Progress toward TMD factorization at next-to-leading power

11th workshop of the APS Topical Group on Hadronic Physics (GHP2025), Anaheim, CA, March 14 - 16, 2025

Leonard Gamberg
Penn State University- Berks
March 16, 2025

with
- Paul Anderson, Nobuo Sato, Wally Melnitchouk, Richard Whitehill (JAM Collaboration)
- Zhongbo Kang, Ding-Yu Shao, John Terry, Fany Zhao (ongoing)

PennState Berks

Outline NLP unpolarized observables

- Discussion of *NLP* effects in SIDIS

- Beyond *LP* factorization **Collinear P_T TMD and or**

- **The observable** $R_{SIDIS} = \frac{\sigma_L}{\sigma_T} \sim \frac{F_{UU,L}}{F_{UU,T}}$ **Feynman "Photon-Hadron Phys." 1972, Ravndal, PLB 1973**

- **The observable** $\langle \cos\phi \rangle$ **Georgi & Cahn, PRL 1978, PLB 1978**

  Critique of the perturbative QCD calculation of azimuthal dependence in leptoproduction
  emphasize importance intrinsic k_T the early days/birth of TMD physics

Leads to..

1. **The challenge of matching "low" to "high" transverse momentum spectrum** q_T or P_{hT}
2. **Factorization BUT @ NLP order** α_s **… issues … necessary (but not sufficient) consistency checks**
3. **Ongoing work**

Importance of NLP TMDs & Factorization

- Importance of NLP "TMD-like" observables underscored while suppressed by $(M/Q)^n$ wrt LP observables
  - NLP/SLP TMDs as sizable as leading-power TMDs in situations where Q not that large… e.g. the kinematics of fixed-target experiments
- Their understanding is required for a complete description of "benchmark processes" SIDIS, DY & e^+e^-…
- Are of interest offer a mechanism to probe physics of quark-gluon-quark correlations, provide novel information about the partonic structure of hadrons; only recently unexplored beyond α_s
  - Such correlations as quantum interference effects, related to average transverse forces acting on partons inside (polarized) hadrons as well as other phenc

- Experimental information from SIDIS on effects related to subleading TMDs is & has been available DESY/Zeus, Fermi-LAB, HERMES, COMPASS, JLab
- Opportunity for EIC with its large kinematical coverage & for SoLID TMD program for making further groundbreaking progress in this area

  - **NB: Iff factorization can be established beyond "tree level" @ next to leading order**
    -Global analysis in terms of NLP TMDs

TMD fact at NLP w & w/o polarization (Literature)

Preprints: JLAB-THY-23-3780, LA-UR-21-20798, MIT-CTP/5386

TMD Handbook

Renaud Boussarie[1], Matthias Burkardt[2], Martha Constantinou[3], William Detmold[4], Markus Ebert[4,5], Michael Engelhardt[2], Sean Fleming[6], Leonard Gamberg[7], Xiangdong Ji[8], Zhong-Bo Kang[9], Christopher Lee[10], Keh-Fei Liu[11], Simonetta Liuti[12], Thomas Mehen[13], Andreas Metz[3], John Negele[4], Daniel Pitonyak[14], Alexei Prokudin[7,16], Jian-Wei Qiu[16,17], Abha Rajan[12,18], Marc Schlegel[2,19], Phiala Shanahan[4], Peter Schweitzer[20], Iain W. Stewart[4], Andrey Tarasov[21,22], Raju Venugopalan[18], Ivan Vitev[10], Feng Yuan[23], Yong Zhao[24,4,18]

10 - Subleading TMDs

L. Gamberg, A. Metz, I. Stewart

From a historical perspective it is very interesting that the subleading-power $\cos\phi_h$ azimuthal modulation of the unpolarized SIDIS cross section was important for the development of the TMD field, since one of the earliest discussions of transverse parton momenta in DIS is related to this observable [290, 291, 1237]; see also Sec. 5.1 for more details. Generally, although suppressed by Λ/Q with respect to leading-power observables, subleading TMD observables are typically not small, especially in the kinematics of fixed-target experiments. In fact, the first-ever observed SSA in SIDIS was a sizeable power-suppressed longitudinal target SSA for pion production from the HERMES Collaboration [480]. Those measurements, which triggered many theoretical studies and preceded the first measurements of the (leading-power) Sivers and Collins SSAs, were critical for the growth of TMD-related research.

e-Print:2304.03302 [hep-ph]

[290] R. N. Cahn, *Azimuthal Dependence in Leptoproduction: A Simple Parton Model Calculation*, *Phys. Lett. B* **78** (1978) 269.

[1237] F. Ravndal, *On the azimuthal dependence of semiinclusive, deep inelastic electroproduction cross-sections*, *Phys. Lett. B* **43** (1973) 301.

[480] HERMES collaboration, A. Airapetian et al., *Observation of a single spin azimuthal asymmetry in semiinclusive pion electro production*, *Phys. Rev. Lett.* **84** (2000) 4047

TMD factorization @NLP w & w/o polarization (vast subject & incomplete ...Literature)

F. Rivindal PLB 1972
Georgi Politzer PRL 1978
Cahn PLB 1978 (response to Georgi Politzer PRL 1978)

A.Kotzinian NPB (1994)
J. Levelt, P.Mulders Phys. Rev. D(1994)
R. Tangerman, P. Mulders hep-ph/9408305 [hep-ph] (1994)
P.Mulders, R. Tangerman, NPB 461(1996)
D. Boer, P. Mulders, Phys.Rev.D 57 (1998)
L. Gamberg, D. Hwang, A Metz, M. Schlegel, PLB 639 (2006), uncanceled rapidity div. @tw3-factorization
Boer Vogelsang DY PRD 2006
Koike Nagashima Vogelsang SIDIS NPB 2006 Large P_T
A.Bacchetta, D. Boer, M. Diehl, P. Mulders JHEP (2008) factorization at NLP consistency checks on matching
A.P. Chen, J.P. Ma, Phys. Lett. B 768 (2017)
I. Feige, D.W. Kolodrubetz, I. Moult, I.W. Stewart, J. High Energy Phys. 11 (2017)
I. Balitsky, A. Tarasov, J. High Energy Phys. 07 (2017)
I. Balitsky, A. Tarasov, J. High Energy Phys. 05 (2018)
M.A. Ebert, I. Moult, I.W. Stewart, F.J. Tackmann, G. Vita, H.X. Zhu, J. High Energy Phys. 12 (2018)
M.A. Ebert, I. Moult, I.W. Stewart, F.J. Tackmann, G. Vita, H.X. Zhu, J. High Energy Phys. 04 (2019)
Moult, I.W. Stewart, G. Vita, arXiv:1905 .07411, 201
A. Bacchetta Bozzi, Echevarria, Pisano, Prokudin, Radici, Physics Letters B 797 (2019)
A.Vladimirov Moos, Scimemi, & S.Rodini JHEP 2022
M. Ebert A. Gao I. Stewart JHEP 06 (2022)
S. Rodini, A. Vladimirov JHEP 08 (2022)
L.Gamberg, Z.Kang, D.Shao, J.Terry, F.Zhao arXiv: e-Print:221.13209 (2022)
I.Balitsky, JHEP 03 (2023) and 2024
L.Gamberg, Z.Kang, D.Shao, J.Terry, F.Zhao in prep 2024
Also Spin transverse spin-dependence Qui Sterman collinear higher twist 1991 NLB
X. Ji, J.W. Qiu, W. Vogelsang, and F. Yuan, Phys.Rev.Lett. 97 (2006), Phys.Lett.B 638 (2006),Phys.Rev.D 73 (2006)

Challenges of *SLP/NLP* TMDs

NLP TMD observables challenging in comparison to the current state-of-the-art of leading power observables
Treatments in the literature are mostly limited to a tree-level formalism until recently

First studies beyond tree level : "Matches & Mis-matches" **Bacchetta et al. JHEP 2008, Chen et al. PLB 2017

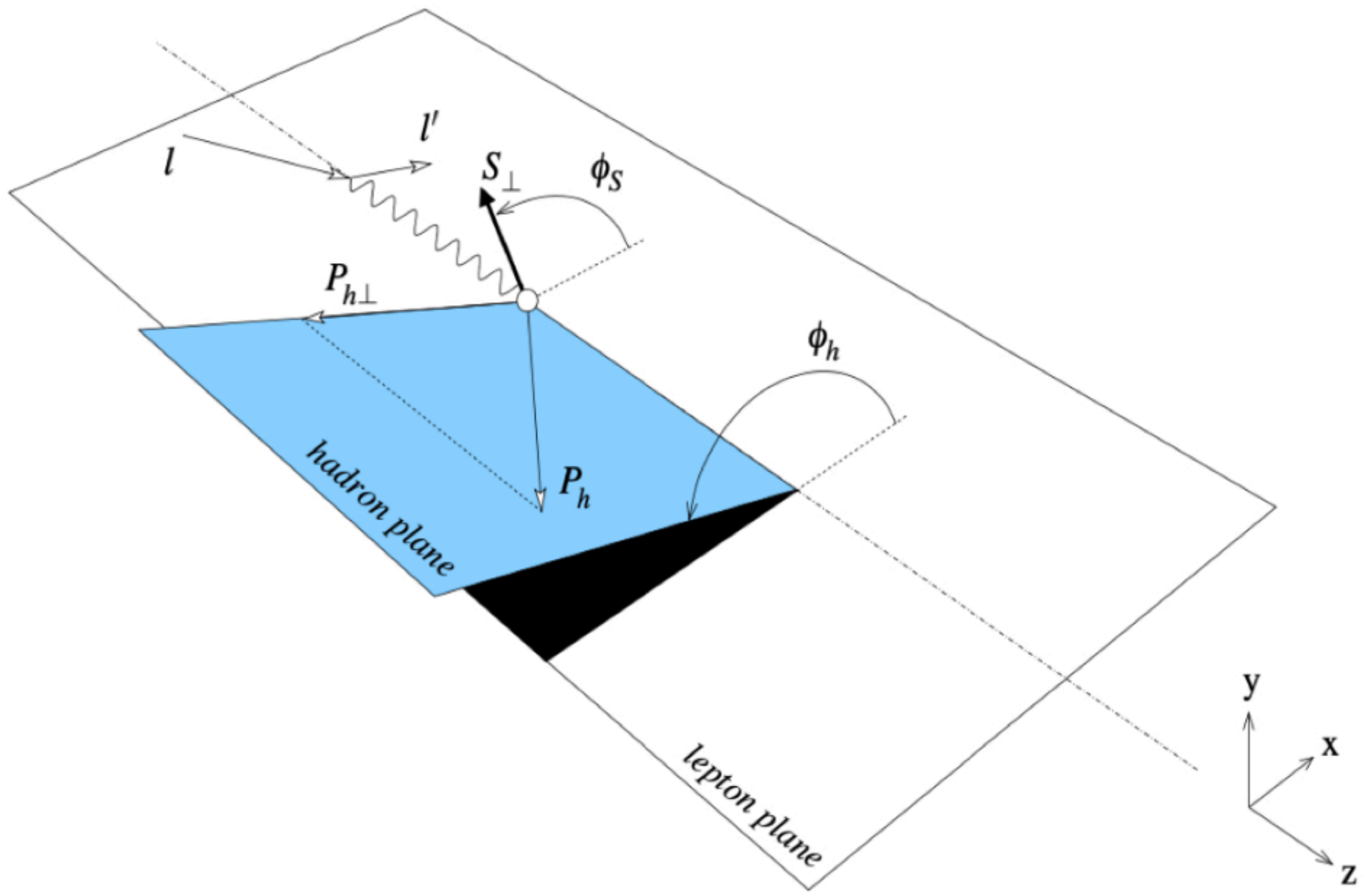
More recently

A.P. Chen, J.P. Ma, PLB (2017)
Bacchetta et al. PLB 2019
MIT group, Gao, Ebert, Stewart JHEP 2022
Gamberg, Kang, Shao, Terry, Zhao arXiv: e-Print:221.13209
Vladimirov, Rodini, Scimemi, Moos, JHEP 2021, 2022, JHEP 2023, PRD 2024
Balitsky 2023 rapidity only TMD evolution

- ***In arXiv: e-Print:221.13209* present a systematic procedure for stress testing TMD factorization for DY & SIDIS at NLP *using CSS formalism* which addresses disagreements in the literature**

TMD Factorization & $P_\perp$ Collinear Factorization

- TMD: applicable $\Lambda_{QCD} \sim P_{h\perp} \ll Q$ Collinear: applicable $P_{h\perp} \sim Q \gg \Lambda_{QCD}$
- $P_{h\perp} \sim \mathbf{k}_T$ or $\mathbf{p}_T$ intrinsic transverse momentum partons CS described via TMDs
- $P_{h\perp} \gg \mathbf{k}_T$ or $\mathbf{p}_T$ generated transverse momentum in the final state as perturbative radiation & non-perturbative structure is given by collinear pdfs & FFs



$$E'E_h \frac{d\sigma_{ep\to e'hX}}{d^3l'\,d^3P_h} \approx \hat{\sigma}_{eq\to e'q'} \otimes f_1 \,\tilde{\otimes}\, D_{h/q'}.$$

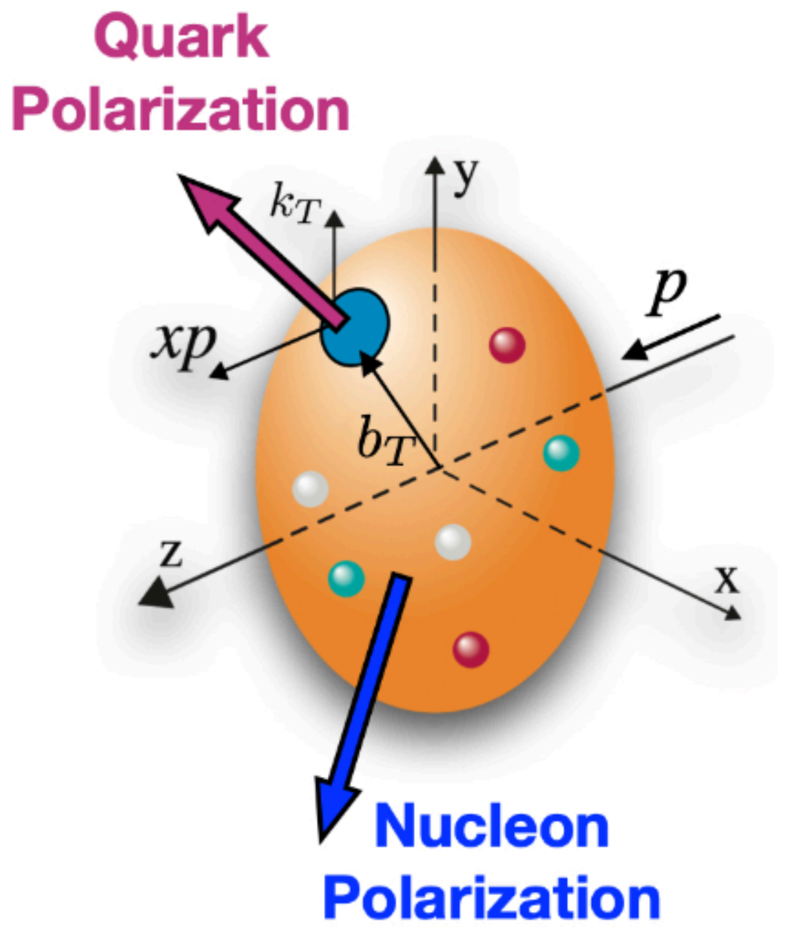
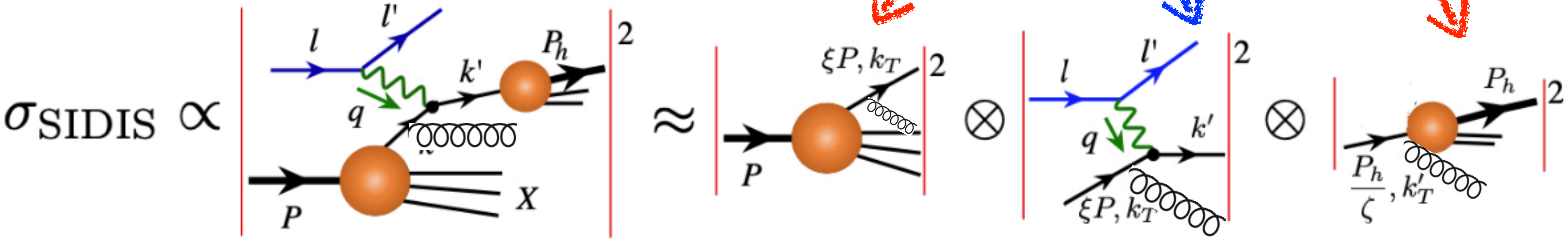


Figure 1.1: Illustration of the mo- spin variables probed on distributions.



R_{SIDIS} *NLP* TMDs?

- NLP TMD observables challenging in comparison to the current "state-of-the-art" of LP observables

- Treatments in literature mostly limited to tree-level formalism (until recently ...)

- The best known of these (& relatively not well understood) is the ratio of longitudinal and transverse cross section (SIDIS) role of longitudinal and transverse photon: power supressed $(m/Q)^2$

Feynman 1972 Photon Hadron Interactions, Ravndal PLB 1973:

$$R = \frac{\sigma_L}{\sigma_T} = \frac{4\left(m^2 + \langle p_\perp^2 \rangle\right)}{Q^2} \sim \frac{F_{UU,L}}{F_{UU,T}}$$

where $\langle p_\perp^2 \rangle$ is the intrinsic parton transverse momentum

... For details see Cahn 1989 PRD often assumed that $F_{UU,L}$ is negligible at low transverse momentum

R_{SIDIS} NLP TMDs?

