Proton GPDs from Lattice QCD

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In collaboration with:

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A. Metz, F. Steffens, S. Mukherjee, Y. Zhao**

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Outline

❖ Work extends the approach for the unpolarized and axial cases

❖ Theoretic Formulation

PHYSICAL REVIEW D **106,** 114512 (2022)

Generalized parton distributions from lattice QCD with asymmetric momentum transfer: Unpolarized quarks

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Generalized parton distributions from lattice QCD with asymmetric momentum transfer: Axial-vector case

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❖ Lattice Formulation (focus of this talk)

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❖ Theoretic Formulation

❖ Lattice Formulation (focus of this talk)

- ❖ Background
- ❖ Lattice Methodology
- ❖ Results
  - Matrix Elements
  - Lorentz invariant amplitudes
  - Quasi-GPDs
  - Light-Cone GPDs

Generalized Parton Distributions

❖ **GPDs are rich in information:**

- Reflect spatial distribution of partons in transverse plane
- Hadron mechanical properties are stored in GPDs
- Information on spin

❖ **… but not well studied:**

- extracted from off-forward kinematic (unlike PDFs)
- Multi-variable quantities; dependence upon $x,\ t$ and ξ (unlike PDFs)
- Inferred from Compton form factors from experimental data (e.g., DVCS)
- Other processes proposed (**SDHEP [J. Qiu et al, arXiv:2205.07846]**) still require theoretical developments

❖ **Transversity proton GPDs:**

- Four GPDs: $H_T,\ E_T,\ \widetilde{H}_T,\ \widetilde{E}_T$

$$F^{[i\sigma^{j+}\gamma_5]}_{\lambda,\lambda'}(z,\Delta,P) = -i\epsilon^{-+ij}\bar{u}(p',\lambda')\left[i\sigma^{+i}H_T + \frac{\gamma^+\Delta^i_\perp - \Delta^+\gamma^i_\perp}{2M}E_T + \frac{P^+\Delta^i_\perp - P^i_\perp\Delta^+}{M^2}\widetilde{H}_T + \frac{\gamma^+P^i_\perp - P^+\gamma^i_\perp}{M}\widetilde{E}_T\right]u(p,\lambda)$$

Methodology on the Lattice

❖ Choice of frame:

- Symmetric: $\vec{p_i} = P_3\hat{z} - \vec{\Delta}/2, \quad \vec{p_f} = P_3\hat{z} + \vec{\Delta}/2$
- Asymmetric: $\vec{p}_i = P_3\hat{z} - \vec{\Delta}, \ \vec{p}_f = P_3\hat{z}$

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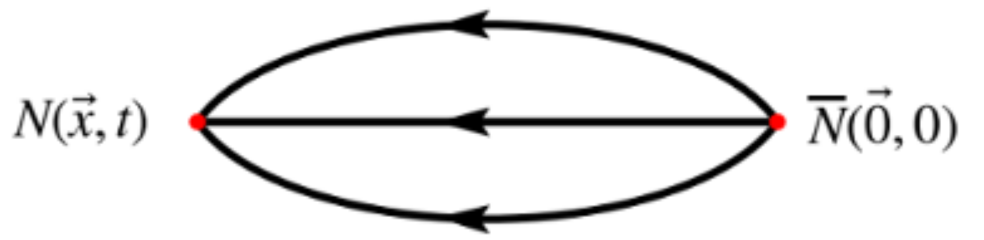
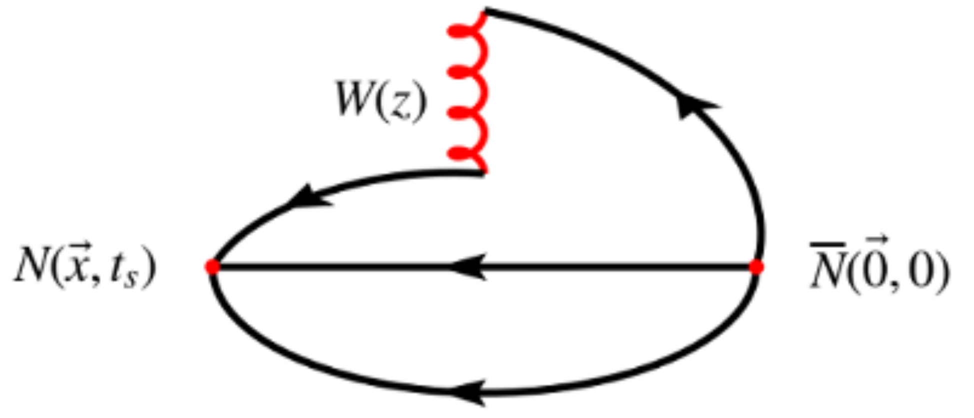
❖ Calculate the 2-, 3-point correlation functions in the frame chosen



$\langle N(P) \,|\, N(P) \rangle$



$\langle N(P_f) \,|\, \bar{\Psi}(z) i\sigma^{\mu\nu}\gamma_5 \mathscr{W}(z,0)\Psi(0) \,|\, N(P_i) \rangle$

Methodology on the Lattice

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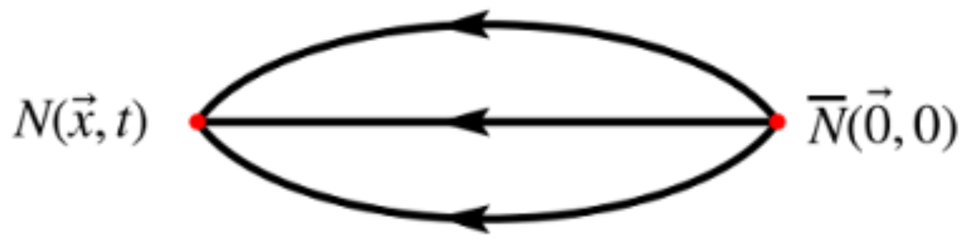

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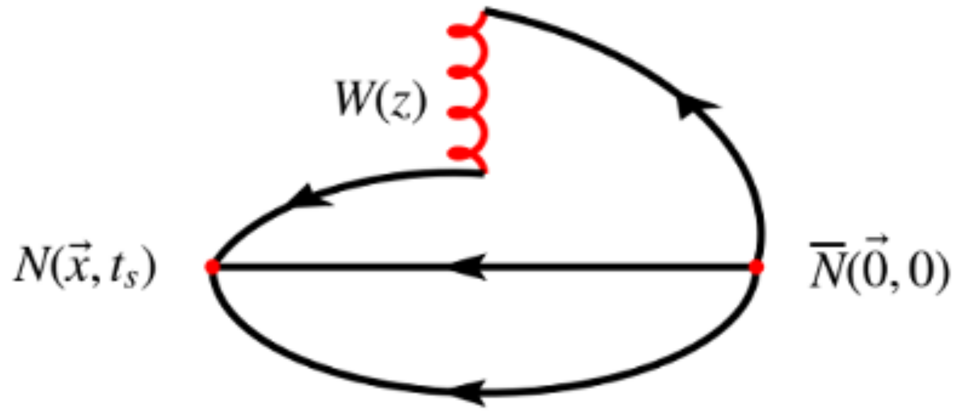

$\langle N(P_f) | \bar{\Psi}(z) i\sigma^{\mu\nu}\gamma_5 \mathscr{W}(z,0)\Psi(0) | N(P_i) \rangle$

❖ Take an appropriate ratio of the 2- and 3-point correlation functions and isolate the ground state (plateau fit)

$$R_\mu(\Gamma_\kappa, z, p_f, p_i; t_s, \tau) = \frac{C_\mu^{3\text{pt}}(\Gamma_\kappa, z, p_f, p_i; t_s, \tau)}{C^{2\text{pt}}(\Gamma_0, p_f; t_s)} \sqrt{\frac{C^{2\text{pt}}(\Gamma_0, p_i, t_s - \tau) C^{2\text{pt}}(\Gamma_0, p_f, \tau) C^{2\text{pt}}(\Gamma_0, p_f, t_s)}{C^{2\text{pt}}(\Gamma_0, p_f, t_s - \tau) C^{2\text{pt}}(\Gamma_0, p_i, \tau) C^{2\text{pt}}(\Gamma_0, p_i, t_s)}} \xrightarrow[\tau \gg a]{t_s - \tau \gg a} \Pi_\mu(\Gamma_\kappa, z, p_f, p_i; t_s)$$

