Quantum algorithms for high energy evolution

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SURGE COLLABORATION

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Motivation

- JIMWLK equation: small-x observables, saturation
- LL JIMWLK → Langevin formulation
- Computationally expensive process
- NLL JIMWLK lacks a Langevin formulation
- Not all observables can be studied

Need a new method to simulate JIMWLK

J.P.Blaizot,E.Iancu,H.Weigert
K.Rummukainen and H.Weigert

Quantum algorithms for JIMWLK

Aim: Develop quantum algorithms for JIMWLK evolution

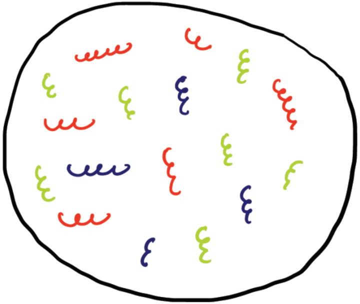
- JIMWLK evolution for density matrices $\leftrightarrow$ Lindblad evolution
- Lindblad evolution in open quantum systems: Widely simulated
- NLL JIMWLK can be simulated in principle

N.Armesto, F.Domniniguez, A.Kovner, M.Lublinsky, V.Skokov (2019)
M.Li, A. Kovner (2020)

What is JIMWLK

Distribution of color charges:

$$\mathbf{W_Y[j]}$$



Color charge configuration $\mathbf{j}$

What is JIMWLK?

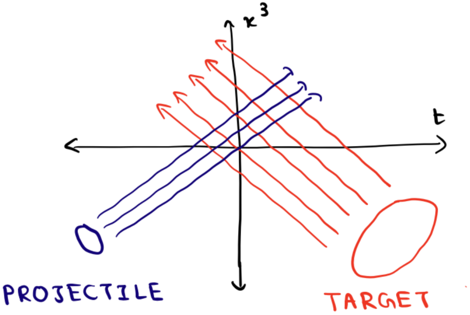
Evolution of target color charge density $W_Y[j]$ with rapidity Y

Scattering of a projectile:

$$\langle S \rangle_Y = \sum_j S(j)\, W_Y[j]$$

Rapidity Evolution:

$$\partial_Y W_Y[j] = H_{JIMWLK} W_Y[j]$$



Evolution due to quantum effects

A.Kovner (2005)
A.Kovner, M.Lublinsky, Y.Mulian (2014)

Usual method: Langevin Evolution

- Evolve the projectile instead:

$$\partial_Y \langle S \rangle_Y = \langle H_{JIMWLK} S \rangle_Y$$

- Written as a Langevin equation
- Evolution as a random walk:

$$S_Y \to S_{Y+\delta Y}(\xi)$$

- Average over gaussian random variables ξ,

$$S_{Y+\delta Y} = \langle S_{Y+\delta Y}(\xi) \rangle_\xi$$

J.P.Blaizot, E.Iancu, H.Weigert
K.Rummukainen and H.Weigert

Problems with the approach

- Needs to be iterated: Computationally expensive
- Averaging gives rise to statistical errors
- No langevin formulation for NLO JIMWLK
- Cannot compute off-diagonal observables

Solution: Look at Lindblad-JIMWLK correspondence

Hamiltonian evolution

$$\boxed{\partial_t \rho_U(t) = -[H, \rho_U]}$$

Lindbladian Evolution

$$\partial_t \rho_A(t) = -[H, \rho_A] + \sum_\alpha Q_\alpha \rho_A(t) Q_\alpha^\dagger - \frac{1}{2}\{Q_\alpha^\dagger Q_\alpha, \rho(t)\}$$

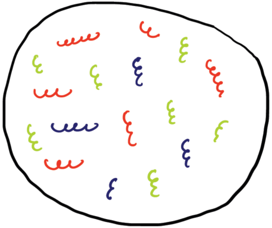
Jump operator (Q_α): Change in A states due to B