In 21st century interpretation in semi-inclusive DIS (SIDIS)-TMD physics

$$R = \frac{\sigma_L}{\sigma_T} \to \frac{F_{UU,L}}{F_{UU,T}}$$

$$\frac{d\sigma}{dx\, dy\, dz\, d\phi_S\, d\phi_h\, dP_{h\perp}^2} = \frac{\alpha^2}{xQ^2}\,\frac{y}{2\,(1-\varepsilon)}\left\{F_{UU,T} + \varepsilon\, F_{UU,L} + \quad \ldots\right.$$

Mulders & Tangerman 1995
Bacchetta et al. JHEP 2007

$$F_{UU,T} = C[f_1 D_1]\,, \quad F_{UU,L} = C[\,.\,?\,.\,]$$

$$\mathcal{C}\big[w\, f\, D\big] = x \sum_a e_a^2 \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, \delta^{(2)}\big(\boldsymbol{p}_T - \boldsymbol{k}_T - \boldsymbol{P}_{h\perp}/z\big)\, w(\boldsymbol{p}_T, \boldsymbol{k}_T)\, f^a(x, p_T^2)\, D^a(z, k_T^2)$$

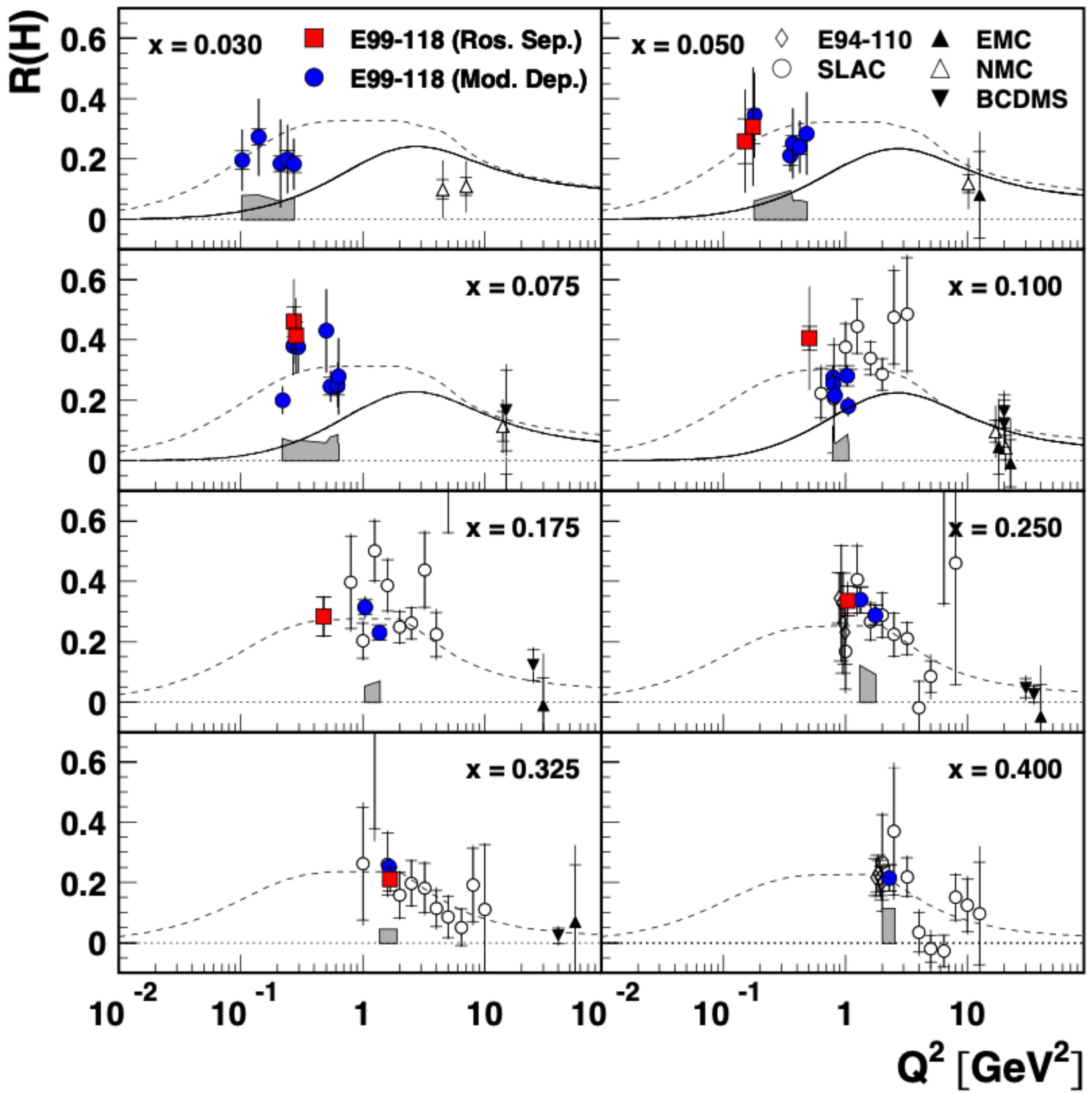
In 20th century interpretation in collinear inclusive DIS physics

Recall the inclusive cross section can be expressed in terms of σ_T and σ_L and or the structure functions $F_L(F_2 \,\&\, F_1)$ & $F_T(F_1)$, the cross sections via absorption of transverse and longitudinal photons:

JLab exp. (E99-118) Prl 2007

$$R = \frac{\sigma_T}{\sigma_L} = \frac{F_L}{F_T} = \frac{1}{2xF_1}\left\{F_2\left(1+\frac{4x^2M^2}{Q^2}\right) - 2xF_1\right\}$$

$$\lim_{Q^2\to\infty} R \approx \frac{4x^2M^2}{Q^2} \to 0\,,$$ Zero in the scaling limit



Comparison of the values of $R(x, Q^2)$ for hydrogen from the JLab exp. (E99-118) to results of other exps.

R_{SIDIS} *NLP* TMDs ?

- R_{sidis} esitmate sizable contribution up to 20%
  Bacchetta et al. (MAP) JHEP **10**, 127 (2022)

$$\frac{d\sigma}{dx\, dy\, dz\, d\phi_S\, d\phi_h\, dP_{h\perp}^2} = \frac{\alpha^2}{xQ^2} \frac{y}{2(1-\varepsilon)} \Big\{ F_{UU,T} + \varepsilon F_{UU,L} +$$

- ε ratio of longitudinal and transverse photon flux ...

$$\varepsilon = \frac{1 - y - \frac{1}{4}\gamma^2 y^2}{1 - y + \frac{1}{2}y^2 + \frac{1}{4}\gamma^2 y^2}, \quad \gamma^2 \equiv \frac{4M^2x^2}{Q^2}$$

- Findings demonstrate $F_{UU,L}$ can't be ignored, is substantial & essential for an accurate interpretation of $F_{UU,T}$ which is associated with leading twist TMDs.

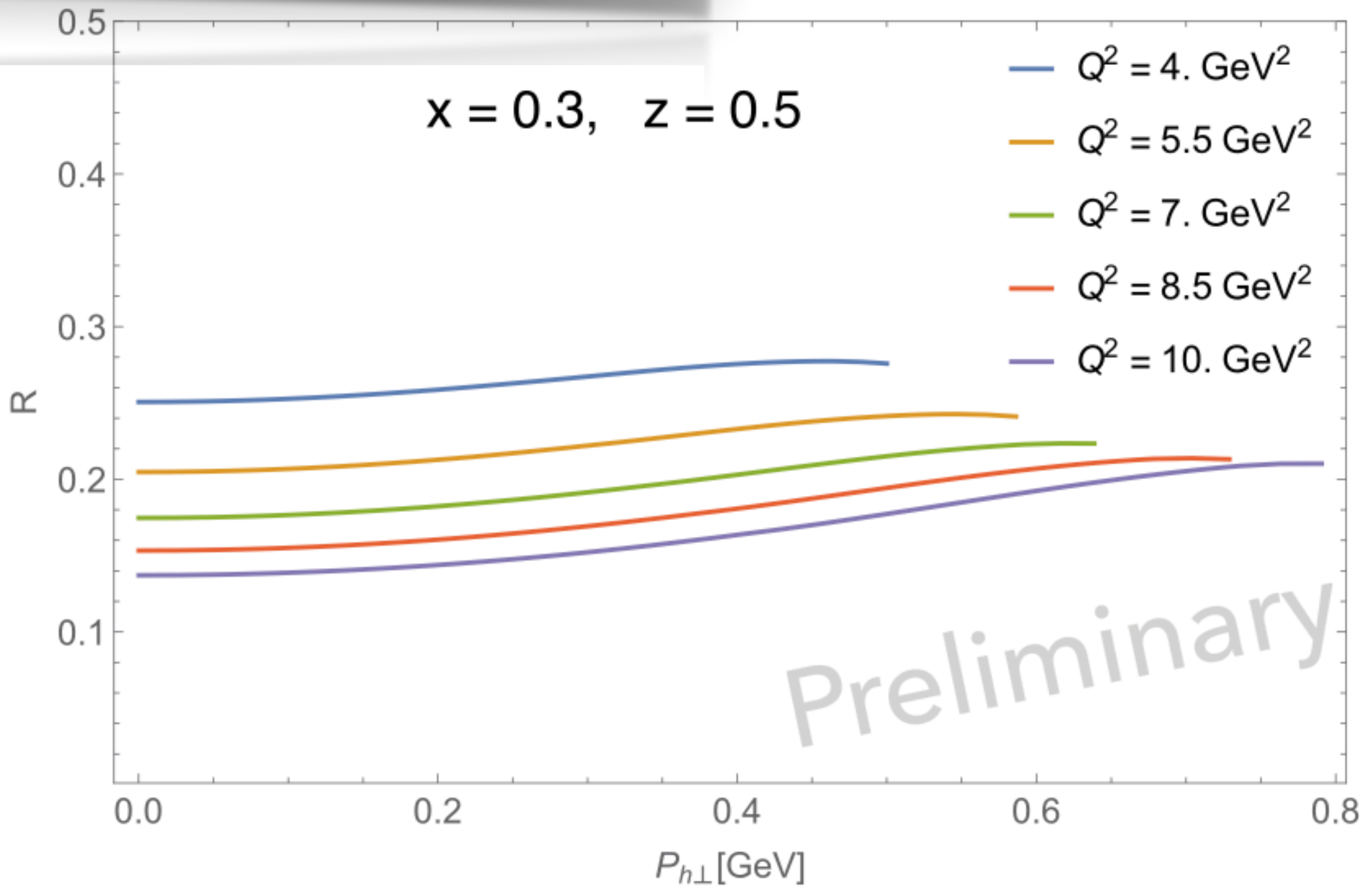


Fig. 18 Estimate of $R_{\mathrm{SIDIS}} = F_{UU,L}/F_{UU,T}$ versus the hadron transverse momentum $P_T\,(P_{hT})$ at fixed values of x and z and for different values of Q^2, compatible with JLab22 kinematics, using MAP22 TMD analysis [134]

- ***What is the physics here?!***

Context TMD Correlator at tree level "twist 4"

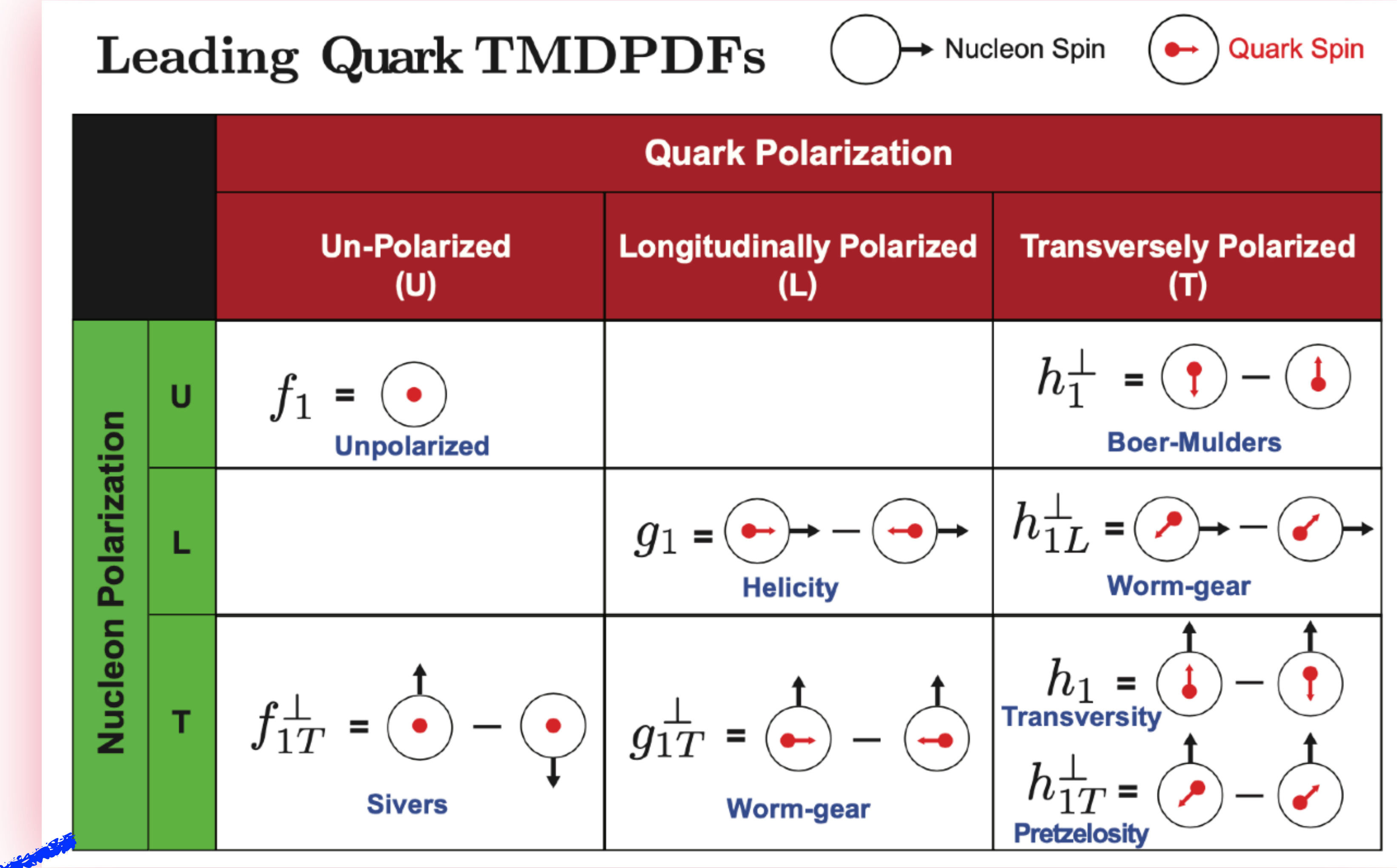
K. Goeke, A. Metz, M. Schlegel PLB 2005

NNLP:

$$\Phi^{[\gamma^-]} = \frac{M^2}{(P^+)^2}\left[f_3(x,\vec{k}_T^2) - \frac{\varepsilon_T^{ij} k_{Ti} S_{Tj}}{M} f_{3T}^{\perp}(x,\vec{k}_T^2) \right]$$

Correlator at tree level @ "twist" 4

previously of academic interest



LP & NLP :

$$\Phi^{[\gamma^+]} = f_1(x,\vec{k}_T^2) - \frac{\varepsilon_T^{ij} k_{Ti} S_{Tj}}{M} f_{1T}^{\perp}(x,\vec{k}_T^2),$$

$$\Phi^{[\gamma^i]} = \frac{M}{P^+}\left[\frac{k_T^i}{M}\left(f^{\perp}(x,\vec{k}_T^2) - \frac{\varepsilon_T^{jk} k_{Tj} S_{Tk}}{M} f_T^{\perp\prime}(x,\vec{k}_T^2) \right) + \quad \ldots \right.$$

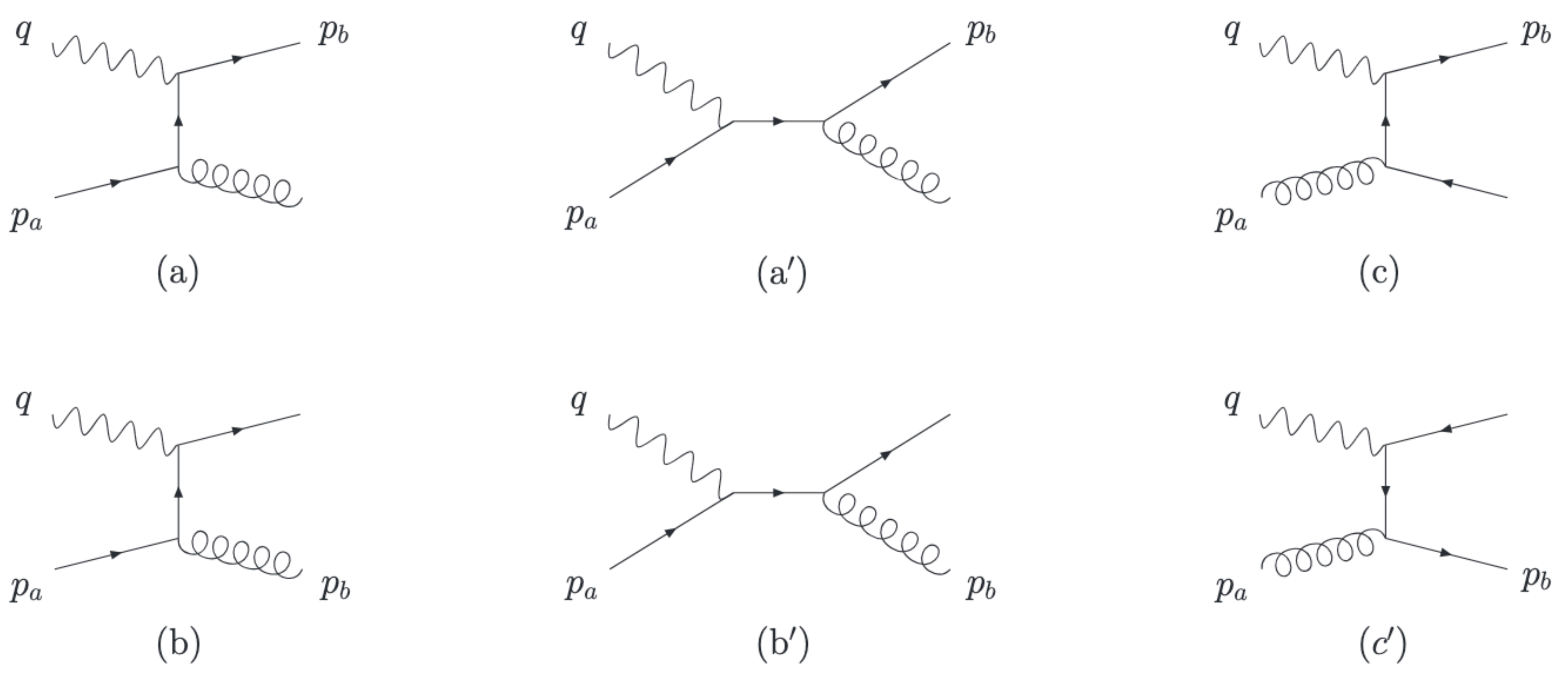
Subleading Quark TMDPDFs

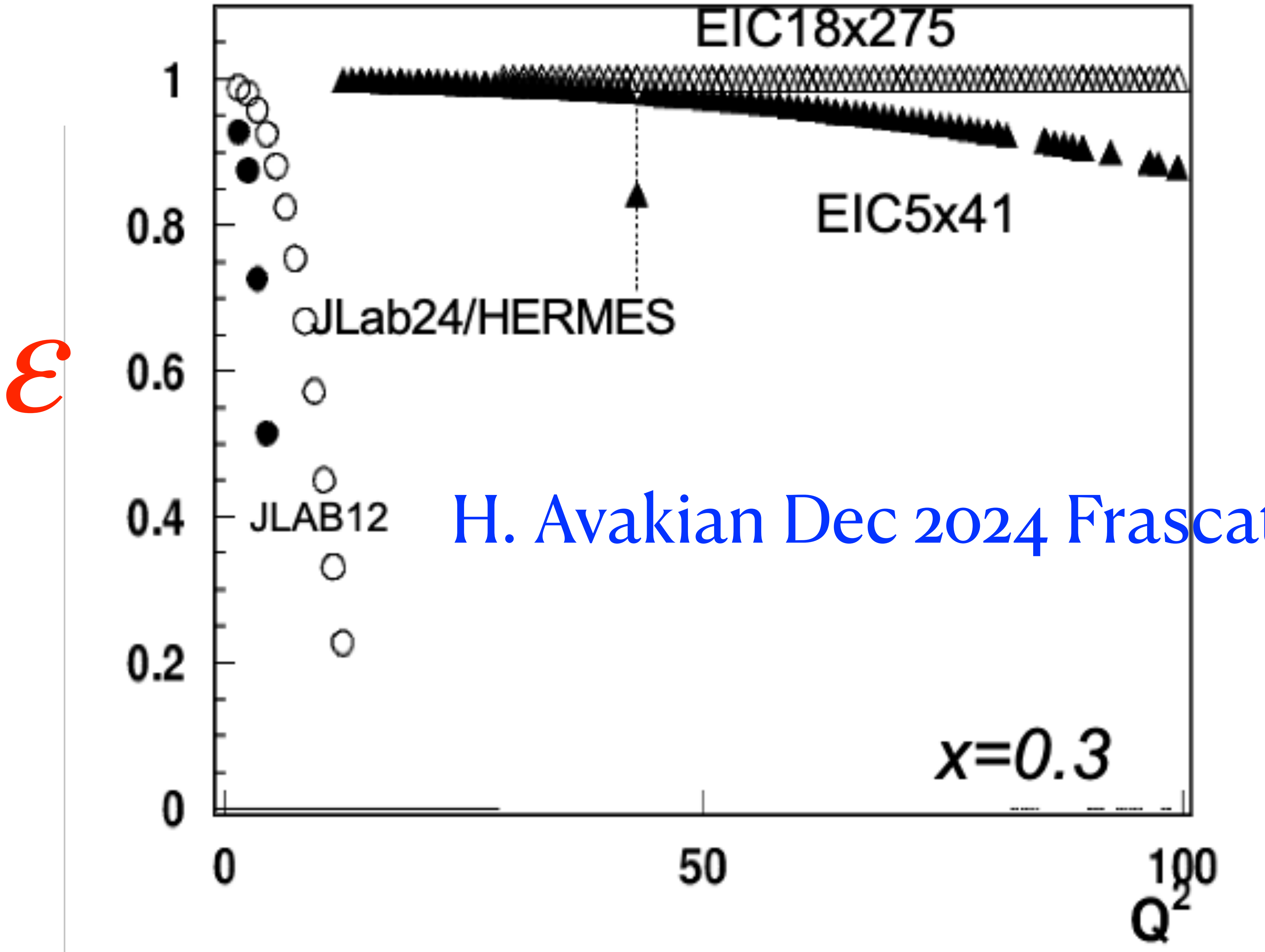
Nucleon Polarization	Quark Chirality: Chiral Even	Quark Chirality: Chiral Odd
U	$f^{\perp}, g^{\perp}$	$e\,,\ h$
L	$f_L^{\perp}, g_L^{\perp}$	e_L, h_L
T	$f_T, f_T^{\perp}, g_T, g_T^{\perp}$	$e_T, e_T^{\perp}, h_T, h_T^{\perp}$