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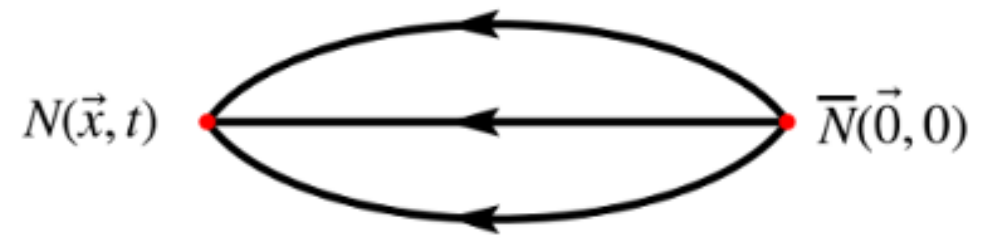
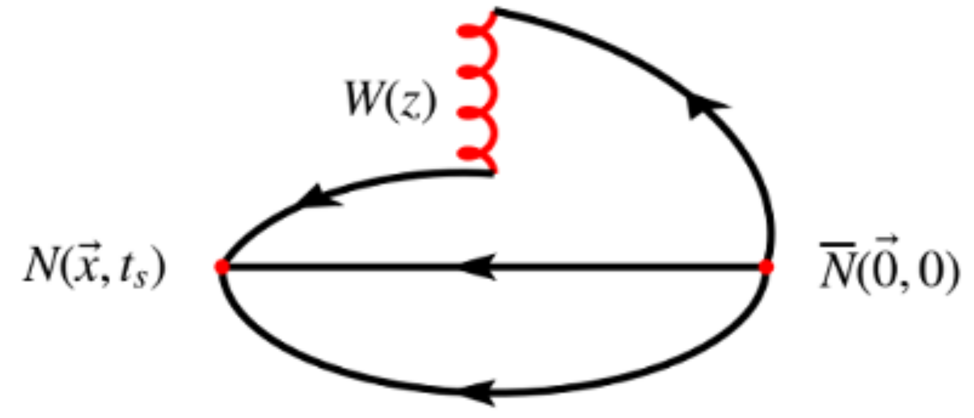


$\langle N(P)|N(P)\rangle$

❖ Calculate the 2-, 3-point correlation functions in the frame chosen



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❖ Parameterize the matrix elements in terms of (Lorentz invariant) amplitudes

$$F_{\lambda,\lambda'}^{[i\sigma^{\mu\nu}\gamma_5]} = P^{[\mu}z^{\nu]}A_1 + \frac{P^{[\mu}\Delta^{\nu]}}{M^2}\gamma_5 A_2 + z^{[\mu}\Delta^{\nu]}\gamma_5 A_3 + \gamma^{[\mu}\left(\frac{P^{\nu]}}{M}A_4 + Mz^{\nu]}A_5 + \frac{\Delta^{\nu]}}{M}A_6\right)\gamma_5 + M\gamma_\alpha z^\alpha\left(P^{[\mu}z^{\nu]}A_7 + \frac{P^{[\mu}\Delta^{\nu]}}{M^2}A_8 + z^{[\mu}\Delta^{\nu]}A_9\right) + i\sigma^{\mu\nu}\gamma_5 A_{10}$$

$$+ i\epsilon^{\mu\nu Pz}A_{11} + i\epsilon^{\mu\nu z\Delta}A_{12}$$

Methodology on the Lattice

❖ Choice of frame:

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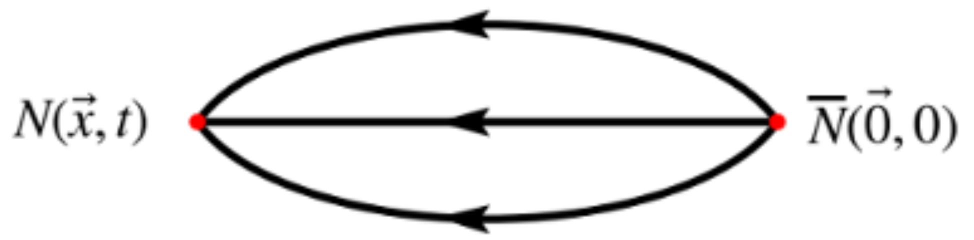

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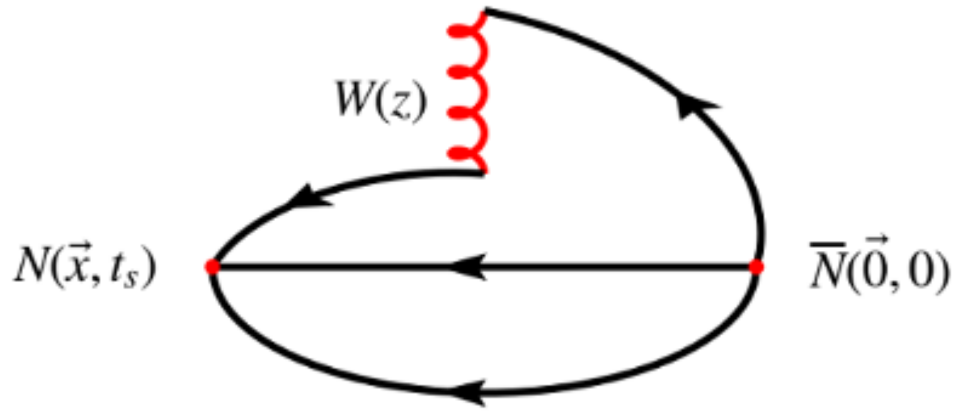

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$$+ i\epsilon^{\mu\nu P z}A_{11} + i\epsilon^{\mu\nu z\Delta}A_{12}$$

→ 12 linearly independent Lorentz invariant amplitudes

Methodology on the Lattice

- Equate and relate the amplitude and quasi-GPD decomposition

$$F^{[i\sigma^{j+}\gamma_5]}_{\lambda,\lambda'}(z,\Delta,P) = -i\epsilon^{-+ij}\bar{u}(p',\lambda')\left[i\sigma^{+i}H_T + \frac{\gamma^+\Delta^i_\perp - \Delta^+\gamma^i_\perp}{2M}E_T + \frac{P^+\Delta^i_\perp - P^i_\perp\Delta^+}{M^2}\widetilde{H}_T + \frac{\gamma^+P^i_\perp - P^+\gamma^i_\perp}{M}\widetilde{E}_T\right]u(p,\lambda)$$

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Standard Definitions

$$\mathscr{H}^{s/a}_T = -2A_{T2}\left(1-\frac{\bar{P}^2}{m^2}\right) + A_{T4} - zA_{T8}\left(\frac{E_f^2 - E_i^2}{2P_3}\right) + A_{T10}$$

$$\mathscr{E}^{s/a}_T = 2A_{T2} - A_{T4} + zA_{T8}\left(\frac{E_f^2 - E_i^2}{2P_3}\right)$$

$$\tilde{\mathscr{H}}^{s/a}_T = -A_{T2} - zA_{T12}\frac{m^2}{P_3}$$

$$\tilde{\mathscr{E}}^{s/a}_T = -2A_{T6} - zA_{T8}\frac{(E_f + E_i)^2}{2P_3}$$

Substitute $\bar{P}^{s/a}$ and $\Delta^{s/a}$ respectively

LI Definitions

$$\mathscr{H}^{LI}_T = -2A_{T2}\left(1-\frac{\bar{P}^2}{m^2}\right) + A_{T4} + A_{T10}$$

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❖ Renormalization functions: RI-MOM

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- Renormalization functions: RI-MOM
- Transform from position to momentum space (Backus-Gilbert)

[Backus & Gilbert, Geophysical Journal International 16, 169 (1968)]

Methodology on the Lattice

❖ Equate and relate the amplitude and quasi-GPD decomposition

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❖ Renormalization functions: RI-MOM

❖ Transform from position to momentum space (Backus-Gilbert) [Backus & Gilbert, Geophysical Journal International 16, 169 (1968)]

❖ Extract light cone-GPDs using one-loop matching formalism [Liu, et al., Phys. Rev. D 100, 034006 (2019)]

Decomposition

Symmetric frame ($\xi = 0$)

$$\Pi^s_{01}(\Gamma_0) = iK\left(-\frac{\Delta_1 P_3^2}{4m^3}A_{T4} + \frac{\Delta_1 P_3}{4m}zA_{T5} + \frac{(E+m)\Delta_1}{4m^2}A_{T10}\right)$$

$$\Pi^s_{03}(\Gamma_0) = iK\left(\frac{P_3(\Delta_1^2+\Delta_2^2)}{4m^3}A_{T6} - \frac{E(E(E+m)-P_3^2)}{2m^2}zA_{T11}\right)$$

