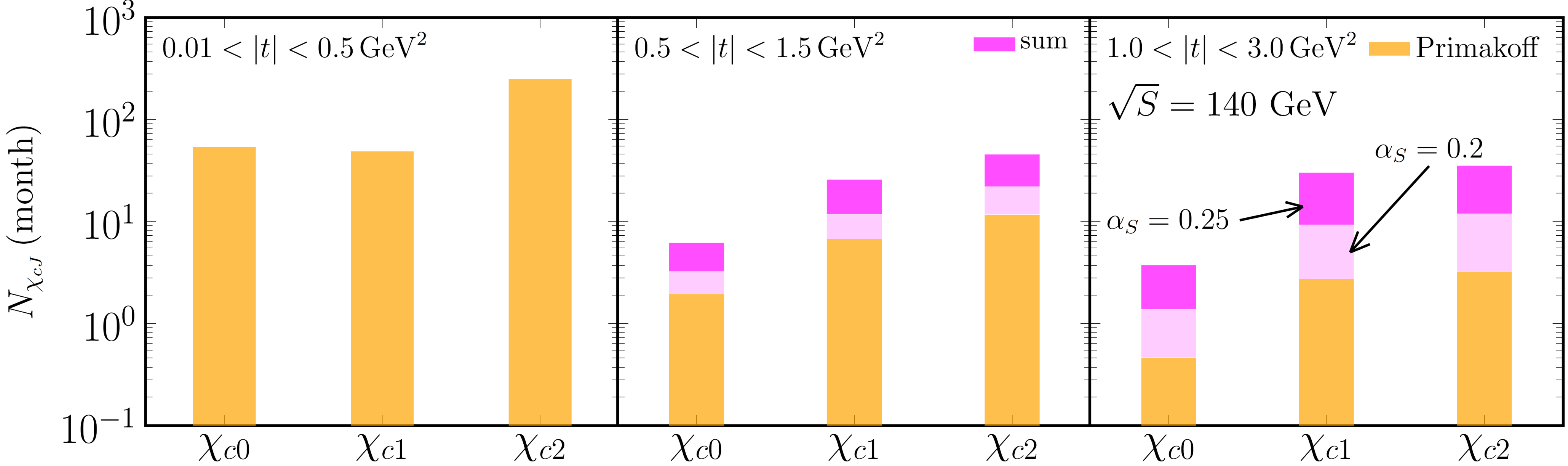
Odderon: **weak t-dependence**

$d\sigma/d|t| \propto |t| \cdot \exp(-B|t|)$, $B \approx 1$ GeV^{-2}

Pomeron: **strong t-dependence**

$d\sigma/d|t| \propto \exp(-B|t|)$, $B \approx 4$ GeV^{-2}

Expected number of events at the EIC



- * detection channel: χ_{cJ} -> $J/\psi\gamma$, J/ψ -> e^+e^- / $\mu^+\mu^-$
- * we predict **excess** odderon events over Primakoff background
- * for χ_{c1} (30% BR to $J/\psi + \gamma$): with EIC luminosity 10^{34} cm^{-2} s^{-1} expect ~20 events/month (only Primakoff: ~7 events/month) in $0.5 < |t| < 1.5$ window

Summary

* Eikonal Odderon exchange is an intriguing prediction of QCD with Nc ≥ 3 colors !

* $\langle A^{+\,3} \rangle \neq 0 \;\longrightarrow$ evidence for non-Gaussian color fluctuations in the proton at sub-femtometer scales !

* magnitude, t-dependence, and phase (relative to γ exchange) offer most valuable insight into the structure of the proton
(not predicted by general arguments based on analyticity & unitarity)

* high luminosity EIC may be able to discover “hard Odderon” (C, P odd eikonal gluon exchanges)

Thank you !