Opportunity R_{SIDIS} & large P_T

- However @ large P_T , $F_{UU,L} \sim F_{UU}^{\cos 2\phi_h}$ — see Bacchetta et al. JHEP 2008 "Matches & Mis-matches": in principle hard gluon raditation — "collinear P_T factorization applies CSS 1985 Catani et al. 1997-2015, Nadolsky, Vogelsang Koike … many others

$$\frac{d\sigma}{dx\,dy\,dz\,d\phi_S\,d\phi_h\,dP_{h\perp}^2} = \frac{\alpha^2}{xQ^2}\frac{y}{2(1-\varepsilon)}\Big\{F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\varepsilon(1+\varepsilon)}\cos\phi_h\,F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}$$





H. Avakian Dec 2024 Frascati

TMD Factorization & $P_\perp$ Collinear Factorization

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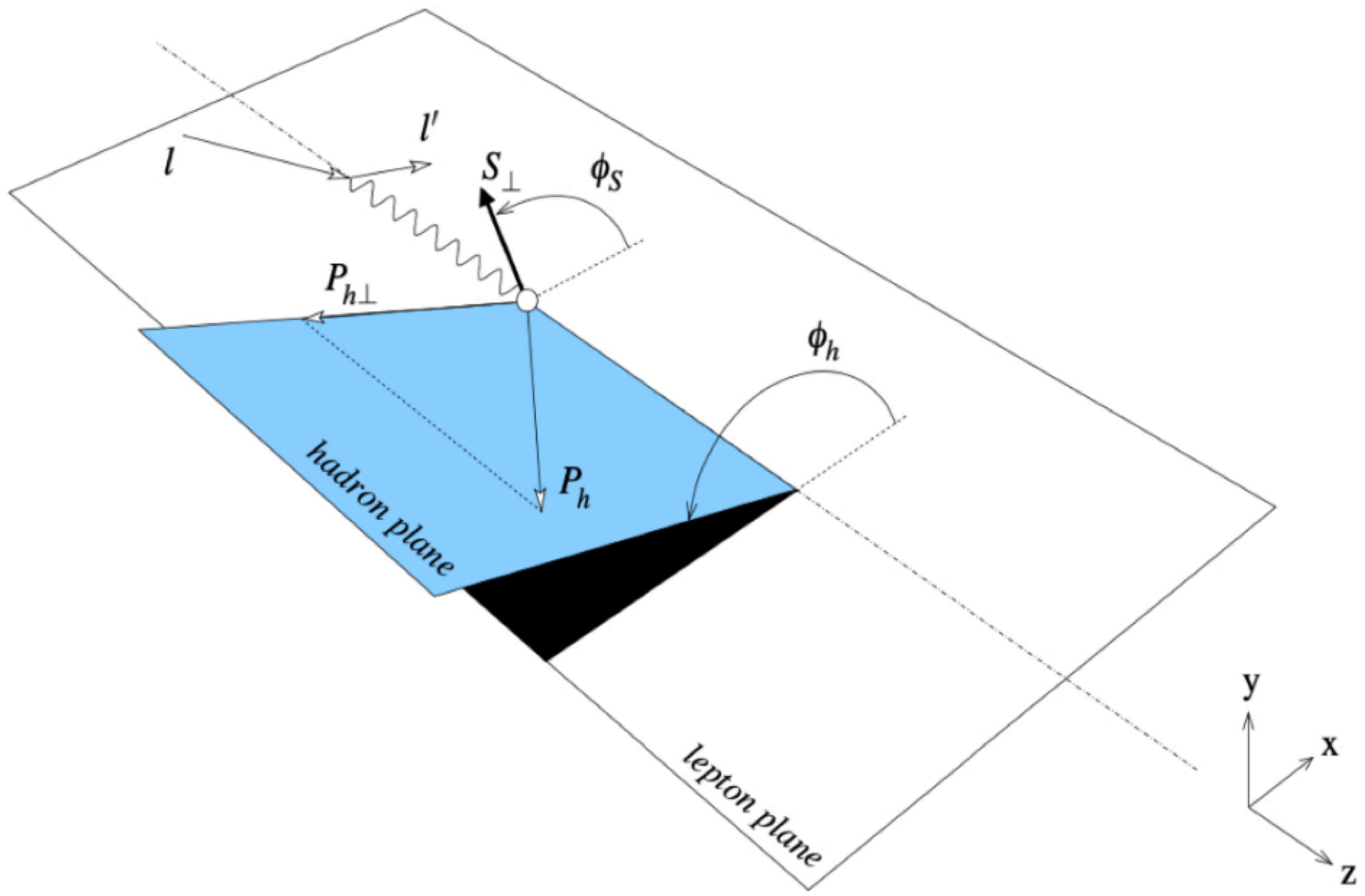
- $P_{h\perp} \sim \mathbf{k}_T$ or $\mathbf{p}_T$ intrinsic transverse momentum partons CS described via TMDs
- $P_{h\perp} \gg \mathbf{k}_T$ or $\mathbf{p}_T$ generated transverse momentum in the final state as perturbative radiation & non-perturbative structure is given by collinear pdfs & FFs



$$E' E_h \frac{d\sigma_{ep\to e'hX}}{d^3 l' d^3 P_h} \approx \hat{\sigma}_{eq\to e'q'} \otimes f_1 \widetilde{\otimes} D_{h/q'}.$$

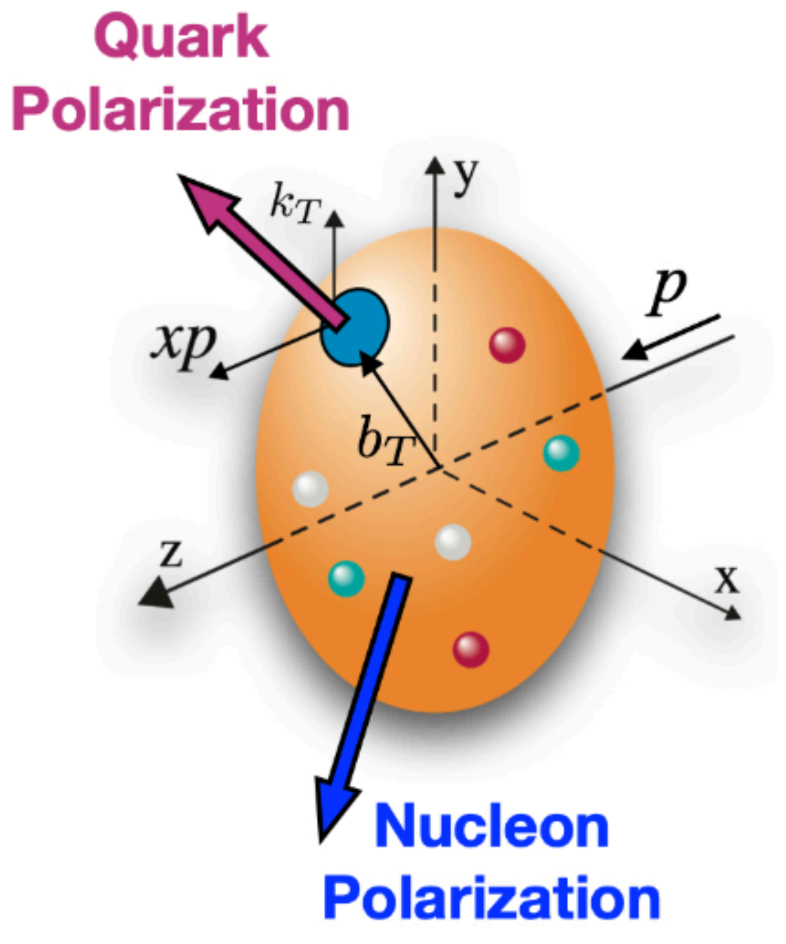
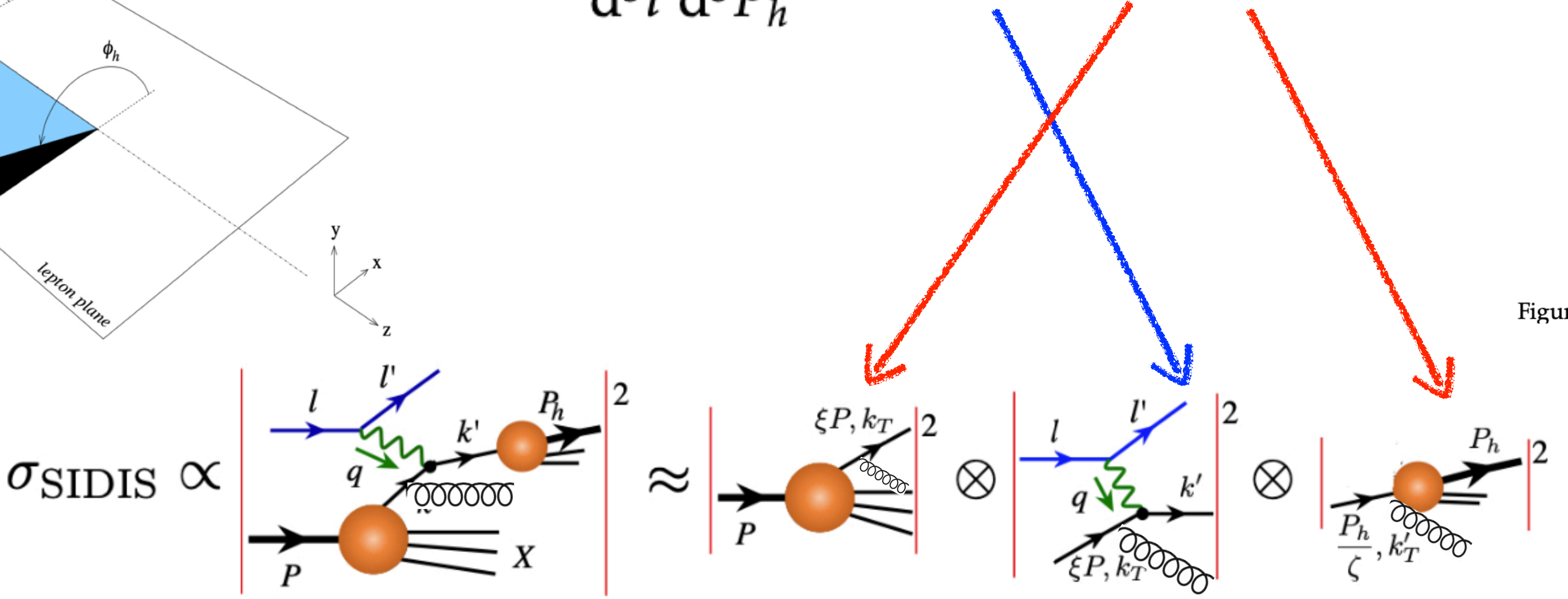


Figure 1.1: Illustration of the mo- spin variables probed on distributions.



R_{SIDIS} & σ_L at large p_T

$$\frac{d\sigma}{dx\,dy\,dz\,dq_T^2\,d\varphi} = \frac{\pi\alpha^2 yz}{4Q^2}\Big[\underbrace{\overbrace{\sinh^2\vartheta F_{UU,L}}^{\sigma_L} - \overbrace{\frac{1}{2}(2+\sinh^2\vartheta)F_{UU,T}}^{\sigma_T}}_{\sigma_0} - \underbrace{\sinh 2\vartheta F_{UU}^{\cos\varphi}}_{\sigma_1}\cos\varphi + \underbrace{\frac{1}{2}\sinh^2\vartheta F_{UU}^{\cos 2\varphi}}_{\sigma_2}\cos 2\varphi\Big]$$

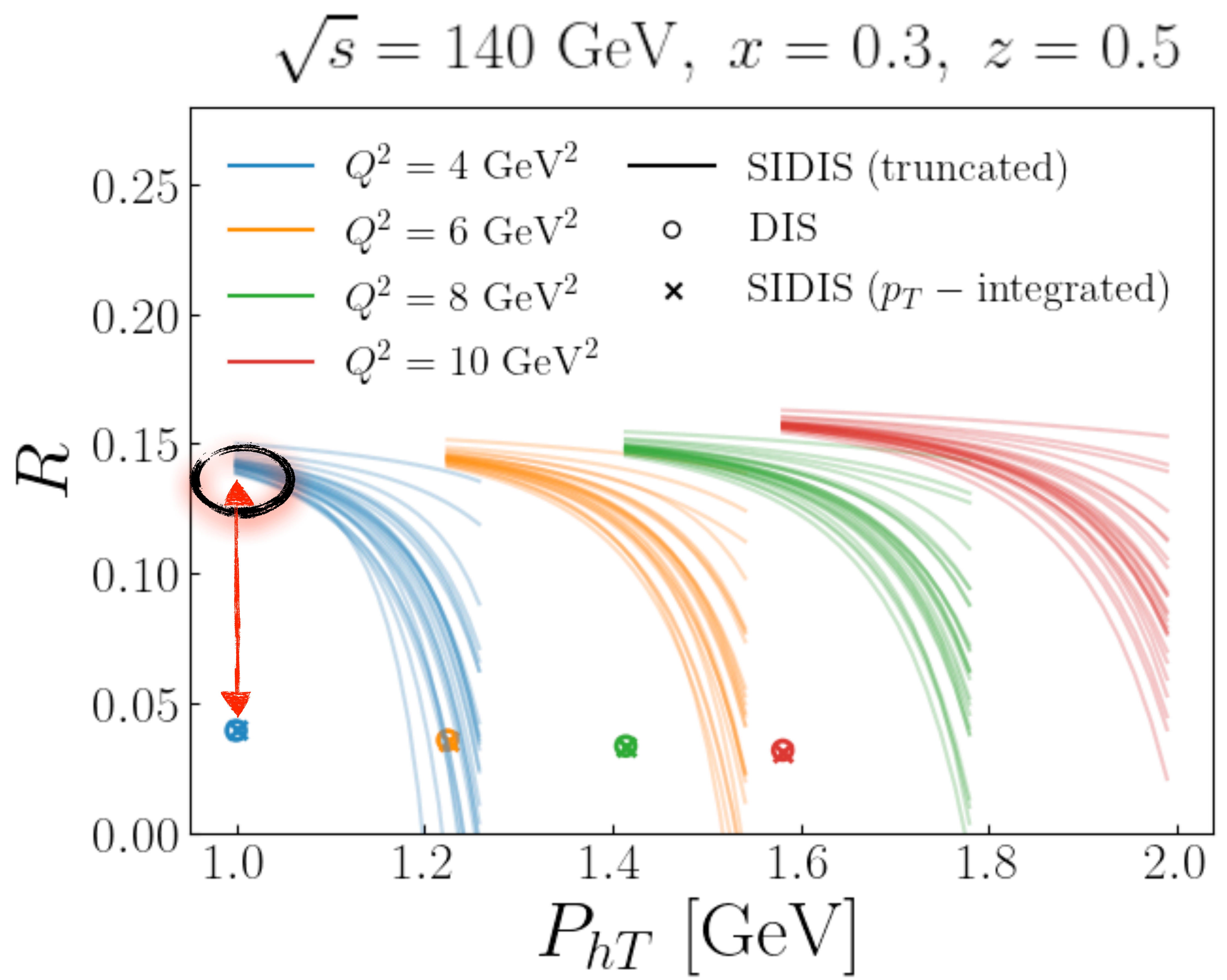
e.g.

$$F_{UU,T} = \frac{1}{Q^2}\frac{\alpha_s}{(2\pi z)^2}\sum_a x e_a^2 \int_x^1 \frac{d\hat{x}}{\hat{x}}\int_z^1 \frac{d\hat{z}}{\hat{z}}\,\delta\left(\frac{q_T^2}{Q^2} - \frac{(1-\hat{x})(1-\hat{z})}{\hat{x}\hat{z}}\right) \times \left[f_1^a\left(\frac{x}{\hat{x}}\right)D_1^a\left(\frac{z}{\hat{z}}\right)C_{UU,T}^{(\gamma^* q\to qg)} + f_1^a\left(\frac{x}{\hat{x}}\right)D_1^g\left(\frac{z}{\hat{z}}\right)C_{UU,T}^{(\gamma^* q\to gq)} + f_1^g\left(\frac{x}{\hat{x}}\right)D_1^a\left(\frac{z}{\hat{z}}\right)C_{UU,T}^{(\gamma^* g\to q\bar{q})}\right]$$

Comments:

- q_{Tmax} associated with large x implies R_{SIDIS} can provide further constraints on large x behavior if pdfs

- Truncated moment is sig. larger than P_T integrated SIDIS—indication of a TMD contribution?



gluon contribute large uncertainty @ hi-x (see delta function)

