Exploring Hadron Structure Through Monte-Carlo Fits and Model Calculations

Chris Cocuzza

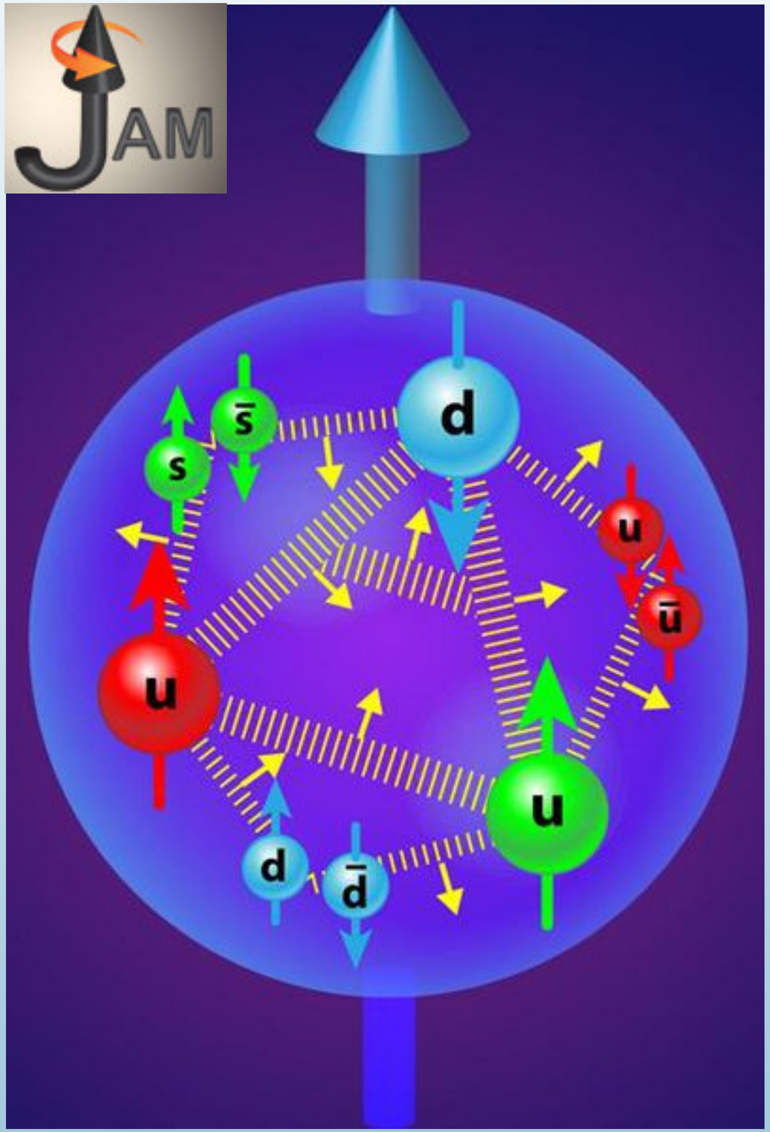
www.jlab.org/theory/jam

March 16, 2025

JAM Collaboration

3-dimensional structure of nucleons:

- Parton distribution functions (PDFs)
- Fragmentation functions (FFs)
- Transverse momentum dependent distributions (TMDs)
- Generalized parton distributions (GPDs)



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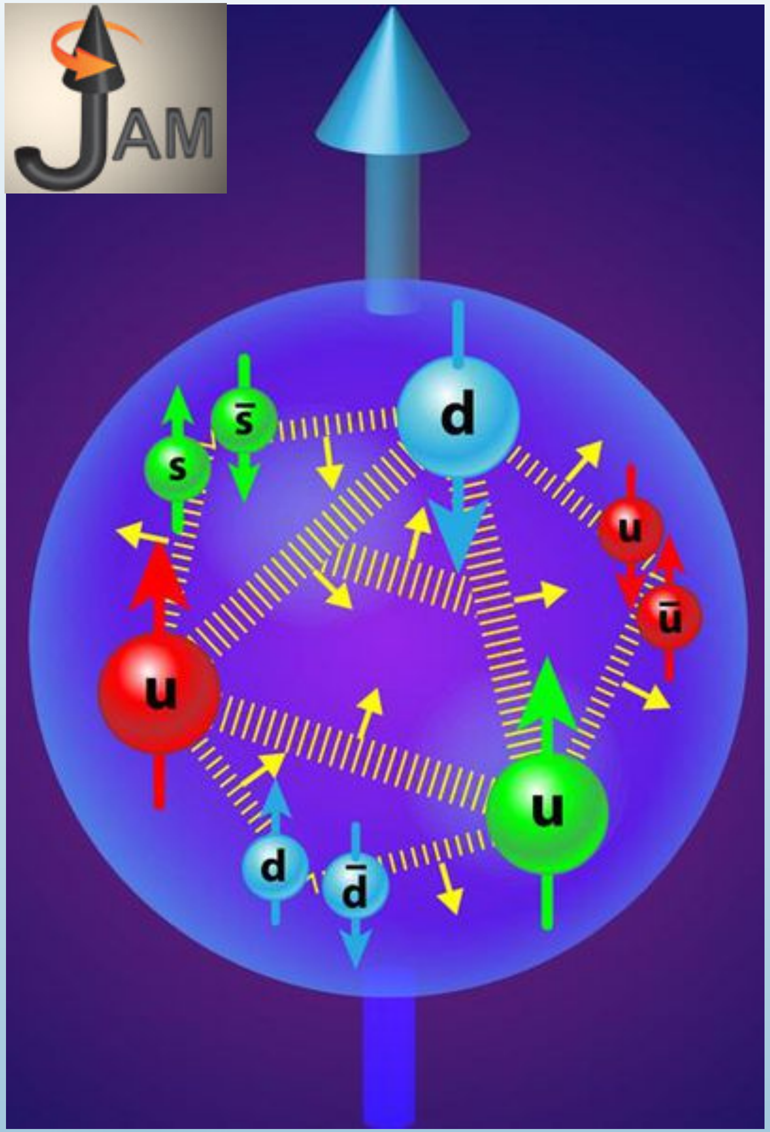


- Collinear factorization in perturbative QCD
- Simultaneous determinations of PDFs, FFs, etc.
- Monte Carlo methods for Bayesian inference

Hadron Structure

Global QCD Analysis

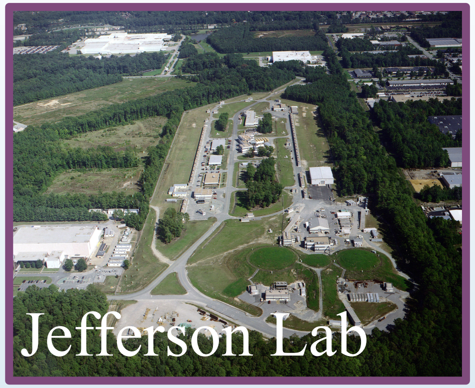
Hadron Structure

Global QCD Analysis

Introduction

Hadron Structure

Global QCD Analysis



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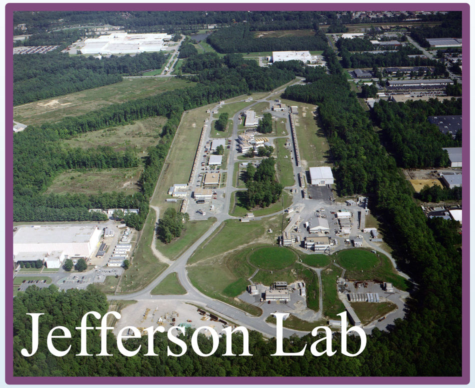
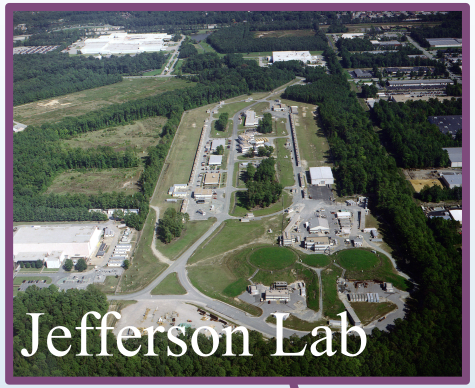


Global QCD Analysis

Hadron Structure

USQCD

Extended Twisted Mass Collaboration



Hadron Structure

Global QCD Analysis

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Introduction

Hadron Structure

$$\frac{\mathrm{d}}{\mathrm{d}\ \ln(\mu^2)} f_i(x,\mu) = \sum_j \int_x^1 \frac{dz}{z} P_{ij}(z,\mu) f_j(\frac{x}{z},\mu)$$

Param. + Evolve + Factorization

$$\sigma = \sum_{i,j} H_{ij} \otimes f_i \otimes f_j$$

Global QCD Analysis

USQCD

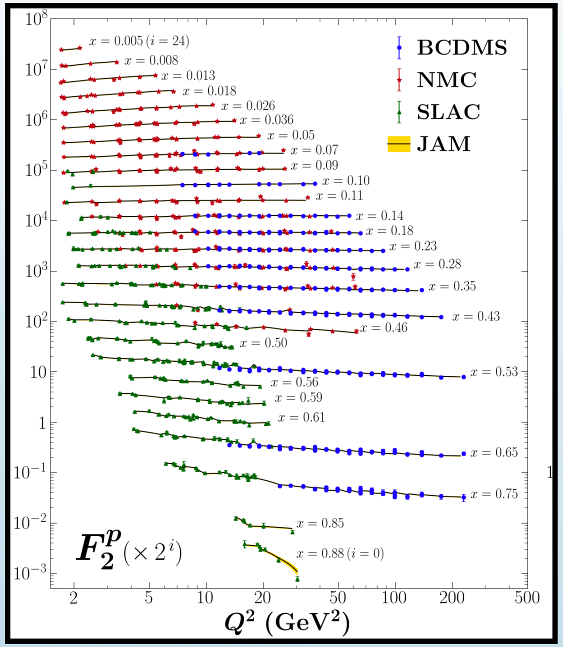
Extended Twisted Mass Collaboration

$$\chi^2(\boldsymbol{a}) = \sum_{i,e}\left(\frac{d_{i,e} - \sum_k r_e^k \beta_{i,e}^k - T_{i,e}(\boldsymbol{a})/N_e}{\alpha_{i,e}}\right)^2 + \sum_k \left(r_e^k\right)^2 + \left(\frac{1-N_e}{\delta N_e}\right)^2$$

χ^2 Minimization

$$\mathcal{L}(\boldsymbol{a}, \text{data}) = \exp\left(-\frac{1}{2}\chi^2(\boldsymbol{a}, \text{data})\right)$$

$$\mathcal{P}(\boldsymbol{a}|\text{data}) \sim \mathcal{L}(\boldsymbol{a}, \text{data})\, \pi(\boldsymbol{a})$$



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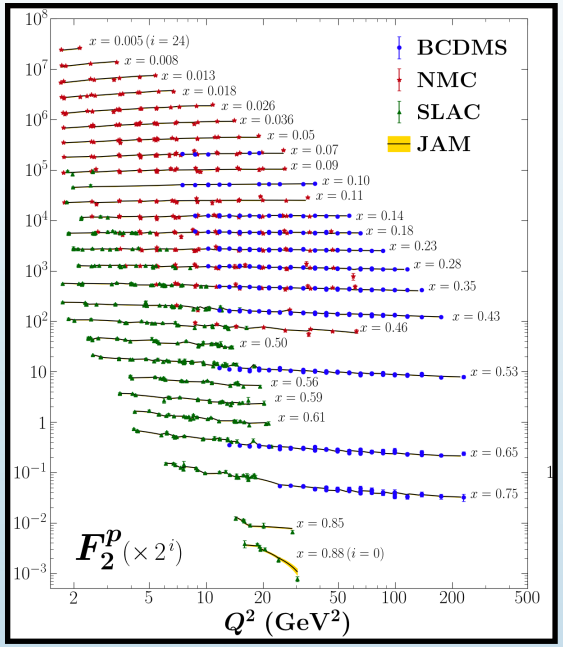

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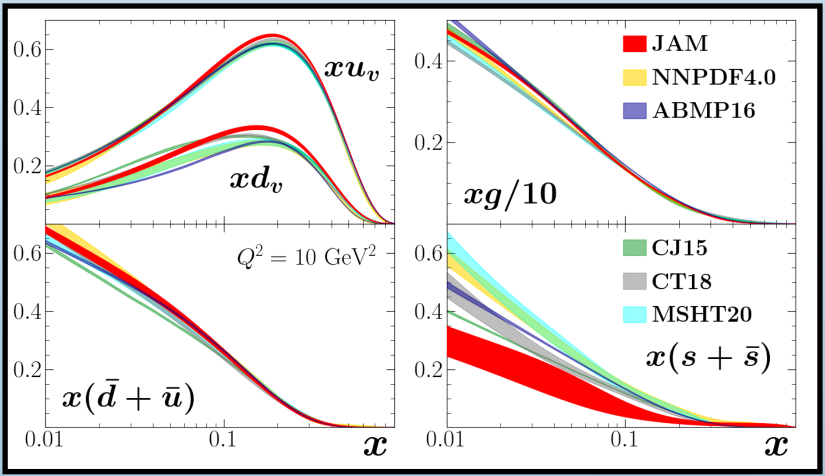

Data Resampling

$$\tilde{\sigma} = \sigma + N(0,1)\ \alpha$$

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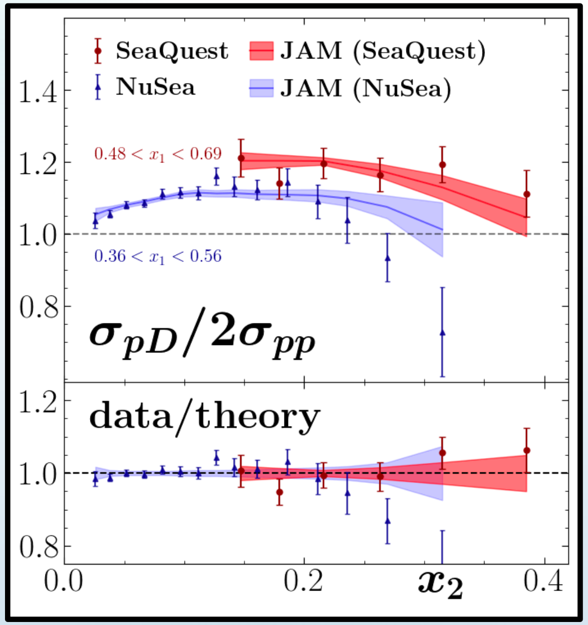
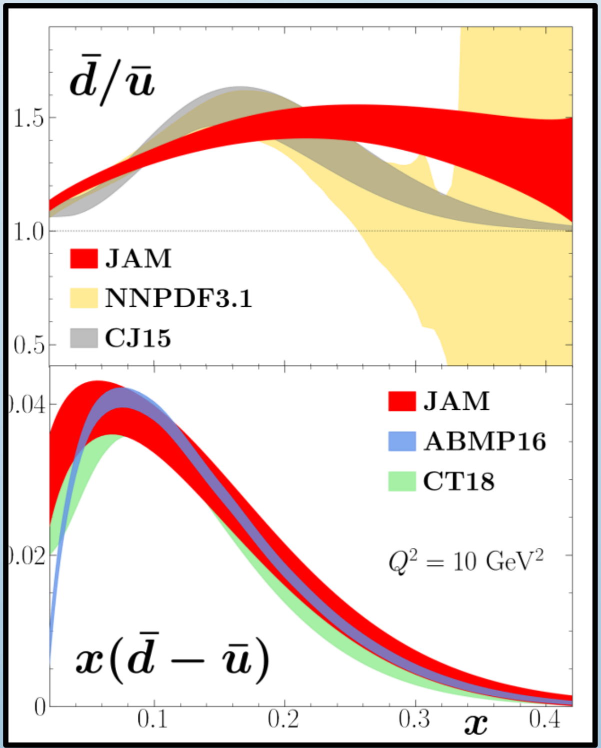
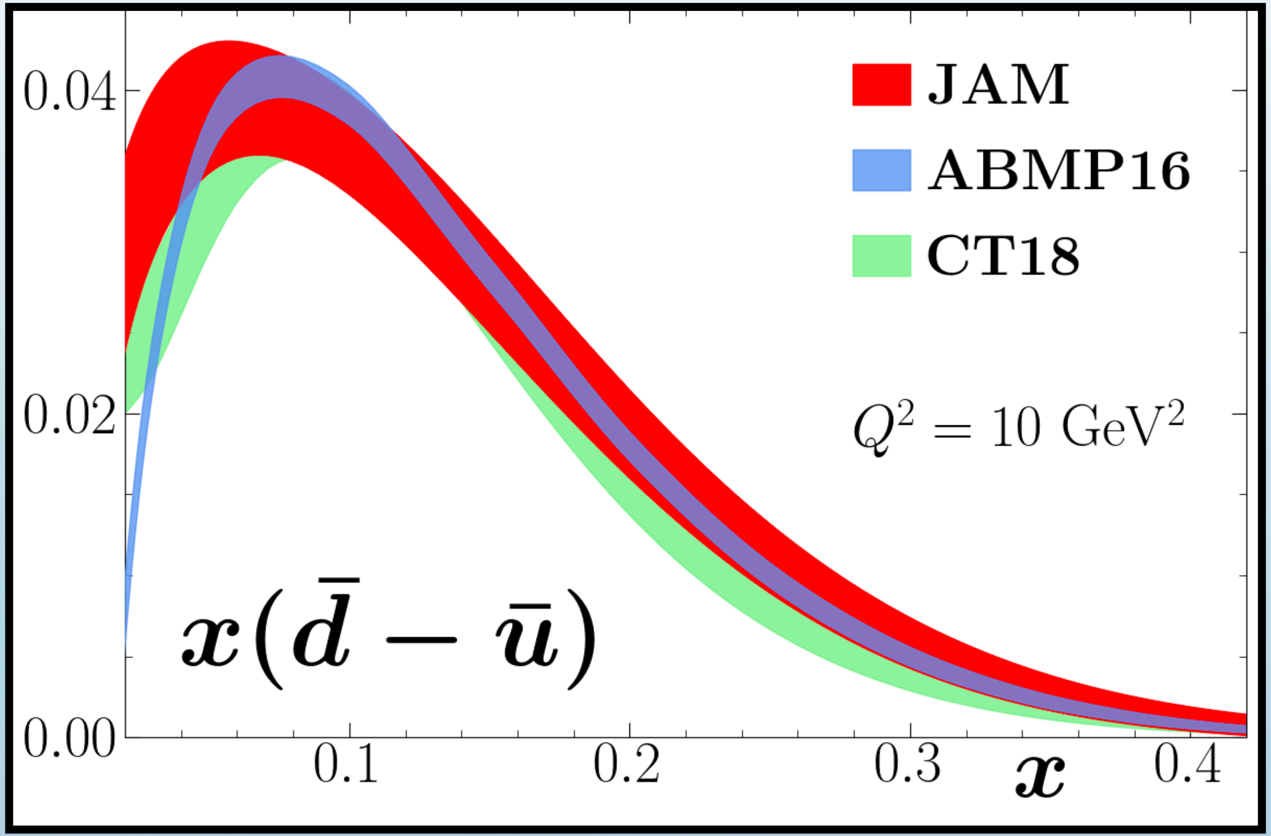
3. Extraction of Nuclear Effects
4. Helicity Parton Distribution Functions
5. Di-Hadron Production and Transversity Parton Distribution Functions
6. Summary and Outlook





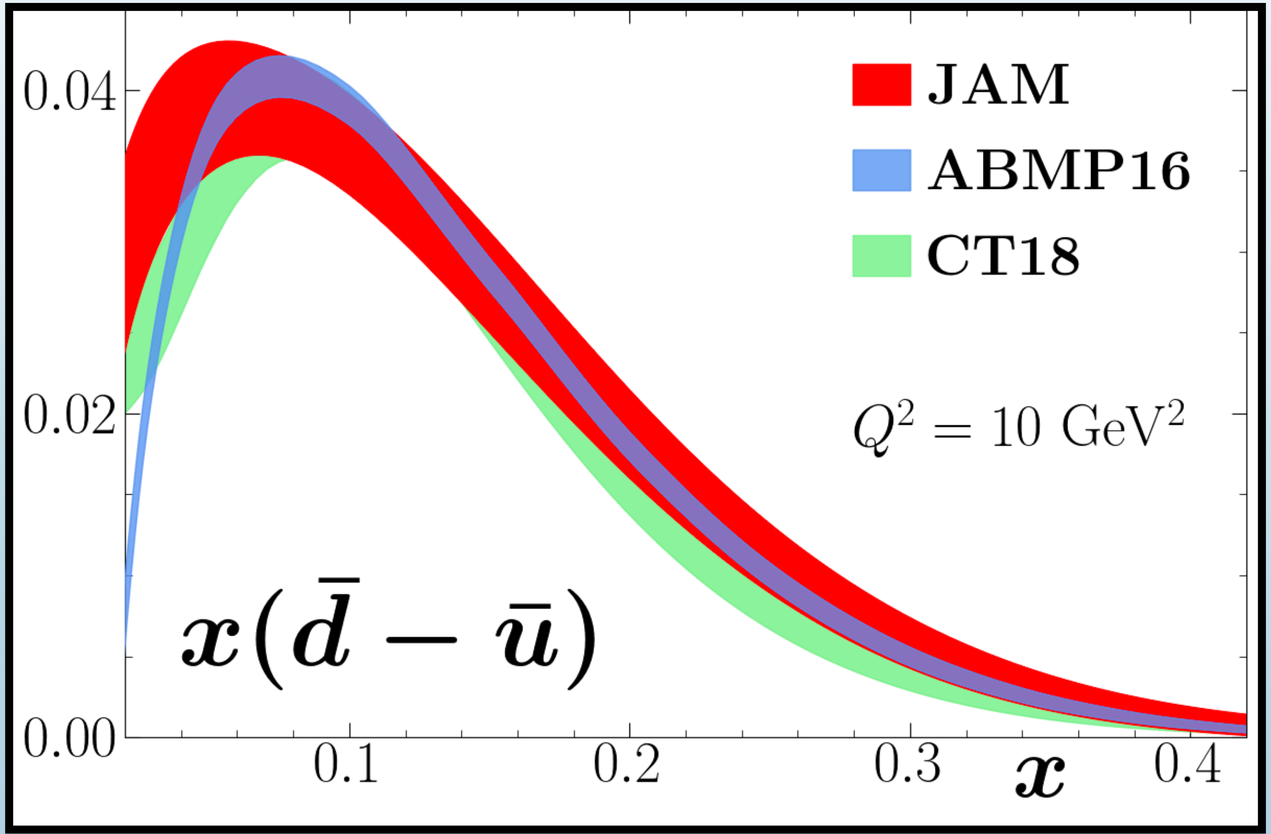
C. Cocuzza, W. Melnitchouk, A. Metz, and N. Sato, Phys. Rev. D. **104**, 074031 (2021)

Introduction to Sea Asymmetry



Unpolarized

Introduction to Sea Asymmetry



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Cannot be explained from gluons splitting into quark-antiquark pairs

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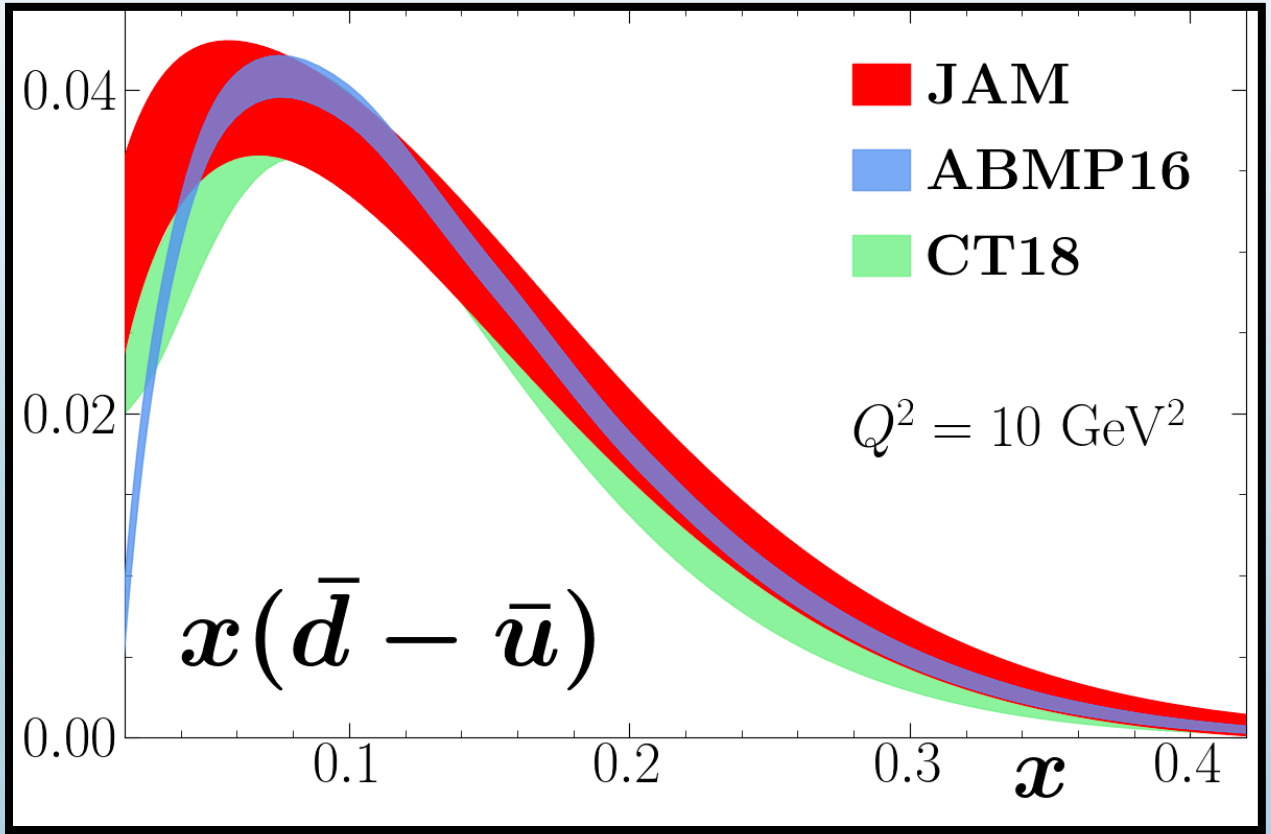

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Meson Cloud Models
Chiral Soliton Models
Statistical Models

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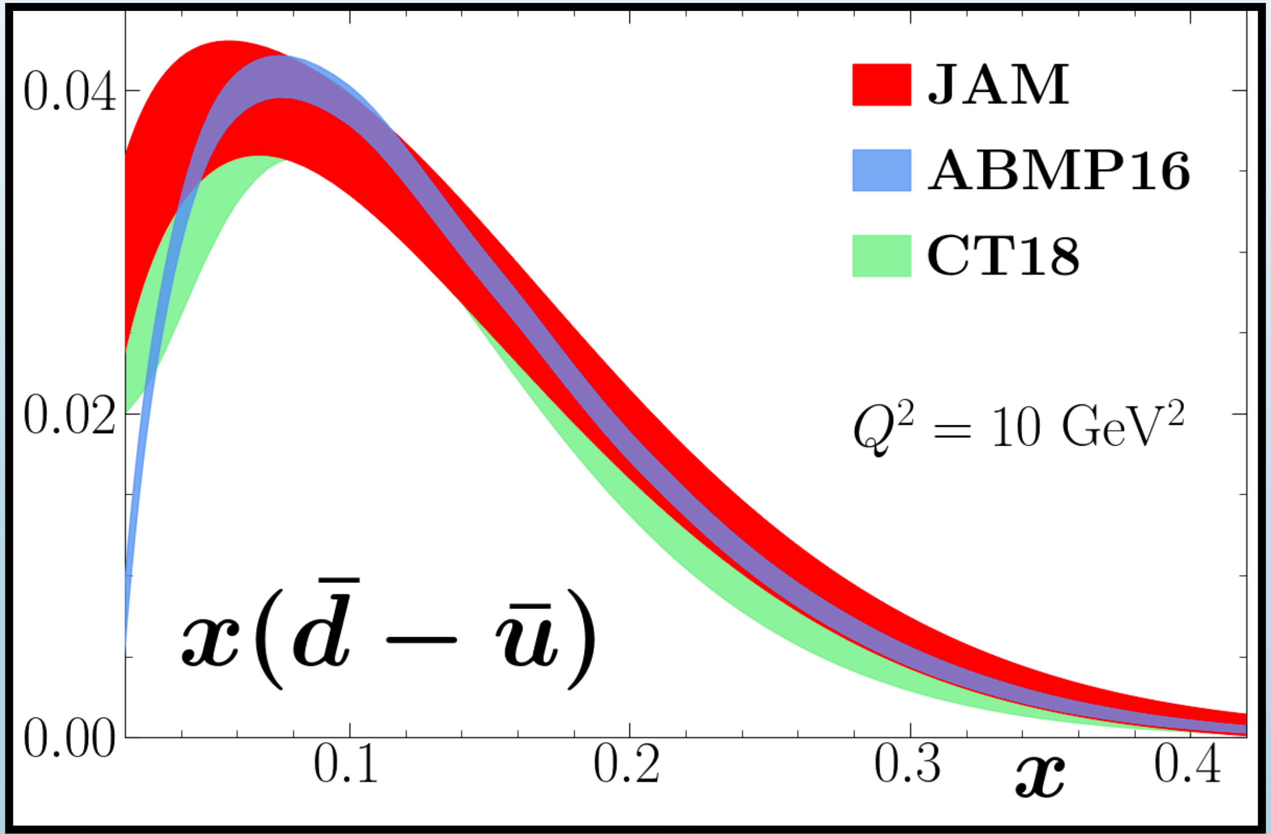

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Chiral Soliton Models
Statistical Models

Questions at high $x > 0.2$ and for helicity asymmetry

Kinematic Coverage (Spin-Averaged)

Deep Inelastic Scattering	BCDMS, NMC, SLAC, HERA, Jefferson Lab	3863	points
Drell-Yan	Fermilab E866, E906	205	points
W/Z Boson Production	CDF/D0, STAR, LHCb, CMS	153	points
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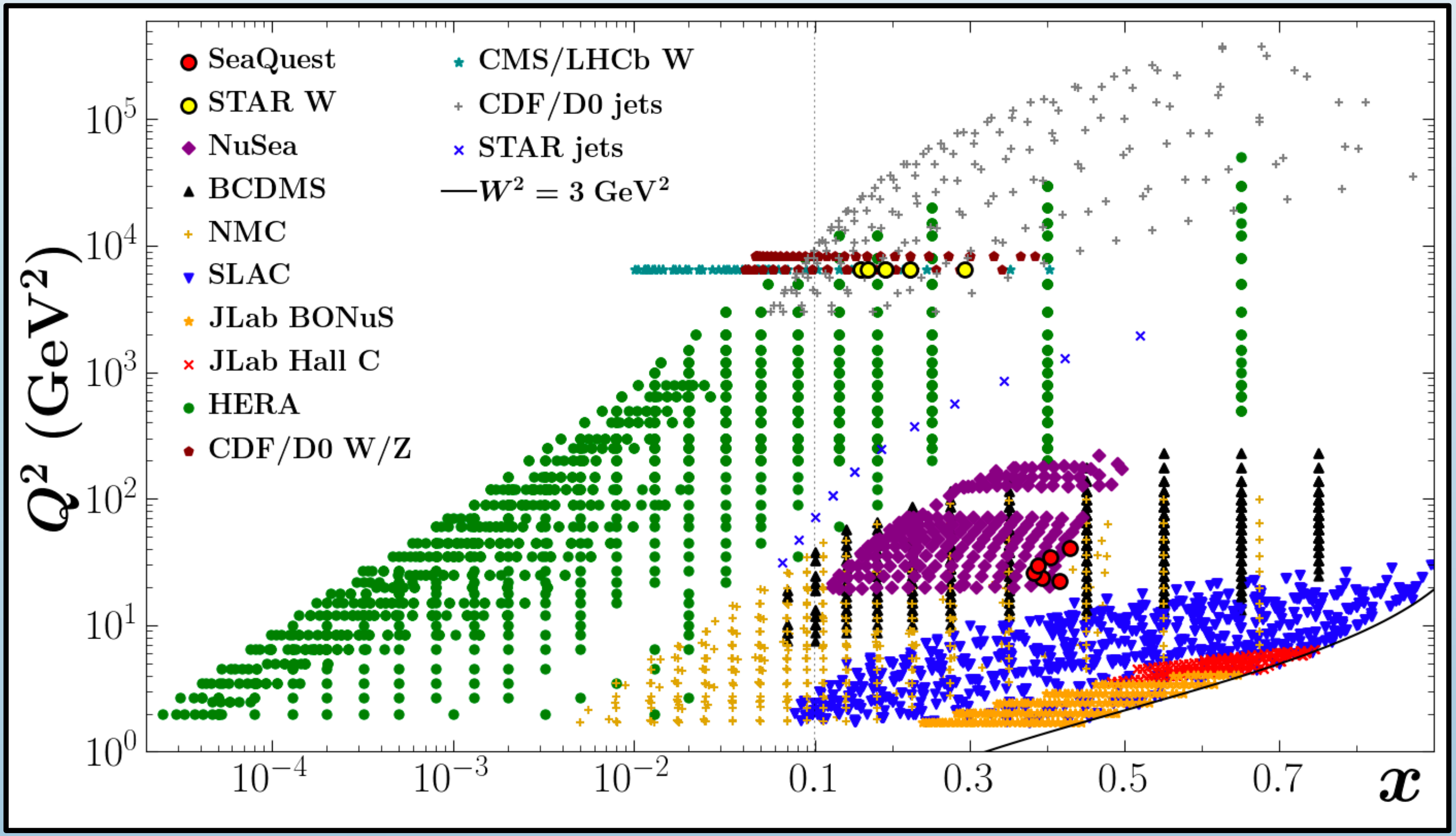

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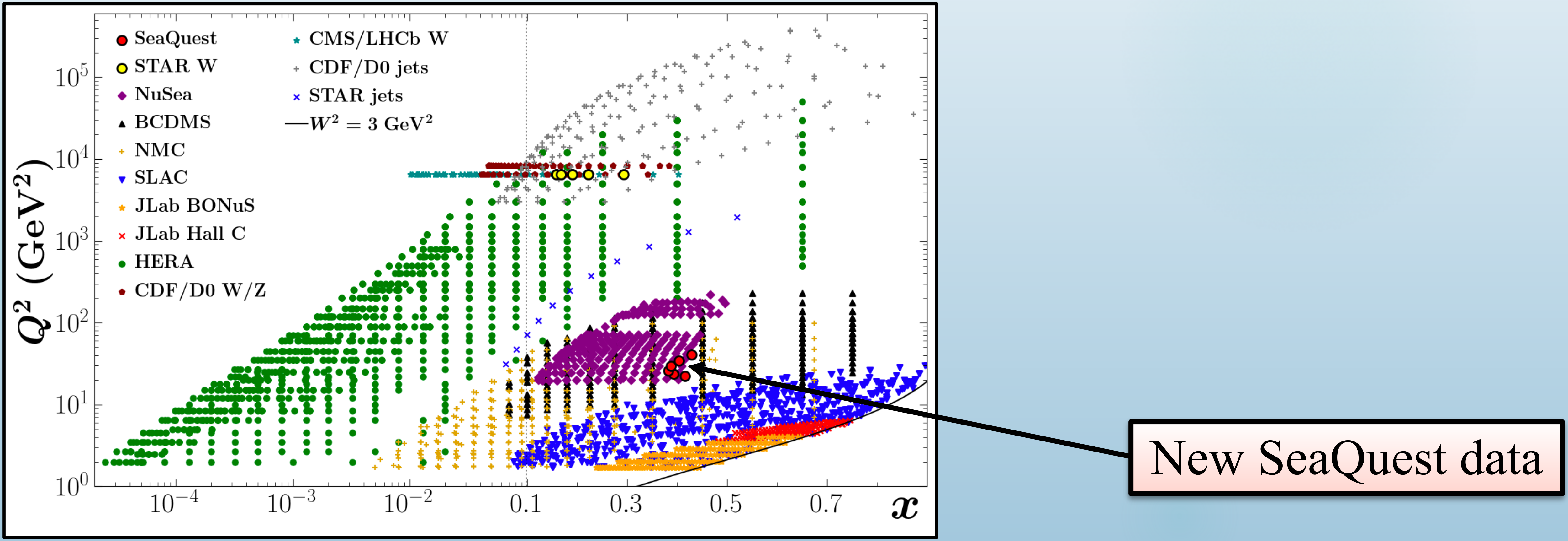

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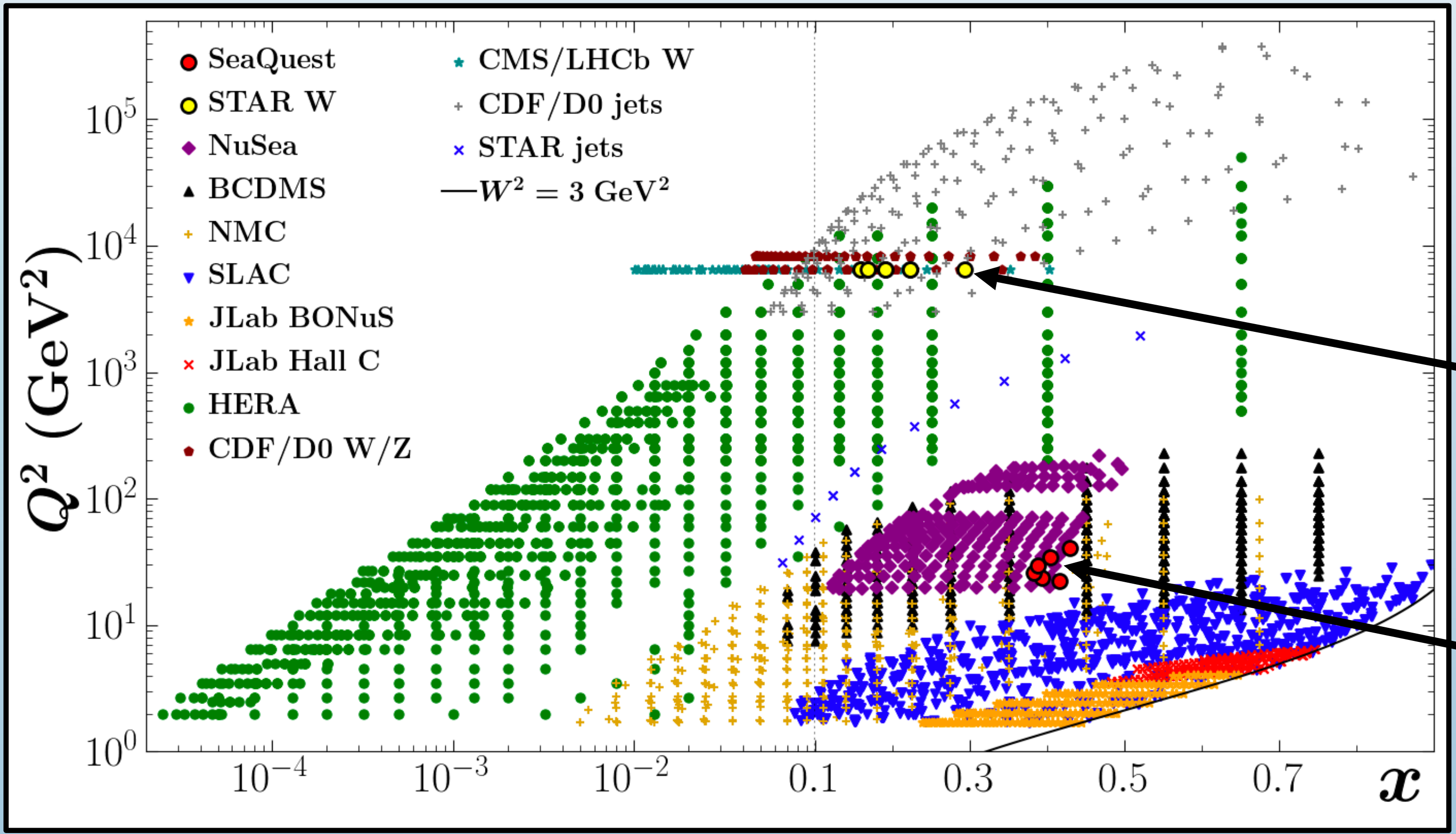
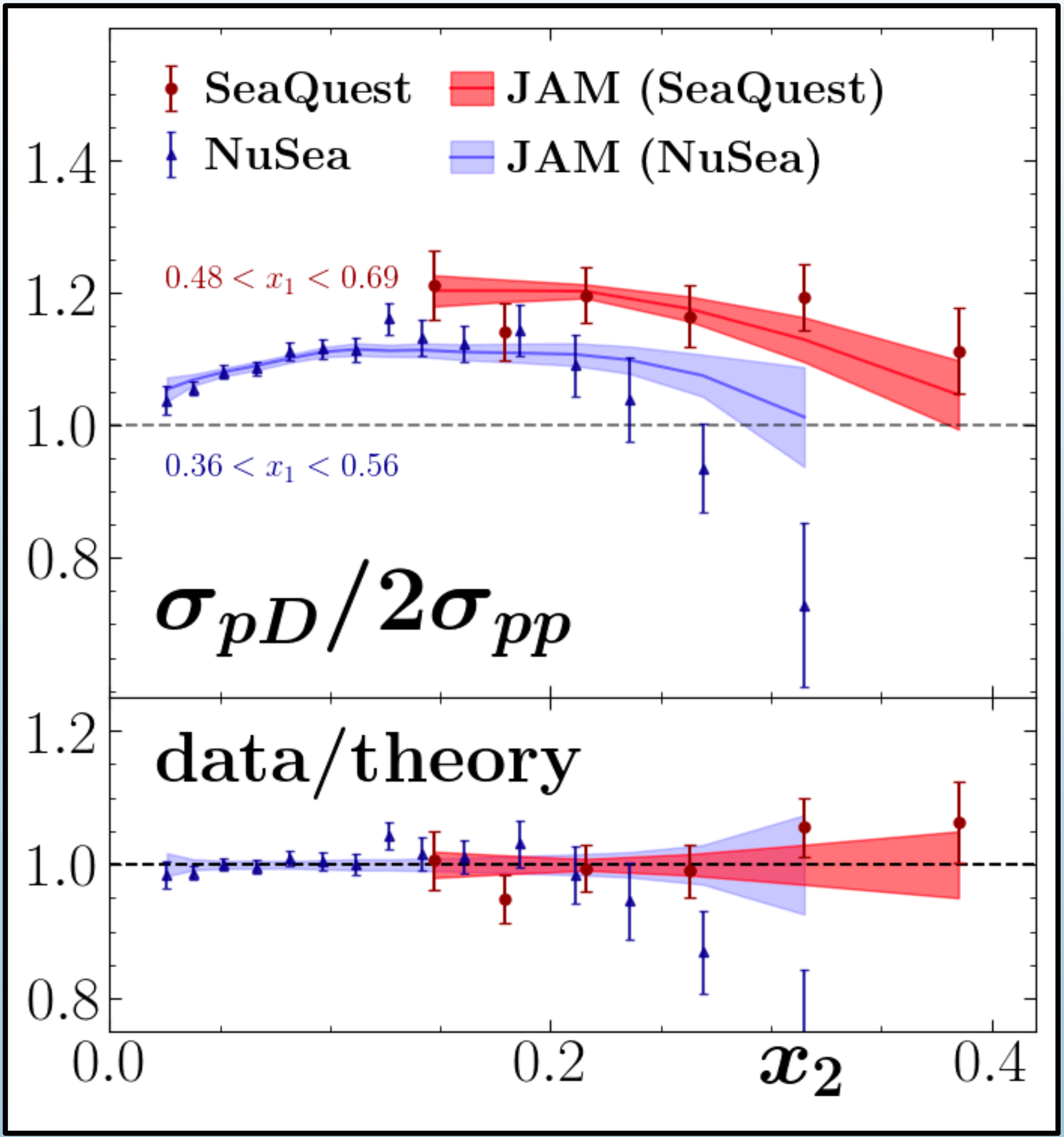


New STAR data

New SeaQuest data

SeaQuest and NuSea Quality of Fit



$$\left.\frac{\sigma_{pD}}{2\sigma_{pp}}\right|_{x_1 \gg x_2} \approx \frac{1}{2}\left[1 + \frac{\bar{d}(x_2)}{\bar{u}(x_2)}\right]$$

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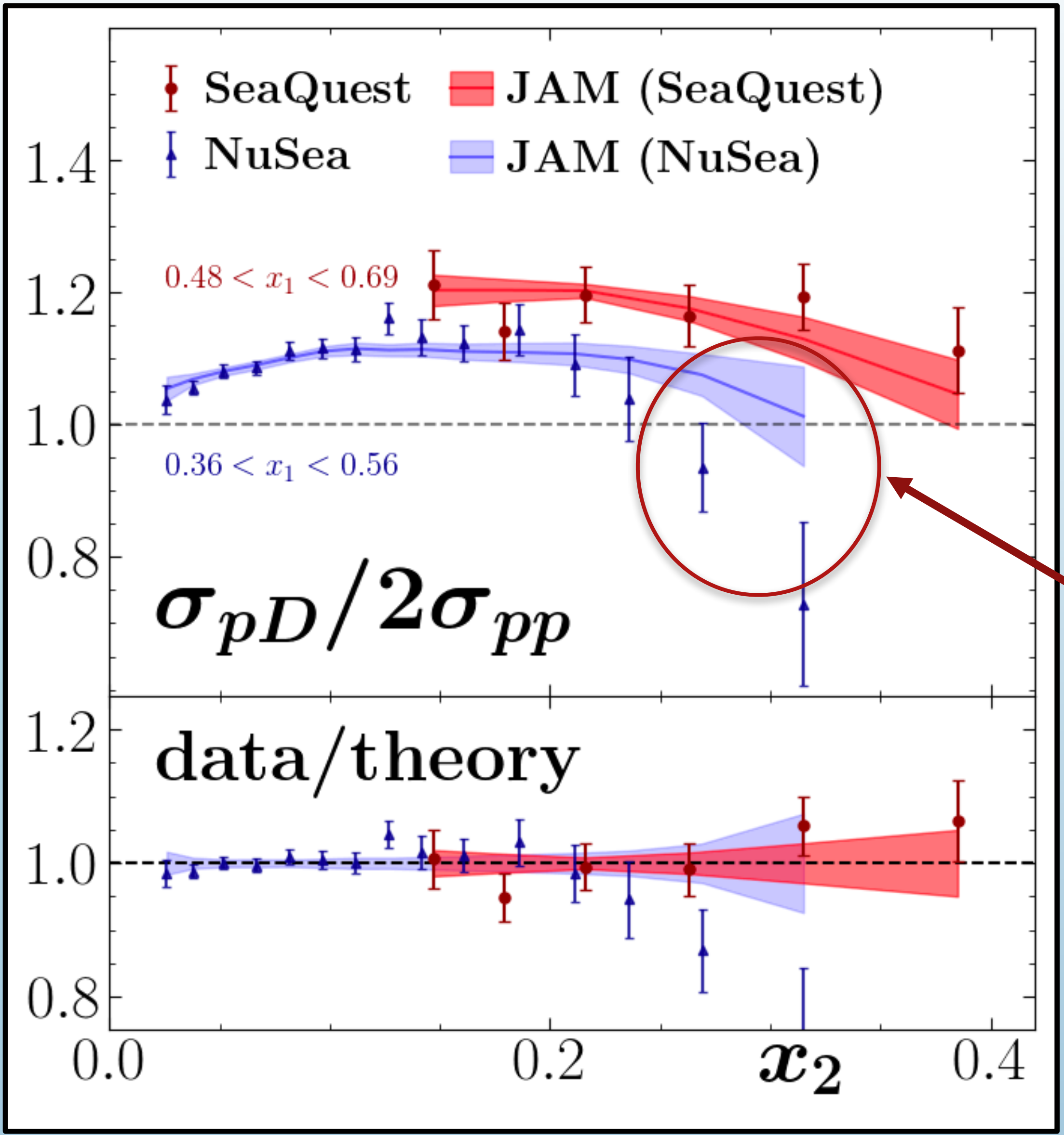

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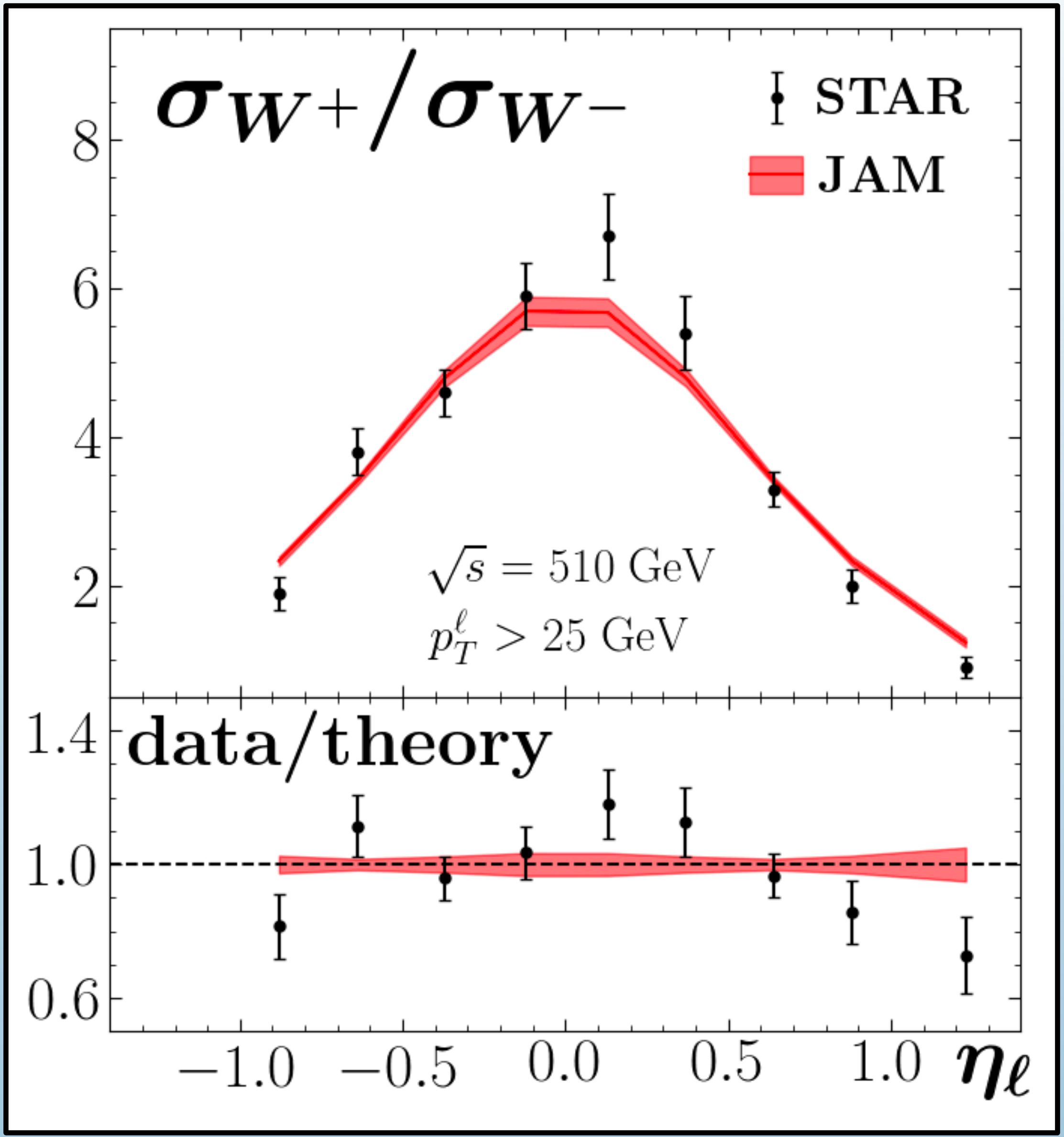
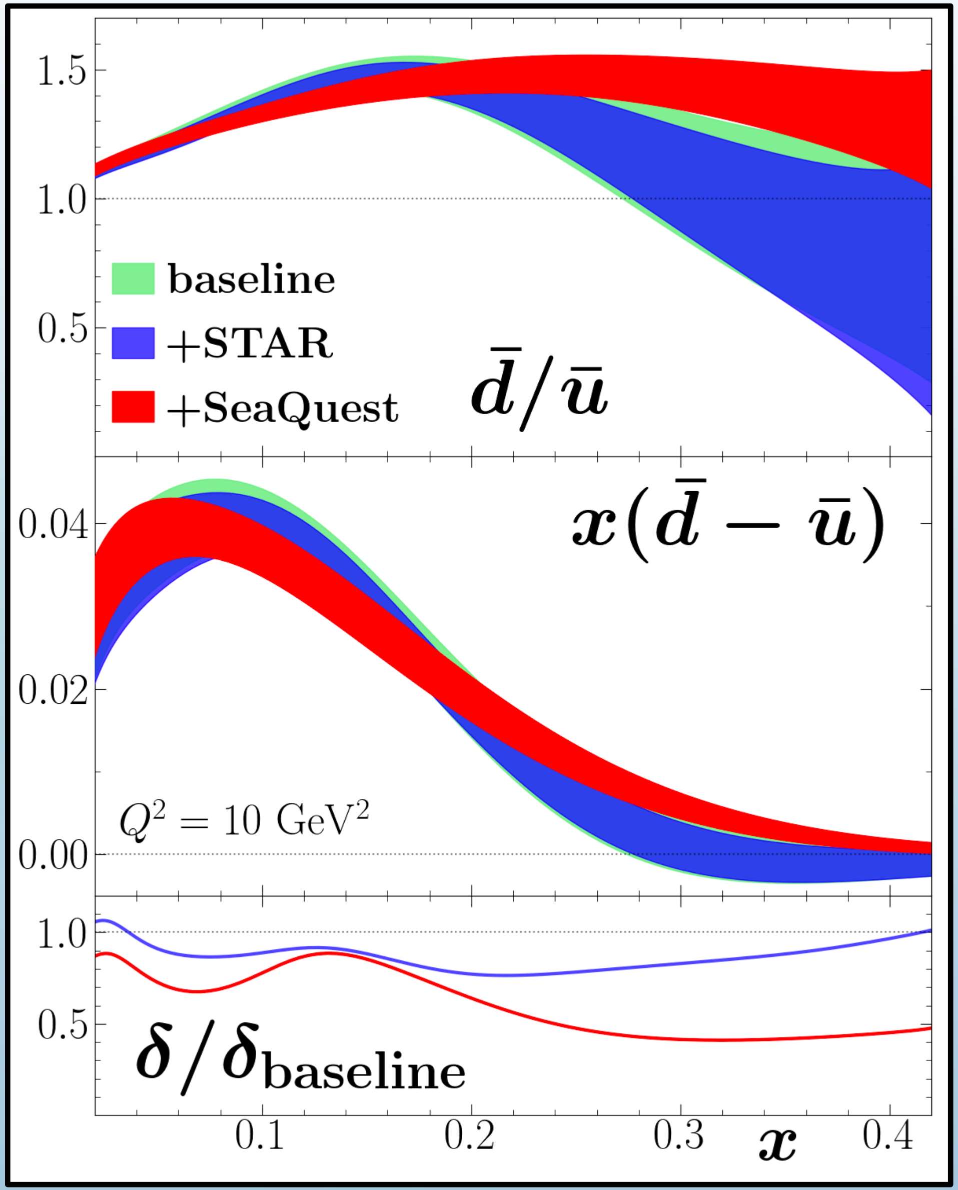
Well-known tension between NuSea and SeaQuest

STAR Quality of Fit



Impact from STAR and SeaQuest



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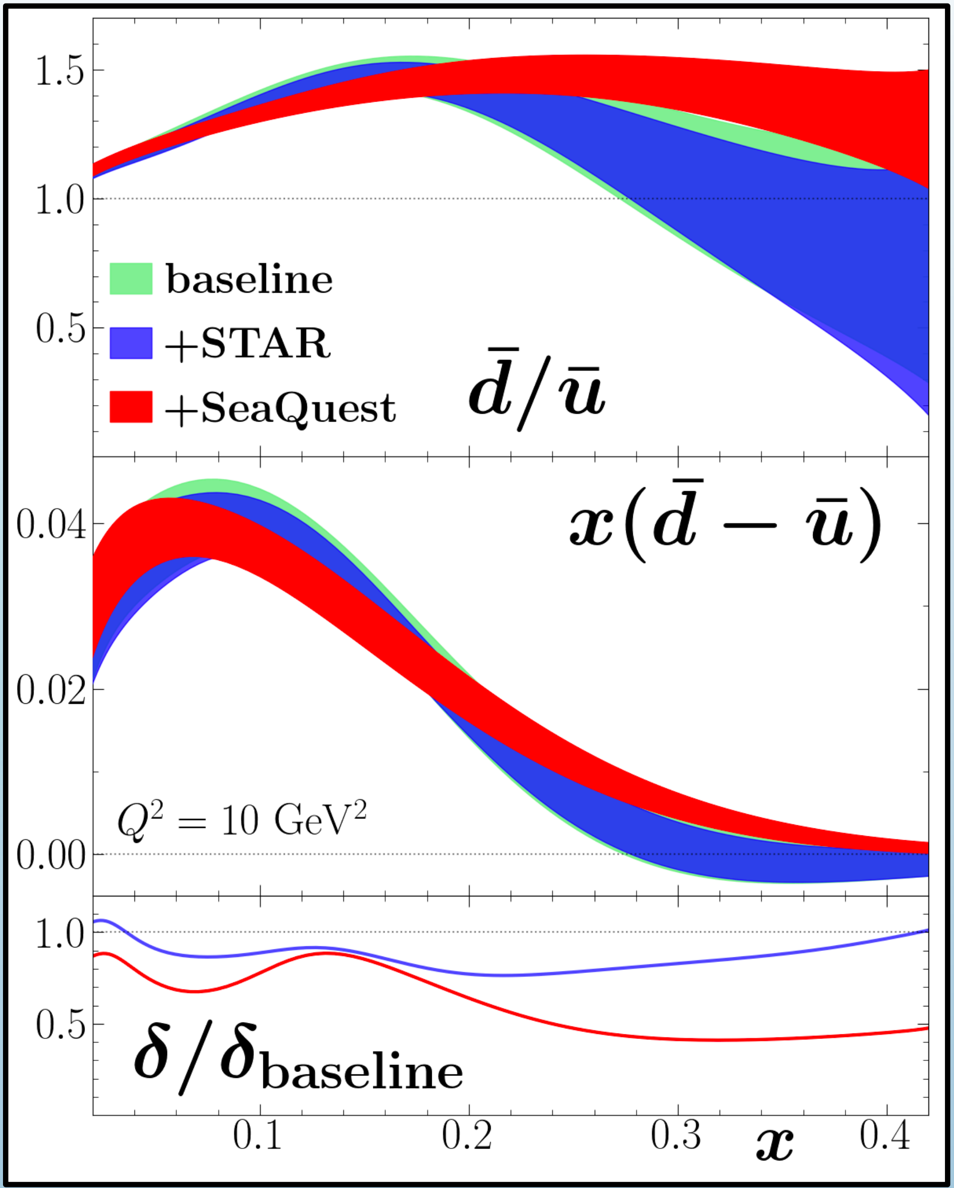

STAR: Moderate reduction of uncertainties

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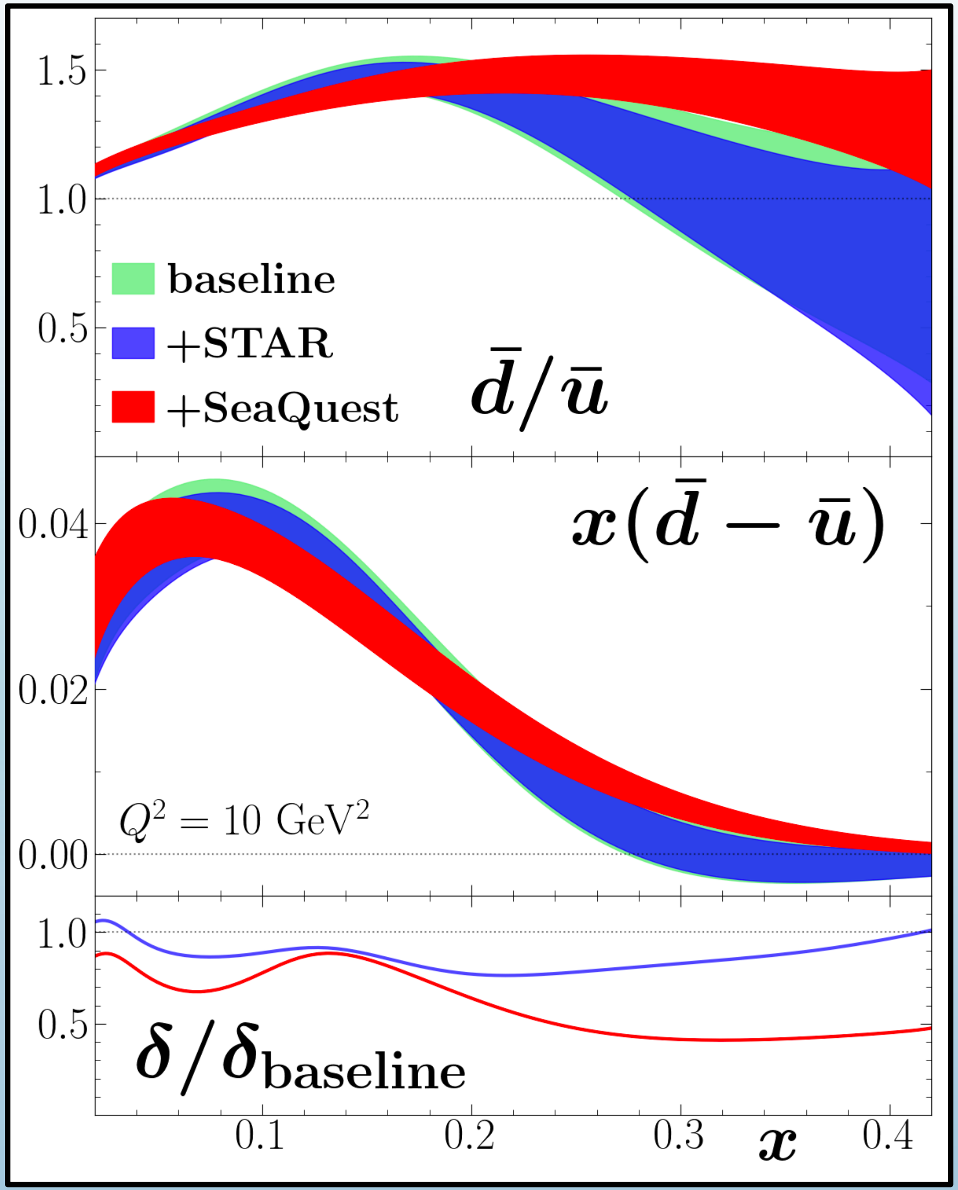

STAR: Moderate reduction of uncertainties

SeaQuest: Large reduction of uncertainties, $\bar{d}/\bar{u} > 1$ up to $x \approx 0.4$

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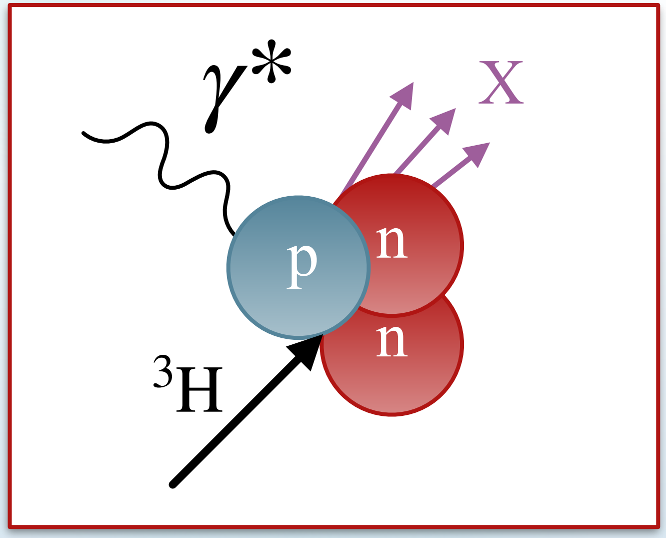

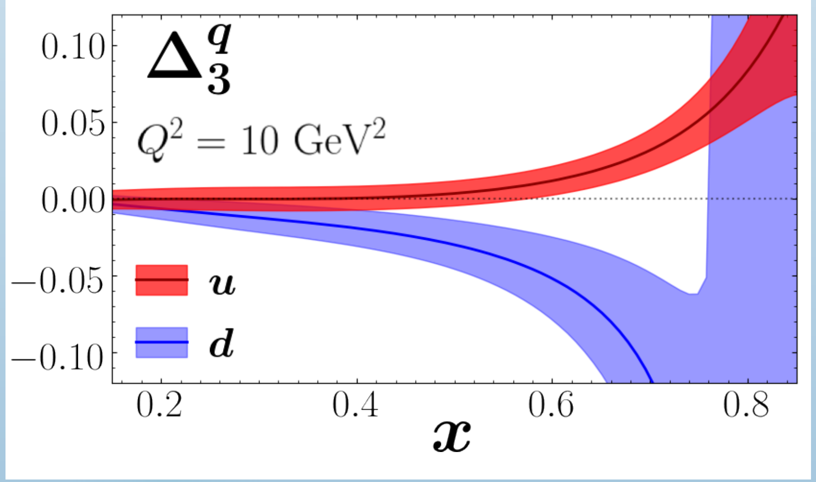

C. Cocuzza, C. E. Keppel, W. Melnitchouk, A. Metz, N. Sato, and A. W. Thomas, Phys. Rev. Lett. **127**, 242001 (2021)

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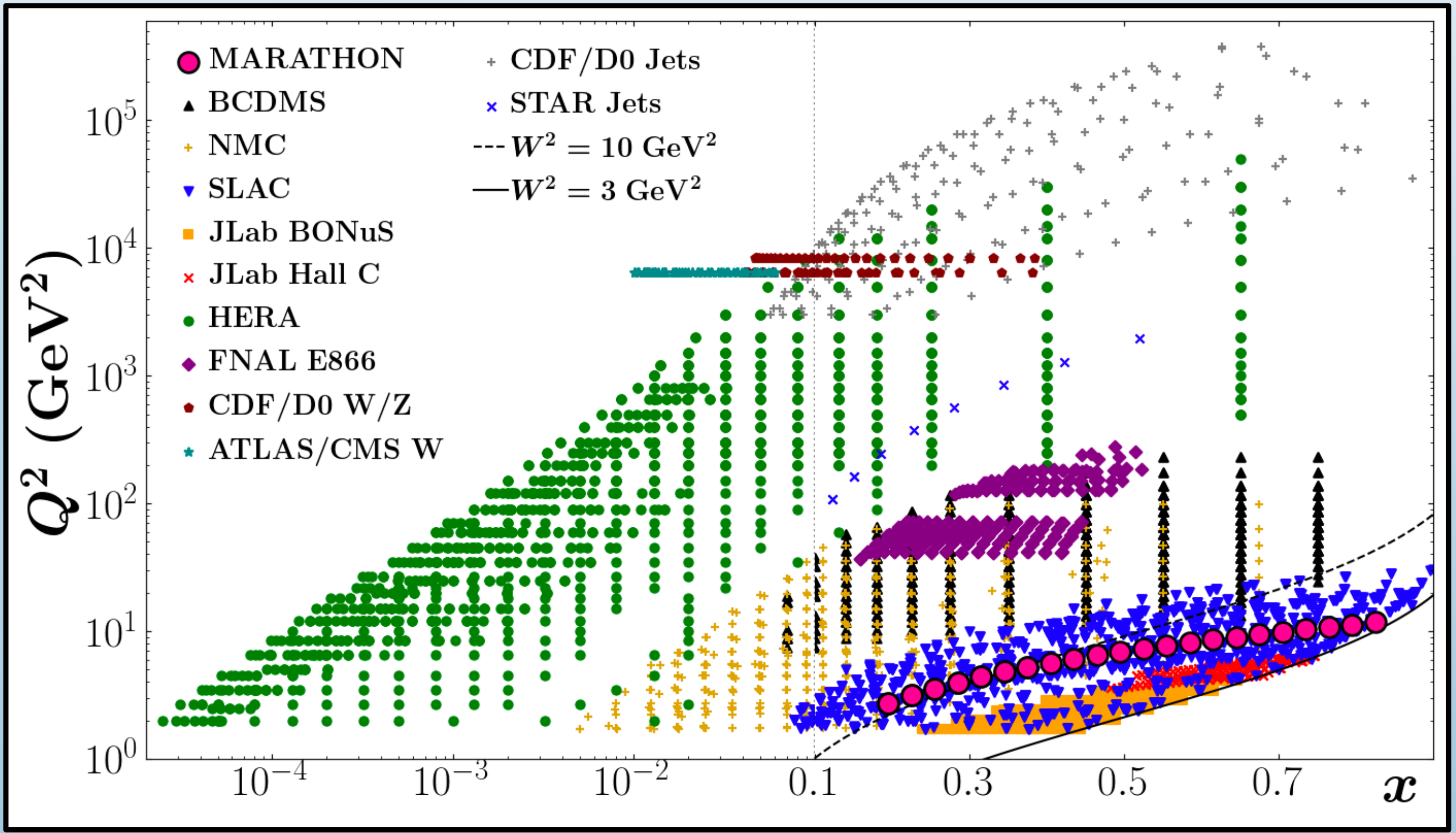

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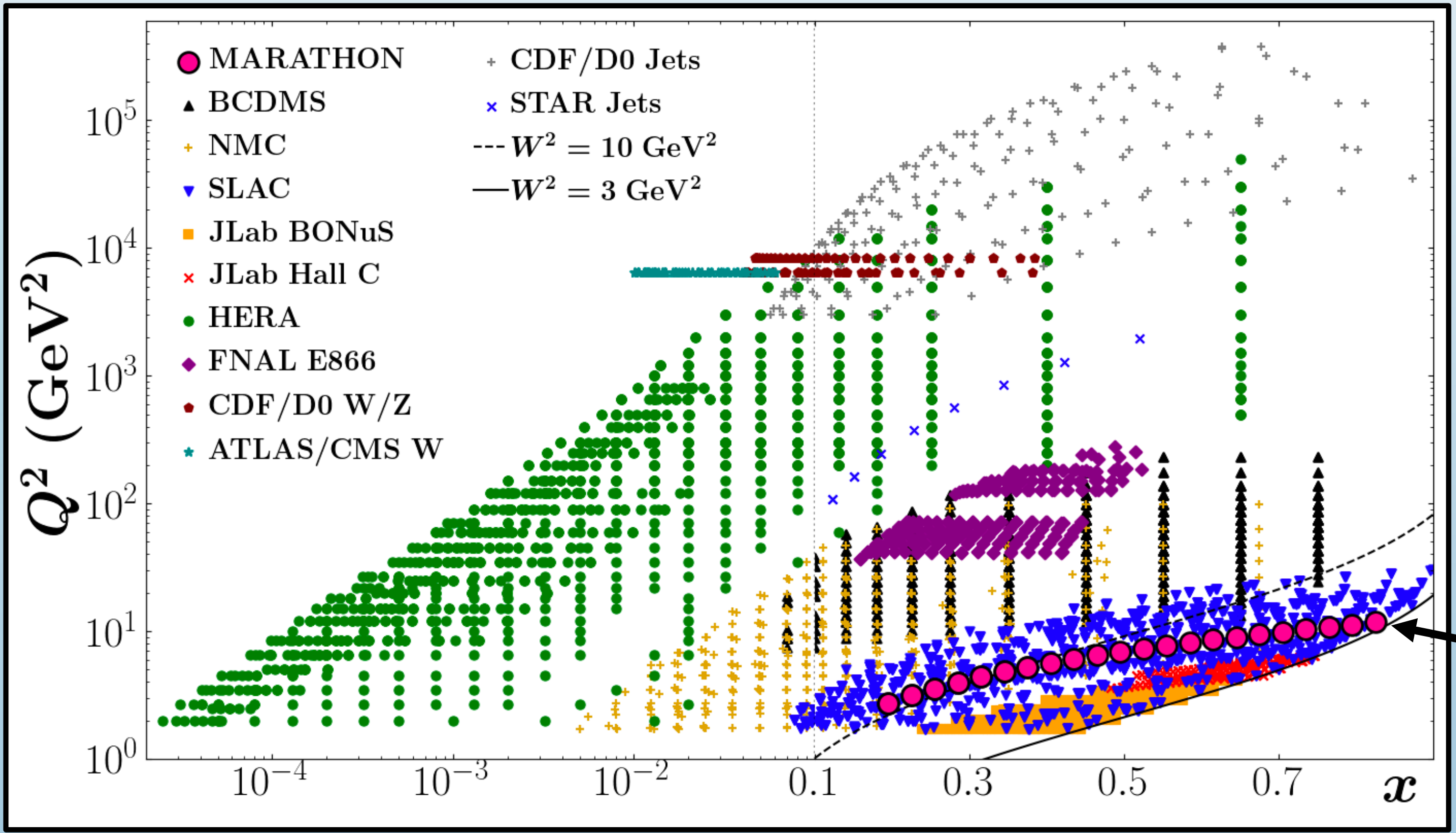


New MARATHON data

Impact from MARATHON

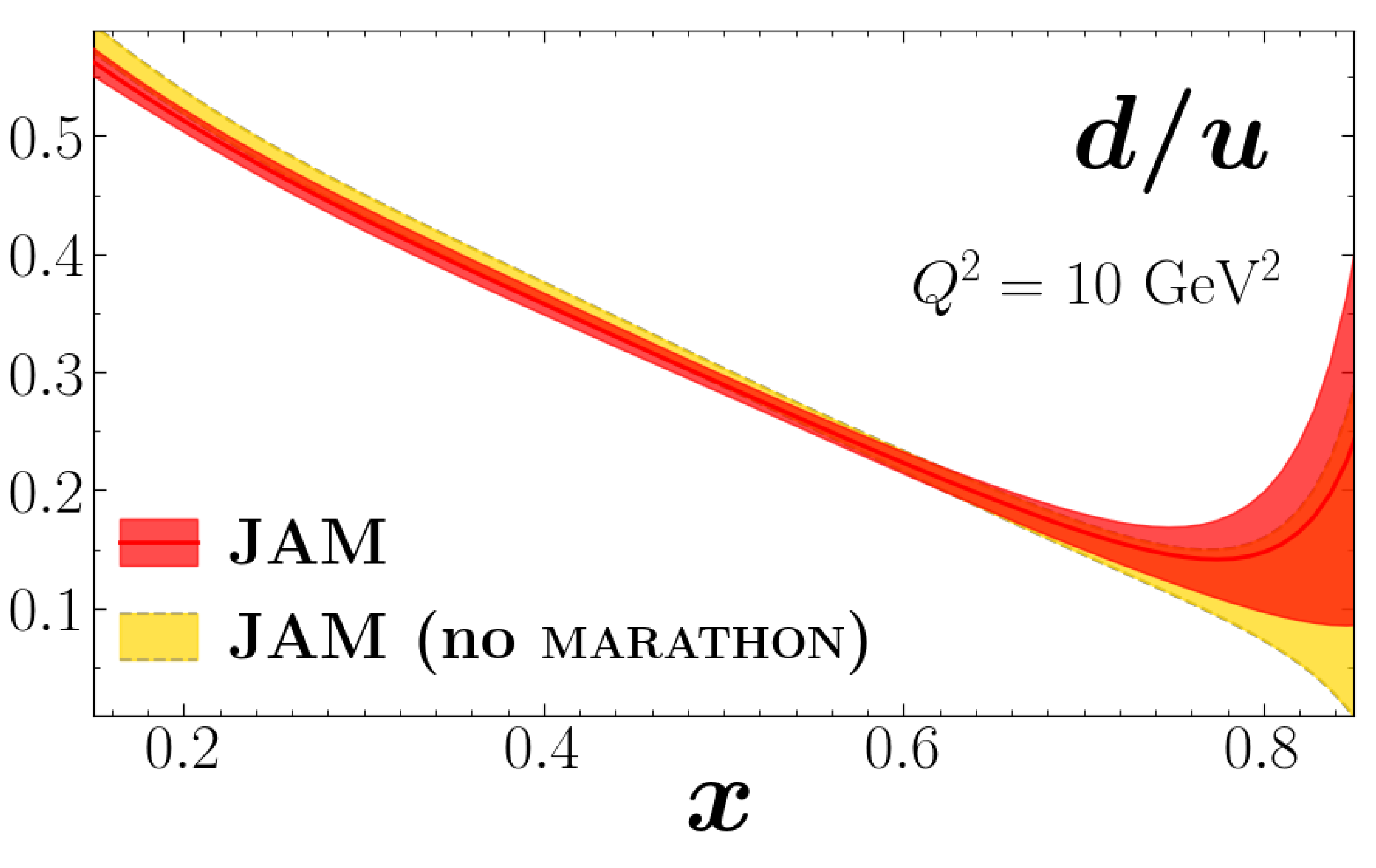
Me**A**surement of the F_2^n/F_2^p, d/u **RA**tios and $A = 3$ EMC Effect in Deep Inelastic Electron Scattering Off the **T**ritium and **H**elium Mirr**O**r **N**uclei

d/u Ratio ?