Backup Slides

Illustration: JIMWLK / MV type source averages

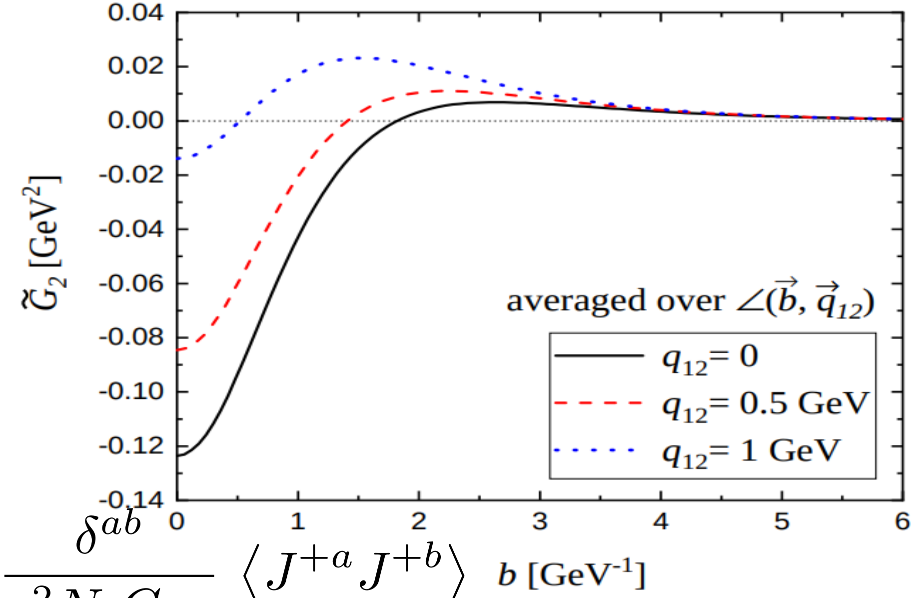
$$\langle \cdots \rangle \sim \int \mathcal{D}J^+ \exp\left(-\frac{\delta^{ab}J^{+a}J^{+b}}{2\mu^2} + \frac{d^{abc}J^{+a}J^{+b}J^{+c}}{\kappa} + \cdots\right)$$

for J/ψ

for χ_c

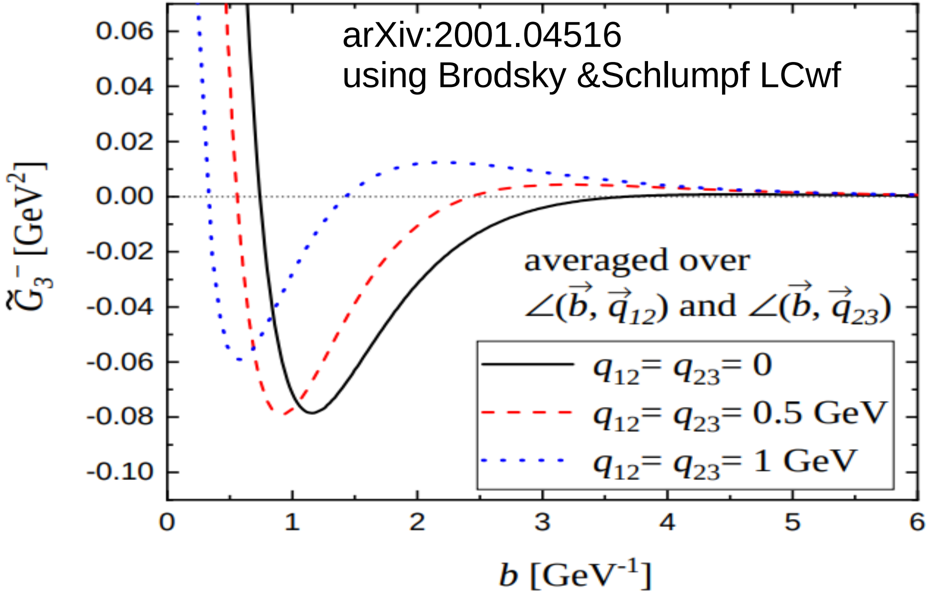
Color charge fluctuations in the proton ≈ Gaussian ?

Not at moderately small x:



$$\frac{\delta^{ab}}{g^2 N_c C_{2F}} \langle J^{+a} J^{+b} \rangle$$

$$\frac{d^{abc}}{g^3 N_c C_{3F}} \langle J^{+a} J^{+b} J^{+c} \rangle$$



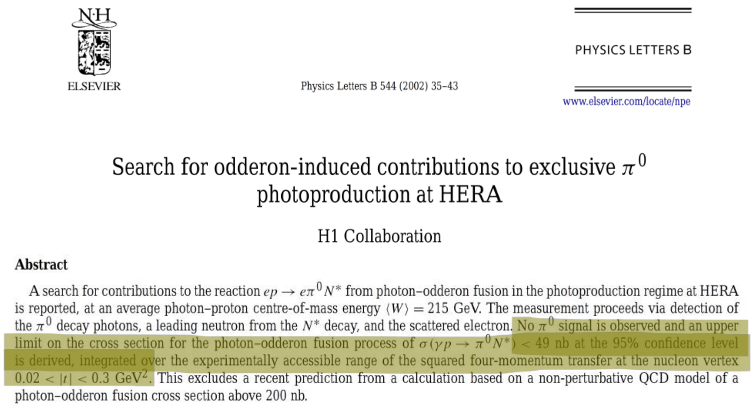
arXiv:2001.04516
using Brodsky &Schlumpf LCwf