Asymmetric frame ($\xi = 0$)

$$\Pi^a_{01}(\Gamma_0) = iK\left(\frac{(m-E_f)(m+E_f)\Delta_1}{4m^3}A_{T4} + \frac{P_3\Delta_1}{4m}zA_{T5} + \frac{(E_f+m)\Delta_1}{4m^2}A_{T10}\right)$$

$$\Pi^a_{03}(\Gamma_0) = iK\left(\frac{(E_f-E_i)(E_f+E_i)P_3}{8m^3}A_{T4} + \frac{(E_i^2-E_f^2)P_3}{4m^3}A_{T6} + \frac{(E_i-E_f)P_3}{4m^2}A_{T10} + \frac{(E_f+E_i)(E_f-E_i-2m)(E_f+m)}{8m^2}zA_{T11} - \frac{(E_f-E_i)(E_f-E_i-2m)(E_f+m)}{4m^2}zA_{T12}\right)$$

Decomposition

Symmetric frame ($\xi = 0$)

$$\Pi^s_{01}(\Gamma_0) = iK\left(-\frac{\Delta_1 P_3^2}{4m^3}A_{T4} + \frac{\Delta_1 P_3}{4m}zA_{T5} + \frac{(E+m)\Delta_1}{4m^2}A_{T10}\right)$$

$$\Pi^s_{03}(\Gamma_0) = iK\left(\frac{P_3(\Delta_1^2+\Delta_2^2)}{4m^3}A_{T6} - \frac{E(E(E+m)-P_3^2)}{2m^2}zA_{T11}\right)$$

Asymmetric frame ($\xi = 0$)

$$\Pi^a_{01}(\Gamma_0) = iK\left(\frac{(m-E_f)(m+E_f)\Delta_1}{4m^3}A_{T4} + \frac{P_3\Delta_1}{4m}zA_{T5} + \frac{(E_f+m)\Delta_1}{4m^2}A_{T10}\right)$$

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$$\Pi_i^s \neq \Pi_i^a$$

Decomposition

Symmetric frame ($\xi = 0$)

$$\Pi^s_{01}(\Gamma_0) = iK\left(-\frac{\Delta_1 P_3^2}{4m^3}A_{T4} + \frac{\Delta_1 P_3}{4m}zA_{T5} + \frac{(E+m)\Delta_1}{4m^2}A_{T10}\right)$$

$$\Pi^s_{03}(\Gamma_0) = iK\left(\frac{P_3(\Delta_1^2 + \Delta_2^2)}{4m^3}A_{T6} - \frac{E(E(E+m) - P_3^2)}{2m^2}zA_{T11}\right)$$

Coefficients become more complicated

Asymmetric frame ($\xi = 0$)

Amplitudes frame independent

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$$\Pi_i^s \neq \Pi_i^a \qquad A^s_{Ti} = A^a_{Ti}$$

Decomposition

Symmetric frame ($\xi = 0$)

$$\Pi^s_{01}(\Gamma_0) = iK\left(-\frac{\Delta_1 P_3^2}{4m^3}A_{T4} + \frac{\Delta_1 P_3}{4m}zA_{T5} + \frac{(E+m)\Delta_1}{4m^2}A_{T10}\right)$$

$$\Pi^s_{03}(\Gamma_0) = iK\left(\frac{P_3(\Delta_1^2 + \Delta_2^2)}{4m^3}A_{T6} - \frac{E(E(E+m) - P_3^2)}{2m^2}zA_{T11}\right)$$

Coefficients become more complicated

Amplitudes frame independent

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★ **Frame dependence of matrix elements due to kinematic coefficients of A_{Ti}** ★

Lattice Setup

- $N_f = 2 + 1 + 1$ Twisted mass fermions with a clover term

Parameters							
Ensemble	β	a [fm]	volume $L^3 \times T$	N_f	m_π [MeV]	Lm_π	L [fm]
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N/A	±1.25	(0,0,0)	0	0	2	329	16	10528
symm	±0.83	(±2,0,0), (0,±2,0)	0.69	0	8	67	8	4288
symm	±1.25	(±2,0,0), (0,±2,0)	0.69	0	8	249	8	15936
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symm	±1.25	(±2, ±2, 0)	1.38	0	16	224	8	28672
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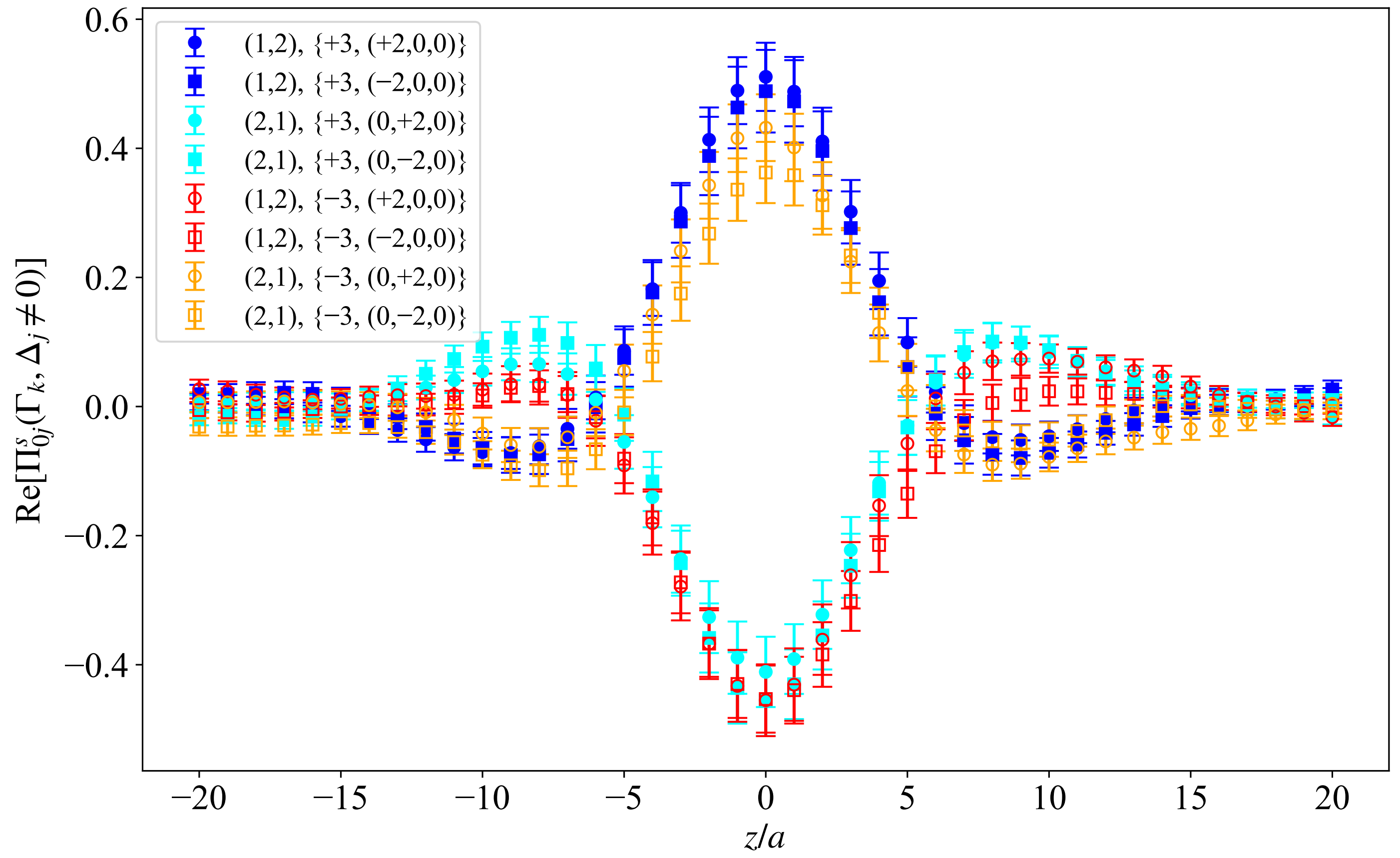
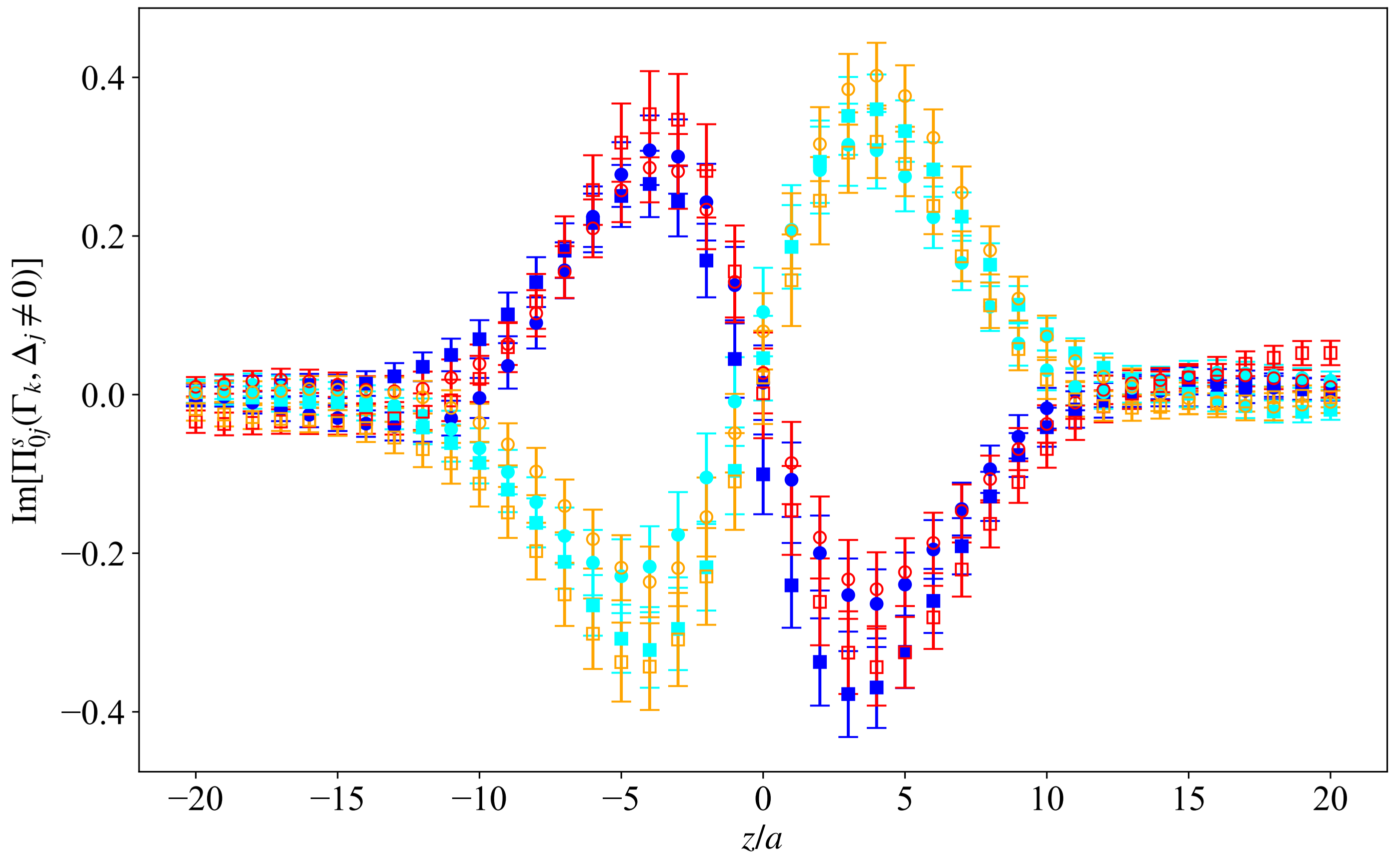
❖ Strategy: decomposition of amplitudes for each kinematic setup $(\pm P_3, \pm \vec{\Delta}, \pm z)$