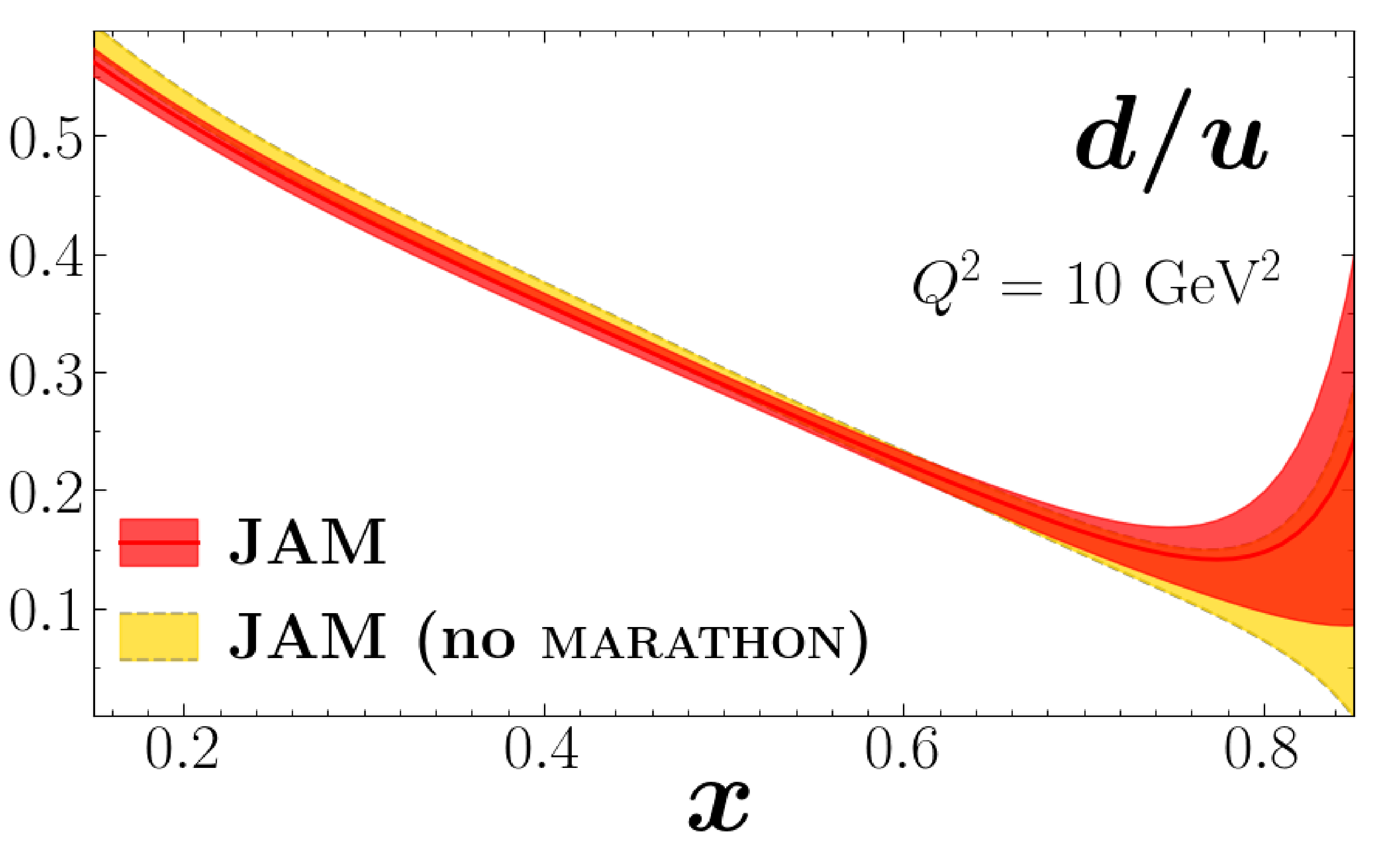
F_2^n/F_2^p Ratio ?

$A = 3$ EMC Effects ?

Impact on d/u

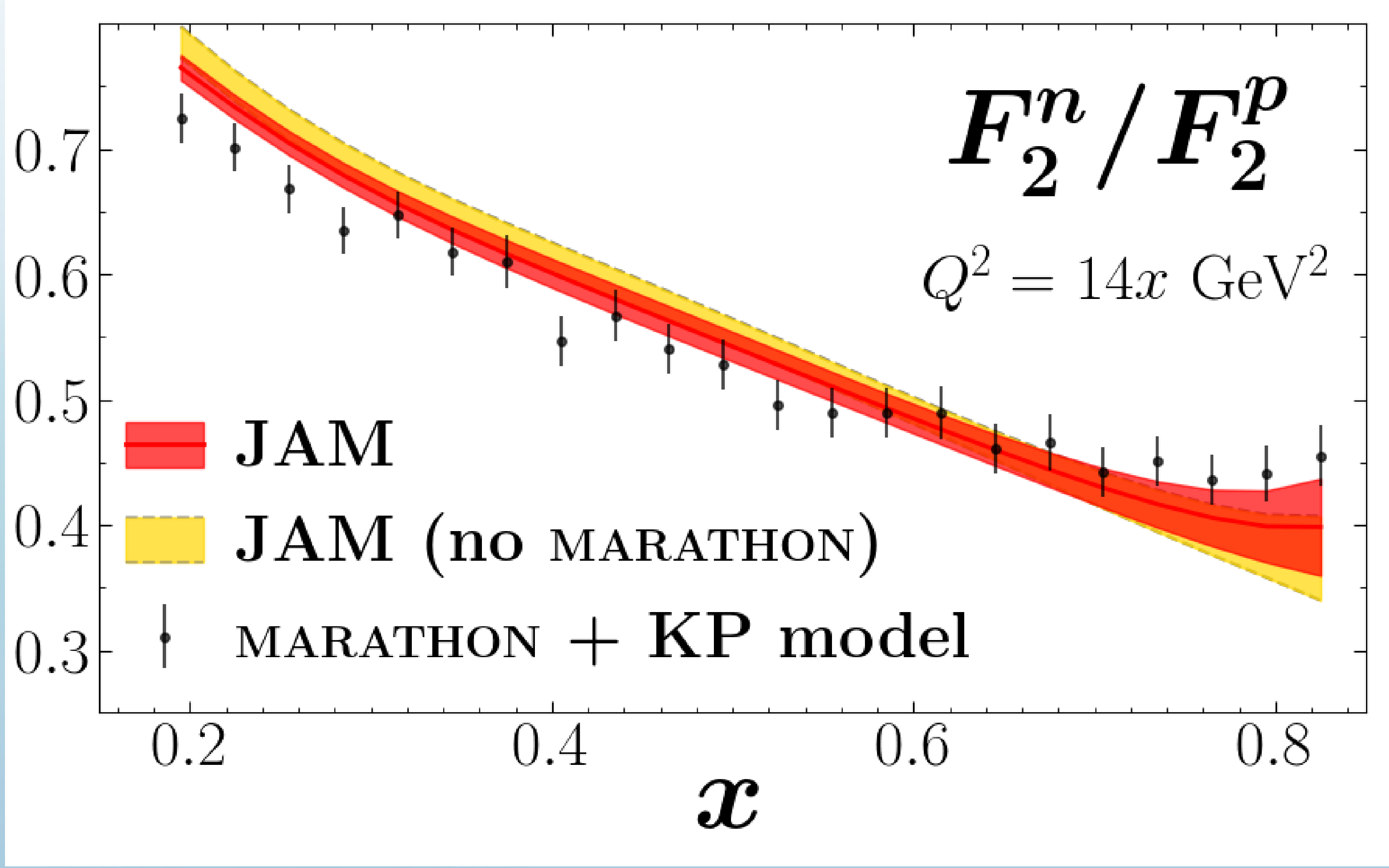


Impact on d/u



d/u ratio largely constrained by W boson production data (mostly Tevatron)

Impact on F_2^n/F_2^p



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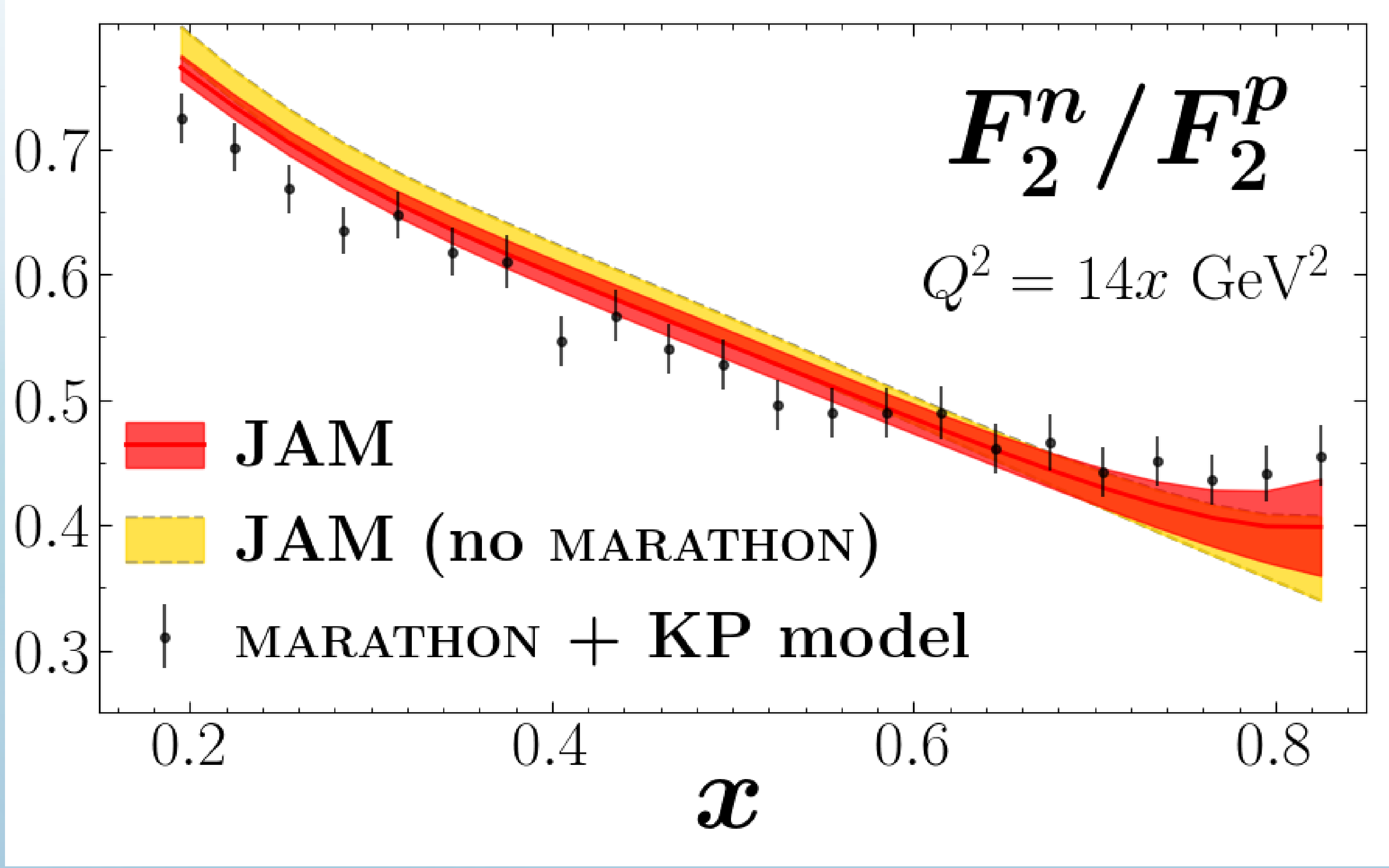

Slight shift towards MARATHON + KP model result

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d/u Ratio	~
F_2^n/F_2^p Ratio	~
$A = 3$ EMC Effects	?

Isospin Symmetry

How to relate quarks between protons and neutrons?

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? Asymmetric Nuclei (^{3}He, ^{3}H, ^{197}Au)

Isovector Effect

I. C. Cloet, W. Bentz and A. W. Thomas, Phys. Rev. Lett. **102**, 252301 (2009)

Mediated by $I_3 = 1$ mesons, dependent on third component of isospin

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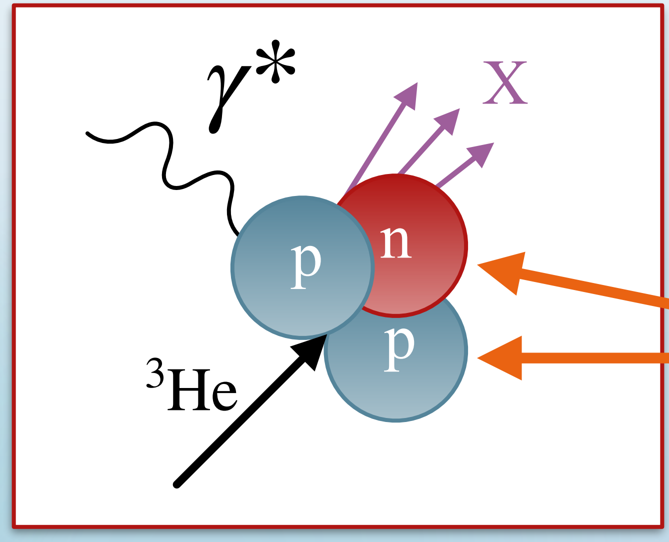

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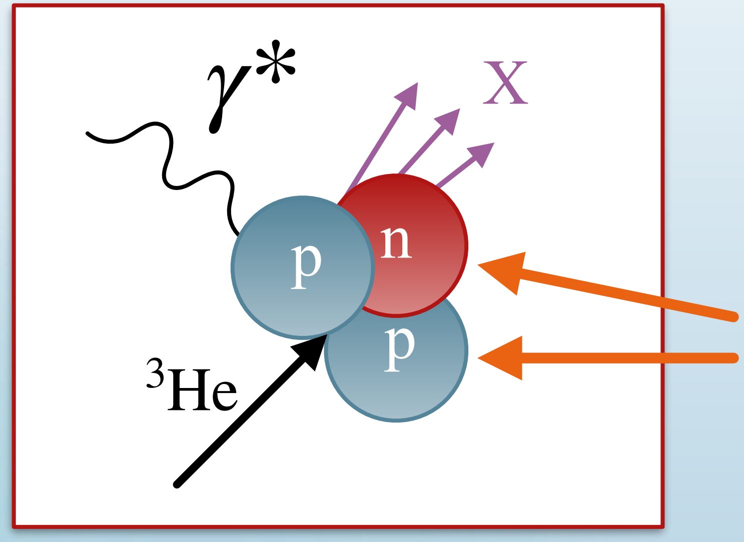

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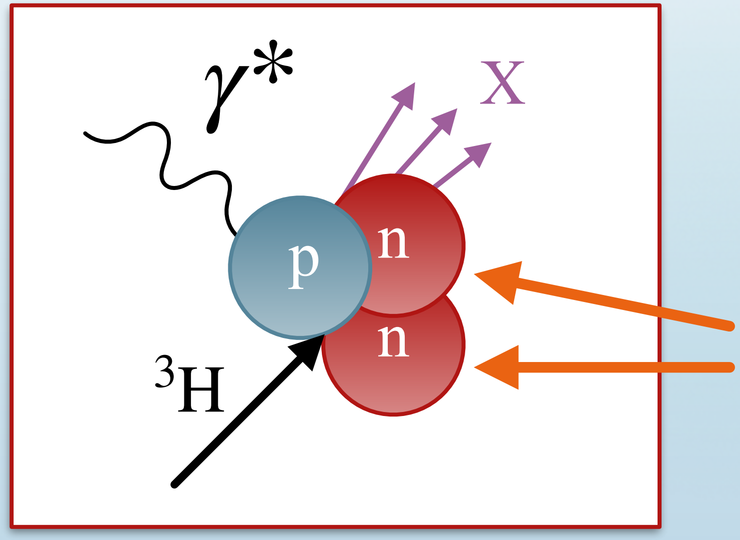

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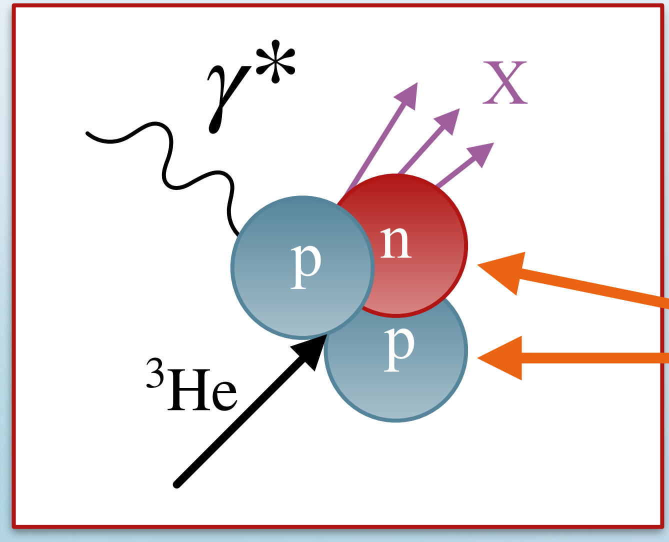

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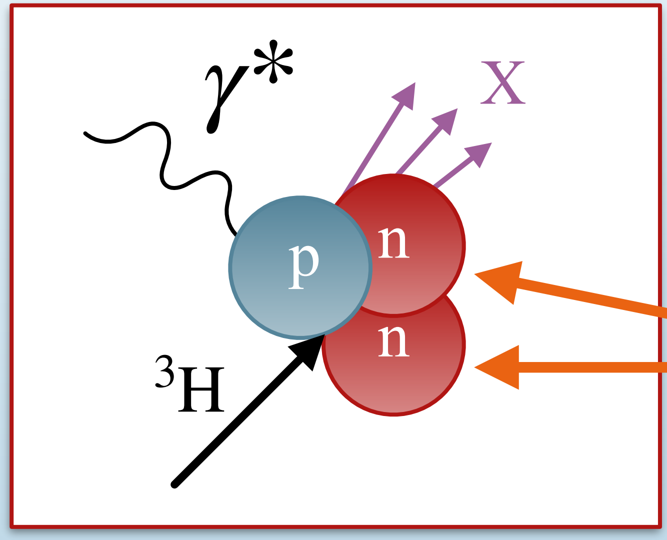


Spectators
$I_3 = -1$

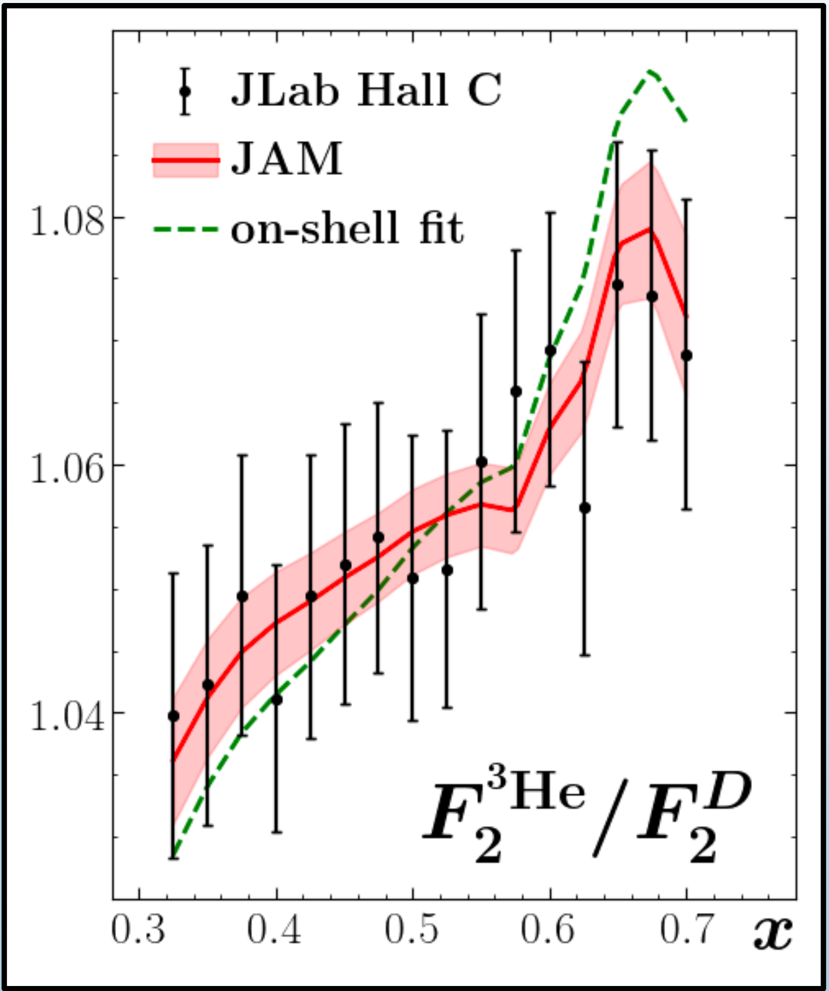
Parameterize phenomenologically:

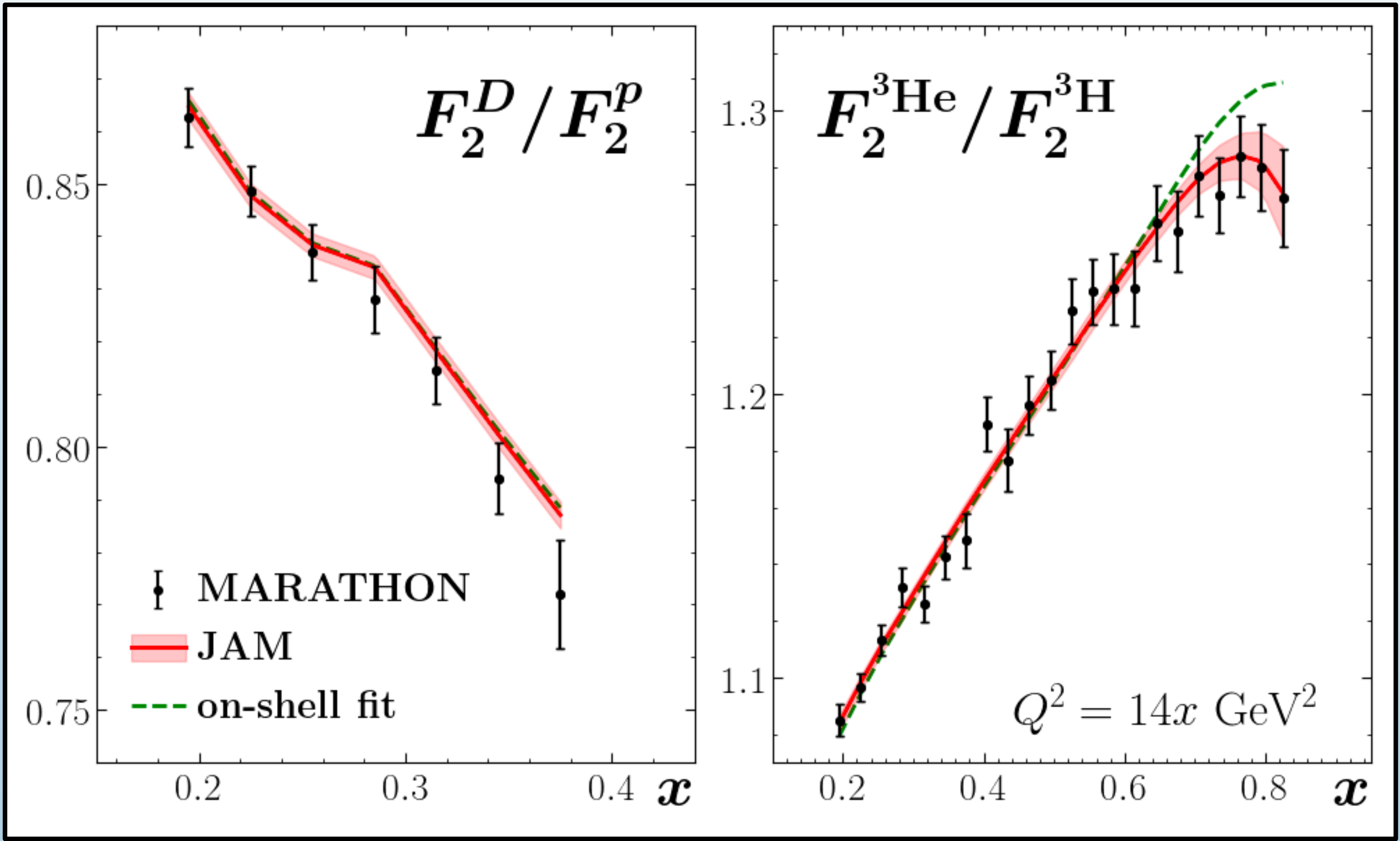
$$\tilde{q}_{N/A}(p^2) = q_N + \nu(p^2)\ \delta q_{N/A} + \ldots$$

Virtuality

$$\nu(p^2) = (p^2 - M^2)/M^2 \ll 1$$

Data vs. Theory

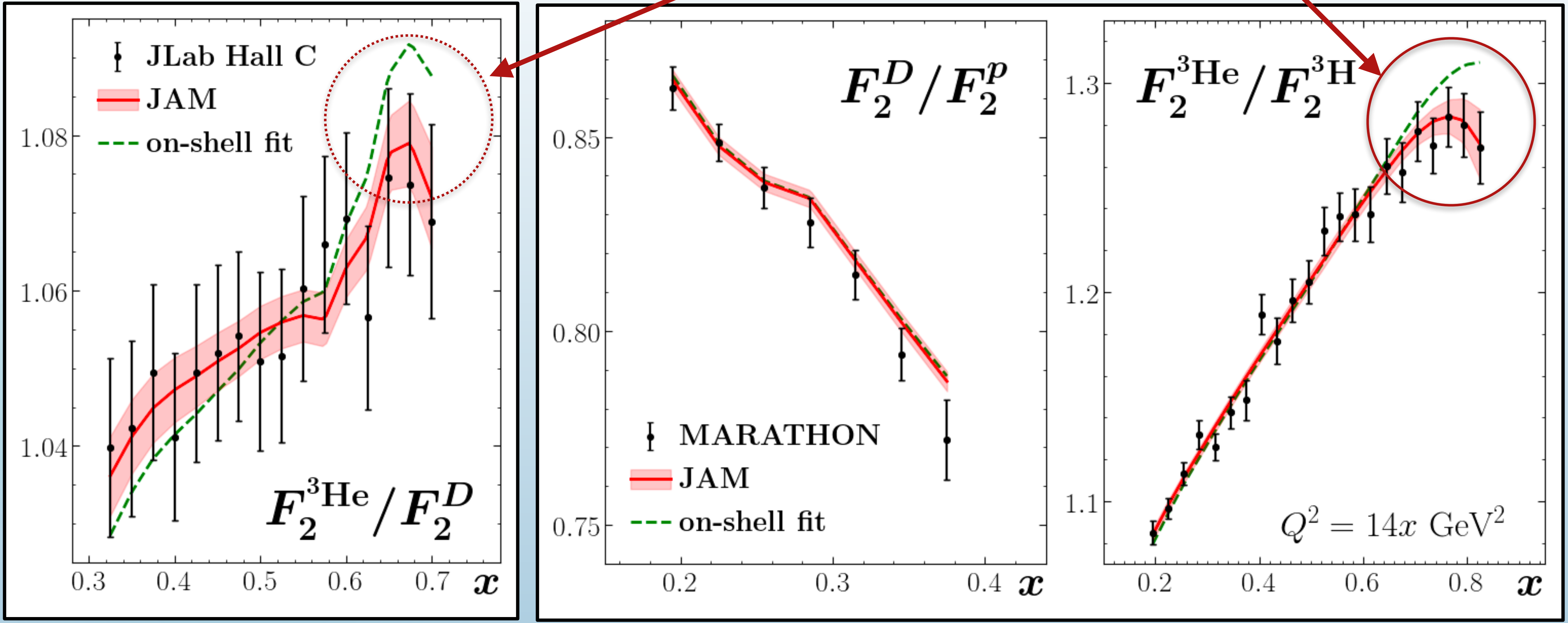




First global QCD analysis of JLab ^{3}He/D and MARATHON data

Data vs. Theory

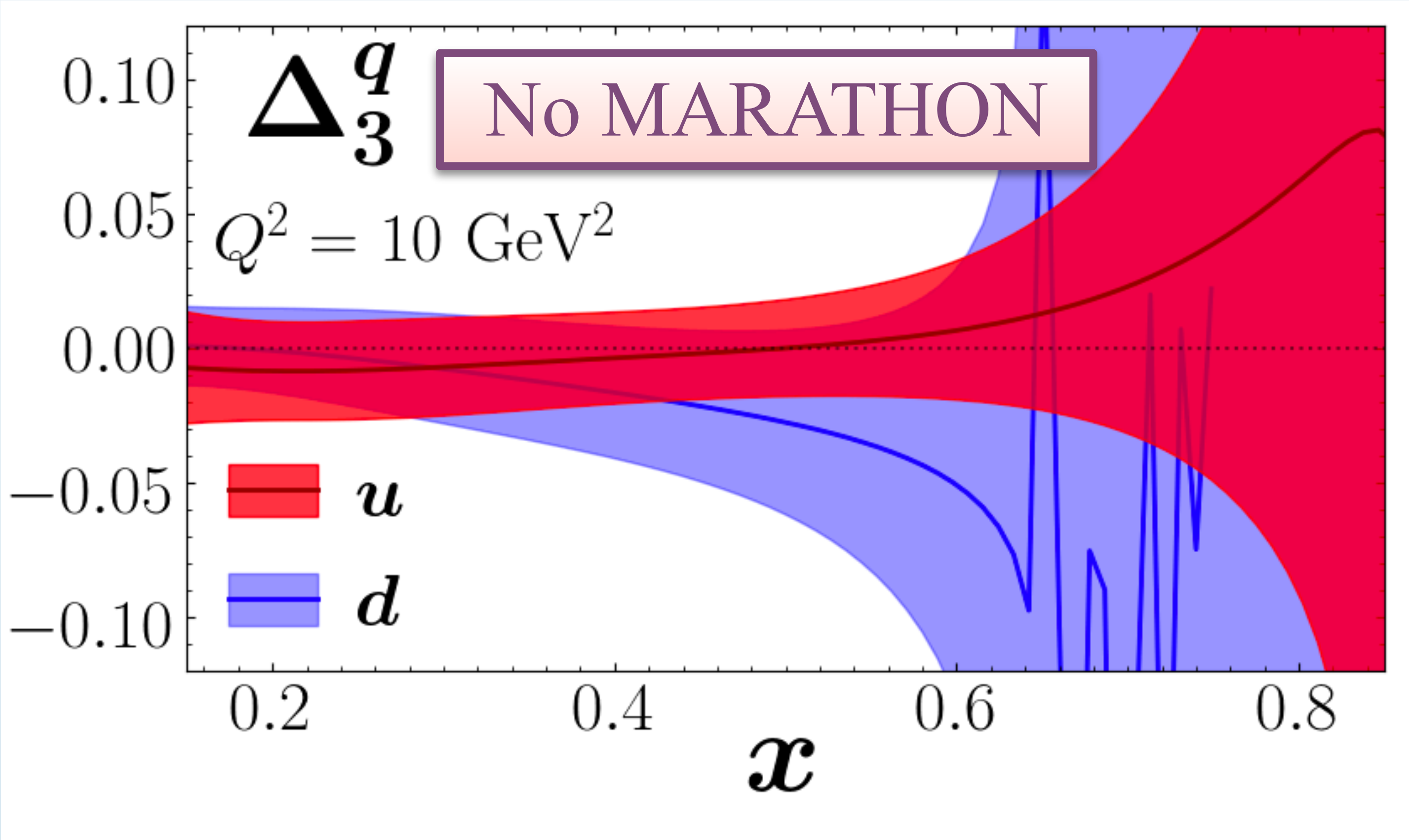
Suggestion of off-shell effects!



First global QCD analysis of JLab ^{3}He/D and MARATHON data

Isovector Extraction

$$\Delta_3^q \equiv \frac{q_{p/^3\mathrm{H}} - q_{p/^3\mathrm{He}}}{q_{p/^3\mathrm{H}} + q_{p/^3\mathrm{He}}}$$



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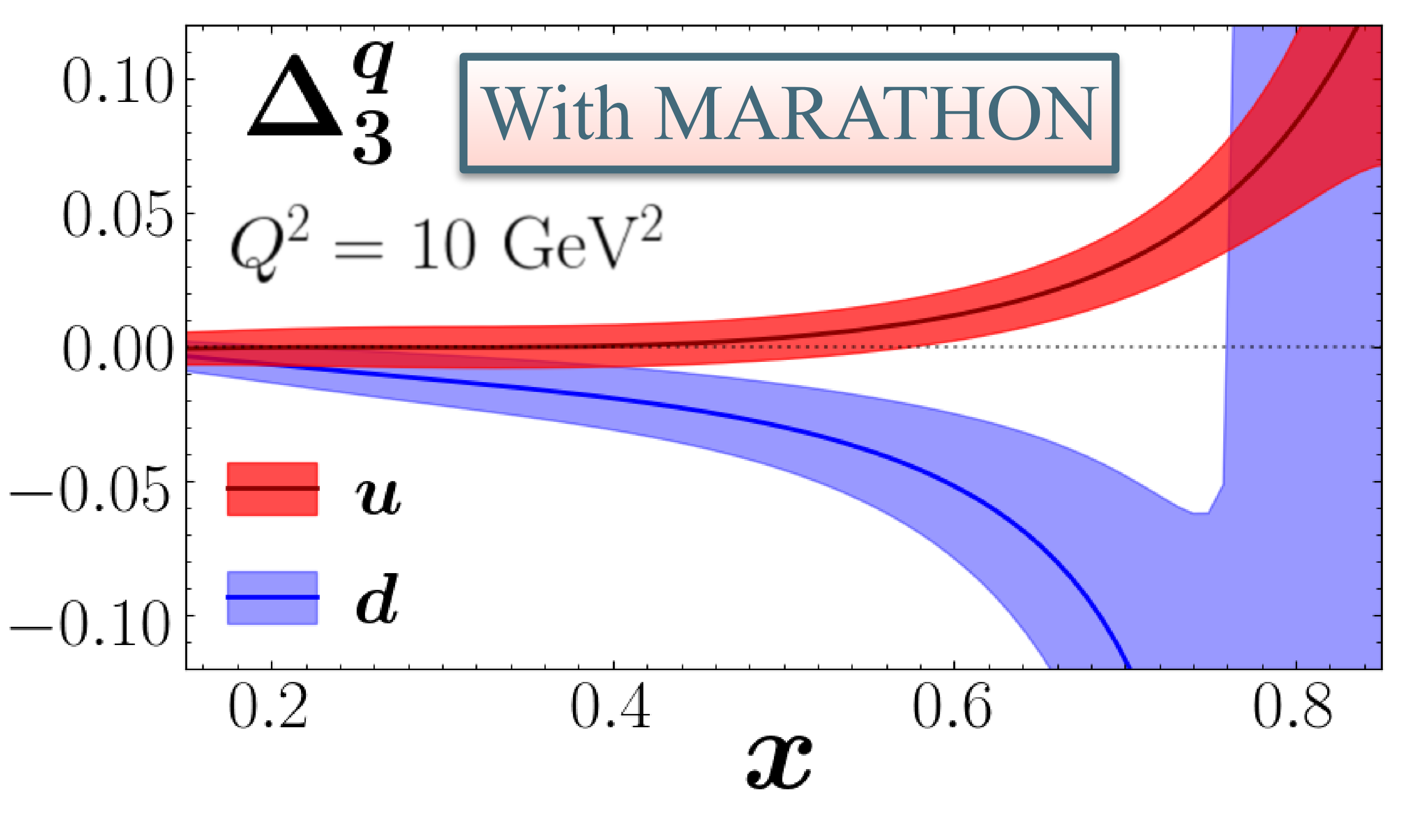
Signal for non-zero effect above $x \gtrsim 0.4$!



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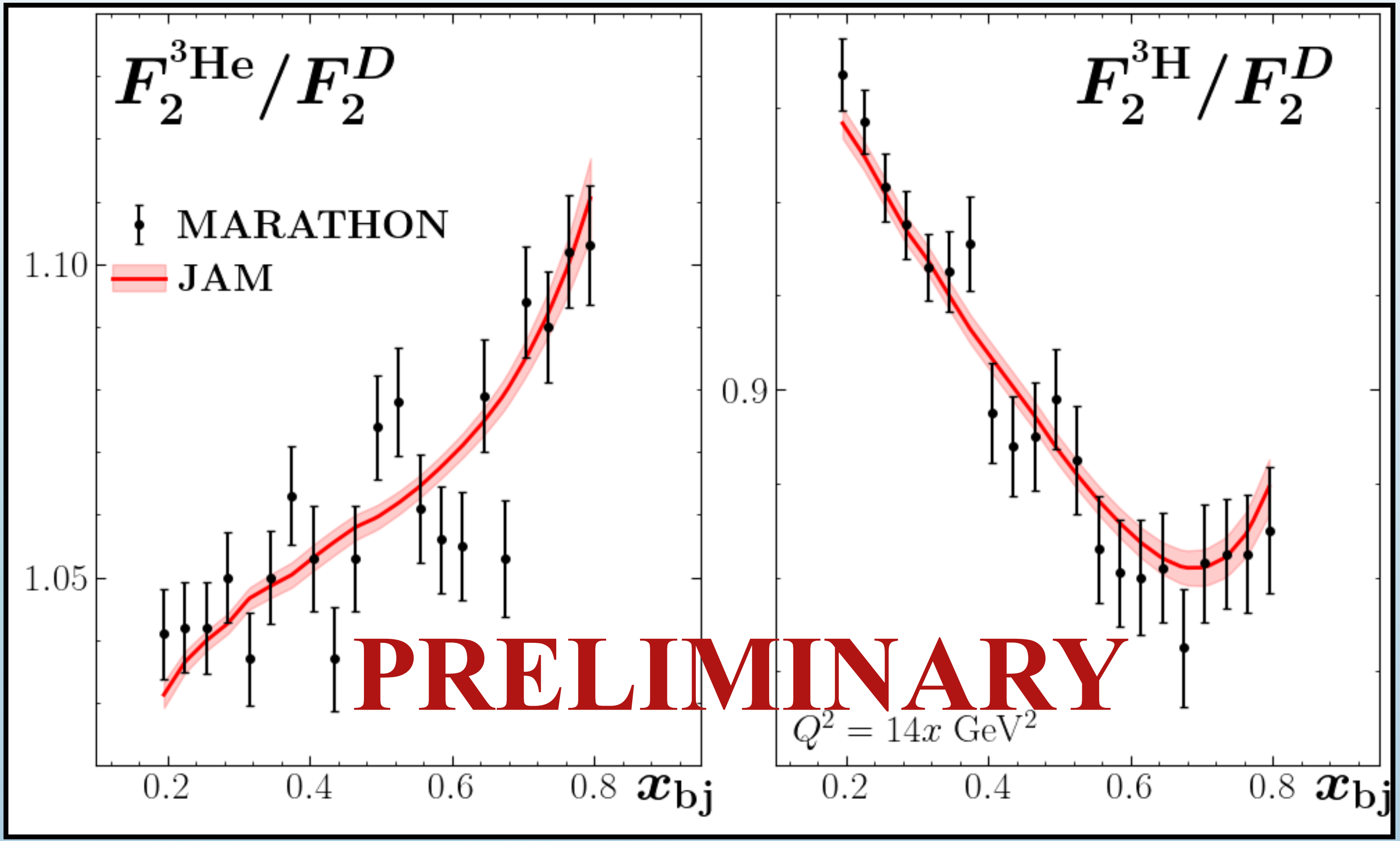
d/u Ratio	~
F_2^n/F_2^p Ratio	~
$A = 3$ EMC Effects	✓

Future Work



MARATHON released new results on ^{3}He/D and ^{3}H/D very recently. We are able to fit this data well

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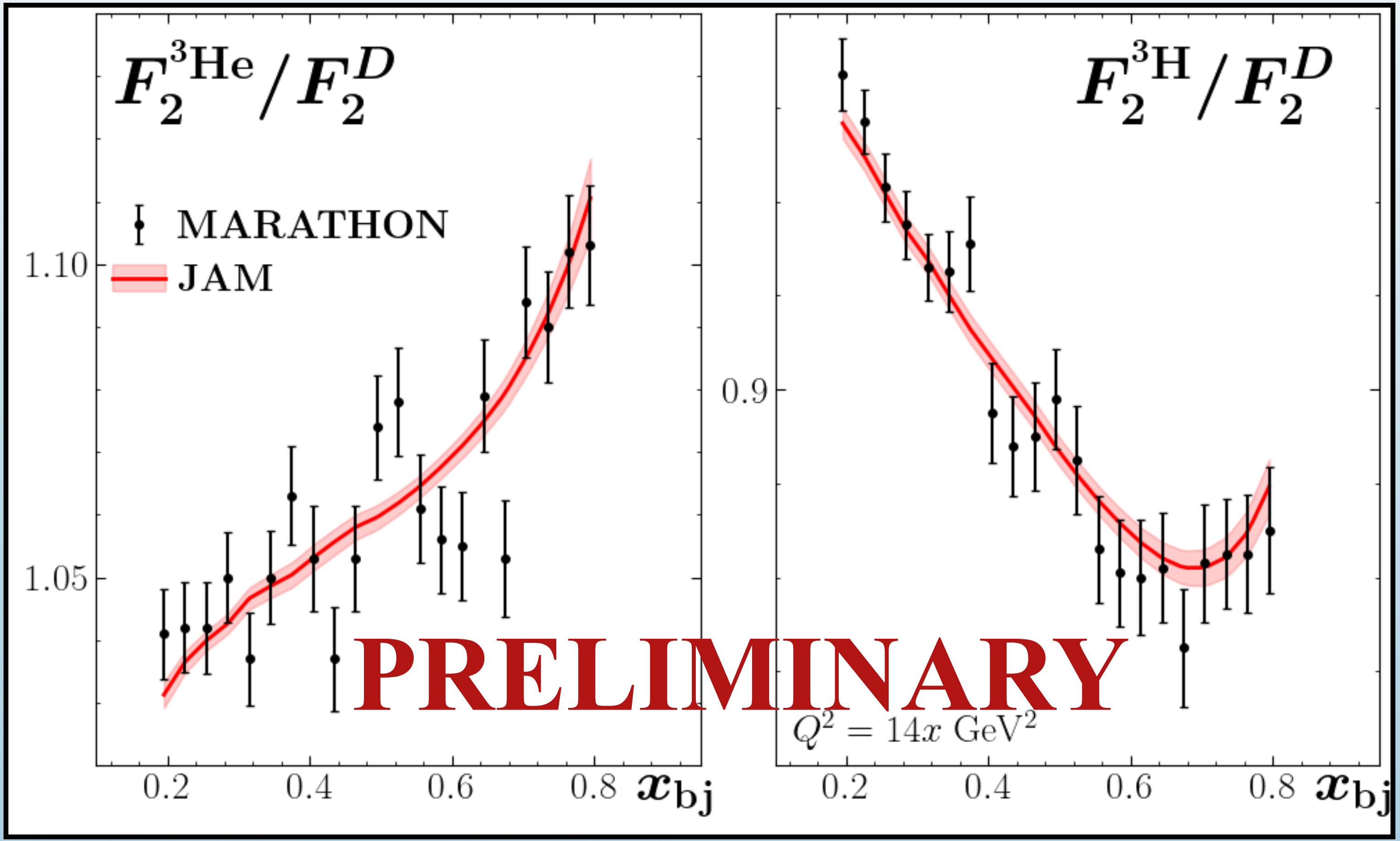

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Preliminary results show that it reinforces our previous findings

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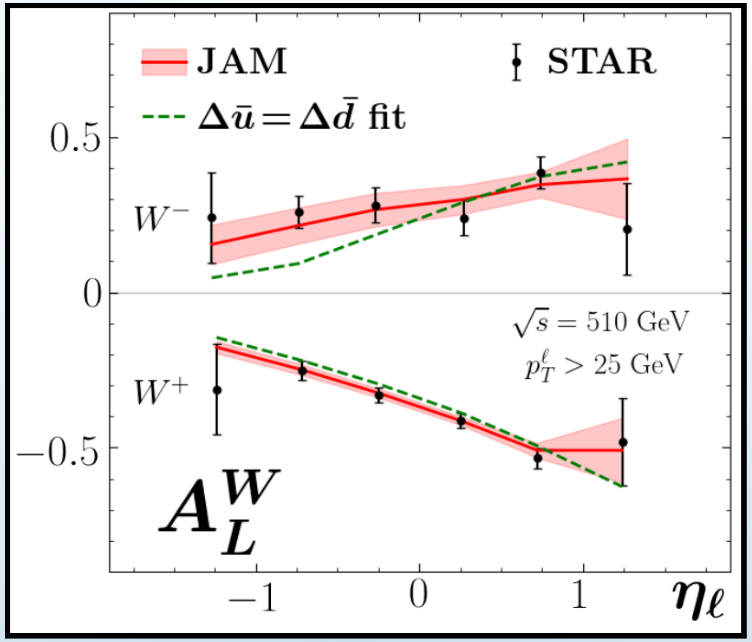
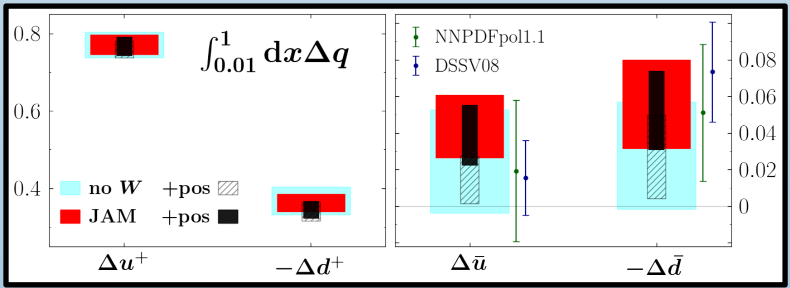
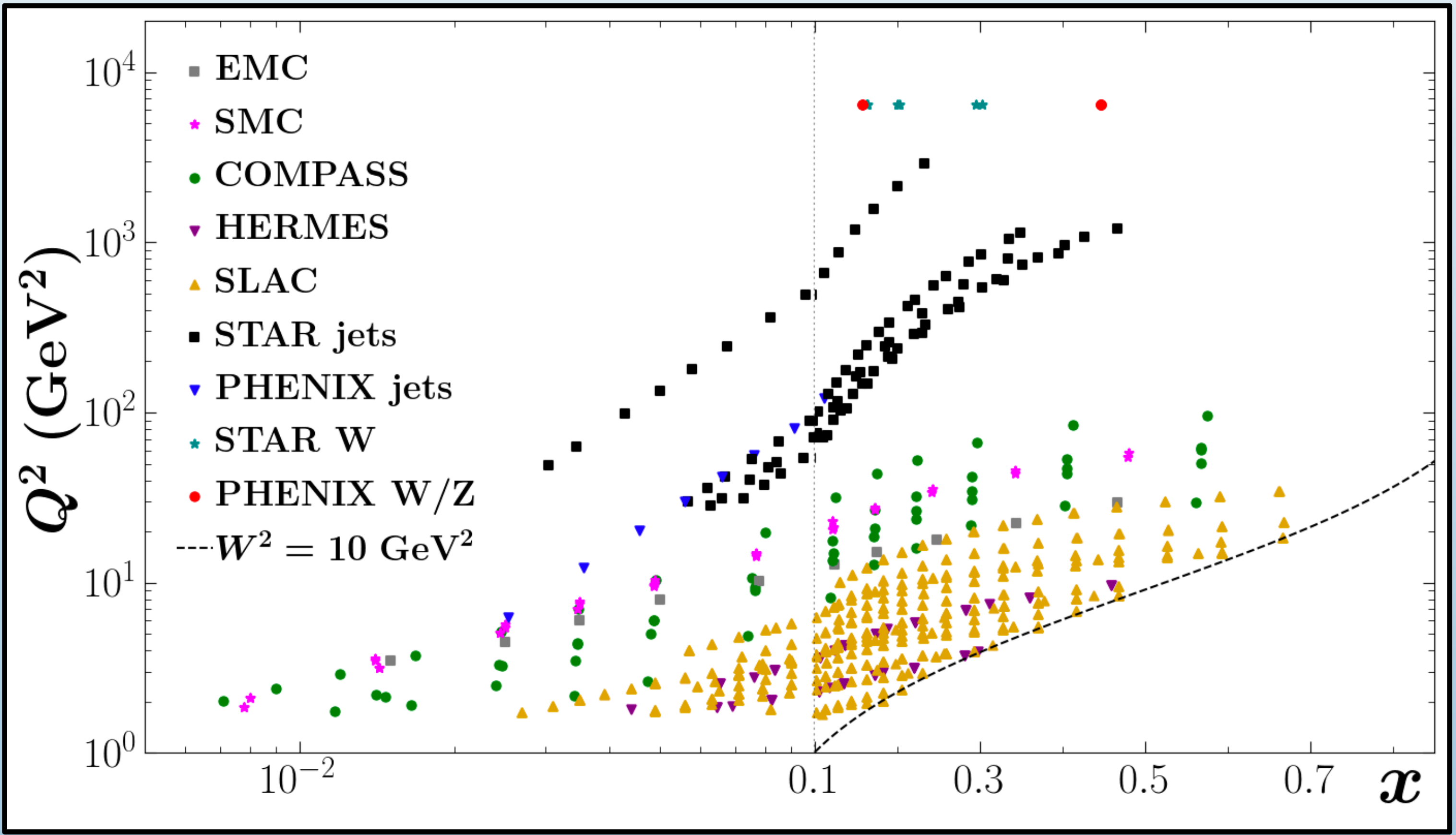




C. Cocuzza, W. Melnitchouk, A. Metz, and N. Sato, Phys. Rev. D. **106**, L031502 (2022)

Kinematic Coverage (Helicity)

Deep Inelastic Scattering	COMPASS, EMC, HERMES, SLAC, SMC	365 points
Semi-Inclusive DIS	COMPASS, HERMES, SMC	231 points
W/Z Boson Production	STAR, PHENIX	18 points
Jets	STAR, PHENIX	61 points



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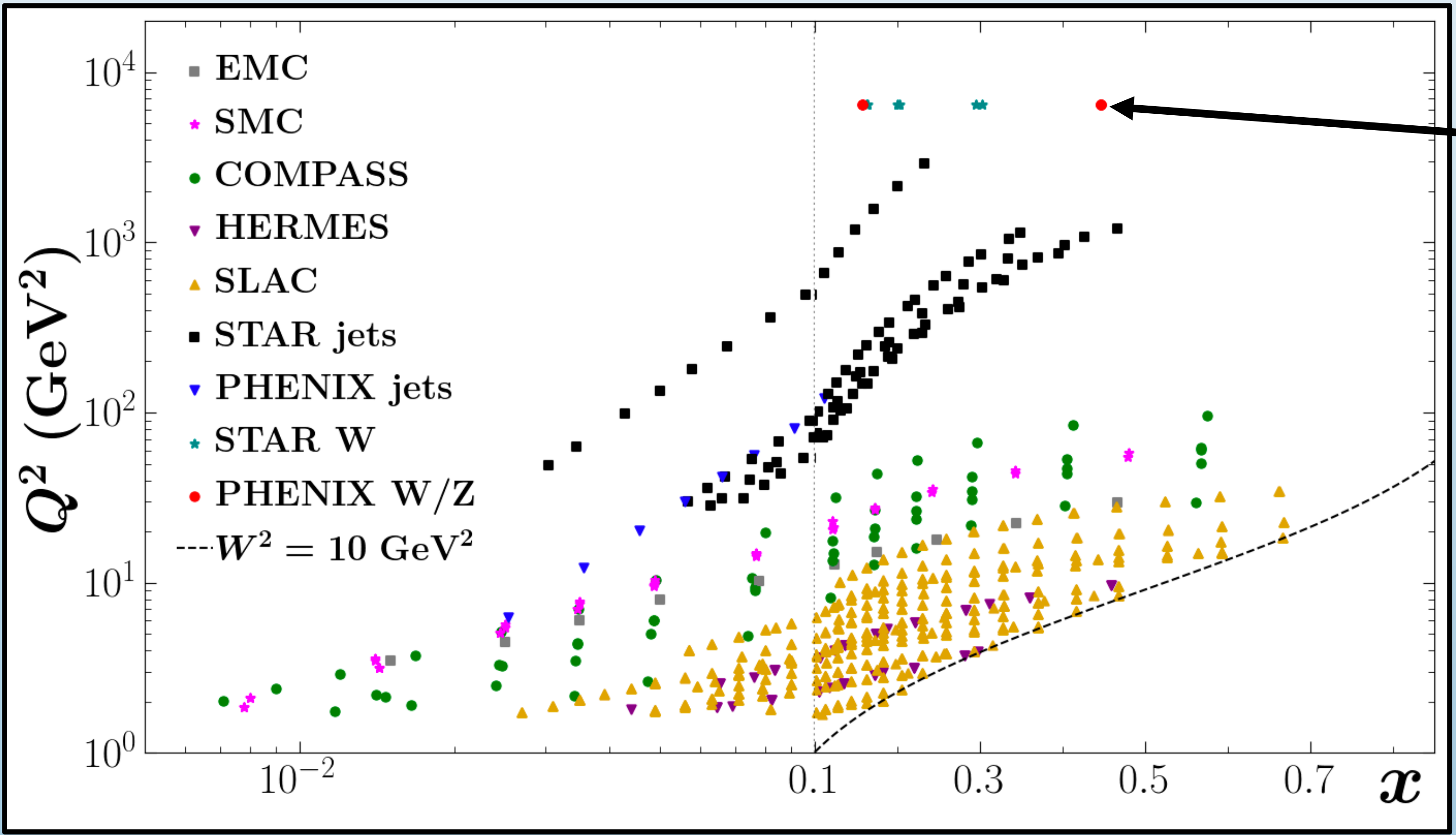

STAR + PHENIX
W/Z Production

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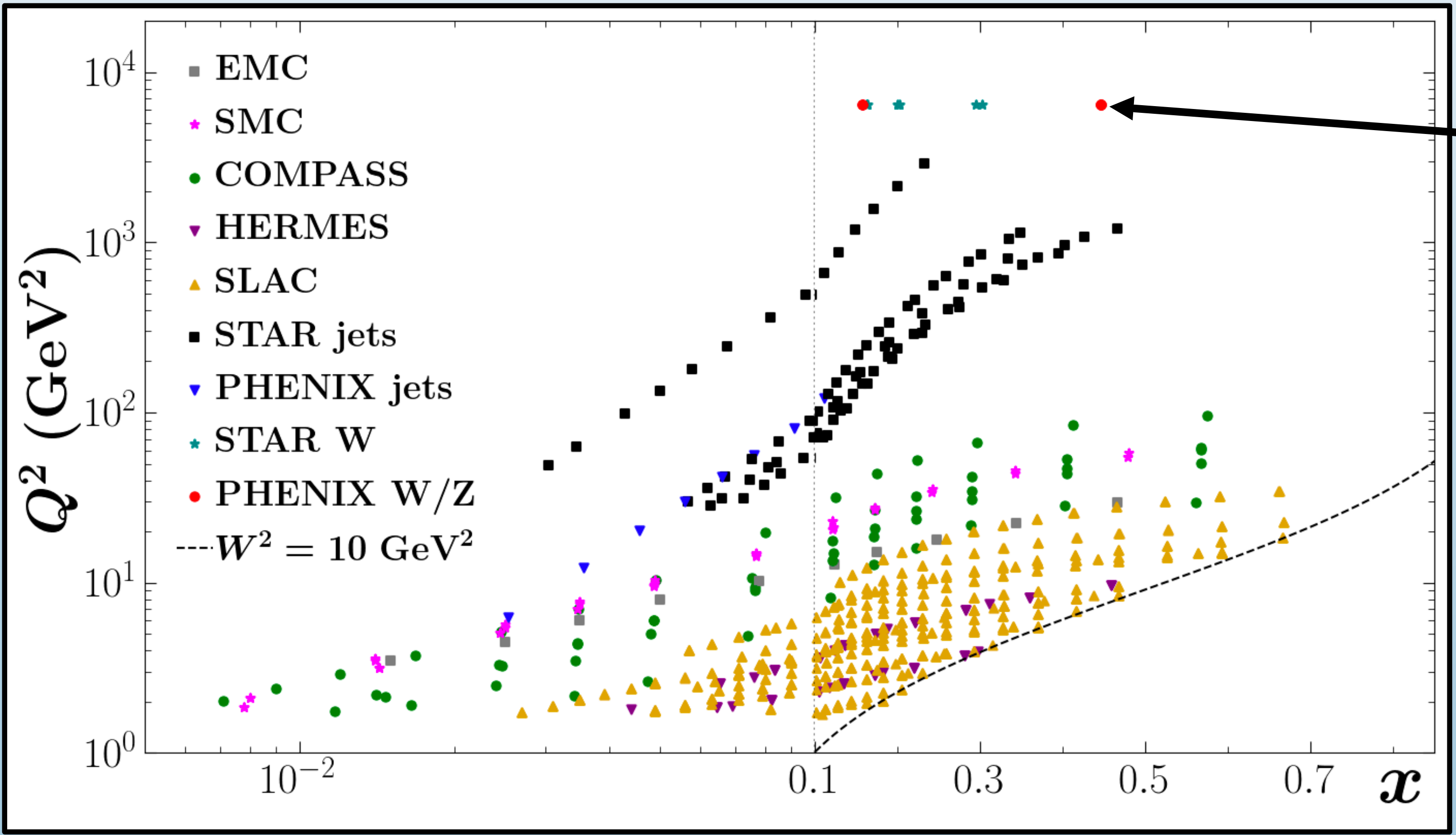
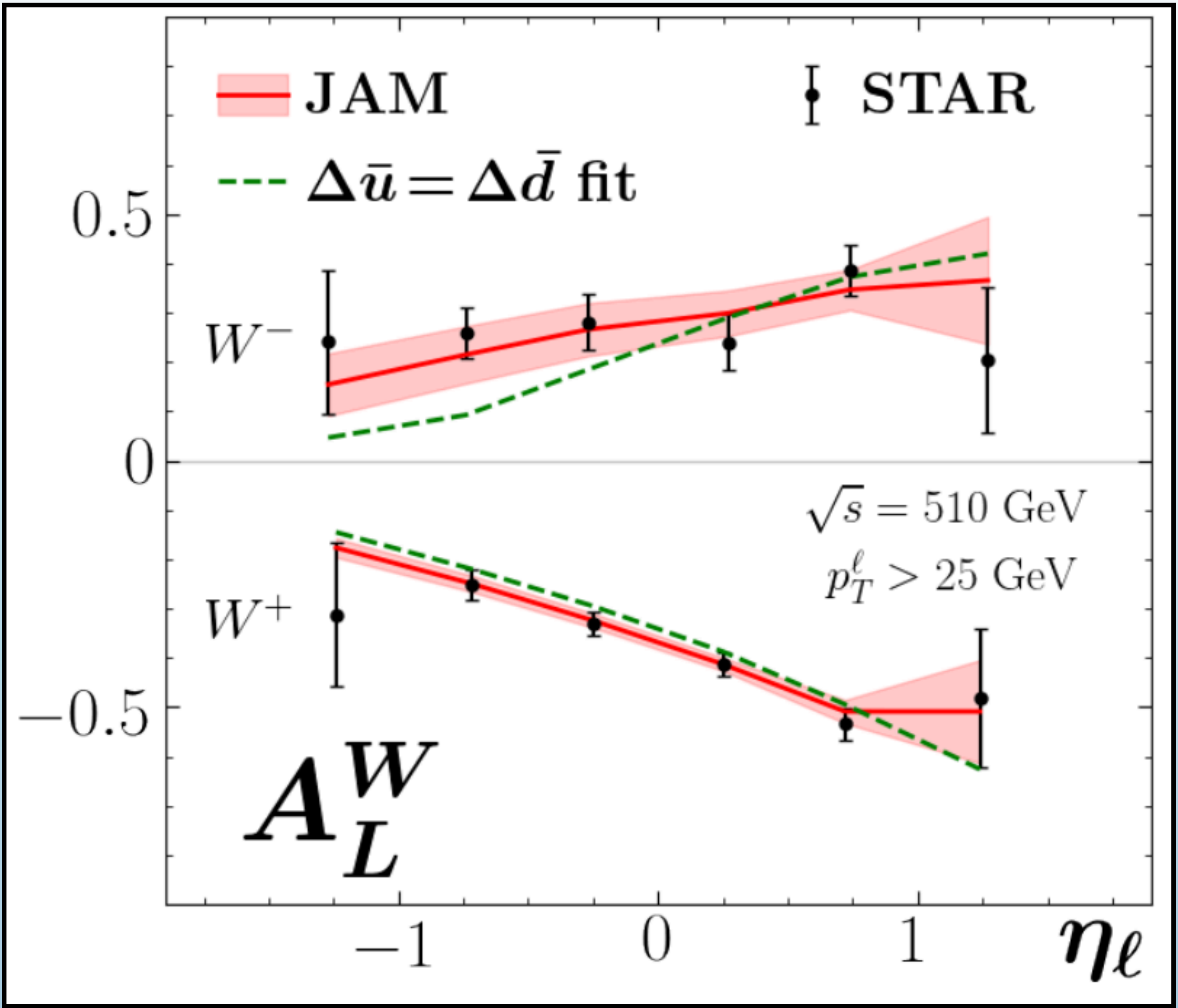


STAR + PHENIX W/Z Production

Simultaneous extraction of spin-averaged PDFs, helicity PDFs, and FFs

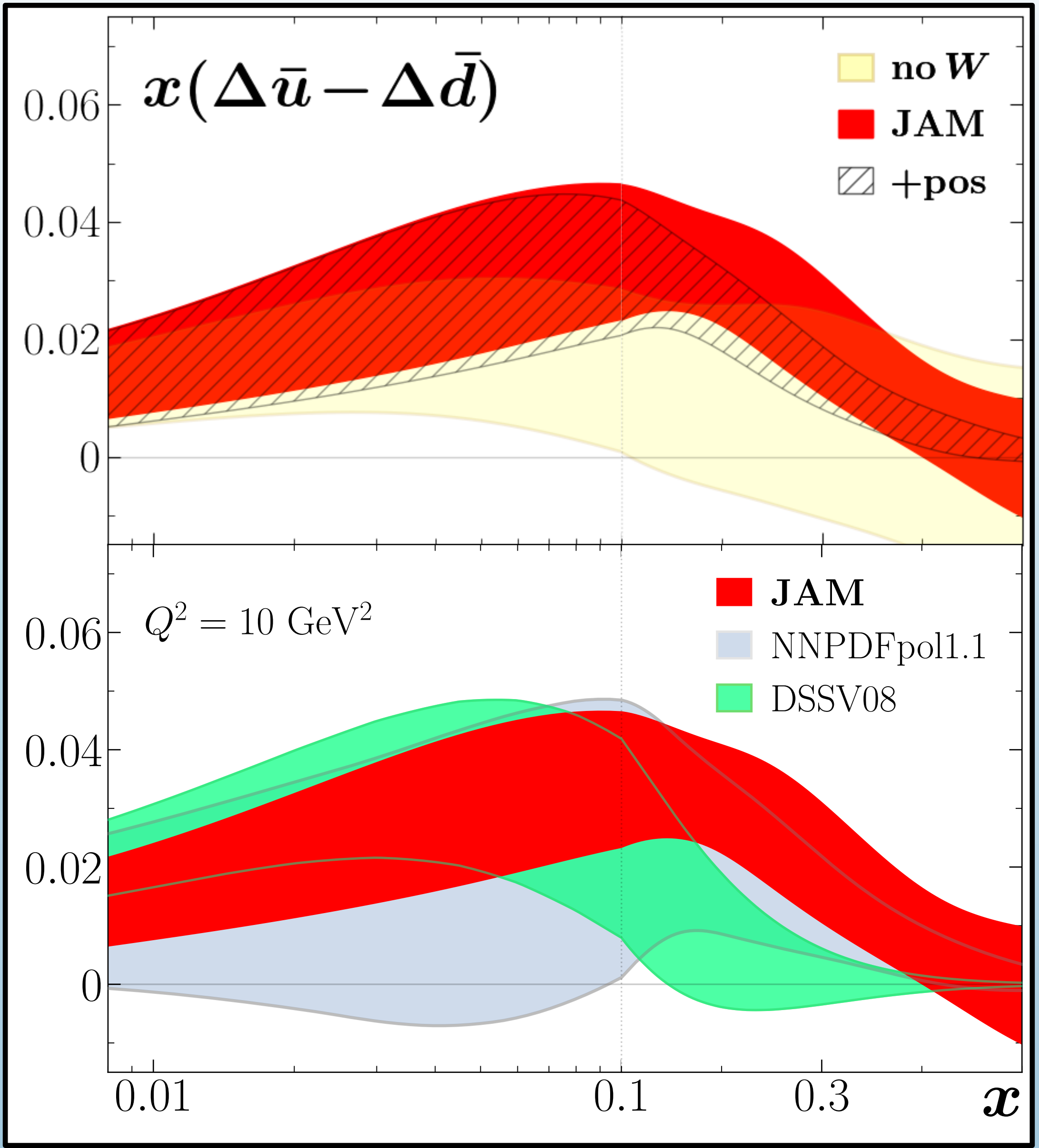
STAR Quality of Fit



$$A_L^{W^+}(y_W) \propto \frac{\Delta\bar{d}(x_1)u(x_2) - \Delta u(x_1)\bar{d}(x_2)}{\bar{d}(x_1)u(x_2) + u(x_1)\bar{d}(x_2)}$$

$$A_L^{W^-}(y_W) \propto \frac{\Delta\bar{u}(x_1)d(x_2) - \Delta d(x_1)\bar{u}(x_2)}{\bar{u}(x_1)d(x_2) + d(x_1)\bar{u}(x_2)}$$

Resulting Asymmetry



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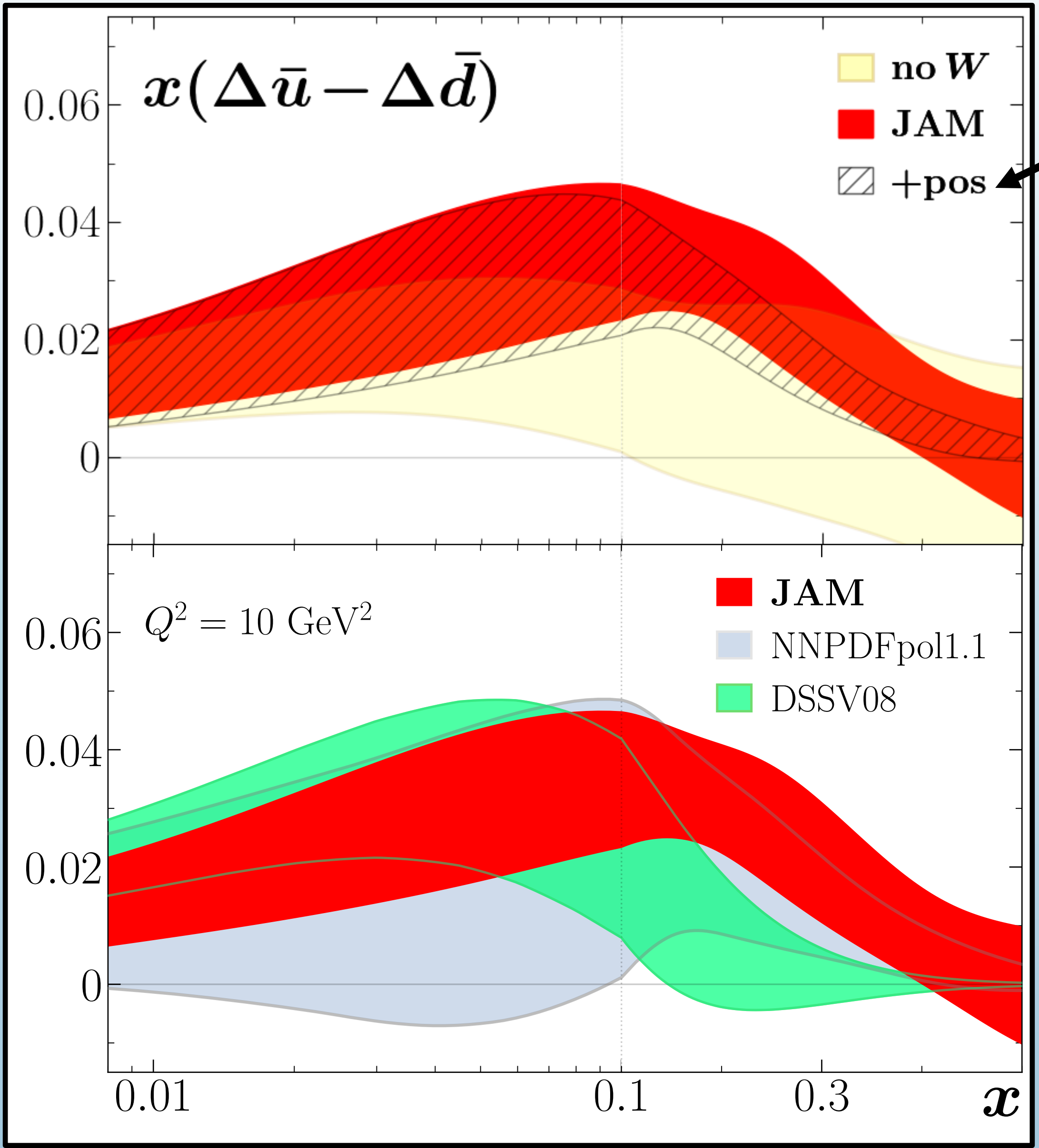
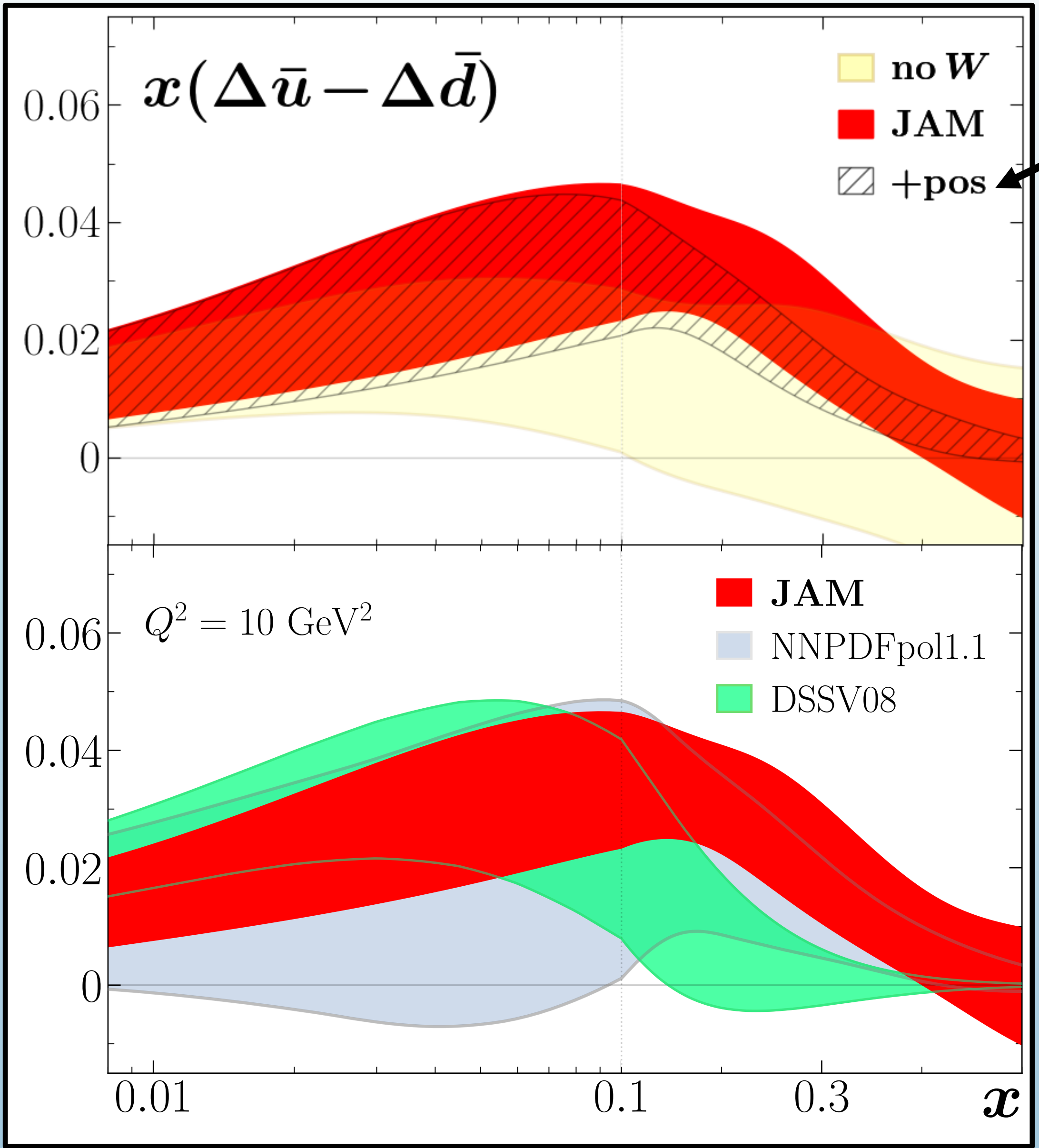