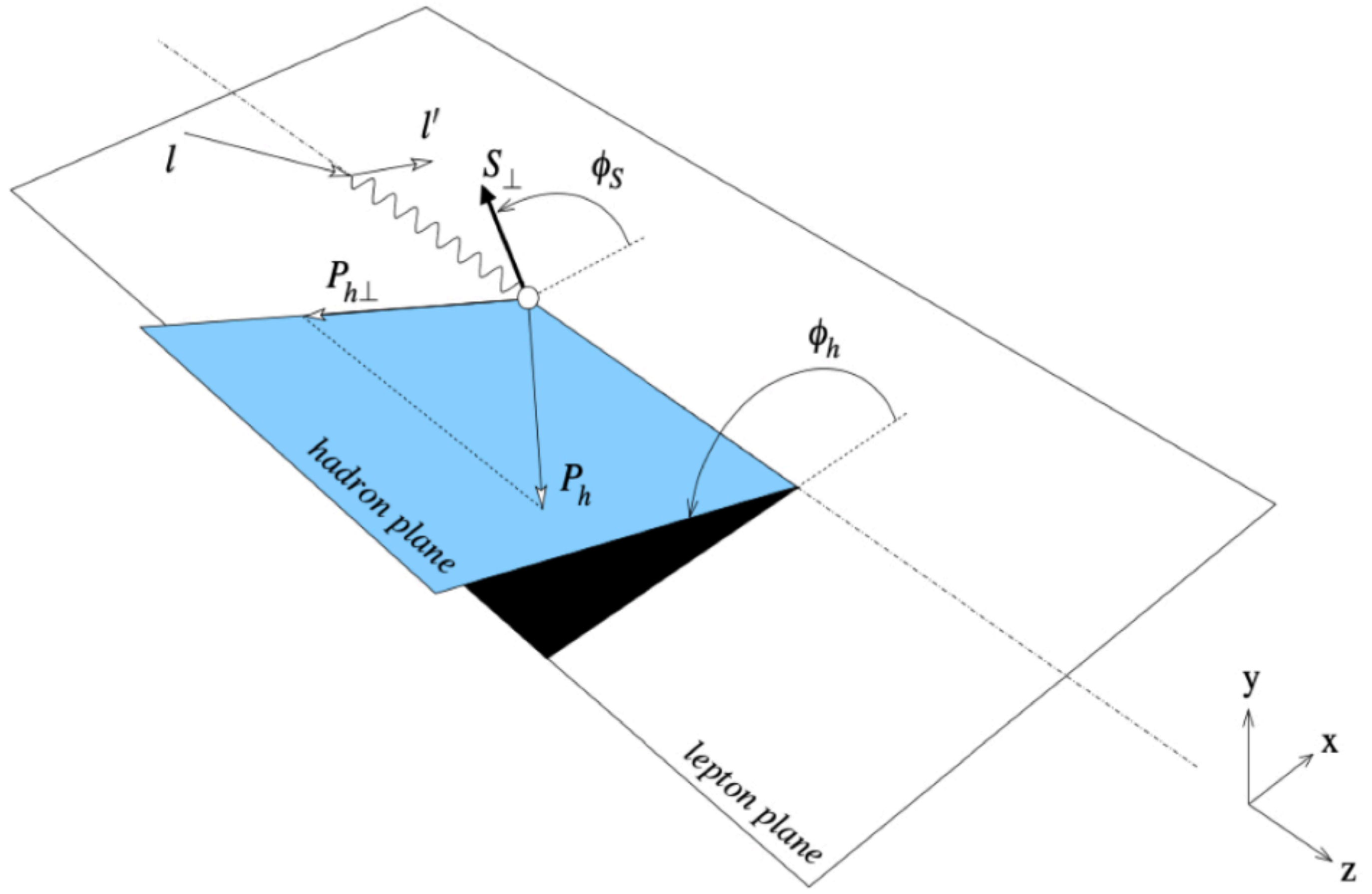
$g \to 0$ ie gluon PDF set to zero

TMDs @ "twist-3 " NLP

The beginning of TMD physics? $\langle \cos\phi \rangle$

- **Georgi Politzer, PRL 1978** "Measurement $\langle \cos\phi \rangle$ provides clean test of predictions of PQCD

- ~12-15% …clean test of QCD "…since such effects would not arise as a result of limited transverse momentum associated with confined quarks…"



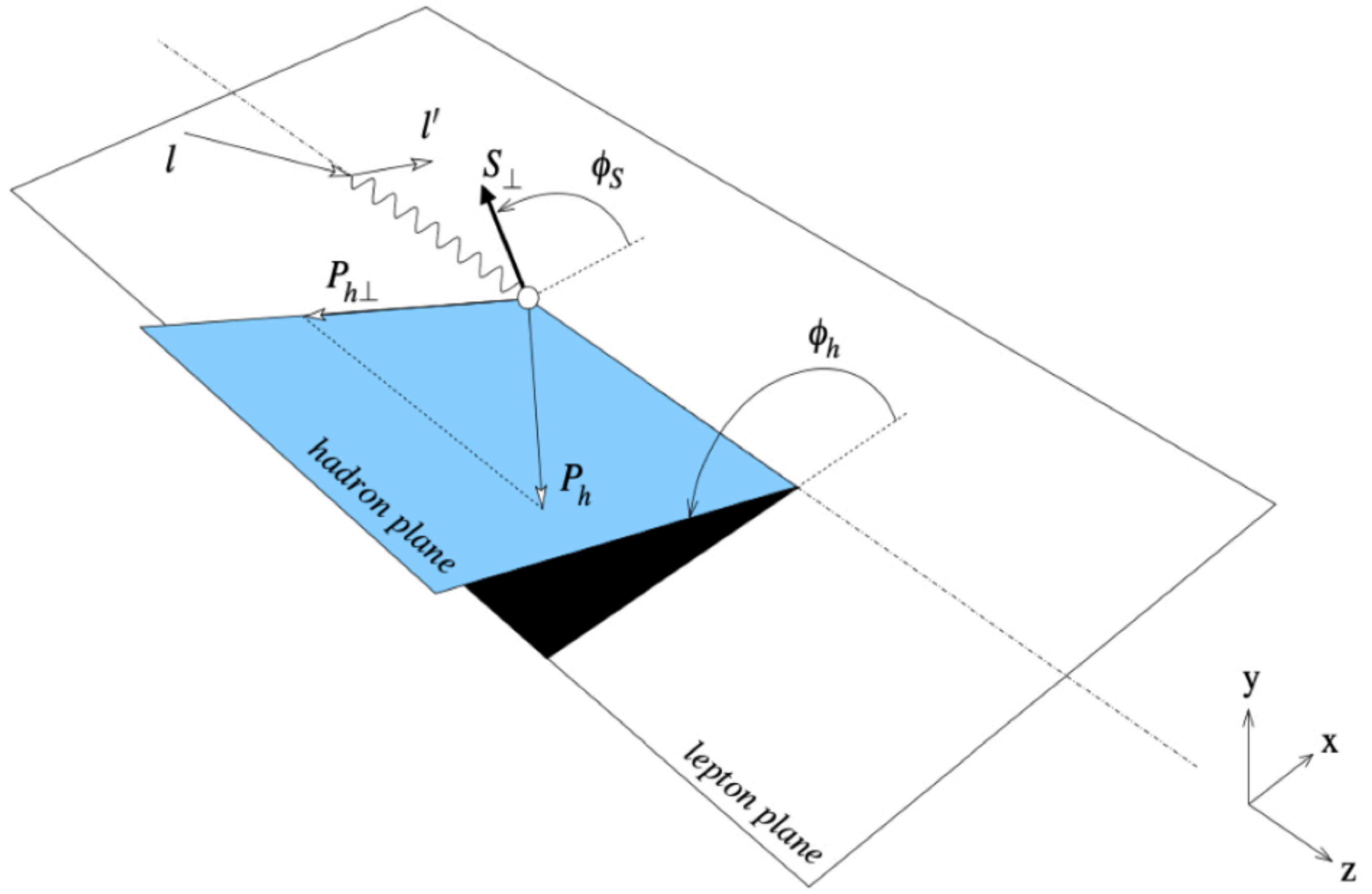
- **Cahn, PLB 1978, (& earlier paper by Ravndal, PLB 1972)**
  Critique QCD calculation of azimuthal dependence
  emphasize importance intrinsic $\boldsymbol{k}_T$ …

- "…Results cast doubt on the utility of such azimuthal asymmetry as a clean test of quantum chromodynamics " (i.e. of G&P 78)

TMDs @ "twist-3 " NLP

The beginning of TMD physics? $\langle \cos\phi \rangle$

- **<u>Georgi Politzer, PRL 1978</u>** "Measurement $\langle \cos\phi \rangle$ provides clean test of predictions of PQCD
Performed QCD analysis of hard gluon radiation in SIDIS: predict absolute value of final state hadron's $\boldsymbol{P}_T$ *or* $P_{h\perp}$, and the angular distribution relative to lepton scattering plane



- **<u>Cahn, PLB 1978, (& earlier paper by Ravndal, PLB 1972)</u>**
Critique QCD calculation of azimuthal dependence
emphasize importance intrinsic $\boldsymbol{k}_T$ …

The observable $\langle \cos\phi \rangle$

No assumption of mechanism

$$\frac{d\sigma}{dx\,dy\,dz\,d\phi_S\,d\phi_h\,dP_{h\perp}^2} = \frac{\alpha^2}{xQ^2}\frac{y}{2(1-\varepsilon)}\Big\{F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}$$

$$\int d\sigma^{(1)}\,\cos\phi = \int d^2P_T\,\cos\phi\,\frac{d\sigma}{dx_H\,dy\,dz_H\,d^2P_T}$$

SIDIS Kinematics dictionary

$$Q^2 = -q^2\,, \quad \mathbf{P}_T = \mathbf{P}_{2T}\,, \quad \phi\,,$$

$$x_H = \frac{Q^2}{2P_1\cdot q}\,, \quad y = \frac{P_1\cdot q}{P_1\cdot k_1}\,, \quad z_H = \frac{P_1\cdot P_2}{P_1\cdot q}\,,$$

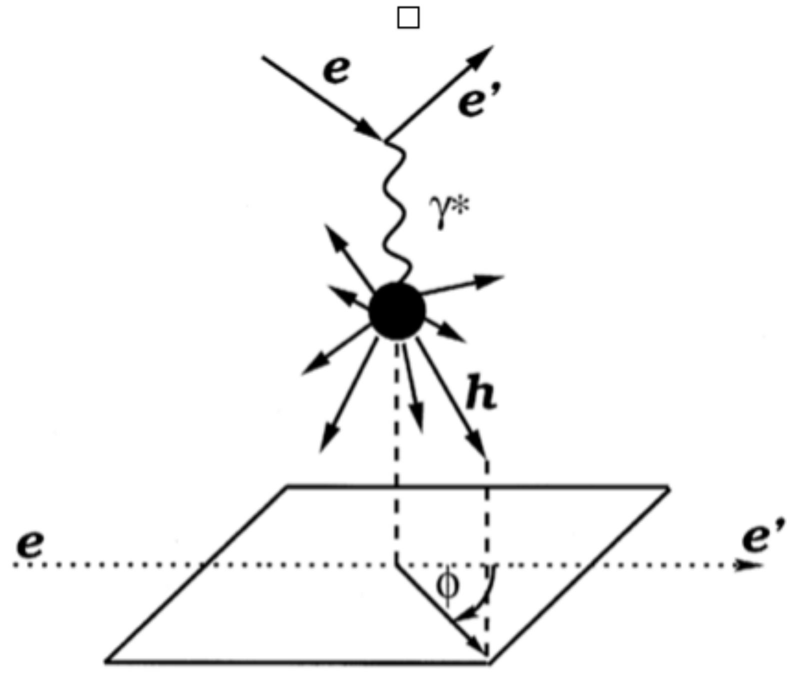
and the parton variables

$$x = \frac{x_H}{\xi} = \frac{Q^2}{2p_1\cdot q}\,, \quad z = \frac{z_H}{\xi'} = \frac{p_1\cdot p_2}{p_1\cdot q}\,.$$

Clean tests of QCD ?

PHYSICAL REVIEW LETTERS

VOLUME 40 | 2 JANUARY 1978 | NUMBER 1



Clean Tests of Quantum Chromodynamics in μp Scattering

Howard Georgi
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and

H. David Politzer
California Institute of Technology, Pasadena, California 91125

(Received 25 October 1977)

Hard gluon bremsstrahlung in μp scattering produces final-state hadrons with a large component of momentum transverse to the virtual-photon direction. Quantum chromodynamics can be used to predict not only the absolute value of the transverse momentum, but also its angular distribution relative to the muon scattering plane. The angular correlations should be insensitive to nonperturbative effects.

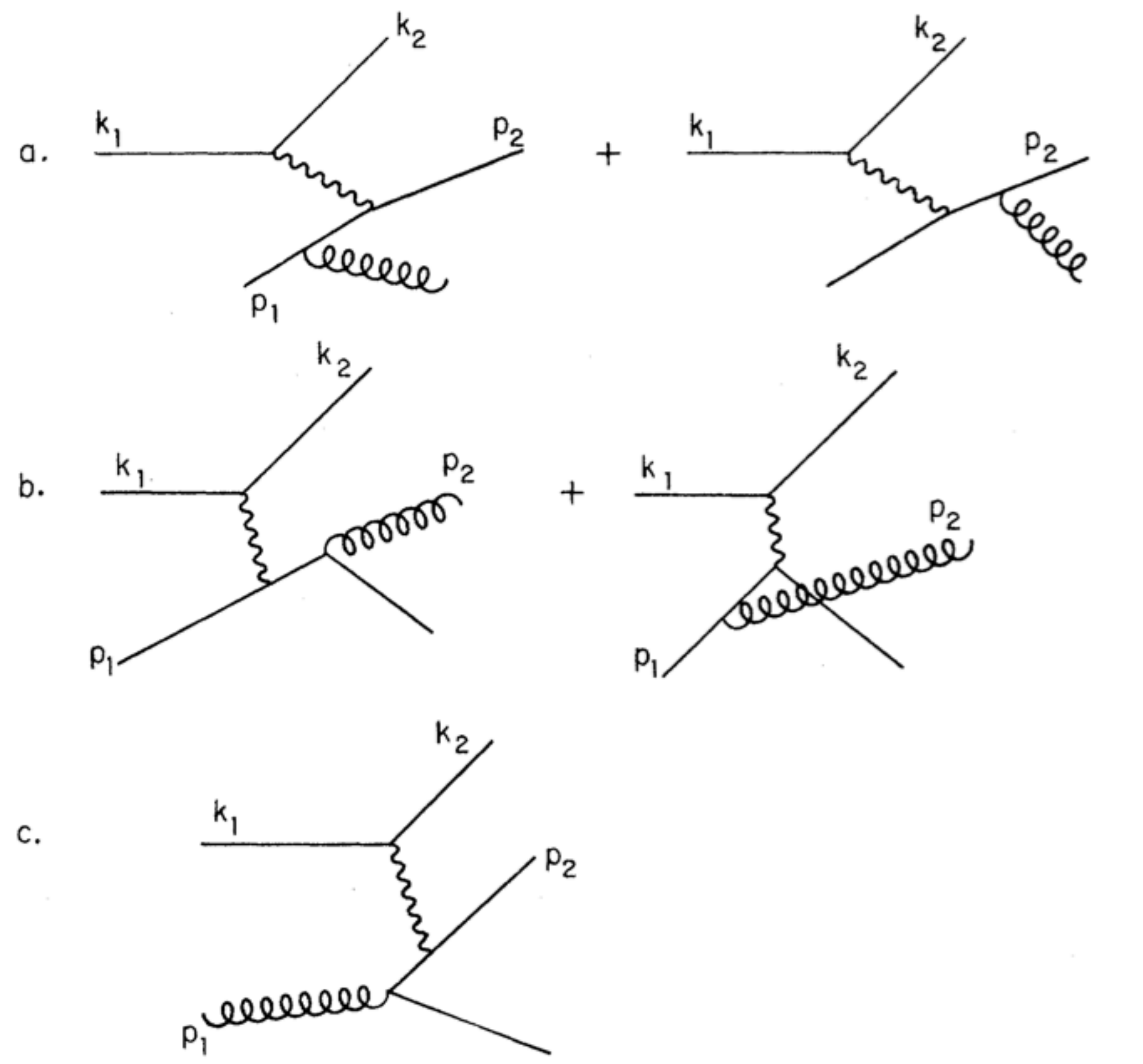


FIG. 1. Diagrams contributing to semi-inclusive μ-parton scattering to first order in α_s. k (p) denotes muon (parton) momentum. The wavy line is a virtual photon. The curly line is a gluon.

Pert. QCD $\alpha_s = g^2/4\pi$

$$\langle \cos \varphi \rangle_{ep} = -\frac{\alpha_s}{2} \kappa \sqrt{1-z}\, \frac{(2-y)\sqrt{1-y}}{1+(1-y)^2}$$

Cahn intrinsic k_T

Volume 78B, number 2,3 PHYSICS LETTERS 25 September 1978

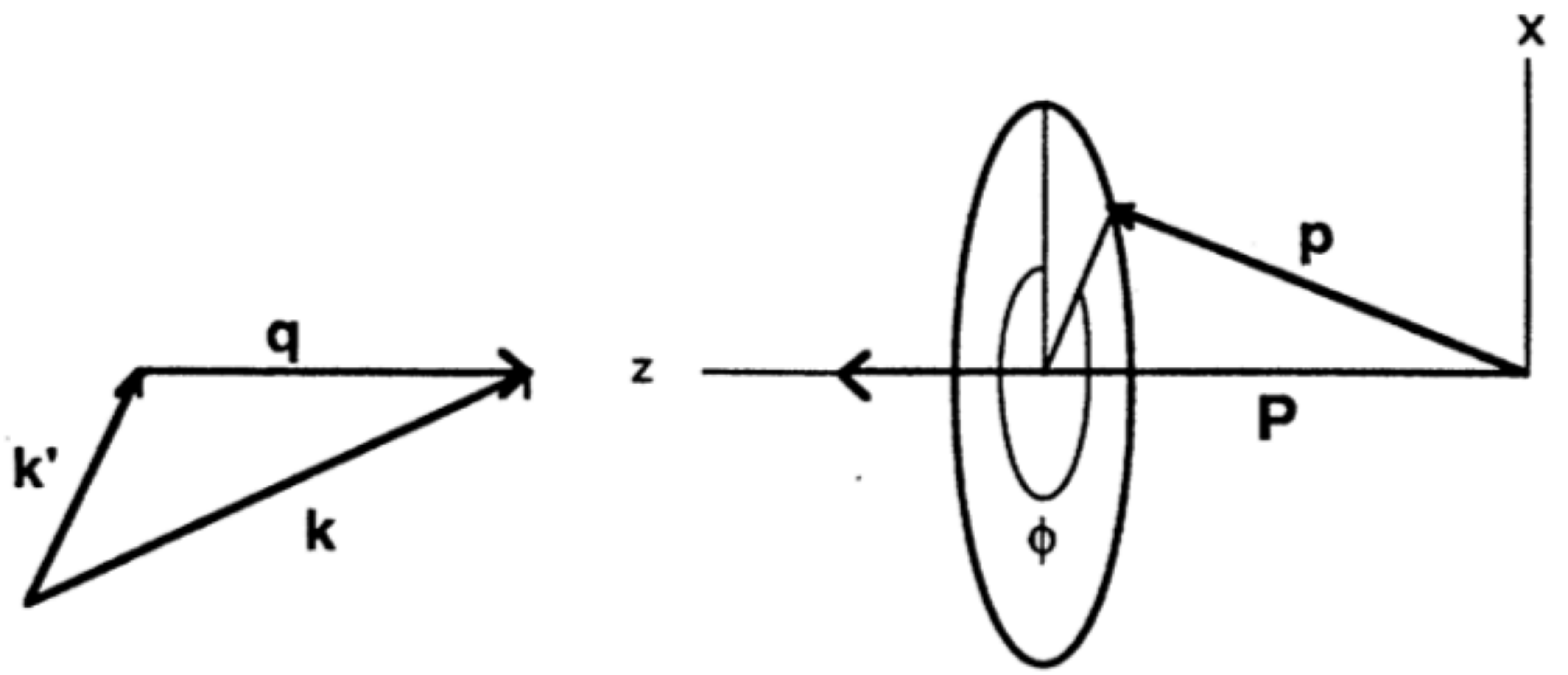
AZIMUTHAL DEPENDENCE IN LEPTOPRODUCTION: A SIMPLE PARTON MODEL CALCULATION☆

Robert N. CAHN
Department of Physics, University of Michigan, Ann Arbor, MI 48109, USA

Received 5 June 1978

Simple parton model argument allowing for transverse momentum in Mandelstam variables...