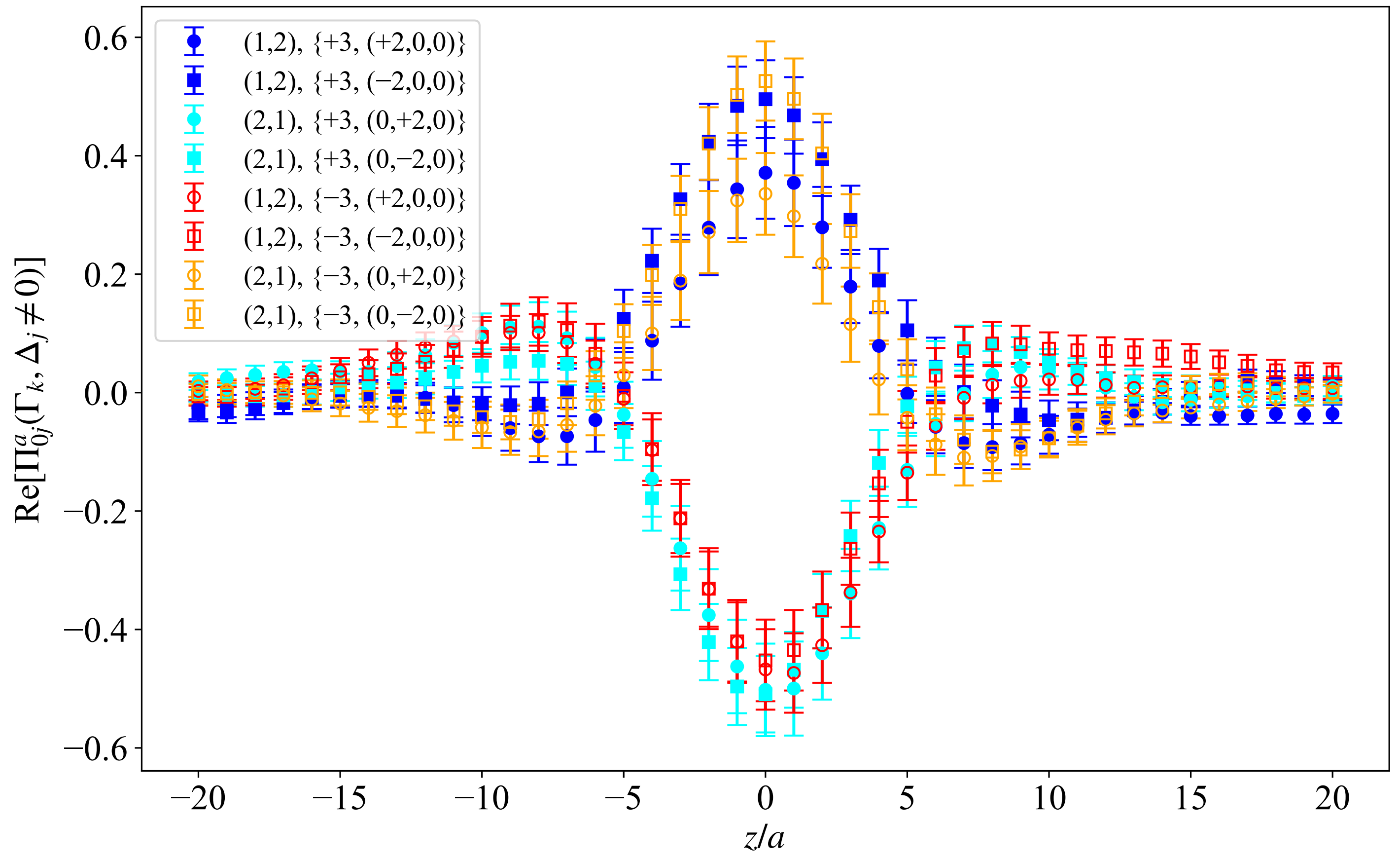
❖ Exploitation of A_{Ti} symmetry properties with respect to $(\pm P_3, \pm \vec{\Delta}, \pm z)$