Positivity Constraints:
$|\Delta f(x, Q^2)| < f(x, Q^2)$

Resulting Asymmetry



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Can $\overline{\text{MS}}$ parton distributions be negative?
Alessandro Candido, Stefano Forte and Felix Hekhorn

Positivity and renormalization of parton densities
John Collins, Ted C. Rogers, Nobuo Sato

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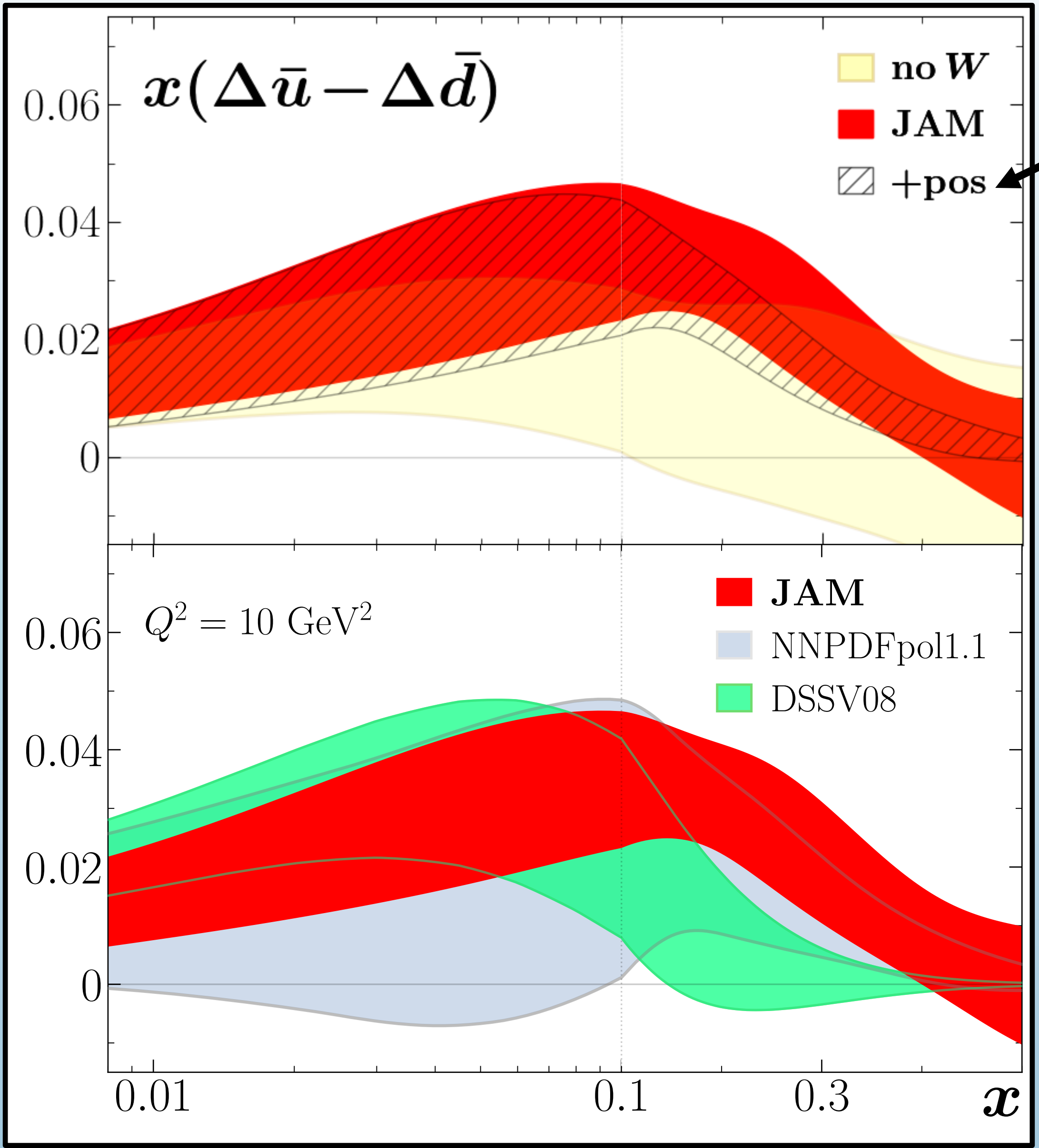

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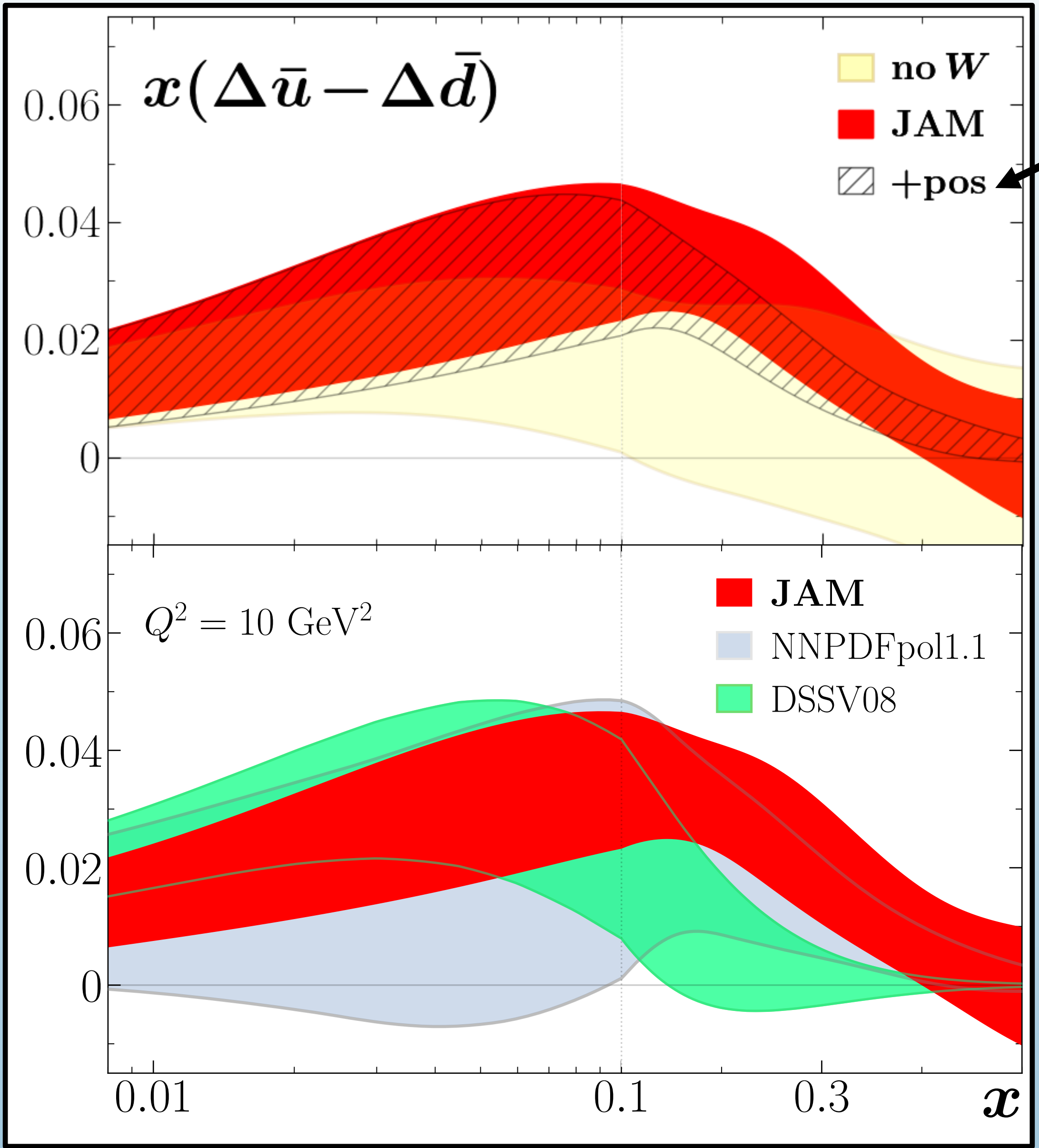

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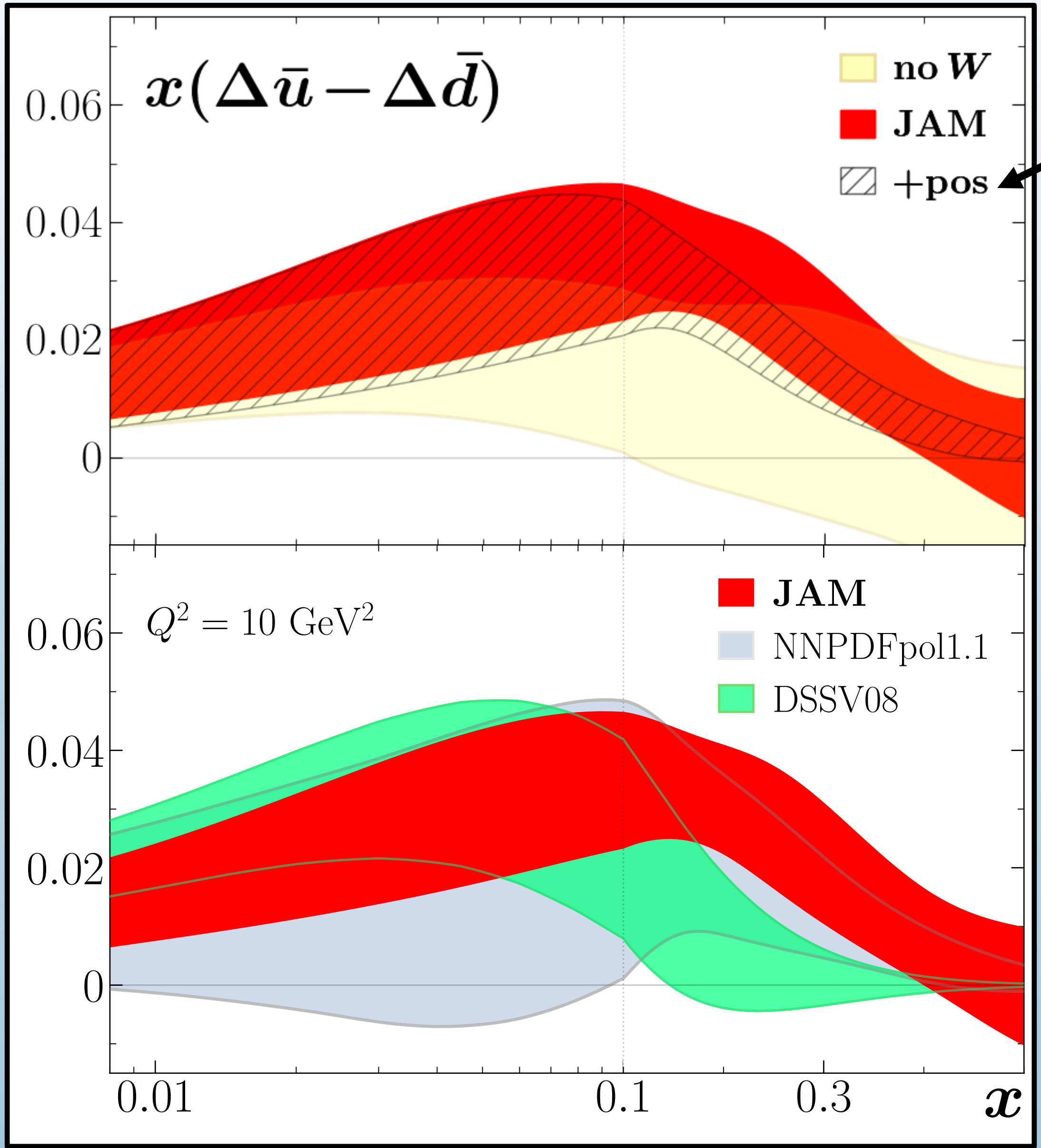

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DSSV08 shows positive asymmetry at low $x < 0.1$

NNPDF shows hint of positive asymmetry at intermediate x

Our result is strongly positive in both regions of x

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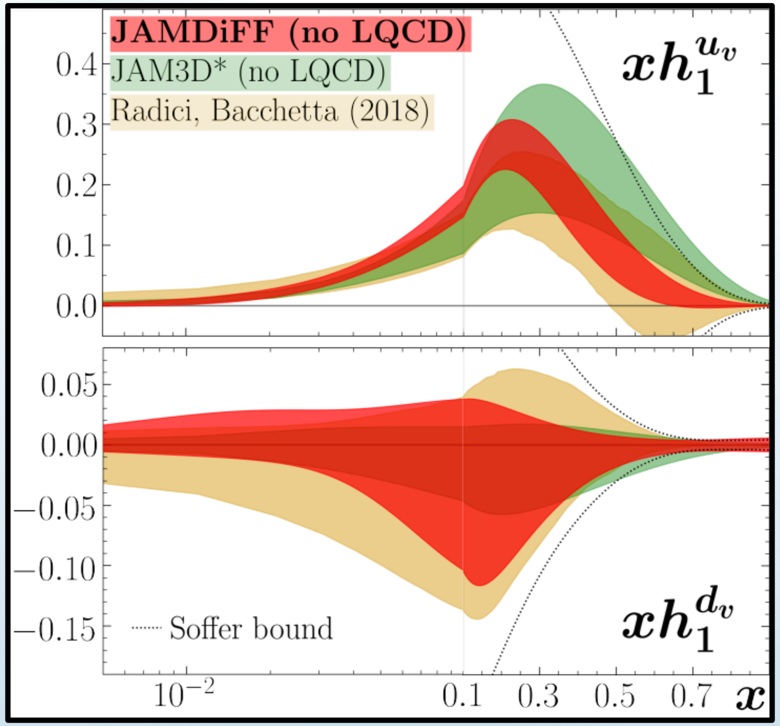
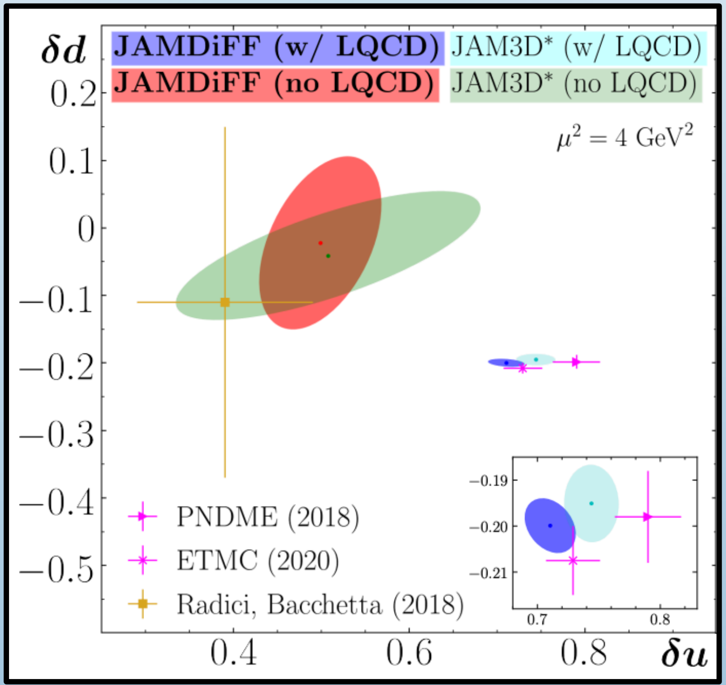
C. Cocuzza, A. Metz, D. Pitonyak, A. Prokudin, N. Sato, and R. Seidl, Phys. Rev. Lett. **132**, 091901 (2024)

C. Cocuzza, A. Metz, D. Pitonyak, A. Prokudin, N. Sato, and R. Seidl, Phys. Rev. D **109**, 034024 (2024)





Approaches to Extract Transversity

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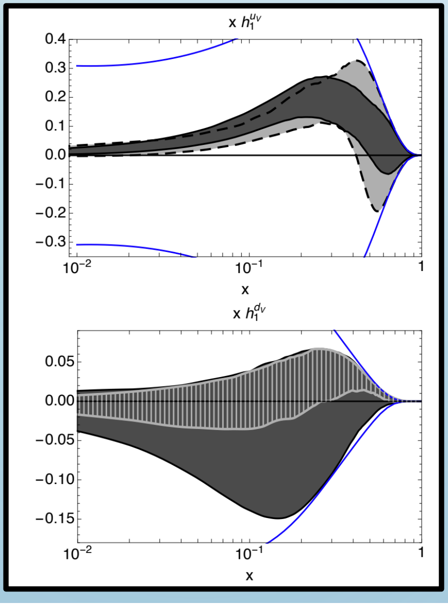
Dihadron Frag.

- Radici + Bacchetta (RB18)
- Benel + Courtoy + Ferro-Hernandez (2020)



M. Radici and A. Bacchetta,
Phys. Rev. Lett. **120**, no. 19, 192001 (2018)

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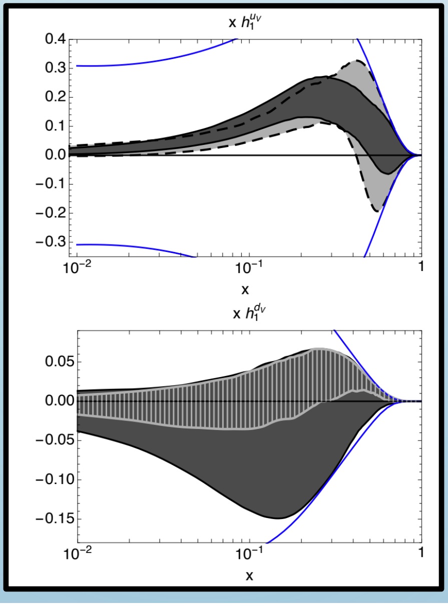

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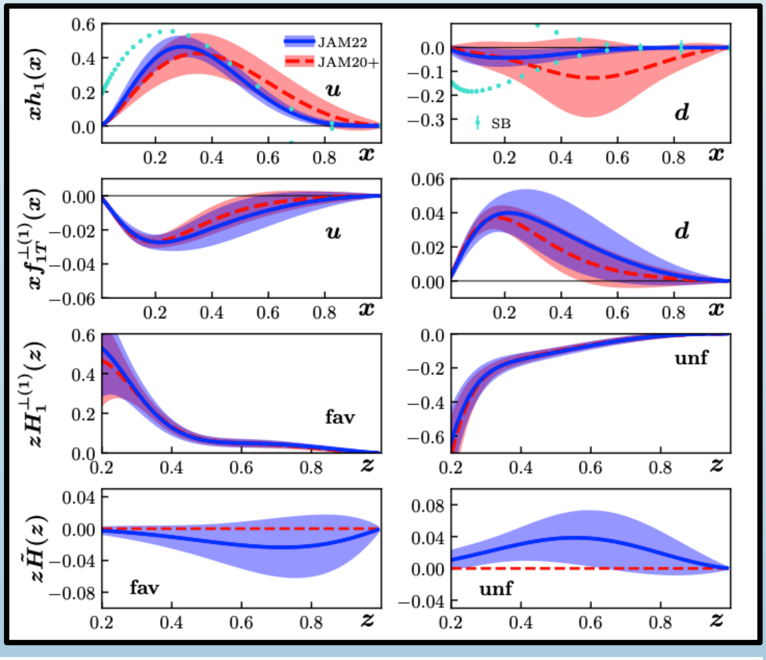
TMD + Collinear Twist-3

- JAM3D



L. Gamberg *et al.*, Phys. Rev. D **106**, no. 3, 034014 (2022)

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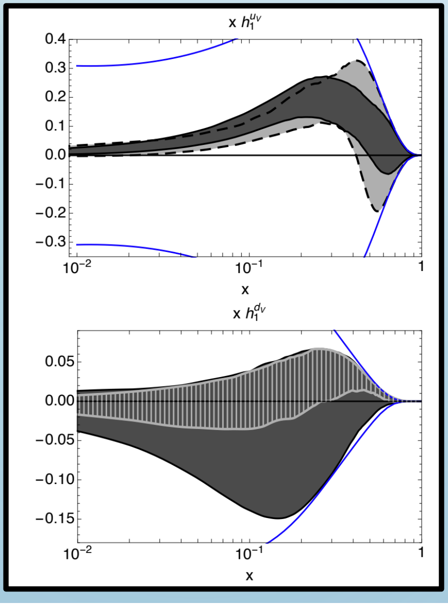

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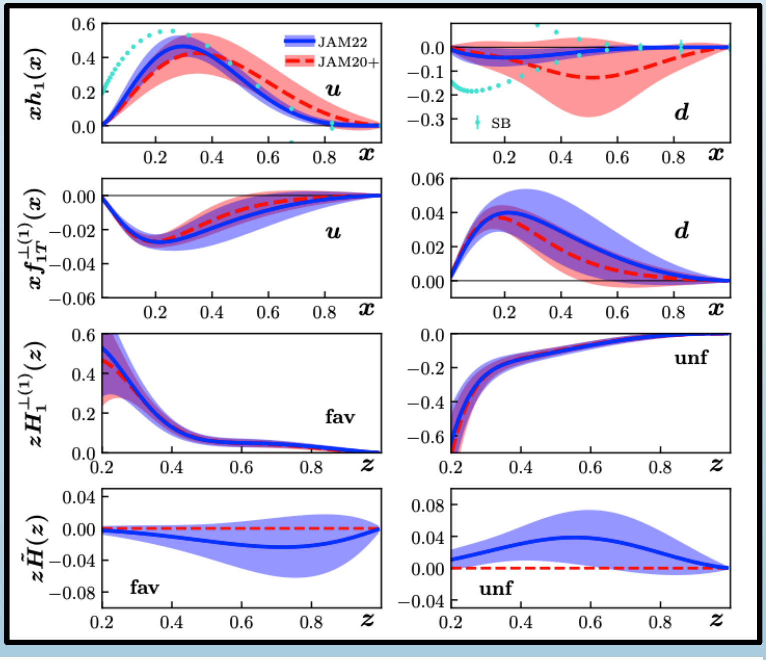
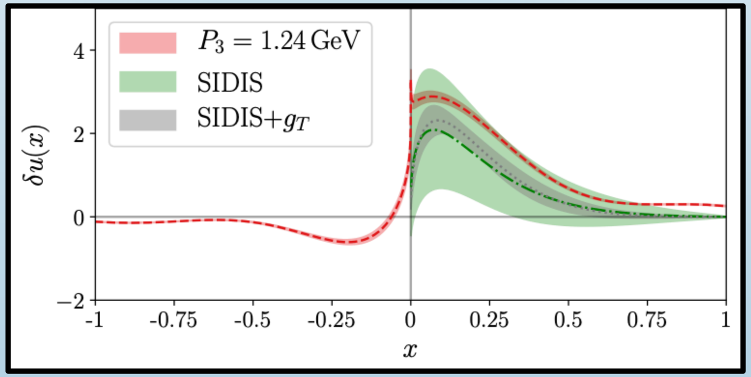


L. Gamberg *et al.*, Phys. Rev. D **106**, no. 3, 034014 (2022)

Lattice QCD

- ETMC Collaboration
- PNDME Collaboration
- LHPC Collaboration

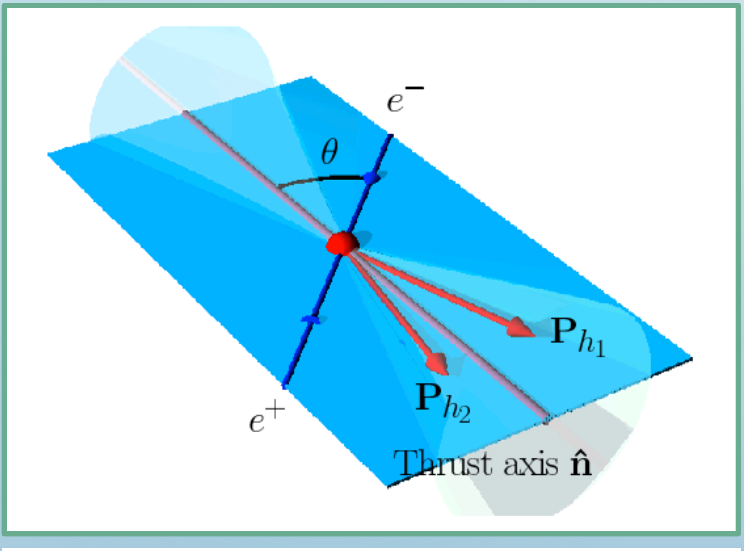


C. Alexandrou *et al.*, Phys. Rev. D **104**, no. 5, 054503 (2021)

JAM Global Analysis in the collinear DiFF Approach

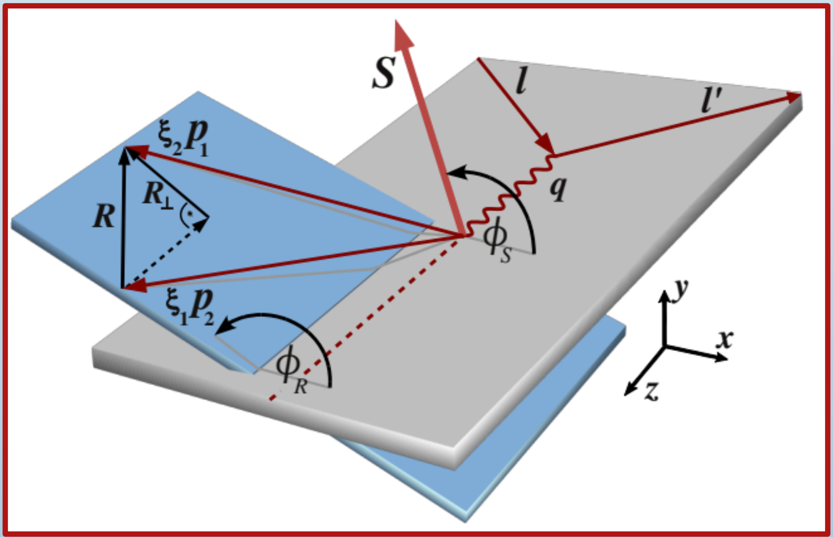
First *simultaneous* extraction of $\pi^+\pi^-$ DiFFs (D_1^q), IFFs ($H_1^{\sphericalangle,q}$), and transversity PDFs (h_1^q) at LO

Semi-Inclusive Annihilation



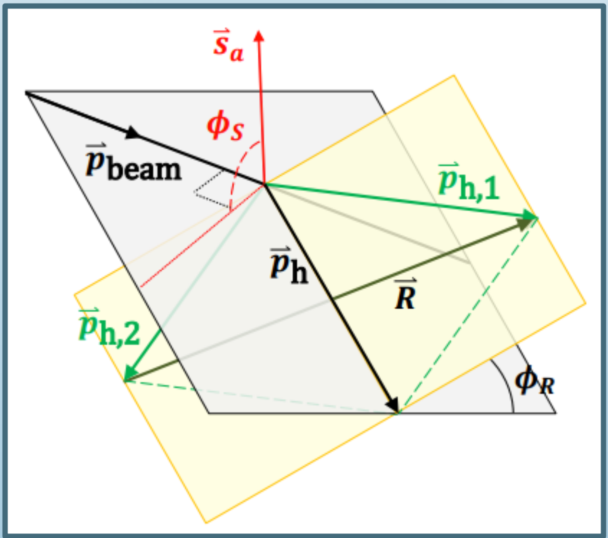
R. Seidl *et al.*, Phys. Rev. D **96**, no. 3, 032005 (2017)

Semi-Inclusive Deep Inelastic Scattering



C. Adolph *et al.*, Phys. Lett. B **713**, 10-16 (2012)

Proton-Proton Collisions



L. Adamczyk *et al.*, Phys. Rev. Lett. **115**, 242501 (2015)

Tensor Charges

$$\delta u \equiv \int_0^1 \mathrm{d}x(h_1^u - h_1^{\bar{u}}),$$

$$\delta d \equiv \int_0^1 \mathrm{d}x(h_1^d - h_1^{\bar{d}}),$$

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Tensor
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QCD Pheno for Transversity

Tensor Charges

Anselmino, *et al.* (2007, 2009, 2013, 2015);
Goldstein, *et al.* (2014);
Kang, *et al.* (2016);
D'Alesio, *et al.* (2020);
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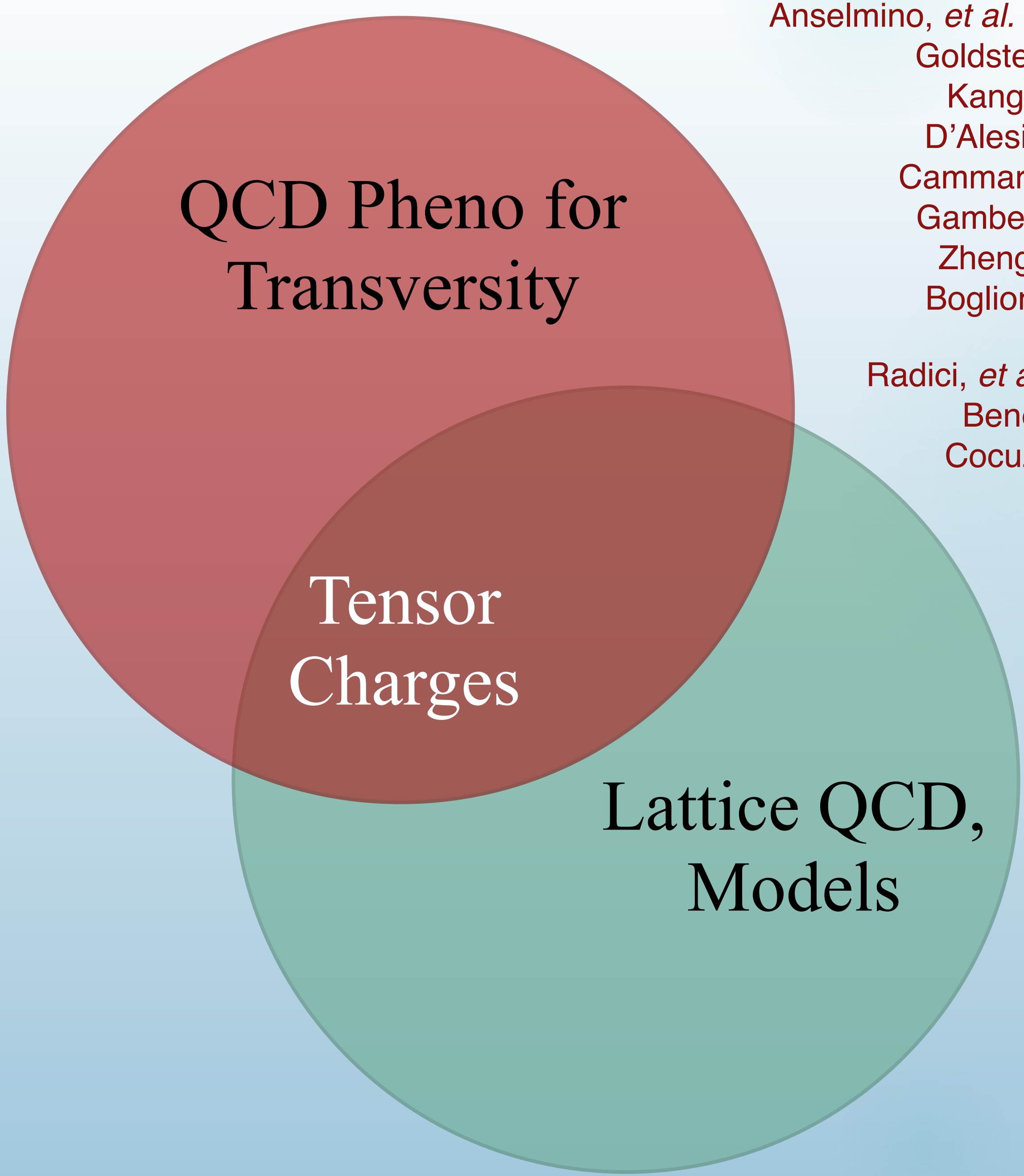

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He, Ji (1995);
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Gamberg, Goldstein (2001);
Pasquini, *et al.* (2005);
Wakamatsu (2007);
Lorce (2009);
Gupta, *et al.* (2018);
Yamanaka, *et al.* (2018);
Hasan, *et al.* (2019);
Alexandrou, *et al.* (2019, 2023);
Yamanaka, *et al.* (2013);
Pitschmann, *et al.* (2015);
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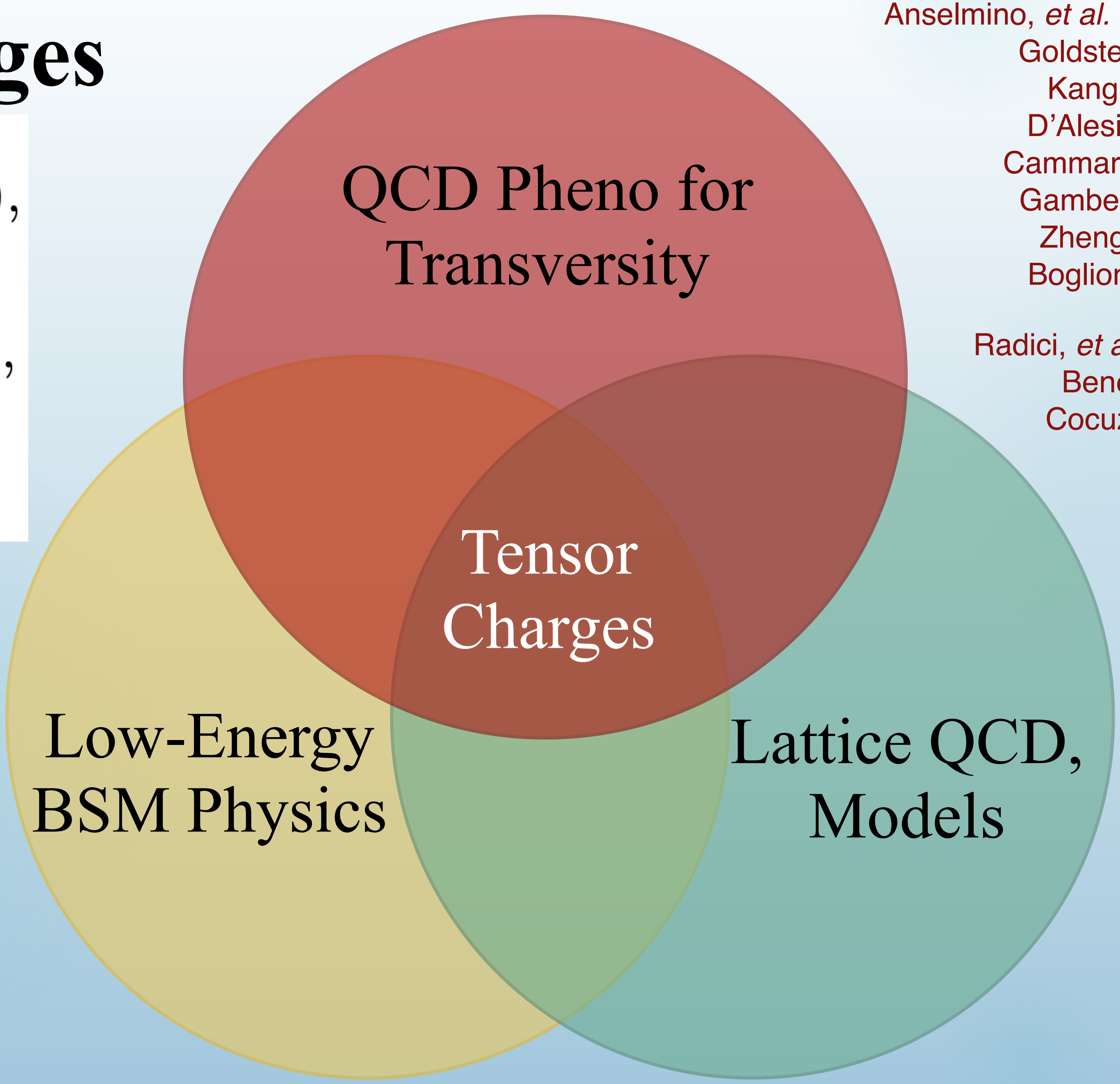

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Herczeg (2001);
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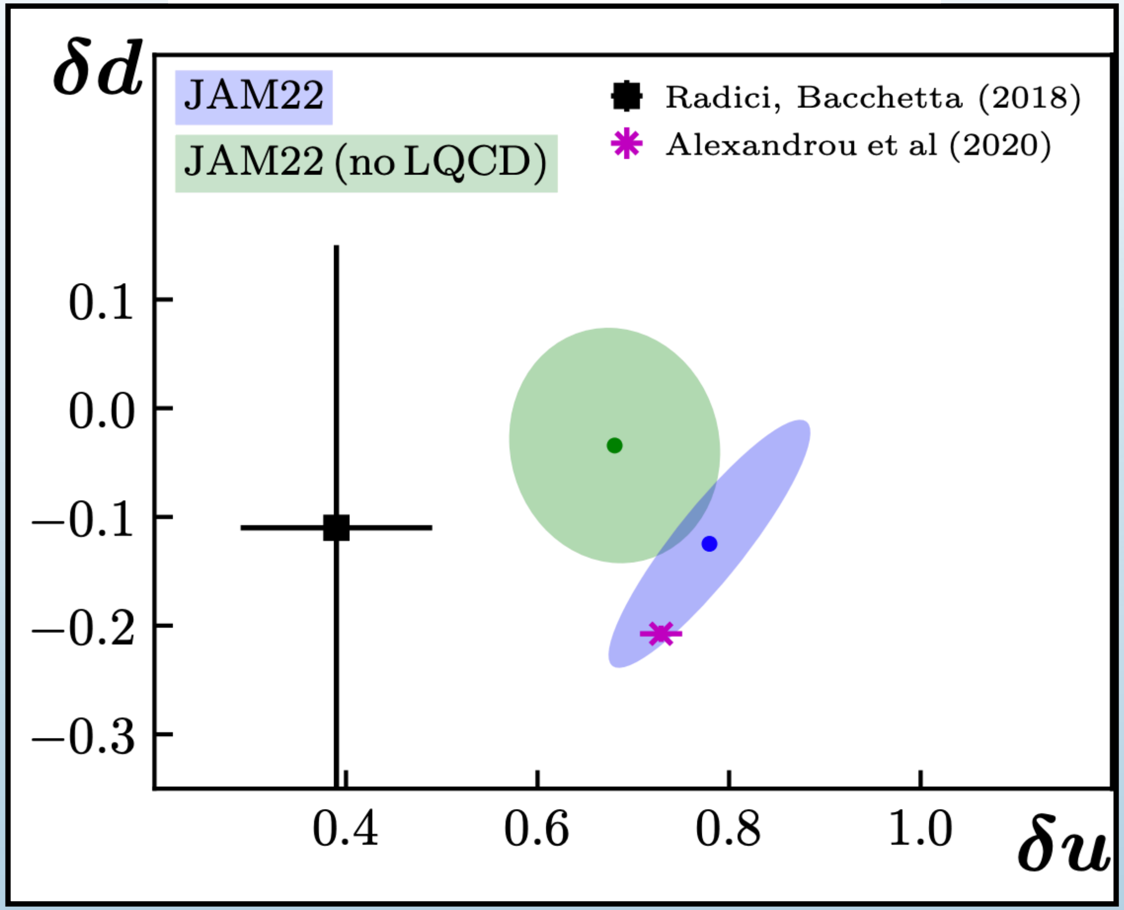
The Transverse Spin Puzzle?

L. Gamberg *et al.*, Phys. Rev. D **106**, no. 3, 034014 (2022)



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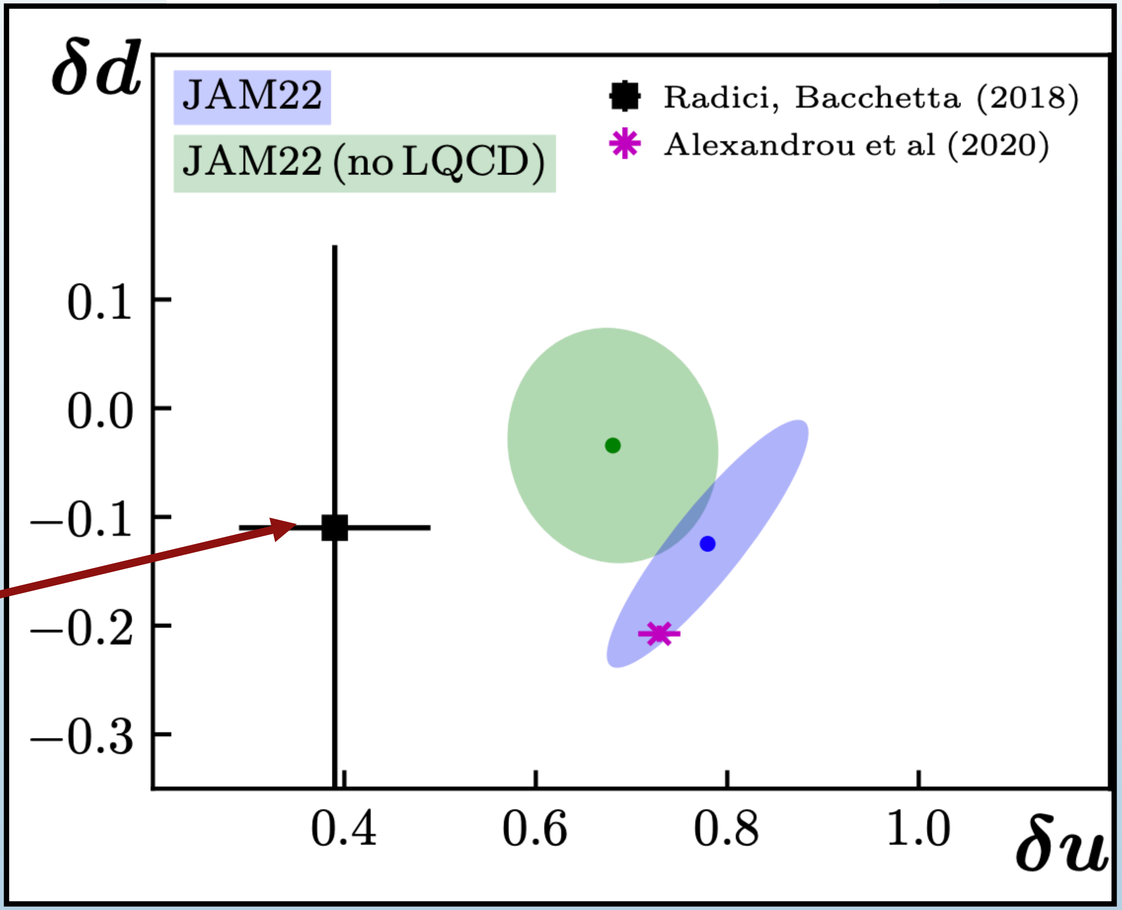

RB18

The Transverse Spin Puzzle?

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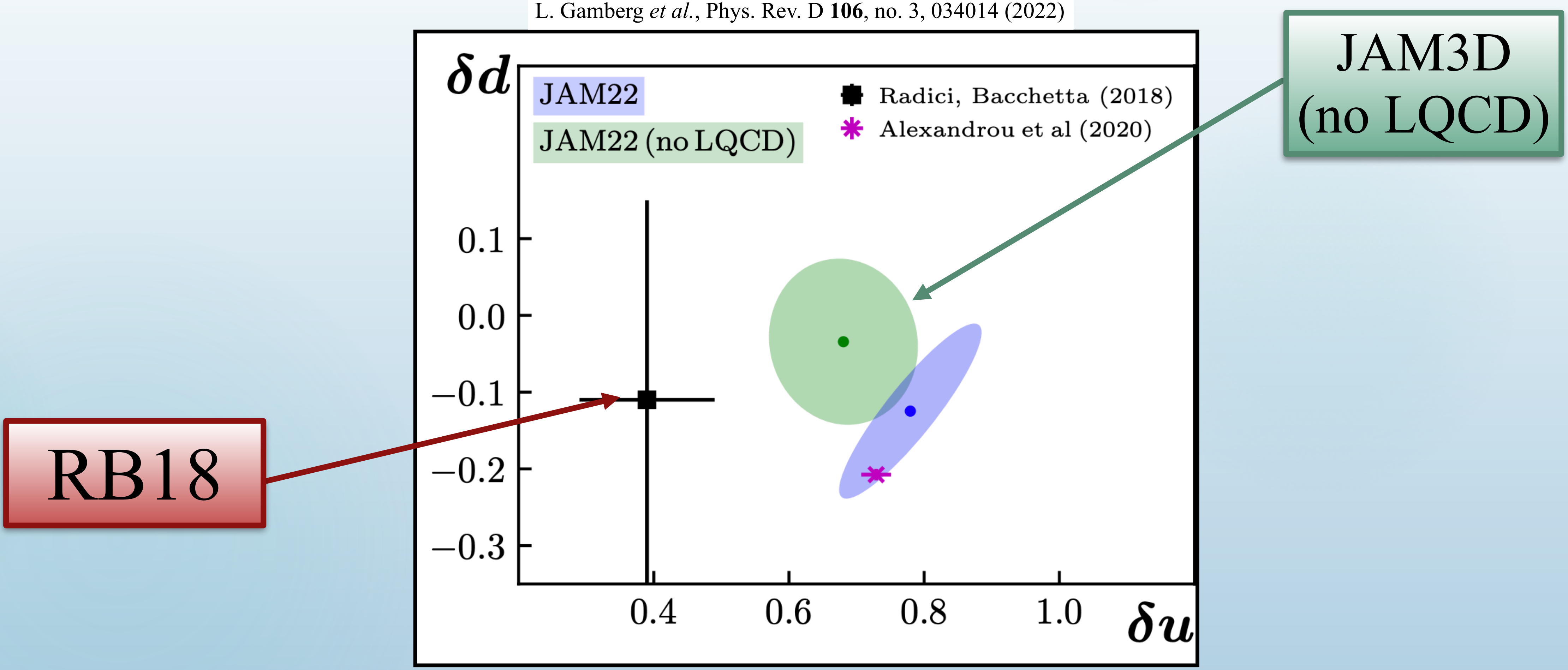

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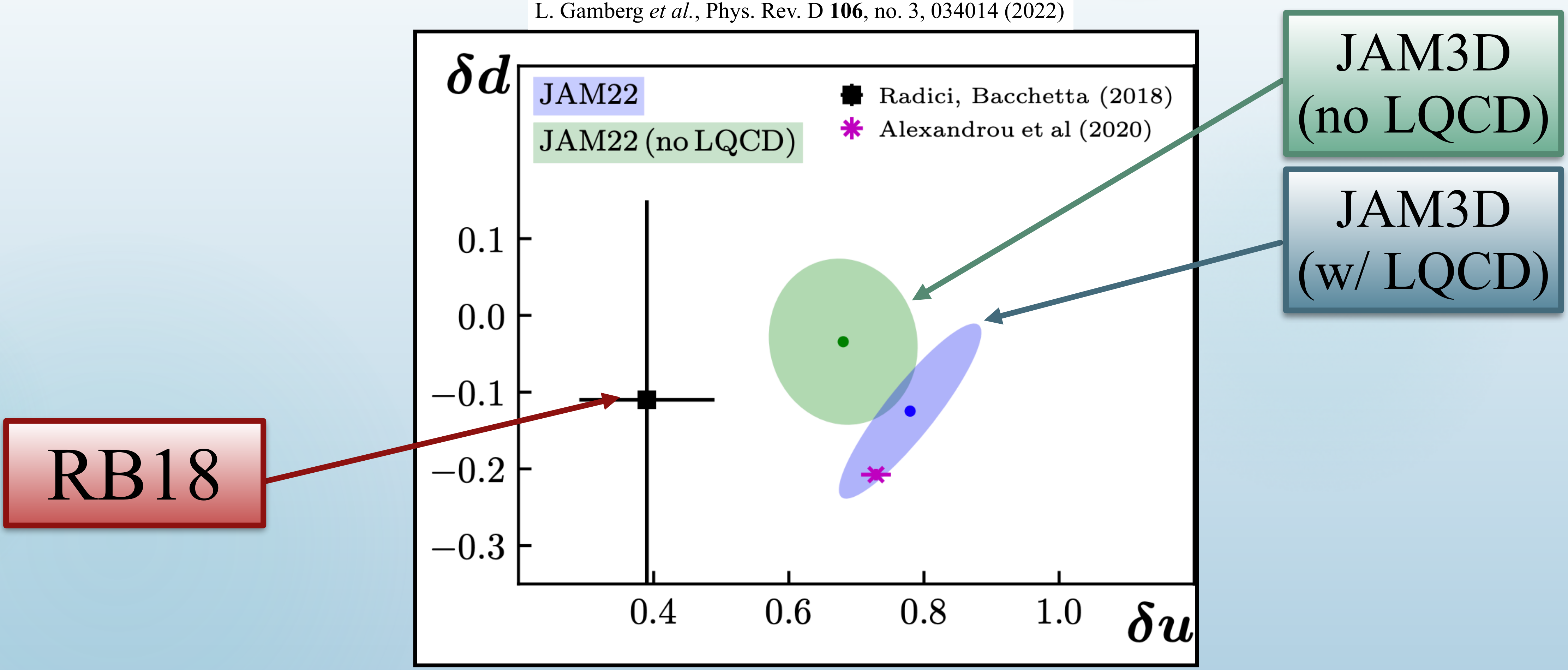

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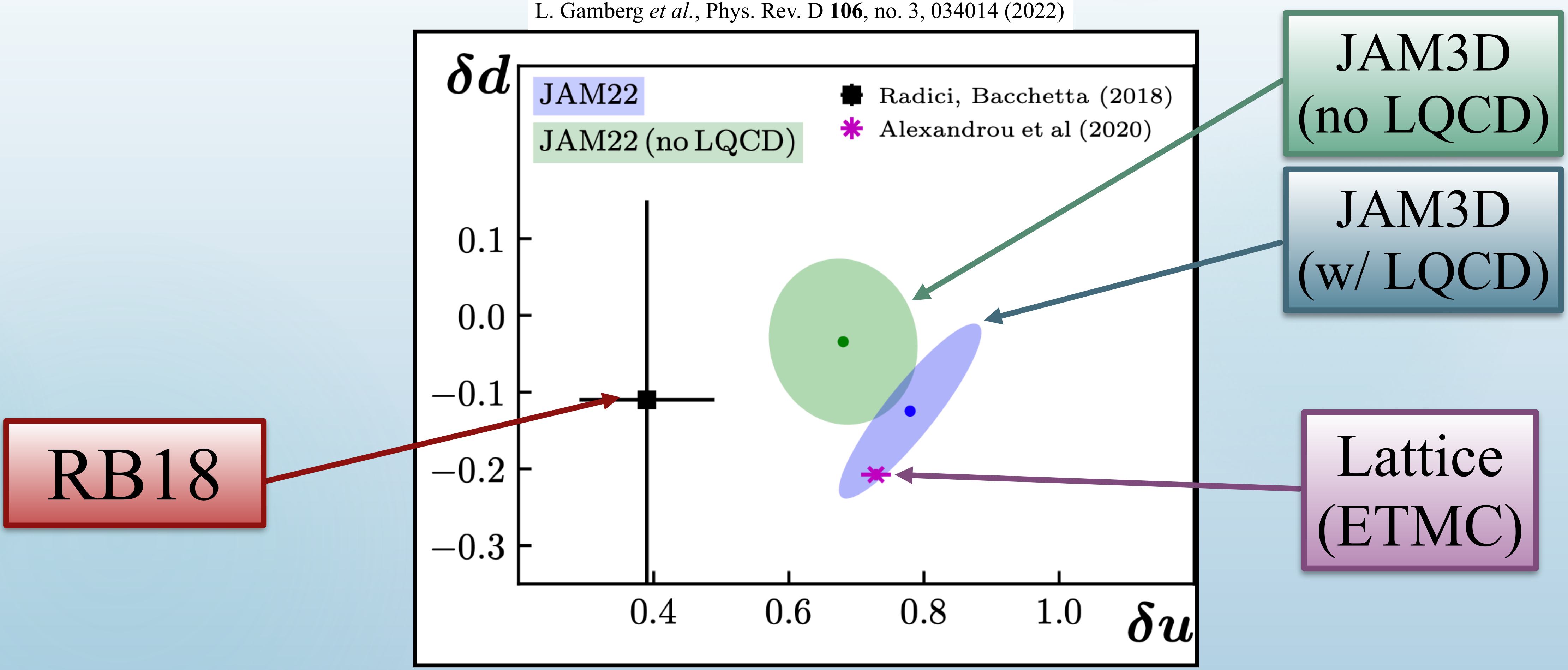

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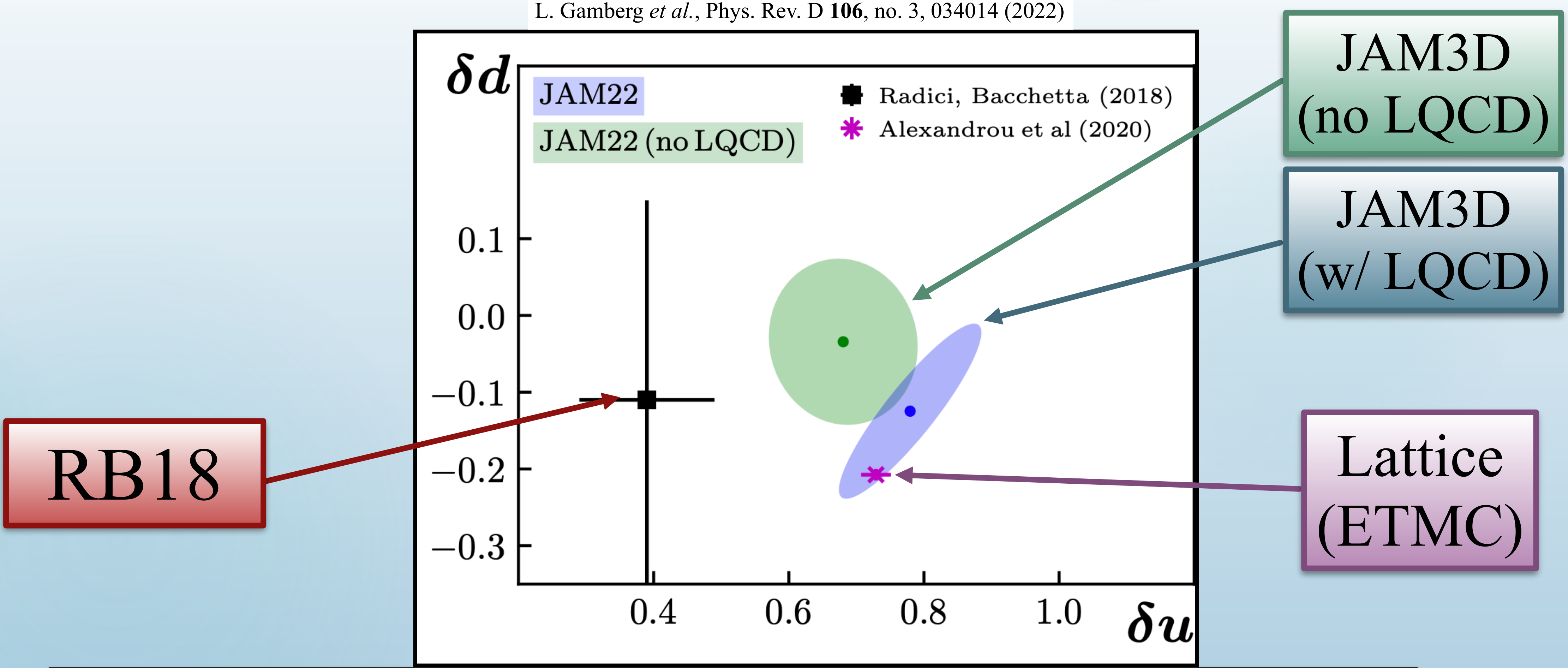
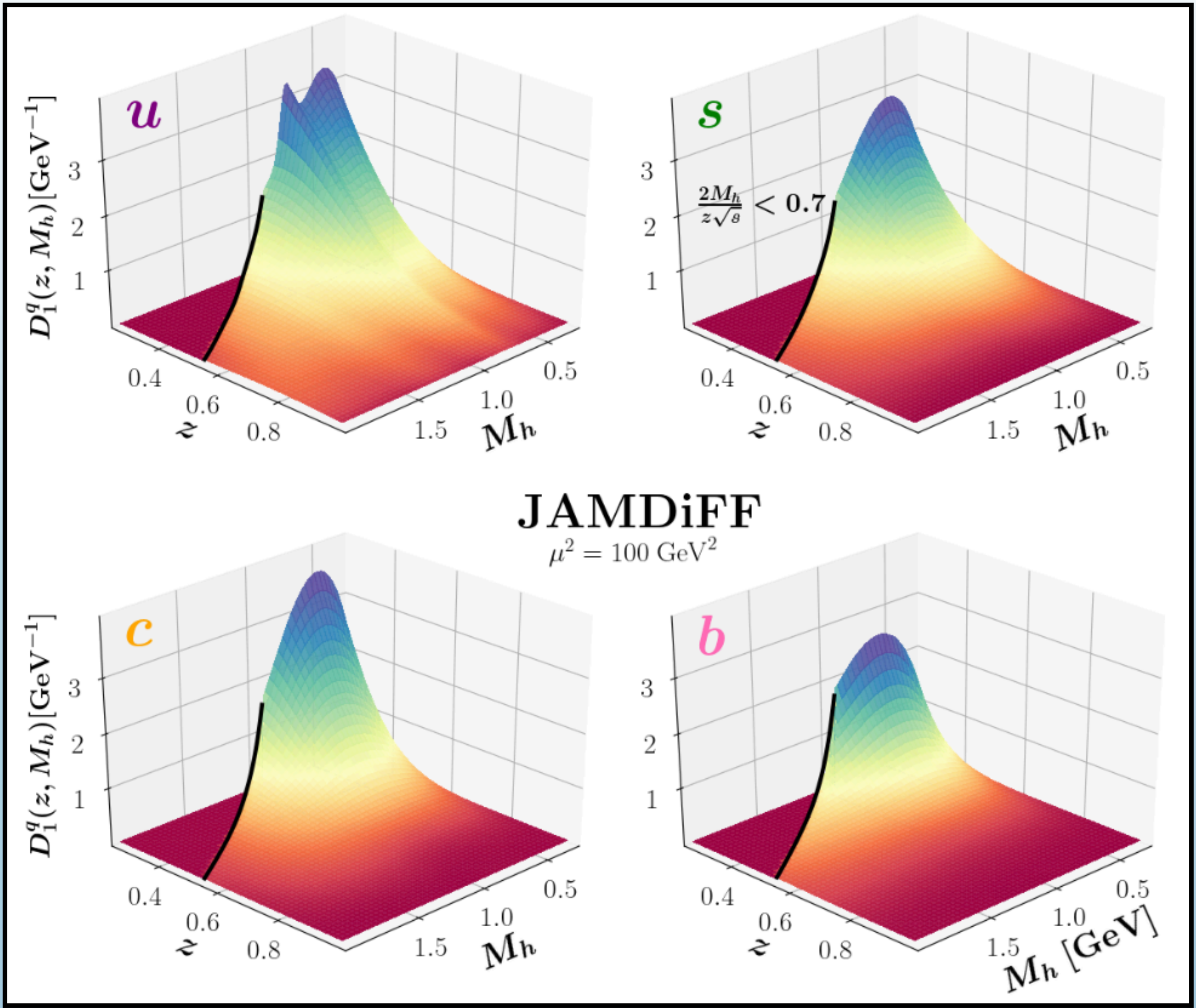
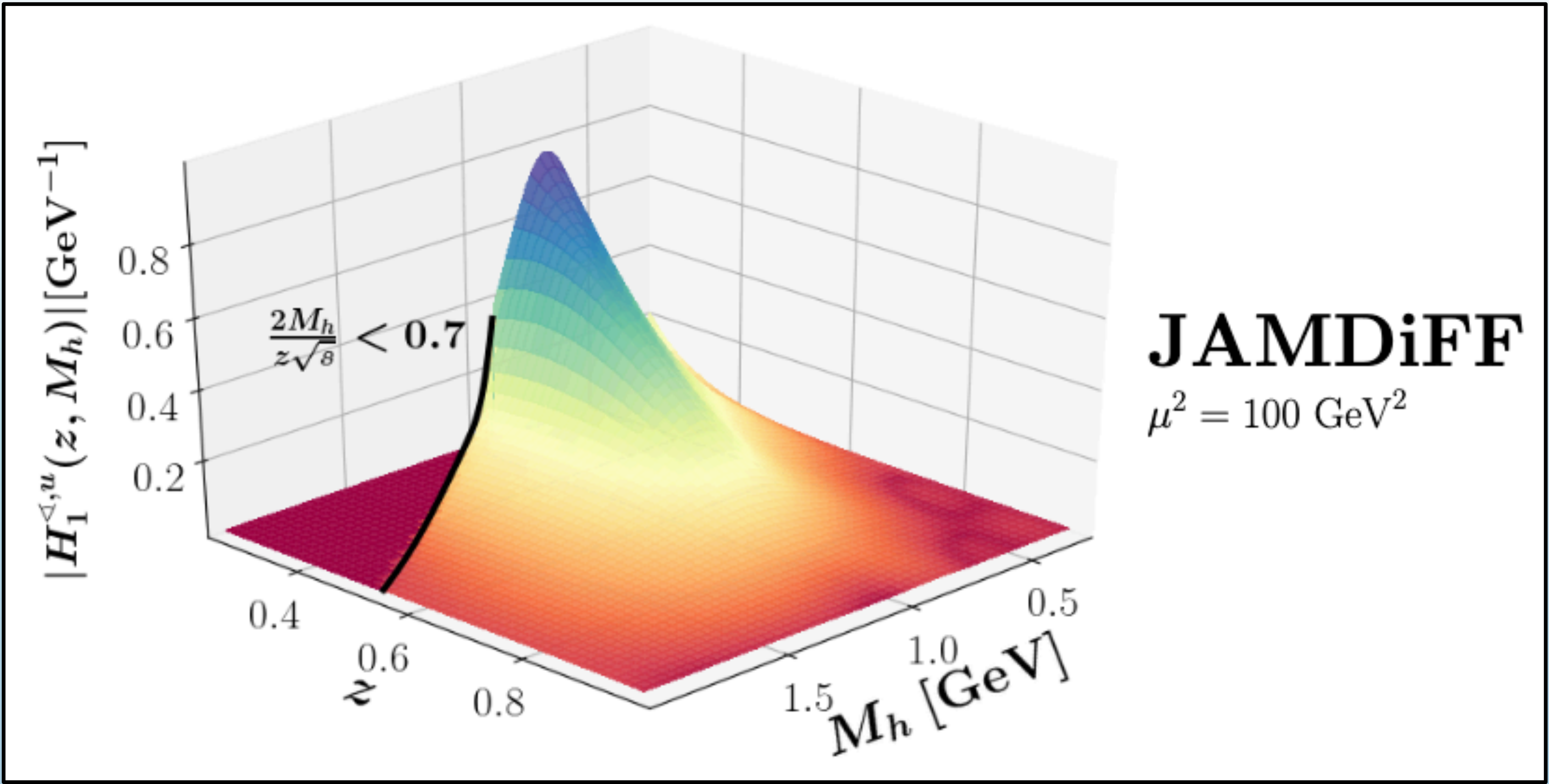


Large disagreements between three approaches…
Can this be solved?