Semi-inclusive leptoproduction, $\ell + p \to \ell' + h + X$, is considered in the naive parton model. The scattered parton shows an azimuthal asymmetry about the momentum transfer direction. Simple derivations for the effects in ep, νp and $\bar{\nu}$p scattering are given. Reduction of the asymmetry due to fragmentation of partons into hadrons is estimated. The results cast doubt on the utility of such azimuthal asymmetry as a clean test of quantum chromodynamics.



$$\sigma_{ep} \propto \hat{s}^2 + \hat{u}^2 \propto \left[1 - \frac{2p_\perp}{Q}\sqrt{1-y}\,\cos\phi\right]^2 + (1-y)^2\left[1 - \frac{2p_\perp}{Q\sqrt{1-y}}\cos\phi\right]^2$$

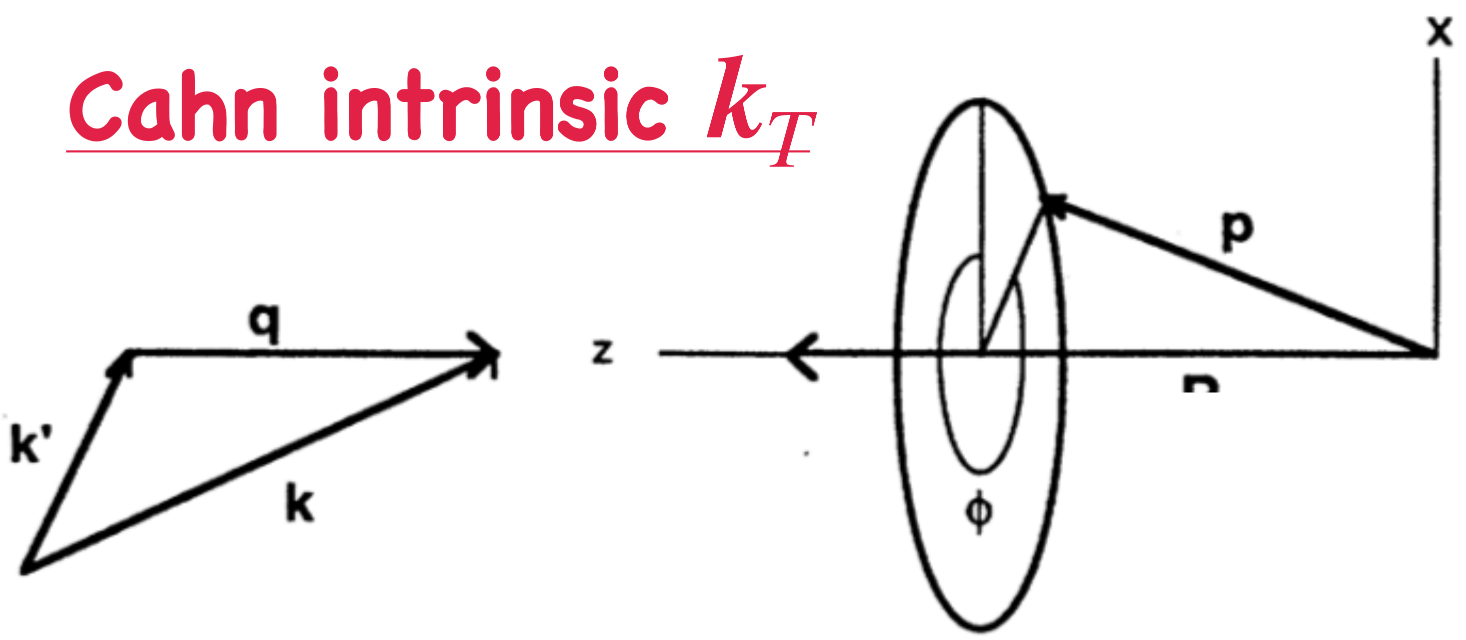
NLP ! $\frac{p_\perp}{Q}$

$$\langle\cos\phi\rangle_{ep} = -\left[\frac{2p_\perp}{Q}\right]\frac{(2-y)\sqrt{1-y}}{1+(1-y)^2}$$

Two mechanisms ? *Matching*…

Factorization & Matching collinear to TMD unpolarized/angle independent Collins Soper Sterman NPB 1985

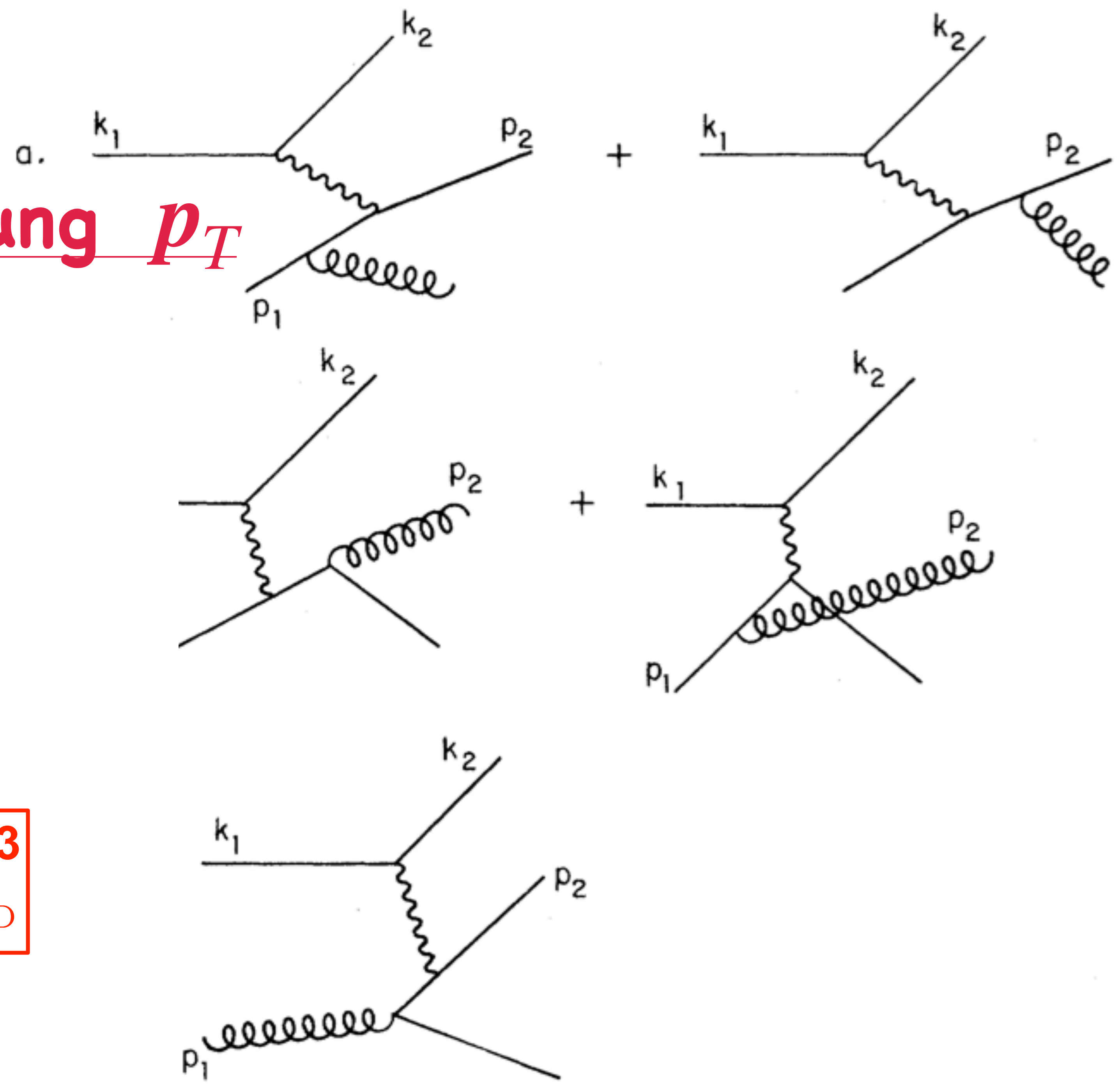
Cahn intrinsic k_T

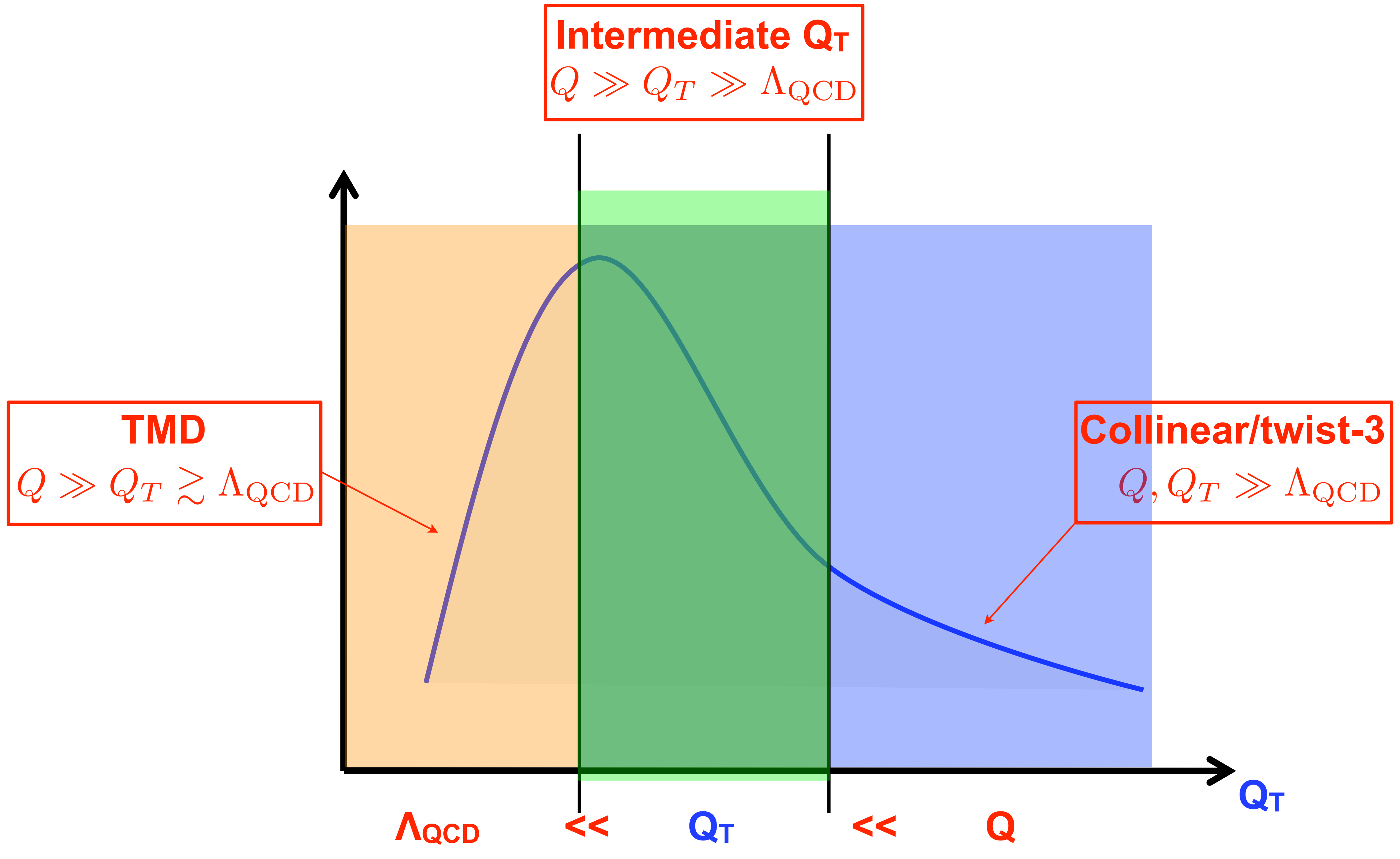


- **"TMD" region**

$(p_T \sim k_T) \sim q_T \ll Q$

Georgi & Politzer
hard gluon bremsstrahlung p_T





- **"Collinear " region**

$\Lambda_{qcd} \ll q_T \sim Q$

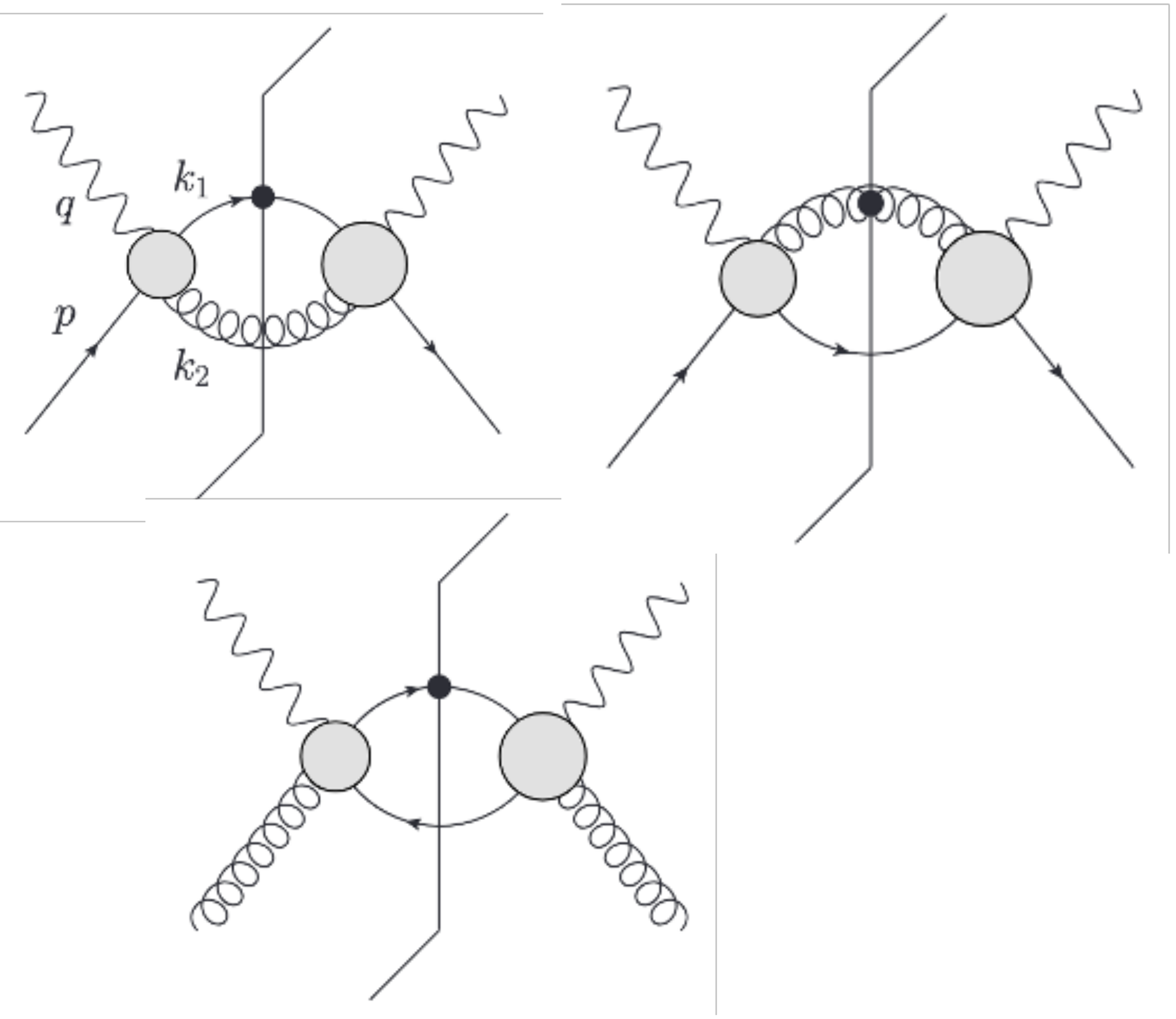
A comprehensive study of matching the hi & low Q_T in the overlap region
in SIDIS was carried out by JHEP (2008) Bacchetta et al.
where attention was given to azimuthal and polarization dependence

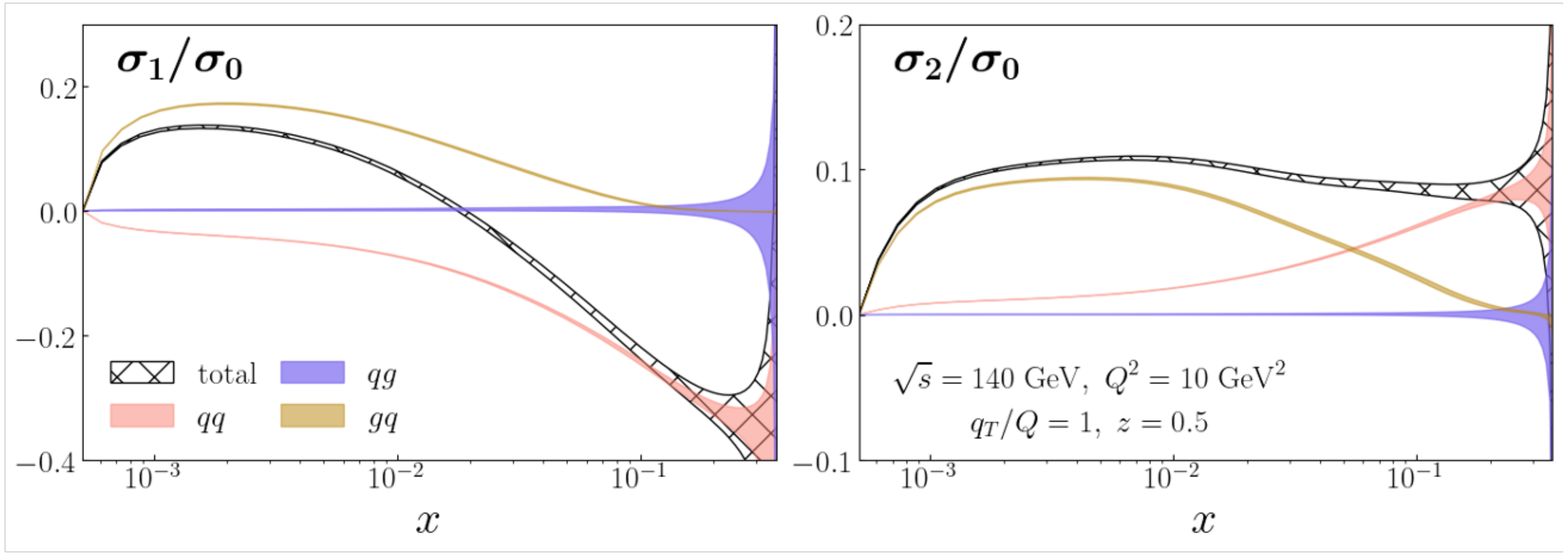
We have explored lg q_T angular modulations Cahn effect & $\cos 2\phi_h$

$$\frac{d\sigma}{dx\,dy\,dz\,dq_T^2\,d\varphi} = \frac{\pi\alpha^2 yz}{4Q^2}\Big[\underbrace{\sinh^2\vartheta F_{UU,L} - \frac{1}{2}(2+\sinh^2\vartheta)F_{UU,T}}_{\sigma_0} - \underbrace{\sinh 2\vartheta F_{UU}^{\cos\varphi}}_{\sigma_1}\cos\varphi + \underbrace{\frac{1}{2}\sinh^2\vartheta F_{UU}^{\cos 2\varphi}}_{\sigma_2}\cos 2\varphi\Big]$$

e.g.

$$F_{UU,T} = \frac{1}{Q^2}\frac{\alpha_s}{(2\pi z)^2}\sum_a x\,e_a^2\int_x^1\frac{d\hat{x}}{\hat{x}}\int_z^1\frac{d\hat{z}}{\hat{z}}\,\delta\left(\frac{q_T^2}{Q^2}-\frac{(1-\hat{x})(1-\hat{z})}{\hat{x}\,\hat{z}}\right)\times\left[f_1^a\left(\frac{x}{\hat{x}}\right)D_1^a\left(\frac{z}{\hat{z}}\right)C_{UU,T}^{(\gamma^* q\to qg)} + f_1^a\left(\frac{x}{\hat{x}}\right)D_1^g\left(\frac{z}{\hat{z}}\right)C_{UU,T}^{(\gamma^* q\to gq)} + f_1^g\left(\frac{x}{\hat{x}}\right)D_1^a\left(\frac{z}{\hat{z}}\right)C_{UU,T}^{(\gamma^* g\to q\bar{q})}\right]$$





w/ P. Anderson, W. Melntitchok, N. Sato, R. Whitehill (JAM)