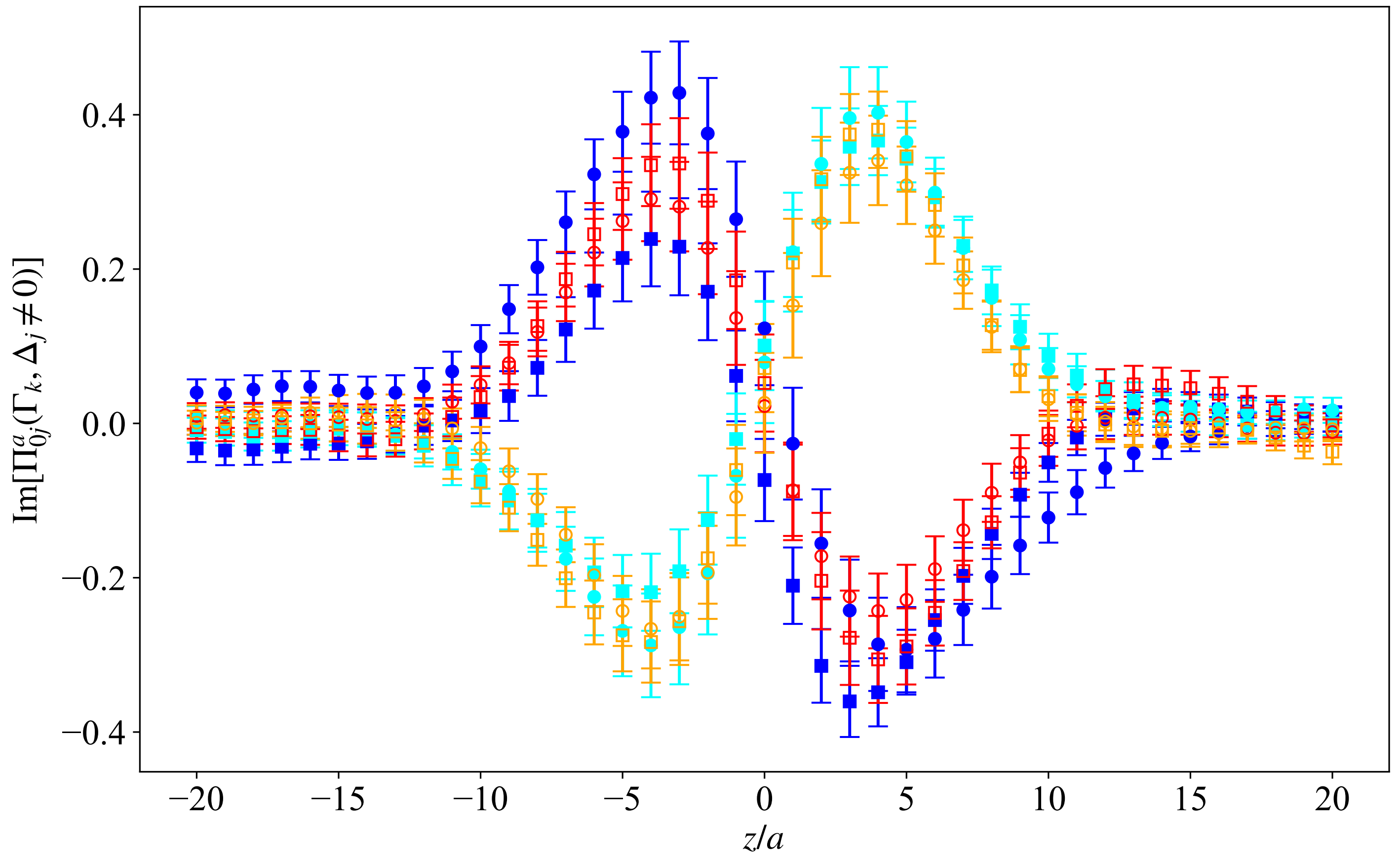
Matrix Elements: $\Pi^{s/a}_{0j}(\Gamma_k,\ \Delta_j \neq 0)$

$|P_3| = 1.25\ \text{GeV} \qquad -t = 0.69\ \text{GeV}$









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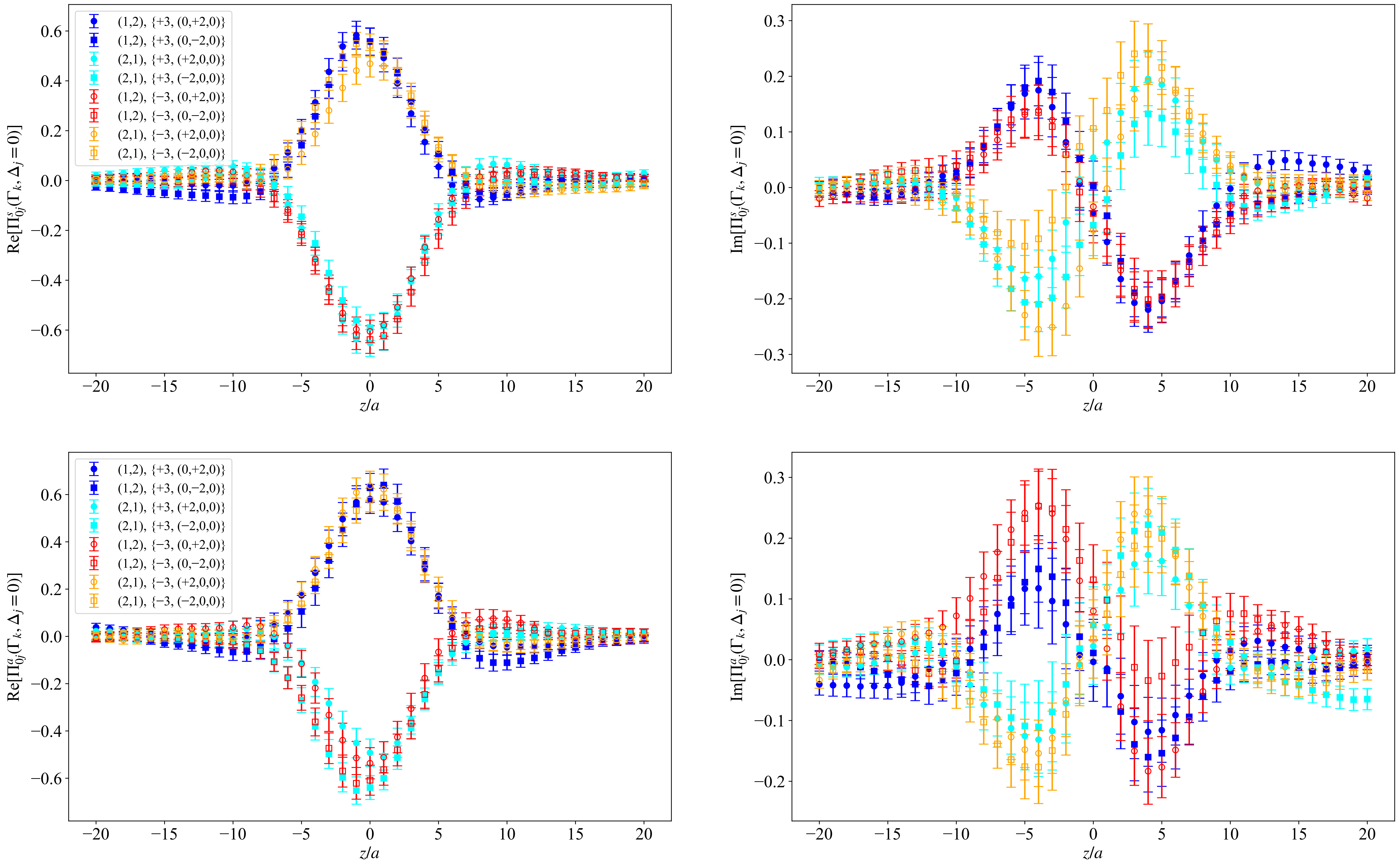

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We reduce the data to 12 independent Matrix Elements to disentangle the A_i

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- Disentangling amplitudes gets more difficult in (any) non-symmetric frame compared to the symmetric one
- Amplitudes are frame independent by construction
- Coefficients are frame dependent