Extracted DiFFs (3D)

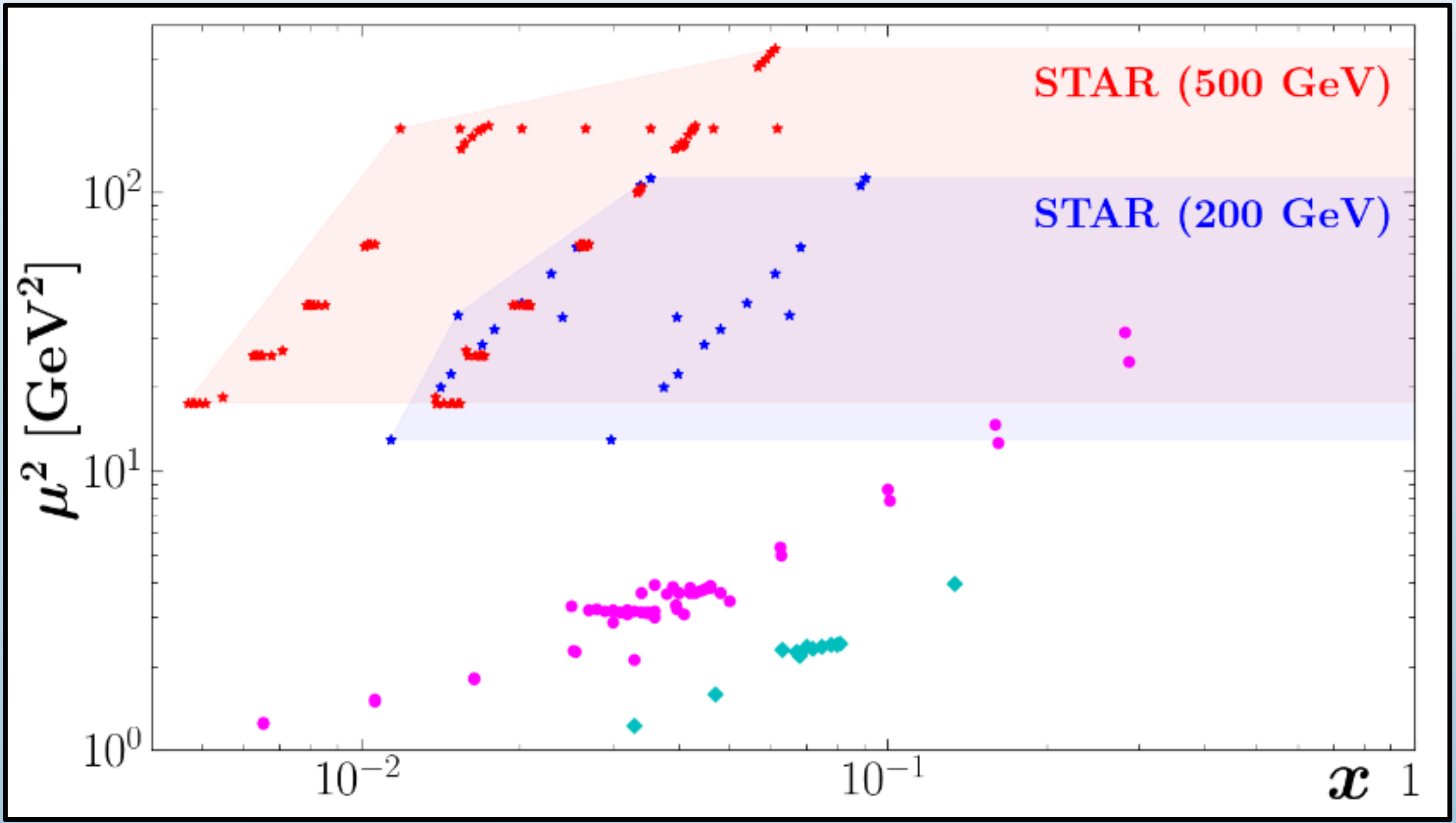


Extracted IFFs (3D)



Data for PDFs

Process	Collaborations	Points
SIDIS (p, D)	COMPASS, HERMES	64
Proton-Proton	STAR	269



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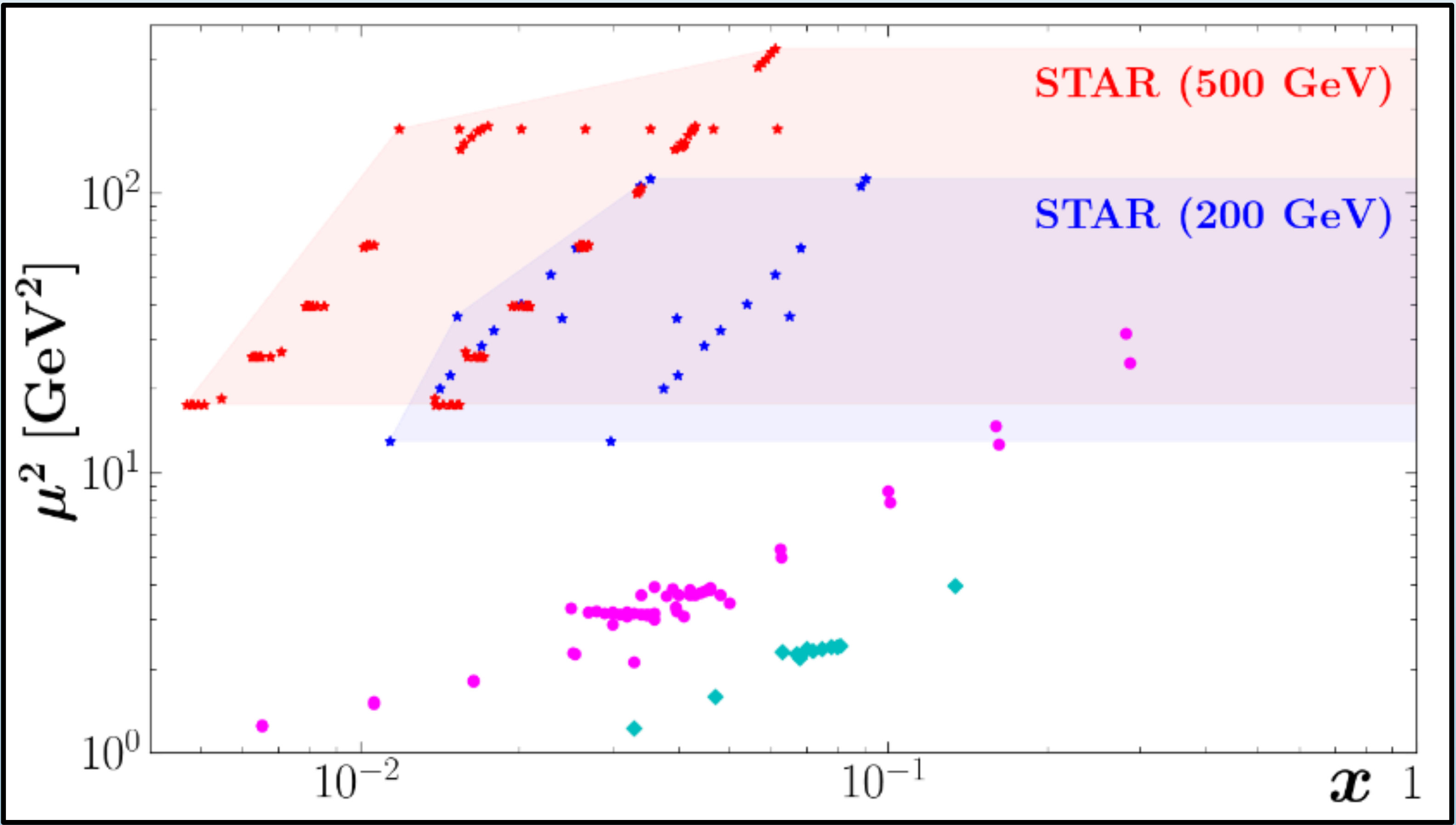

Parameterization Choices

3 independent observables
3 independent functions

$$h_1^{u_v}$$
$$h_1^{d_v}$$
$$h_1^{\bar{u}} = -h_1^{\bar{d}}$$

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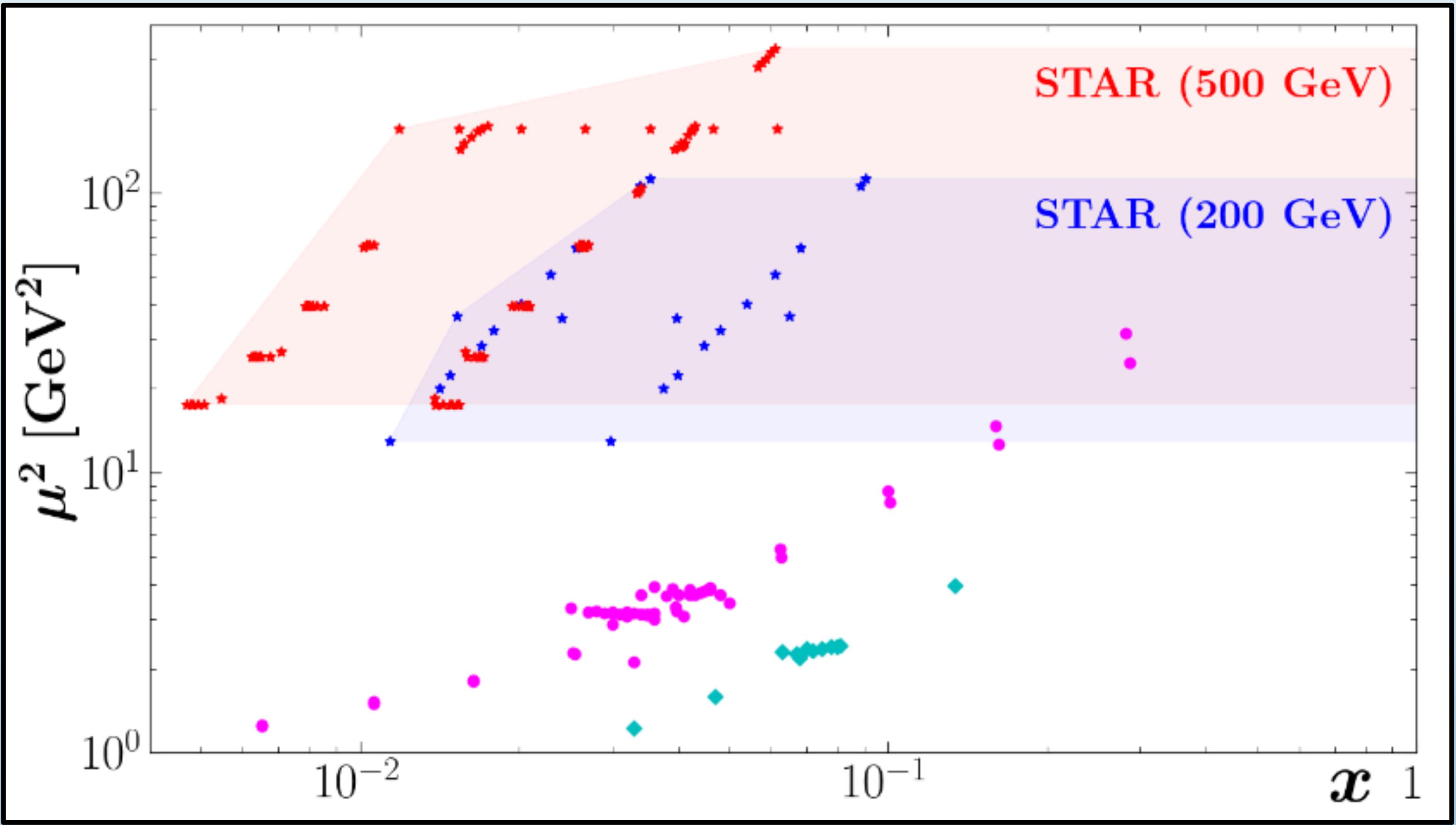

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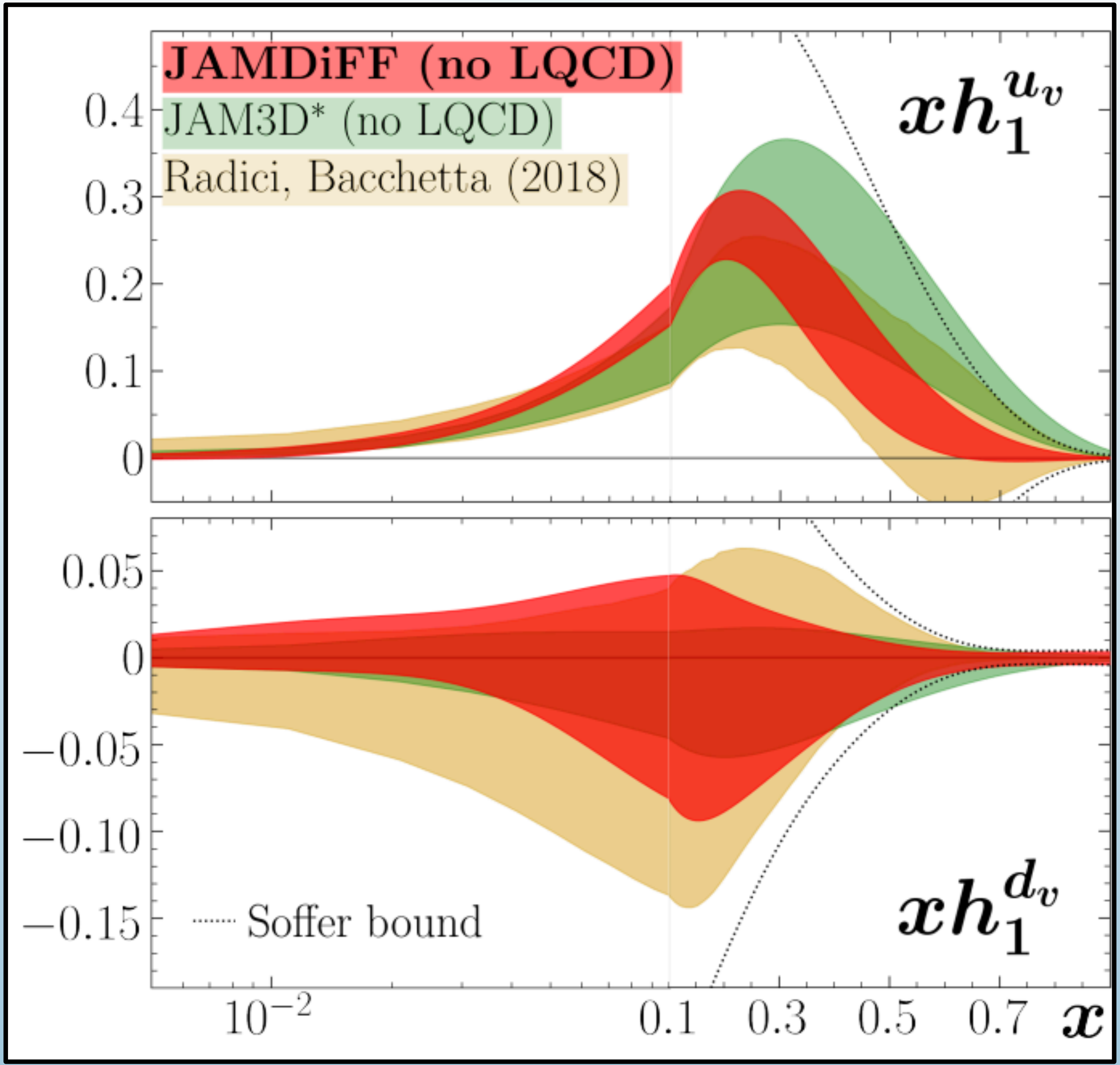
Prediction from large-N_c limit

P. V. Pobylitsa, arXiv:hep-ph/0301236 (2003)

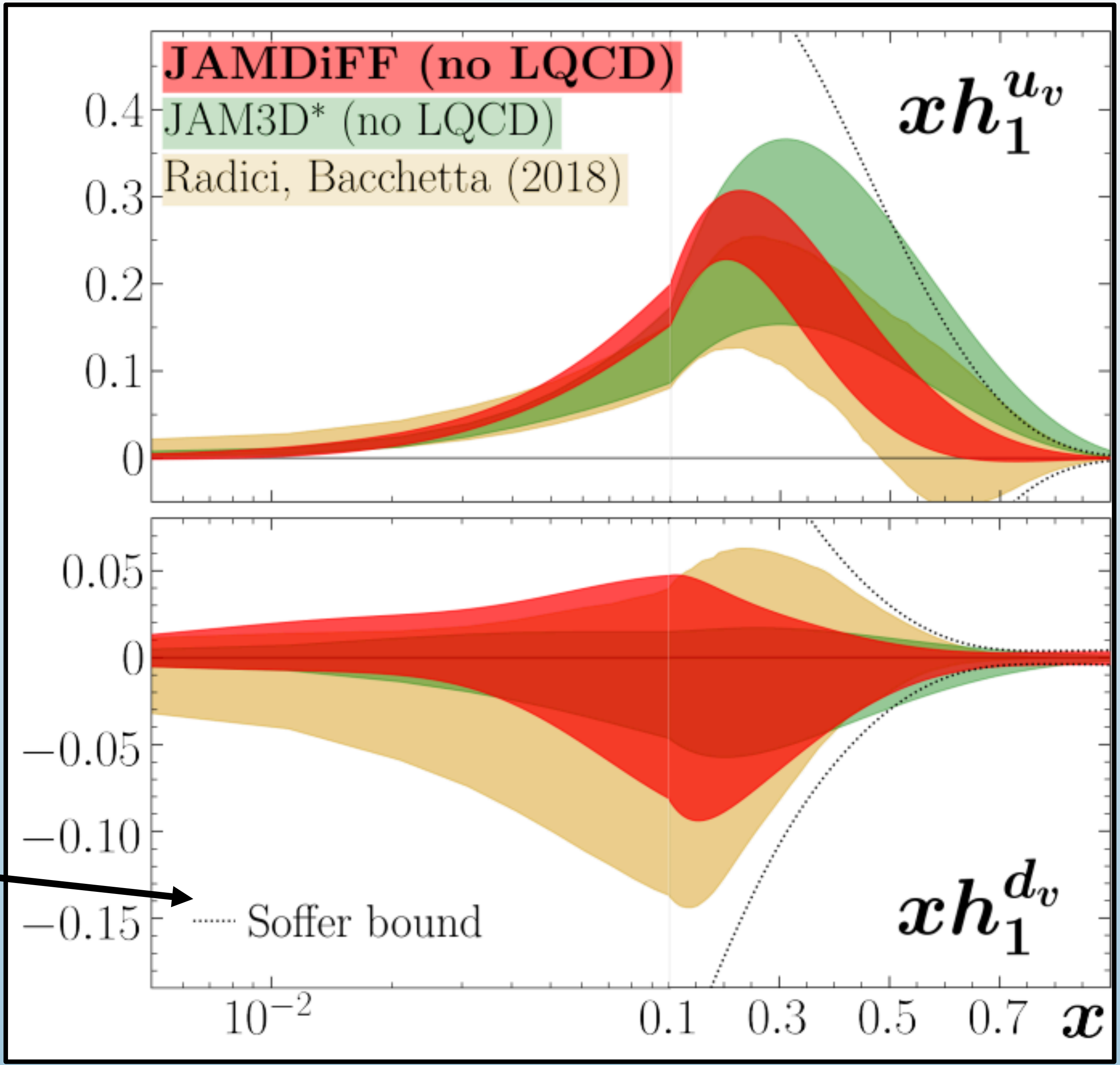
Transversity PDFs



Transversity PDFs

Soffer Bound: $|h_1^q| < \frac{1}{2}[f_1^q + g_1^q]$

J. Soffer, Phys. Rev. Lett. **74**, 1292-1294 (1995)



Transversity PDFs

JAM3D* = JAM3D-22 (no LQCD)
+ Antiquarks w/ $\bar{u} = -\bar{d}$
+ small-x constraint (see slide 27)

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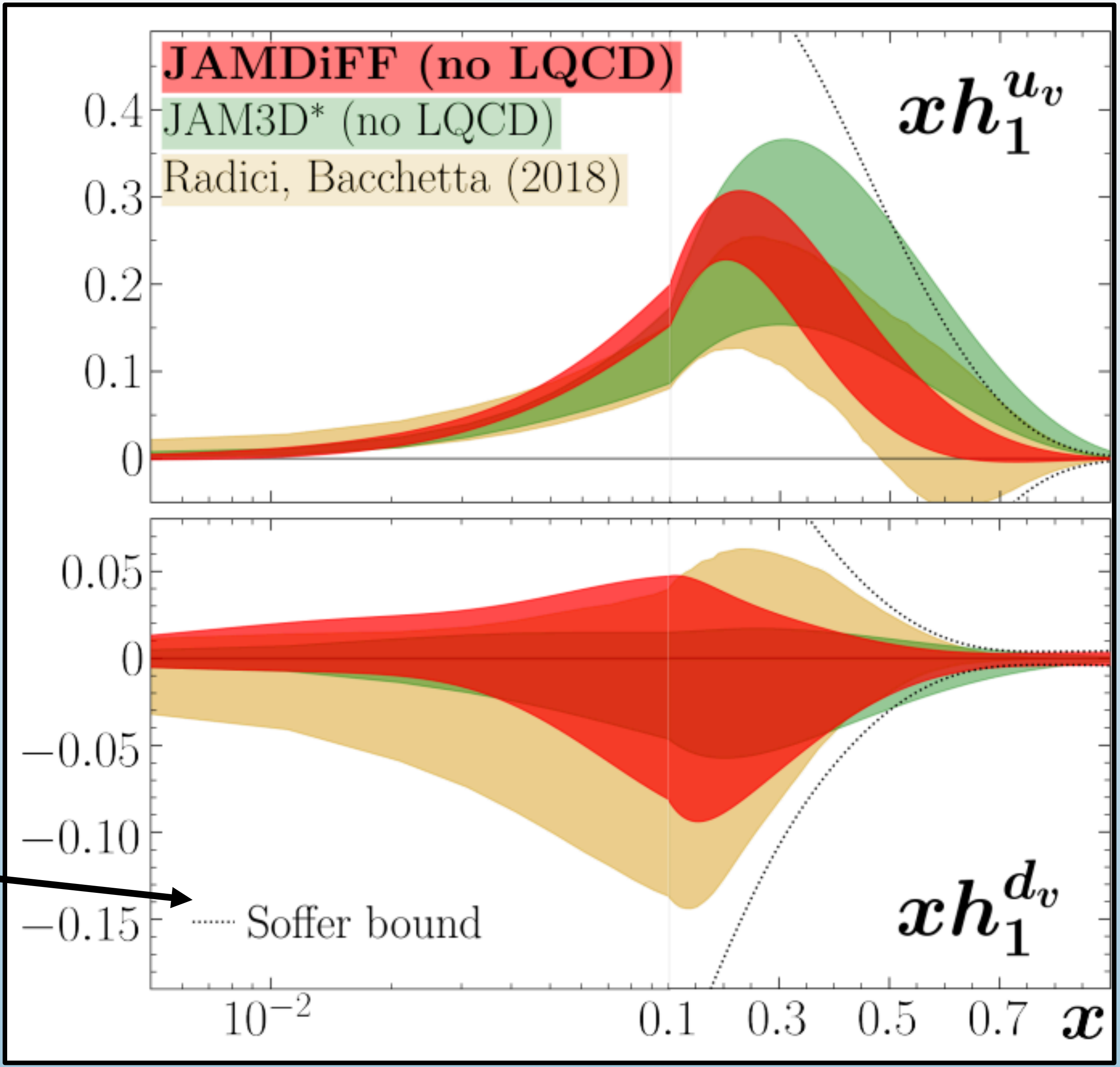

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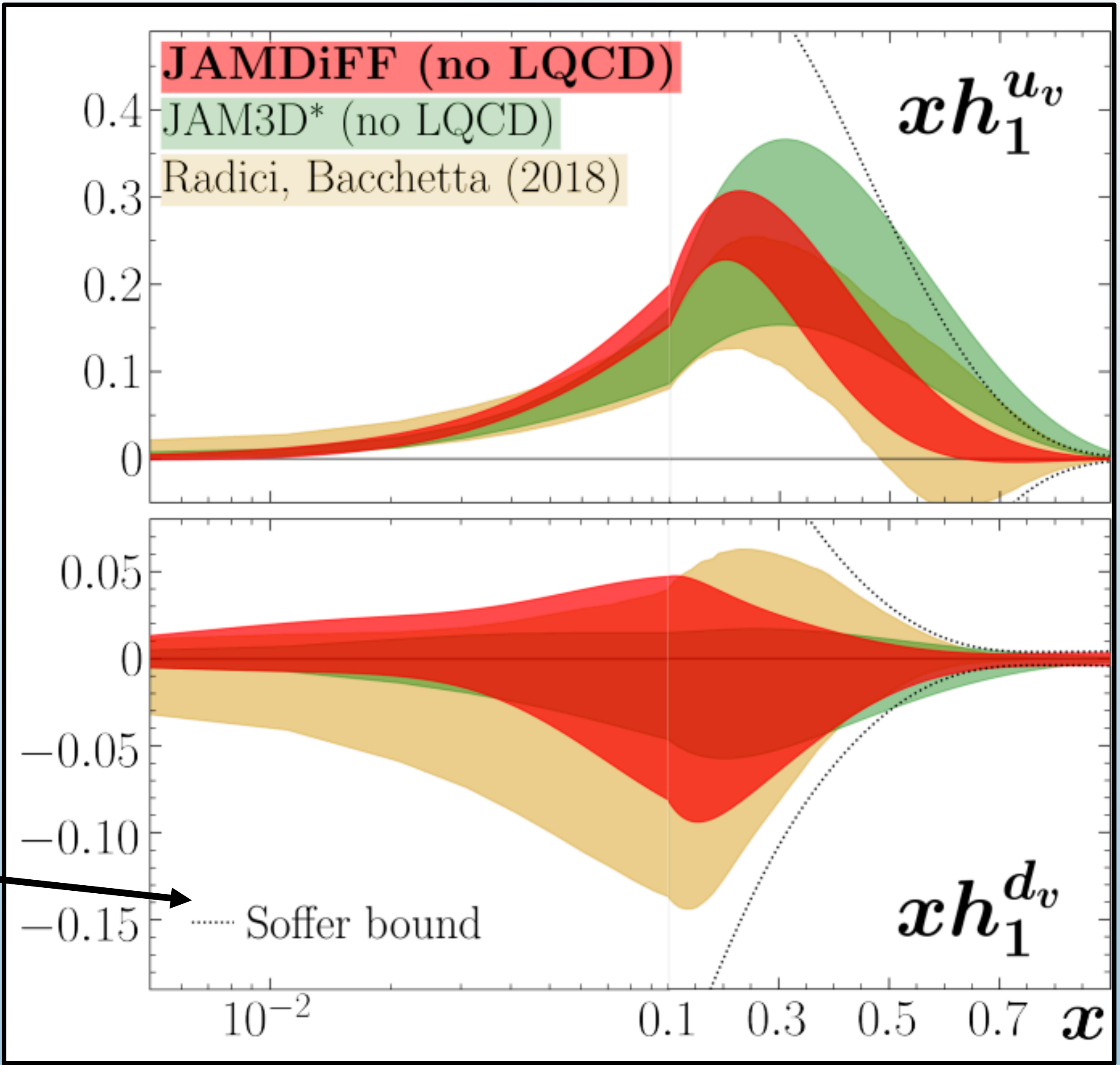
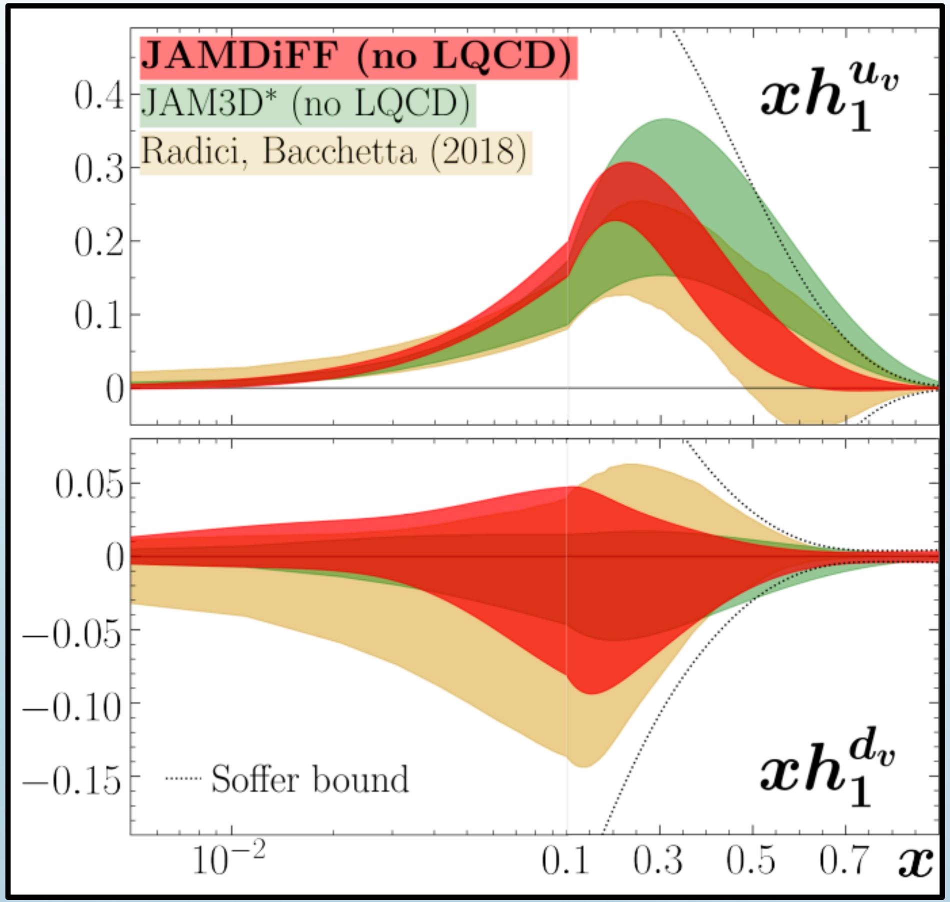
Agreement between all three analyses within errors

Soffer Bound: $|h_1^q| < \frac{1}{2}\left[f_1^q + g_1^q\right]$

J. Soffer, Phys. Rev. Lett. **74**, 1292-1294 (1995)



Controlling Extrapolation



$$\delta u \equiv \int_0^1 \mathrm{d}x (h_1^u - h_1^{\bar{u}}),$$

$$\delta d \equiv \int_0^1 \mathrm{d}x (h_1^d - h_1^{\bar{d}}),$$

$$g_T \equiv \delta u - \delta d,$$

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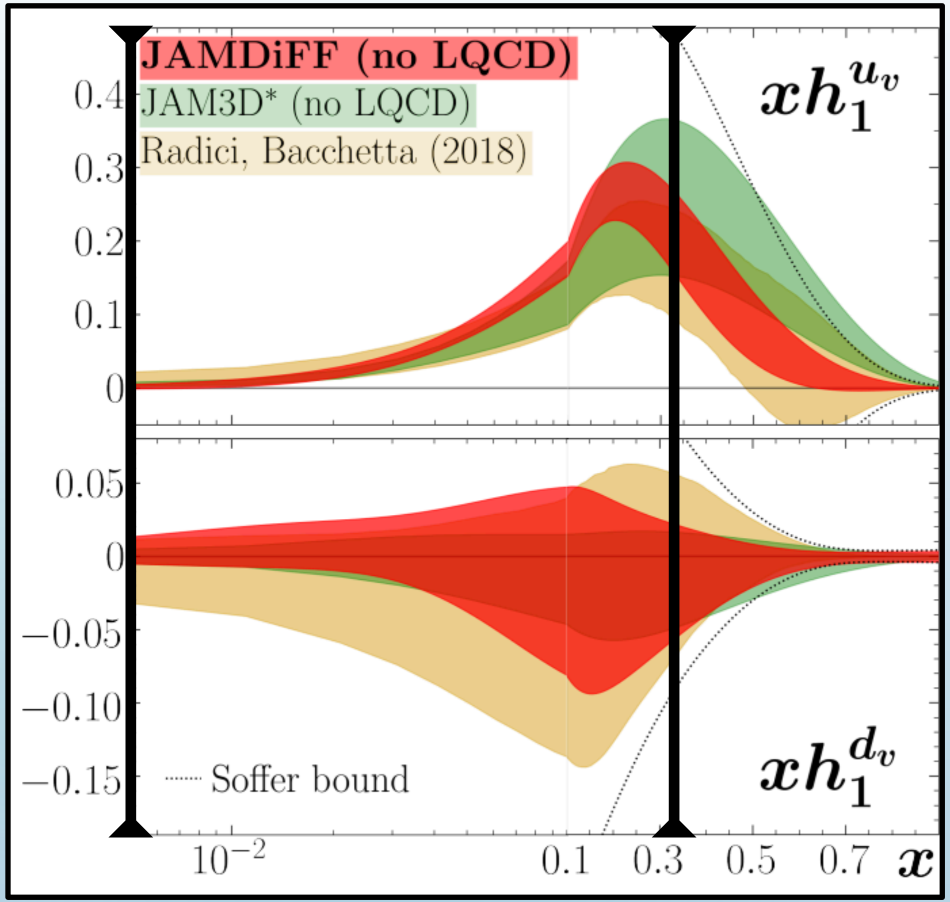

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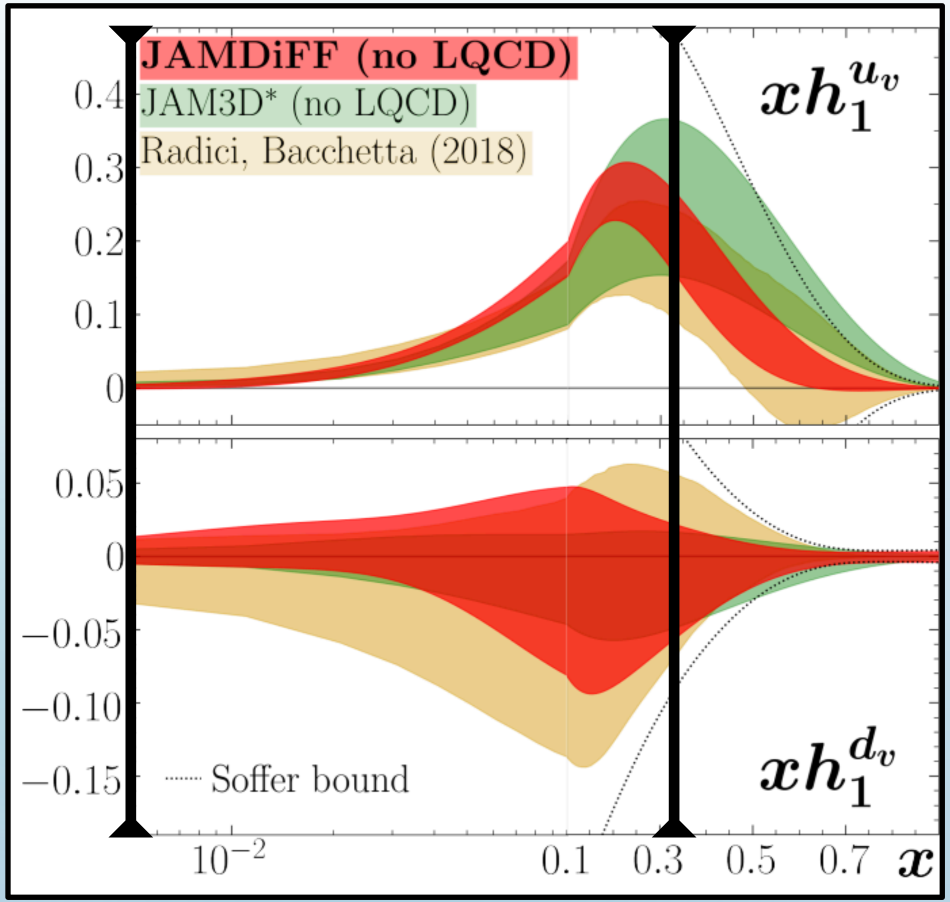
Measured Region

Controlling Extrapolation



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Large $x \gtrsim 0.3$

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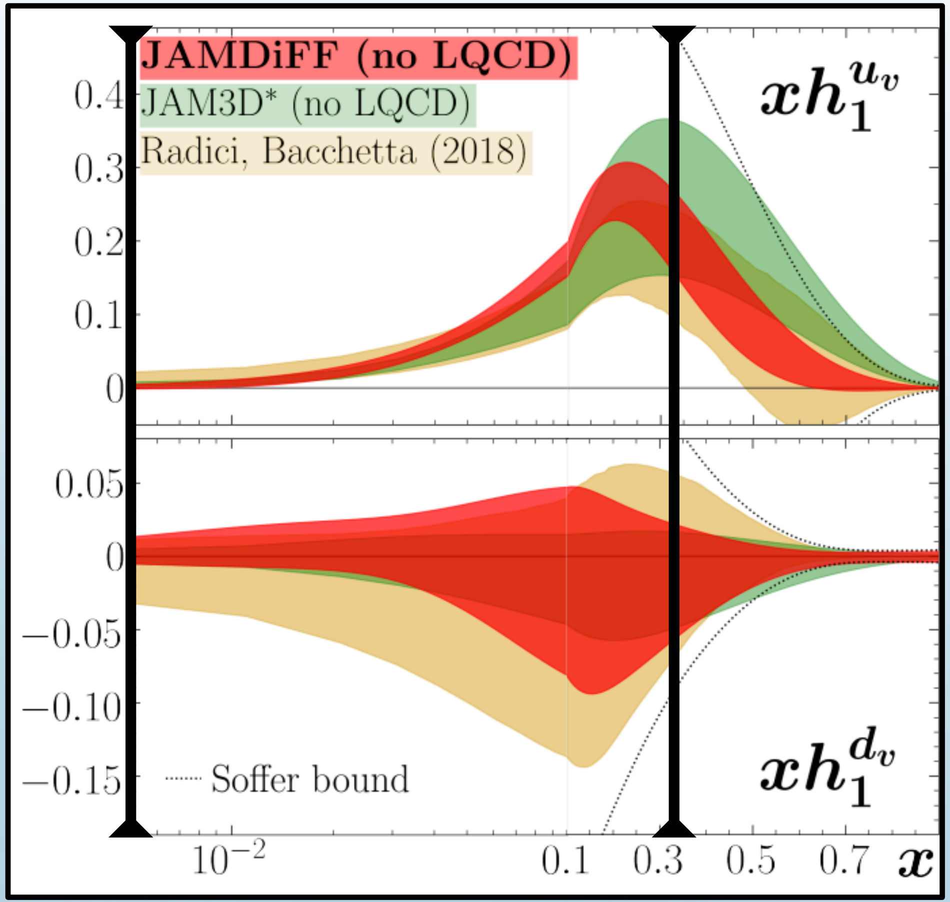
Controlling Extrapolation



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Soffer Bound: $|h_1^q| < \frac{1}{2}\left[f_1^q + g_1^q\right]$

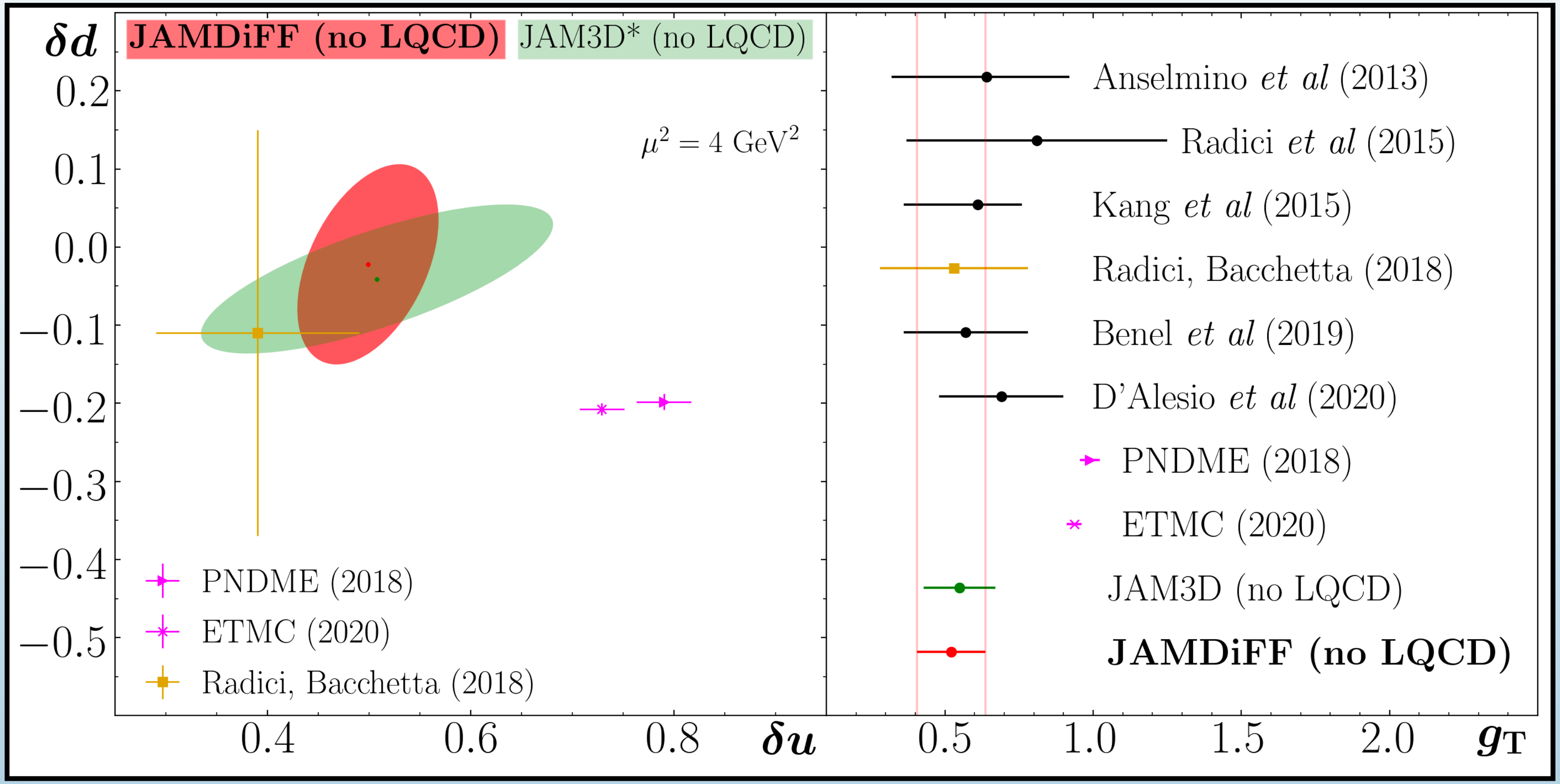
J. Soffer, Phys. Rev. Lett. **74**, 1292-1294 (1995)

Small $x \lesssim 0.005$

$$h_1^q \xrightarrow[x \to 0]{} x^{\alpha_q} \qquad \alpha_q = 1 - 2\sqrt{\frac{\alpha_s N_c}{2\pi}} \approx 0.17 \pm 0.085$$

Y. V. Kovchegov and M. D. Sievert, Phys. Rev. D **99**, 054033 (2019)

Tensor Charges



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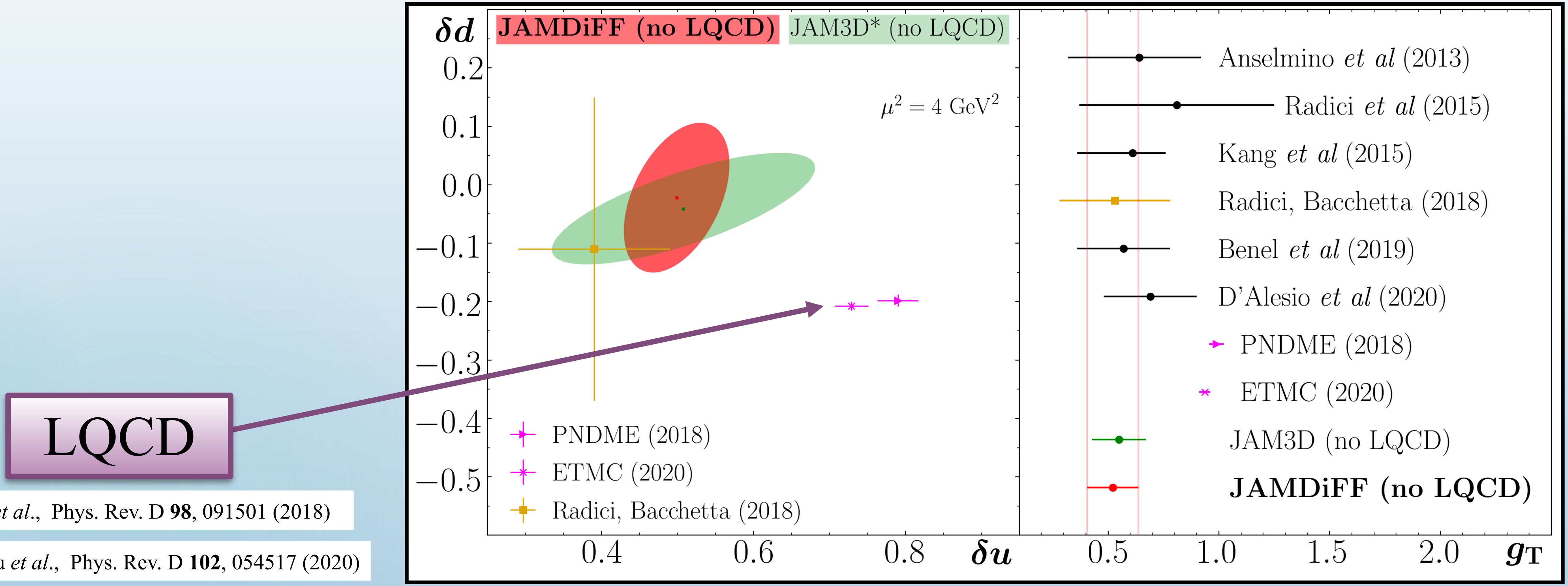
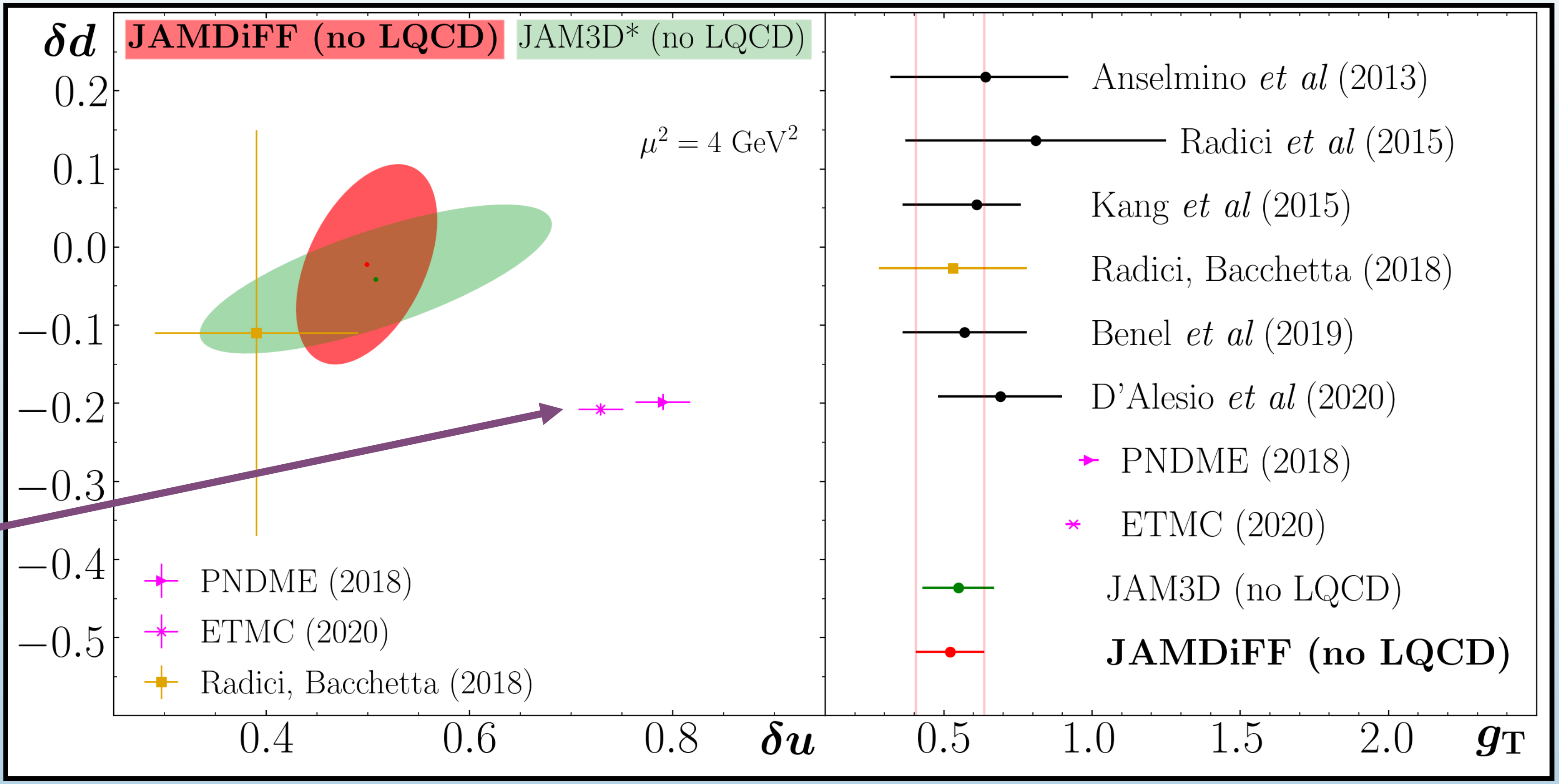


R. Gupta *et al.*, Phys. Rev. D **98**, 091501 (2018)

C. Alexandrou *et al.*, Phys. Rev. D **102**, 054517 (2020)

Tensor Charges



LQCD

R. Gupta *et al.*, Phys. Rev. D **98**, 091501 (2018)

C. Alexandrou *et al.*, Phys. Rev. D **102**, 054517 (2020)

Consistent with RB18 and JAM3D* (no LQCD).
What happens if we include LQCD in the fit?

Quality of Fit

Experiment	N_{dat}	χ^2_{red} w/ LQCD	χ^2_{red} no LQCD
Belle (cross section) [63]	1094	1.01	1.01
Belle (Artru-Collins) [92]	183	0.74	0.73
HERMES [72]	12	1.13	1.10
COMPASS (p) [71]	26	1.24	0.75
COMPASS (D) [71]	26	0.78	0.76
STAR (2015) [94]	24	1.47	1.67
STAR (2018) [64]	106	1.20	1.04
ETMC δu [28]	1	0.71	—
ETMC δd [28]	1	1.02	—
PNDME δu [25]	1	8.68	—
PNDME δd [25]	1	0.04	—
Total χ^2_{red} (N_{dat})		**1.01** (1475)	**0.98** (1471)

Quality of Fit

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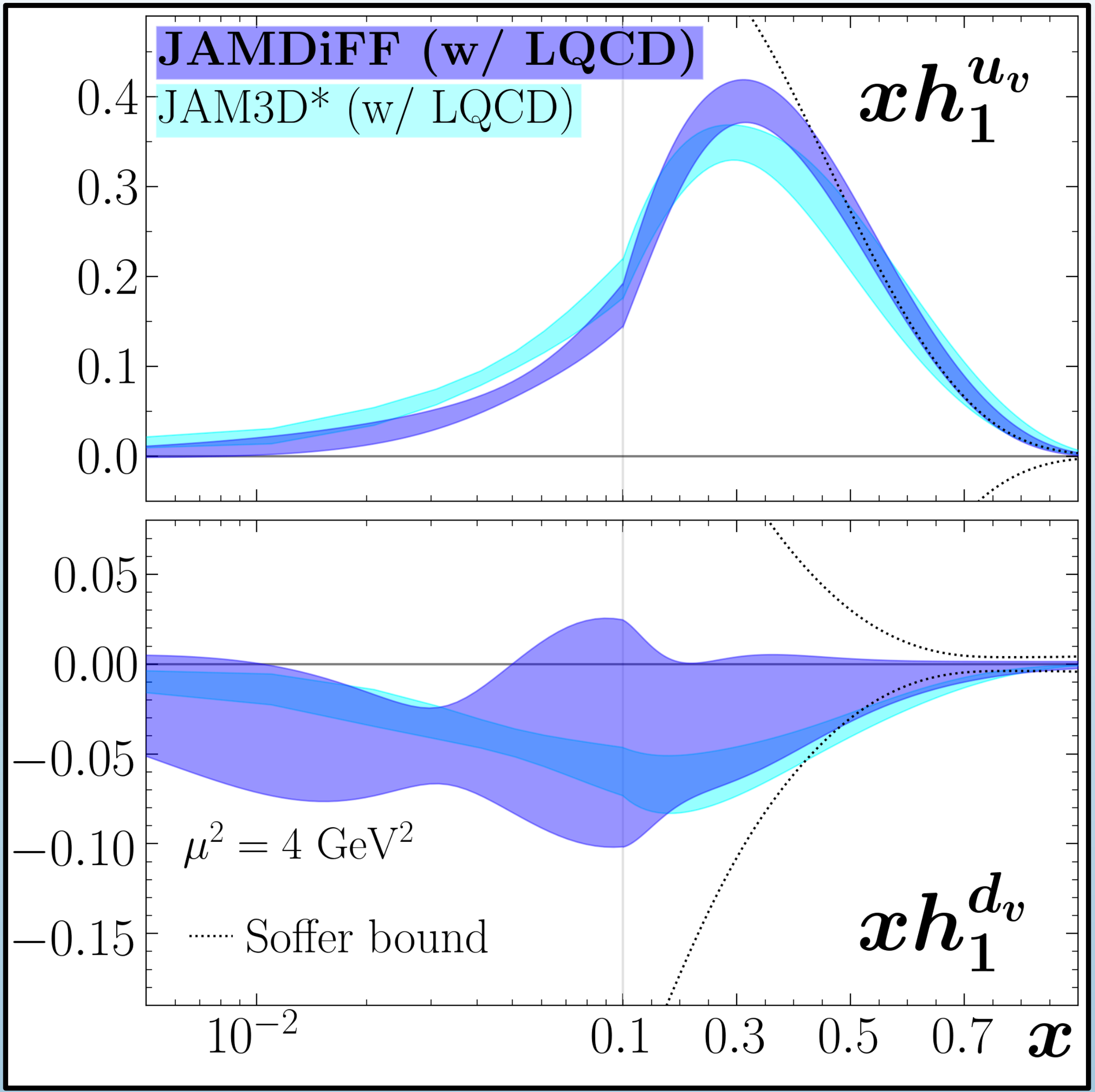
Physical Pion Mass

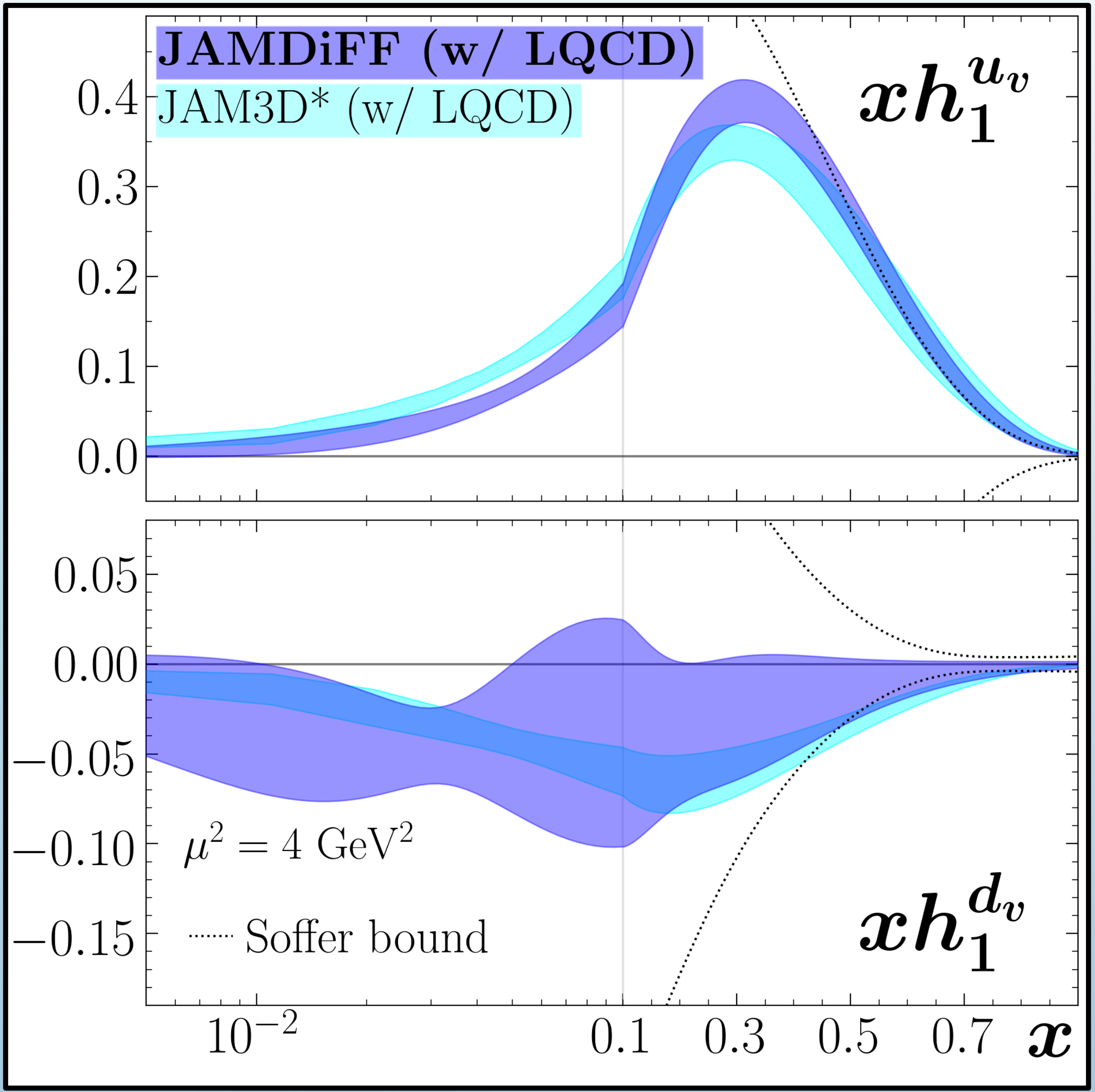
$N_f = 2 + 1 + 1$

Use δu and δd instead of g_T

Transversity PDFs (w/ LQCD)

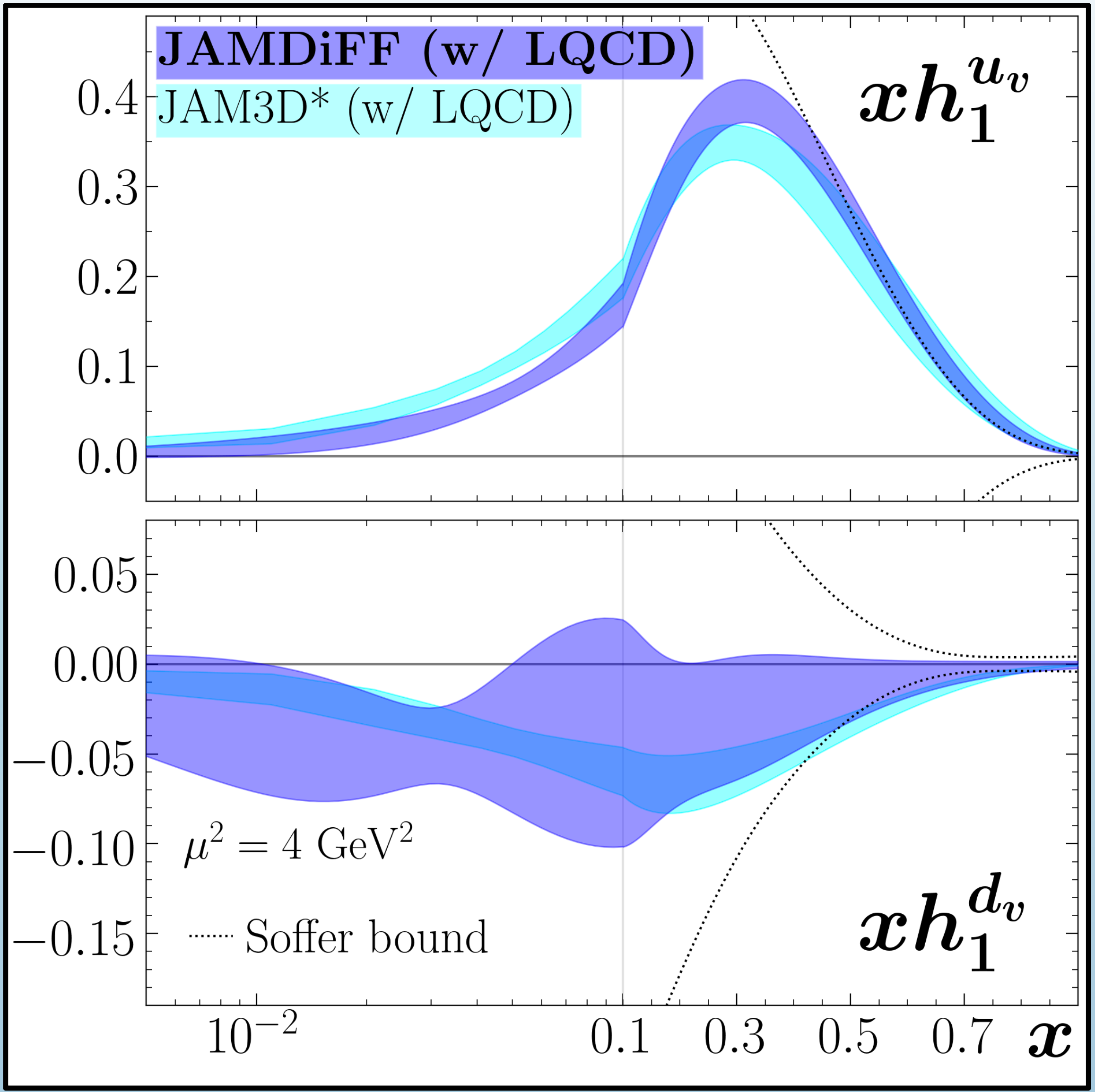


Transversity PDFs (w/ LQCD)



JAM3D* = JAM3D-22 (w/ LQCD)
+ Antiquarks w/ $\bar{u} = -\bar{d}$
+ small-x constraint (see slide 27)
+ δu, δd from ETMC & PNDME
(instead of g_T from ETMC)

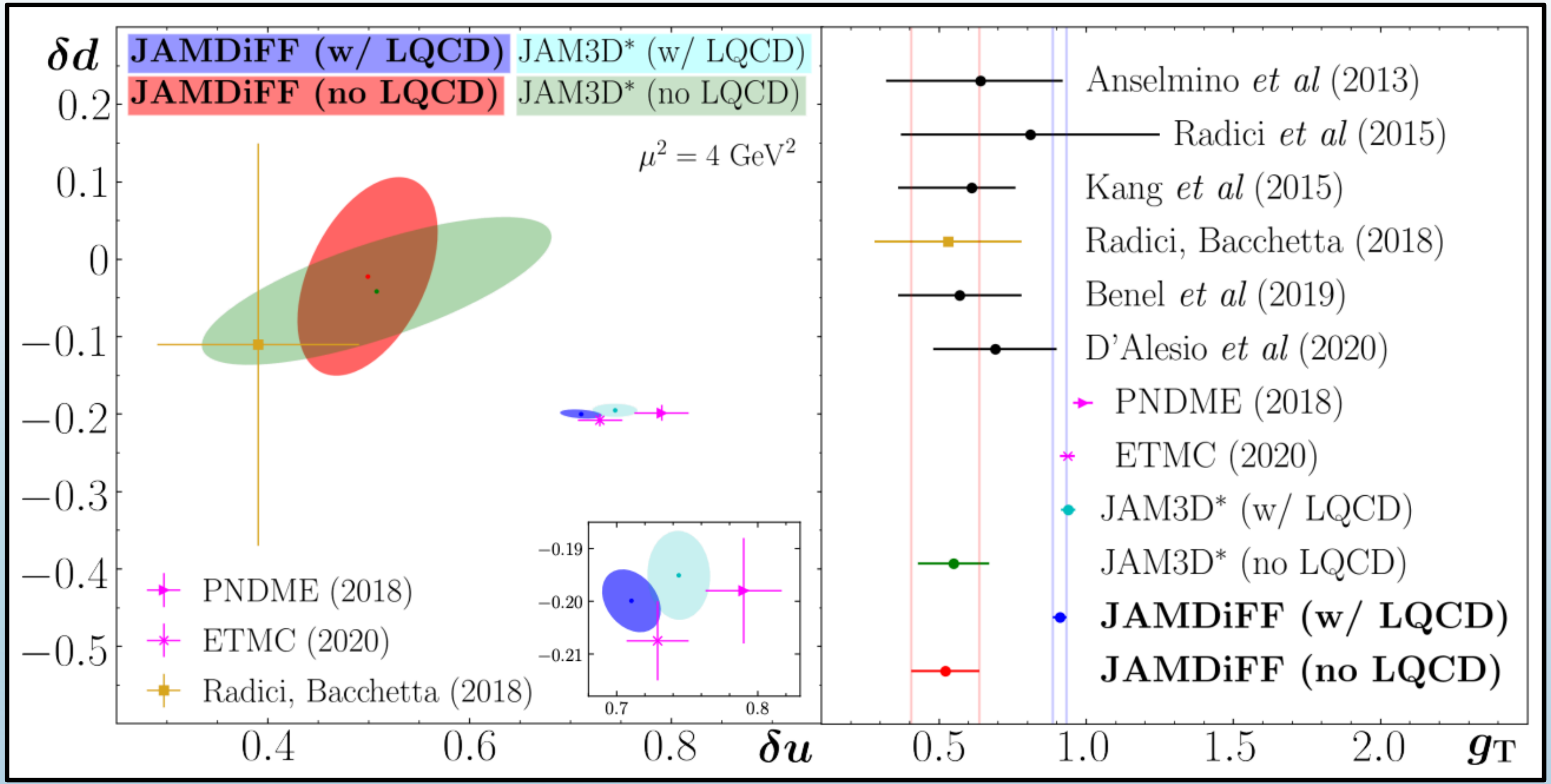
Transversity PDFs (w/ LQCD)



JAM3D* = JAM3D-22 (w/ LQCD)
+ Antiquarks w/ $\bar{u} = -\bar{d}$
+ small-x constraint (see slide 27)
+ $\delta u, \delta d$ from ETMC & PNDME
(instead of g_T from ETMC)

JAMDiFF (w/ LQCD) and JAM3D* (w/ LQCD) largely agree

Tensor Charges (w/ LQCD)



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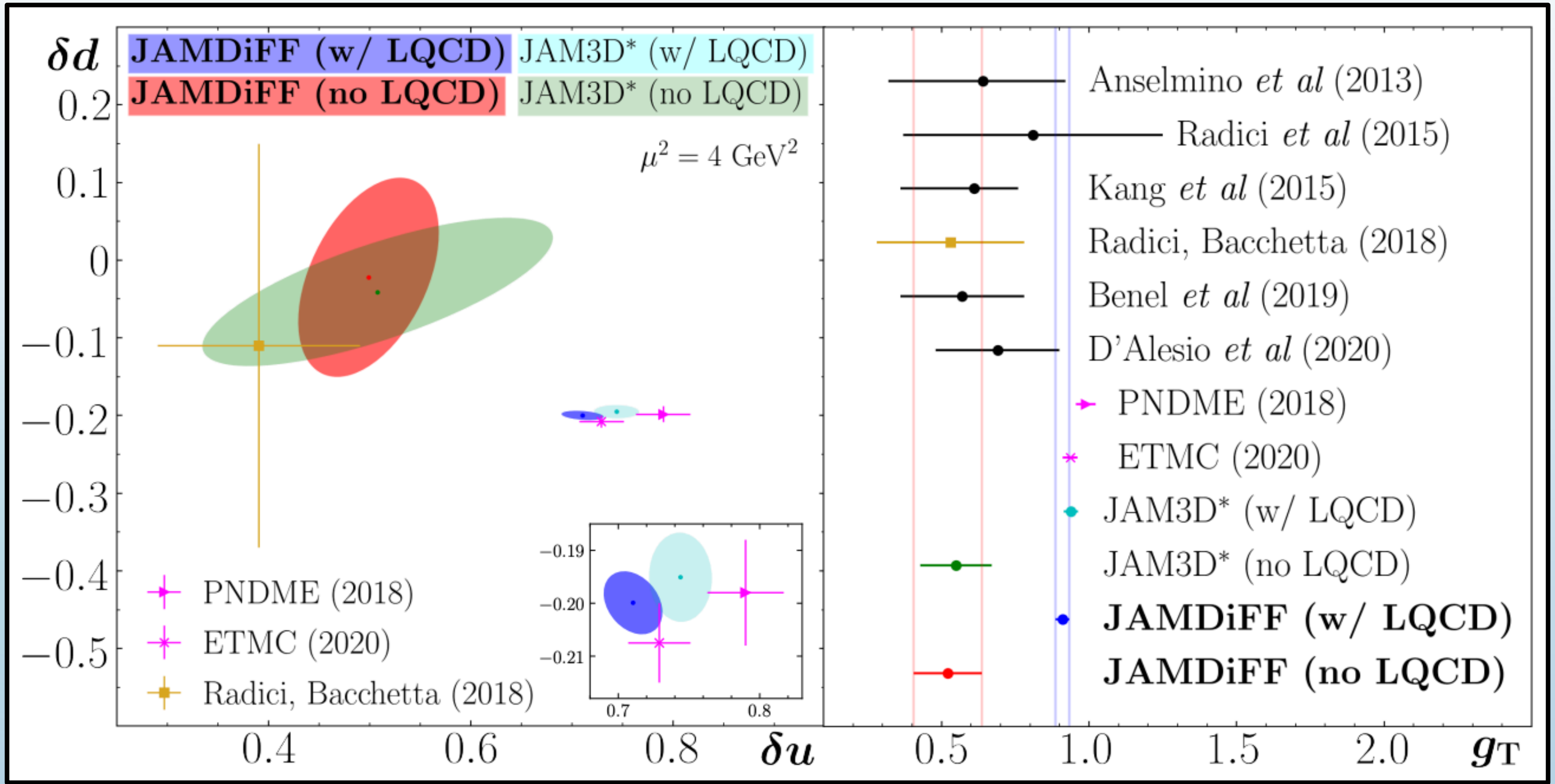
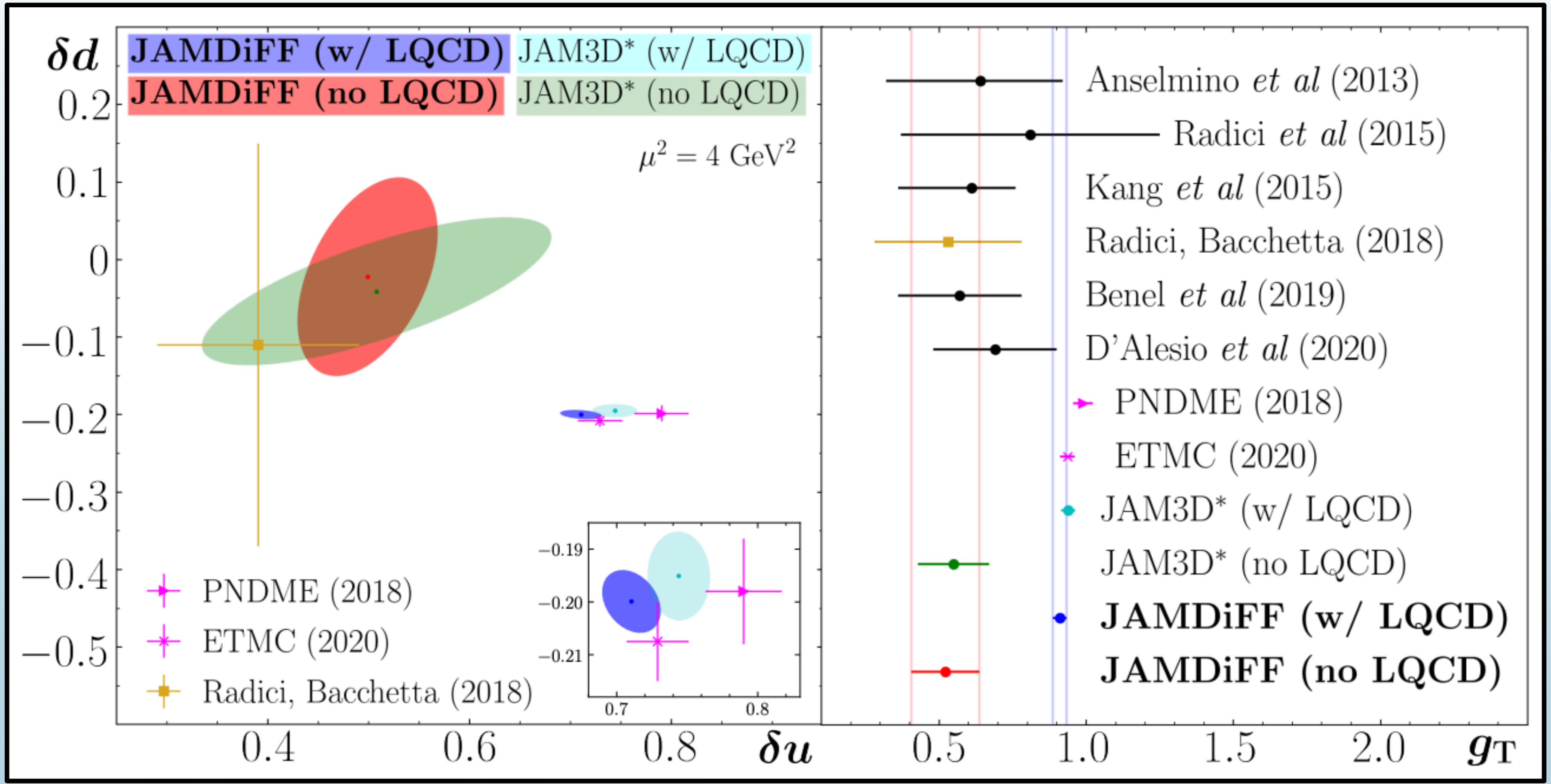


Noticeable shift from including lattice data

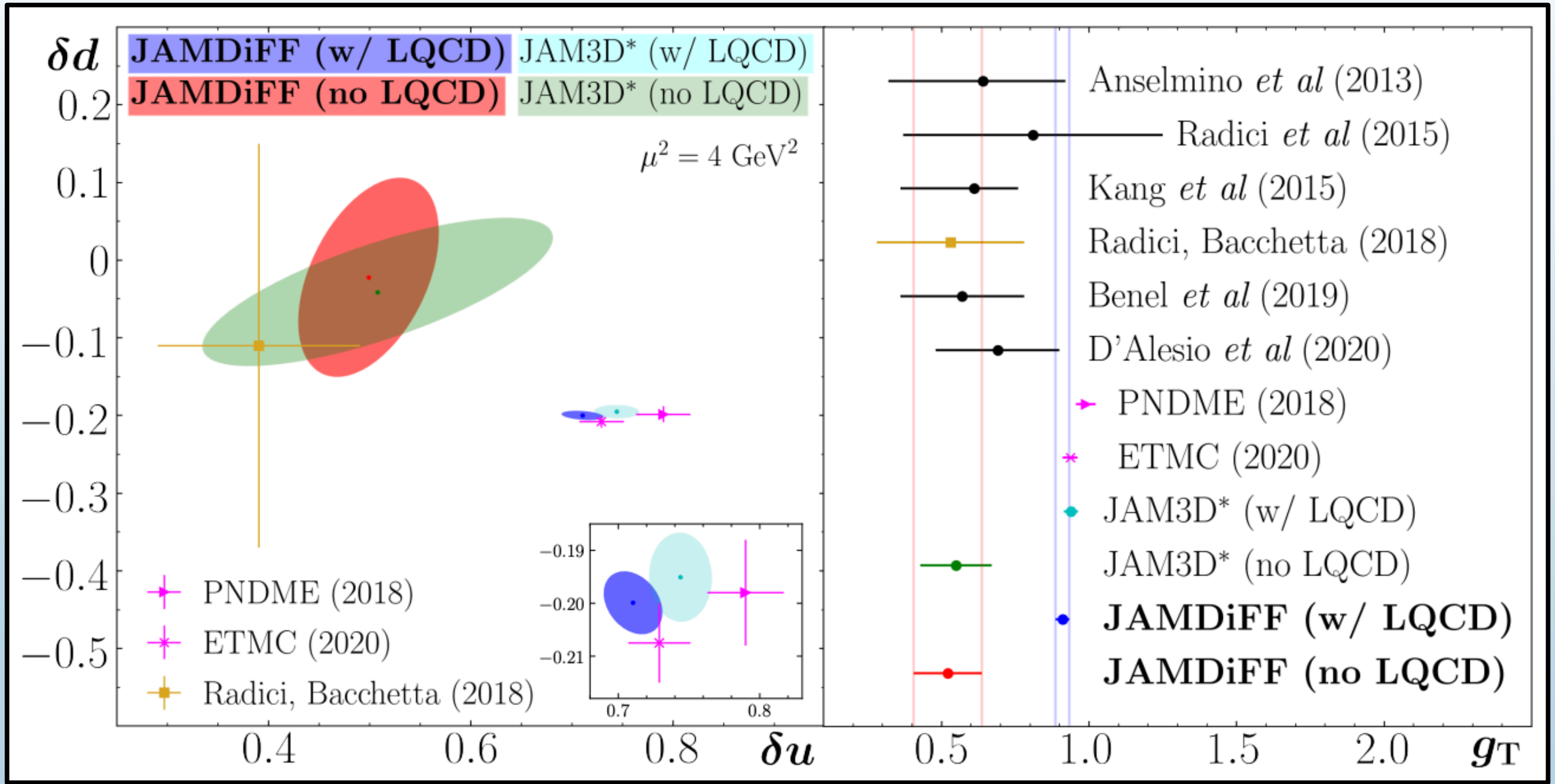
Tensor Charges (w/ LQCD)



Likelihood function $\mathscr{L} = \exp(-\chi^2/2)$ does not guarantee that errors overlap when using Monte Carlo method

Noticeable shift from including lattice data

Tensor Charges (w/ LQCD)



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M.N. Constantini *et al.*, JHEP **12**, 064 (2024)

N.T. Hunt-Smith *et al.*, Comput. Phys. Commun. **296**, 109059 (2024)

N. T. Hunt-Smith *et al.*, Phys. Rev. D **106**, 036003 (2022)

Noticeable shift from including lattice data

Currently looking into Markov Chain Monte Carlo to better assess uncertainties.

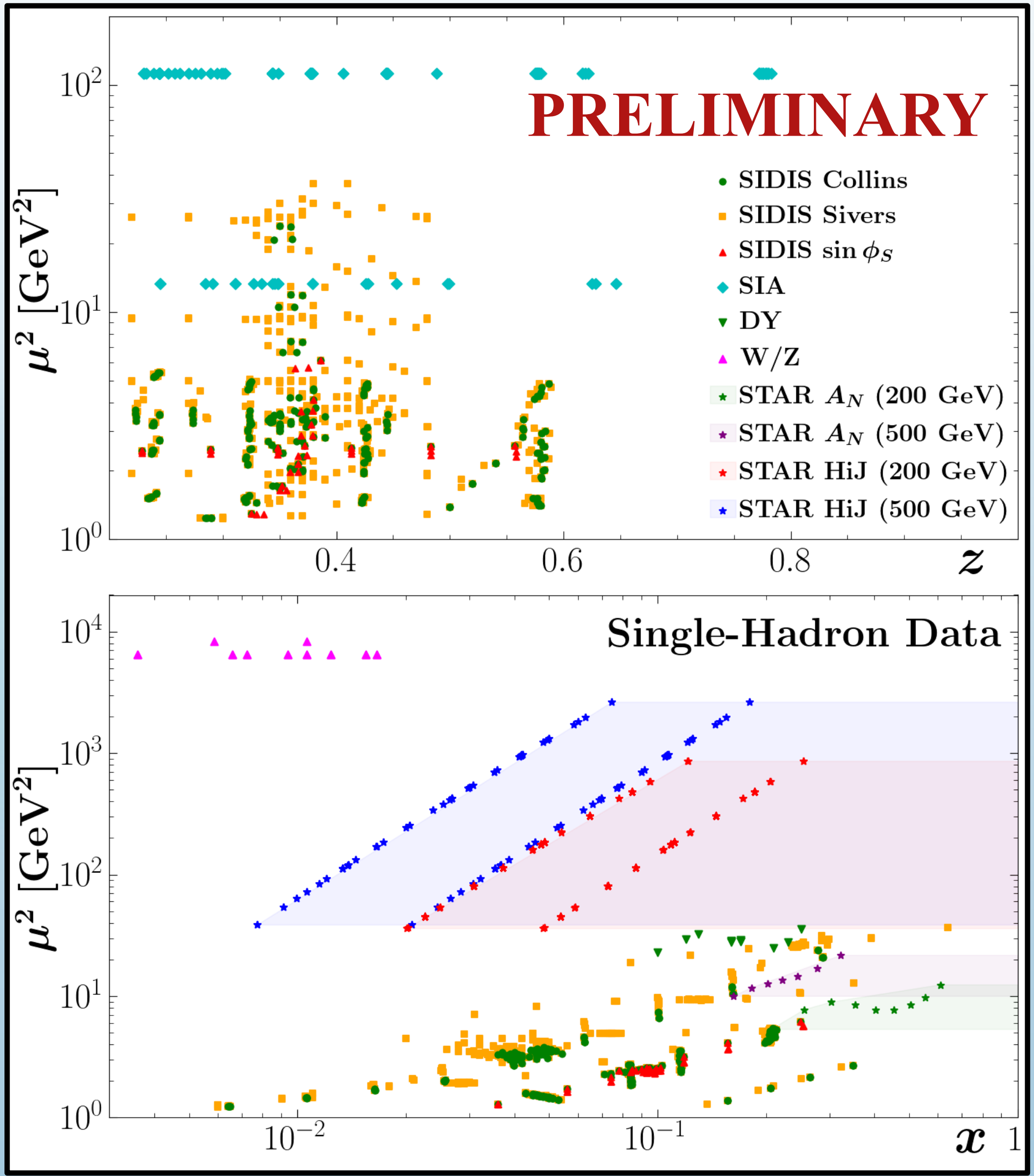
Future Work

Currently working on including DiFF data, TMD data, and LQCD calculations into a single global QCD analysis.

