Toward extracting scattering phase shift from integrated correlation function in lattice QCD

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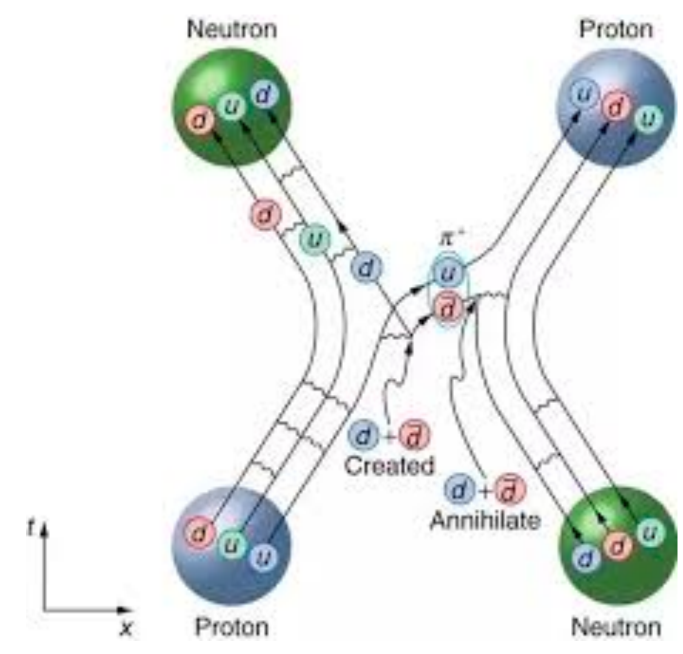
The 11th Workshop of the APS Topical Group on Hadronic Physics, Anaheim, CA
Mar 14-16, 2025

Major tasks in Nuclear/Hadron physics:

- Understand and extract few-body dynamics, such as scattering phase shift

- Understand resonances and bound states



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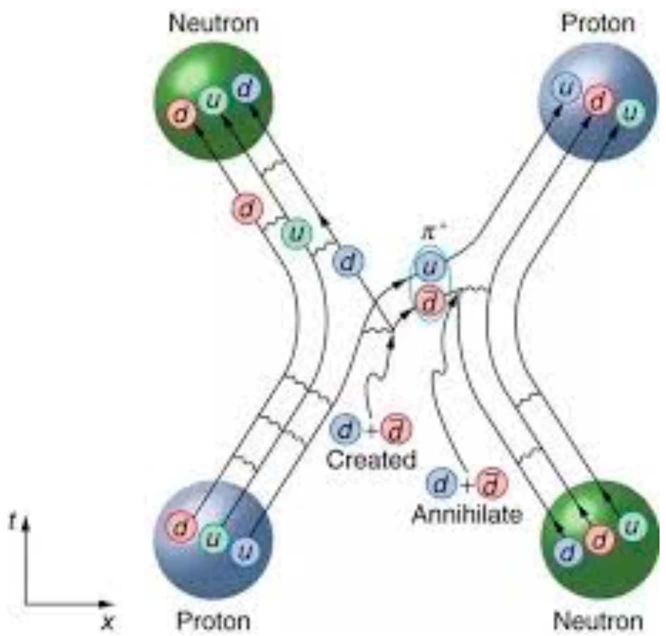

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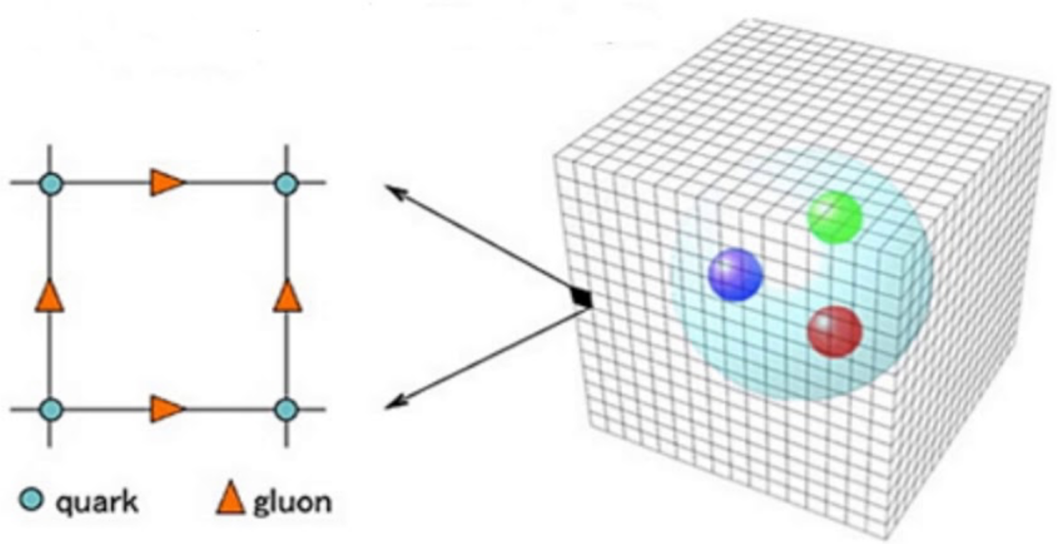

☑ Periodic box in LQCD

Lattice QCD: one route to non-perturbative dynamics

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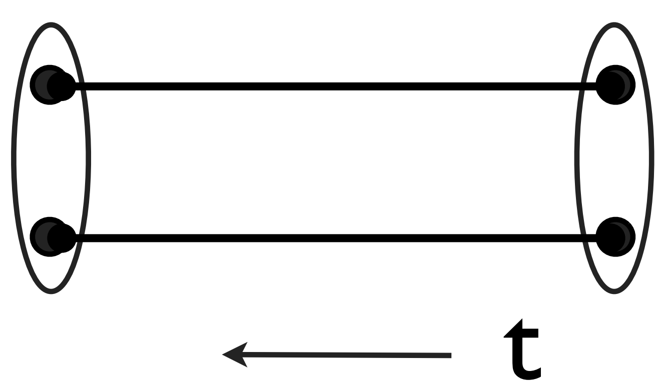

Lattice QCD: one route to non-perturbative dynamics

$$\langle 0|\mathcal{O}(t)\mathcal{O}^\dagger(0)|0\rangle \propto \int \mathcal{D}[U]\mathcal{D}[\psi,\bar{\psi}]e^{-S_G[U]-S_F[\psi,\bar{\psi},U]}\mathcal{O}(t)\mathcal{O}^\dagger(0)$$

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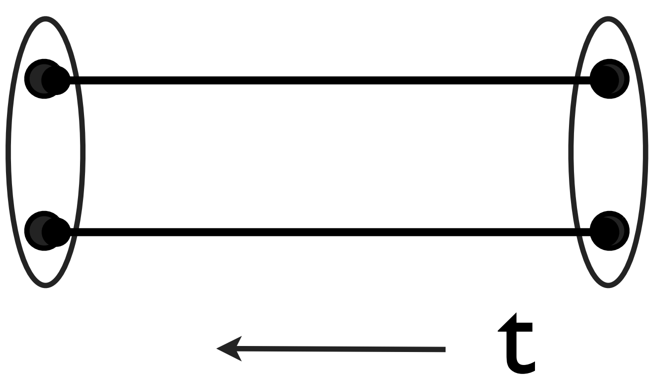

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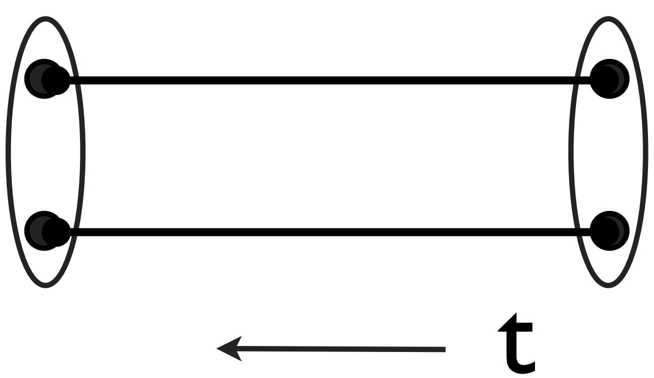