Cahn intrinsic k_T

$$\frac{d\sigma}{dx\,dy\,dz\,d\phi_S\,d\phi_h\,dP_{h\perp}^2} = \frac{\alpha^2}{xQ^2}\frac{y}{2\,(1-\varepsilon)}\left\{F_{UU,T} + \varepsilon F_{UU,L} + \sqrt{2\,\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} + \varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}\right.$$

TMD Collaboration

Preprints: JLAB-THY-23-3780, LA-UR-21-20798, MIT-CTP/5386

TMD Handbook

Renaud Boussarie[1], Matthias Burkardt[2], Martha Constantinou[3], William Detmold[4], Markus Ebert[4,5], Michael Engelhardt[2], Sean Fleming[6], Leonard Gamberg[7], Xiangdong Ji[8], Zhong-Bo Kang[9], Christopher Lee[10], Keh-Fei Liu[11], Simonetta Liuti[12], Thomas Mehen[13], Andreas Metz[3], John Negele[4], Daniel Pitonyak[14], Alexei Prokudin[7,16], Jian-Wei Qiu[16,17], Abha Rajan[12,18], Marc Schlegel[2,19], Phiala Shanahan[4], Peter Schweitzer[20], Iain W. Stewart[4], Andrey Tarasov[21,22], Raju Venugopalan[18], Ivan Vitev[10], Feng Yuan[23], Yong Zhao[24,4,18]

10 - Subleading TMDs

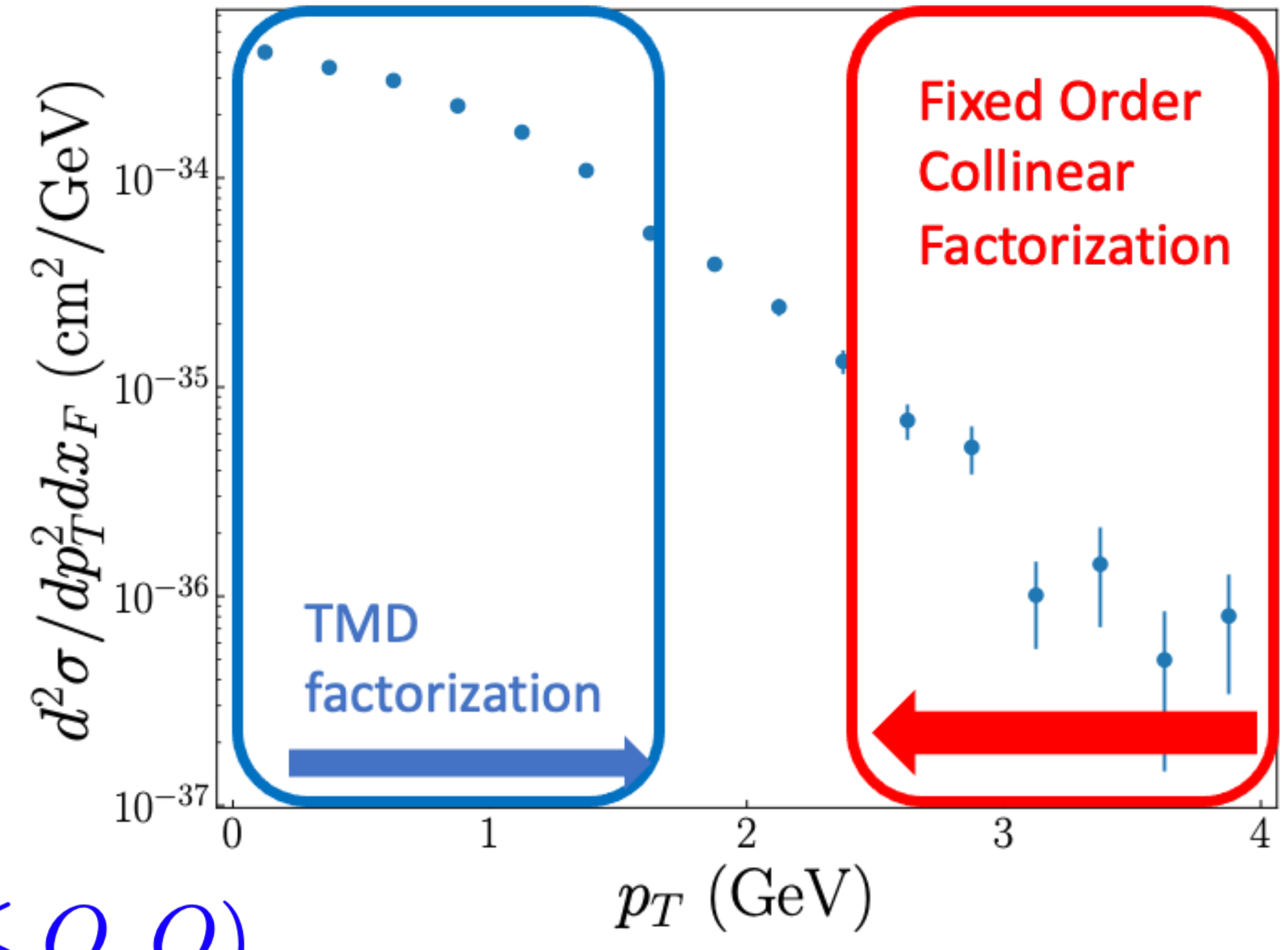
L. Gamberg, A. Metz, I. Stewart

Subleading Quark TMDPDFs

		Quark Chirality	
		Chiral Even	Chiral Odd
Nucleon Polarization	U	$f^\perp,\ g^\perp$	$e\ ,\ h$
	L	$f_L^\perp\ ,\ g_L^\perp$	$e_L\ ,\ h_L$
	T	$f_T\ ,\ f_T^\perp,\ g_T,\ g_T^\perp$	$e_T\ ,\ e_T^\perp,\ h_T,\ h_T^\perp$

"Mis"-Matches Factorization @ sub-leading power

- **Factorization & Matching collinear to TMD unpolarized/angle independent Collins Soper Sterman NPB 1985**



- **Cross section in terms of different "regions"**
- W valid for $q_T \sim k_T \ll Q$ TMD factorization
- FO valid for $k_T \ll p_T \sim Q$ Collinear factorization
- AY subtracts d.c. & in principle,

  $AY \to W,\ p_T \to \infty$ and $AY \to FO, p_T \to 0$

$$\frac{d\sigma(m \lesssim q_T \lesssim Q, Q)}{dy\,dq^2\,dp_T^2} = W(p_T, Q) + FO(p_T, Q) - AY(p_T, Q) + O\left(\frac{m}{Q}\right)^c$$

- **Bacchetta, Boer, Diehl, Mulders JHEP (2008) Cahn Effect:**
  Mis-match/inconsistency breakdown of TMD factorization at NLP?

$$\langle \cos\phi \rangle = \frac{\int d\sigma^{(0)}\ \cos\phi + \int d\sigma^{(1)}\ \cos\phi}{\int d\sigma^{(0)} + \int d\sigma^{(1)}}$$

Attempt to match the high-q_T result for $F_{UU}^{\cos\phi_h}$ to low-q_T result at intermediate q_T consistency check on factorization framework that extends CSS to NLP

Conjecture on matching $\langle \cos\phi_h \rangle$ (see Bacchetta et al 2019 PLB)

??? $\quad W \rightarrow AY \leftarrow FO \quad$???

$$\mathcal{C}\left[w f D\right] = \sum_a x e_a^2 \int d^2\boldsymbol{p}_T \, d^2\boldsymbol{k}_T \, d^2\boldsymbol{l}_T \, \delta^{(2)}(\boldsymbol{p}_T - \boldsymbol{k}_T + \boldsymbol{l}_T + \boldsymbol{q}_T) \;\times w(\boldsymbol{p}_T, \boldsymbol{k}_T) \, f^a(x, p_T^2) \, D^a(z, k_T^2) \, U(l_T^2)$$

schematically

$$F_{UU}^{\cos\phi} = C\left[\frac{\hat{h}\cdot p_\perp}{Q} f_1(x,p_\perp)\frac{\tilde{D}^\perp(z,k_\perp)}{z} S(l_\perp) + \frac{\hat{h}\cdot k_\perp}{Q} x f^\perp(x,p_\perp) D_1(z,k_\perp) S(l_\perp)\right]$$

Method: get AY term from TMD and match to AY from FO

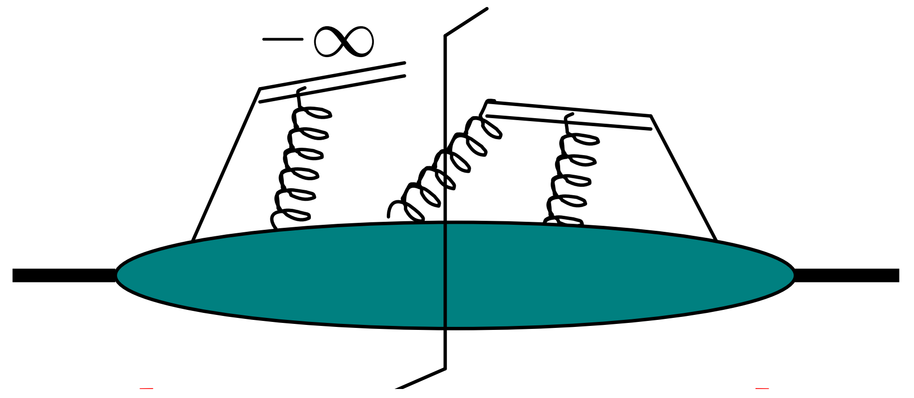
let one of $p_\perp, k_\perp \, l_\perp \rightarrow q_\perp$ *large*, others small

"Mis"-matches Factorization @ sub-leading power

$$\langle \cos\phi \rangle = \frac{\int d\sigma^{(0)} \cos\phi + \int d\sigma^{(1)} \cos\phi}{\int d\sigma^{(0)} + \int d\sigma^{(1)}}$$

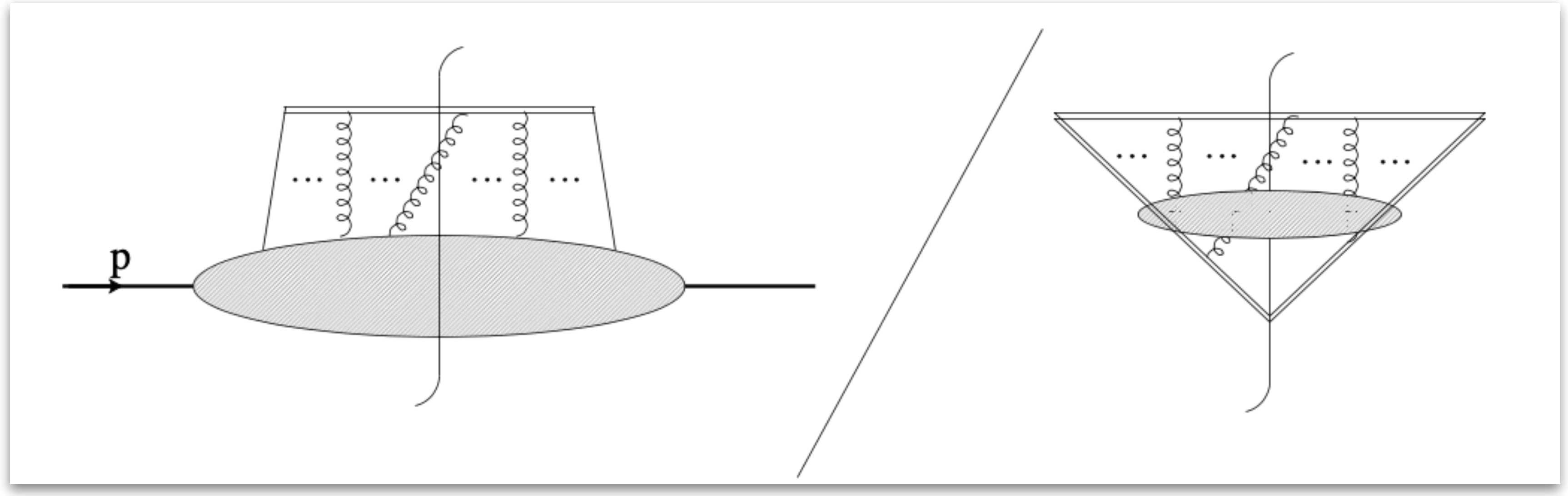
To cure mismatch, Bacchetta et al PLB (2019) speculated that soft factor subtraction from leading power (LP) TMD same as NLP TMDs

What's the soft factor ???
Advertisement TMD Handbook 2023 e-Print:2304.03302 [hep-ph]
Collins QCD book 2011, Aybat Rogers 2011 PRD, Echevarria et al. 2012 JHEP



$$\tilde{f}^{\rm sub}_{j/H}(x, b_T; \mu, y_n) = \lim_{\substack{y_A \to +\infty \\ y_B \to -\infty}} \underbrace{\tilde{f}^{\rm unsub}_{j/H}(x, b_T; \mu, y_P - y_B)} \sqrt{\frac{\tilde{S}(b_T; y_A, y_n)}{\tilde{S}(b_T; y_A, y_B)\tilde{S}(b_T; y_n, y_B)}} \times UV_{renorm}$$

Soft factor subtractior



1) cancel rapidity divergences in "unsubtracted" TMDs
2) separate "right & left" movers i.e. full factorization
3) remove double counting of momentum regions

To address these subtleties fresh look TMD @ at *NLO* & *NLP* factorization

Exploit the "diaginositc tool" of using Fierz decomp & "good and bad" LC quark fields

Focus on NLO soft factor calculation

L.Gamberg, Z.Kang, D.Shao, J.Terry, F.Zhao arXiv: e-Print:221.13209 (2022) & in prep 2025

Challenges of *SLP/NLP* TMDs

Various sources for power suppressed terms identified and discussed in the literature from

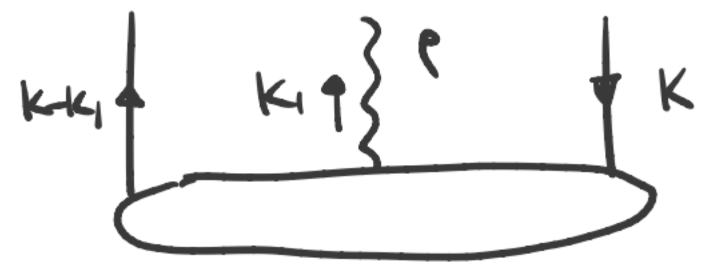
Tree level Studies, Mulders, Tangerman (1996), Bacchetta et al. JHEP (2007)

- Corrections associated w/ kinematic prefactors contractions between the leptonic & hadronic tensors, referred to as ***kinematic power corrections***

- Another involve subleading terms in quark-quark correlators involving Dirac structures that differ from LP referred to as ***intrinsic power corrections***— *e.g. Cahn function* $f^{\perp}(x, k_T)$, $e(x, k_T)$ …

$$\Gamma_a \in \Big\{ \underbrace{\frac{\not{n}}{4}, \frac{\not{n}\gamma^5}{4}, \frac{i}{4}\sigma^{i+}\gamma^5}_{\text{LP}}, \underbrace{\frac{1}{2}, \frac{\gamma^5}{2}, \frac{\gamma^i}{2}, \frac{\gamma^i\gamma^5}{2}, \frac{i}{2}\sigma^{ij}\gamma^5, \frac{i}{4}\sigma^{+-}\gamma^5}_{\text{NLP}} + ... \Big\}$$

- Another from hadronic matrix elements of (interaction dependent) quark-gluon-quark operators, referred to ***dynamic power corrections*** multi-parton *qgq* correlators



All three distributions are not required to span the NLP cross section due to EOM

$$\Phi^{\text{int}}_{q/P\,jj'}(x, \mathbf{k}_\perp, \mathbf{S}) = \Phi^{\text{kin}}_{q/P\,jj'}(x, \mathbf{k}_\perp, \mathbf{S}) + \Phi^{\text{dyn}}_{q/P\,jj'}(x, \mathbf{k}_\perp, \mathbf{S})$$

[8]Gamberg, Kang, Shao, Terry, Zhao 2022

Factorization at sub-leading power … revisit Tree level

$$\frac{d\sigma}{dx\,dy\,d\Psi\,dz\,d^2P_{h\perp}} = \kappa\frac{\alpha_{\rm em}^2}{4Q^4}\frac{y}{z}L_{\mu\nu}W^{\mu\nu}$$

- "TMD" region $(p_T \sim k_T) \sim q_T \ll Q$

$$W_{\mu\nu} = \frac{1}{(2\pi)^4}\sum_X \int d^4x\, e^{-iqx}\langle P|J_\mu^\dagger(0)|h,X\rangle\langle h,X|\,J_\nu(x)|P\rangle\,,$$

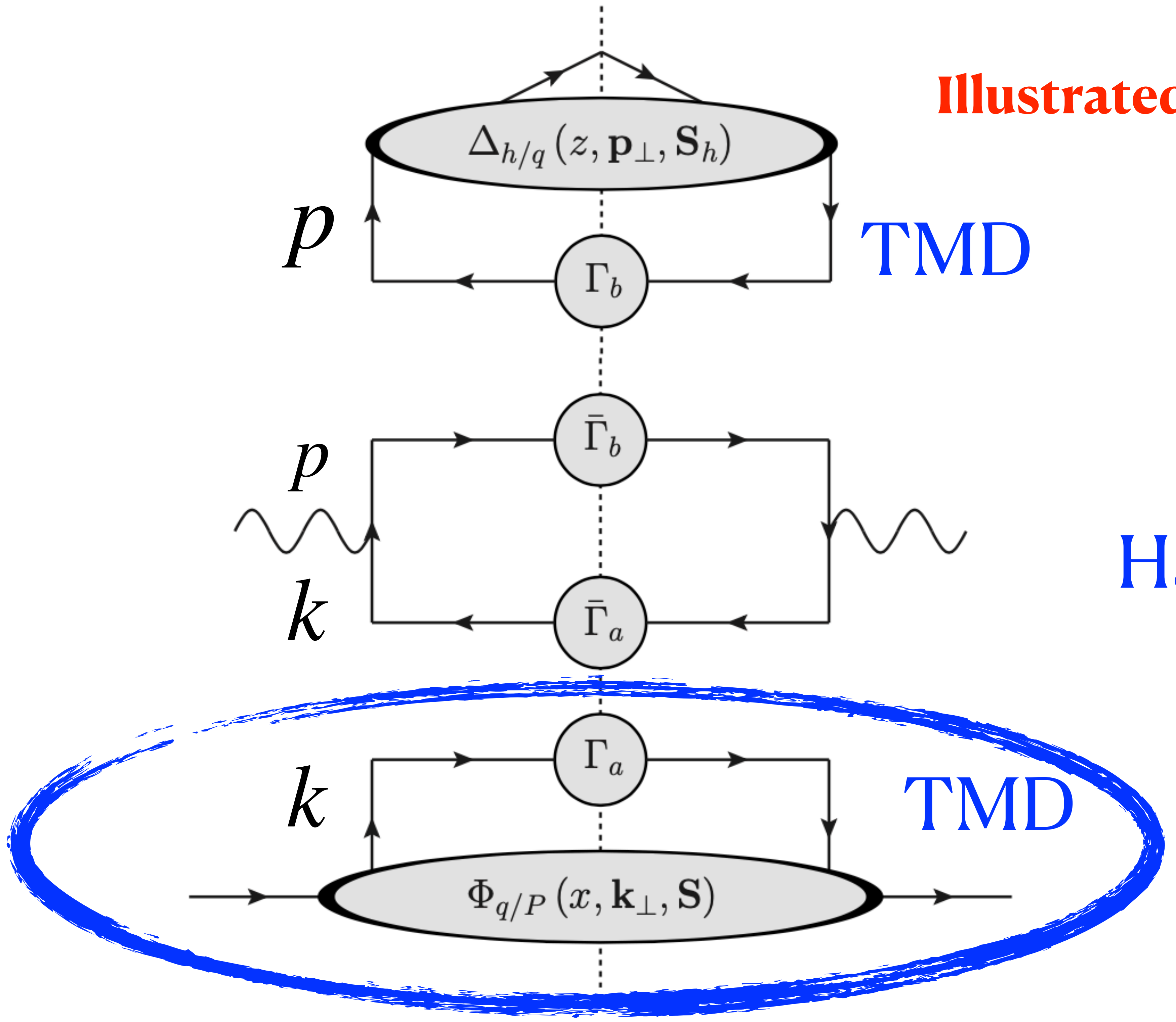
$$J_\mu(x) = J_\mu^{(2)}(x) + J_\mu^{(3)}(x)$$