The ultimate global QCD analysis for transversity!

JAM3D +
JAMDiFF
=
JAM**3D**i**FF**

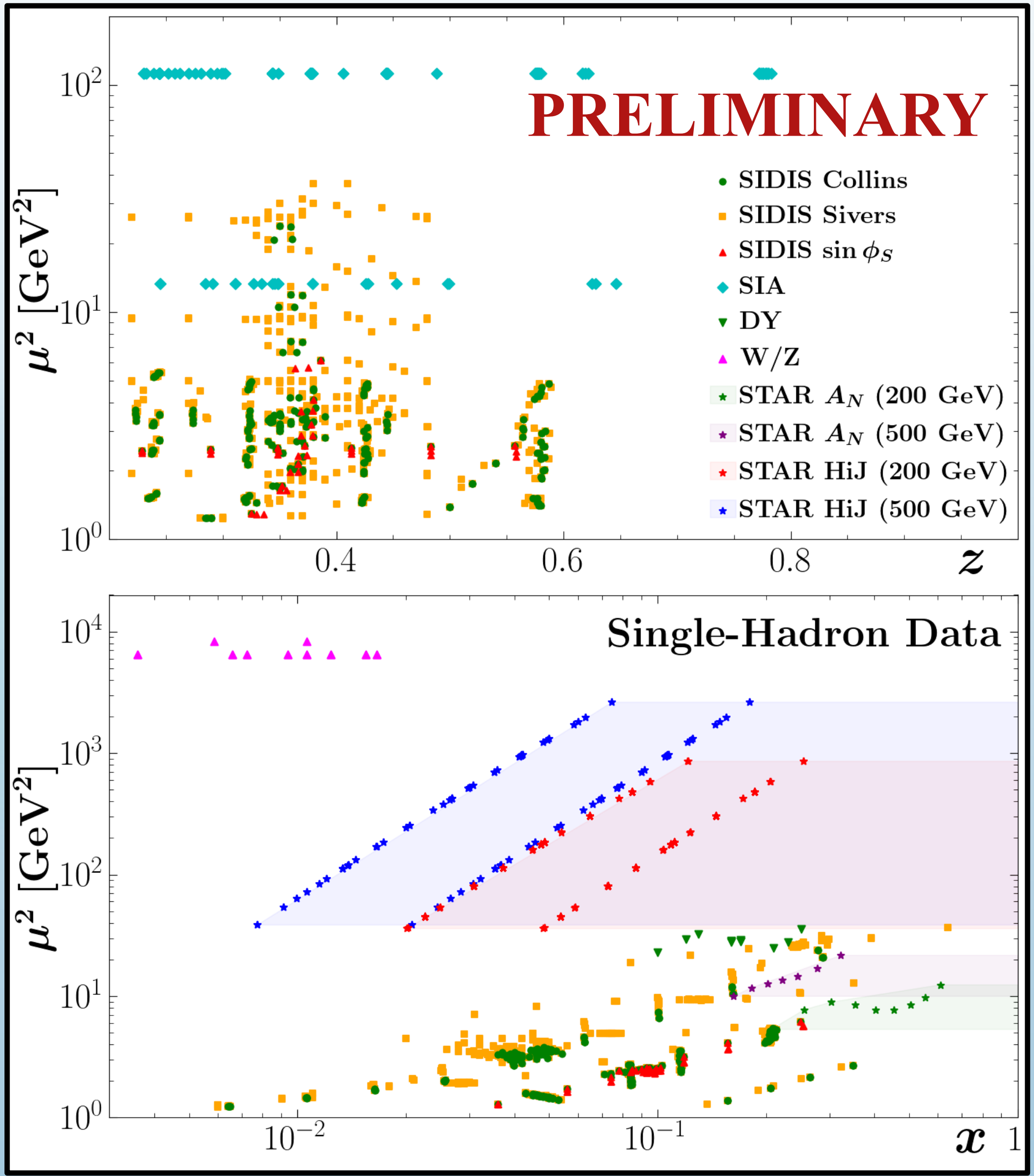
Kinematics and Functions



Process	Collaborations	Points
SIA	BaBaR, Belle, BESIII	176
SIDIS Asym.	COMPASS, HERMES	525
DY	COMPASS	15
W/Z	STAR	17
pp AN	STAR, AnDY	44
Hadron-in-jet	STAR	708

PRELIMINARY

Kinematics and Functions



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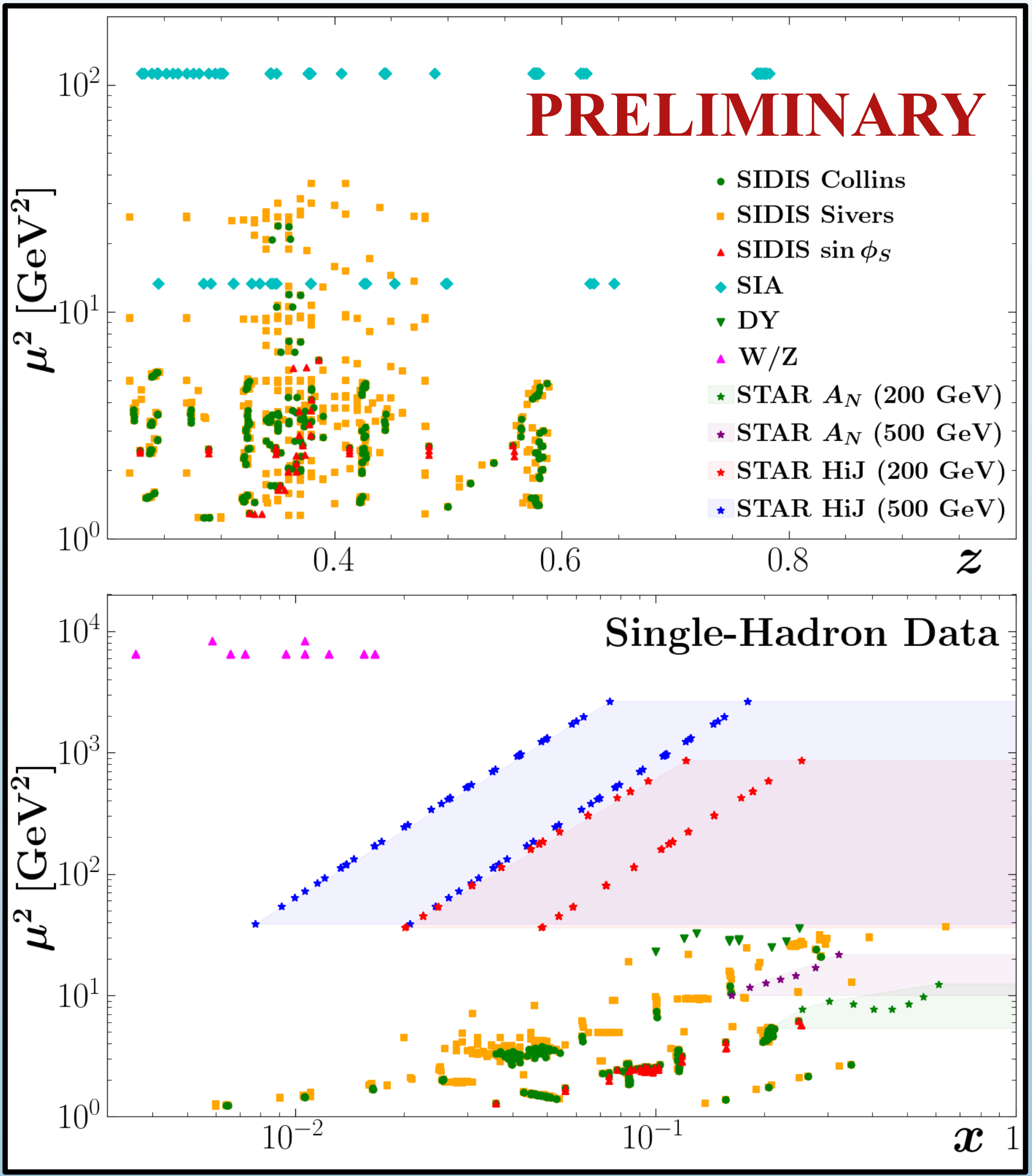
PRELIMINARY

Transversity $h_1 : u, d, \bar{u}, \bar{d}$ + widths

Kinematics and Functions



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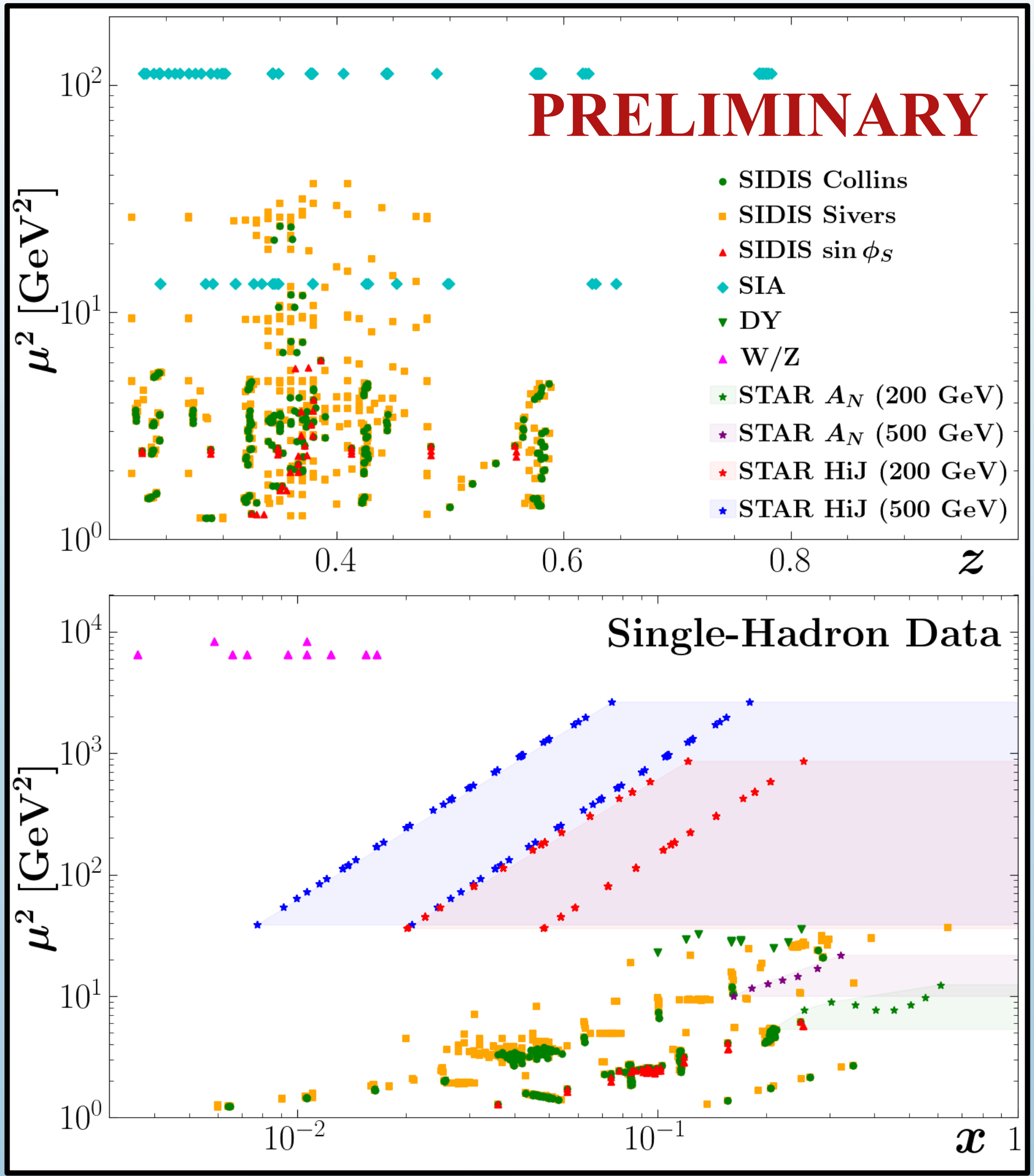
PRELIMINARY

Transversity $h_1 : u, d, \bar{u}, \bar{d} \ +$ widths

Sivers $f_{1T}^{\perp(1)} : u, d, \bar{u}, \bar{d}, s, \bar{s} \ +$ widths

Kinematics and Functions



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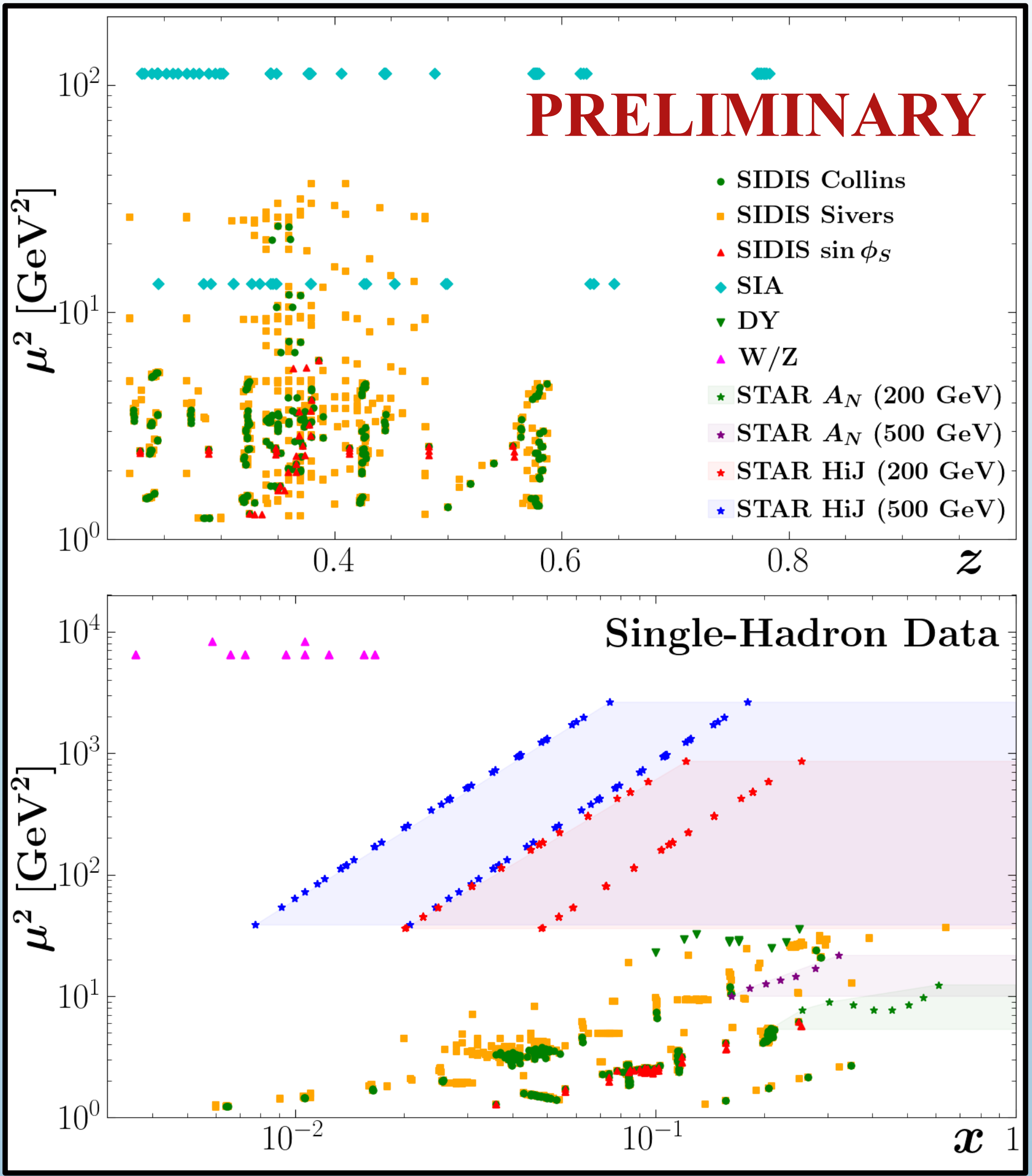
Collins (pion) $H_1^{\perp(1)} : \text{fav.}, \text{unfav.} \ + \ \text{widths}$

Kinematics and Functions



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PRELIMINARY

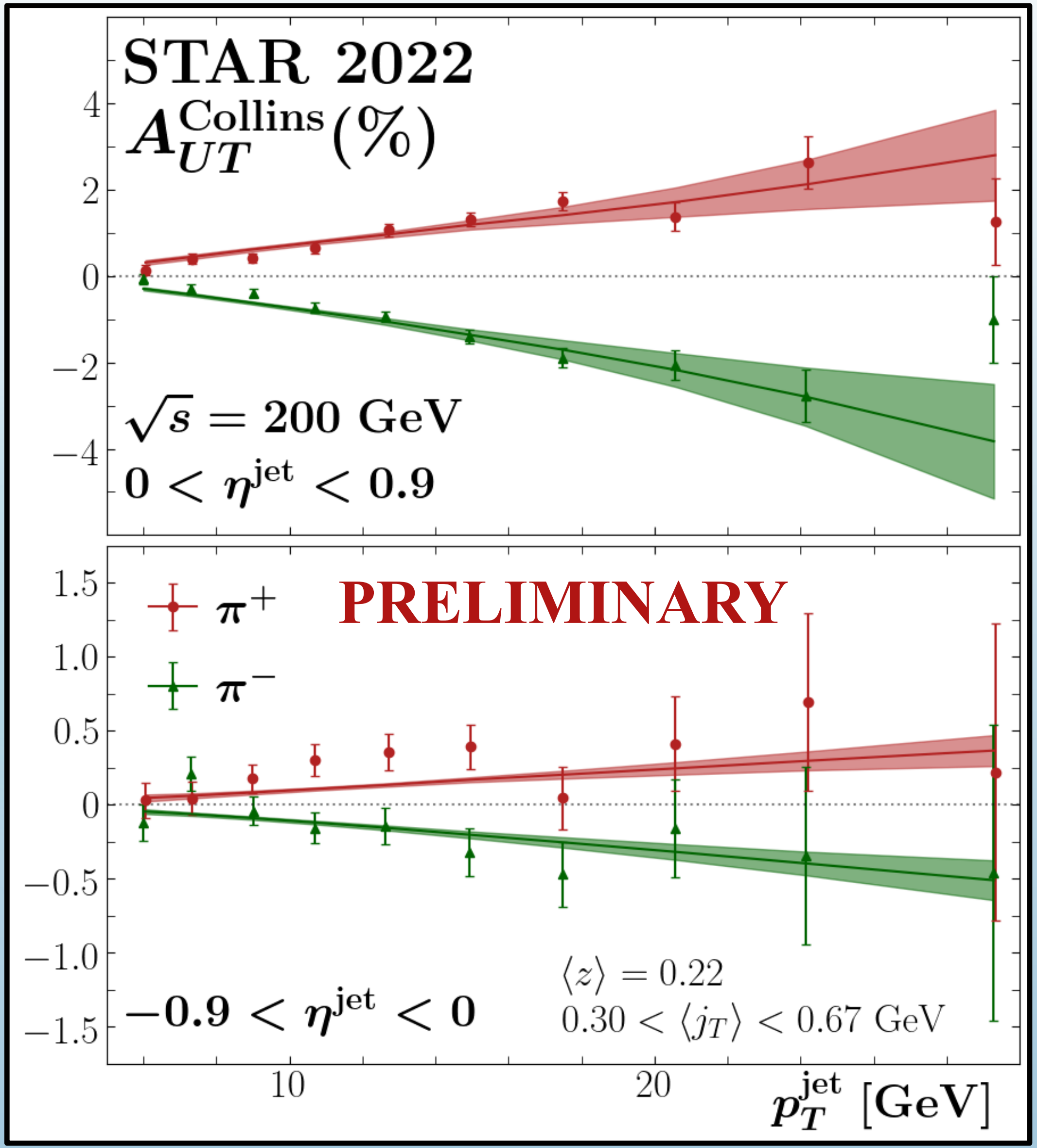
Transversity h_1 : $u, d, \bar{u}, \bar{d}$ + widths

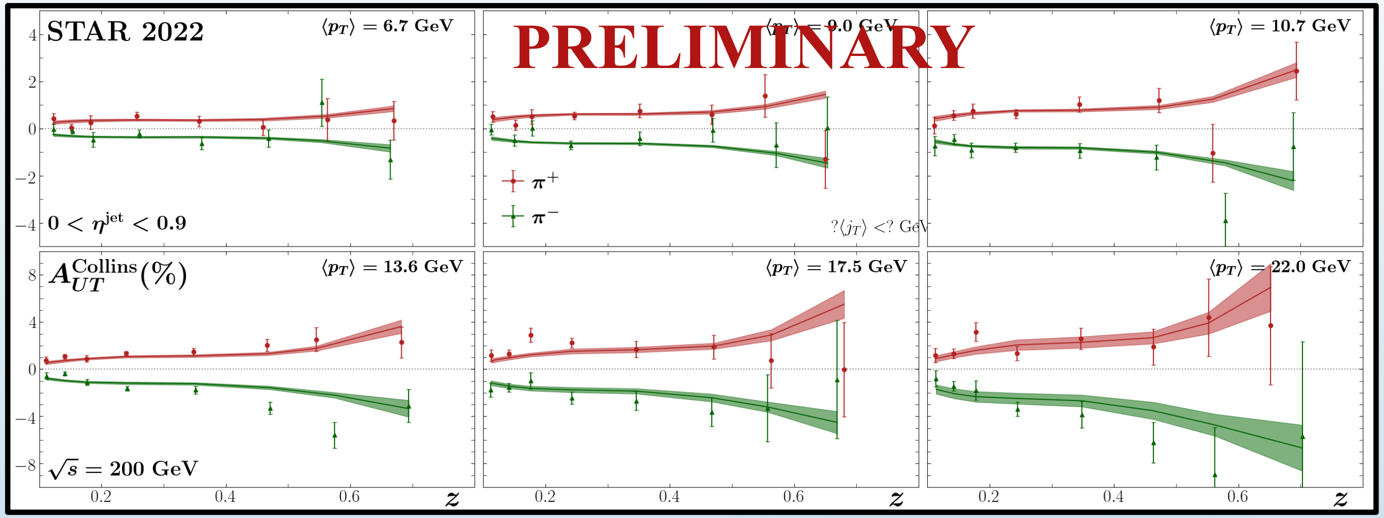
Sivers $f_{1T}^{\perp(1)}$: $u, d, \bar{u}, \bar{d}, s, \bar{s}$ + widths

Collins (pion) $H_1^{\perp(1)}$: fav., unfav. + widths

Twist-3 FF (pion) $\tilde{H}$: fav., unfav.

Hadron-in-jet





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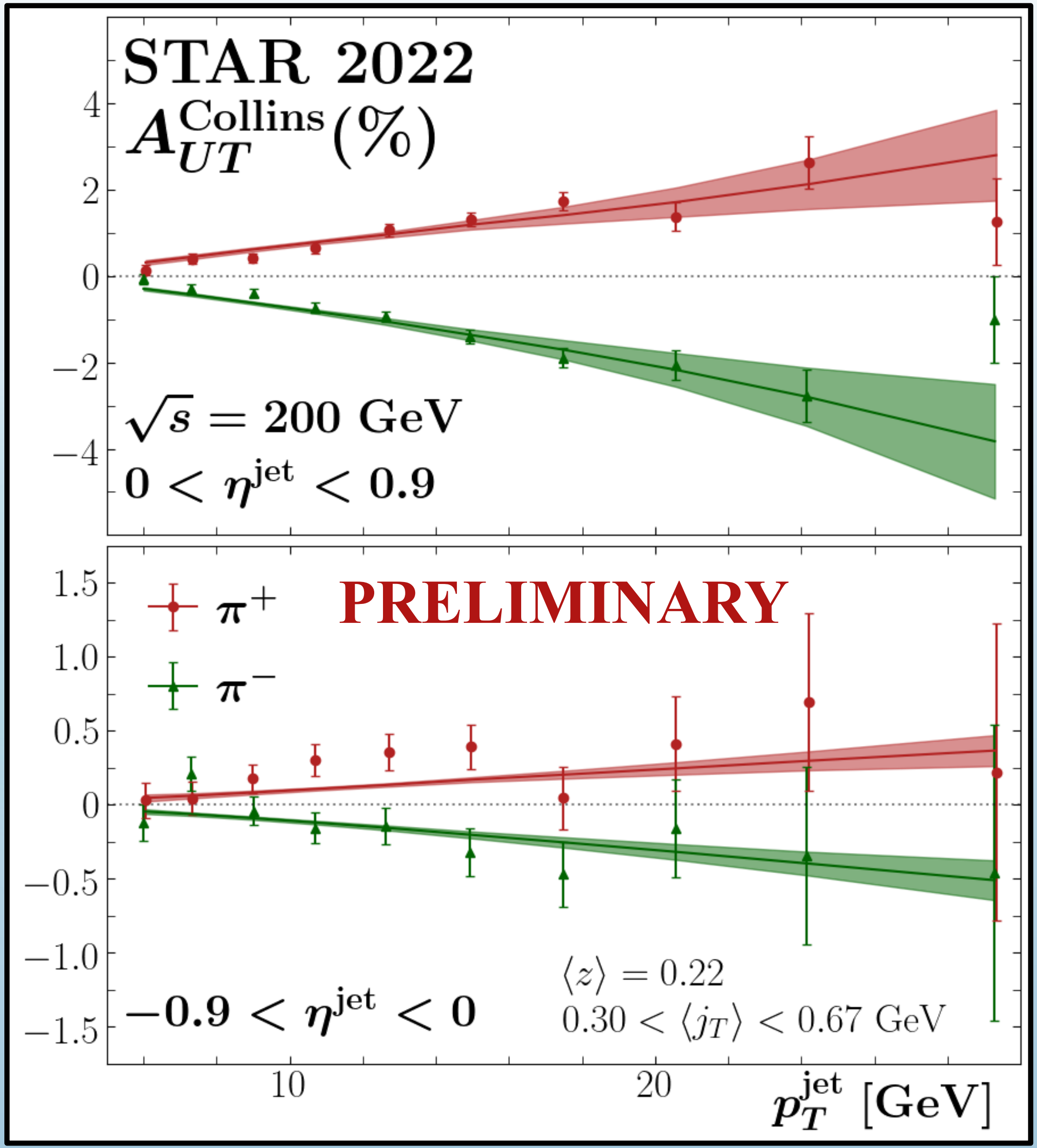

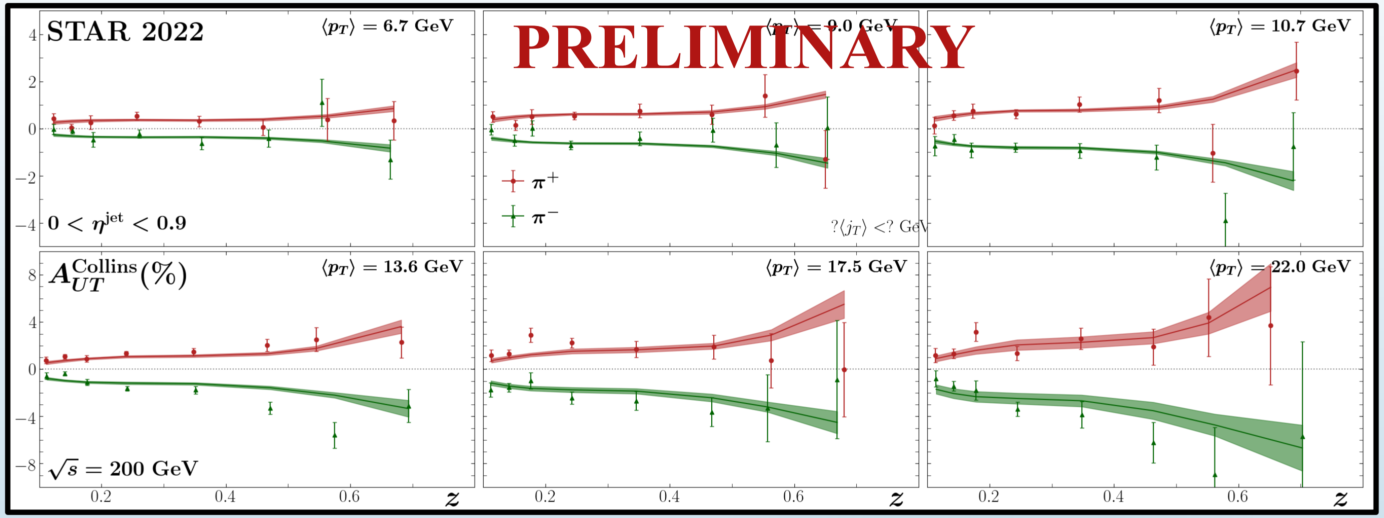

First global QCD analysis to include Hadron-in-jet data!

Quality of Fit and Inclusion of LQCD

Process	Points	chi2 (no LQCD)	chi2 (w/ LQCD)
SIA	176	1.09	1.15
SIDIS	1050	1.38	1.38
DY	15	0.24	0.24
W/Z	17	1.71	1.68
pp AN	44	1.89	1.80
Hadron-in-jet	708	1.03	1.03
LQCD	4	—	0.92
TOTAL	2014	1.24	1.24

PRELIMINARY

PRELIMINARY

Quality of Fit and Inclusion of LQCD

Process	Points	chi2 (no LQCD)	chi2 (w/ LQCD)
SIA	176	1.09	1.15
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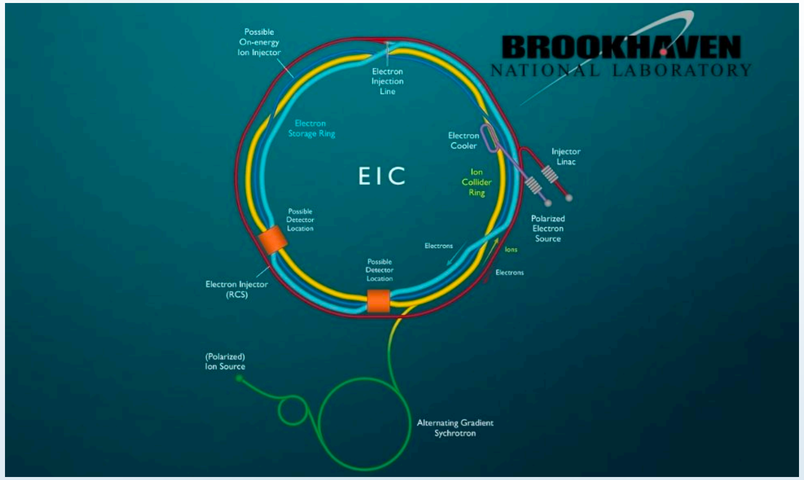
Hadron-in-jet	708	1.03	1.03
LQCD	4	—	0.92
TOTAL	2014	1.24	1.24

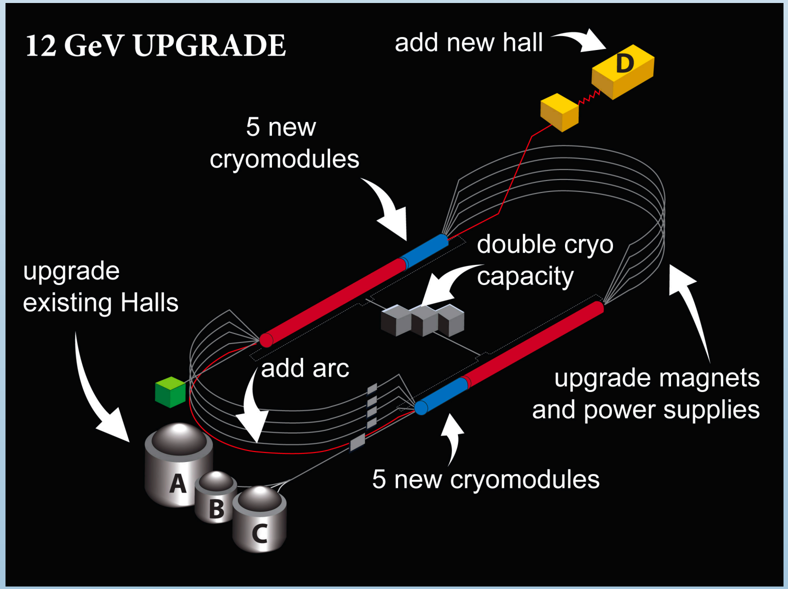
PRELIMINARY

PRELIMINARY

Inclusion of LQCD barely affects description of JAM3D data!

1. Introduction
2. Spin-Averaged Parton Distribution Functions
3. Extraction of Nuclear Effects
4. Helicity Parton Distribution Functions
5. Di-Hadron Production and Transversity Parton Distribution Functions
6. **Summary and Outlook**

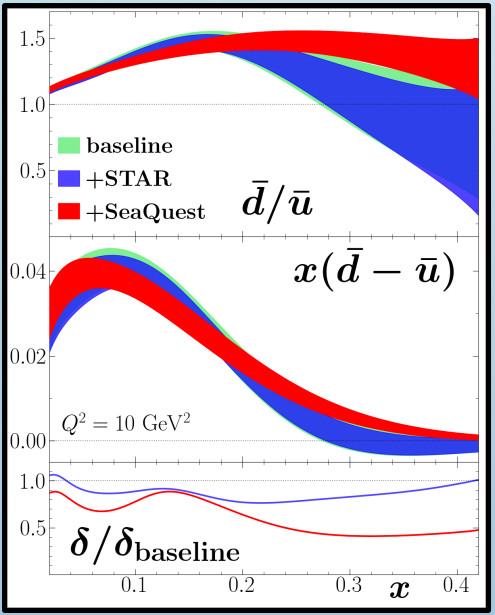


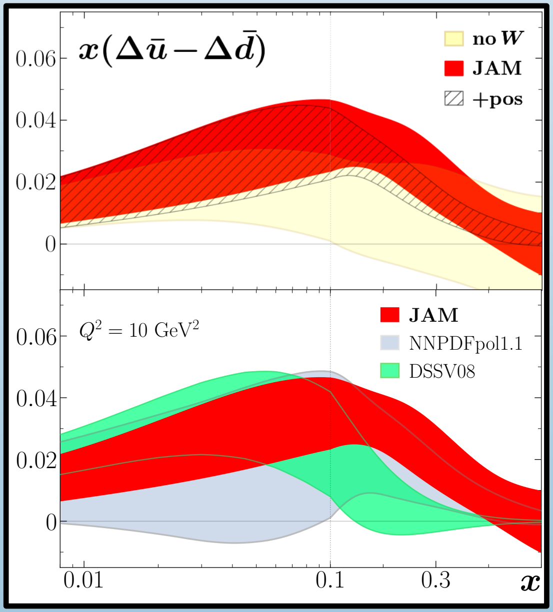


Summary

Summary

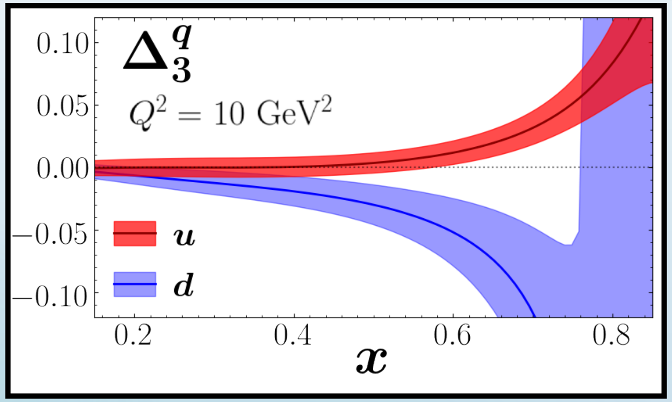
Sea Asymmetries





Summary

Isovector EMC Effect



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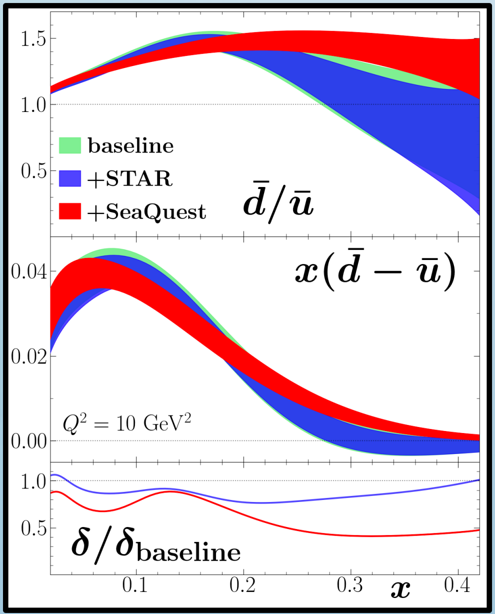

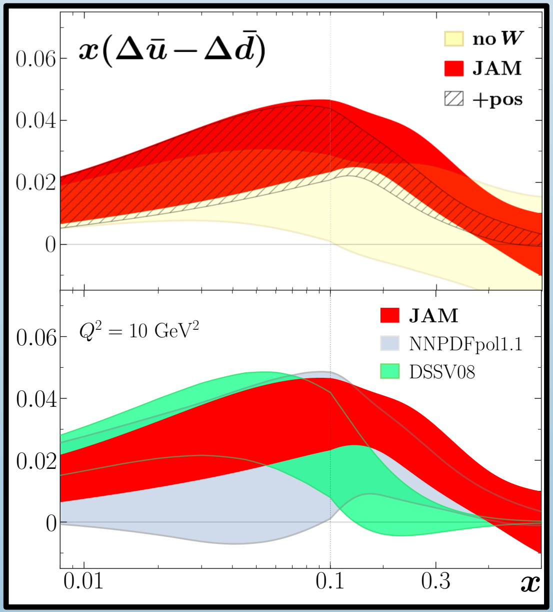

Summary

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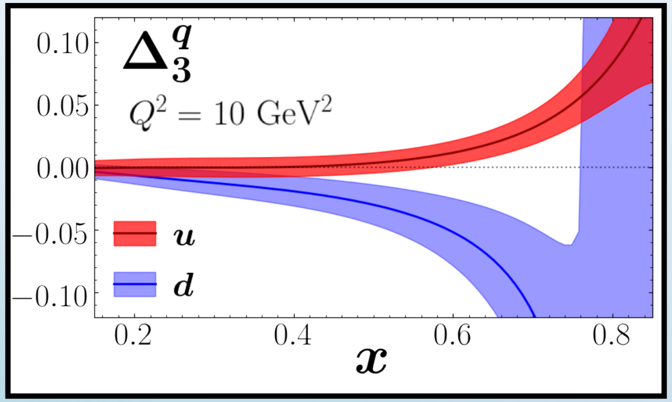
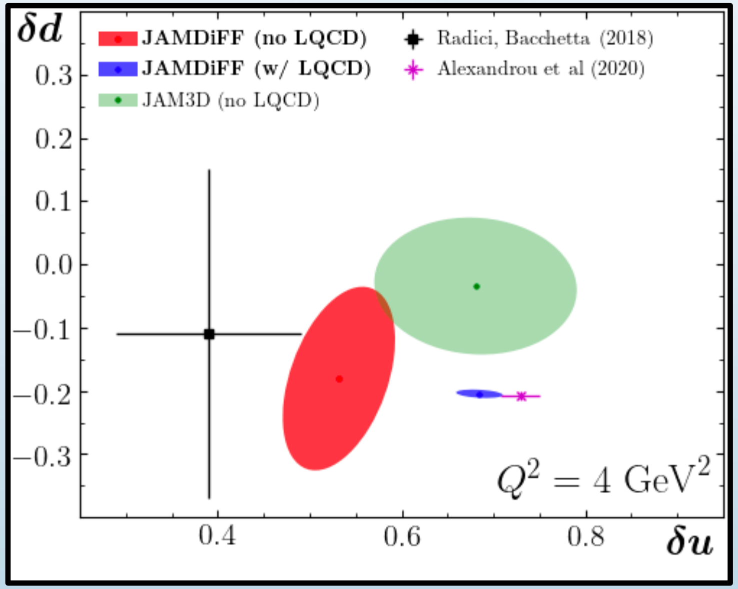


Transverse Spin Puzzle



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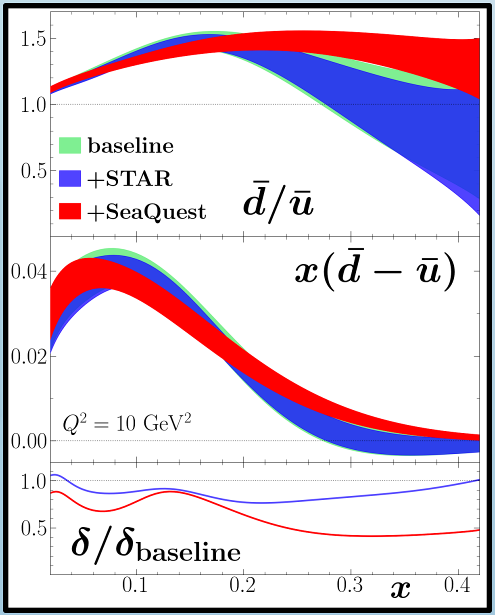
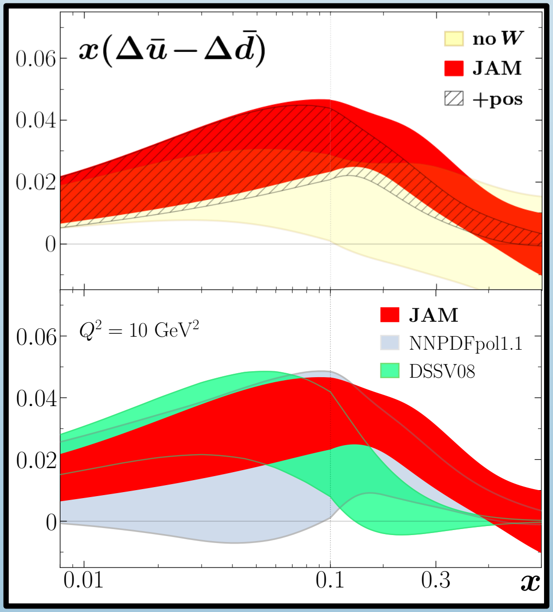
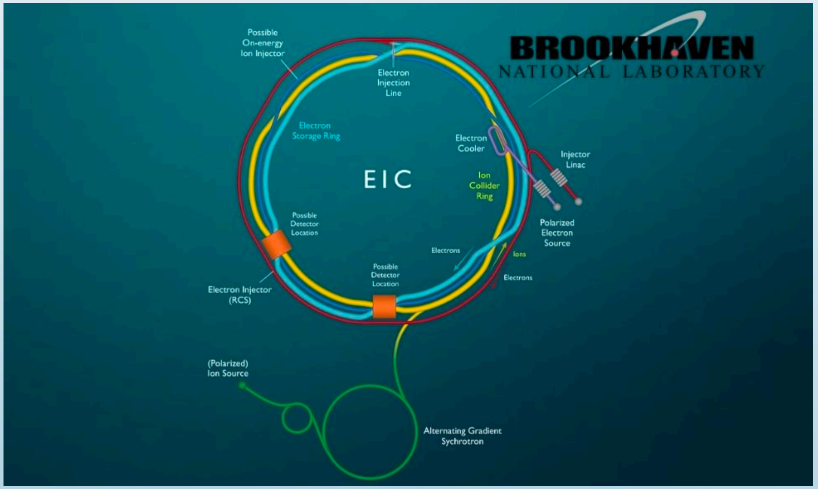




Electron Ion Collider (EIC) + JLab 12 GeV Upgrade

First polarized electron-ion collider



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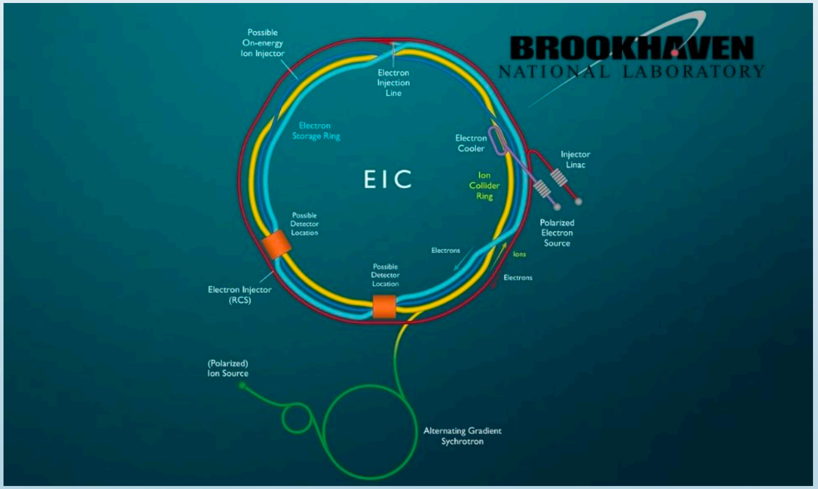
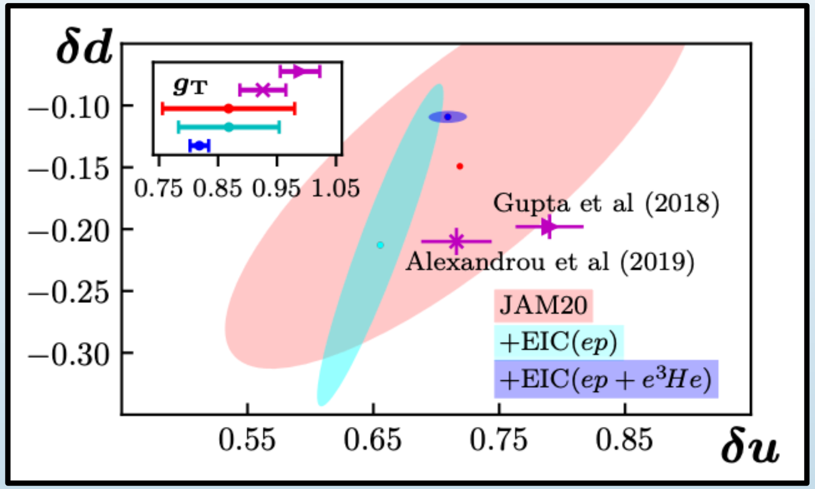


Tensor charges



L. Gamberg *et al.*, Phys. Lett. B **816**, 136255 (2021)

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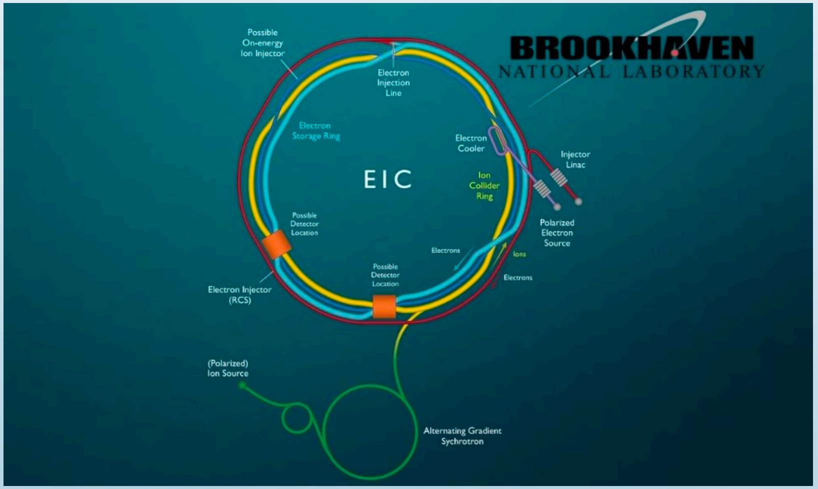

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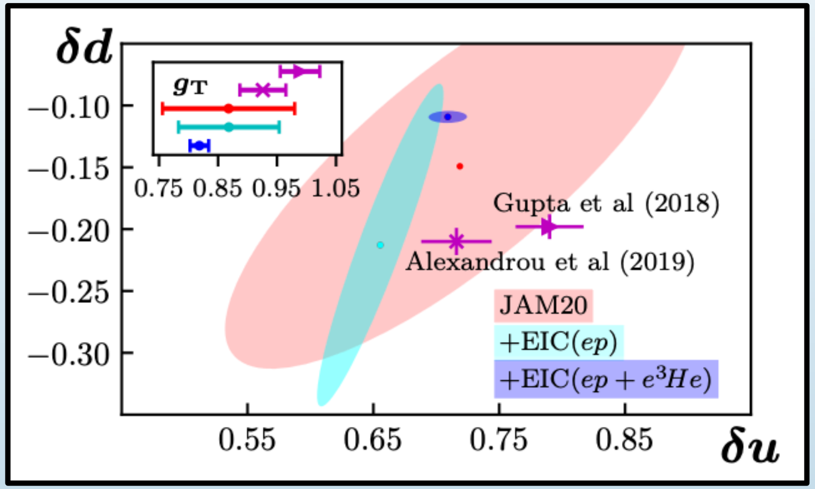

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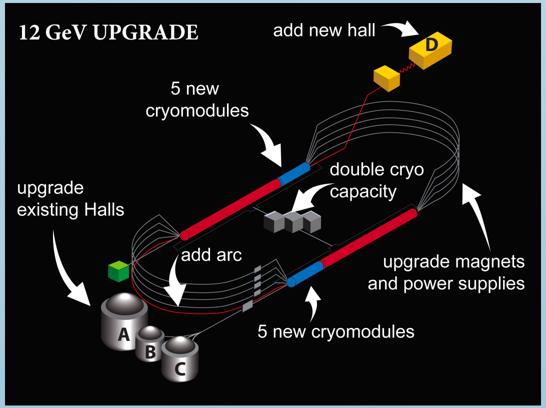
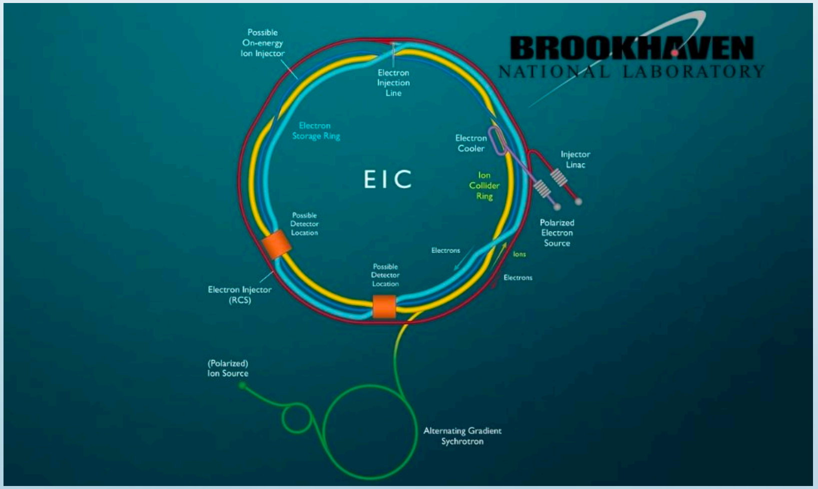


MARATHON data on ^{3}He/D and ^{3}H/D
+ Spectator tagged DIS + precise high x DIS data

Electron Ion Collider (EIC) + JLab 12 GeV Upgrade

First polarized electron-ion collider



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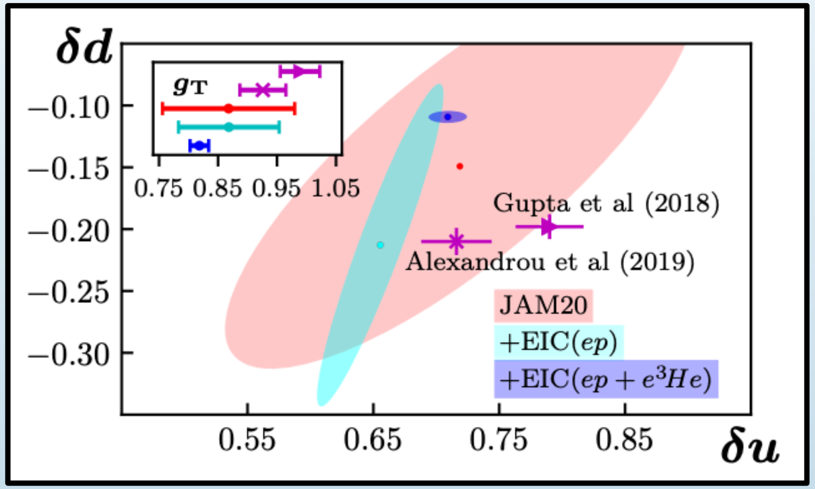
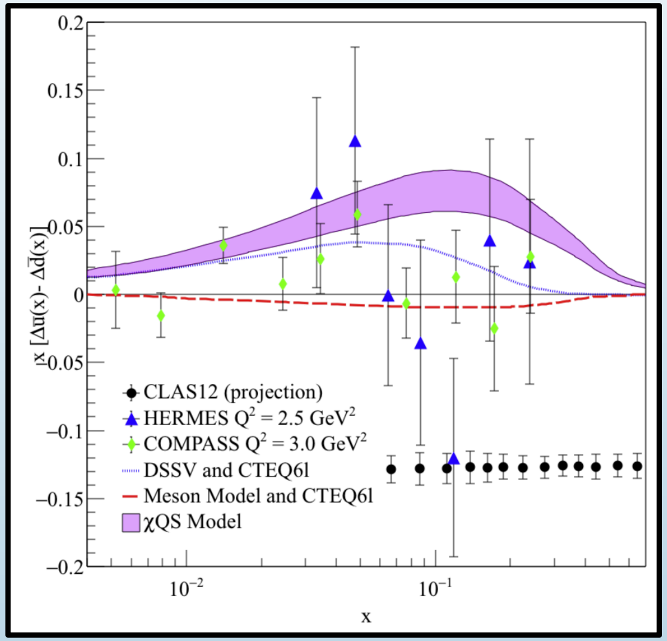
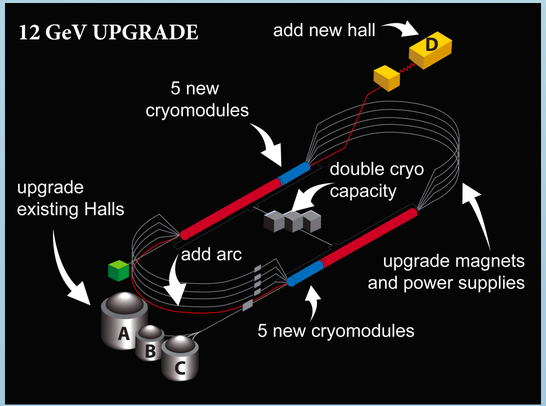


L. Gamberg *et al*., Phys. Lett. B **816**, 136255 (2021)

Polarized SIDIS



D. F. Geesaman and P. E. Reimer, Rep. Prog. Phys. **82**, 046301 (2019)



MARATHON data on $^3He/D$ and $^3H/D$ + Spectator tagged DIS + precise high x DIS data

Collaboration

Andreas Metz

Shohini Bhattacharya

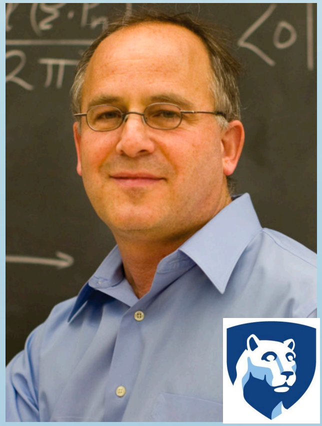
Nobuo Sato

Wally Melnitchouk

Daniel Pitonyak

Alexey Prokudin

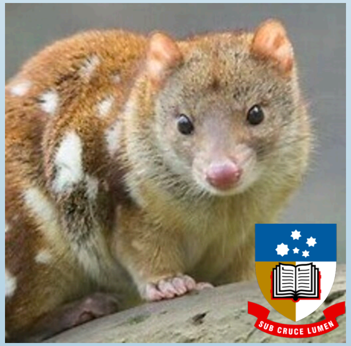
Leonard Gamberg



Hanjie Liu

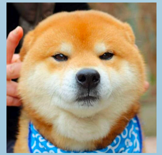
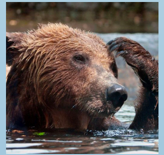
Ralf Seidl

Anthony Thomas



Thia Keppel

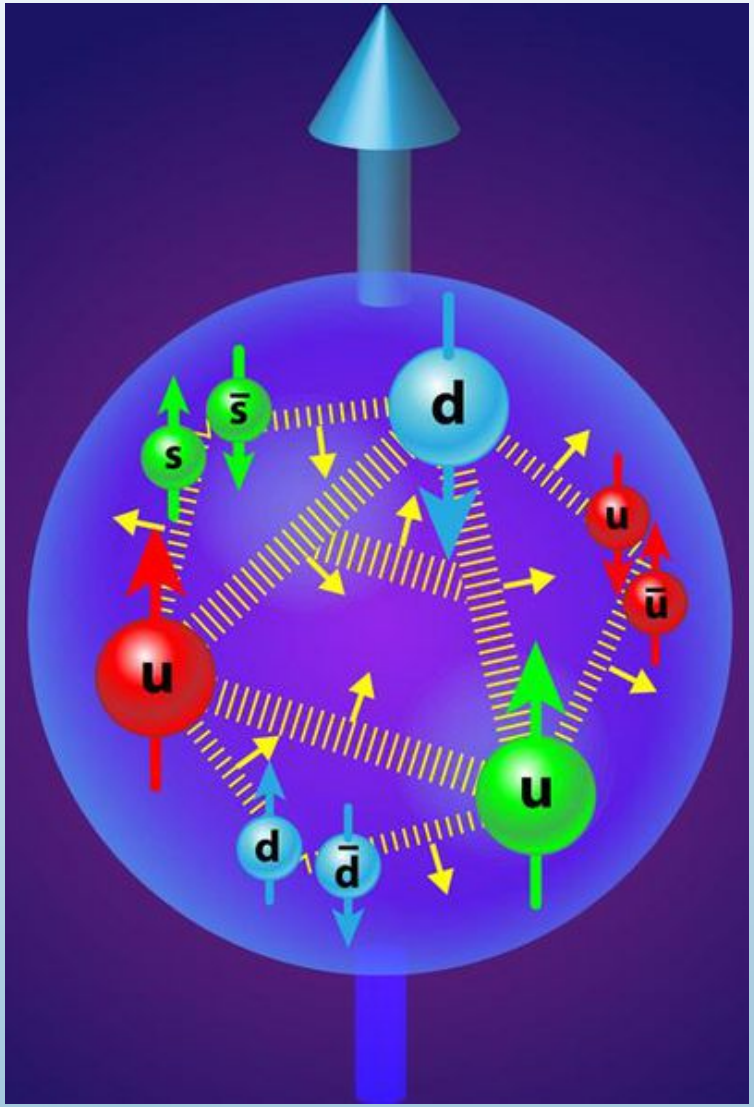
Thank you to Yiyu Zhou and Patrick Barry for helpful discussions

Extra

Internal Structure of Hadrons

Hadrons (such as protons) are composed of partons (quarks and gluons), bound by the strong interaction [Quantum Chromodynamics (QCD)]



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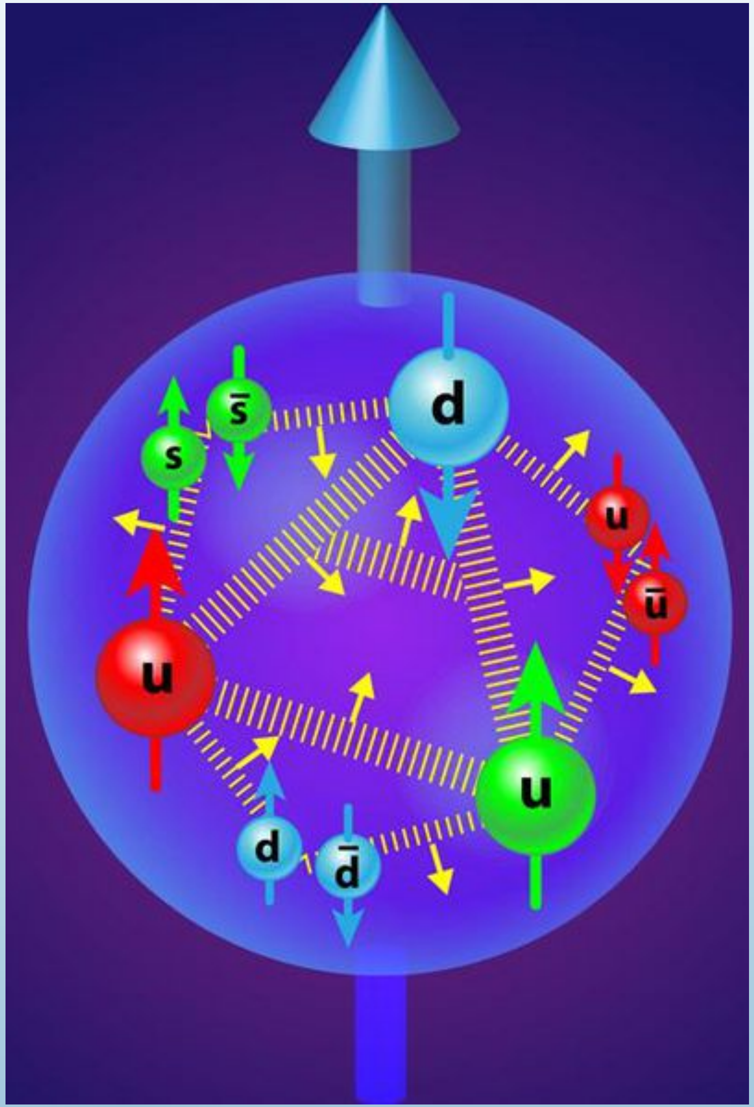

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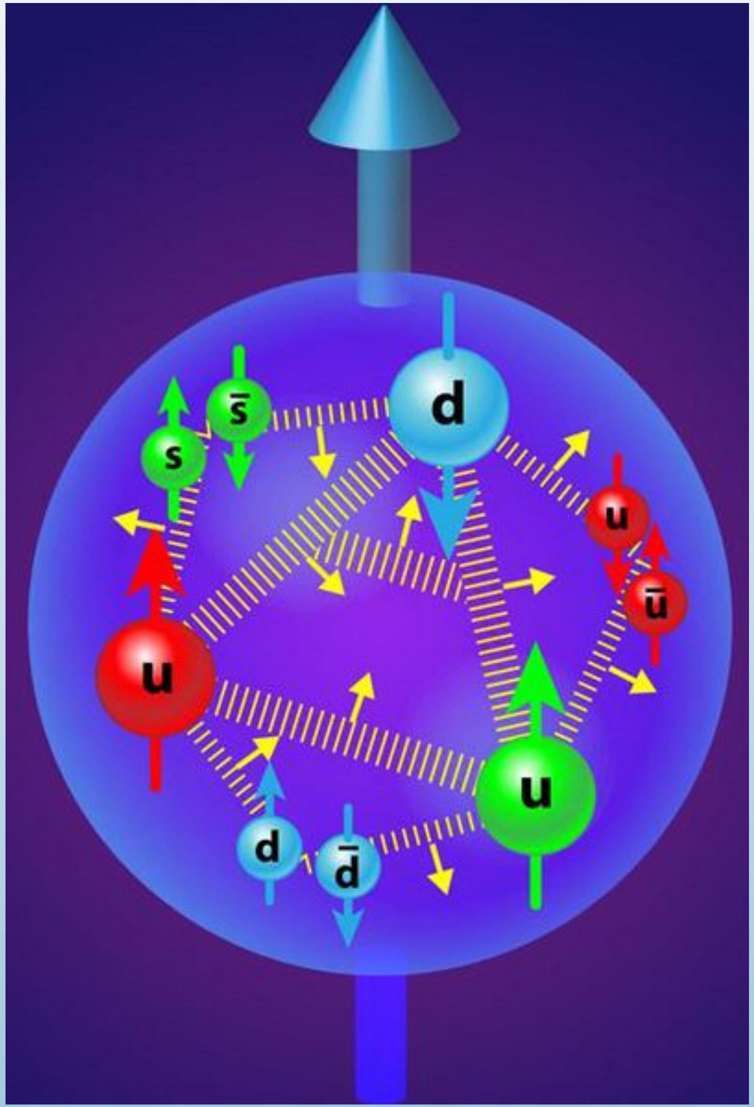
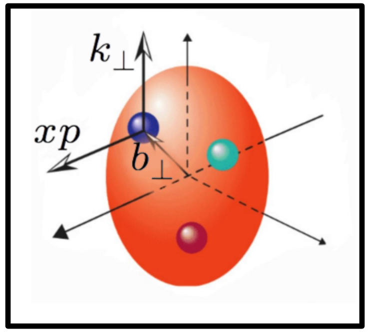
Information can be gained through model calculations and experiments acting as high energy probes



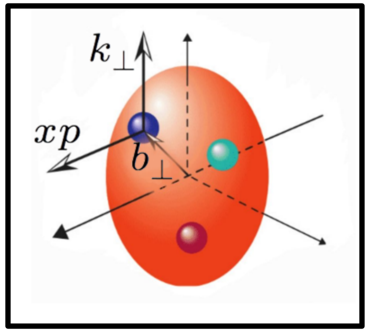
Partonic Functions



Partonic Functions

x: Momentum fraction (parton/hadron)

$\overrightarrow{k}_{\perp}$: Transverse momentum



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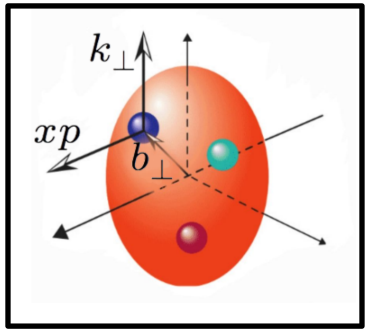

ξ and $\vec{\Delta}_\perp$ describe change in hadron's momentum

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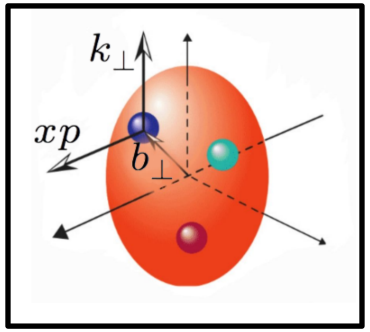
Generalized Transverse Momentum Dependent Distribution (GTMD)

$W(x, \overrightarrow{k}_{\perp}, \xi, \overrightarrow{\Delta}_{\perp})$



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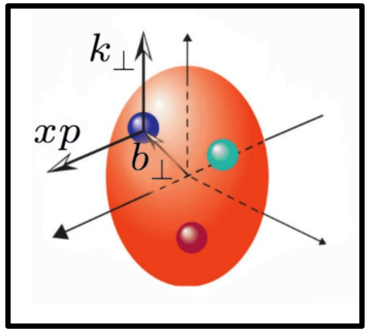

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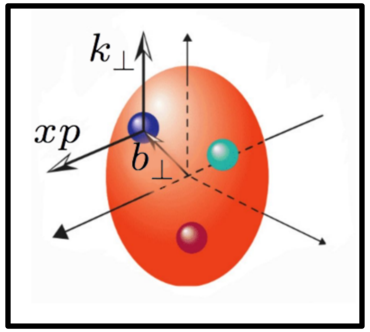

$f(x, \vec{k}_\perp)$

Transverse Momentum Dependent Distribution (TMD)

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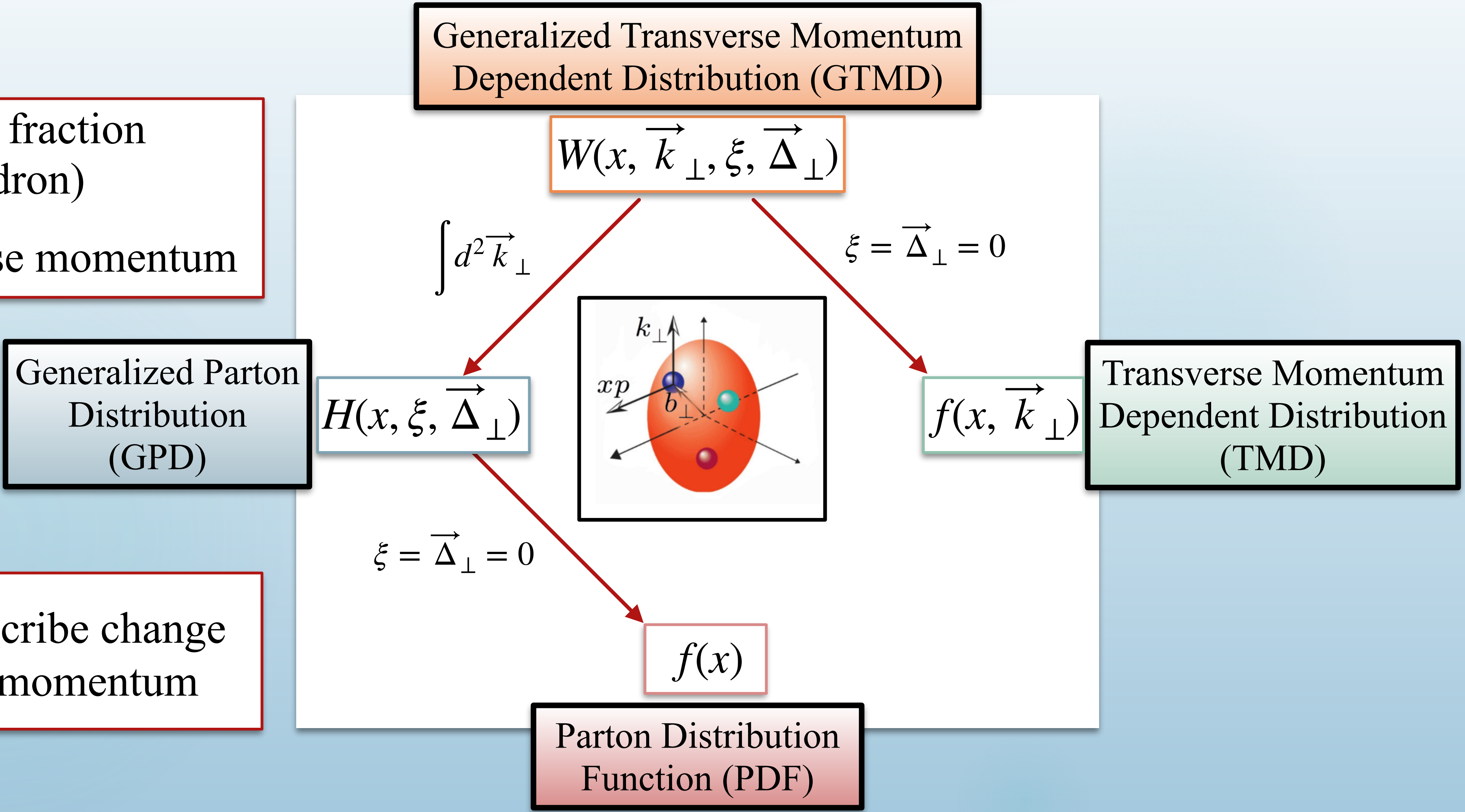

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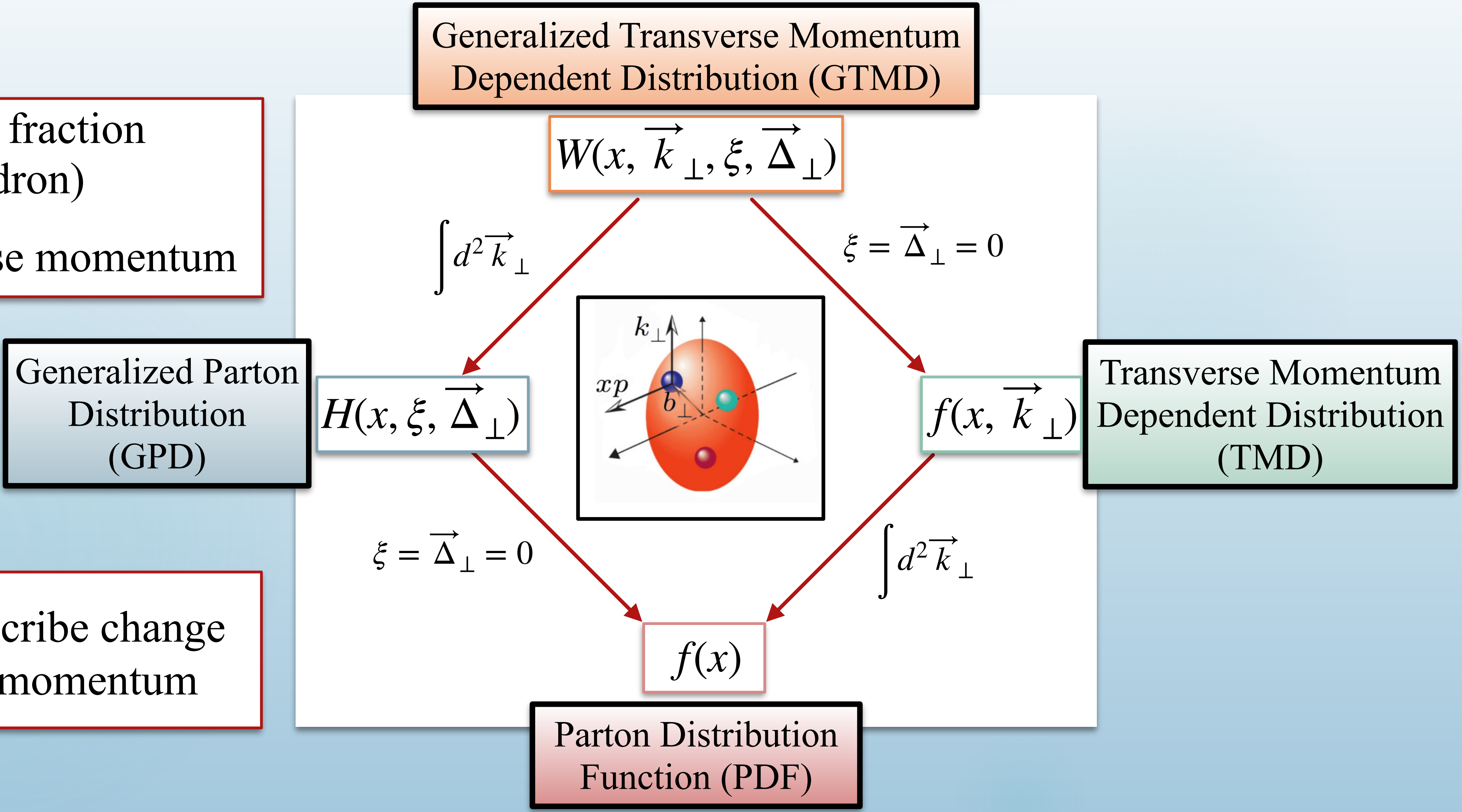