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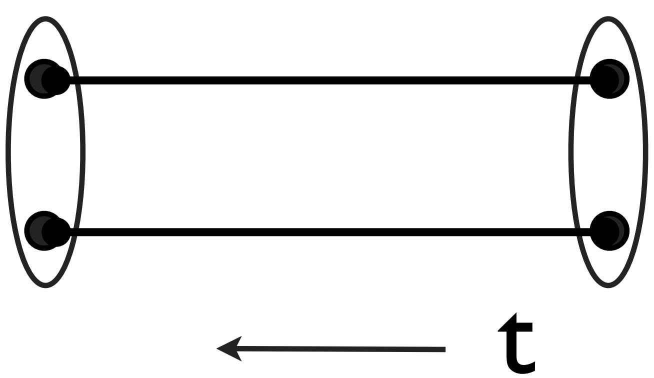
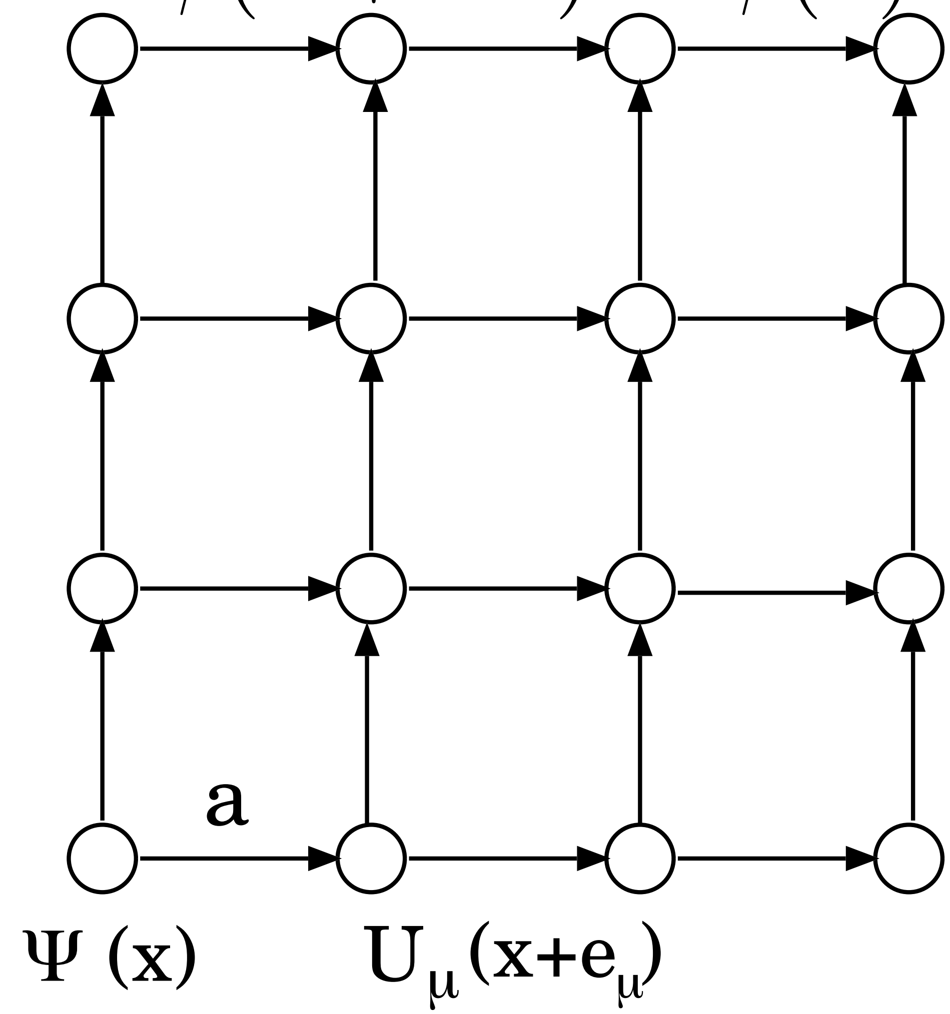


$$\psi(x+nL) = \psi(x)$$



$$U_\mu(n) = e^{iaA_\mu(n)}$$

Spectroscopy from Lattice QCD

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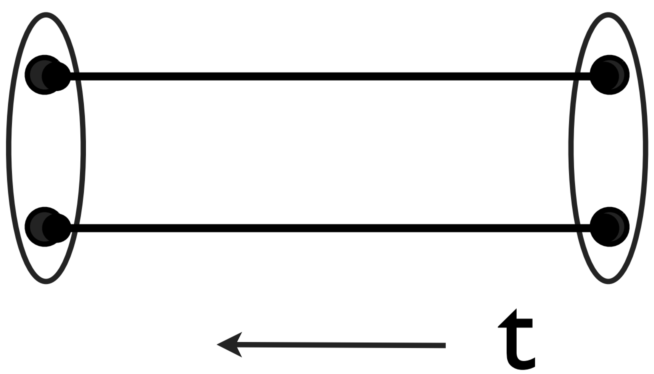

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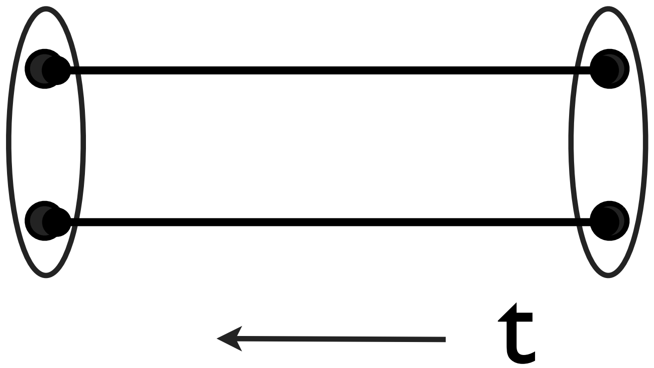

Spectroscopy from Lattice QCD

$$\mathcal{O}(t) = e^{t\hat{H}}\mathcal{O}(0)e^{-t\hat{H}}$$

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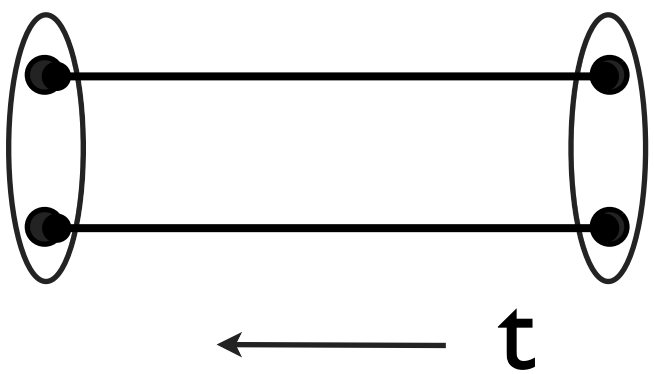

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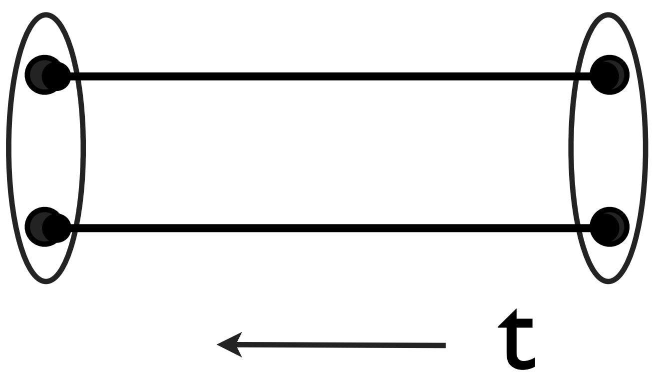

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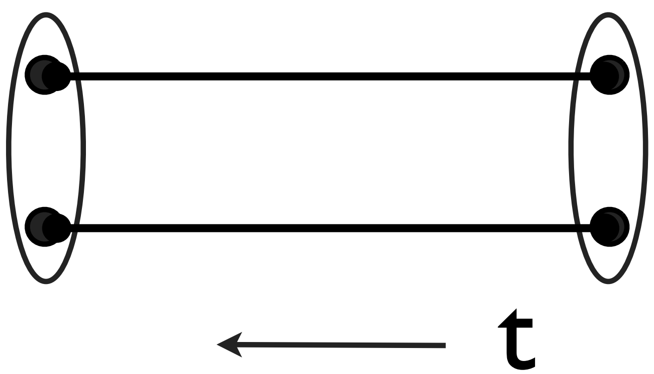

$$\rightarrow |z_0|^2 e^{-E_0 t}$$

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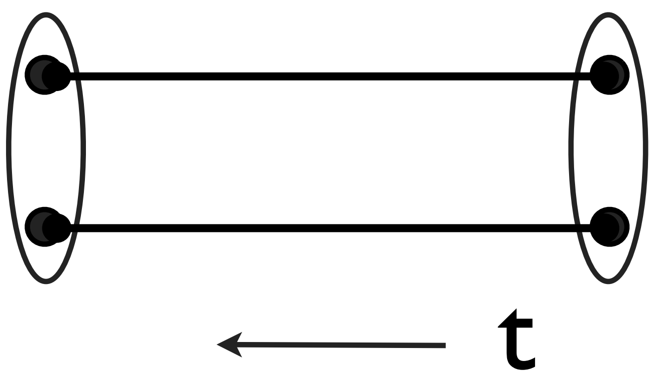