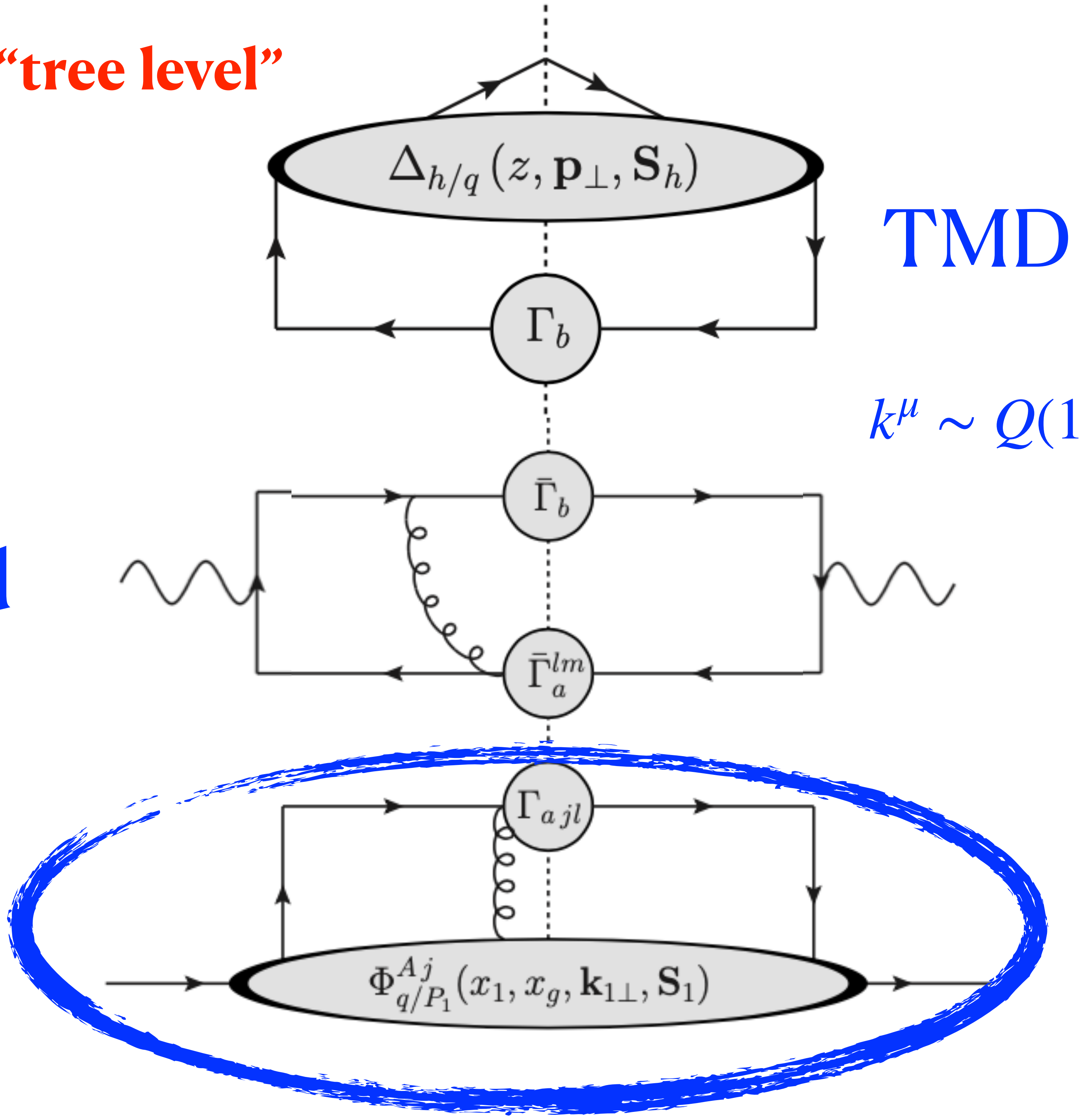
$k^\mu \sim Q(1,\lambda^2,\lambda),\ p^\mu \sim Q(\lambda^2,1,\lambda)$

Working @ NLP, the current contains 3(!) contributions:

- One with 2 partons entering from each correlation function
- Another with 3 partons entering from one correlation function
- **& partonic kinematic power corrections-momentum scaling**

Illustrated at "tree level"





Tree level factorization sub-leading power

Combining contributions and multiplying by leptonic tensor get factorized form,

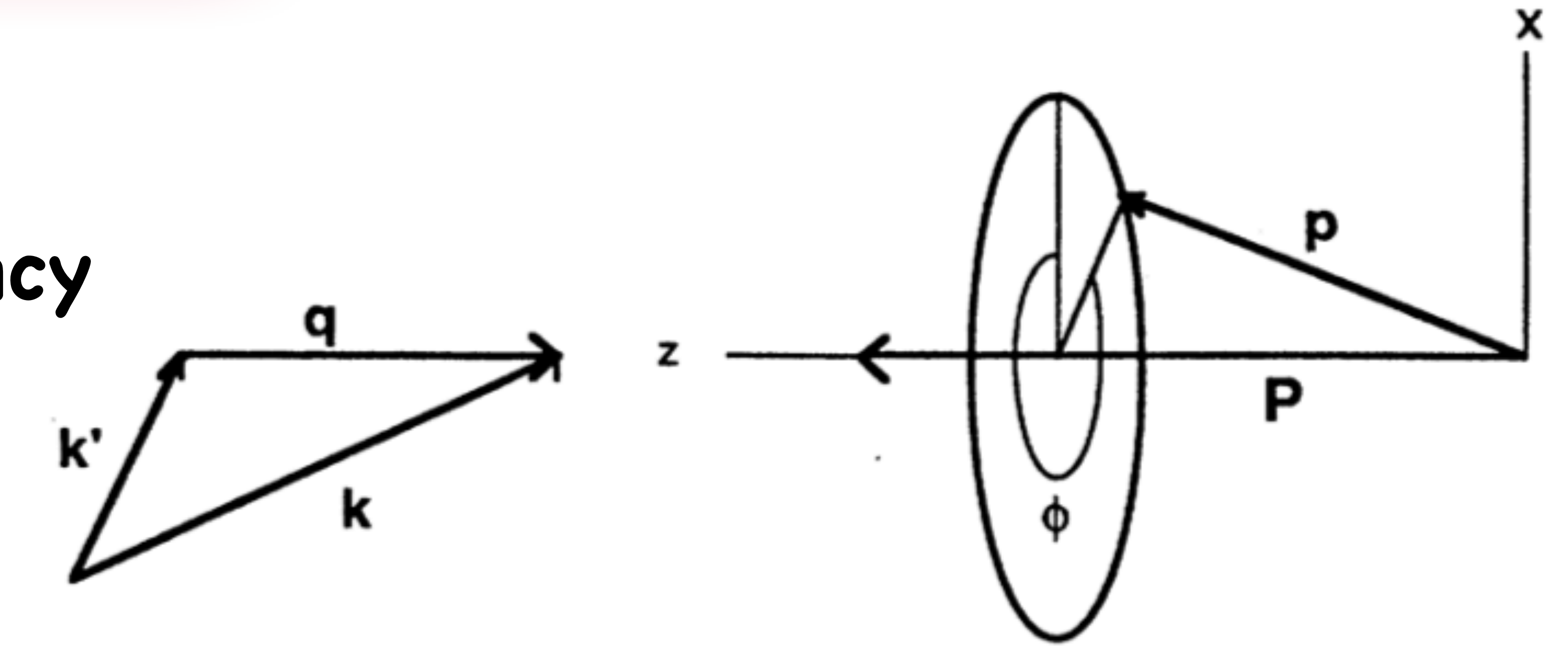
Cahn and more ....

We use "intrinsic & dynamical" basis

$$F^3_{\rm DIS}(x,z,\boldsymbol{P}_{h\perp}) = \mathcal{C}^{\rm DIS}\left[\frac{q_\perp}{Q} f_1\, D_1\right] - \mathcal{C}^{\rm DIS}\left[\left(x\,\frac{\boldsymbol{k}_\perp\cdot\hat{x}}{Q} f^\perp\right) D_1 - f_1\left(\frac{\boldsymbol{p}_\perp\cdot\hat{x}}{zQ} D^\perp\right)\right]$$
$$-\int\frac{dx_g}{x_g}\mathcal{C}^{\rm DIS}_{\rm dyn\, x_g}\left[\left(x\,\frac{\boldsymbol{k}_\perp\cdot\hat{x}}{Q}\tilde{f}^\perp\right) D_1\right] + \int\frac{dz_g}{z_g}\mathcal{C}^{\rm DIS}_{\rm dyn\, z_g}\left[f_1\left(\frac{\boldsymbol{p}_\perp\cdot\hat{x}}{zQ}\tilde{D}^\perp\right)\right],$$

Cahn and more intrinsic k_T

Different setup then Bacchetta et al 2007 allows us to check RG consistency
Gamberg, Kang, Shao, Terry, Zhao arXiv: e-Print:221.13209



Factorization & resummation at *NLO & NLP*

Beyond tree level

- We perform one loop calculation & attempt to establish renormalization group consistency: Regions hard,soft,collinear

$$
\begin{aligned}
F^3_{\mathrm{DIS}}(x,z,\boldsymbol{P}_{h\perp}) = {} & H^{\mathrm{LP}}_{\mathrm{DIS}}(Q;\mu)\,\mathcal{C}^{\mathrm{DIS}}\left[\frac{q_\perp}{Q} f_1\, D_1\, \mathcal{S}^{\mathrm{LP}}\right] \\
& - H^{\mathrm{int}}_{\mathrm{DIS}}(Q;\mu)\,\mathcal{C}^{\mathrm{DIS}}\left[\left(x\,\frac{\boldsymbol{k}_\perp\cdot\hat{x}}{Q} f^\perp D_1 - \frac{\boldsymbol{p}_\perp\cdot\hat{x}}{zQ} f_1\, D^\perp\right)\mathcal{S}^{\mathrm{int}}\right] \\
& - \int \frac{dx_g}{x_g} H^{\mathrm{dyn}}_{\mathrm{DIS}}(x_g,Q;\mu)\,\mathcal{C}^{\mathrm{DIS}}\left[x\,\frac{\boldsymbol{k}_\perp\cdot\hat{x}}{Q}\tilde{f}^\perp D_1\, \mathcal{S}^{\mathrm{dyn}}\right] \\
& + \int \frac{dz_g}{z_g} H^{\mathrm{dyn}}_{\mathrm{DIS}}(z_g,Q;\mu)\,\mathcal{C}^{\mathrm{DIS}}\left[\frac{\boldsymbol{p}_\perp\cdot\hat{x}}{zQ} f_1\, \tilde{D}^\perp \mathcal{S}^{\mathrm{dyn}}\right].
\end{aligned}
$$

- H^{LP}, H^{int} and H^{dynam} represent LP, intrinsic NLP, and dynamic NLP hard functions.
- Additionally, S^{LP}, S^{int} and S^{dyn} denote the LP, intrinsic sub-leading power, and dynamic sub-leading power soft function
- ***NB* if soft factors are different universality, of TMDs breaks down. Global analysis w/ NLP observables problematic**

NLO-calculation-factorization
Necessary but not sufficient condition to establish factorization

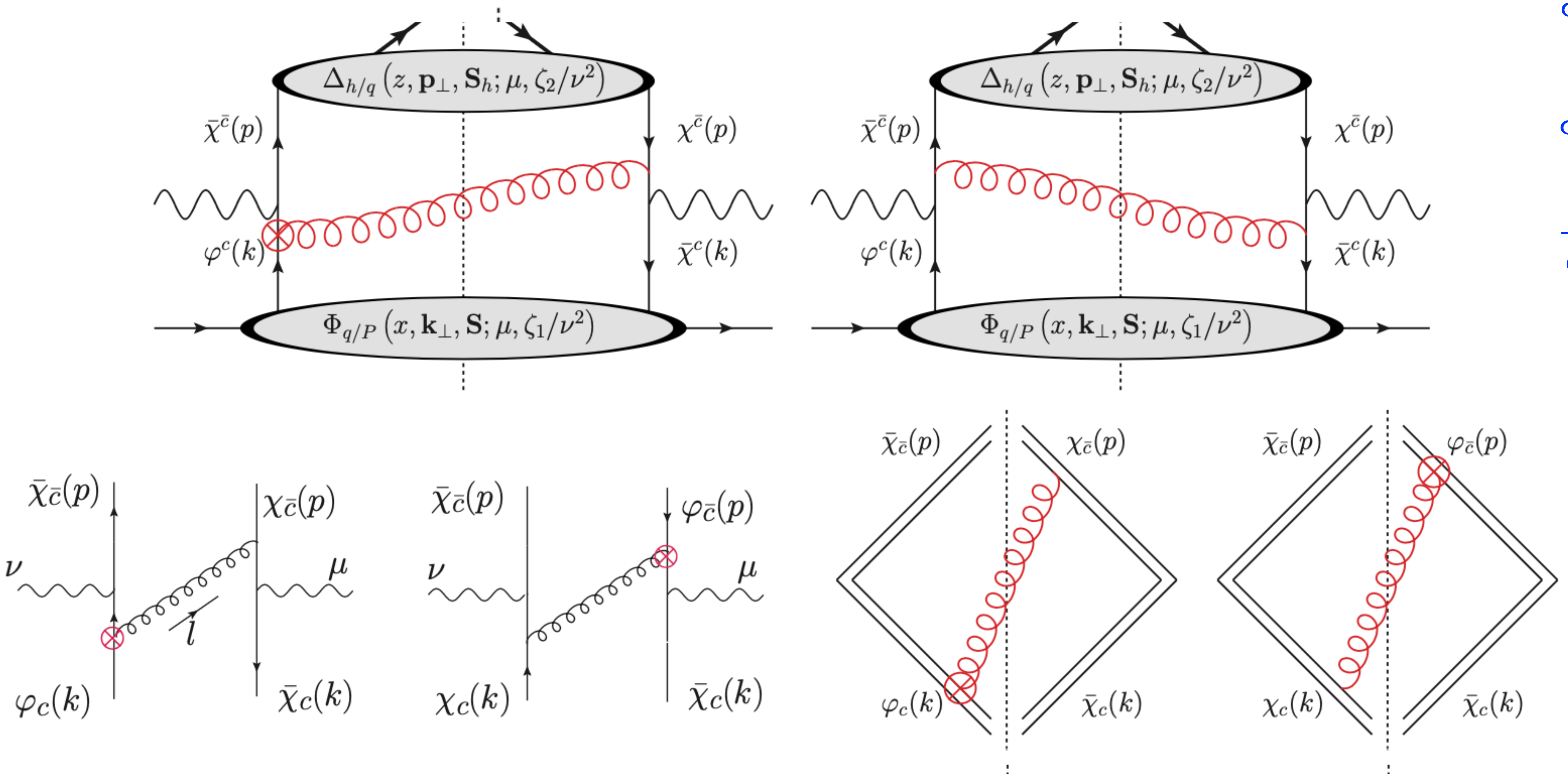
Recipe

- Calculate: soft, collinear (and anti), & hard
- Renormalize
  - Exploit properties of good and bad fields & power counting
- Check renormalization group consistency

NLO Ingredients soft factor

The soft region

The soft function is generated through the emissions of soft gluons in the partonic cross section



$$\hat{\mathcal{S}}^{\rm LP}(b;\mu,\nu) = Z_{\mathcal{S}\,\rm LP}(b;\mu,\nu)\mathcal{S}^{\rm LP}(b;\mu,\nu)$$

$$\hat{\mathcal{S}}^{\rm NLP}(b;\mu,\nu) = Z_{\mathcal{S}\,\rm NLP}(b;\mu,\nu)\mathcal{S}^{\rm NLP}(b;\mu,\nu)$$

$$\frac{\partial}{\partial \ln \mu}\mathcal{S}^{\rm NLP}(b,\mu,\nu) = \Gamma^{\mu}_{\mathcal{S}\,NLP}\mathcal{S}^{\rm NLP}(b,\mu,\nu)$$

$$\frac{\partial}{\partial \ln \nu}\mathcal{S}^{\rm NLP}(b,\mu,\nu) = \Gamma^{\nu}_{\mathcal{S}\,NLP}\mathcal{S}^{\rm NLP}(b,\mu,\nu)$$

$$\Gamma^{\nu}_{\mathcal{S}\,\rm int} = \frac{\partial}{\partial \ln \nu}Z_{\mathcal{S}\,NLP}(b;\mu,\nu)$$

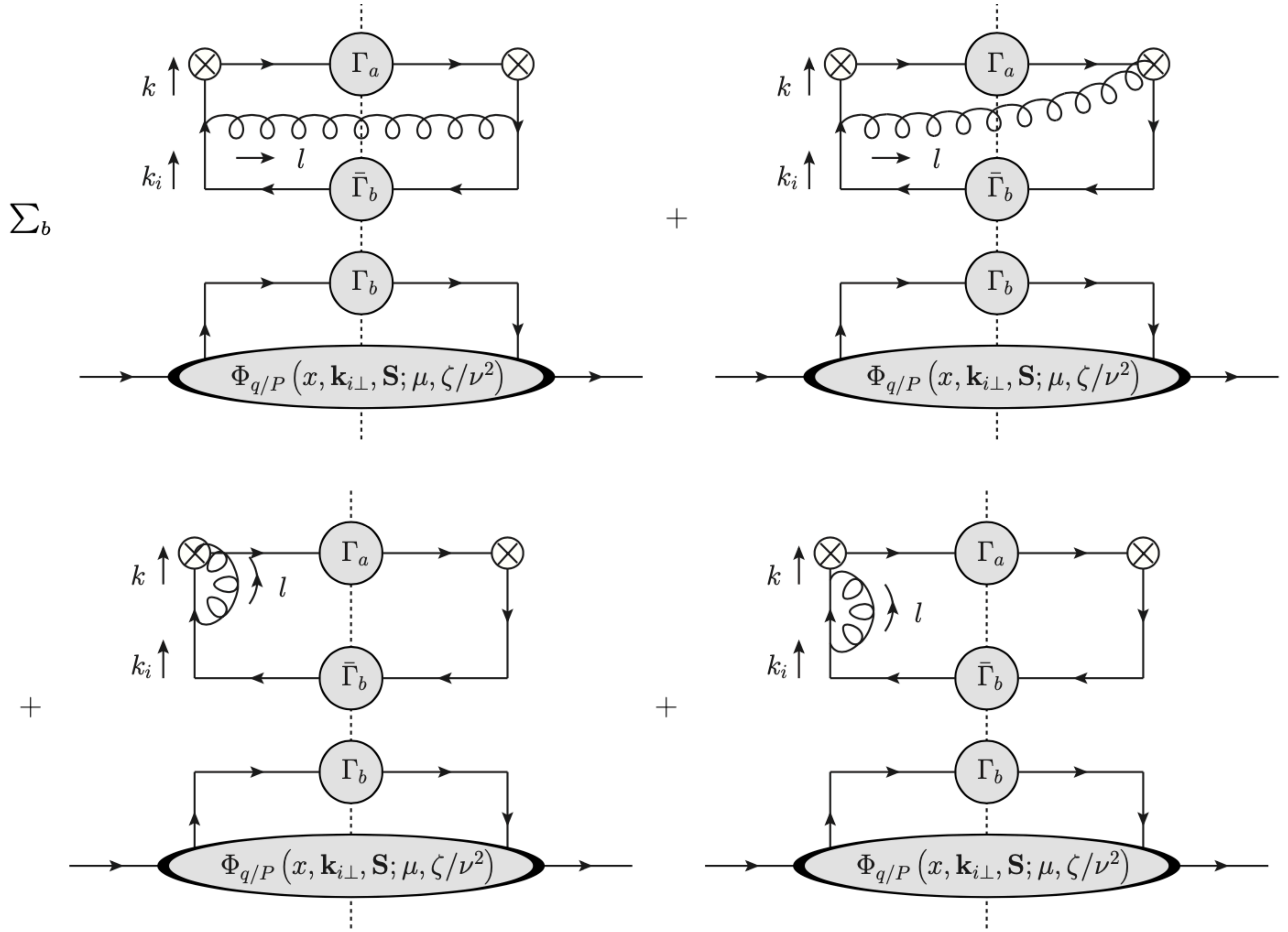
Gamberg, Kang, Shao, Terry, Zhao
arXiv: e-Print:221.13209