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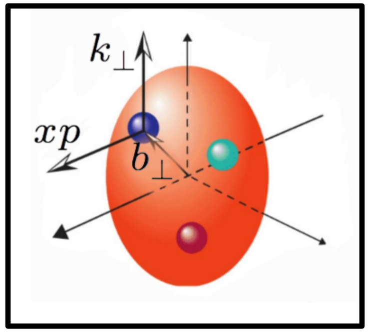

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$f(x)$

Parton Distribution Function (PDF)

ξ and $\vec{\Delta}_\perp$ describe change in hadron’s momentum

Energy scale μ

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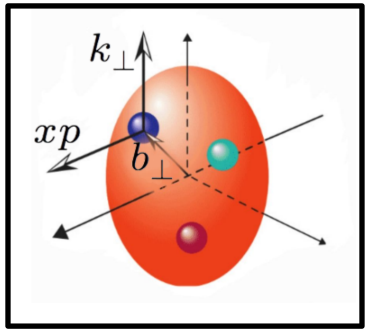

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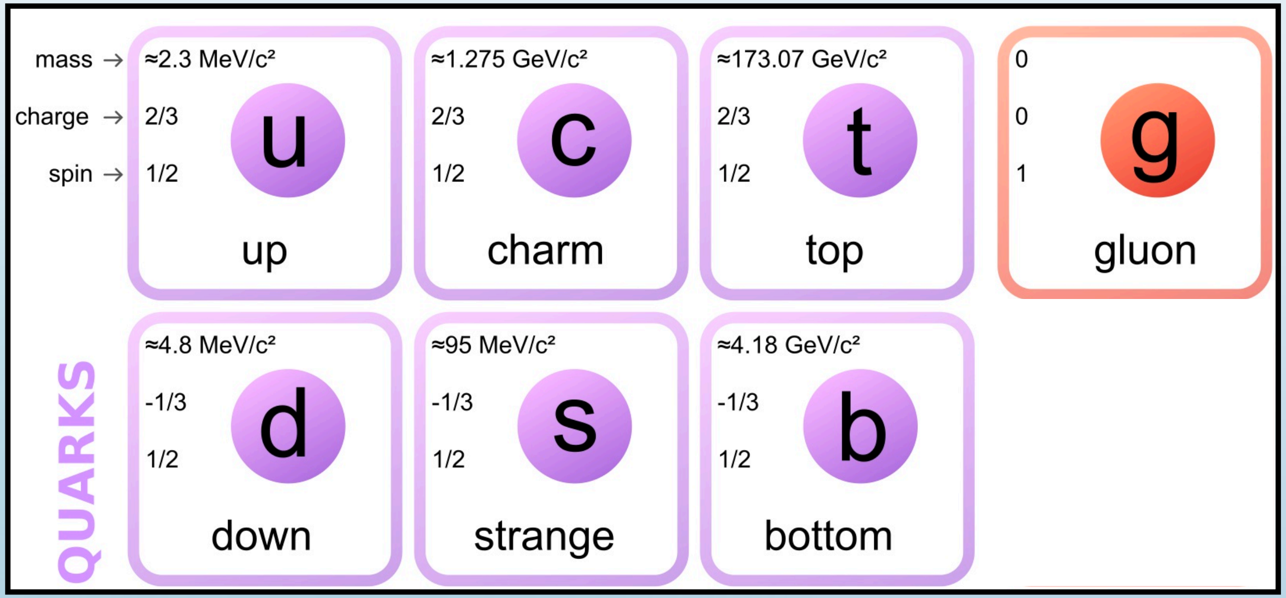
Parton Distribution Functions

PDFs describe the 1-D momentum distributions of quarks, antiquarks, and gluons within a hadron



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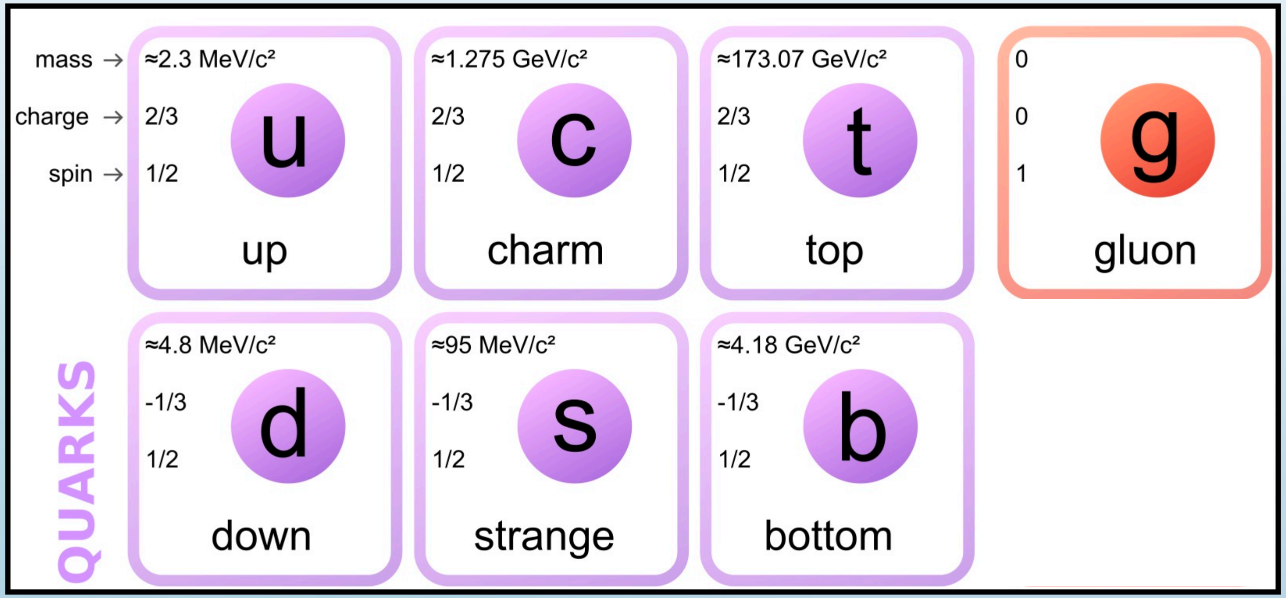

Hadron Spin (relative to momentum)	PDF
Averaged	Unpolarized
Parallel	Helicity
Transverse	Transversity

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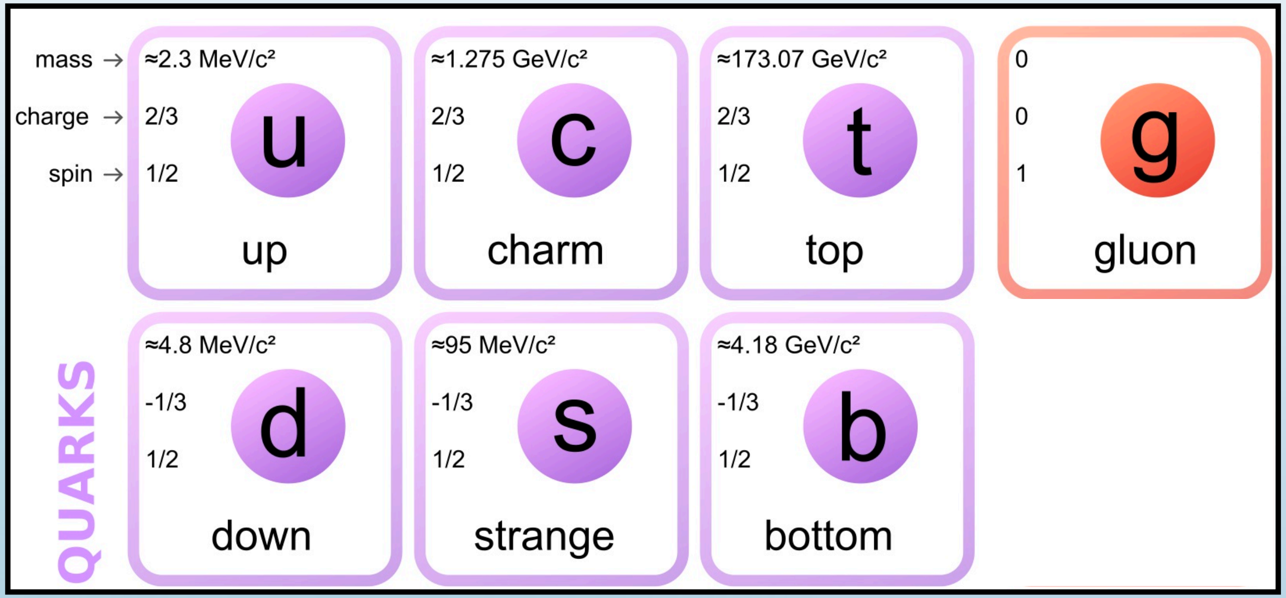

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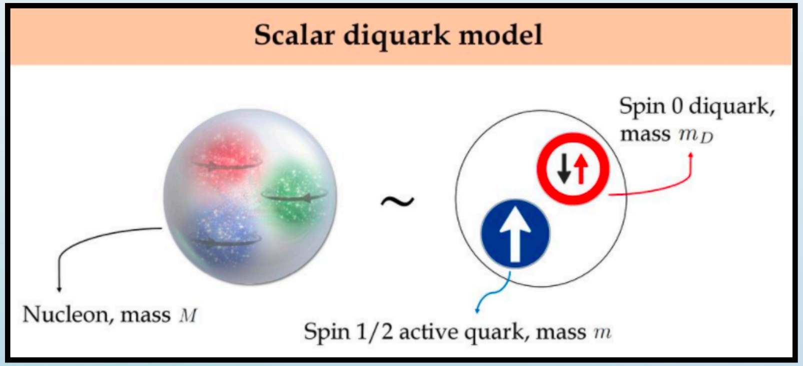
The Question: How do we gain information on partonic functions?

How do we gain information on partonic functions?

Model Calculations



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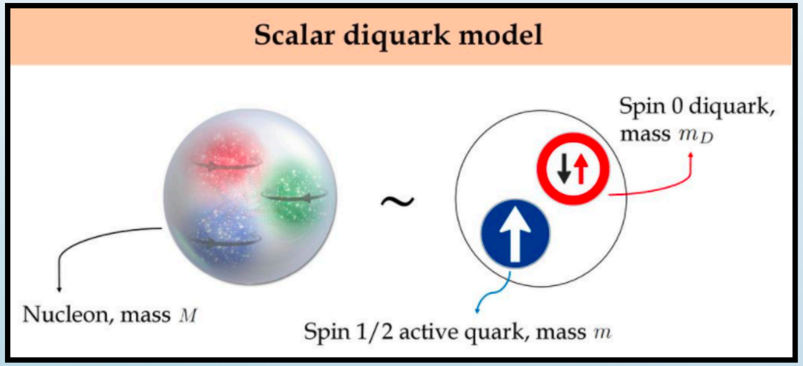

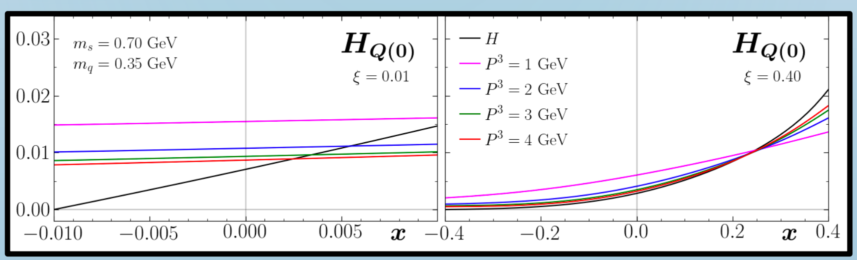

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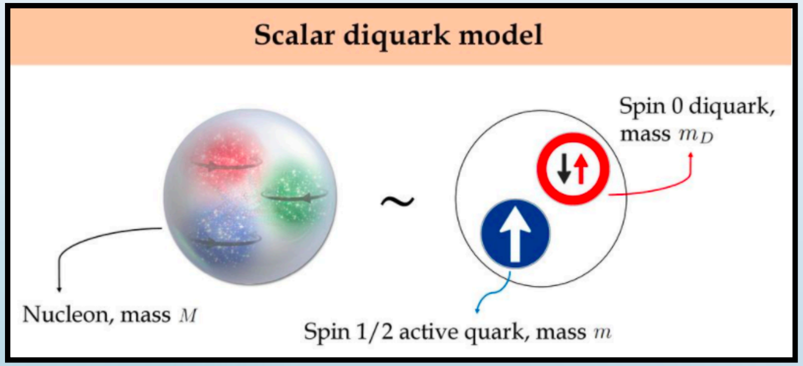
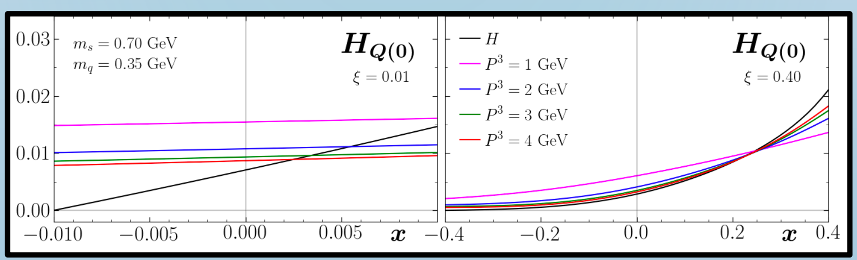
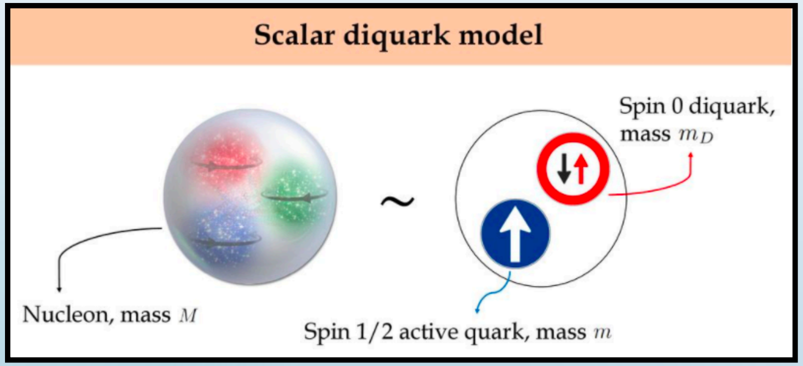


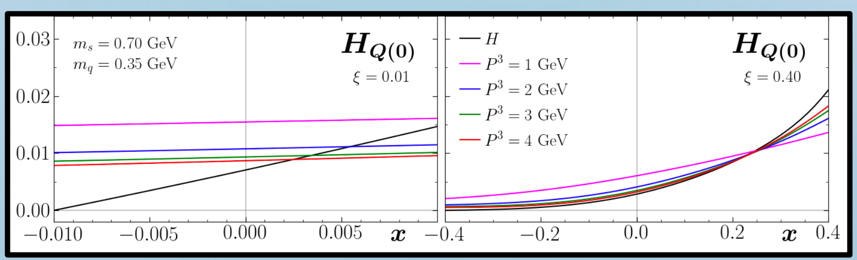


Global QCD Analysis

How do we gain information on partonic functions?

Model Calculations





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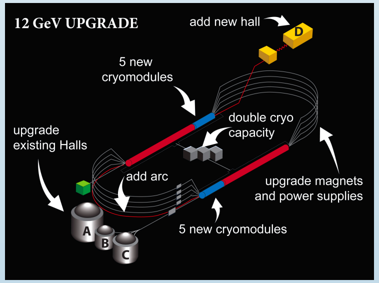
$$\sigma = \sum_{ij} H_{ij} \otimes f_i \otimes f_j$$



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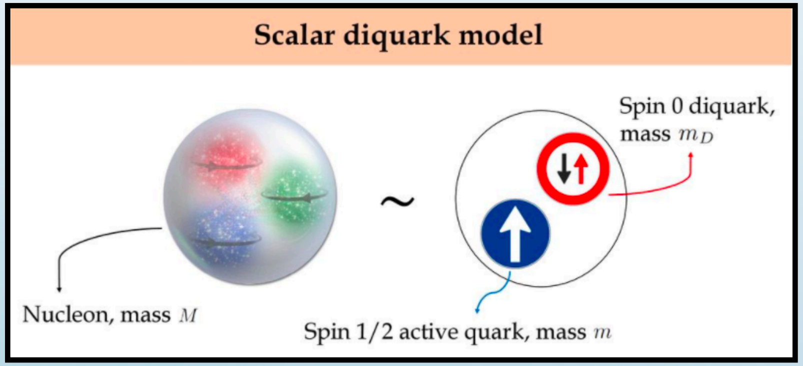

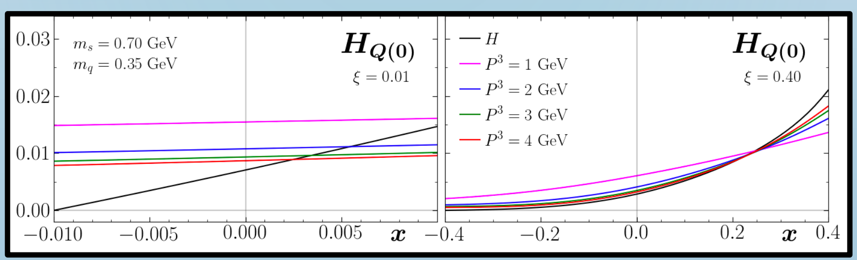

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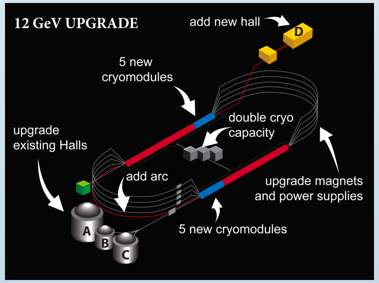

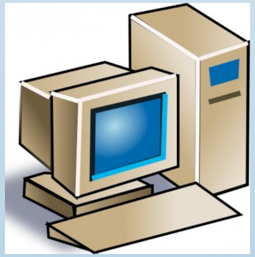

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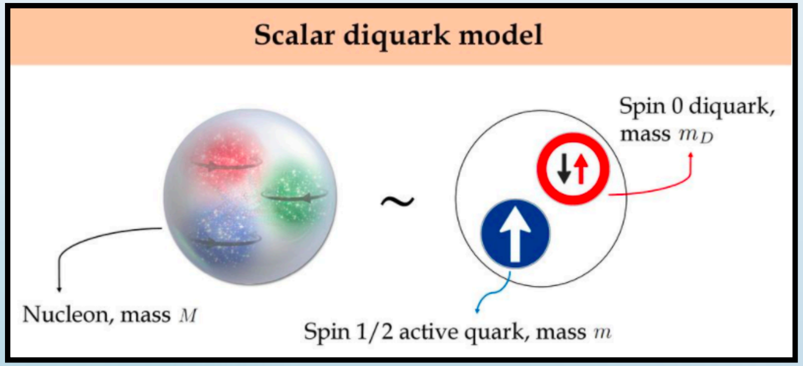

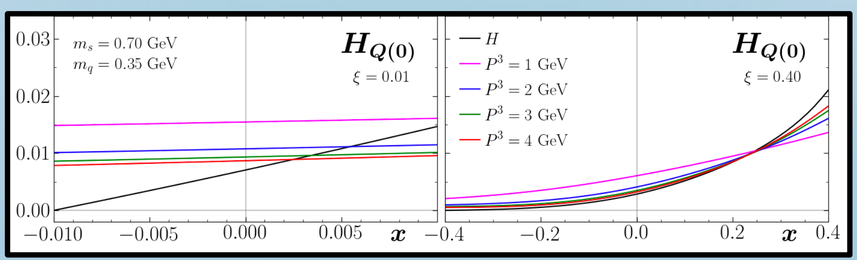

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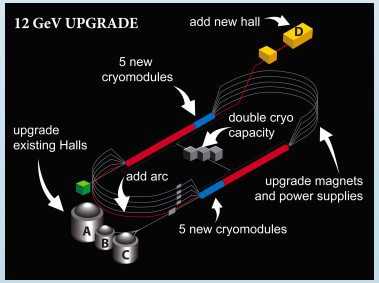

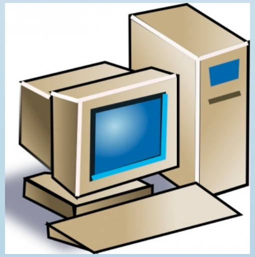

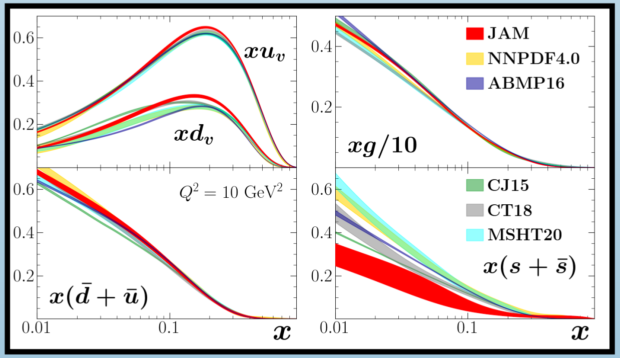

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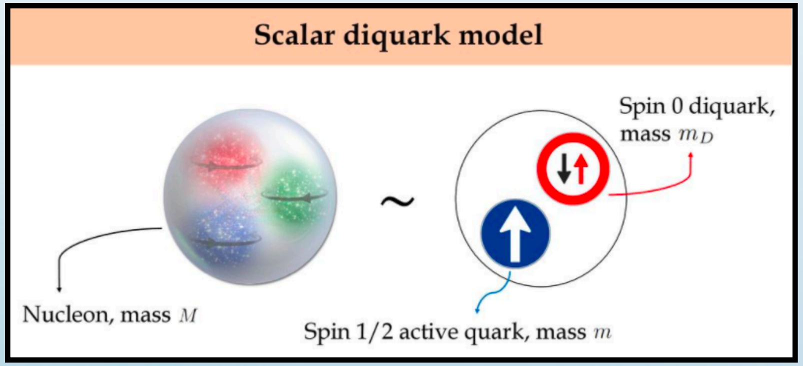

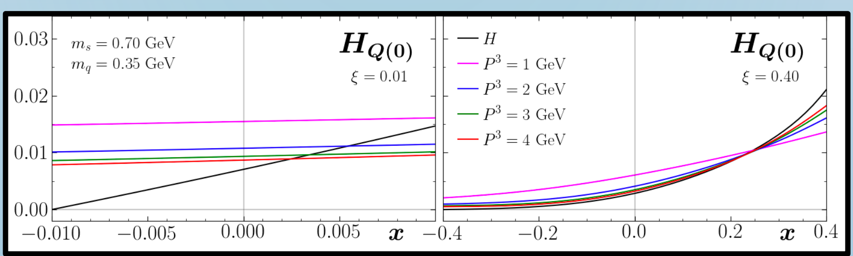

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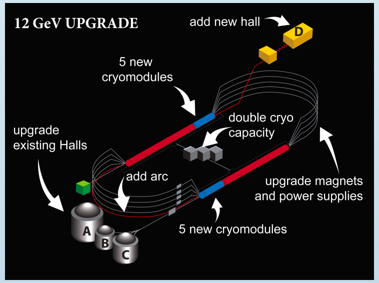

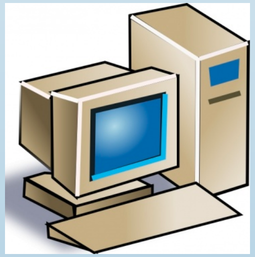

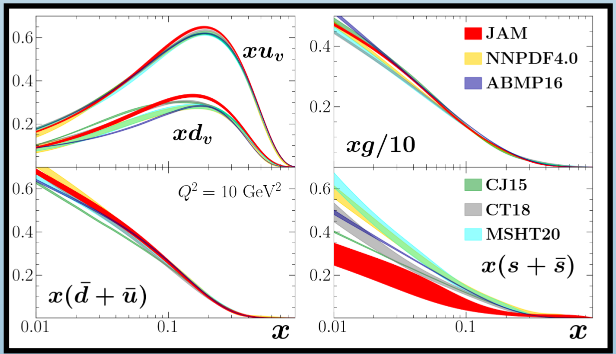

Lattice QCD

How do we gain information on partonic functions?

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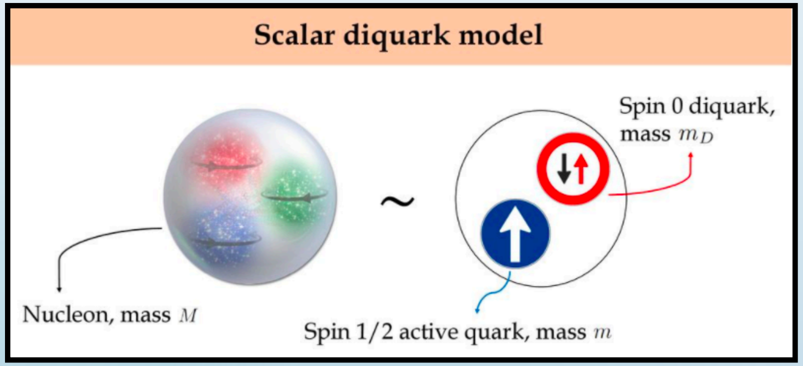

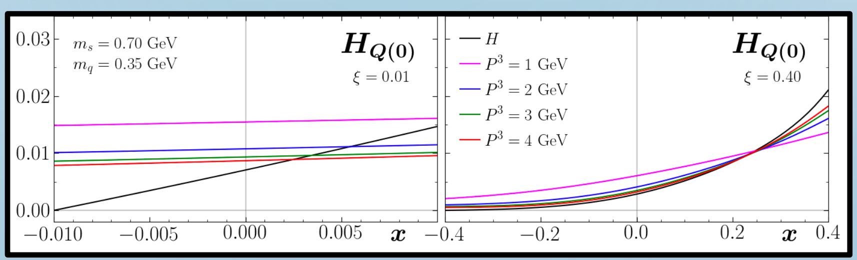

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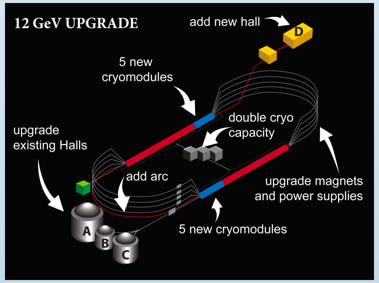
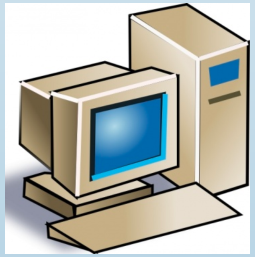
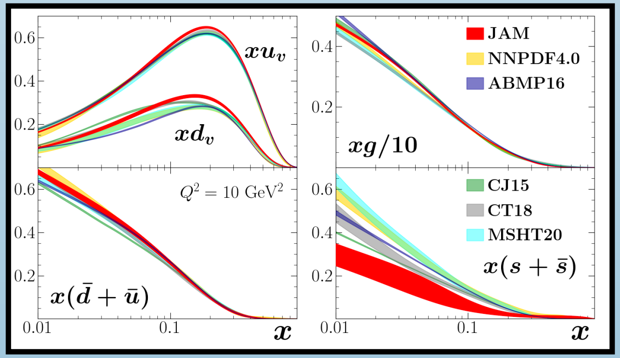




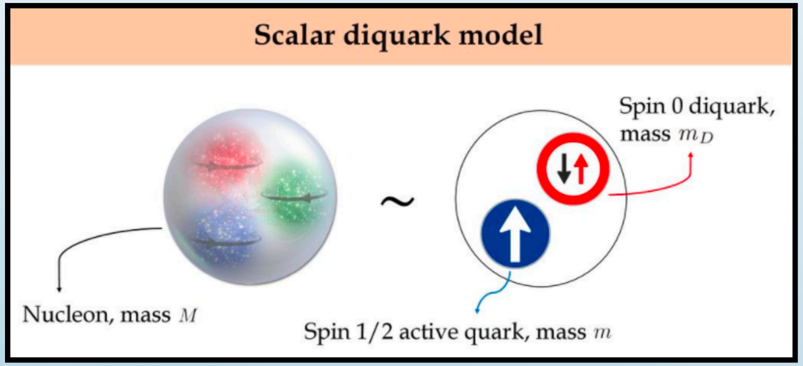


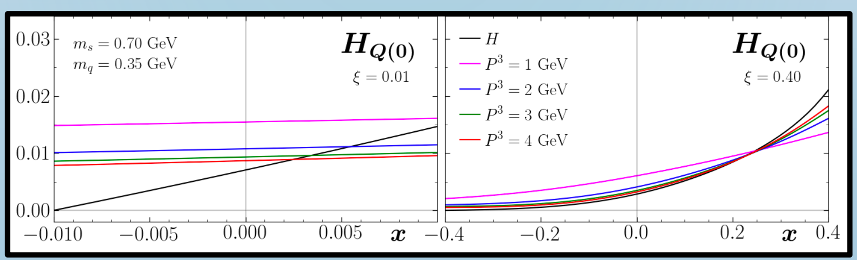
Lattice QCD

$$h_\Gamma^\Upsilon(z, P_3) = \langle N(P_3)|\overline{\psi}(z)\Upsilon\Gamma W(z)\psi(0)|N(P_3)\rangle$$

How do we gain information on partonic functions?

Model Calculations





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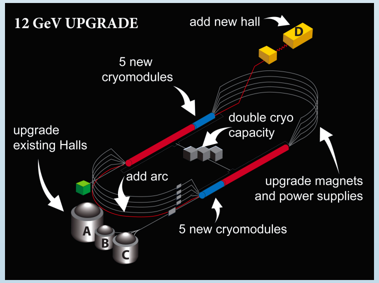

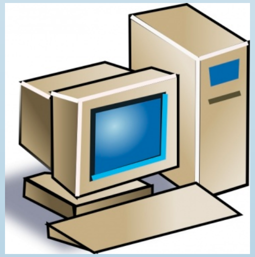

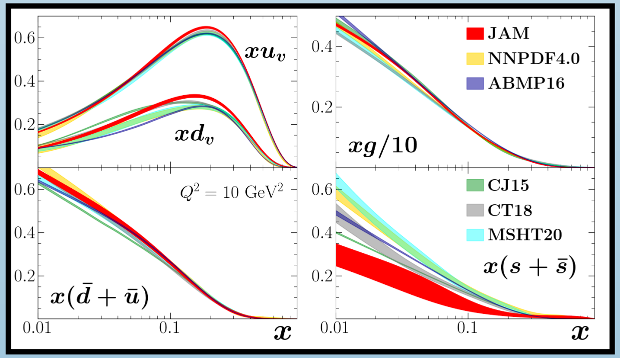

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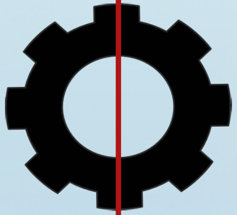
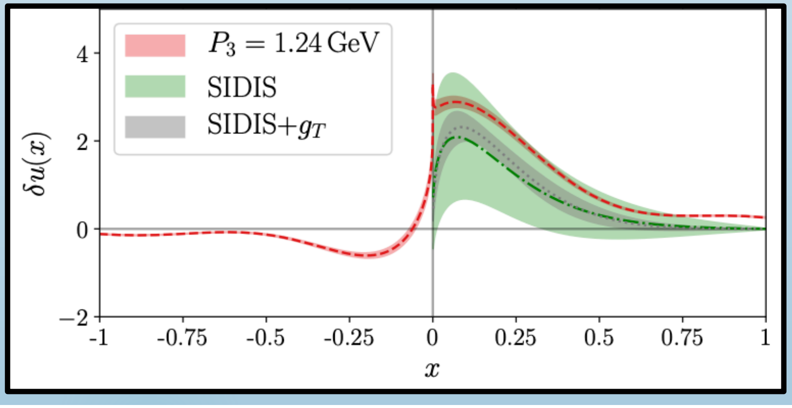
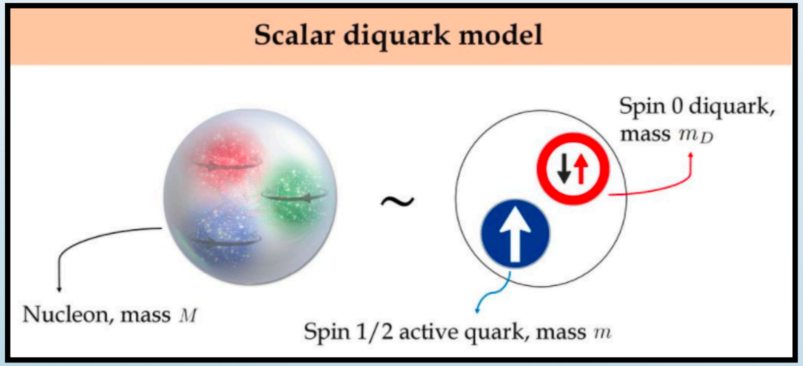


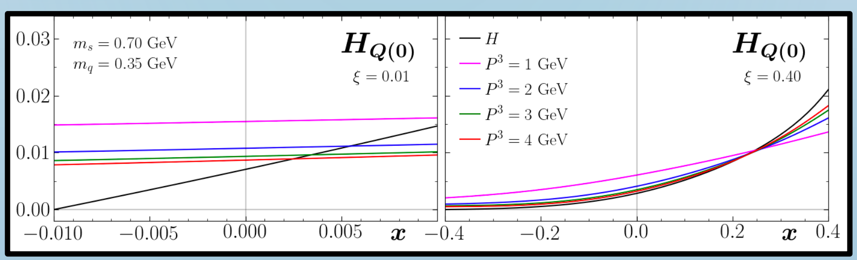


C. Alexandrou *et al.*, Phys. Rev. D **104**, no. 5, 054503 (2021)

How do we gain information on partonic functions?

Model Calculations





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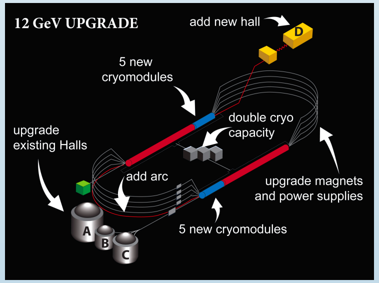
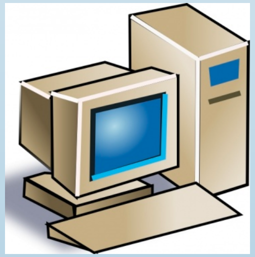
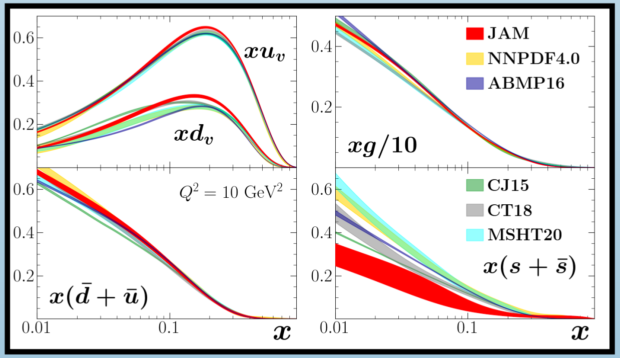


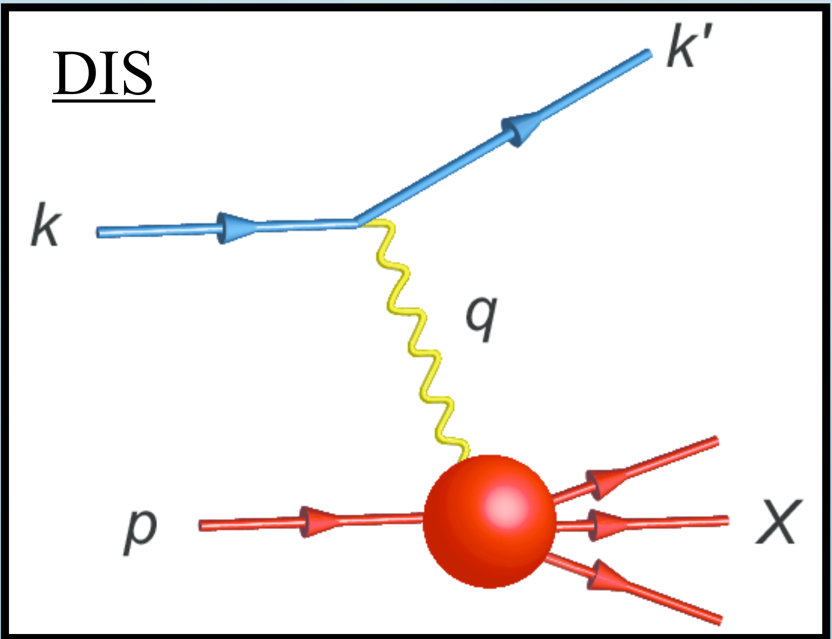


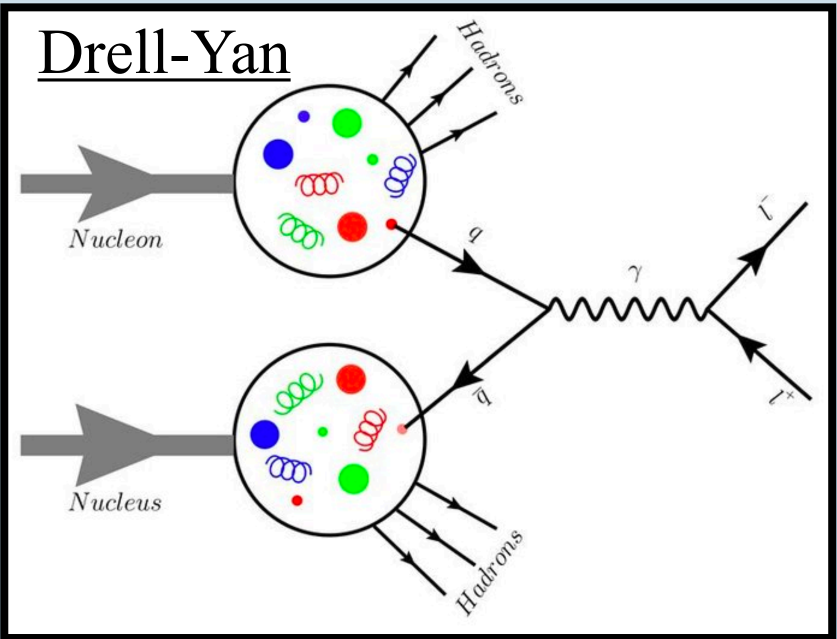


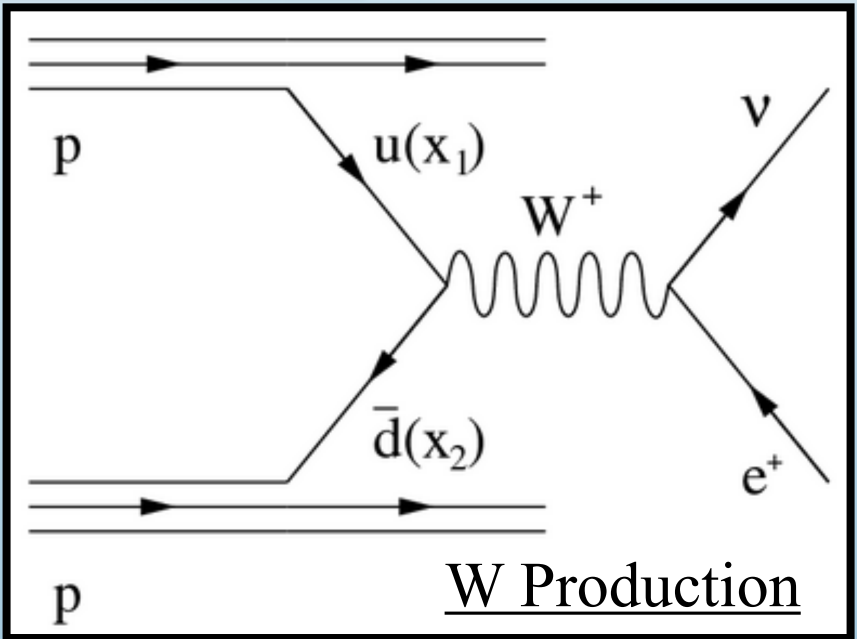
A Global Analysis

Jefferson Lab Angular Momentum Collaboration (JAM)

Functions are extracted using many different processes







Factorization

$$\sigma = \sum_{ij} H_{ij} \otimes f_i \otimes f_j + \mathcal{O}(1/Q)$$

Factorization

Experimentally measured cross-section

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"Hard part" (process dependent)
Cross-section at parton level
Calculated in perturbative QCD

Factorization

Experimentally measured cross-section

"Soft part" (process independent)
Describes internal structure

$$\sigma = \sum_{ij} H_{ij} \otimes f_i \otimes f_j + \mathcal{O}(1/Q)$$

"Hard part" (process dependent)
Cross-section at parton level
Calculated in perturbative QCD

How do global QCD analyses work?

Parameterize PDFs at input scale $Q_0^2 = m_c^2$

$$f_i(x) = N x^\alpha (1-x)^\beta (1 + \gamma\sqrt{x} + \eta x)$$

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Evolve PDFs using DGLAP

$$\frac{\mathrm{d}}{\mathrm{d}\ \ln(\mu^2)} f_i(x,\mu) = \sum_j \int_x^1 \frac{dz}{z} P_{ij}(z,\mu) f_j(\frac{x}{z},\mu)$$

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Calculate Observables

$$d\sigma^{pp} = \sum_{ij} H_{ij}^{pp} \otimes f_i \otimes f_j$$

The χ^2 function

Now that the observables have been calculated…

$$\chi^2(\boldsymbol{a}) = \sum_{i,e} \left(\frac{d_{i,e} - \sum_k r_e^k \beta_{i,e}^k - T_{i,e}(\boldsymbol{a})/N_e}{\alpha_{i,e}} \right)^2 + \sum_k \left(r_e^k\right)^2 + \left(\frac{1 - N_e}{\delta N_e} \right)^2$$

The χ^2 function

Now that the observables have been calculated…

Data

$$\chi^2(\boldsymbol{a}) = \sum_{i,e}\left(\frac{d_{i,e} - \sum_k r_e^k \beta_{i,e}^k - T_{i,e}(\boldsymbol{a})/N_e}{\alpha_{i,e}}\right)^2 + \sum_k \left(r_e^k\right)^2 + \left(\frac{1-N_e}{\delta N_e}\right)^2$$

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Uncorrelated Uncertainties

Correlated Uncertainties

The χ^2 function

Now that the observables have been calculated…

Data

Theory

Normalization

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Uncorrelated Uncertainties

Correlated Uncertainties

Normalization Uncertainty

Bayes' Theorem

Now that we have calculated $\chi^2(\boldsymbol{a}, \text{data})$…

Likelihood Function

$$\mathcal{L}(\boldsymbol{a}, \text{data}) = \exp\left(-\frac{1}{2}\chi^2(\boldsymbol{a}, \text{data})\right)$$

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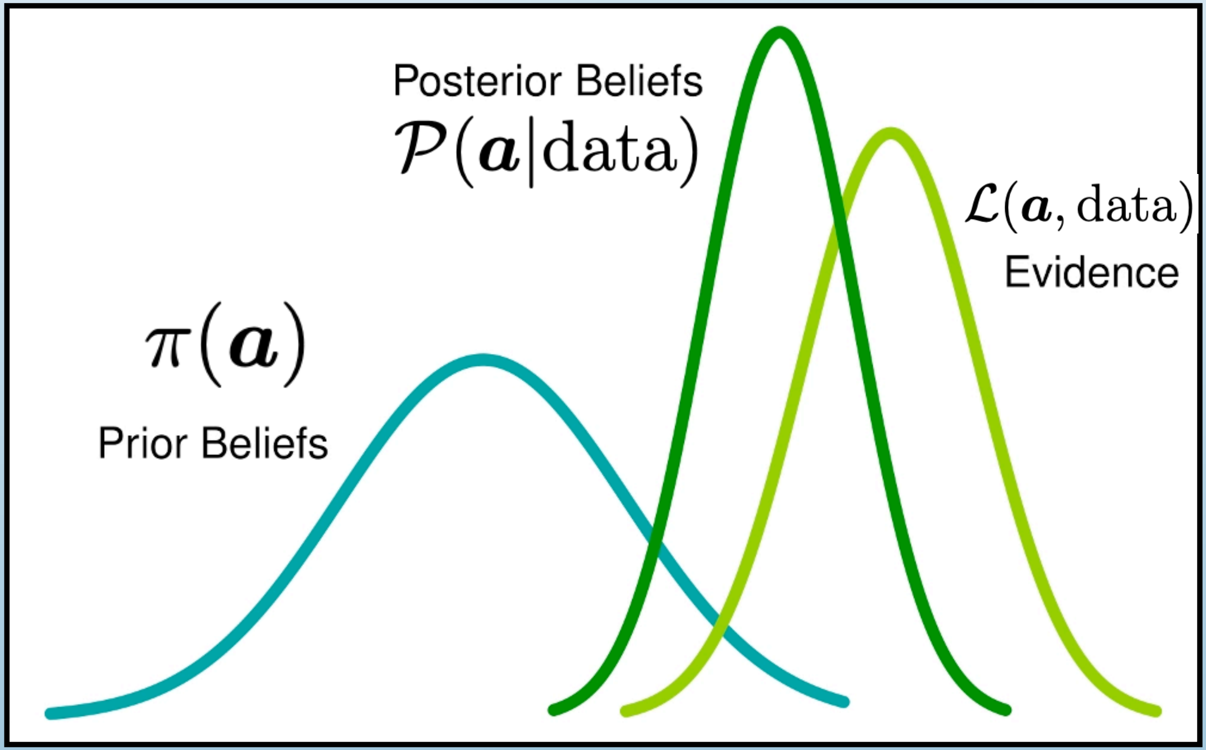
$$\mathcal{P}(\boldsymbol{a}|\text{data}) \sim \mathcal{L}(\boldsymbol{a}, \text{data})\,\pi(\boldsymbol{a})$$



Data Resampling

$$\tilde{\sigma} = \sigma + N(0,1)\ \alpha$$

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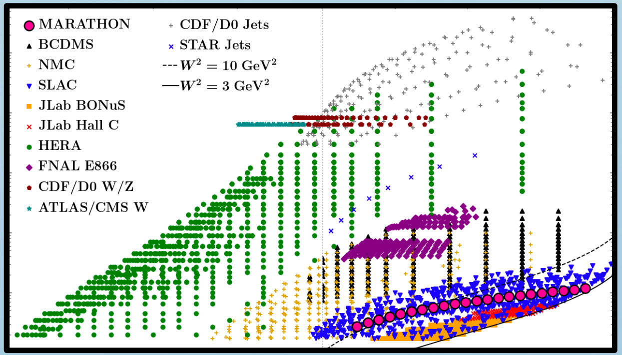
Original Data



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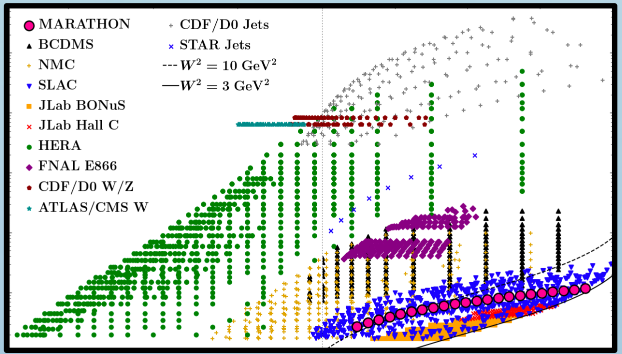

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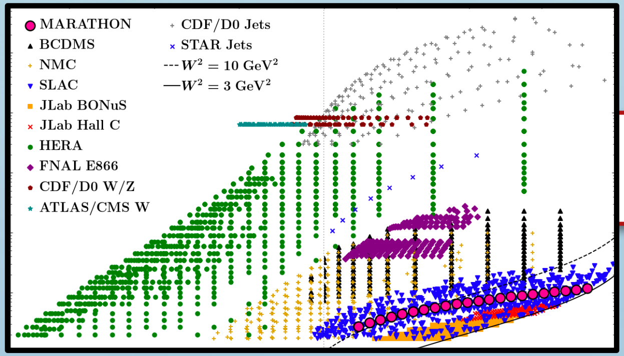
Pseudo-Data → $\tilde{\sigma}$

Data → σ

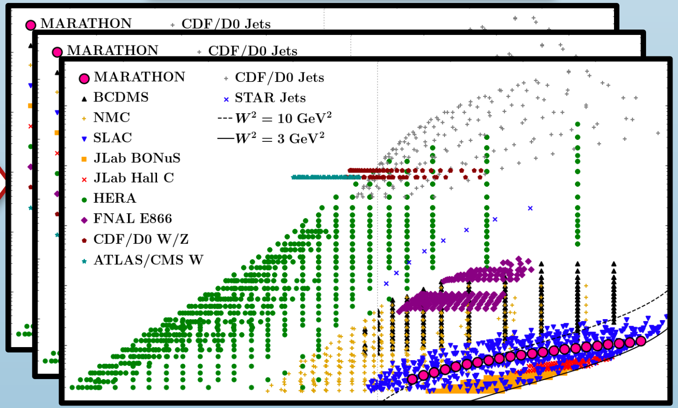
Uncorrelated Uncertainties → α

Original Data



DR

Replica Data



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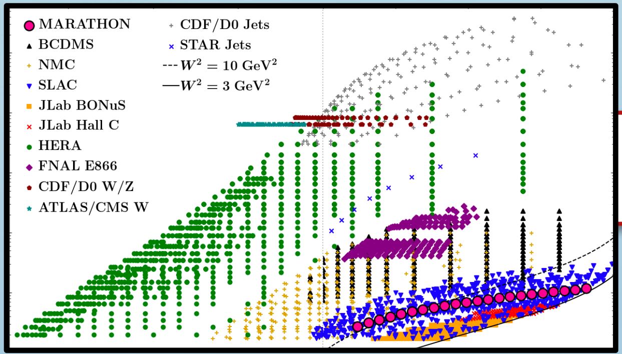
Uncorrelated Uncertainties

Data

Original Data



DR

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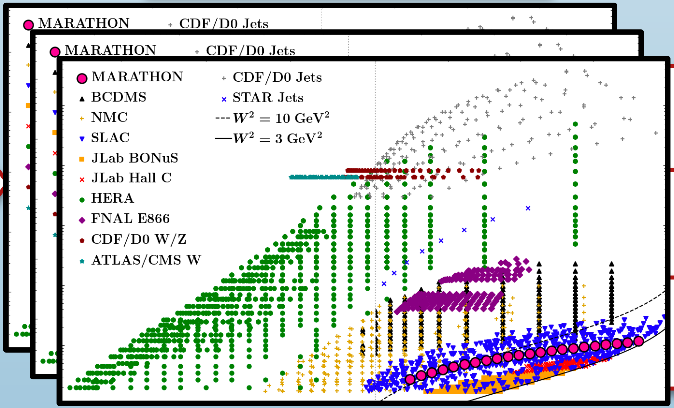
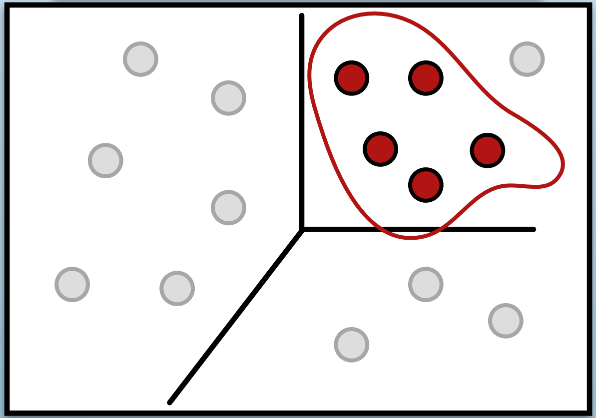


Maximum Likelihood

Maximum Likelihood

Maximum Likelihood

Parameter Space



Error Quantification

For a quantity $O(\boldsymbol{a})$: (for example, a PDF at a given value of (x, Q^2))

$$E[O] = \int d^n a \ \rho(\boldsymbol{a} \,|\, data) \ O(\boldsymbol{a})$$

$$V[O] = \int d^n a \ \rho(\boldsymbol{a} \,|\, data) \ \left[O(\boldsymbol{a}) - E[O]\right]^2$$

Exact, but
$n = \mathcal{O}(100)$!

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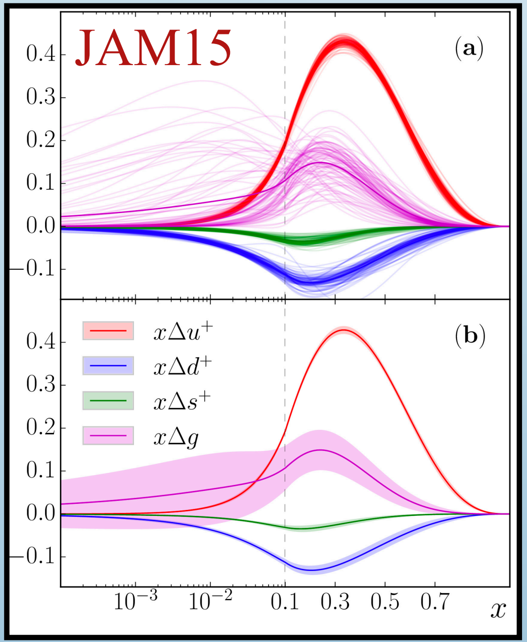
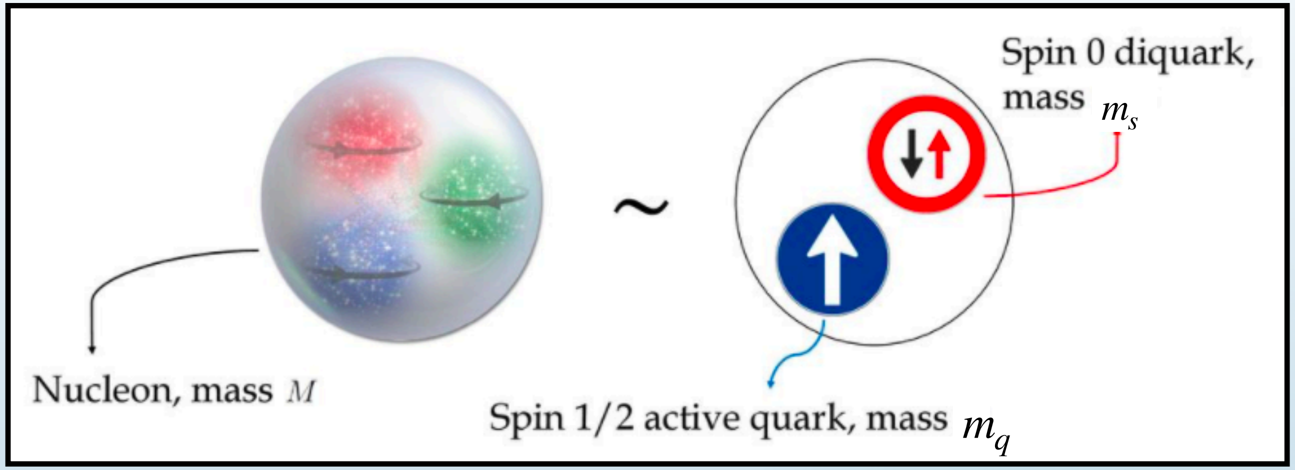


Scalar Diquark Model (SDM)

Model Parameters

$M = 0.939$ GeV
$m_q = 0.35$ GeV
$m_s = 0.70$ GeV



L. Gamberg, Z. B. Kang, I. Vitev, and H. Xing, Phys. Lett. B **743**, 112 (2015)

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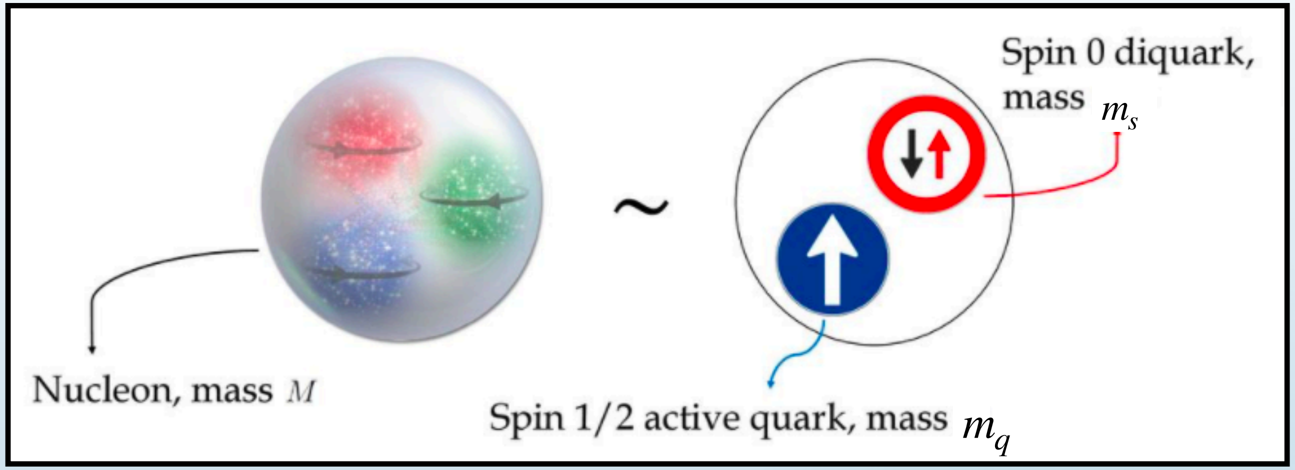

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General Parameters

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Cut-off for $|\vec{k}_\perp|$ integration: $\Lambda = 1$ GeV

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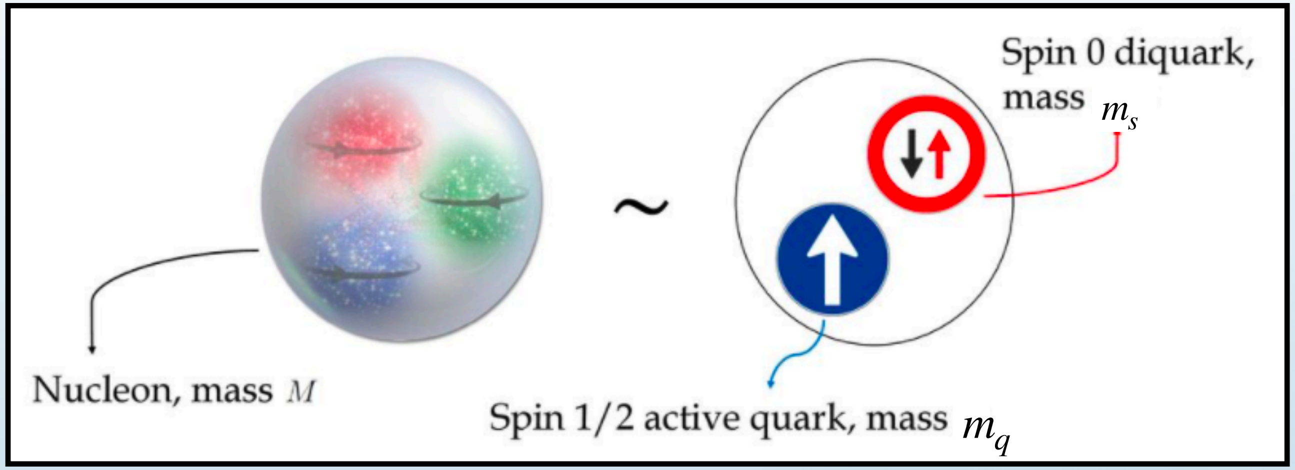

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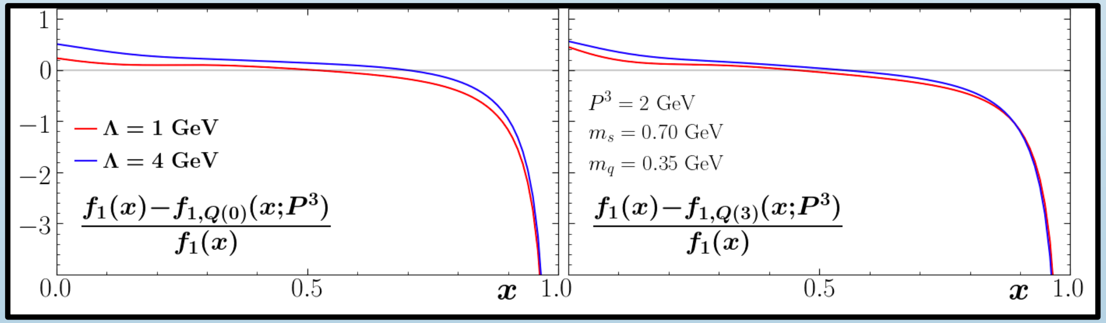
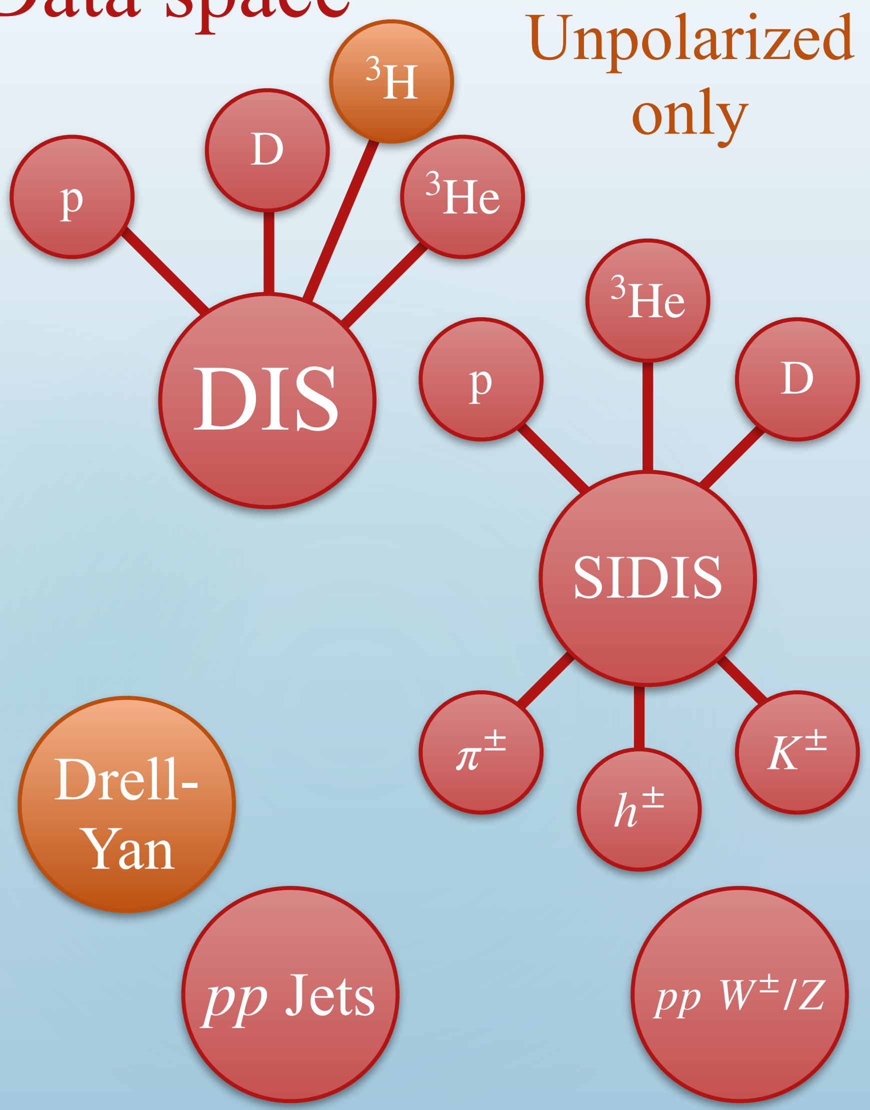


General conclusions hold regardless of parameter choices

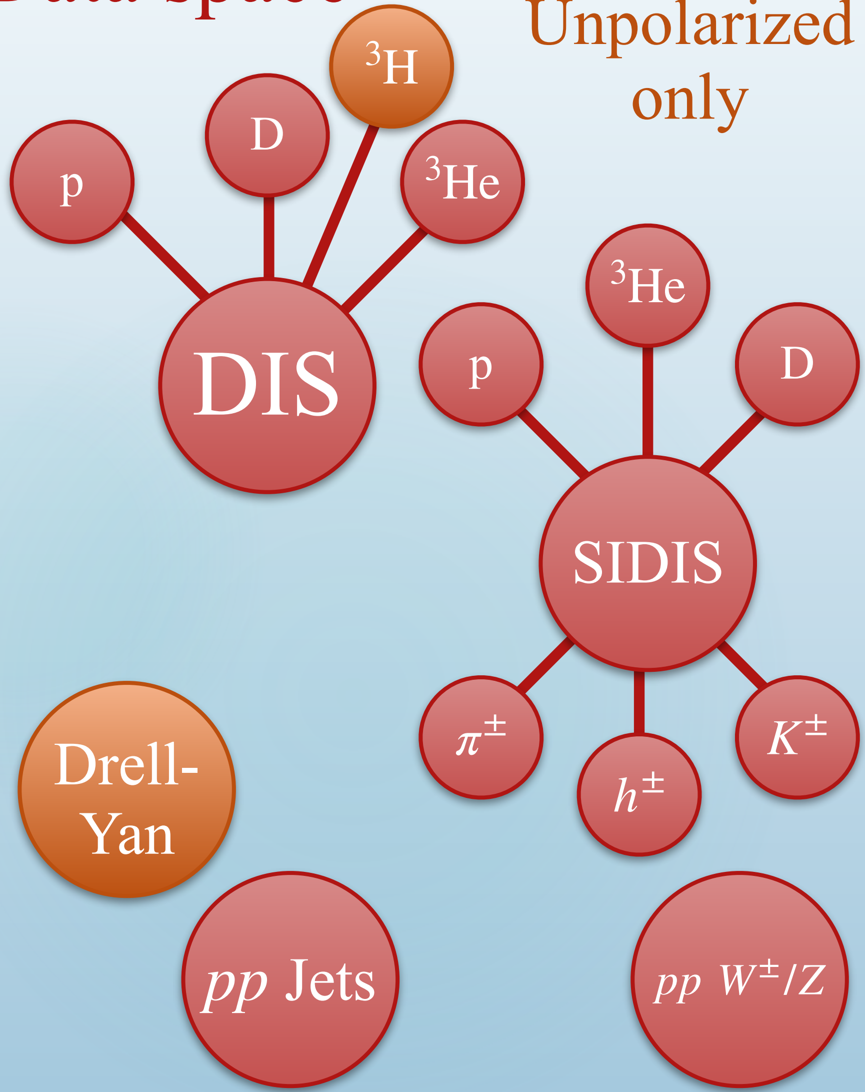
Data space



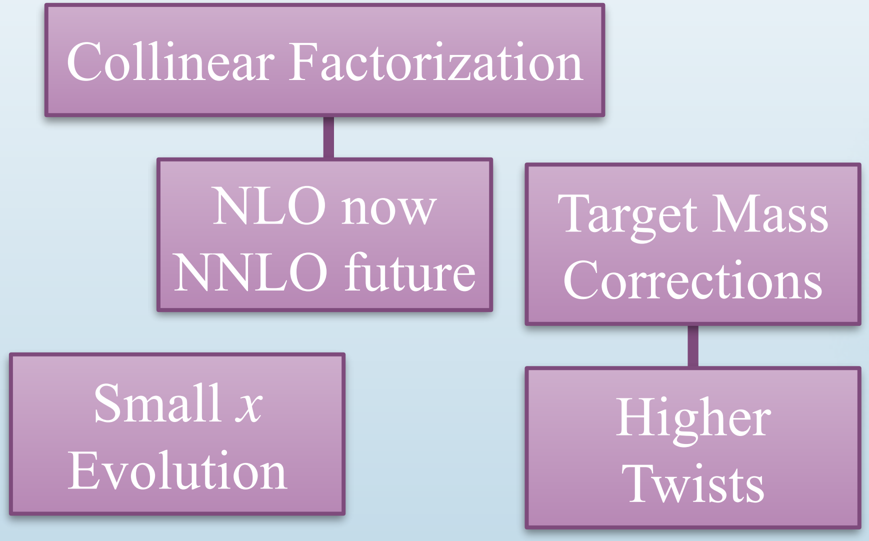
Unpolarized
only
p
D
3H
3He
DIS
3He
p
D
SIDIS
π±
h±
K±
Drell-
Yan
pp Jets
pp W±/Z