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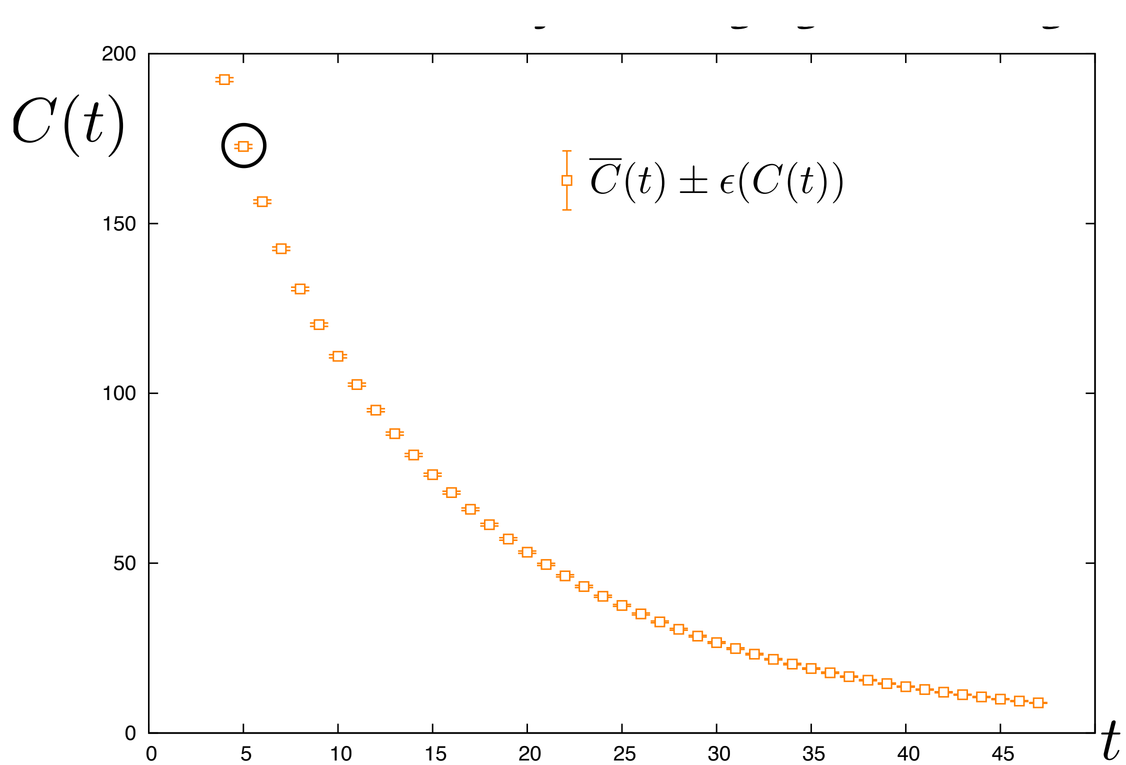

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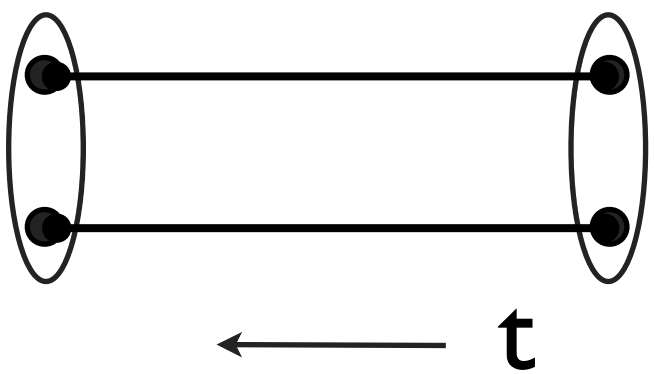
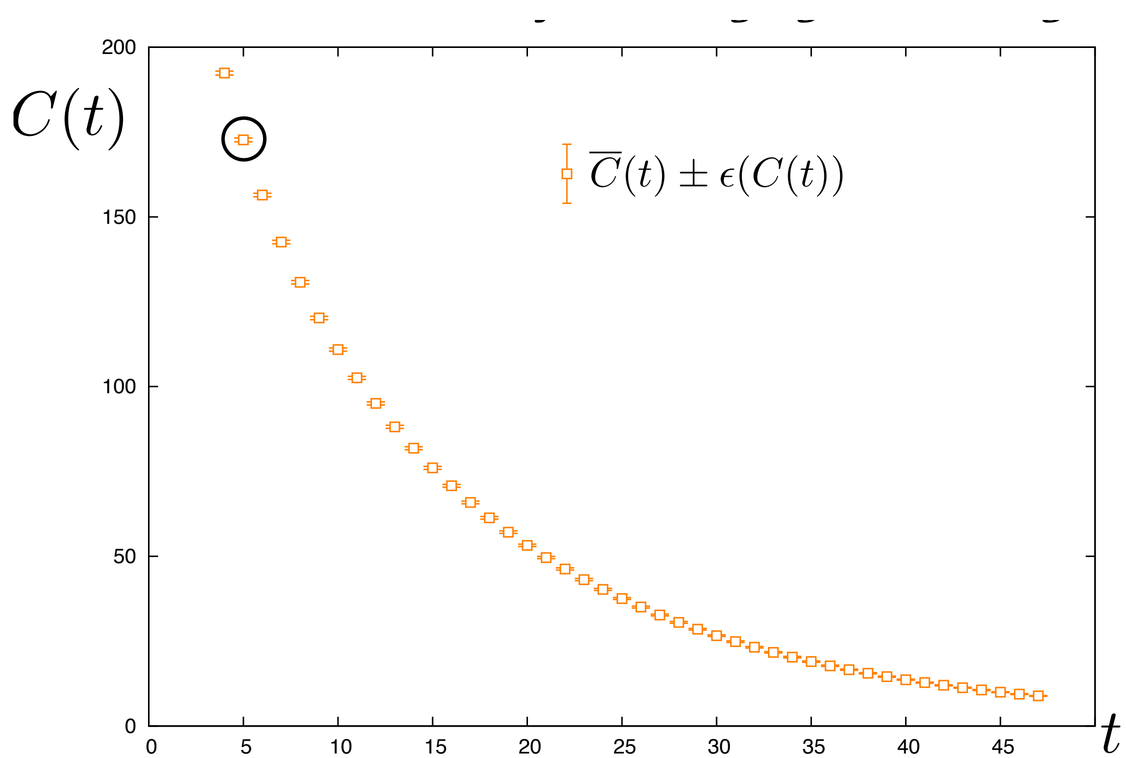
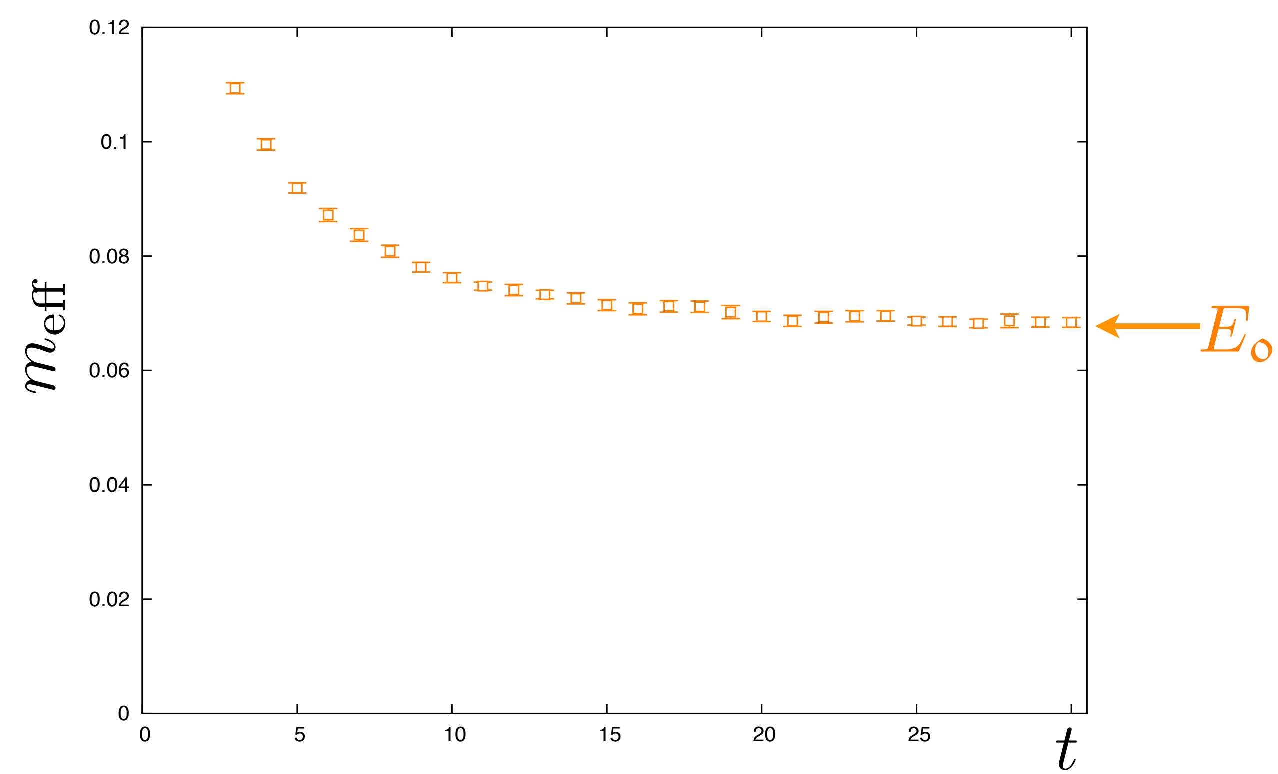