HERA searches for the Odderon

$\sigma(\gamma^* p \text{->} \pi^0 N^*) < 39$ nb

nucleon excited to a P-wave to couple to the quasi-real transverse photon

H1 (2002)



Physics Letters B 544 (2002) 35–43

PHYSICS LETTERS B

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Search for odderon-induced contributions to exclusive π^0 photoproduction at HERA

H1 Collaboration

Abstract

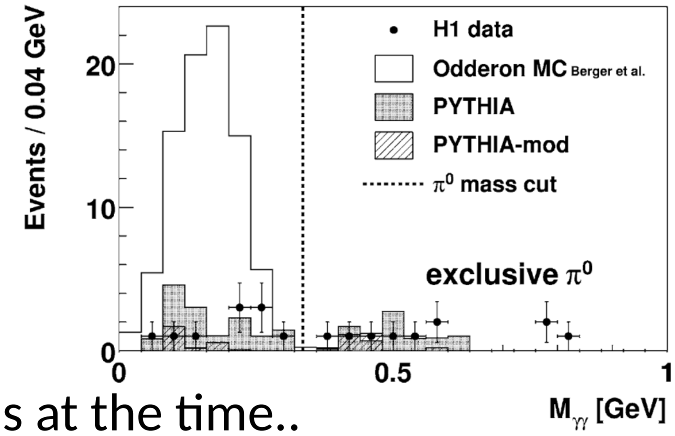
A search for contributions to the reaction $ep \to e\pi^0 N^*$ from photon–odderon fusion in the photoproduction regime at HERA is reported, at an average photon–proton centre-of-mass energy $\langle W \rangle = 215$ GeV. The measurement proceeds via detection of the π^0 decay photons, a leading neutron from the N^* decay, and the scattered electron. No π^0 signal is observed and an upper limit on the cross section for the photon–odderon fusion process of $\sigma(\gamma p \to \pi^0 N^*) < 49$ nb at the 95% confidence level is derived, integrated over the experimentally accessible range of the squared four-momentum transfer at the nucleon vertex $0.02 < |t| < 0.3$ GeV^2. This excludes a recent prediction from a calculation based on a non-perturbative QCD model of a photon–odderon fusion cross section above 200 nb.

HERA kinematics:

$0.02 < |t| < 0.3$ GeV^2

$Q^2 < 0.01$ GeV^2

<W>~200 GeV



about order of magnitude lower than the theory predictions at the time..

Berger (1999)

possible explanations:

-> low-Q^2 -> cannot exclude non-perturbative contributions

-> energy dependence: ~ $W^{0.3}$ but in QCD Odderon is at most a constant with energy (modulo absorptive corrections)

–> coupling to proton at x_0 was overestimated

Collins-Ellis “trick” for γp –> Hp:

$$\mathcal{M}_{\lambda\lambda'}(\gamma p \to \mathcal{H} p) = P_\mu \, \mathcal{M}^{\mu}_{\lambda\lambda'}(\gamma\gamma \to \mathcal{H}) \frac{eF_1(\ell^2)}{\ell^2} \, \frac{1}{q \cdot P} \bar{u}(P') \not{q} \, u(P)$$

In high-E limit:

$$P_\mu \, \mathcal{M}^{\mu}_{\lambda\lambda'}(\gamma\gamma \to \mathcal{H}) \simeq P^+ \mathcal{M}^{-}_{\lambda\lambda'}(\gamma\gamma \to \mathcal{H}) \simeq -\frac{1}{x} \ell_i \mathcal{M}^{i}_{\lambda\lambda'}(\gamma\gamma \to \mathcal{H})$$

since $\ell_\mu \, \mathcal{M}^{\mu}_{\lambda\lambda'}(\gamma\gamma \to \mathcal{H}) = 0$ by QED gauge invariance, and $\ell^\mu \simeq (xP^+, 0, \vec{\ell}_\perp)$

This then leads to (appendix C.1 in arXiv:2402.19134)

$$\frac{\mathrm{d}\sigma(\gamma p \to \mathcal{H} p)}{\mathrm{d}|t|} = \frac{8\pi\alpha(2J+1)F_1^2(t) \; \Gamma(\mathcal{H} \to \gamma\gamma)}{|t| \, M_{\mathcal{H}}^3}$$

at high E and |t| ≪ m_p.