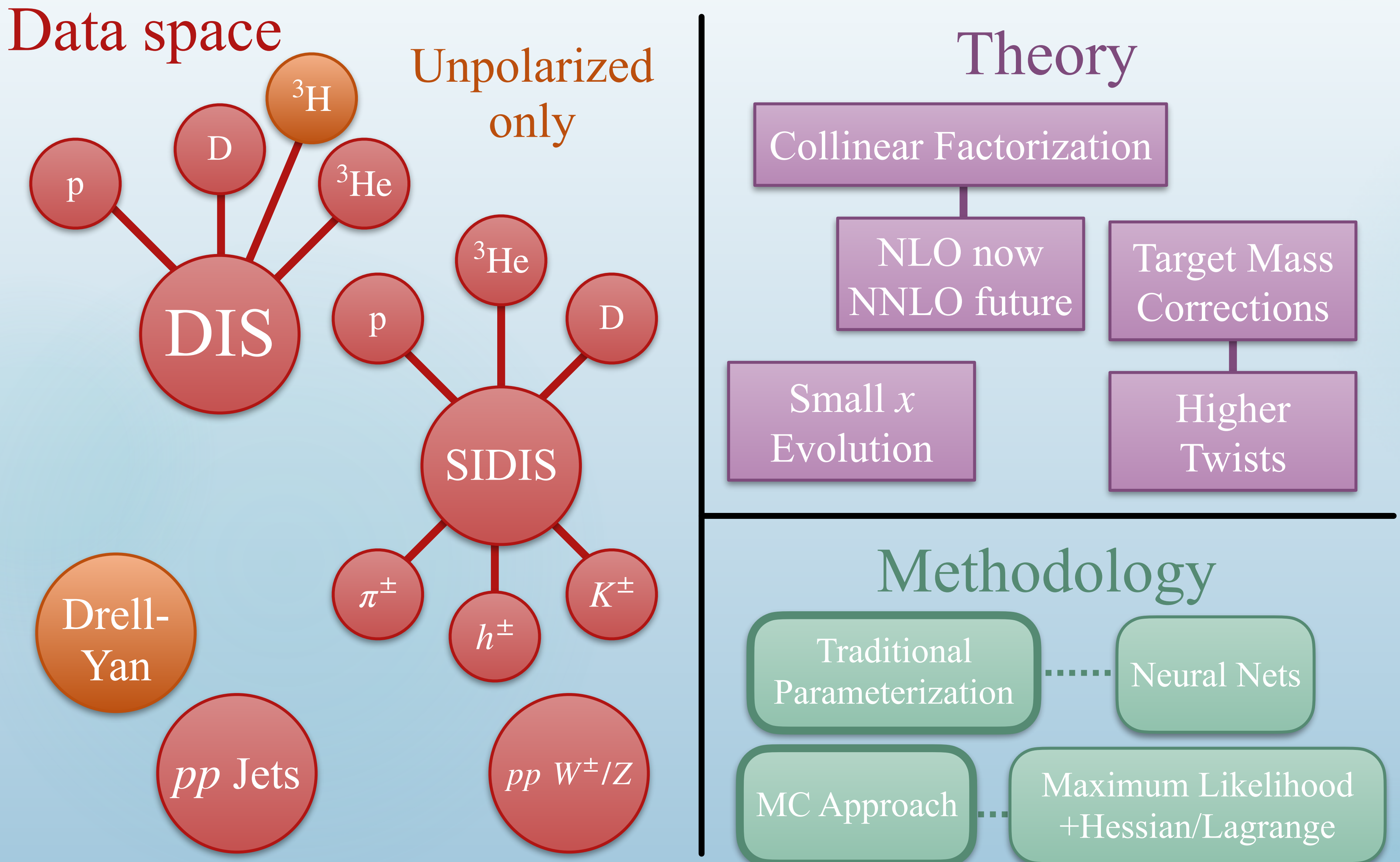
Data space

Unpolarized only



Theory

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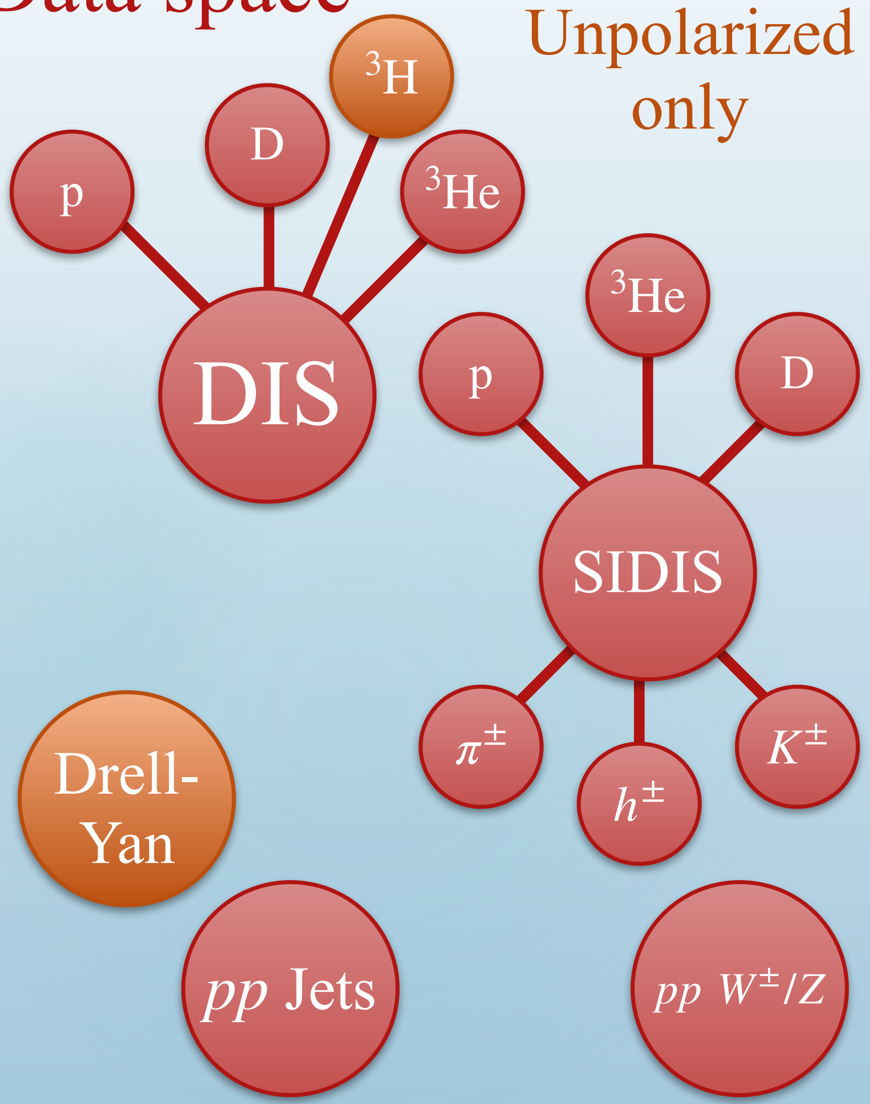
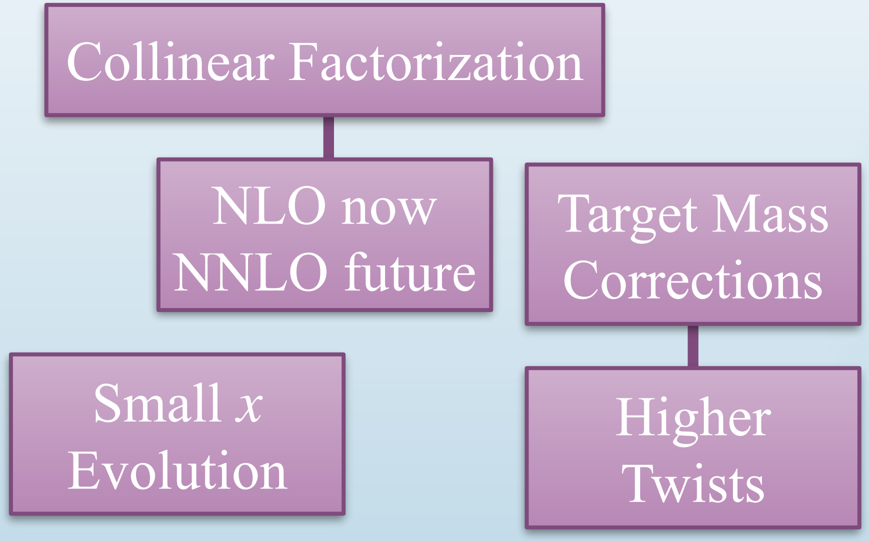
Theory
Collinear Factorization
NLO now NNLO future
Target Mass Corrections
Small x Evolution
Higher Twists
Methodology
Traditional Parameterization
Neural Nets
MC Approach
Maximum Likelihood +Hessian/Lagrange

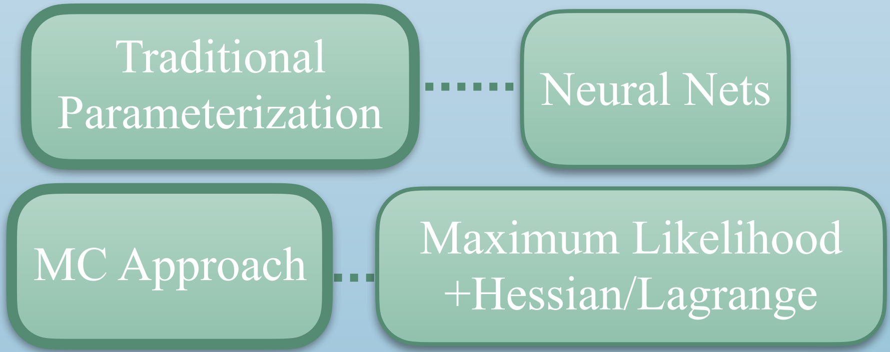
Data space



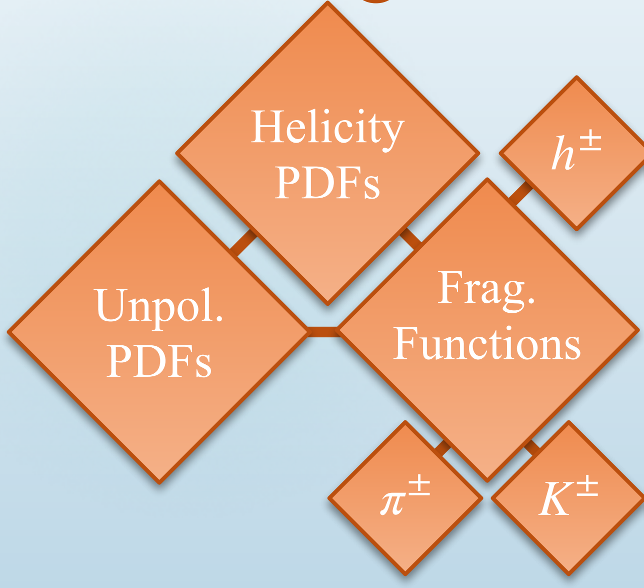
Theory

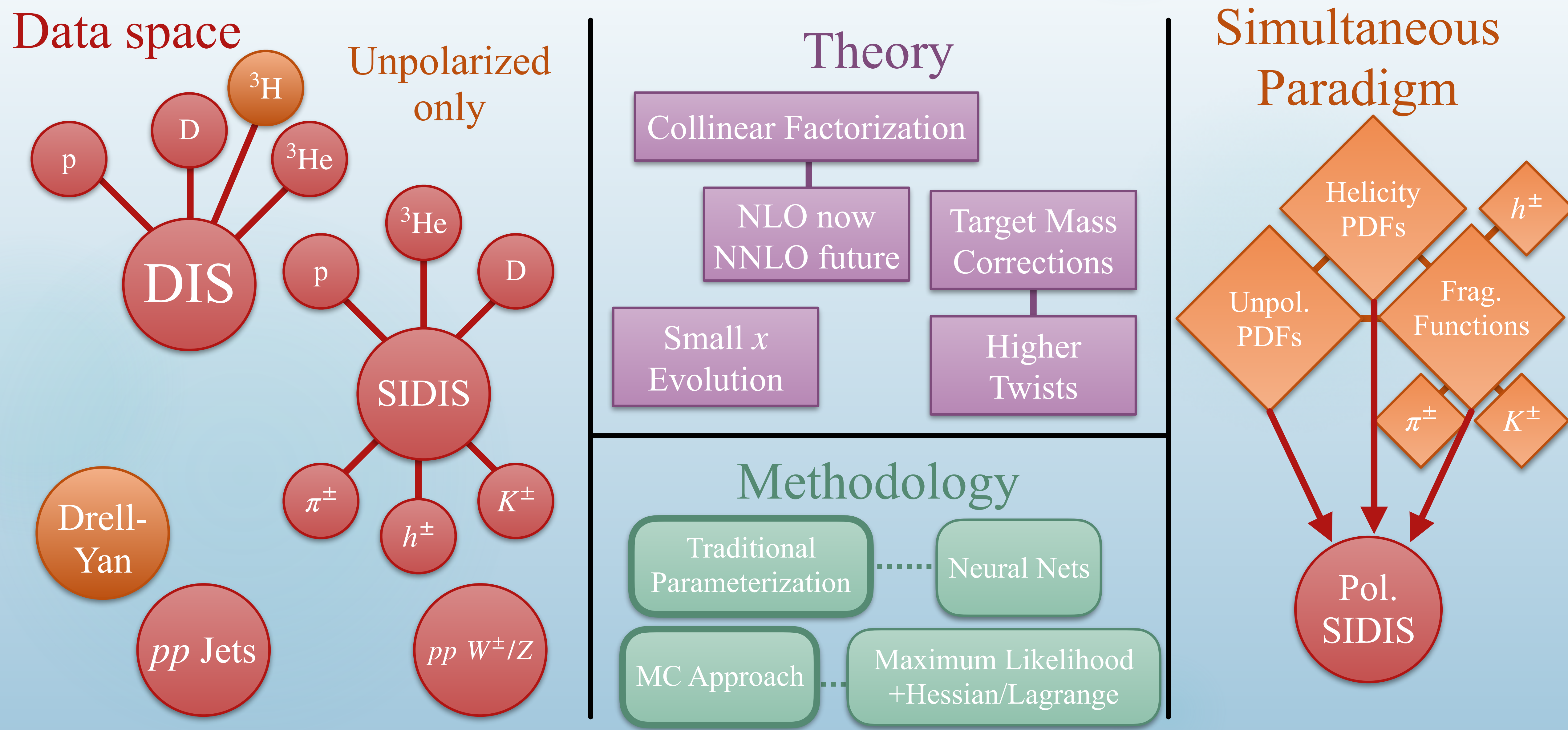


Methodology



Simultaneous Paradigm

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Unpol. PDFs
Frag. Functions
$\pi^\pm$
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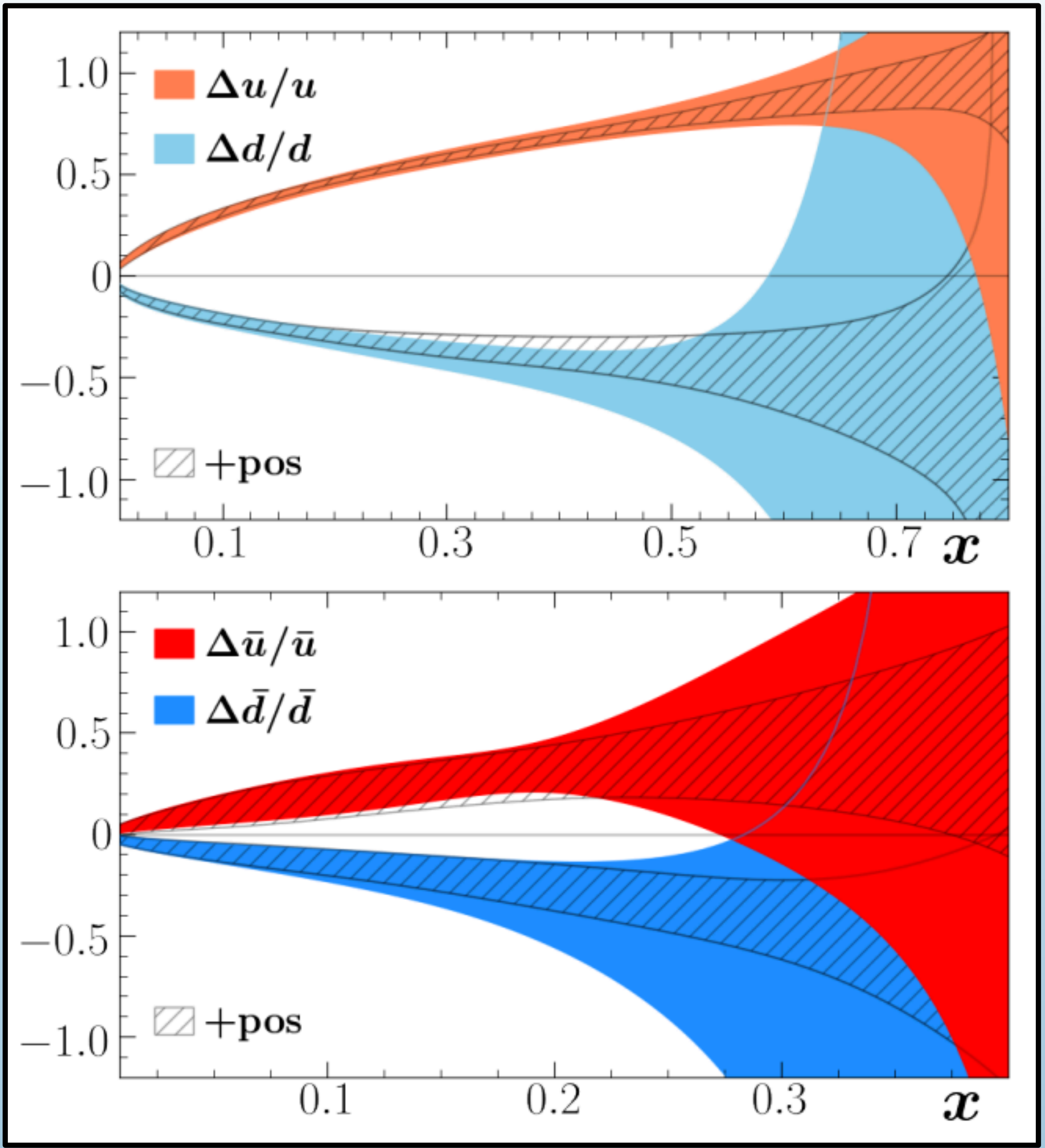
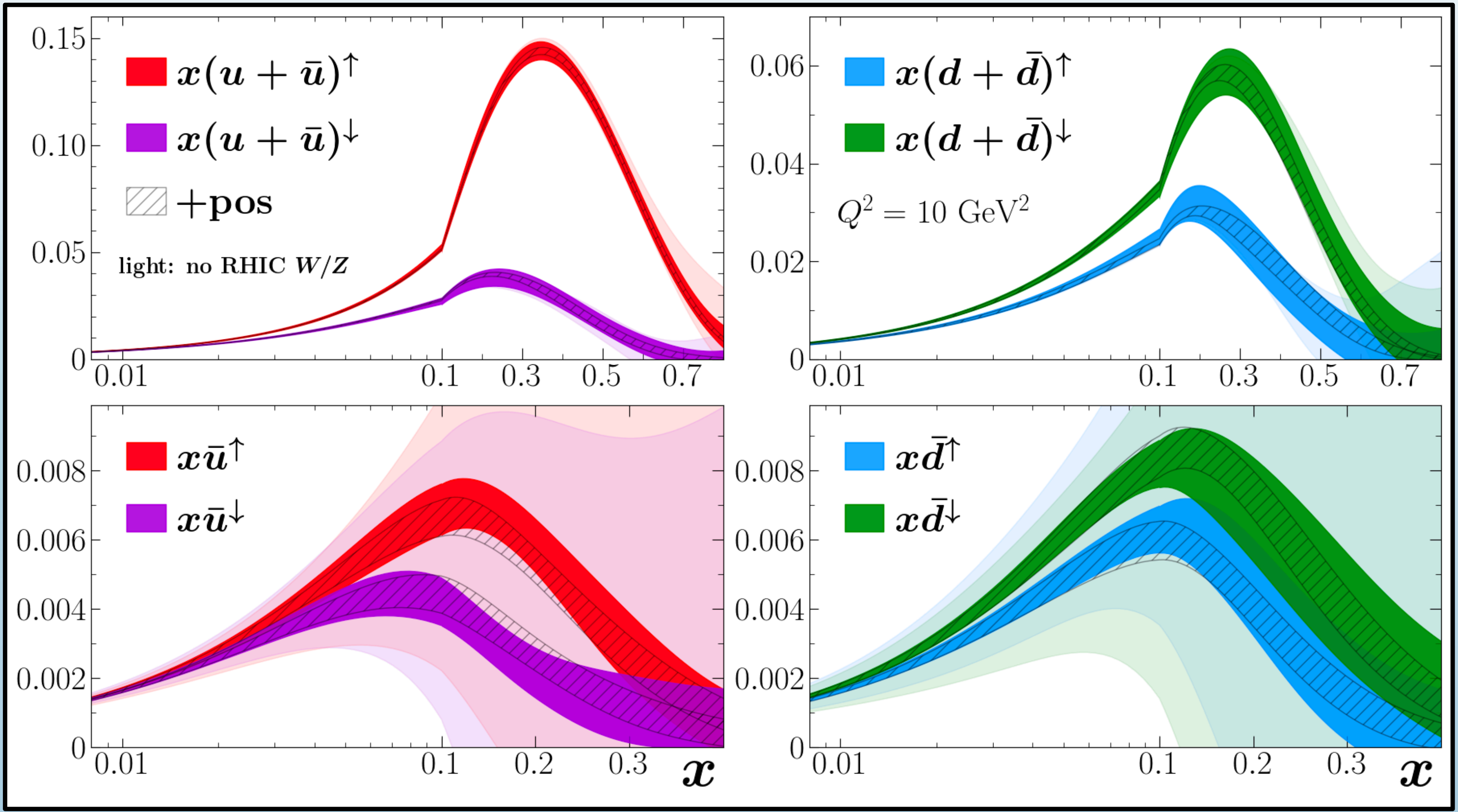
Pol. SIDIS

Quark and Antiquark Polarizations



Spin Up/Down PDFs



$$q^{\uparrow\downarrow} = \frac{1}{2}(q \pm \Delta q)$$

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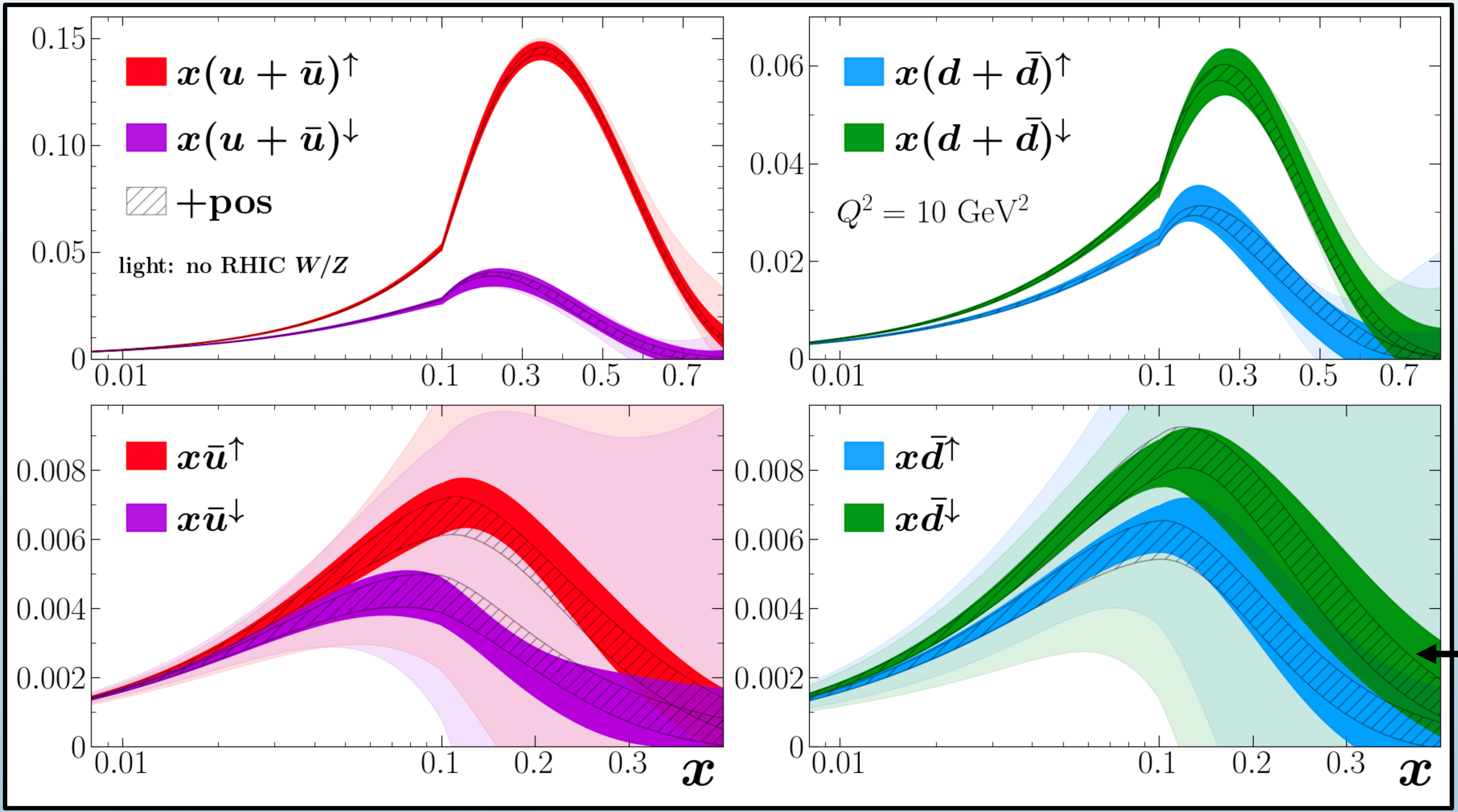

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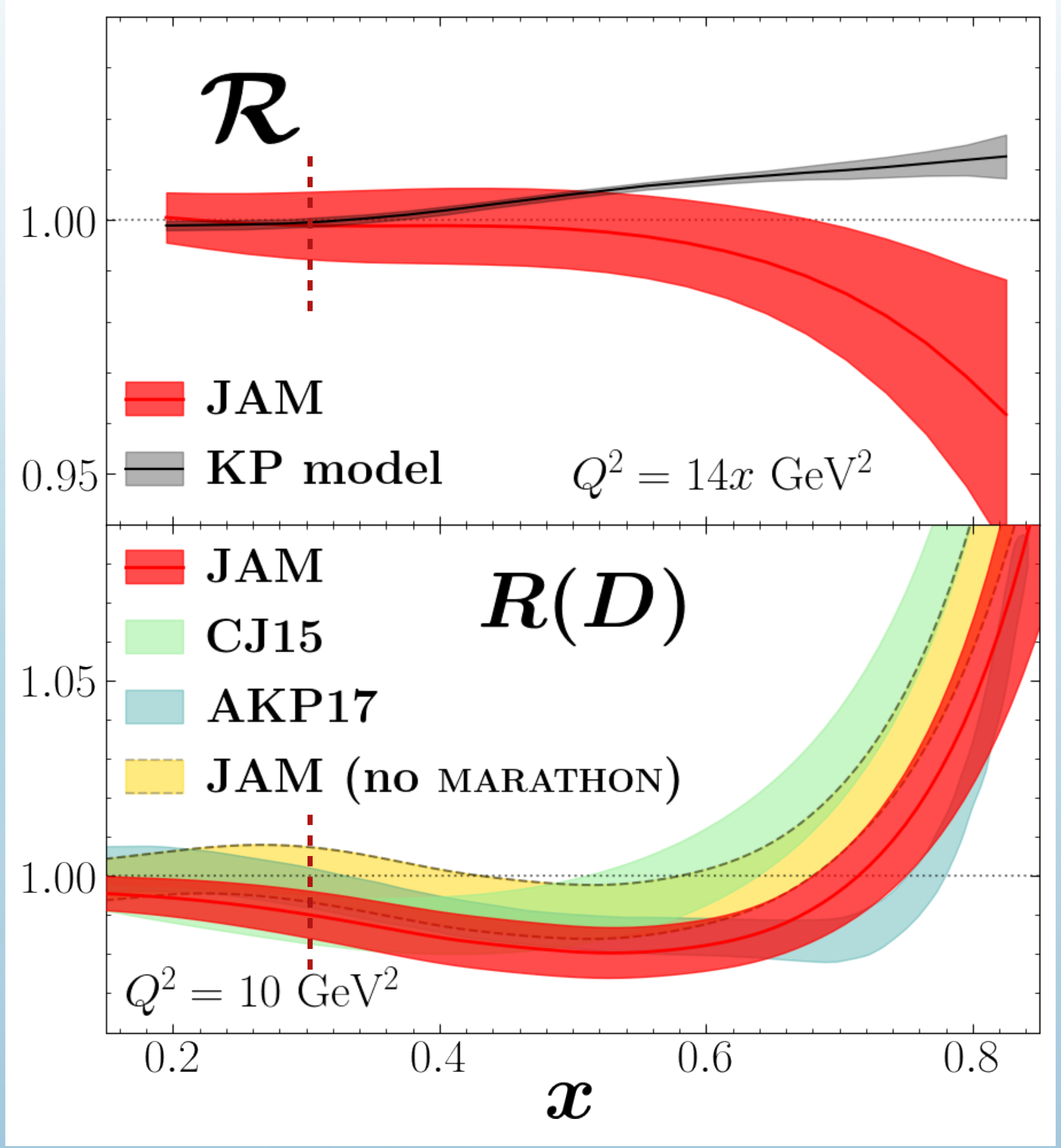
Large impact from RHIC

EMC Ratios



$$R(D) = F_2^D / \left(F_2^p + F_2^n\right)$$

$$R(^3\mathrm{He}) = F_2^{^3\mathrm{He}} / \left(2F_2^p + F_2^n\right)$$

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$$\mathscr{R} = R(^3\mathrm{He}) / R(^3\mathrm{H})$$

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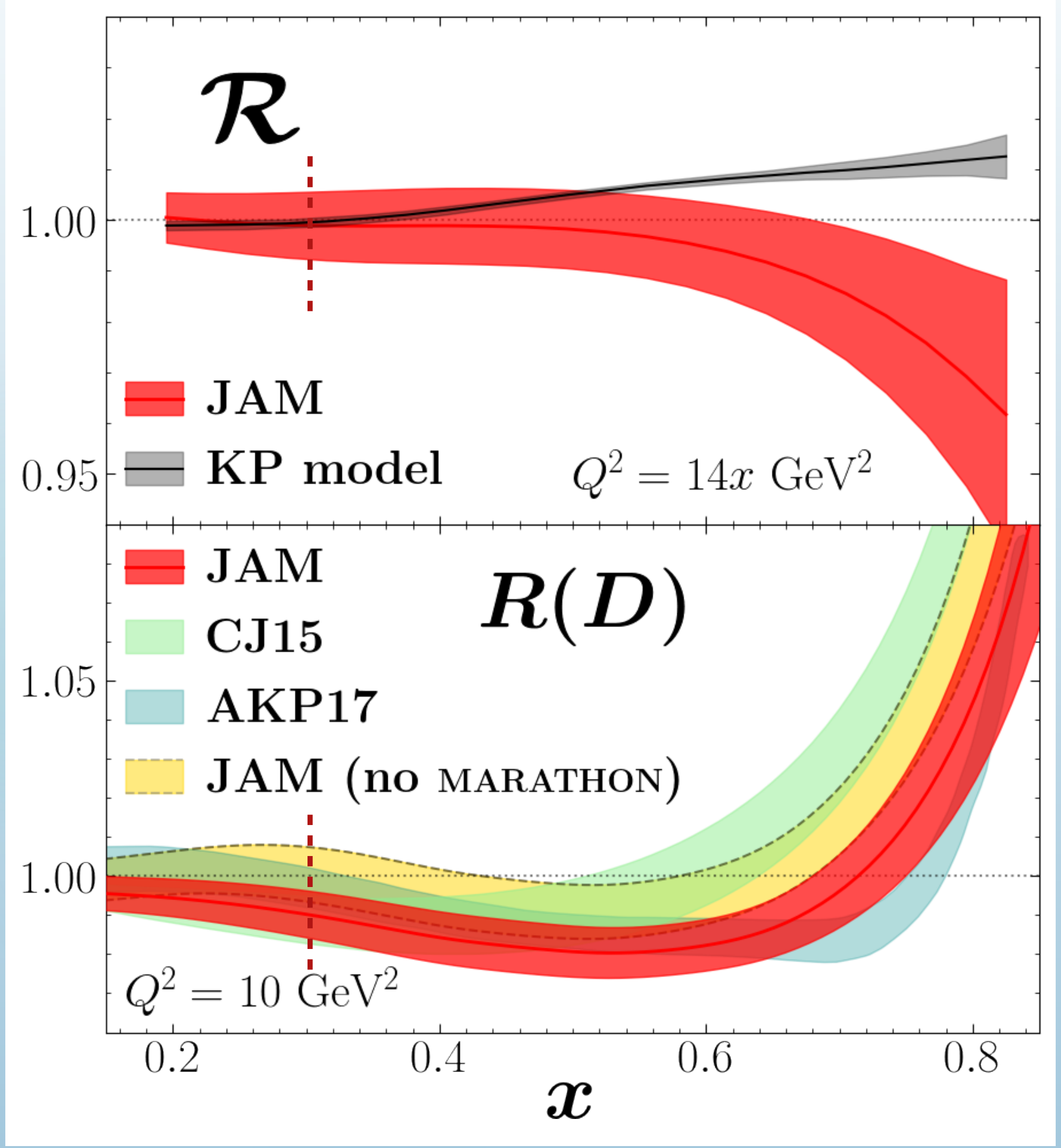

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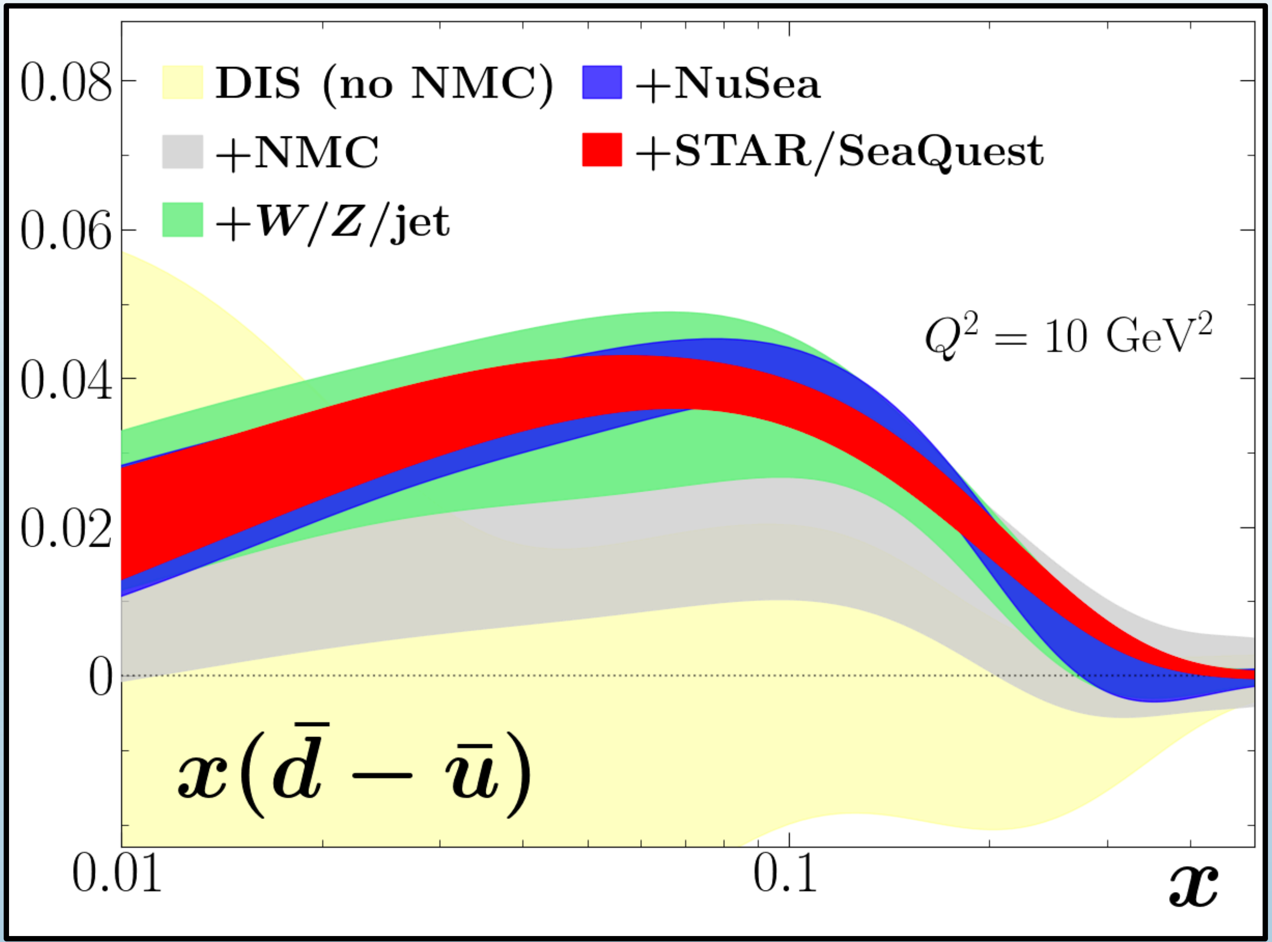
Significant differences between JAM result and KP model result

Sources of Asymmetry



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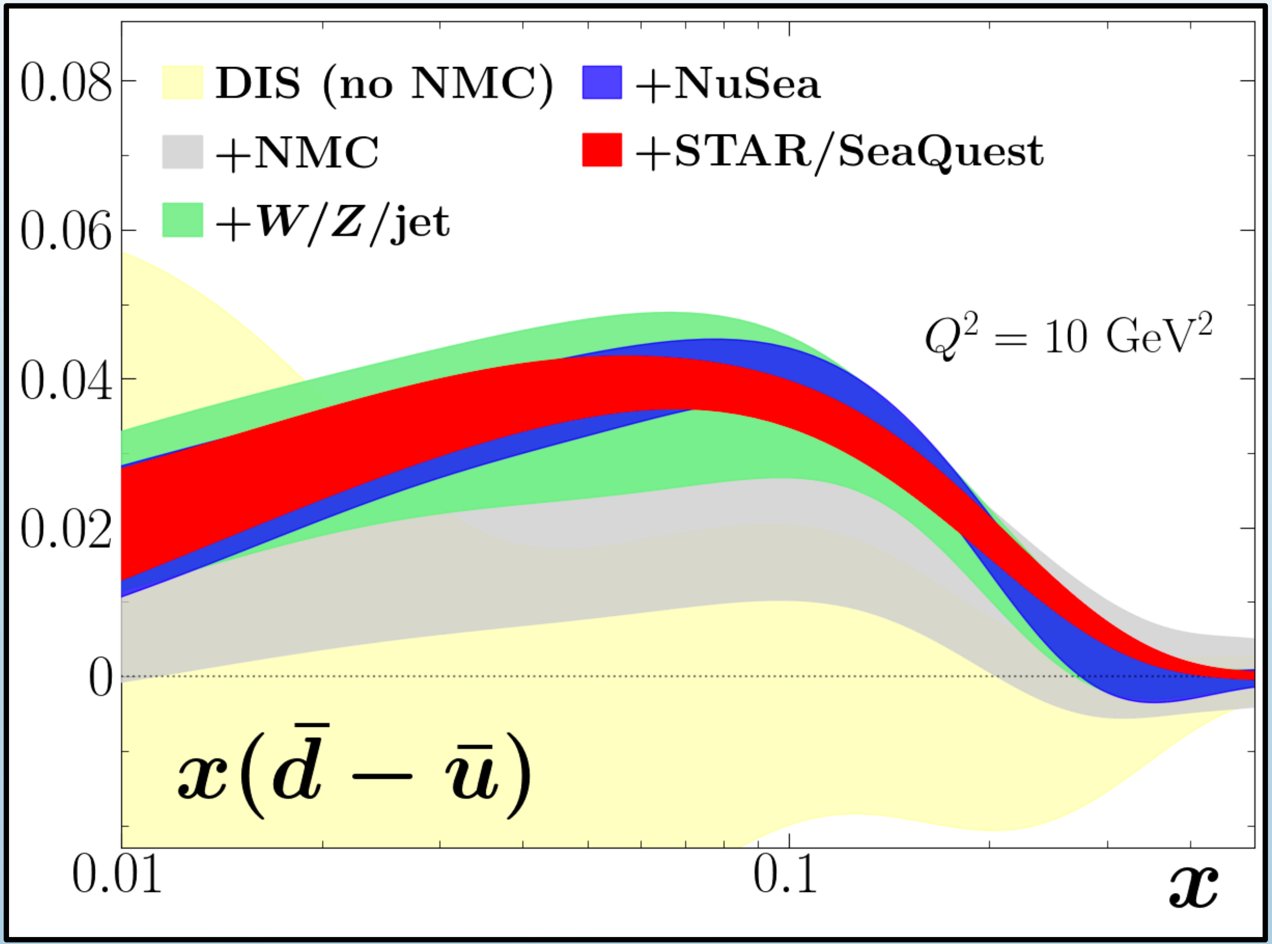
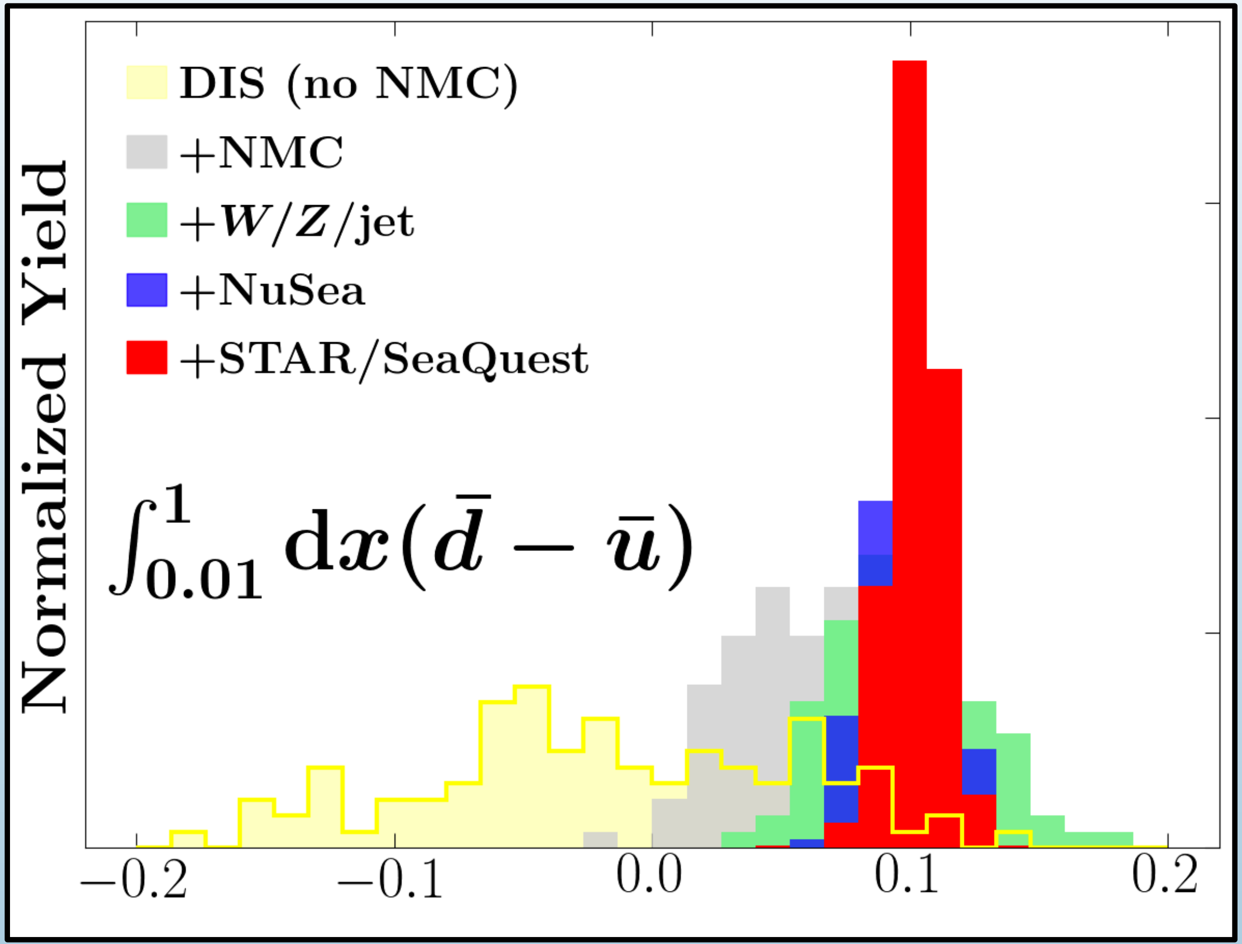
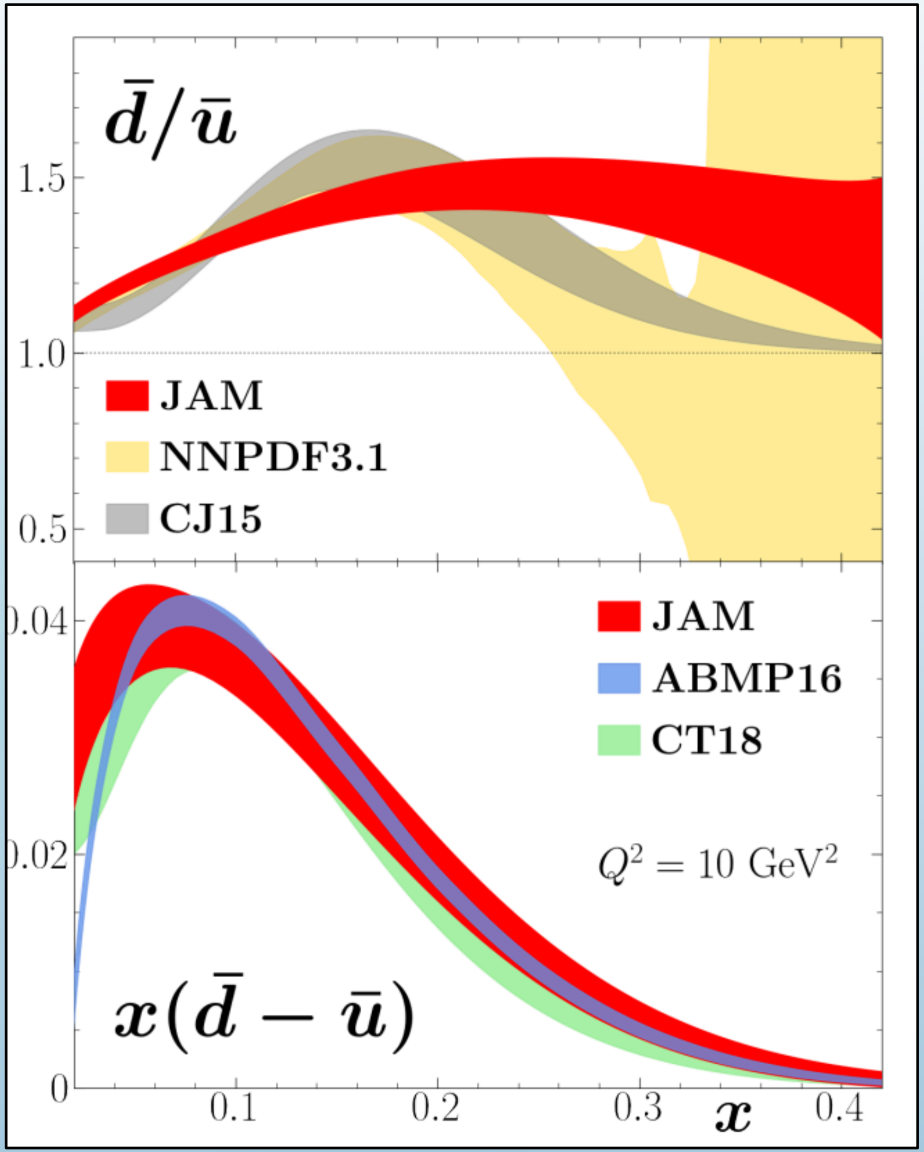




Comparison with Pion Cloud Model



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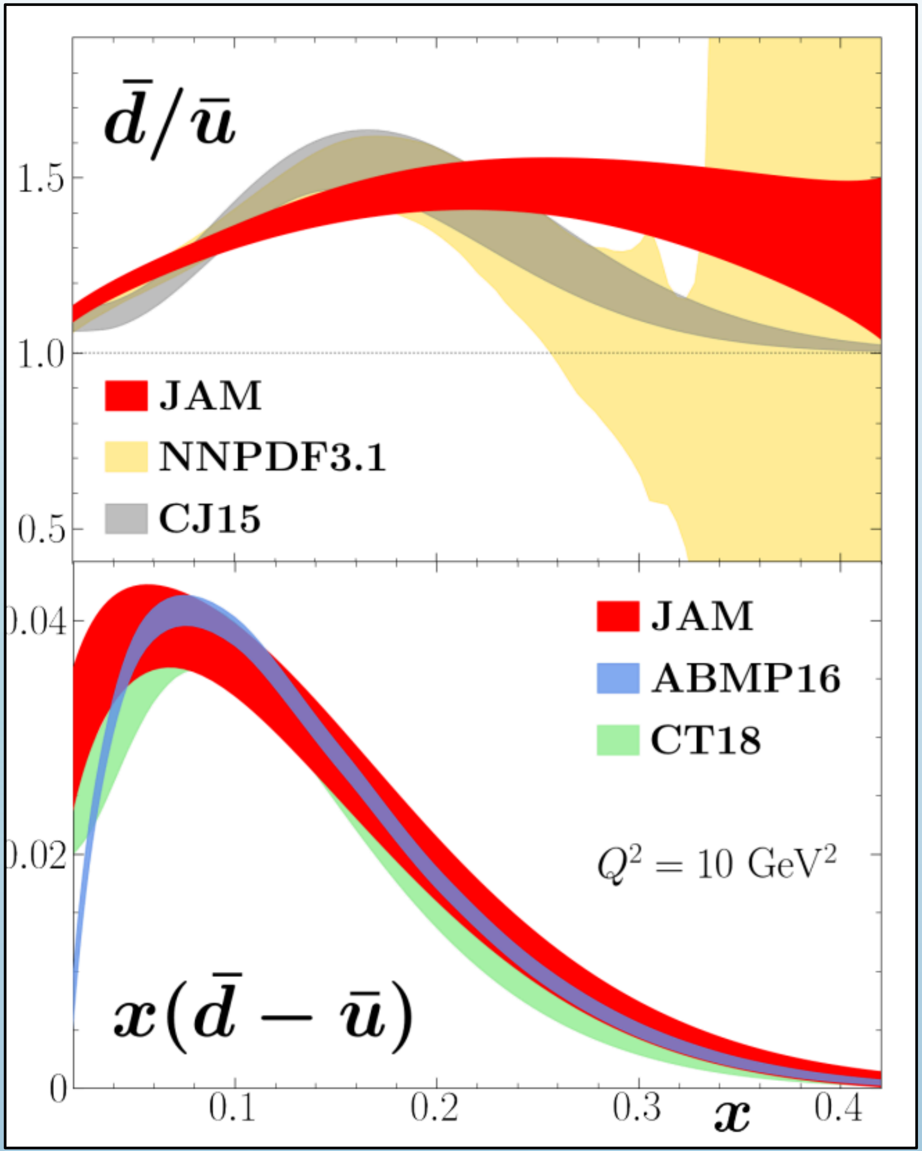

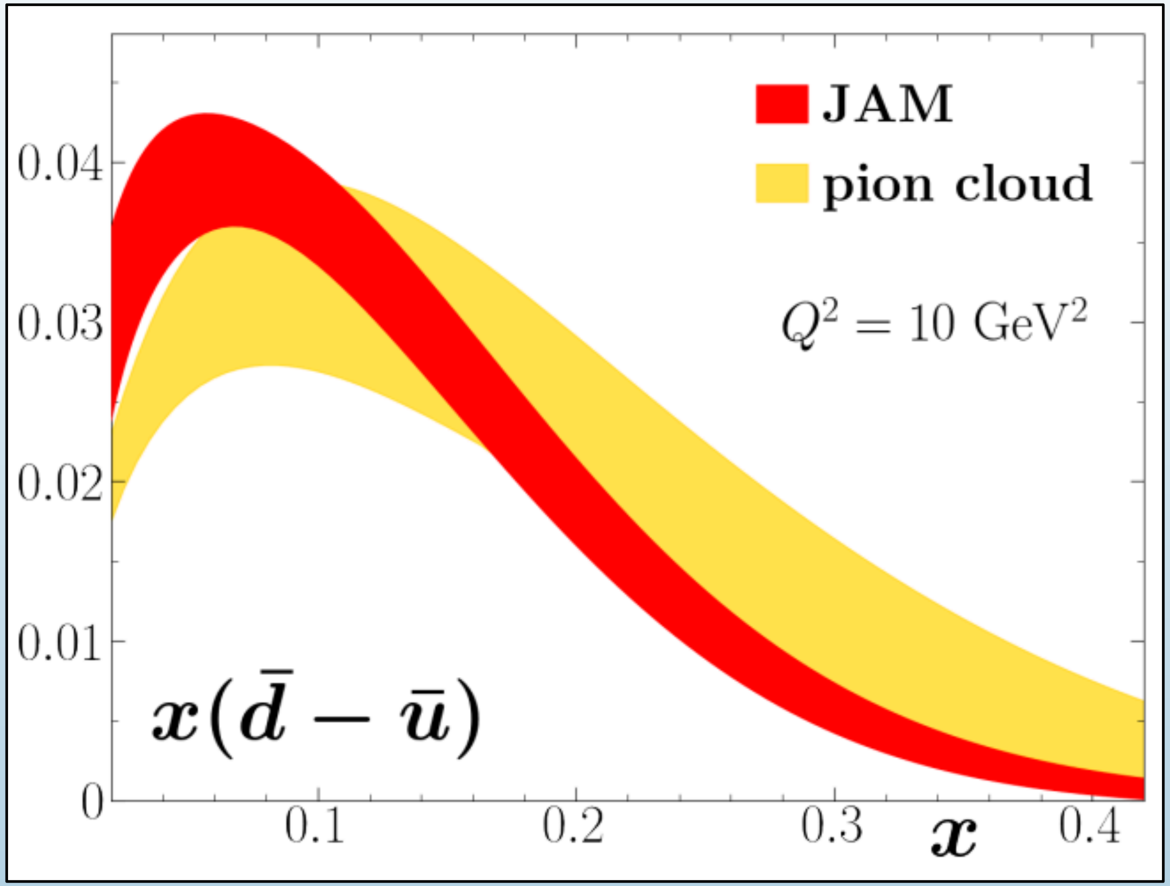

Good agreement with pion cloud model

Symmetries

$$(u, d) \times (p, n) \times (D, {}^3\mathrm{He}, {}^3\mathrm{H}) = 12 \text{ Functions}$$

Symmetries

$\delta u_{p/D}$	$\delta d_{n/D}$
$\delta d_{p/D}$	$\delta u_{n/D}$
$\delta u_{p/^3\mathrm{He}}$	$\delta d_{n/^3\mathrm{H}}$
$\delta u_{p/^3\mathrm{H}}$	$\delta d_{n/^3\mathrm{He}}$
$\delta d_{p/^3\mathrm{He}}$	$\delta u_{n/^3\mathrm{H}}$
$\delta d_{p/^3\mathrm{H}}$	$\delta u_{n/^3\mathrm{He}}$

$$(u, d) \times (p, n) \times (D, {}^3\mathrm{He}, {}^3\mathrm{H}) = 12 \text{ Functions}$$

Symmetries

$$\delta u_{p/D} = \delta d_{n/D}$$

$$\delta d_{p/D} = \delta u_{n/D}$$

$$\delta u_{p/^3\mathrm{He}} = \delta d_{n/^3\mathrm{H}}$$

$$\delta u_{p/^3\mathrm{H}} = \delta d_{n/^3\mathrm{He}}$$

$$\delta d_{p/^3\mathrm{He}} = \delta u_{n/^3\mathrm{H}}$$

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$$(u, d) \times (p, n) \times (D, {}^3\mathrm{He}, {}^3\mathrm{H}) = 12 \text{ Functions}$$

Charge symmetry

Symmetries

$$\delta u_{p/D} = \delta d_{n/D}$$

$$\delta d_{p/D} = \delta u_{n/D}$$

$$\delta u_{p/^3\mathrm{He}} = \delta d_{n/^3\mathrm{H}}$$

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$$\delta d_{p/^3\mathrm{He}} = \delta u_{n/^3\mathrm{H}}$$

$$\delta d_{p/^3\mathrm{H}} = \delta u_{n/^3\mathrm{He}}$$

$$(u, d) \times (p, n) \times (D, {}^3\mathrm{He}, {}^3\mathrm{H}) = 12 \text{ Functions}$$

Charge symmetry

$\delta u_{p/D}$	$\delta u_{p/^3\mathrm{H}}$
$\delta d_{p/D}$	$\delta d_{p/^3\mathrm{H}}$

$\delta u_{p/^3\mathrm{He}}$	$2\delta d_{p/^3\mathrm{He}}$

Symmetries

$$(u,d)\times(p,n)\times(D,{}^3\mathrm{He},{}^3\mathrm{H}) = 12\ \mathrm{Functions}$$

$$\delta u_{p/D} = \delta d_{n/D}$$
$$\delta d_{p/D} = \delta u_{n/D}$$
$$\delta u_{p/^3\mathrm{He}} = \delta d_{n/^3\mathrm{H}}$$
$$\delta u_{p/^3\mathrm{H}} = \delta d_{n/^3\mathrm{He}}$$
$$\delta d_{p/^3\mathrm{He}} = \delta u_{n/^3\mathrm{H}}$$
$$\delta d_{p/^3\mathrm{H}} = \delta u_{n/^3\mathrm{He}}$$

Charge symmetry

$$\delta u_{p/D} \approx \delta u_{p/^3\mathrm{H}}$$
$$\delta d_{p/D} \approx \delta d_{p/^3\mathrm{H}}$$

Isospin Symmetry (Model)

$$\delta u_{p/^3\mathrm{He}} \qquad 2\delta d_{p/^3\mathrm{He}}$$

Symmetries

$$(u,d) \times (p,n) \times (D, {}^3\text{He}, {}^3\text{H}) = 12 \text{ Functions}$$

$$\delta u_{p/D} = \delta d_{n/D}$$

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$$\delta u_{p/D} \approx \delta u_{p/^3\text{H}}$$

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Isospin Symmetry (Model)

$$\delta u$$

$$\delta d$$

Symmetries

$$(u, d) \times (p, n) \times (D, {}^3\text{He}, {}^3\text{H}) = 12 \text{ Functions}$$

$$\delta u_{p/D} = \delta d_{n/D}$$

$$\delta d_{p/D} = \delta u_{n/D}$$

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Isospin Symmetry (Model)

$$\delta u \qquad \delta d$$

$$\delta u_{p/{}^3\text{He}} \approx 2\delta d_{p/{}^3\text{He}}$$

No Isovector (Model)

Symmetries

$\delta u_{p/D} = \delta d_{n/D}$

$\delta d_{p/D} = \delta u_{n/D}$

$\delta u_{p/^3\mathrm{He}} = \delta d_{n/^3\mathrm{H}}$

$\delta u_{p/^3\mathrm{H}} = \delta d_{n/^3\mathrm{He}}$

$\delta d_{p/^3\mathrm{He}} = \delta u_{n/^3\mathrm{H}}$

$\delta d_{p/^3\mathrm{H}} = \delta u_{n/^3\mathrm{He}}$

$(u,d) \times (p,n) \times (D,{}^3\mathrm{He},{}^3\mathrm{H}) = 12$ Functions

Charge symmetry

$\delta u_{p/D} \approx \delta u_{p/^3\mathrm{H}}$

$\delta d_{p/D} \approx \delta d_{p/^3\mathrm{H}}$

Isospin Symmetry (Model)

δu

δd

$\delta u_{p/^3\mathrm{He}} \approx 2\delta d_{p/^3\mathrm{He}}$

No Isovector (Model)

$(\delta u + 2\delta d)/2$

Symmetries

$$\delta u_{p/D} = \delta d_{n/D}$$

$$\delta d_{p/D} = \delta u_{n/D}$$

$$\delta u_{p/^3\mathrm{He}} = \delta d_{n/^3\mathrm{H}}$$

$$\delta u_{p/^3\mathrm{H}} = \delta d_{n/^3\mathrm{He}}$$

$$\delta d_{p/^3\mathrm{He}} = \delta u_{n/^3\mathrm{H}}$$

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$$(u, d) \times (p, n) \times (D, {}^3\mathrm{He}, {}^3\mathrm{H}) = 12 \text{ Functions}$$

Charge symmetry

$$\delta u_{p/D} \approx \delta u_{p/^3\mathrm{H}}$$

$$\delta d_{p/D} \approx \delta d_{p/^3\mathrm{H}}$$

$$\delta u_{p/^3\mathrm{He}} \approx 2\delta d_{p/^3\mathrm{He}}$$

Isospin Symmetry (Model)

$$\delta u$$

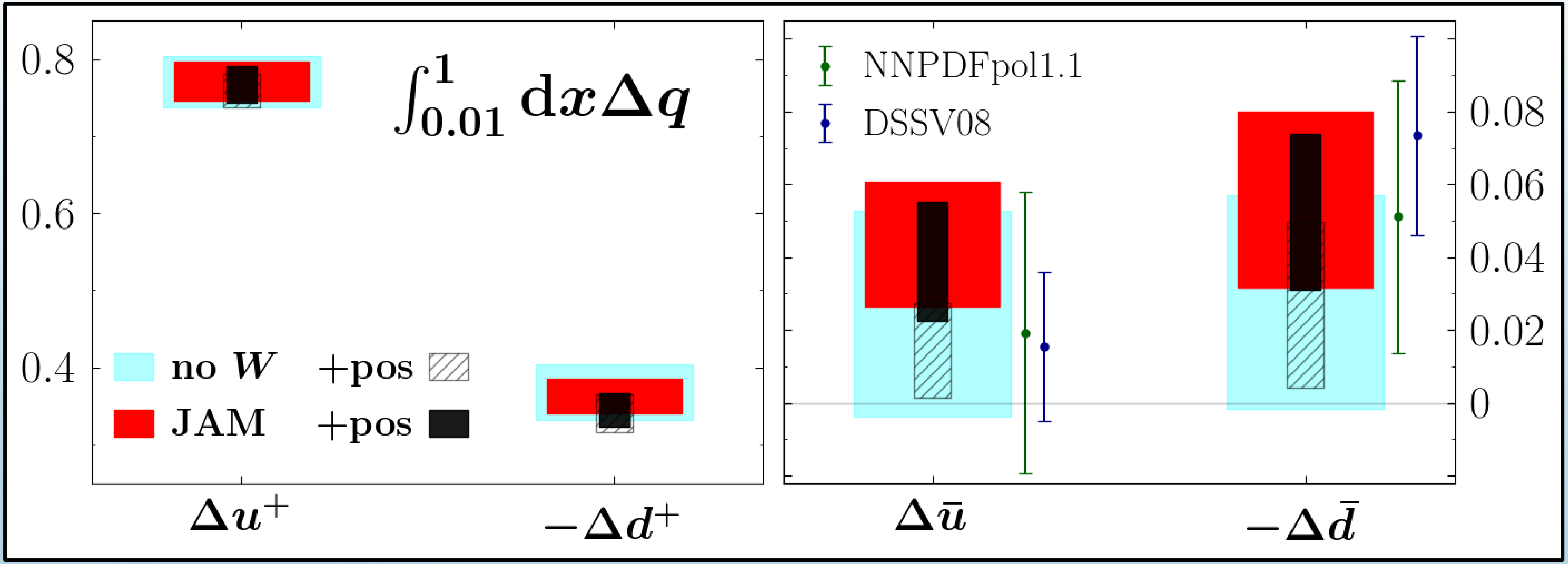
$$\delta d$$

No Isovector (Model)

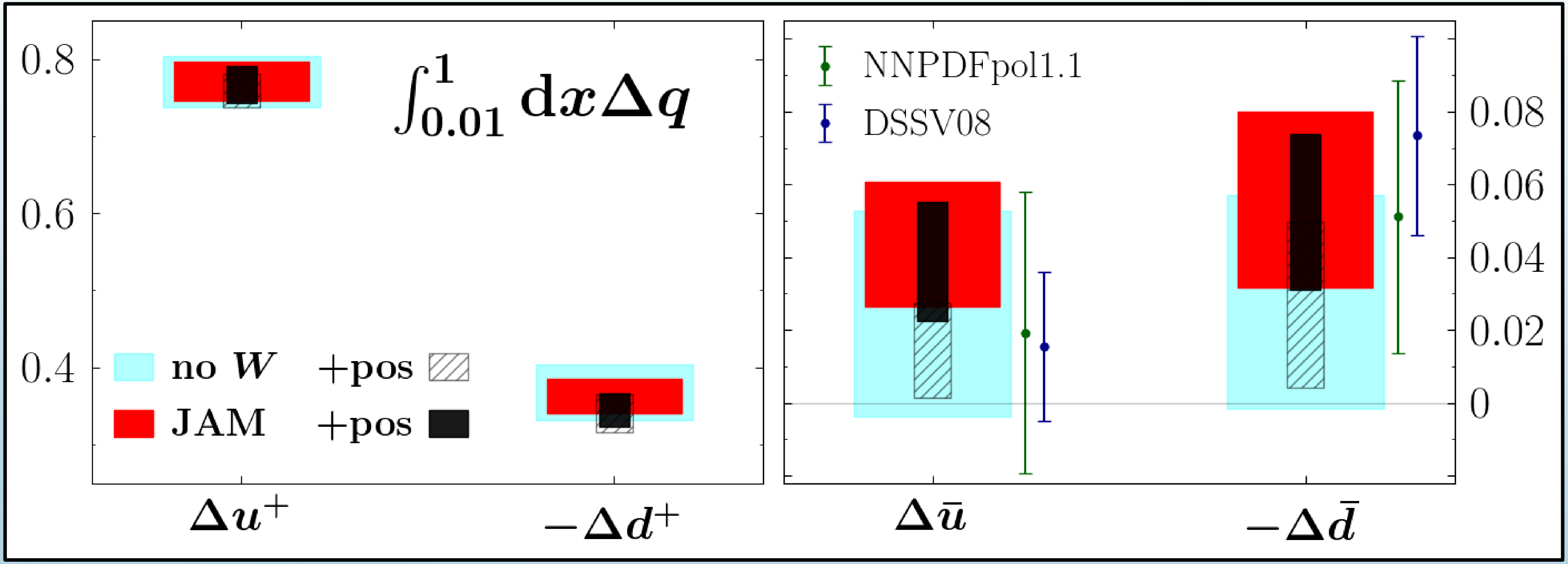
$$(\delta u + 2\delta d)/2$$

Just two functions!