$$m_{\text{eff}}(t) = -\frac{1}{\delta t}\ln\frac{C(t+\delta t)}{C(t)}$$





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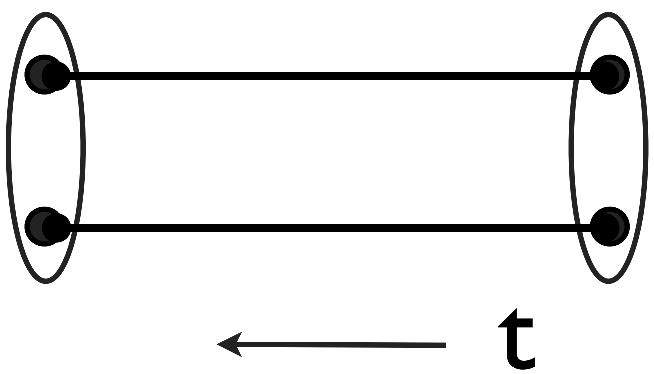

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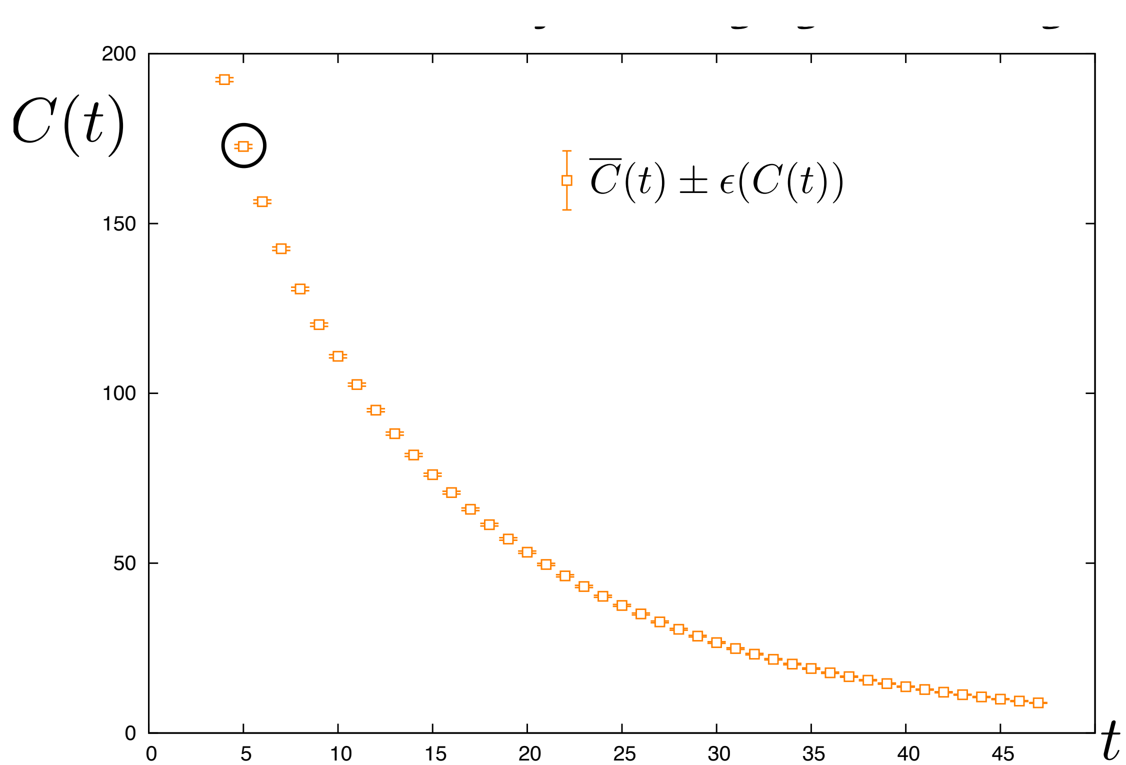
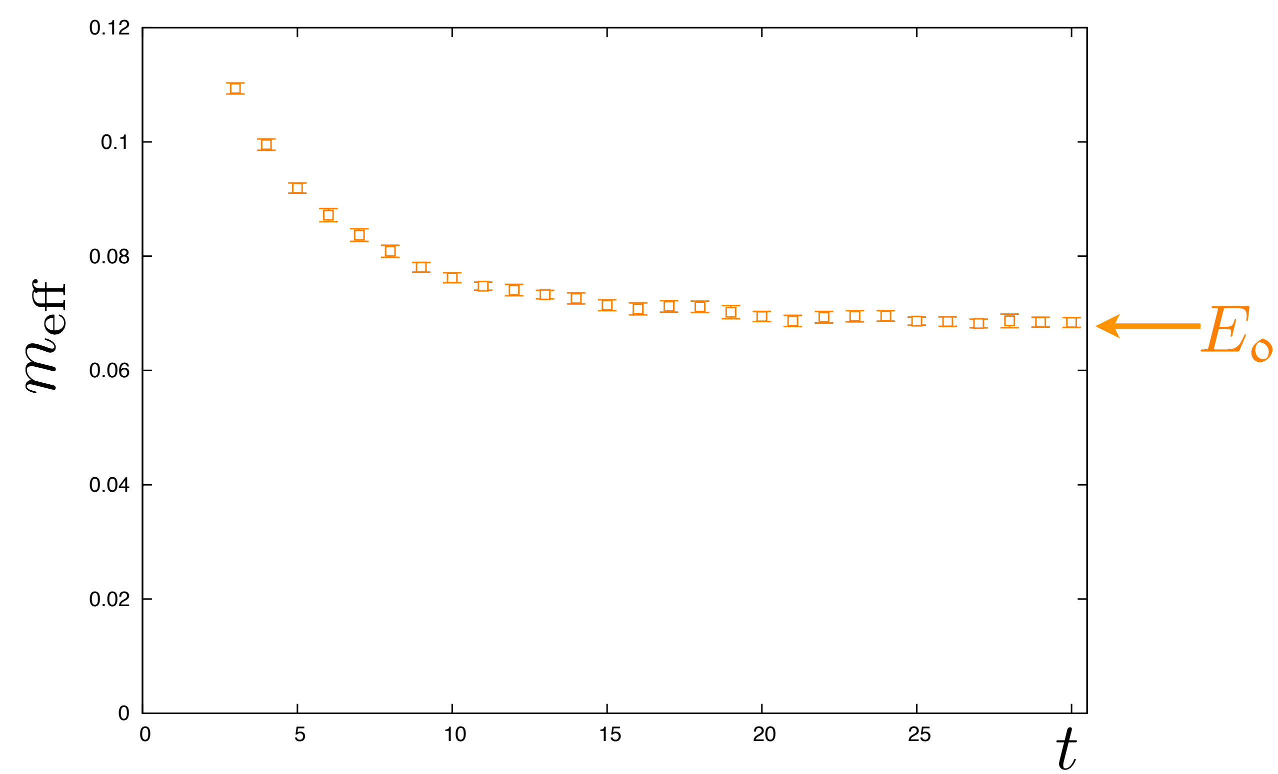
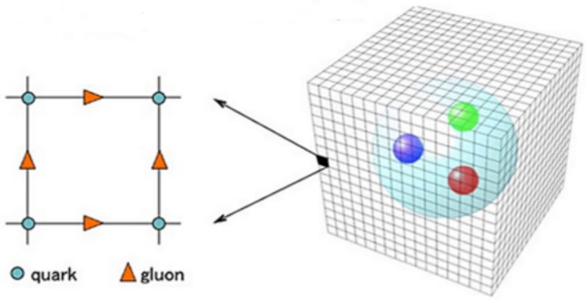