JIMWLK evolution can be written as a Lindblad equation

Gluonic degrees of freedom

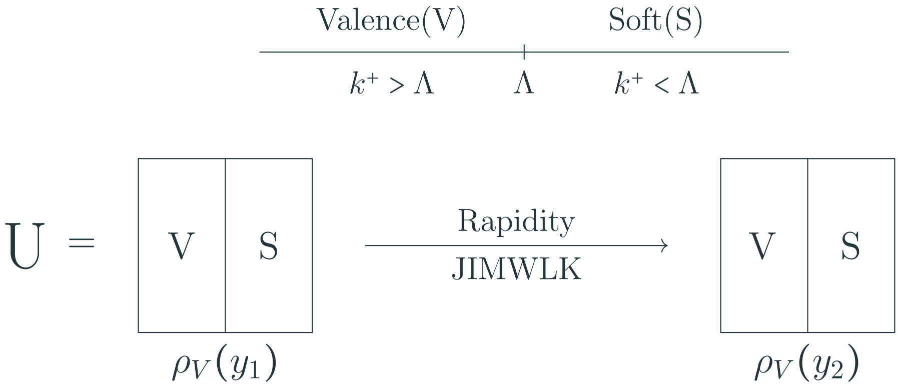
Valence(V)		Soft(S)
$k^+ > \Lambda$	Λ	$k^+ < \Lambda$

Density matrix ρ_v: Valence gluons $|j\rangle$



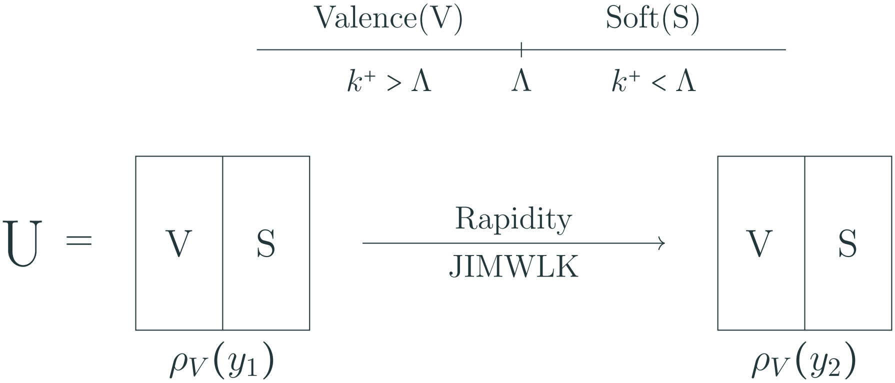
$$\langle j|\rho|j\rangle = W[j]$$

JIMWLK as a Lindblad equation



M. Li, A. Kovner(2020)

JIMWLK as a Lindblad equation



Careful: Only valid in dilute/dense limits

M. Li, A. Kovner(2020)

JIMWLK

$$\boxed{\frac{d}{dY}\hat{\rho}_v = \int \frac{d^2 z_\perp}{2\pi}\Big[\hat{Q}^a[z_\perp], \Big[\hat{Q}^a[z_\perp], \hat{\rho}_v\Big]\Big]}$$

- Hilbert space: Field values $|\alpha^c(x_\perp)\rangle$ generated by $|j\rangle$
- Dilute limit jump operators:

$$Q_i^a[z_\perp] \propto \frac{\delta}{\delta\,\alpha^a(x_\perp)}$$

Functional differential equation acting on density matrices

The method summarized

JIMWLK in 2-dimensions (SU(3))

Reduce dimensions $\downarrow$ 2 points

JIMWLK in 0-dimensions (SU(3))

Reduce generators $\downarrow$ 3 colors

JIMWLK in 0-dimensions (SU(2))

Truncate field space $\downarrow$ 2 field values

QM JIMWLK in 0-dimensions (SU(2))

Discretization I

- Position space discretization(N_p)
- Field space discretization(N_F)
- Reduced color space(N_g)

$$\boxed{\text{Number of states} = (N_F)^{N_p N_g}}$$

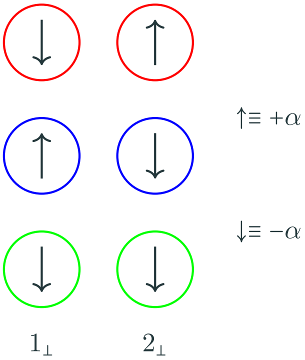
$$\boxed{\text{Number of jump operators} = N_p N_g}$$

Discretization II

- Two points $1_\perp, 2_\perp$: $N_p = 2$
- Two field values $-\alpha, \alpha$: $N_F = 2$
- SU(2) : $N_g = 3$

Number of states: 64

Number of jump operators: 6



Evolution in Quantum computers

$$\frac{d}{dY}\hat{\rho}_v = \sum_{z_\perp} \frac{z_\perp}{2\pi}\left[\hat{Q}^a[z_\perp],\left[\hat{Q}^a[z_\perp],\hat{\rho}_v\right]\right]$$