Transversity Amplitudes

Ratio

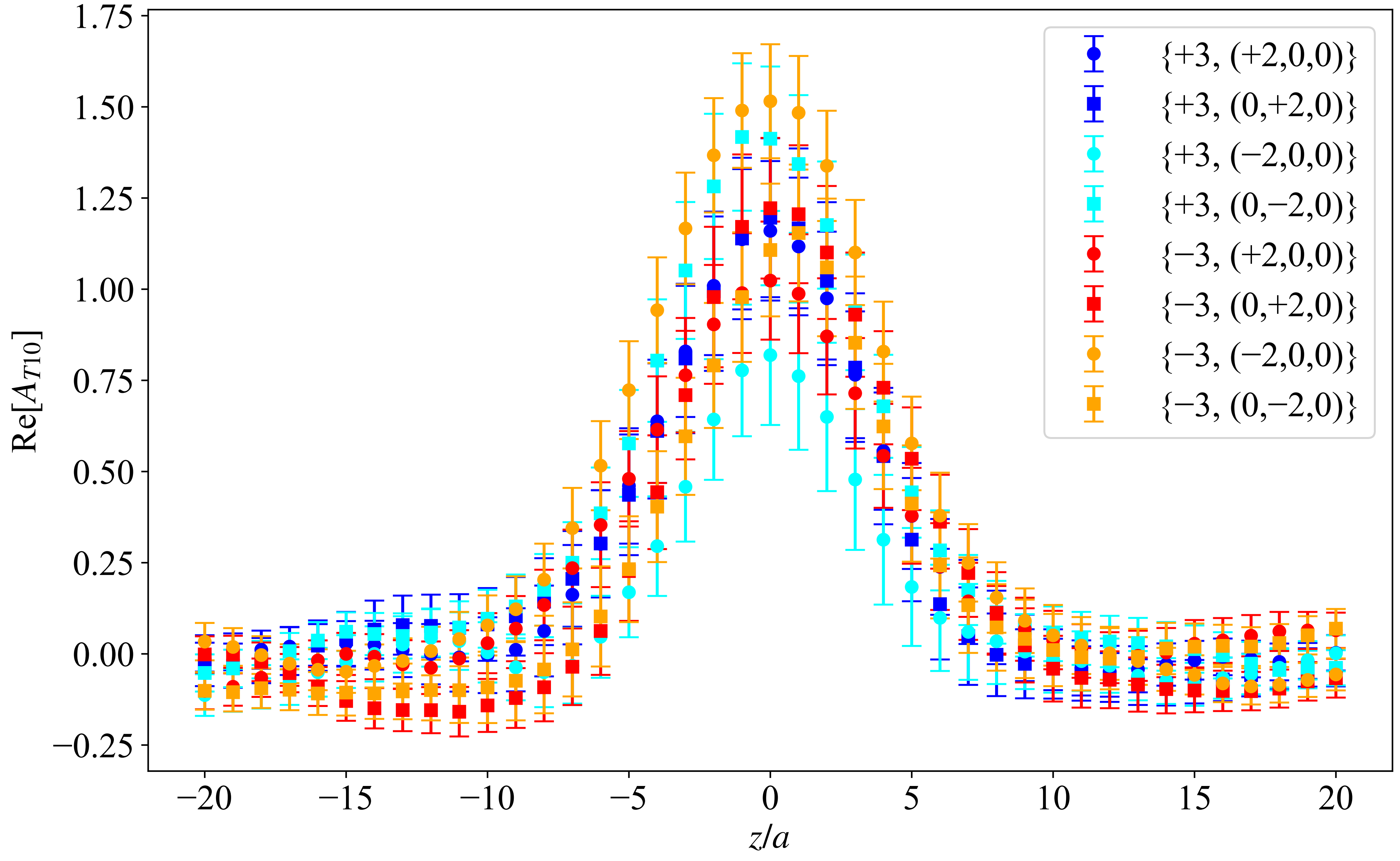
Transversity Amplitudes

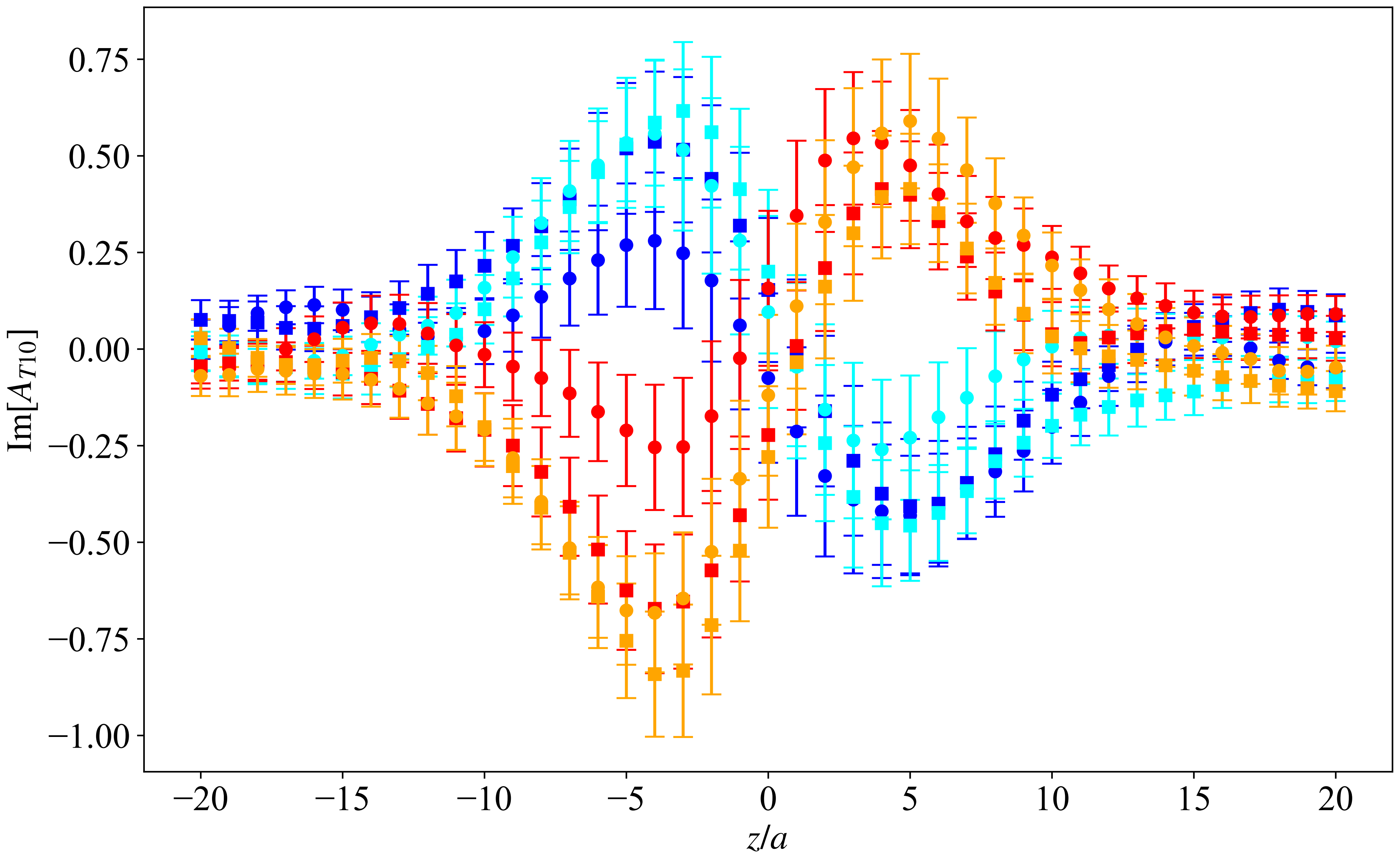
Ratio ➡ *Plateau*

Transversity Amplitudes

Ratio → *Plateau* → *Amplitudes*

Example: A_{T10}





Raw data shows a clear signal

Transversity Amplitudes

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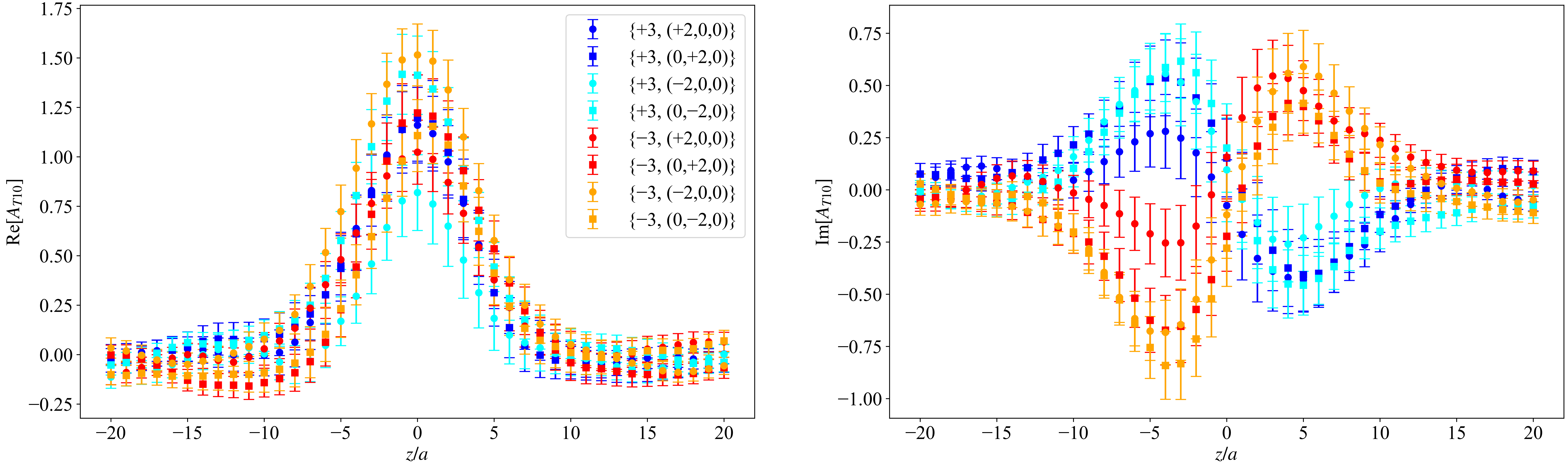