Few-body interaction in a box:



Challenges:

- No asymptotic states
- Stationary solutions instead of scattering solutions
- Discrete energies

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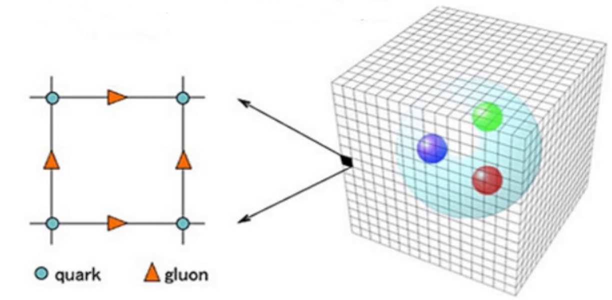

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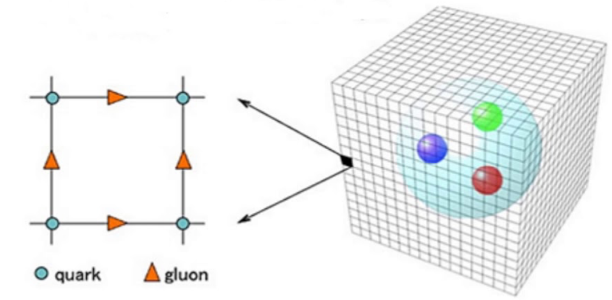

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  - Lusecher formula-like QC as result of factorization of long-range effect and short-range dynamics

$$\det\left[\cot\delta(E) - \mathcal{M}(E)\right] = 0$$

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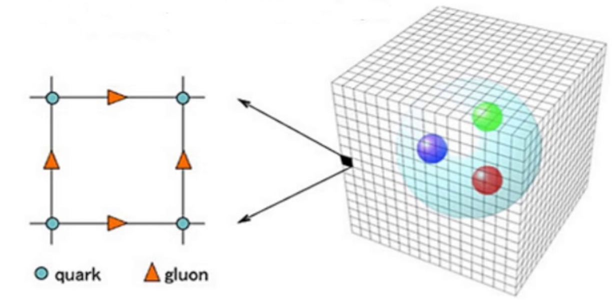

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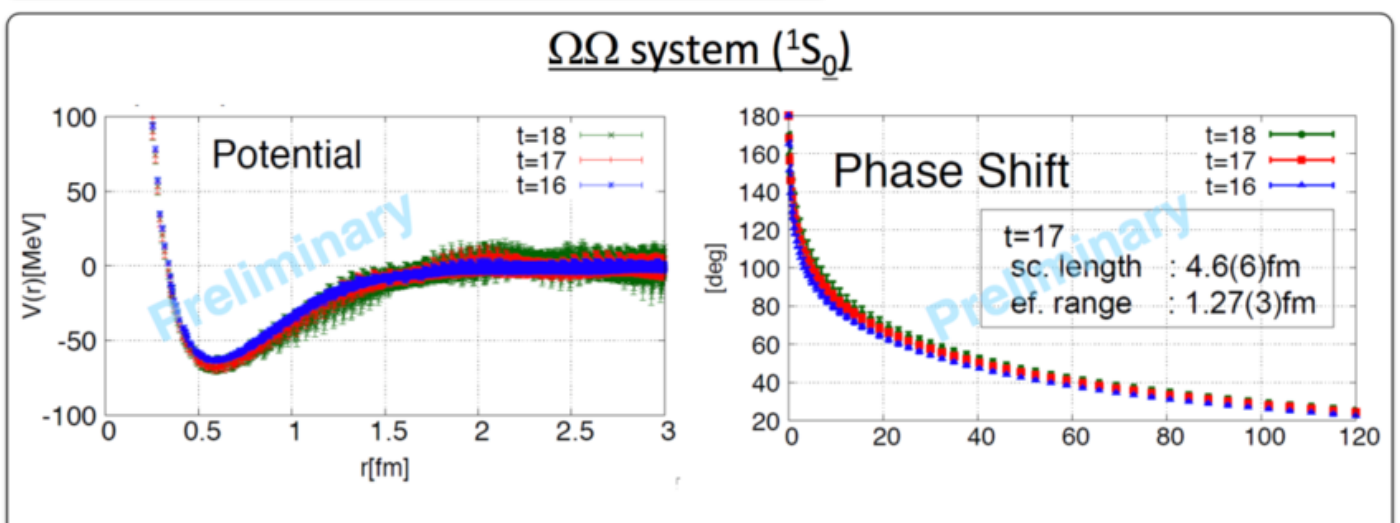
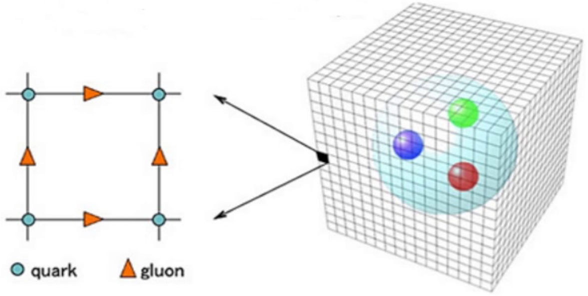
- HAL QCD collaboration potential method

$N_f = 2+1,\ m_\pi = 146\ \text{MeV},\ a \approx 0.085\ \text{fm},\ L \approx 8\ \text{fm}$



EPJ Web of Conferences **175**, 01022 (2018)
Lattice 2017

NN controversy:



- Two-Nucleon correlation function suffer Signal-to-noise ratio issue:

$$\mathcal{R}(t) \overset{t\to\infty}{\sim} e^{-(m_N - \frac{3}{2}m_\pi)t}$$

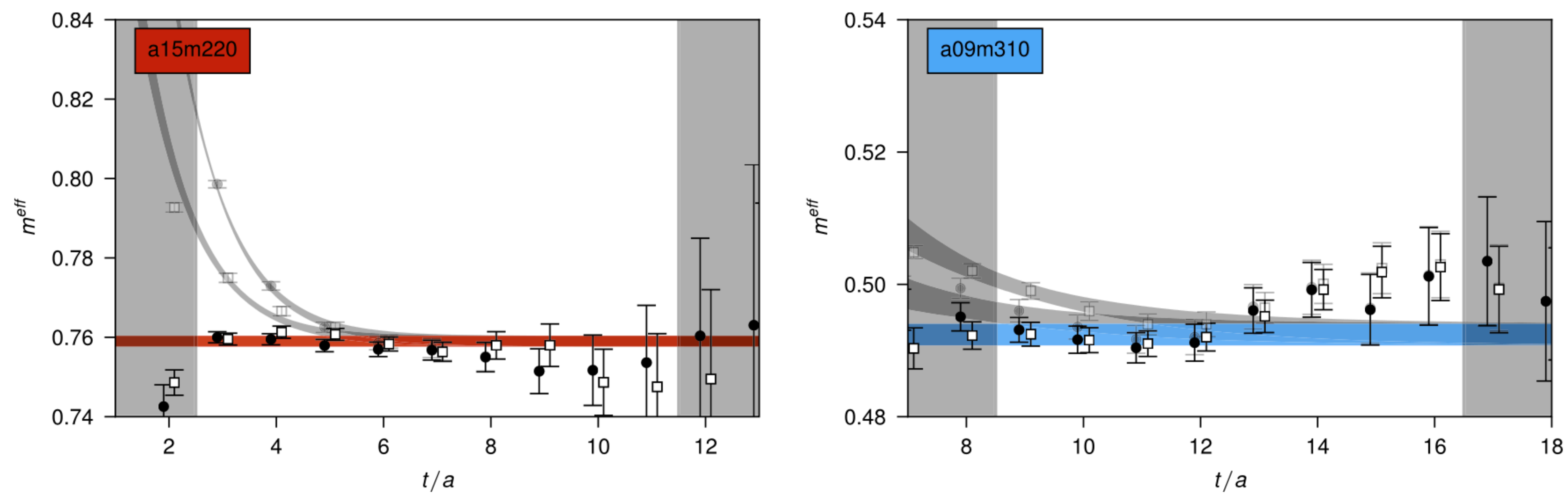


Fig. 1. Effective mass plots of the nucleon from Ref. [64] which suffer from correlated, late time fluctuations, making it more challenging to identify the ground state. The top plot is from a calculation with $a \sim 0.15$ fm and $m_\pi \sim 220$ MeV while the bottom is for $a \sim 0.09$ fm and $m_\pi \sim 310$ MeV.

Progress in Particle and Nuclear Physics 121 (2021) 103888

- Lusecher formula and HAL QCD collaboration potential method disagree on whether or not two-nucleon form a bound state with pion masses as heavy as 800 MeV

Progress in Particle and Nuclear Physics 121 (2021) 103888

- Lusecher formula face difficulties at large volume limit due to increasing density of states

Alternative to Lusecher formula method:

- The difference of integrated correlation functions between interacting and non-interacting systems approaches rapidly to its infinite volume limit which is related to scattering phase shift

$$C(t) - C_0(t) = \sum_n [e^{-\epsilon_n t} - e^{-\epsilon_n^{(0)} t}] \overset{L\to\infty}{\to} \frac{t}{\pi}\int_0^\infty d\epsilon \delta(\epsilon) e^{-\epsilon t}$$

where $$C(t) = \int_0^L dr C(rt; r0)$$

$$C(rt; r'0) = \langle 0| T\left[\widehat{\mathcal{O}}_H(r,t)\widehat{\mathcal{O}}^\dagger_H(r',0)\right]|0\rangle$$

Two-particle creation operator

PHYSICAL REVIEW D **108,** 074504 (2023)

- e-Print: 2402.15628 [hep-lat]

Alternative to Lusecher formula method:

- Exactly solvable model with contact interaction: $\left[-\frac{1}{2\mu}\frac{d^2}{dr^2}+V_0\sum_{n\in\mathbb{Z}}\delta(r+nL)\right]\psi_\epsilon^{(\mathrm{rel})}(r)=\epsilon\psi_\epsilon^{(\mathrm{rel})}(r)$

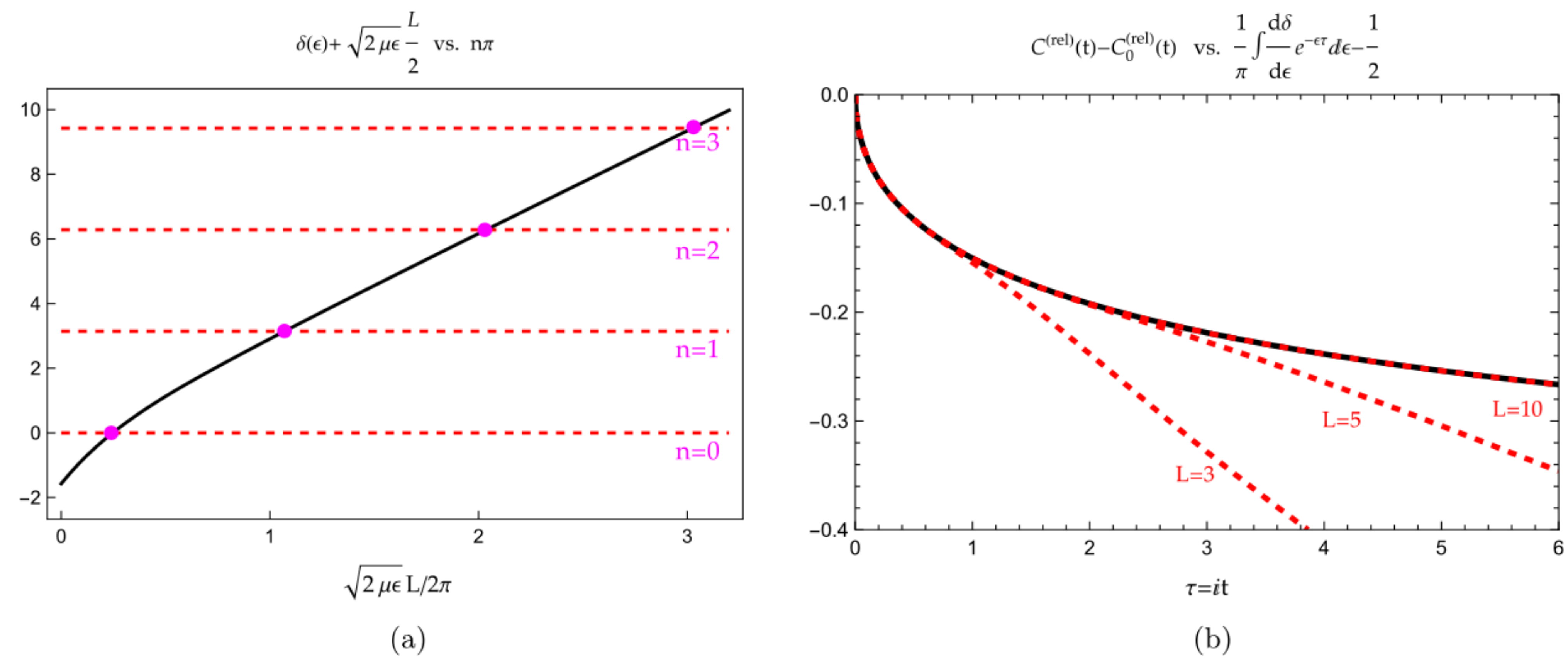


FIG. 1. The energy spectra and difference of integrated correlation function plots for particles interaction in a periodic box: (a) $\delta(\epsilon_n)+\sqrt{2\mu\epsilon_n}\frac{L}{2}$ (solid black) vs $n\pi$ (dashed red) with $L=3$, energy spectra are located at intersection points of black and red curves; (b) $\frac{1}{\pi}\int_0^\infty d\epsilon\frac{d\delta(\epsilon)}{d\epsilon}e^{-\epsilon\tau}-\frac{1}{2}$ (solid black) vs $C^{(\mathrm{rel})}(t)-C_0^{(\mathrm{rel})}(t)$ (dashed red) with $L=3, 5, 10$. The rest of parameters are taken as $V_0=0.5$ and $\mu=1$.