- Non-unitary evolution $\implies$ No direct QC implementation
- Methods available:
  1. Stochastic schrodinger equation
  2. **Effective higher dimensional hamiltonian**
  3. Linear combination of unitaries

J.Li, X.Li(2020)
Z.Ding, X.Li, L.Lin(2024)
A.Schlimgen, K.Head-Marsden, L.M.Sager, P.Narang, D.A.Maziotti(2021)

Evolution in Quantum computers

U = | Valence | Approx Soft |

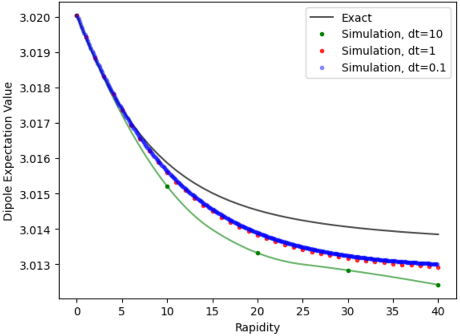
$$\mathcal{H}_U = \begin{pmatrix} 0 & Q_1^\dagger & Q_2^\dagger \\ Q_1 & 0 & 0 \\ Q_2 & 0 & 0 \end{pmatrix} \qquad \mathrm{N_{jump}} = 2$$

$$\rho_{n+1} = \mathrm{Tr}_S\Big[\exp\big(-i\sqrt{\delta Y}\,\mathcal{H}\big)\,\boxed{|0\rangle\langle 0|} \otimes \rho_n \exp\big(i\sqrt{\delta Y}\mathcal{H}\big)\Big]$$

Results

- $\rho_{initial}$: 64×64 matrix
- Pauli matrix expansion of U
- Trotterization
- Simulation using Qiskit

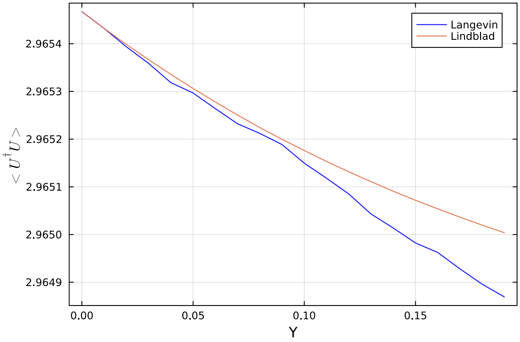
Need to match with Langevin JIMWLK

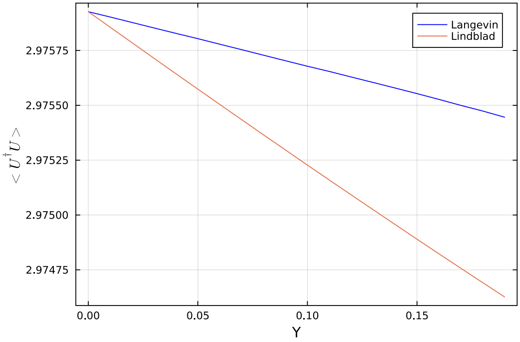


Z.Ding, X.Li, L.Lin (2024)

Matching with Langevin

Match with dilute Langevin JIMWLK

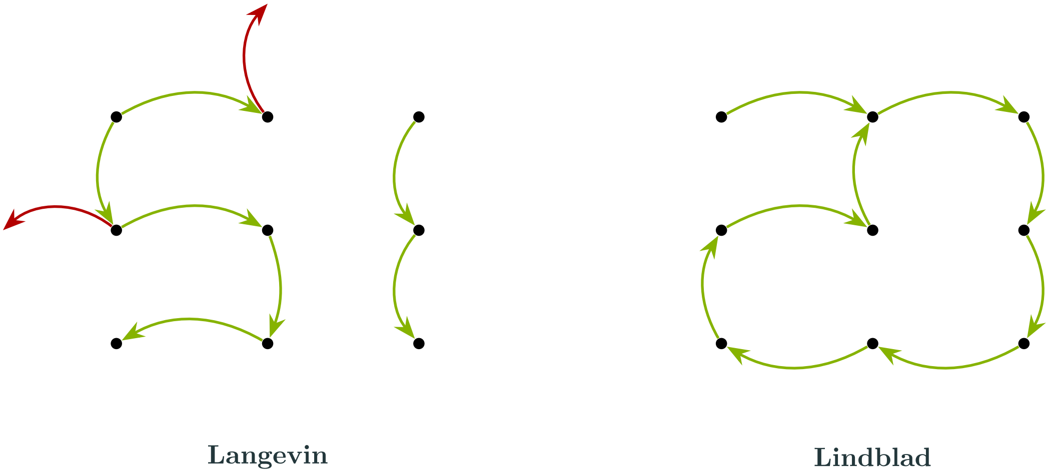




Match not perfect: Discretization effects!!