Raw data shows a clear signal

→ *Average*

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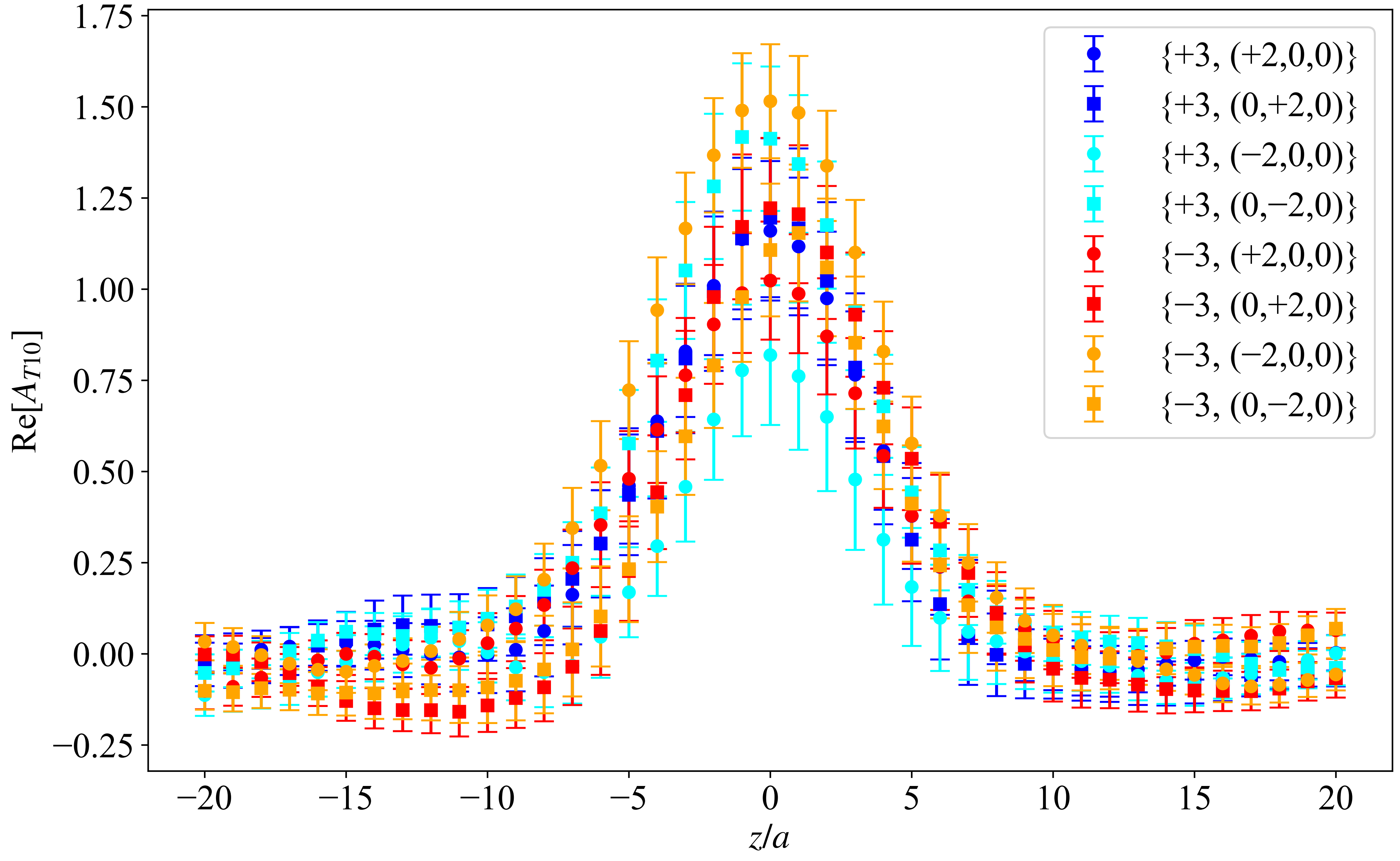

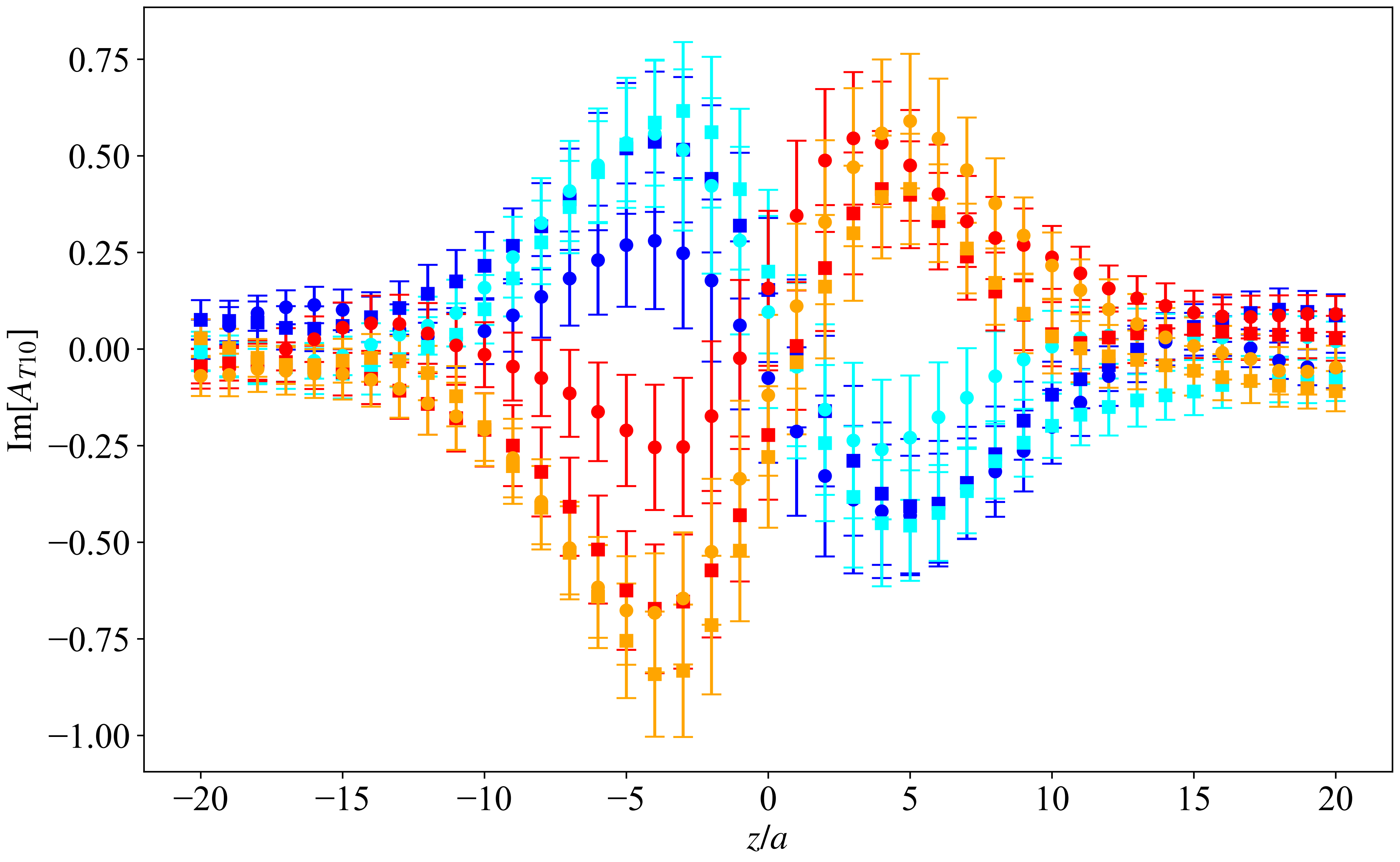

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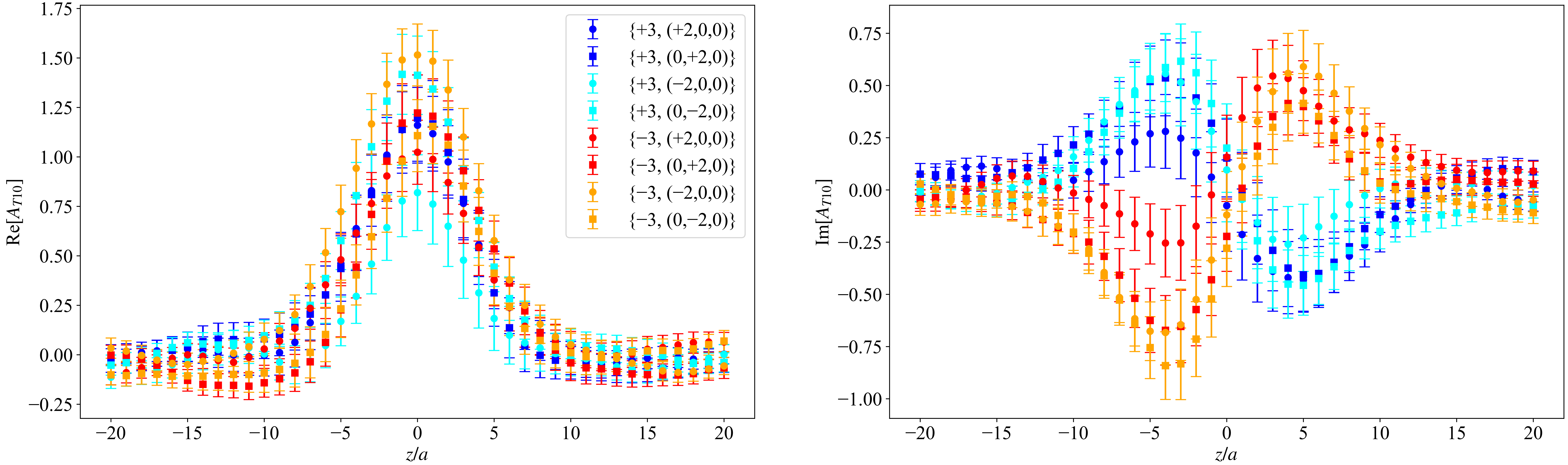