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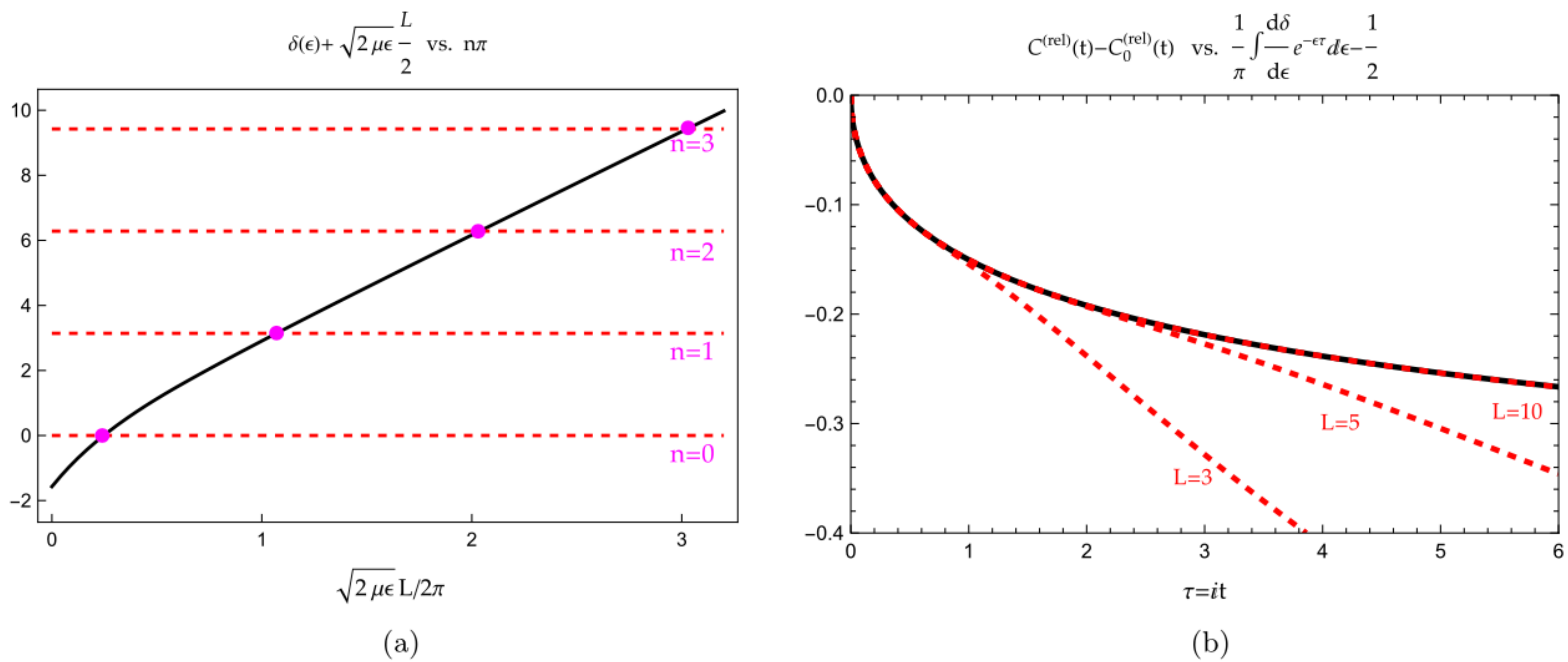
$$\longrightarrow \quad C(t)-C_0(t) \overset{L\to\infty}{\to} \frac{1}{2}\mathrm{erfc}\left(\mu V_0\sqrt{\frac{t}{2\mu}}\right)e^{(\mu V_0)^2\frac{t}{2\mu}}-\frac{1}{2}$$



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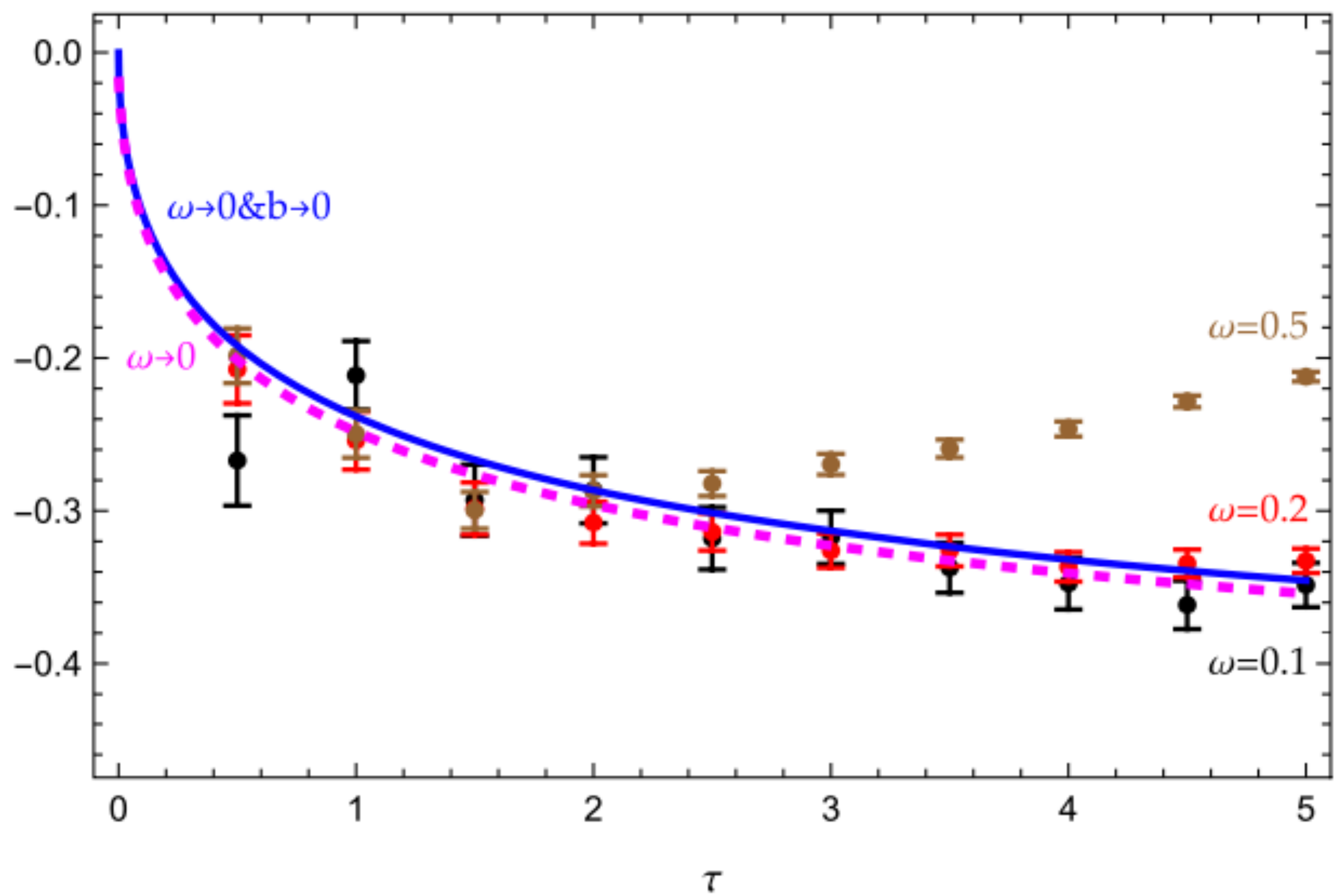
$$\left[-\frac{1}{2\mu}\frac{d^2}{dr^2}+\frac{1}{2}\mu\omega^2 r^2+V(r)\right]\psi_n(r)=\epsilon_n\psi_n(r),$$

where

$$V(r)=\begin{cases}\frac{V_0}{b}, & r\in[-\frac{b}{2},\frac{b}{2}]\\ 0, & \text{otherwise}\end{cases},\quad \overset{b\to 0}{\to} V_0\delta(r).$$

- Monte Carlo simulation test with square well potential in harmonic trap:

$$c(\tau)-c_0(\tau)\ \text{vs.}\ \frac{1}{\pi}\int d\epsilon\frac{d(\delta_+ + \delta_-)}{d\epsilon}e^{-\epsilon\tau}-\frac{1}{2}$$



PHYSICAL REVIEW D **108**, 074504 (2023)

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Alternative to Lusecher formula method:

- Relativistic extension for complex scalar lattice field theory model

$$S_E = -\kappa \sum_{x,t,\hat{n}_x,\hat{n}_t} \hat{\phi}^*(x,t)\hat{\phi}(x+\hat{n}_x, t+\hat{n}_t) + c.c.$$

$$+ (1-2\lambda)\sum_{x,t} |\hat{\phi}(x,t)|^2 + \lambda \sum_{x,t} |\hat{\phi}(x,t)|^4$$

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$$\Delta C(t) \overset{t=-i\tau}{=} \sum_{n=0}^{\infty} \left[\frac{e^{-E_n \tau}}{E_n} - \frac{e^{-E_n^{(0)}\tau}}{E_n^{(0)}} \right]$$