Soft emission from sub-leading fields vanish —> NLO + NLP soft function is half the LP one

$$\Gamma^{\mu}_{\mathcal{S}\,\rm int} = \frac{1}{2}\Gamma^{\mu}_{\mathcal{S}\,\rm LP}\,, \quad \Gamma^{\nu}_{\mathcal{S}\,\rm int} = \frac{1}{2}\Gamma^{\nu}_{\mathcal{S}\,\rm LP}$$

NLO Ingredients collinear factor

Diagrams associated with the evolution of the collinear region



Renormalize TMDs: soft & UV subtraction

$$\hat{\Phi}^{[\Gamma^a]}\left(x, \boldsymbol{b}, \boldsymbol{S}; \mu, \zeta/\nu^2\right) = Z_{\Gamma^a\,\Gamma^b}\left(b, \mu, \zeta/\nu^2\right) \Phi^{[\Gamma^b]0}\left(x, \boldsymbol{b}, \boldsymbol{S}; xP^+\right)$$

$$\Gamma_3^\nu = \frac{\partial}{\partial \ln \nu} Z_{NLP}(b; \mu, \nu)$$

Necessary condition rapidity RG Consistency

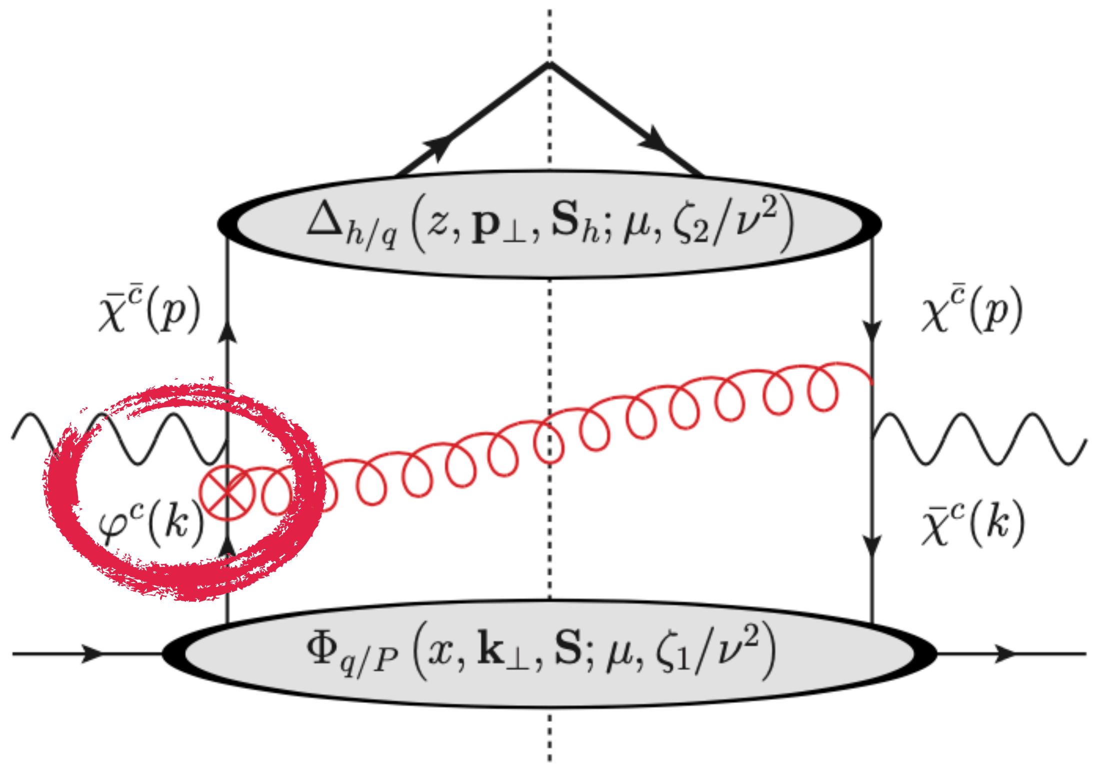
Review Leading power

$$f_1(x,b;\mu,\zeta_1) = f_1(x,b;\mu,\zeta_1/\nu^2)\sqrt{\mathcal{S}^{\rm LP}(b;\mu,\nu)} \longleftrightarrow \Gamma_2^\nu + \frac{1}{2}\Gamma_S^\nu = 0$$

$$D_1(z,b;\mu,\zeta_2) = D_1(z,b;\mu,\zeta_2/\nu^2)\sqrt{\mathcal{S}^{\rm LP}(b;\mu,\nu)} \longleftrightarrow \Gamma_2^\nu + \frac{1}{2}\Gamma_S^\nu = 0$$

Next to leading power

$$\frac{d\sigma}{dP_T^2} \sim \; - H^{\rm int}_{\rm DIS}(Q;\mu)\,\mathcal{C}^{\rm DIS}\left[\left(x\,\frac{\boldsymbol{k}_\perp\cdot\hat{x}}{Q} f^\perp D_1 - \frac{\boldsymbol{p}_\perp\cdot\hat{x}}{zQ} f_1\, D^\perp\right)\mathcal{S}^{\rm int}\right]$$



$$ib^\mu M^2 f^{\perp\,(1)}(x,b;\mu,\zeta_1) = ib^\mu M^2 f^{\perp\,(1)}(x,b;\mu,\zeta_1/\nu^2)\sqrt{\mathcal{S}^{\rm int}(b;\mu,\nu)} \longleftrightarrow \Gamma^\nu_{3\,\rm int} + \frac{1}{2}\Gamma^\nu_{S\,\rm int} = 0$$

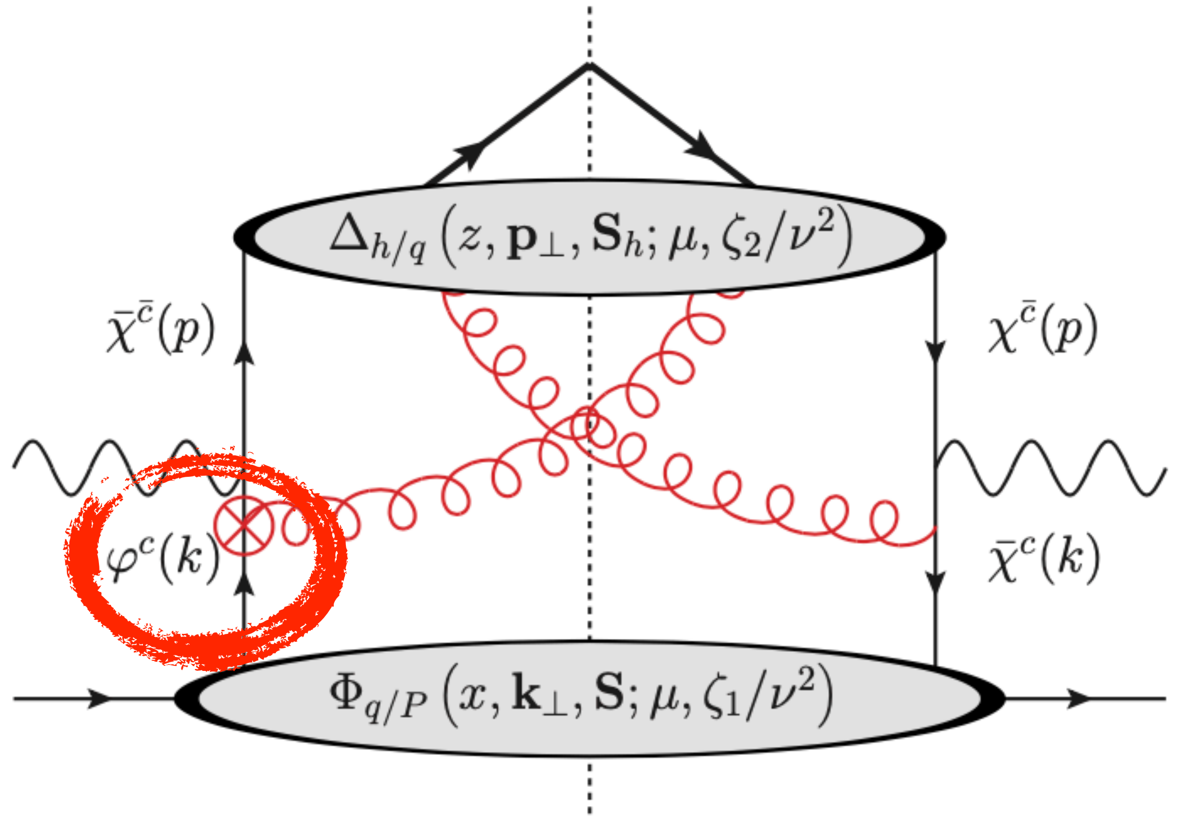
Non-trivial result

However for cross section

!!

$$D_1(z,b;\mu,\zeta_2) = D_1(z,b;\mu,\zeta_2/\nu^2)\sqrt{\mathcal{S}^{\rm LP}(b;\mu,\nu)} \longleftrightarrow \Gamma_2^\nu + \frac{1}{2}\Gamma^\nu_{S\,\rm int} \neq 0$$

!!



$$D_1(z,b;\mu,\zeta_2) = D_1(z,b;\mu,\zeta_2/\nu)\sqrt{S^{int}}\,?? \longleftrightarrow \Gamma^\nu_{2\,\rm mod} + \frac{1}{2}\Gamma^\nu_{S\,\rm int} = 0$$

Necessary condition rapidity RG Consistency

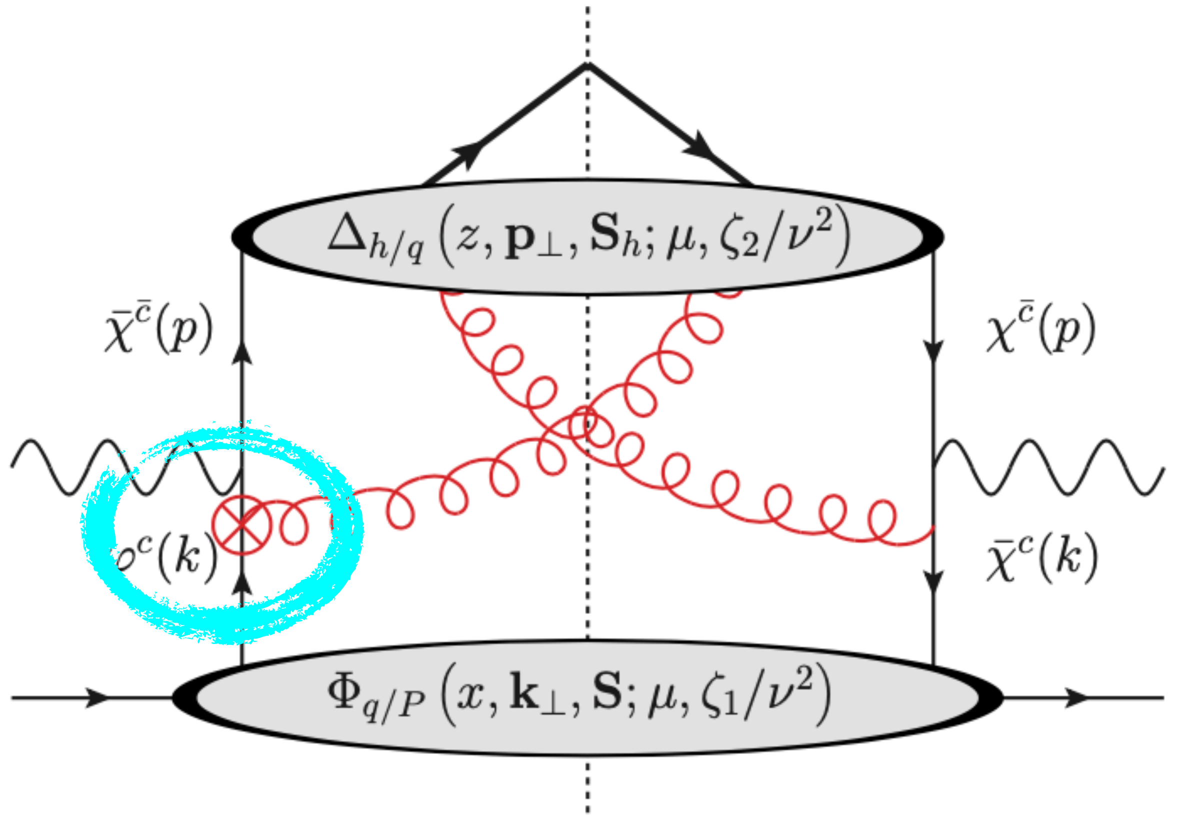
Next to leading power $\frac{d\sigma}{d\ln\nu} = 0$ & $\frac{d\sigma}{d\ln\mu} = 0$

$$- H^{\rm int}_{\rm DIS}(Q;\mu)\, \mathcal{C}^{\rm DIS}\left[\left(x\,\frac{\boldsymbol{k}_\perp\cdot\hat{x}}{Q} f^\perp D_1 - \frac{\boldsymbol{p}_\perp\cdot\hat{x}}{zQ} f_1 D^\perp\right) \mathcal{S}^{\rm int}\right]$$

!!

$$\Gamma^\nu_{\mathcal{S}\,\rm int} + \Gamma^\nu_{3\,\rm int} + \Gamma^\nu_{2\,\rm mod} = 0$$

Have shown



Problem: Breakdown of universality different soft function for D_1 ?!

$$D_1(z,b;\mu,\zeta_2) = D_1(z,b;\mu,\zeta_2/\nu)\sqrt{S^{int}}$$!!

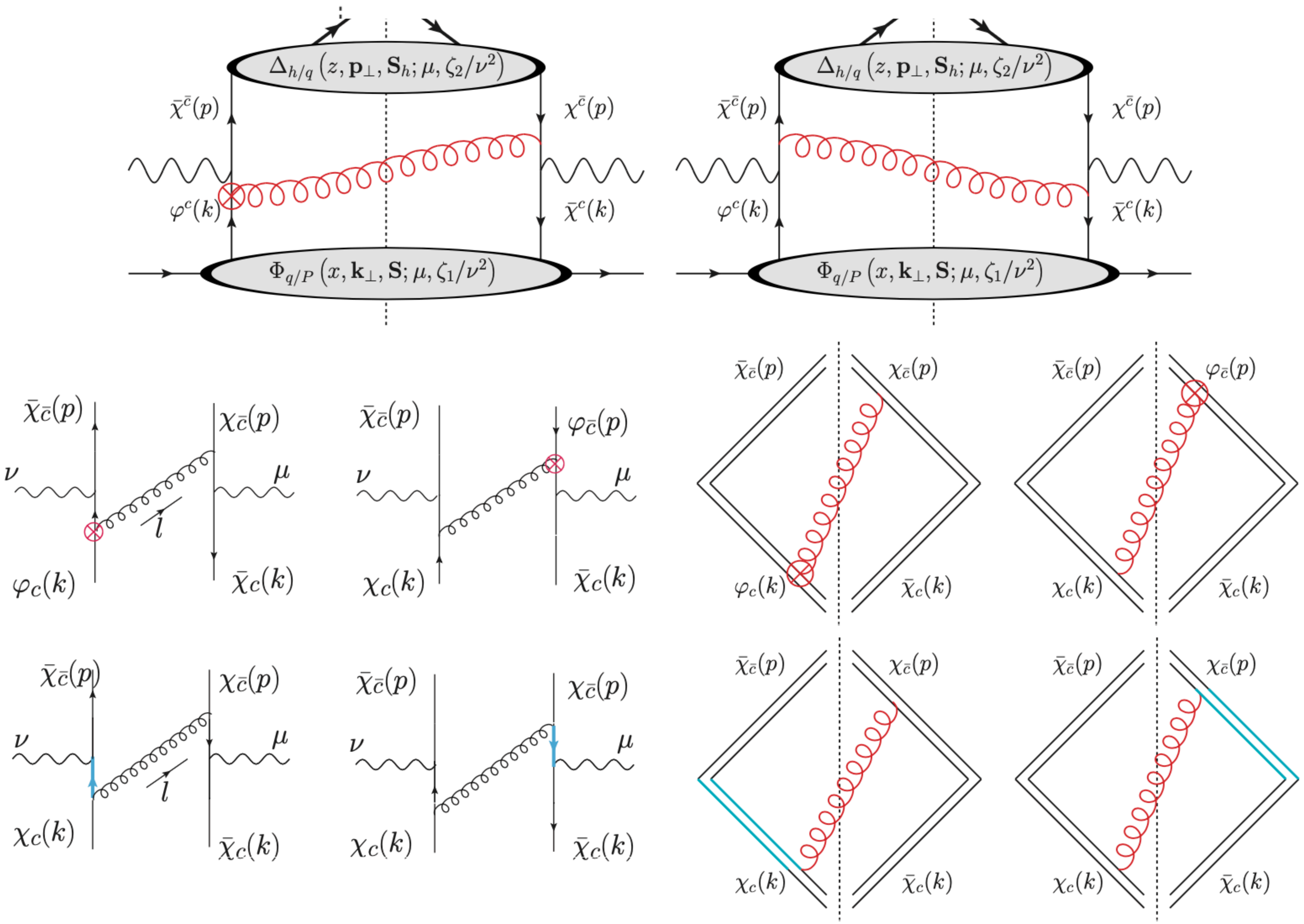
$$D_1(z,b;\mu,\zeta_2) = D_1(z,b;\mu,\zeta_2/\nu^2)\sqrt{\mathcal{S}^{\rm LP}(b;\mu,\nu)}$$

NLP **LP**

Other contributions ? Ingredients soft factor

The soft region

The soft function is generated through the emissions of soft gluons in the partonic cross section



Progress Report
Stay tuned ...

Contributions to the soft factor after applying the eikonal approximation and including the effect from the transverse momentum contributions from the quark propagators.

Summary

We explore NLP $(M/Q)^n$ contributions in large P_T and TMD regions via factorization theorem

- NLP factorization based on "*TMD formalism*"

  —extend the tree level Amsterdam formalism and beyond leading order

  CSS, Ji Ma Yuan, Abyat Rogers, framework vs. SCET and Background Field Methods
- Considier R_{SIDIS} & revisit "Cahn effect" & matching related to early importance intrinsic k_T
  - *"Intrinsic"NLP TMDs related thru EOM in terms "kinematic" & "dynamical"*
- Consider RG consistency of matching to collinear factorization
  - Bacchetta, Boer, Diehl, Mulders JHEP 2008, Bacchetta et al. PLB 2019
- Report progress in this *necessary condition* NLP factorization (not yet sufficient)

- In doing so, we provide the basis for performing global analysis & phenomenology of one the earliest observables used to study intrinsic 3-D momentum structure of the nucleon—Opportunity for EIC study of transverse momentum nucleon structure

Thank You