Proton Spin Contributions

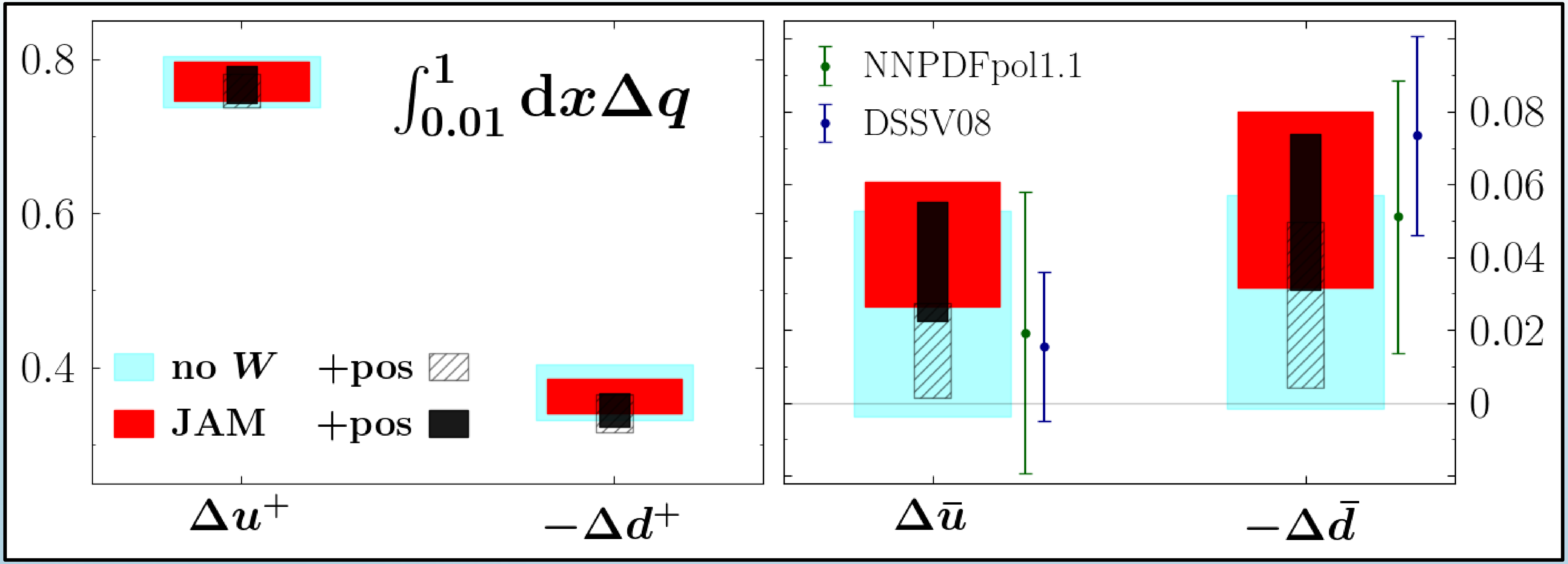


Proton Spin Contributions



Inclusion of RHIC W/Z data shows that $\Delta\bar{u}$ ($\Delta\bar{d}$) contribution is small and positive (negative)

Proton Spin Contributions



Inclusion of RHIC W/Z data shows that $\Delta\bar{u}$ ($\Delta\bar{d}$) contribution is small and positive (negative)

Flavor	JAM moment (truncated)	Lattice Moment (full)	Difference
Δu^+	0.779(34)	0.864(16)	10%
Δd^+	-0.370(40)	-0.426(16)	13%

C. Alexandrou *et al.*, Phys. Rev. D **101**, 094513 (2020)

Checks of Definition

Number density

$$\sum_{h_1 h_2} \int \mathrm{d}z_1 \mathrm{d}z_2 D_1^{h_1 h_2/q}(z_1, z_2) = N^q(N^q - 1)$$

Checks of Definition

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Momentum sum rule

$$\sum_{h_1} \int_0^{1-z_2} \mathrm{d}z_1 \int \mathrm{d}^2 \vec{P}_{1\perp} z_1 D_1^{h_1 h_2/q}(z_1, z_2, \vec{P}_{1\perp}, \vec{P}_{2\perp}) = (1 - z_2) D_1^{h_2/q}(z_2, \vec{P}_{2\perp})$$

D. de Florian and L. Vanni, Phys. Lett. B **578**, 139 (2004)

Checks of Definition

Number density

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D. de Florian and L. Vanni, Phys. Lett. B **578**, 139 (2004)

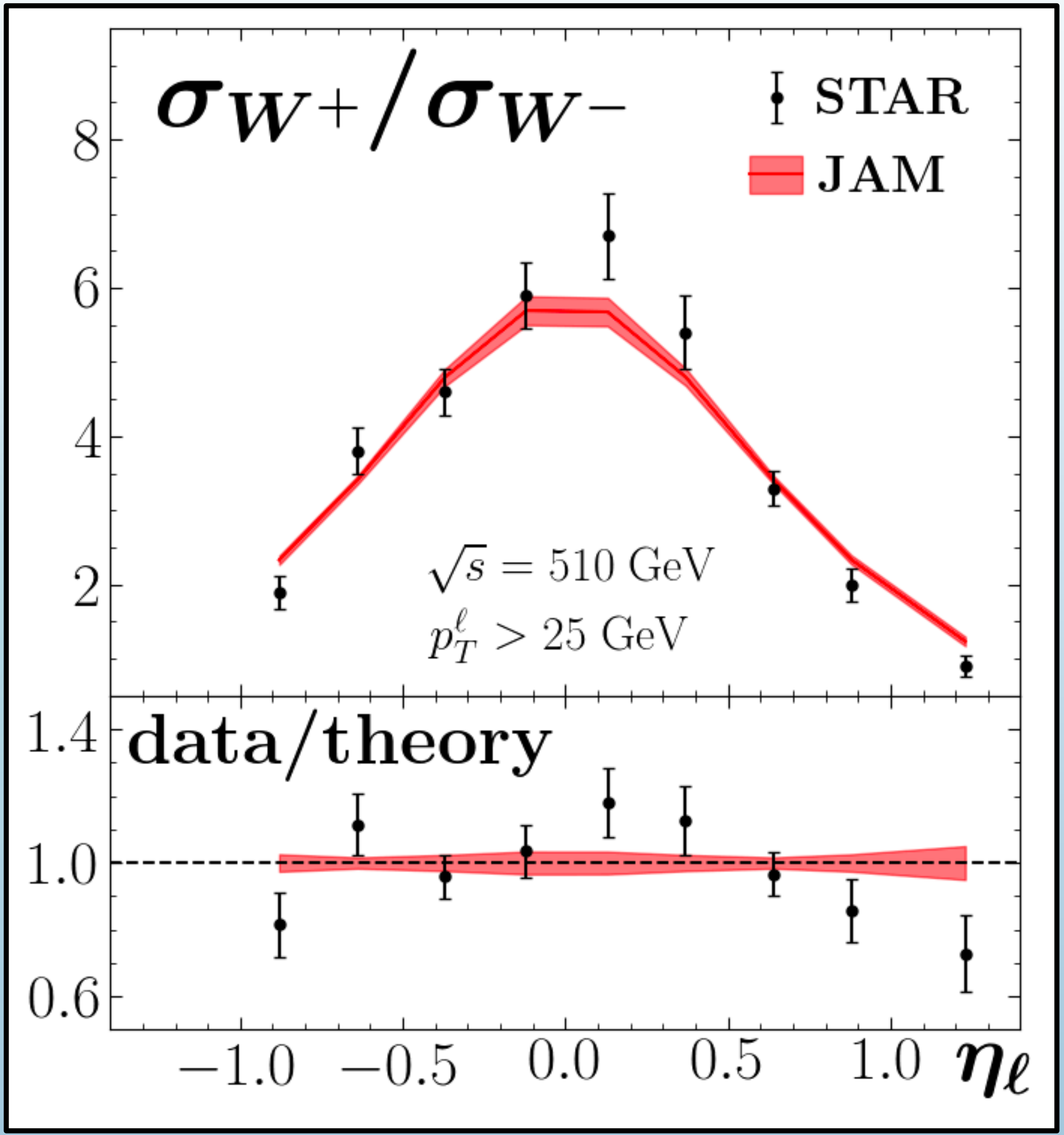
LO cross section for $e^- e^+ \to (h_1 h_2) X$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}z_1 \mathrm{d}z_2} = \sum_{q\bar{q}} \hat{\sigma}^q D_1^{h_1 h_2/q}(z_1, z_2)$$

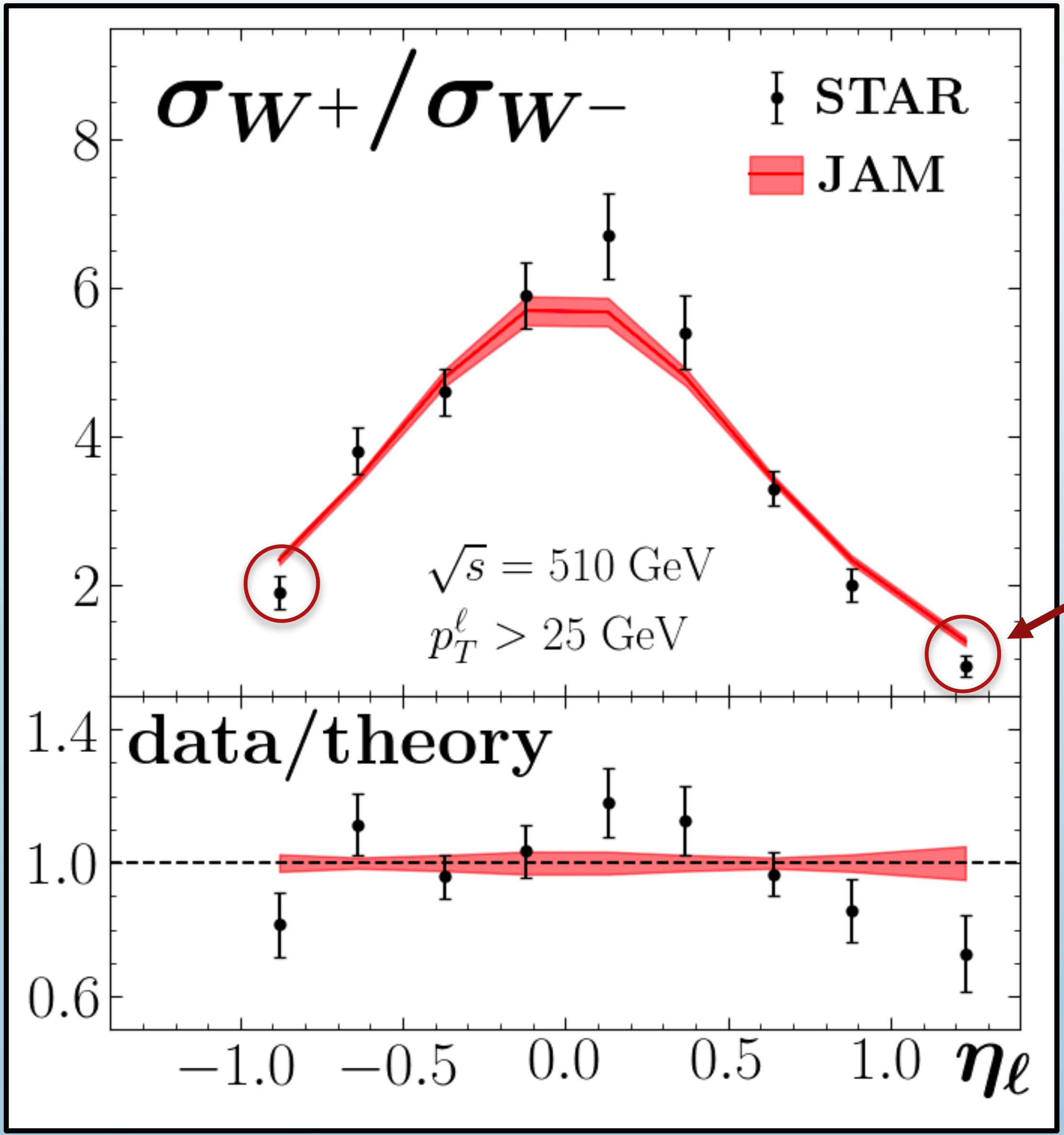
$$\frac{\mathrm{d}\sigma}{\mathrm{d}z \mathrm{d}M_h} = \sum_{q\bar{q}} \hat{\sigma}^q D_1^{h_1 h_2/q}(z, M_h)$$

$$\hat{\sigma}^q = \frac{4\pi e_q^2 \alpha_{\mathrm{em}}^2 N_c}{3Q^2}$$

STAR Difficulties at Extreme Rapidity

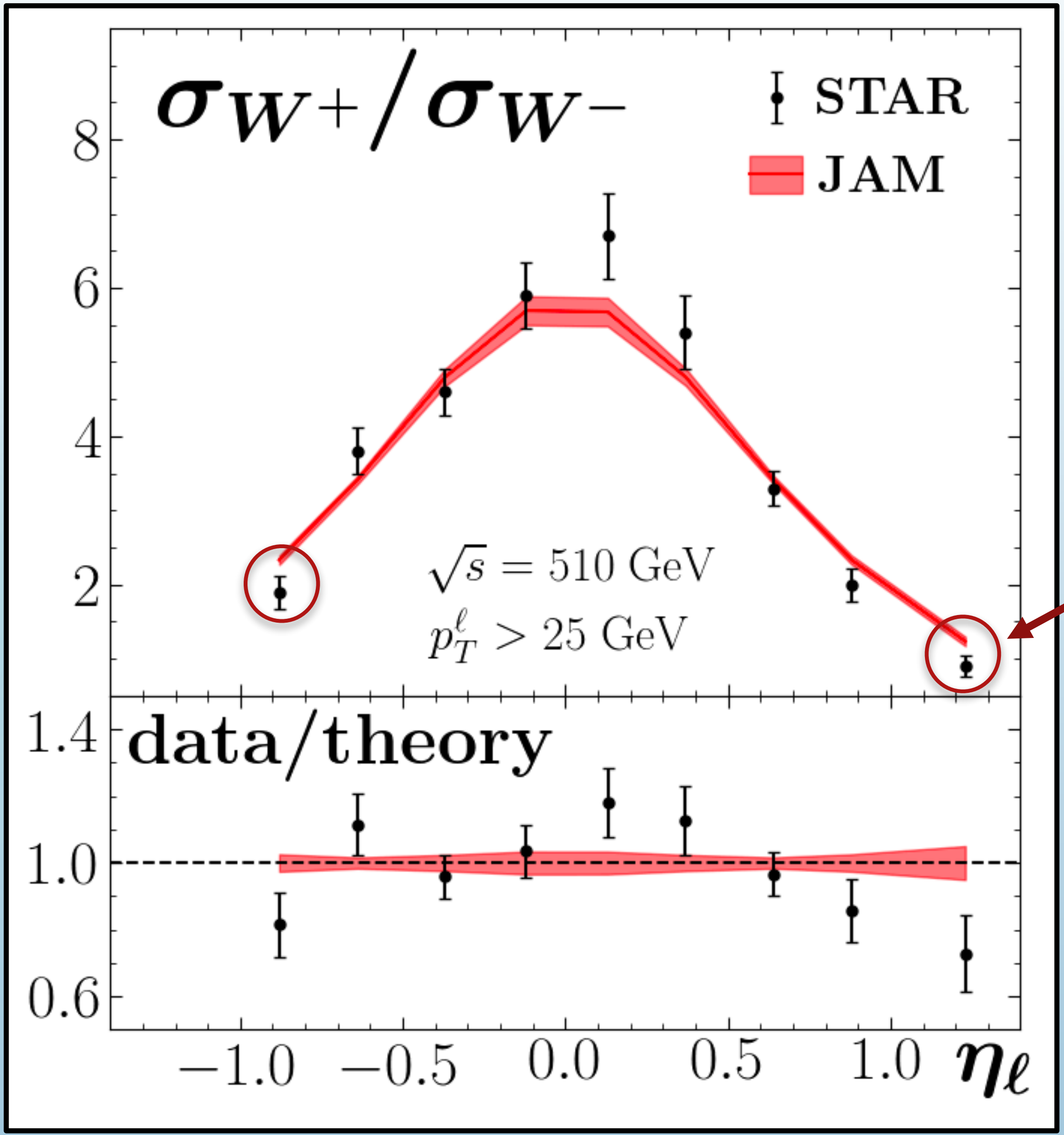


STAR Difficulties at Extreme Rapidity



Difficult to describe at extreme rapidity

STAR Difficulties at Extreme Rapidity



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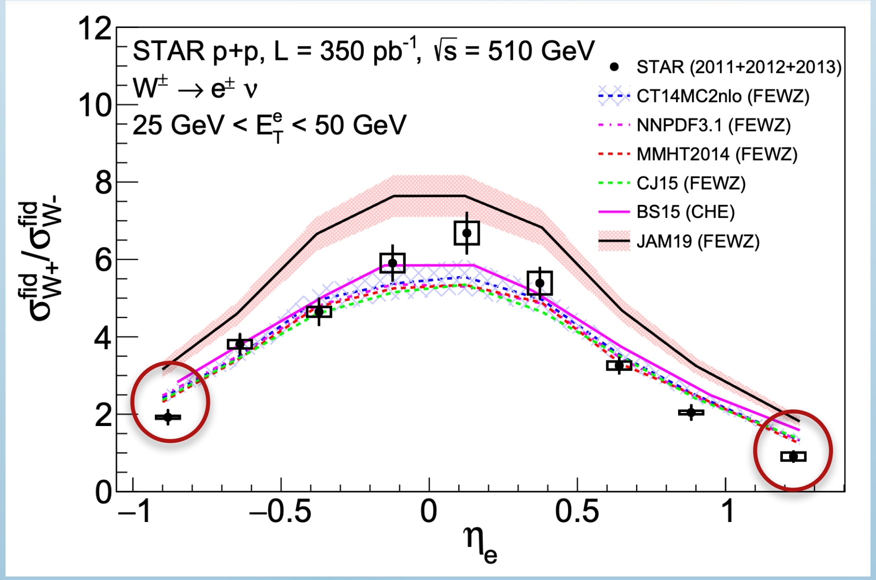

J. Adam *et al*. [STAR], Phys. Rev. D. **103**, 012001 (2021)

Nuclear PDFs

$$q_{N/A}^{(\mathrm{on})}(x, Q^2) = \left[\, f^{N/A} \otimes q_N \,\right]$$

$$q_{N/A}^{(\mathrm{off})}(x, Q^2) = \left[\, \tilde{f}^{N/A} \otimes \delta q_{N/A} \,\right]$$

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Contains Virtuality

$$\nu(p^2) = (p^2 - M^2)/M^2 \ll 1$$

Nuclear PDFs

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Contains Virtuality

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$$\Delta^q_3 \equiv \frac{q_{p/^3\text{H}} - q_{p/^3\text{He}}}{q_{p/^3\text{H}} + q_{p/^3\text{He}}}$$

Nuclear PDFs

$$q_{N/A}^{(\mathrm{on})}(x, Q^2) = [\, f^{N/A} \otimes q_N \,]$$

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Contains Virtuality

$$\nu(p^2) = (p^2 - M^2)/M^2 \ll 1$$

$$\Delta_3^q \equiv \frac{q_{p/^3\mathrm{H}} - q_{p/^3\mathrm{He}}}{q_{p/^3\mathrm{H}} + q_{p/^3\mathrm{He}}}$$

Measures strength of isovector effect

Kinematics and Definitions for DiFFs

$$q(k) \to h_1(P_1) + h_2(P_2) + X$$

$$z_{1,2} = P_{1,2}^- / k^-$$

$$M_h^2 \equiv P_h^2 \equiv (P_1 + P_2)^2$$

$$R \equiv \frac{1}{2}(P_1 - P_2)$$

$$z \equiv z_1 + z_2$$

$$\zeta = \frac{z_1 - z_2}{z}$$

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$$D_1^{h_1 h_2/q}(z_1, z_2, \overrightarrow{P}_{1\perp}, \overrightarrow{P}_{2\perp}) \equiv \frac{1}{64\pi^3 z_1 z_2} \int \frac{\mathrm{d}\xi^+ \mathrm{d}^2 \overrightarrow{\xi}_T}{(2\pi)^3} e^{ik\cdot\xi} \, \mathrm{Tr}\Big[\langle 0 | \psi_q(\xi) | h_1, h_2, X\rangle\langle h_1, h_2, X | \bar{\psi}_q(0) | 0\rangle \gamma^-\Big]_{\xi^-=0}$$

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Needed for number density interpretation

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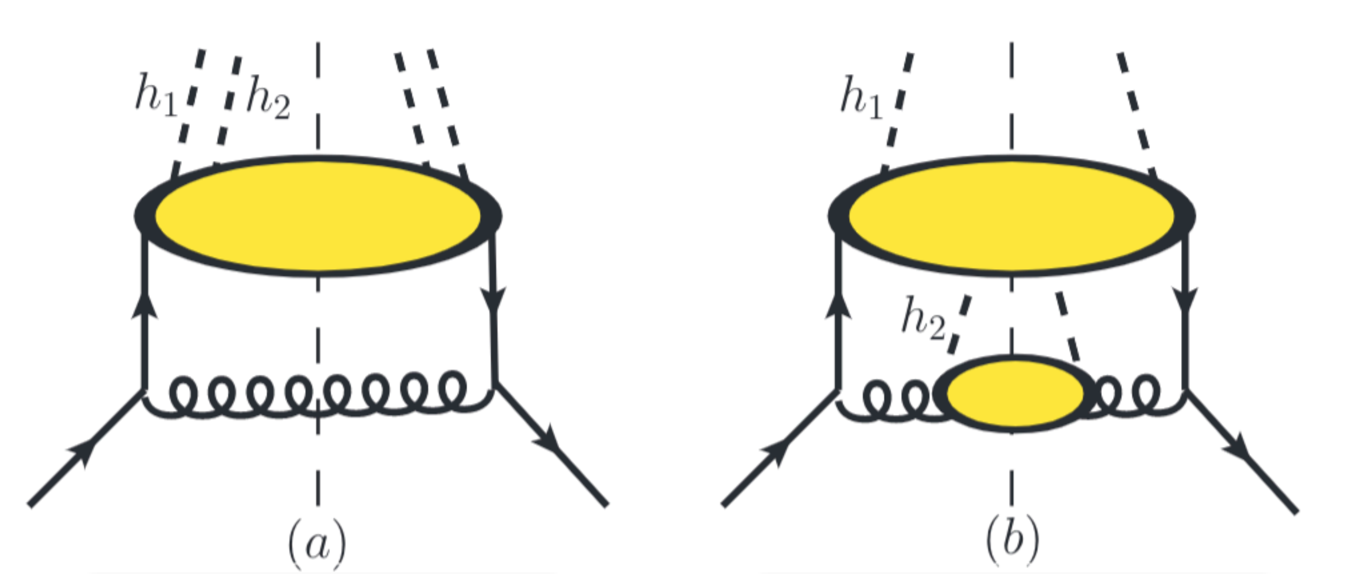
Needed for number density interpretation

Extended DiFFs (extDiFFs) are written in terms of $(z, \xi, \vec{R}_T^2)$

Evolution

Evolution for extDiFFs (quark non-singlet)

$$\frac{\partial}{\partial \ln \mu^2} D_1^{h_1 h_2/q}(z, \zeta, \vec{R}_T^2; \mu) = \int_z^1 \frac{\mathrm{d}w}{w} D_1^{h_1 h_2/q}(\frac{z}{w}, \zeta, \vec{R}_T^2; \mu) P_{q\to q}(w)$$



Homogeneous

Inhomogeneous

Homogeneous term only for extended DiFFs

F. A. Ceccopieri, M. Radici, and A. Bacchetta, Phys. Lett. B **650**, 81 (2007)

Inhomogeneous term exists for $D_1^{h_1 h_2}(z_1, z_2)$

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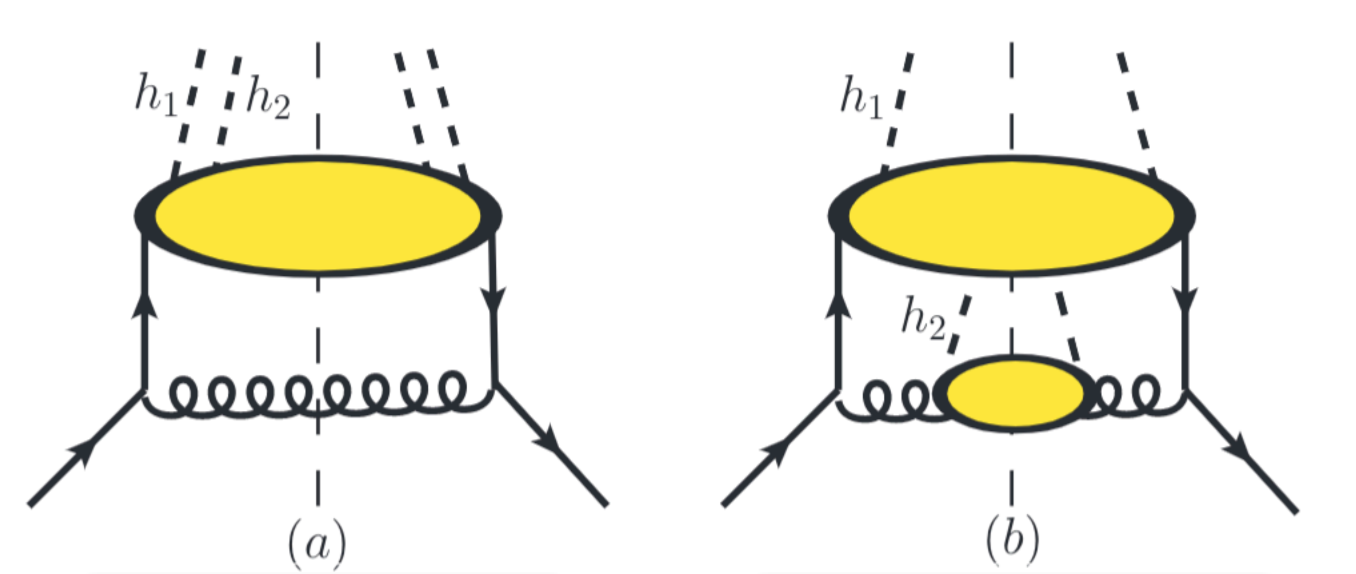

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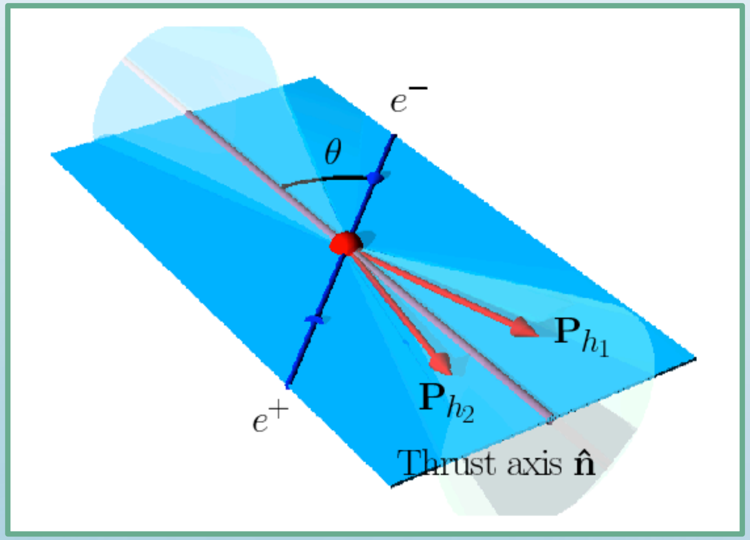
Inhomogeneous term exists for $D_1^{h_1 h_2}(z_1, z_2)$

Analogous derivations done for $D_1^{h_1 h_2/g}$ and $H_1^{\sphericalangle, h_1 h_2/q}$

Observables for DiFFs

SIA Cross Section

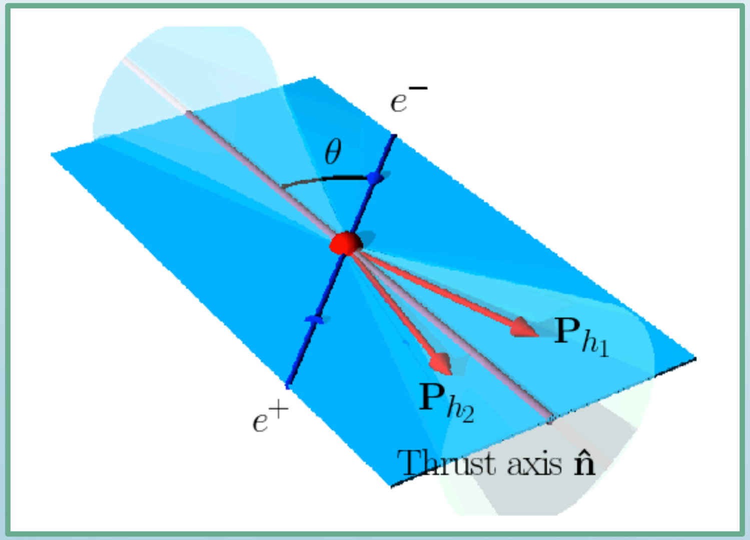


R. Seidl *et al.*, Phys. Rev. D **96**, no. 3, 032005 (2017)

$$\frac{d\sigma}{dz\, dM_h} = \frac{4\pi\alpha_{em}^2}{s} \sum_q e_q^2\, D_1^q(z, M_h)$$

Observables for DiFFs

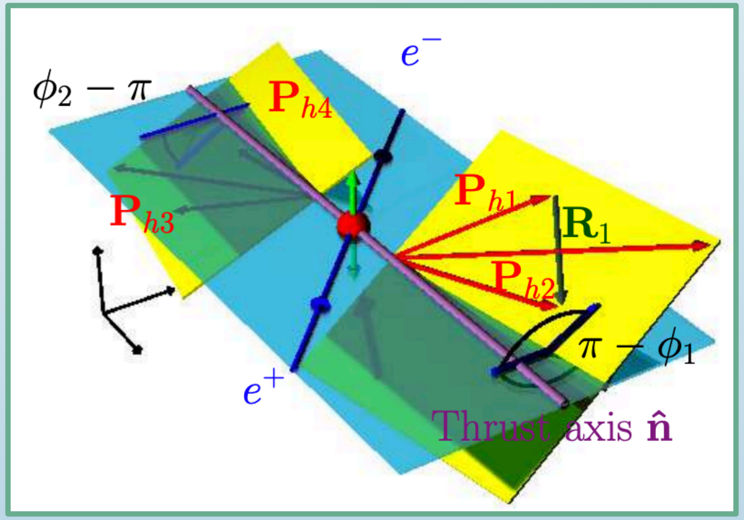
SIA Cross Section



R. Seidl *et al.*, Phys. Rev. D **96**, no. 3, 032005 (2017)

$$\frac{d\sigma}{dz\, dM_h} = \frac{4\pi\alpha_{em}^2}{s} \sum_q e_q^2 D_1^q(z, M_h)$$

SIA Artru-Collins Asymmetry

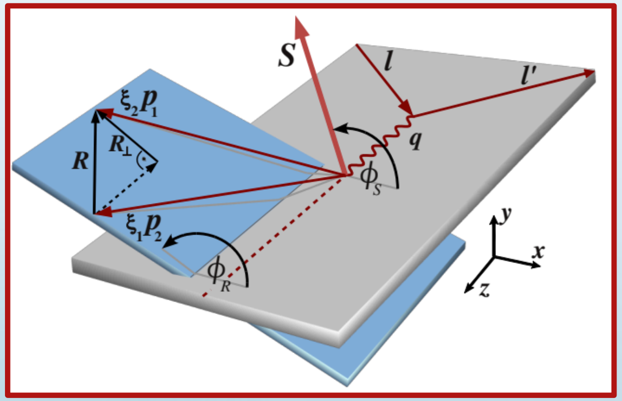


A. Vossen *et al.*, Phys. Rev. Lett. **107**, 072004 (2011)

$$A^{e^+e^-}(z, M_h, \bar{z}, \overline{M}_h) = \frac{\sin^2\theta \sum_q e_q^2 H_1^{\sphericalangle,q}(z, M_h) H_1^{\sphericalangle,\bar{q}}(\bar{z}, \overline{M}_h)}{(1+\cos^2\theta) \sum_q e_q^2 D_1^q(z, M_h) D_1^{\bar{q}}(\bar{z}, \overline{M}_h)}$$

Observables for Transversity PDFs

SIDIS asymmetry (p and D)

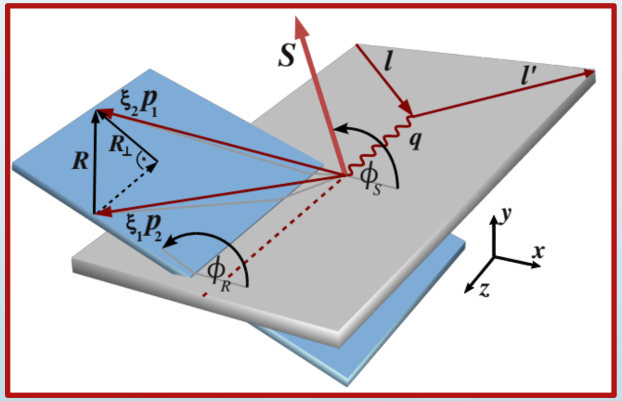


$$A_{UT}^{\text{SIDIS}} = c(y)\frac{\sum_q e_q^2\, h_1^q(x)\, H_1^{\sphericalangle,q}(z, M_h)}{\sum_q e_q^2\, f_1^q(x)\, D_1^q(z, M_h)}$$

C. Adolph *et al.*, Phys. Lett. B **713**, 10-16 (2012)

Observables for Transversity PDFs

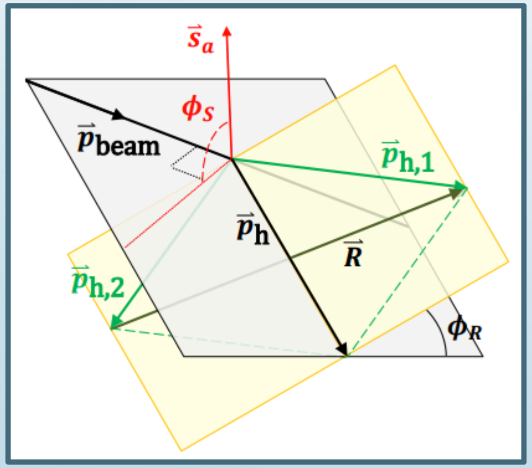
SIDIS asymmetry (p and D)



C. Adolph *et al.*, Phys. Lett. B **713**, 10-16 (2012)

$$A_{UT}^{\text{SIDIS}} = c(y) \frac{\sum_q e_q^2\, h_1^q(x)\, H_1^{\sphericalangle,q}(z, M_h)}{\sum_q e_q^2\, f_1^q(x)\, D_1^q(z, M_h)}$$

pp Asymmetry



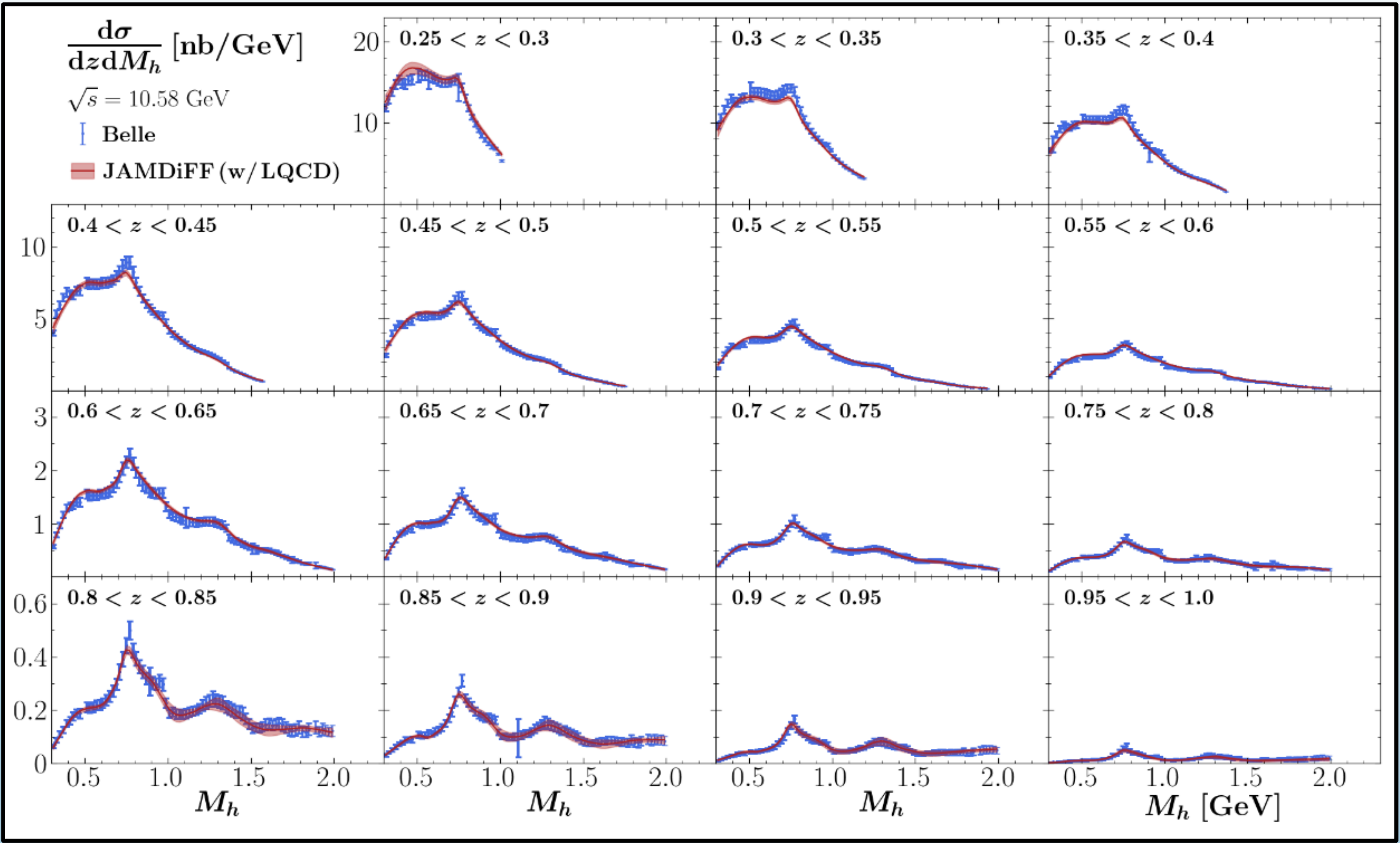
L. Adamczyk *et al.*, Phys. Rev. Lett. **115**, 242501 (2015)

$$A_{UT}^{pp} = \frac{\mathscr{H}(M_h, P_{hT}, \eta)}{\mathscr{D}(M_h, P_{hT}, \eta)}$$

$$\mathscr{H}(M_h, P_{hT}, \eta) = 2P_{hT} \sum_i \sum_{a,b,c} \int_{x_a^{\min}}^1 \mathrm{d}x_a \int_{x_b^{\min}}^1 \frac{\mathrm{d}x_b}{z} f_1^a(x_a)\, h_1^b(x_b) \frac{\mathrm{d}\Delta\hat{\sigma}_{ab^\uparrow \to c^\uparrow d}}{\mathrm{d}\hat{t}}\, H_1^{\sphericalangle,c}(z, M_h)$$

$$\mathscr{D}(M_h, P_{hT}, \eta) = 2P_{hT} \sum_i \sum_{a,b,c} \int_{x_a^{\min}}^1 \mathrm{d}x_a \int_{x_b^{\min}}^1 \frac{\mathrm{d}x_b}{z} f_1^a(x_a)\, f_1^b(x_b) \frac{\mathrm{d}\hat{\sigma}_{ab \to cd}}{\mathrm{d}\hat{t}}\, D_1^c(z, M_h)$$

Quality of Fit (Unpolarized Cross Section)



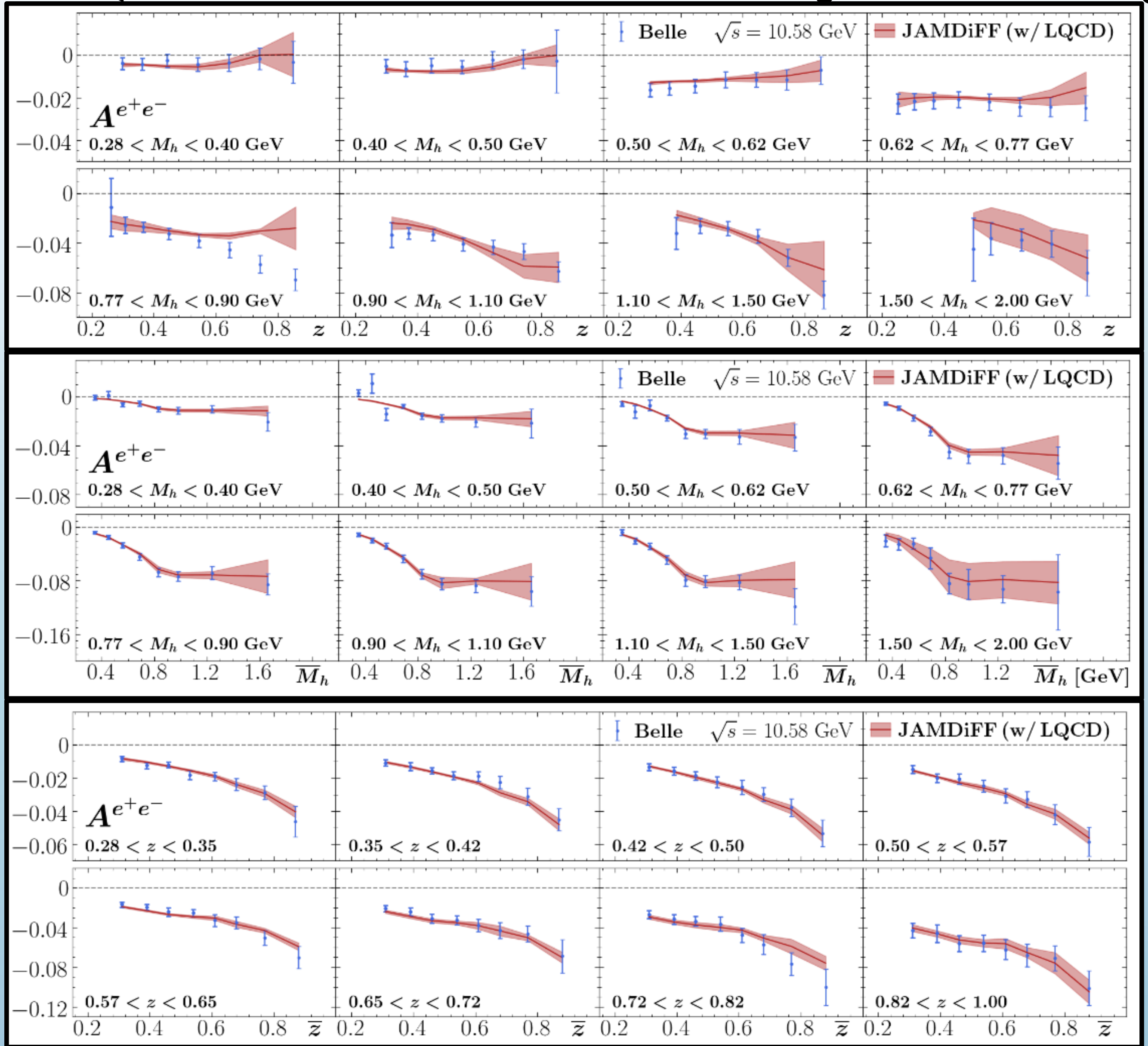
R. Seidl *et al.*, Phys. Rev. D **96**, 032005 (2017)

Quality of Fit (Artru-Collins Asymmetry)

(z, M_h) binning

$(M_h, \overline{M}_h)$ binning

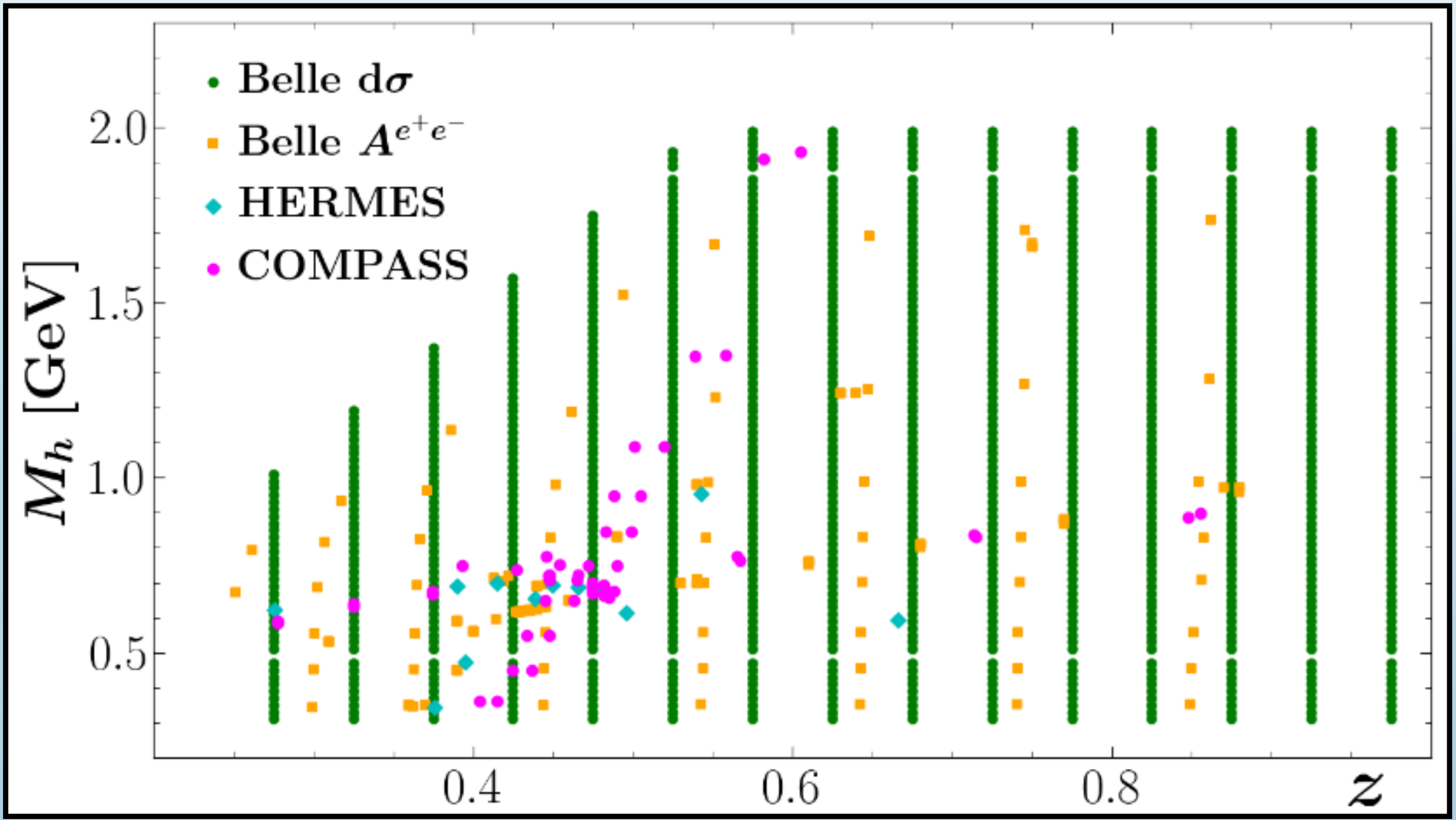
$(z, \bar{z})$ binning



A. Vossen *et al.*,
Phys. Rev. Lett. **107**, 072004 (2011)

Data for DiFFs

SIA cross section	Belle	1094 points
SIA Artru-Collins	Belle	183 points



Data for DiFFs

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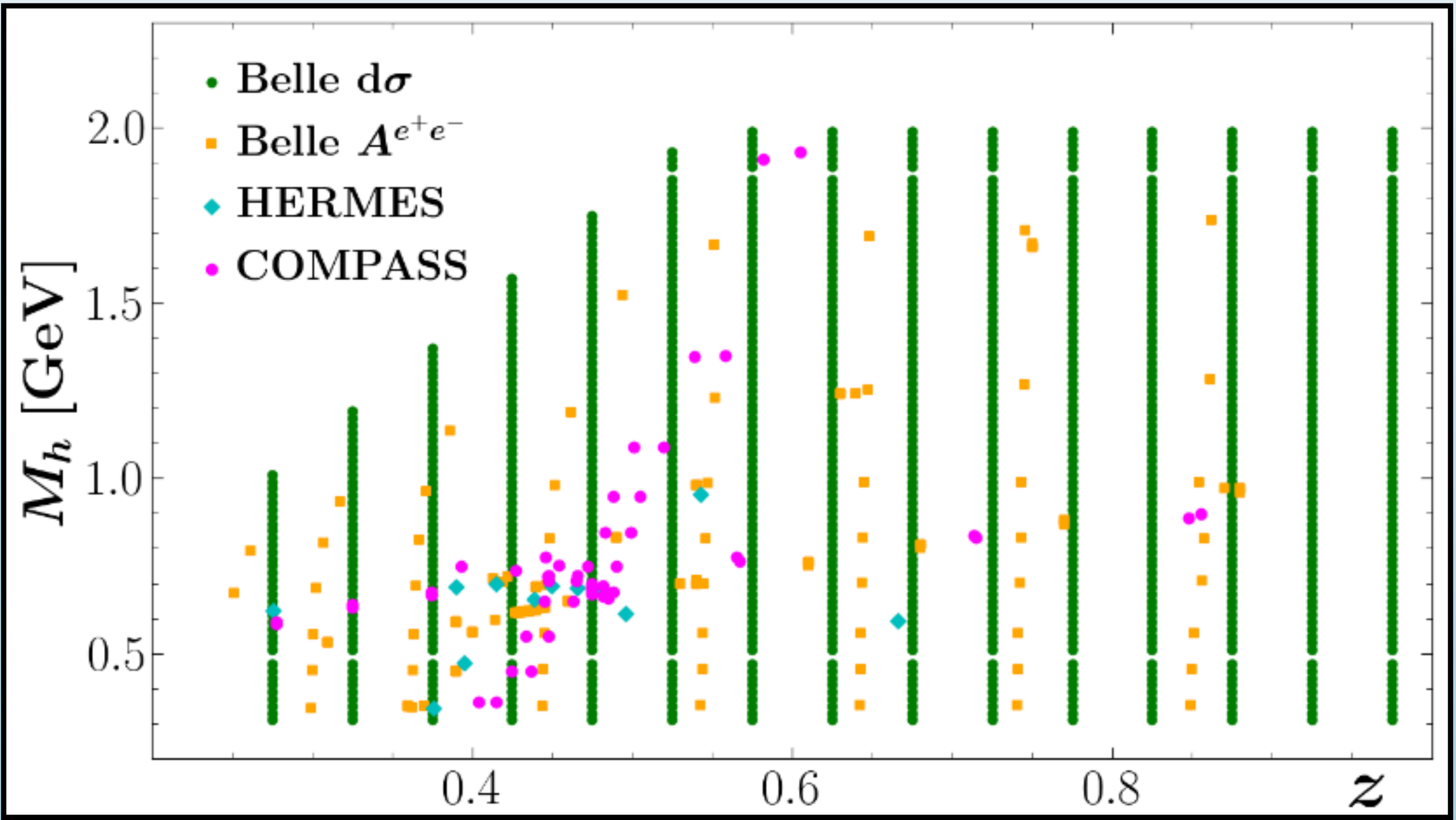
$\pi^+\pi^-$ DiFFs



$$D_1^u = D_1^d = D_1^{\bar{u}} = D_1^{\bar{d}},$$

$$D_1^s = D_1^{\bar{s}}, \quad D_1^c = D_1^{\bar{c}}, \quad D_1^b = D_1^{\bar{b}},$$

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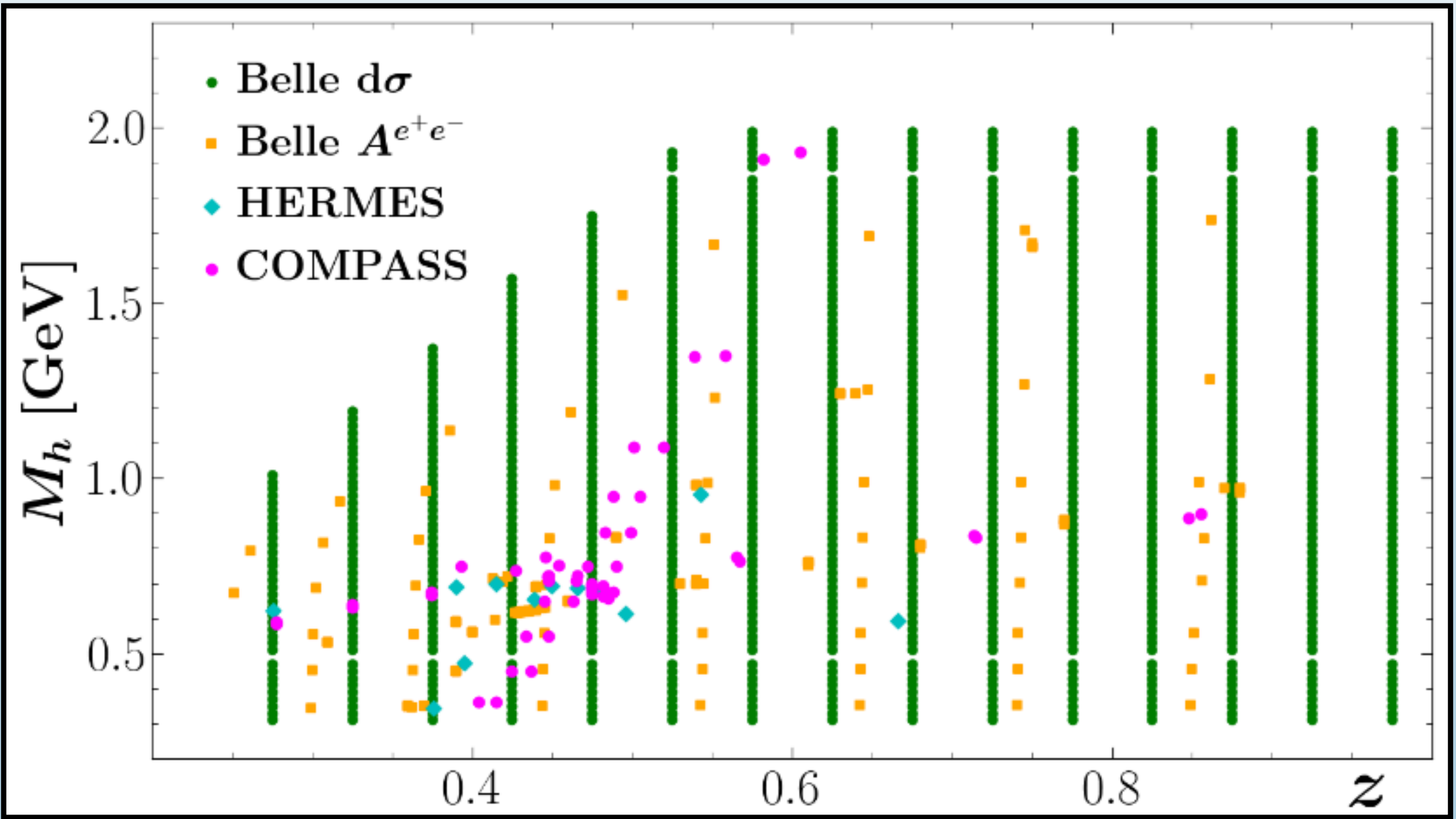

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$$H_1^{\sphericalangle,u} = -H_1^{\sphericalangle,d} = -H_1^{\sphericalangle,\bar{u}} = H_1^{\sphericalangle,\bar{d}},$$

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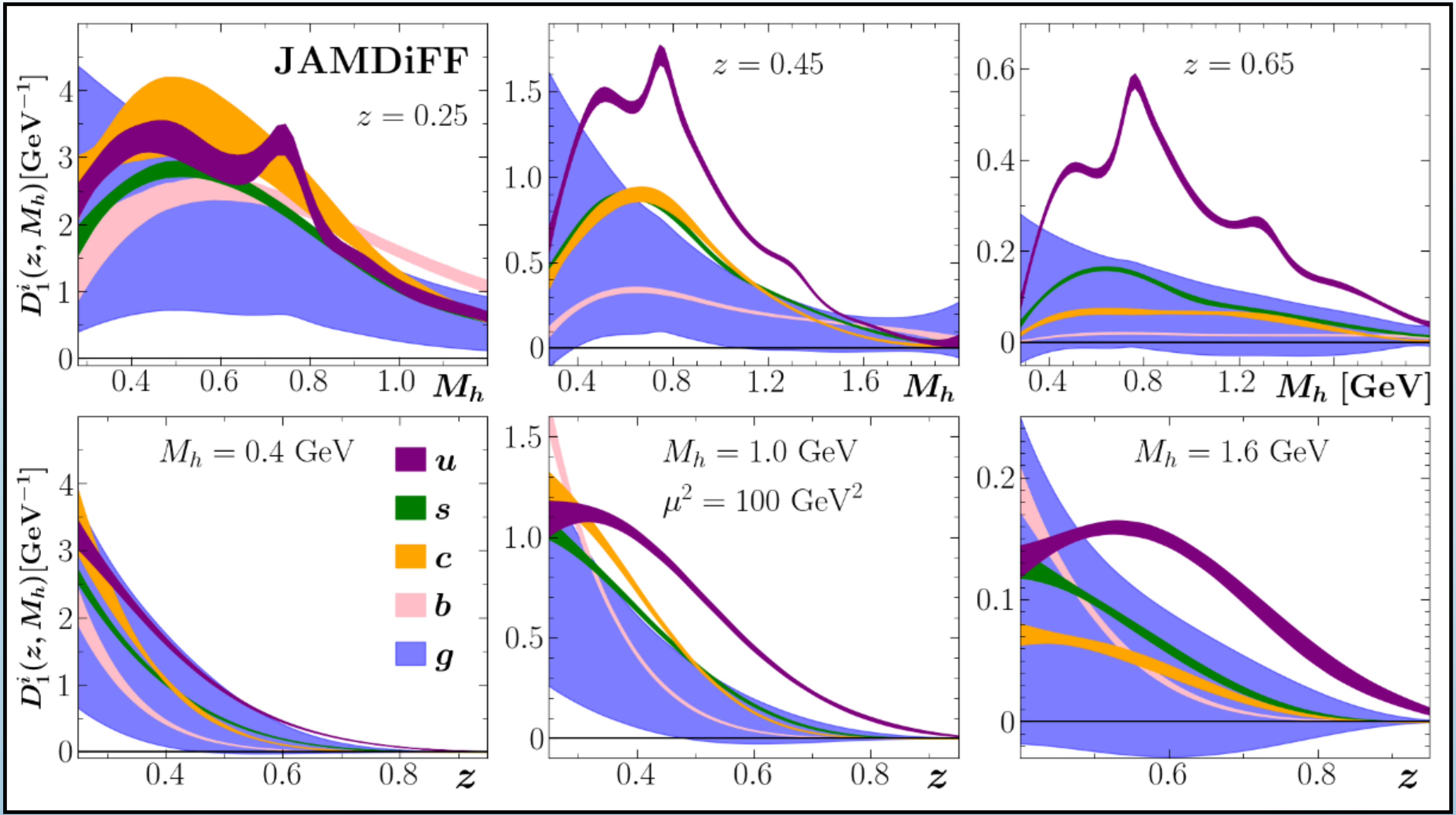
1 independent function

A. Courtoy et al., Phys. Rev. D **85**, 114023 (2012)

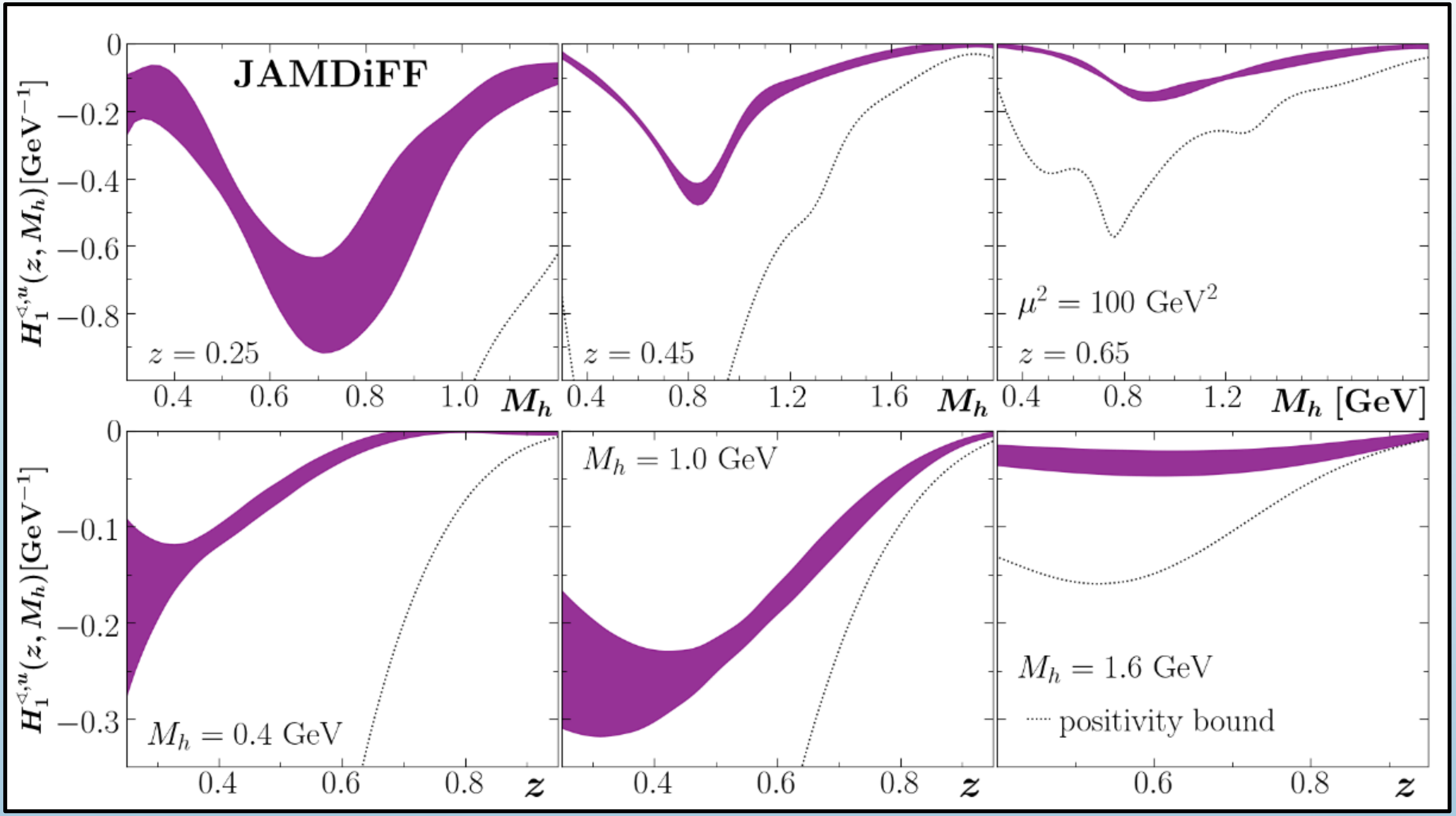
Extracted DiFFs



Bound: $D_1^q > 0$

A. Bacchetta and M. Radici, Phys. Rev. D **67**, 094002 (2003)

Extracted IFFs

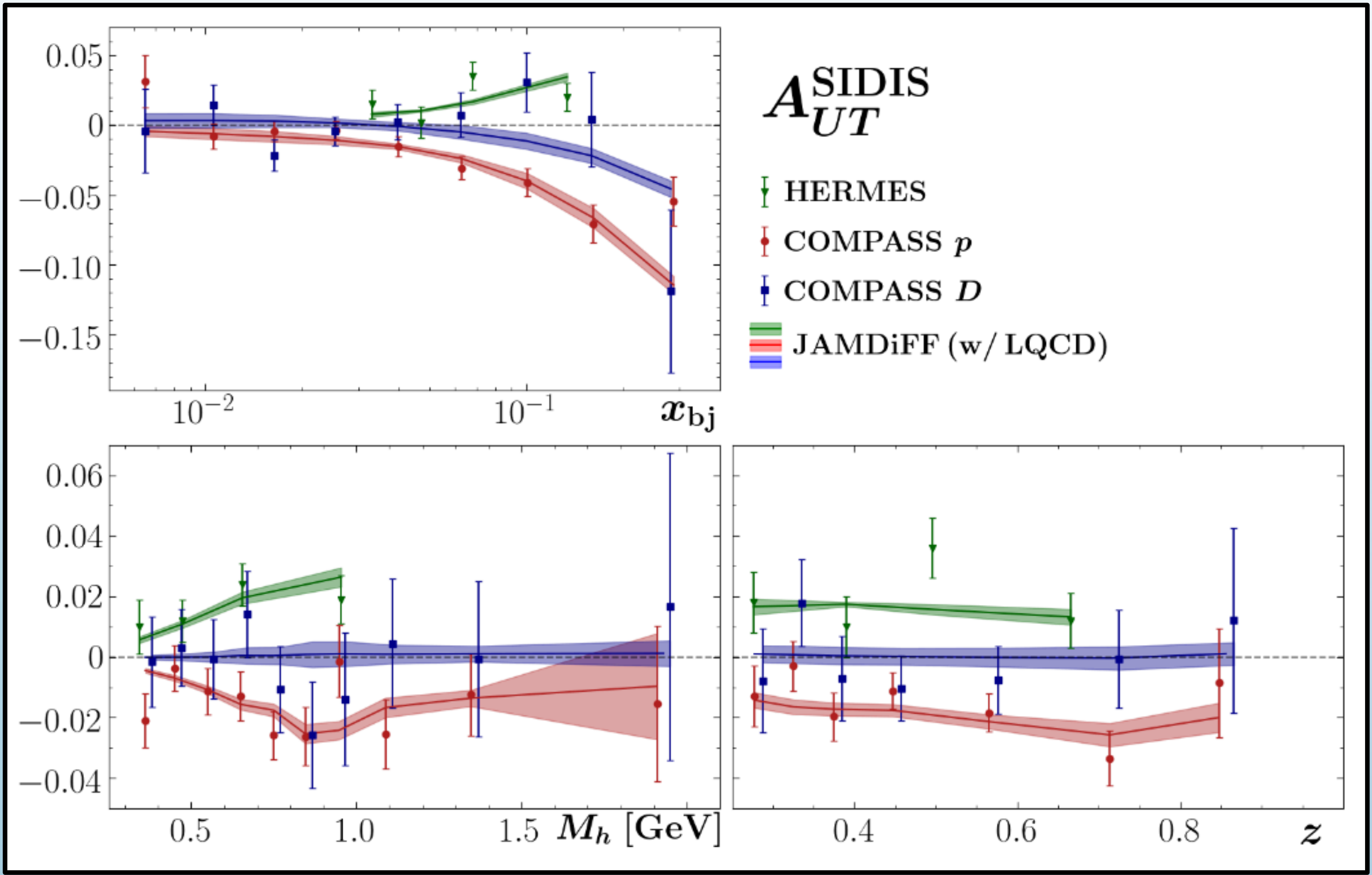


Bound:

$$|H_1^{\sphericalangle,q}| < D_1^q$$

A. Bacchetta and M. Radici, Phys. Rev. D **67**, 094002 (2003)

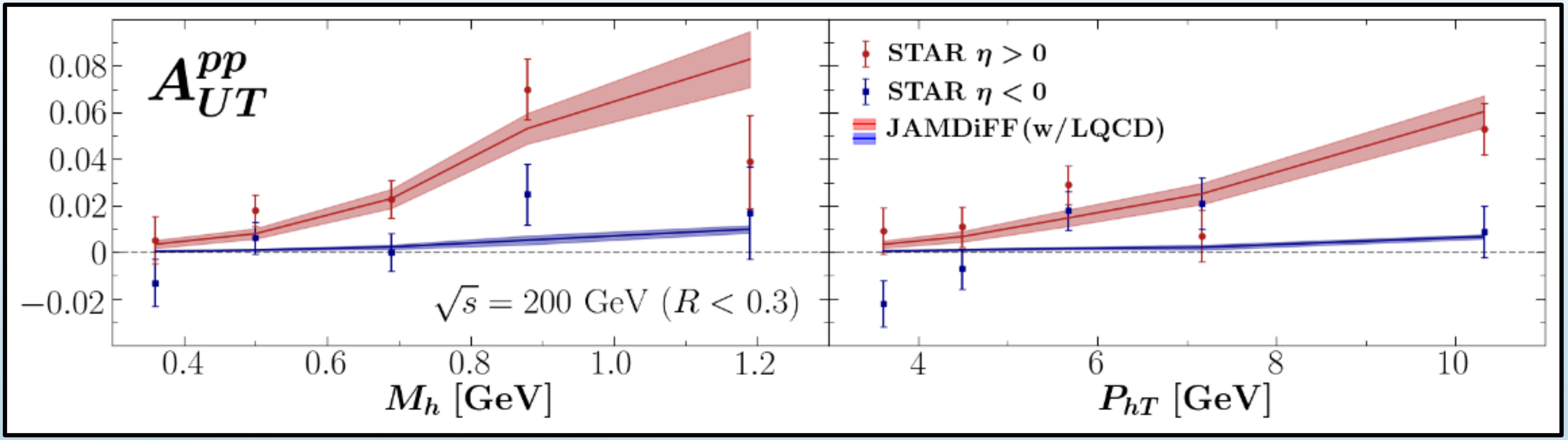
Quality of Fit (SIDIS)

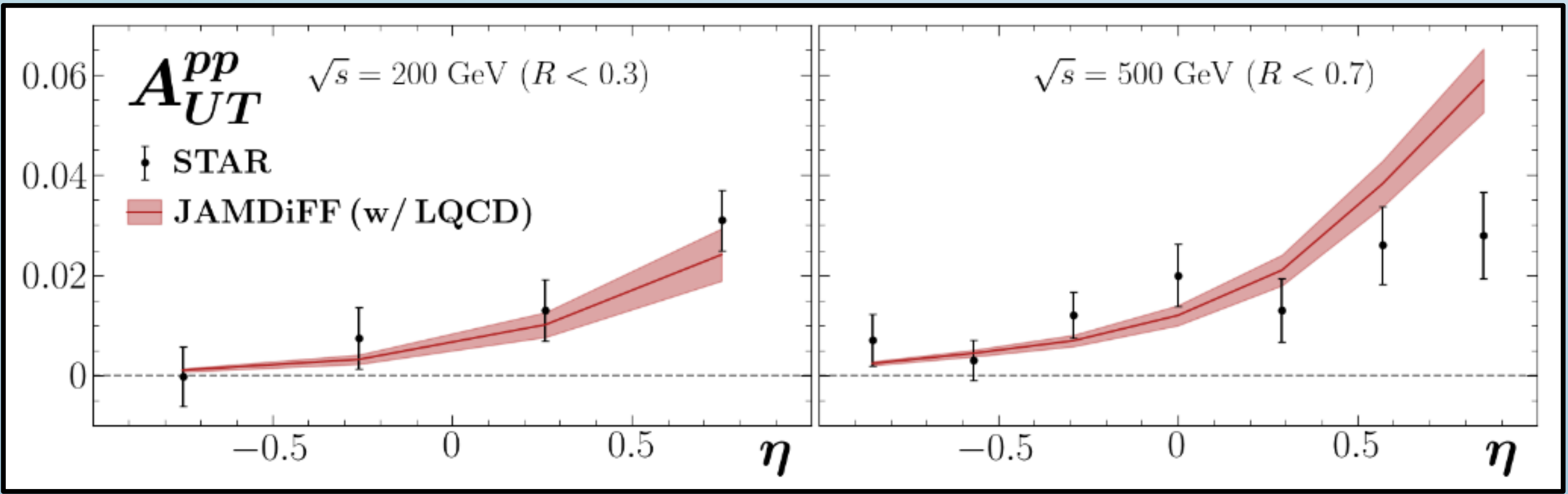


A. Airapetian *et al.*, JHEP **06**, 017 (2008)

COMPASS, arXiv:hep-ph/2301.02013 (2023)

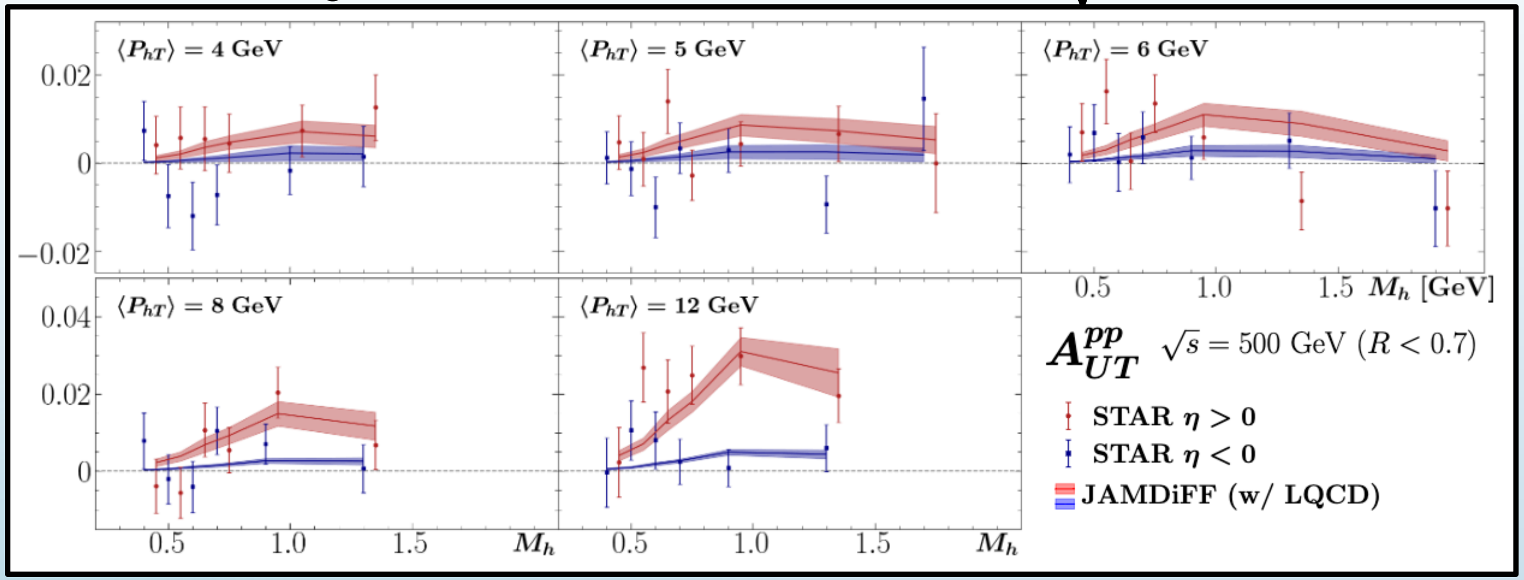
Quality of Fit (STAR $\sqrt{s}$ = 200 GeV)

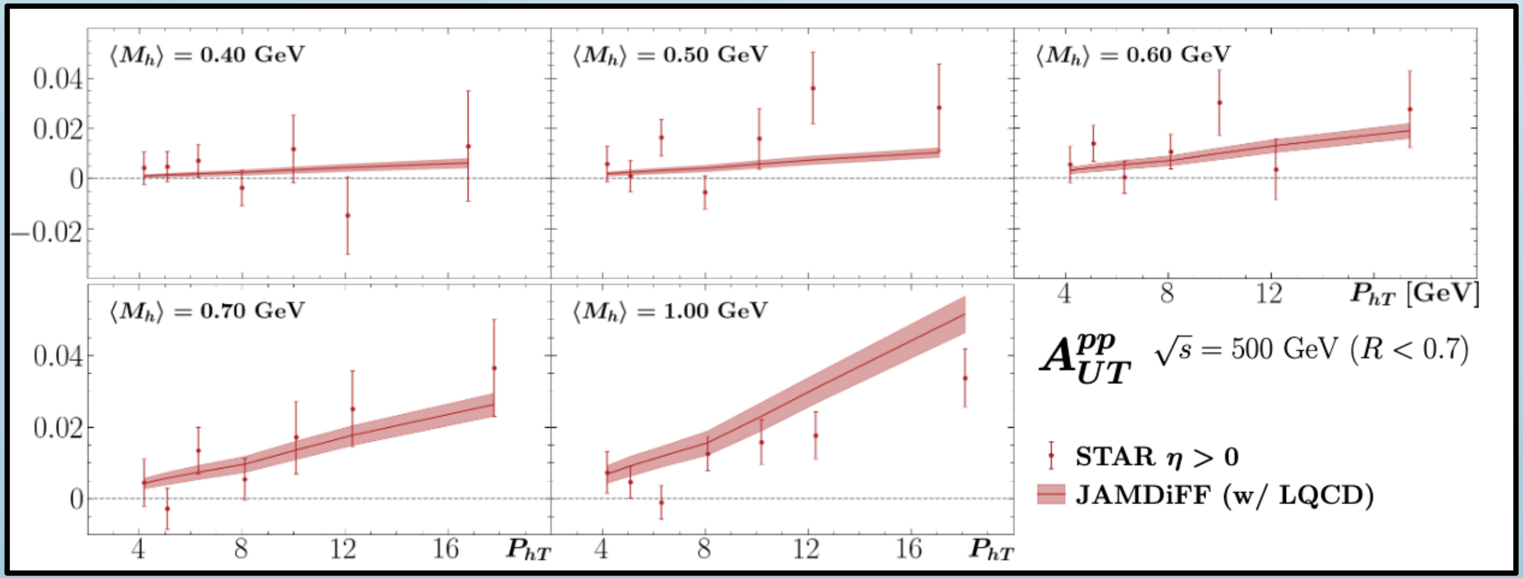




L. Adamczyk *et al.*, Phys. Rev. Lett. **115**, 24501 (2015)

Quality of Fit (STAR $\sqrt{s} = 500$ GeV)





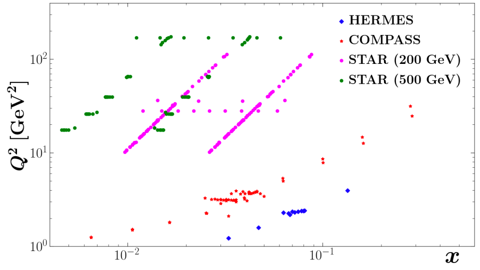
L. Adamczyk *et al.*, Phys. Rev. B **780**, 332-339 (2018)

Quality of Fit

Experiment	N_{dat}	χ^2_{red} w/ LQCD	χ^2_{red} no LQCD
Belle (cross section) [63]	1094	1.01	1.01
Belle (Artru-Collins) [92]	183	0.74	0.73
HERMES [72]	12	1.13	1.10
COMPASS (p) [71]	26	1.24	0.75
COMPASS (D) [71]	26	0.78	0.76
STAR (2015) [94]	24	1.47	1.67
STAR (2018) [64]	106	1.20	1.04
ETMC δu [28]	1	0.71	—
ETMC δd [28]	1	1.02	—
PNDME δu [25]	1	8.68	—
PNDME δd [25]	1	0.04	—
Total χ^2_{red} (N_{dat})		**1.01** (1475)	**0.98** (1471)

Experiment + Lattice + Theory

EXPERIMENT
(measured region)



THEORY
(unmeasured regions)

$$|h_1^q| < \frac{1}{2}\left[f_1^q + g_1^q\right]$$

$$\alpha_q = 1 - 2\sqrt{\frac{\alpha_s N_c}{2\pi}}$$

LATTICE
(full moments)

$$\delta u \equiv \int_0^1 \mathrm{d}x (h_1^u - h_1^{\bar{u}}),$$

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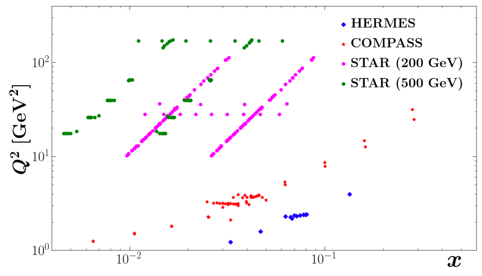

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Presently, trivial to find compatibility between any two

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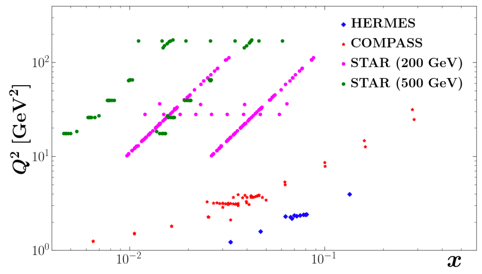

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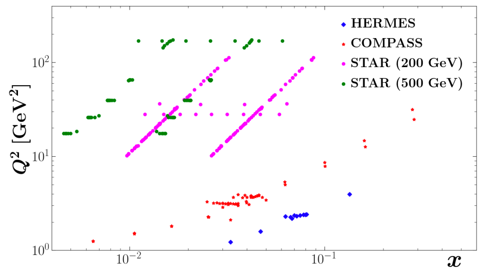

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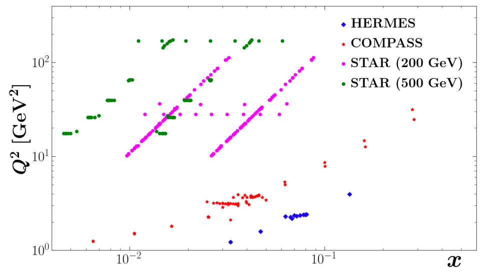

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Presently, trivial to find compatibility between any two

Only meaningful when all three are included

Future of JAM Global QCD Analysis

Improve perturbative accuracy

Spin-Averaged + Helicity PDFs: NLO → NNLO

Transversity PDFs: LO → NLO

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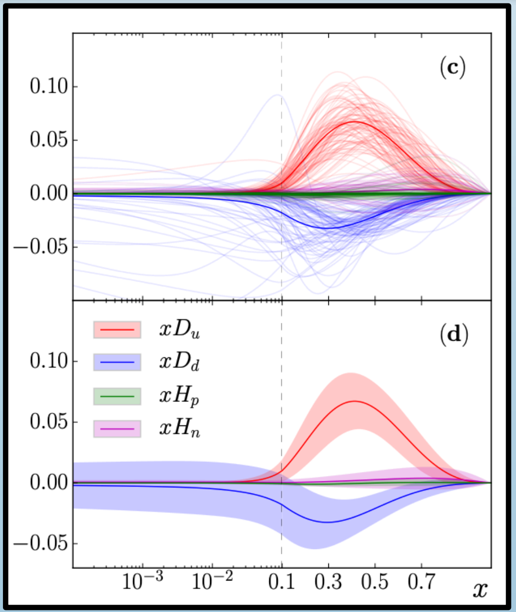
High *x* analysis for polarized data (in progress!)



N. Sato *et al.*, Phys. Rev. D **93**, no. 7, 074005 (2016)

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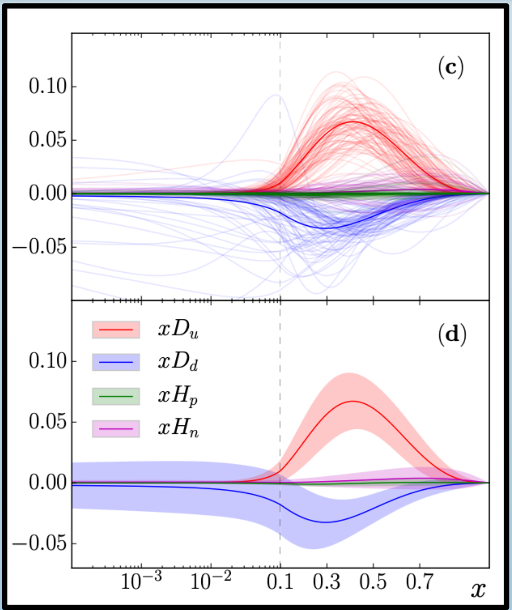

N. Sato *et al.*, Phys. Rev. D **93**, no. 7, 074005 (2016)

Simultaneous fit of DiFF channel + TMD channel + Lattice QCD