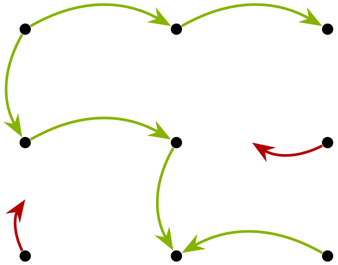
T.Altinoluk, G.Beuf, M.Lublinsky, V.Skokov (2024)

Truncation effect(IR)

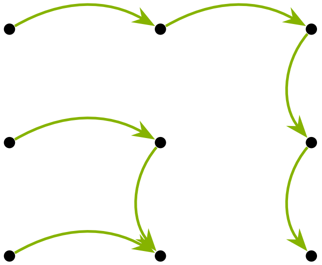


Langevin jumps outside the space near **Edge states**

Coarse graining effect (UV)



Langevin

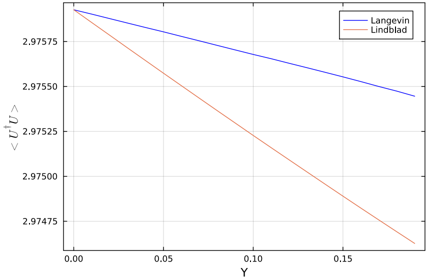


Lindblad

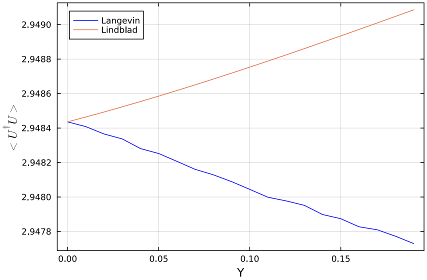
Langevin evolution is not sensitive to **lattice spacing**

Discretization effects

Two types: Coarse Gaining(UV) and Truncation(IR)



Coarse Graining effect



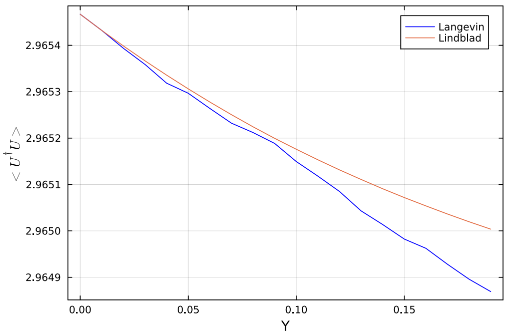
Truncation effect

Sensitivity to discretization of two evolutions different

Avoiding these effects

- **IR effects:** Avoid edge states

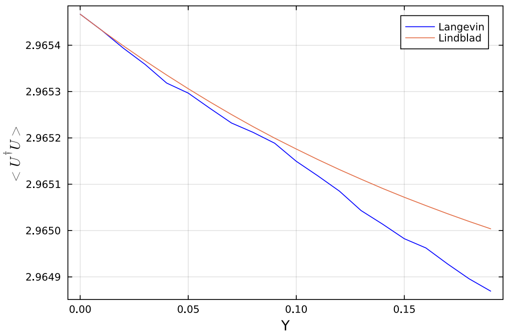
- **UV effects:** Avoid localization



Maybe a change of basis?

Avoiding these effects

- **IR effects:** Avoid edge states
- **UV effects:** Avoid localization



Currently looking at spin motivated basis. Will report in the future!

Summary

JIMWLK in 2-dimensions (SU(3))

Reduce dimensions ↓ 2 points

JIMWLK in 0-dimensions (SU(3))

Reduce generators ↓ 3 colors

JIMWLK in 0-dimensions (SU(2))

Truncate field space ↓ 2 field values

QM JIMWLK in 0-dimensions (SU(2))

In Future

- Use spin motivated basis to reduce discretization effects
- Extend the analysis to more number of transverse points
- Repeat the calculation for more number of colors

Efficiently simulate 2-d JIMWLK with SU(3) on Quantum computers

Thanks for your attention!