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$$C(rt; r'0) = \langle 0 | \mathcal{T} \left[\widehat{\mathcal{O}}(r,t) \widehat{\mathcal{O}}^\dagger(r',0) \right] | 0 \rangle . \qquad \widehat{\mathcal{O}}^\dagger(r,t) = \frac{1}{\sqrt{2L}} \int_0^L dx_2 \phi(r+x_2,t)\phi(x_2,t),$$

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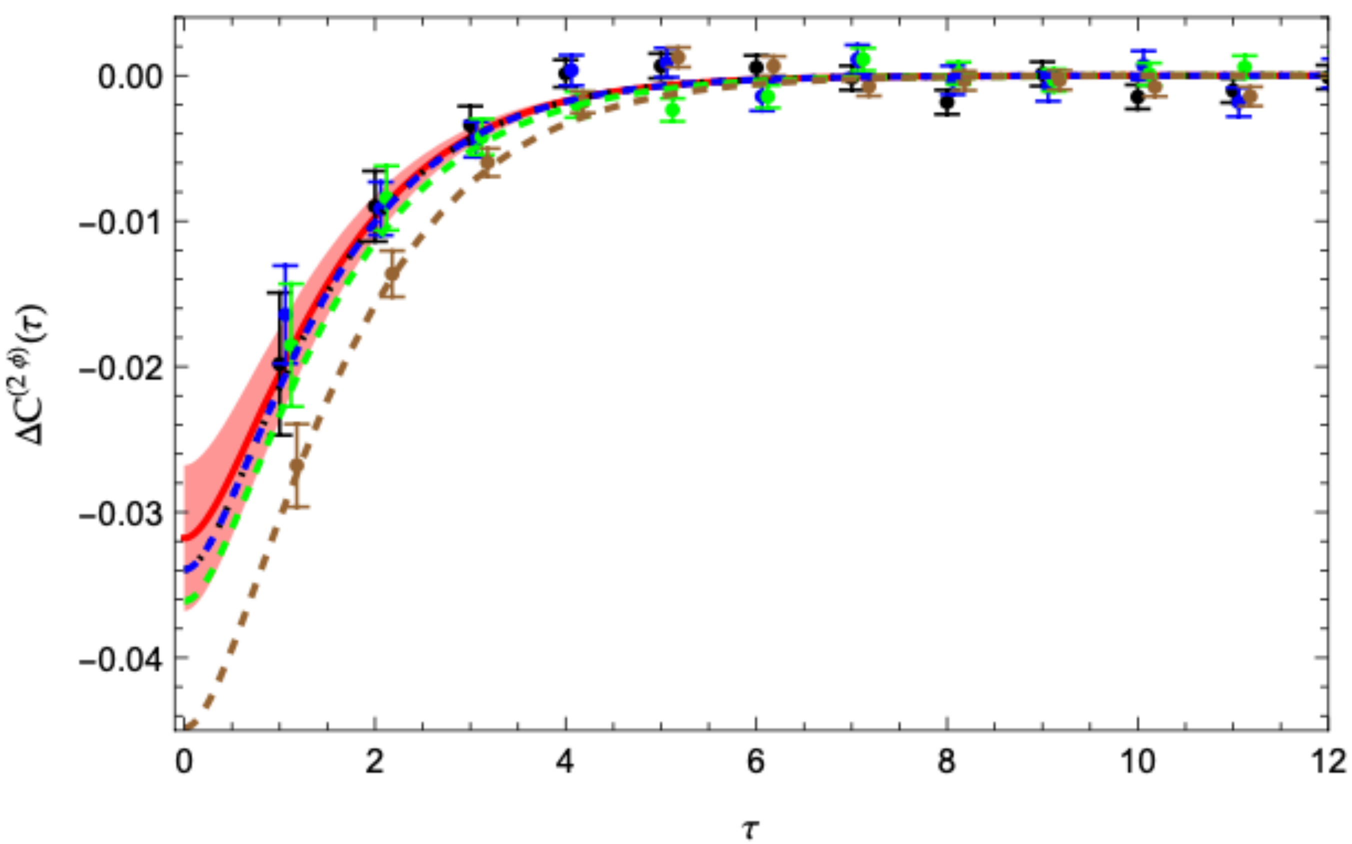
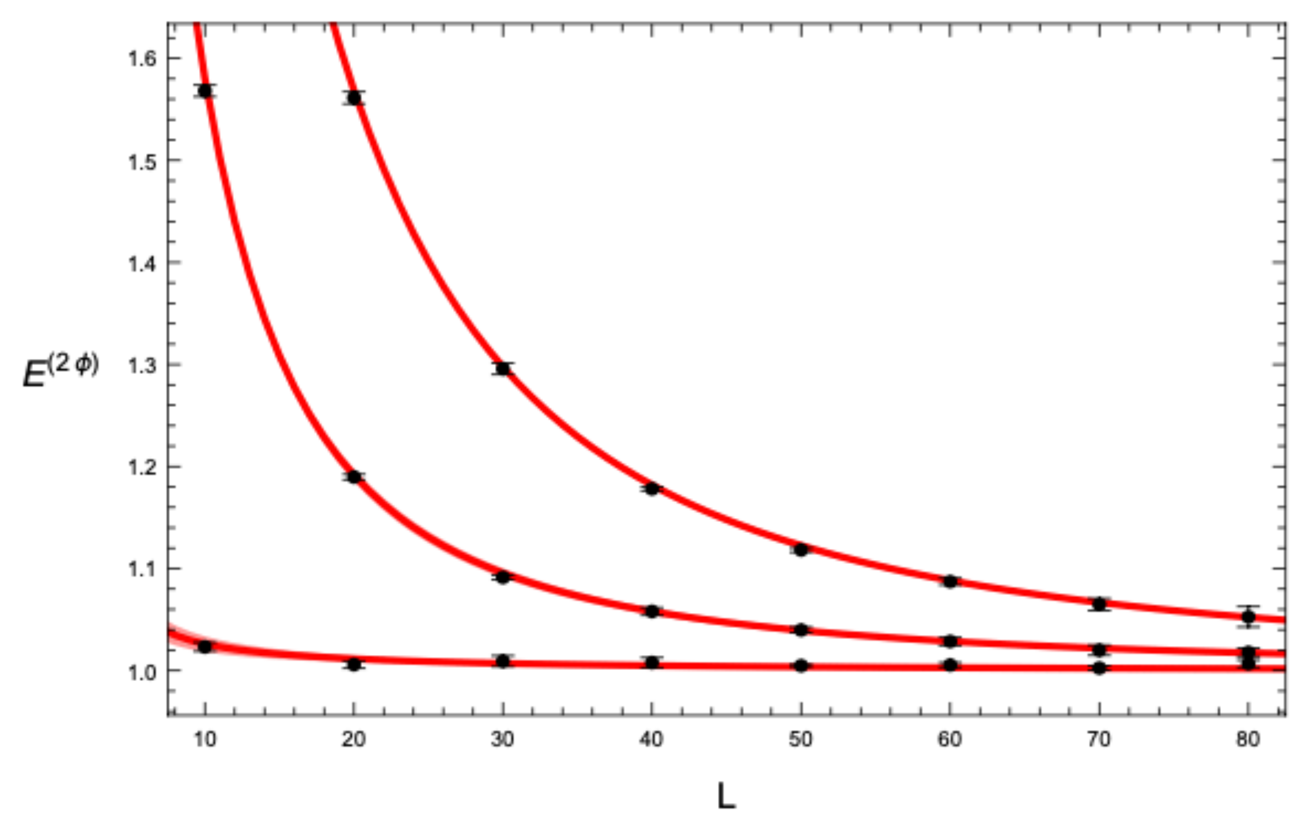


(a) Heavy ϕ field with $m \sim 0.500$ vs. lattice data.

PHYSICAL REVIEW D **110,** 014504 (2024)



(b) $E^{(2\phi)}$ with $m \sim 0.500$ and $V_0 \sim 0.271$

- e-Print: 2402.15628 [hep-lat]

Summary

Alternative approach to Lusecher formula method

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U.S. National Science Foundation

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Outlook

Integrated correlation function approach to scattering phase shifts

- ☑ **Including coupled-channel effect:**

PHYSICAL REVIEW D **111,** 054506 (2025)

Toward extracting scattering phase shift from integrated correlation functions. III. Coupled channels

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PHYSICAL REVIEW D **111**, 054506 (2025)

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PHYSICAL REVIEW D **111**, 054506 (2025)

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- ☑ **Quantum simulation (coming soon).**

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