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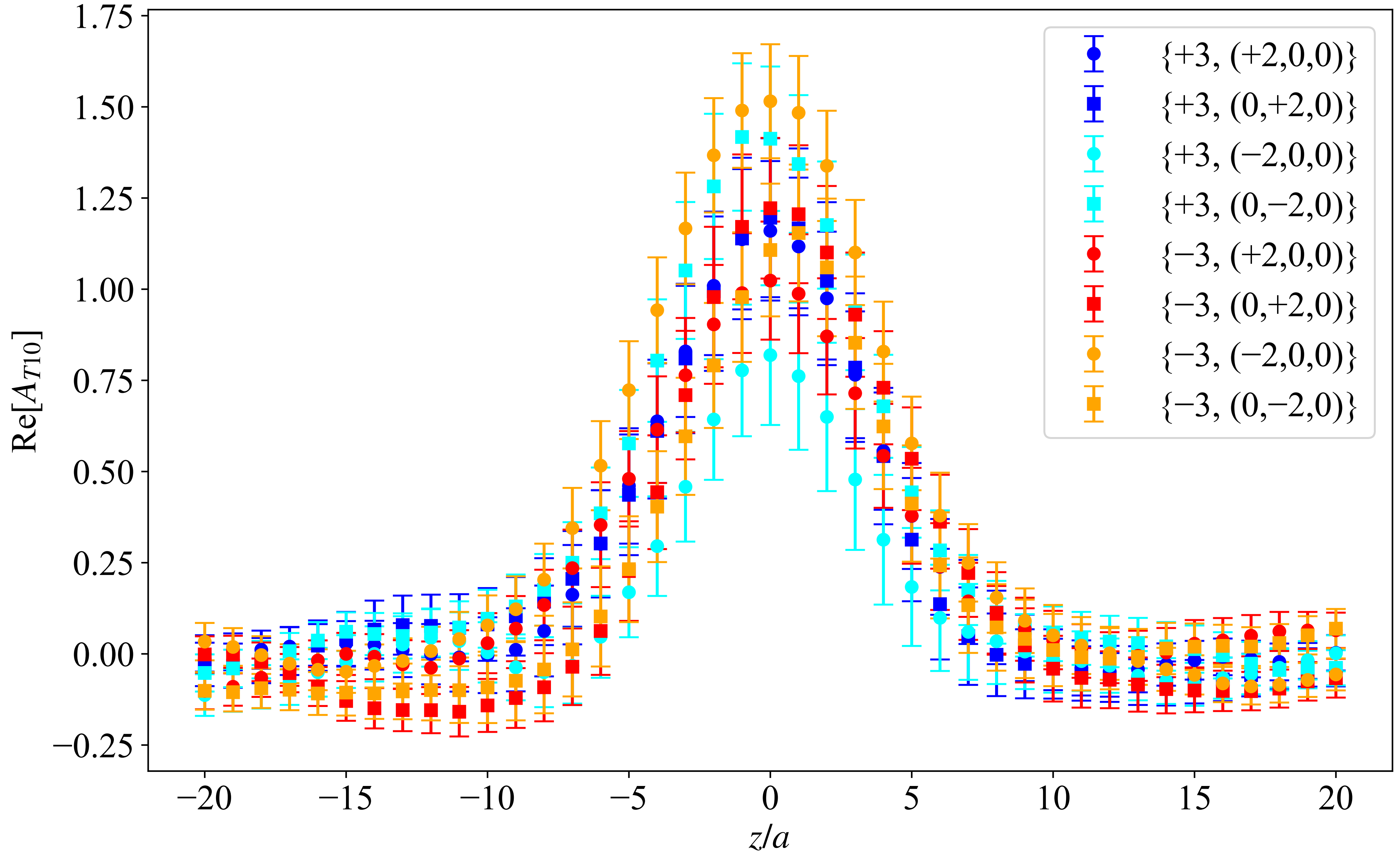

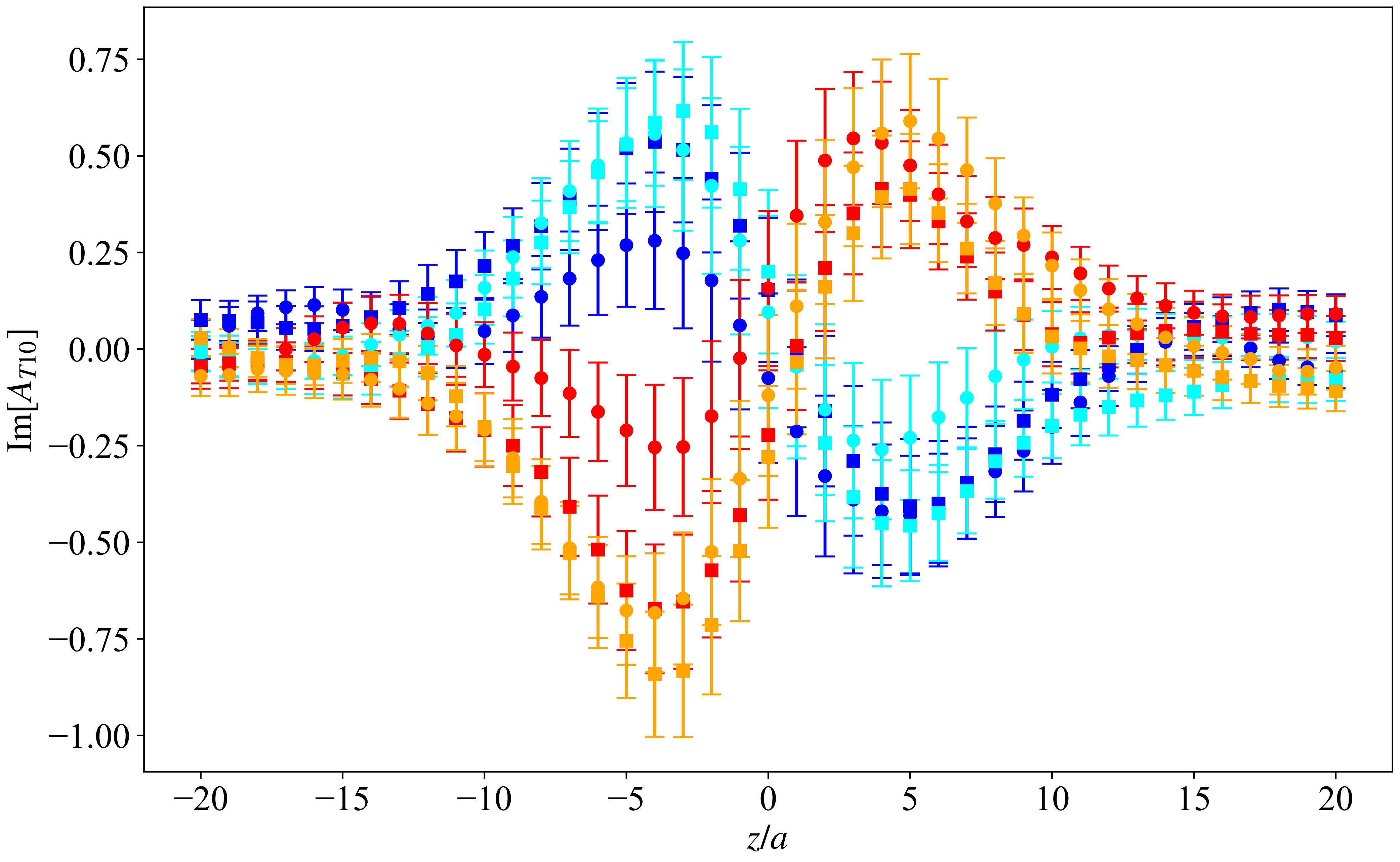

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→ *Average* → *Quasi-GPDs* → *Backus-Gilbert* → *Match*

Amplitude Symmetry

❖ Symmetry properties of amplitudes

$$A^*_{Ti}(-z \cdot P, z \cdot \Delta, \Delta^2, z^2) = A_{Ti}(z \cdot P, z \cdot \Delta, \Delta^2, z^2) \qquad i = 1,\ 2,\ 4,\ 7,\ 8,\ 10,\ 11$$

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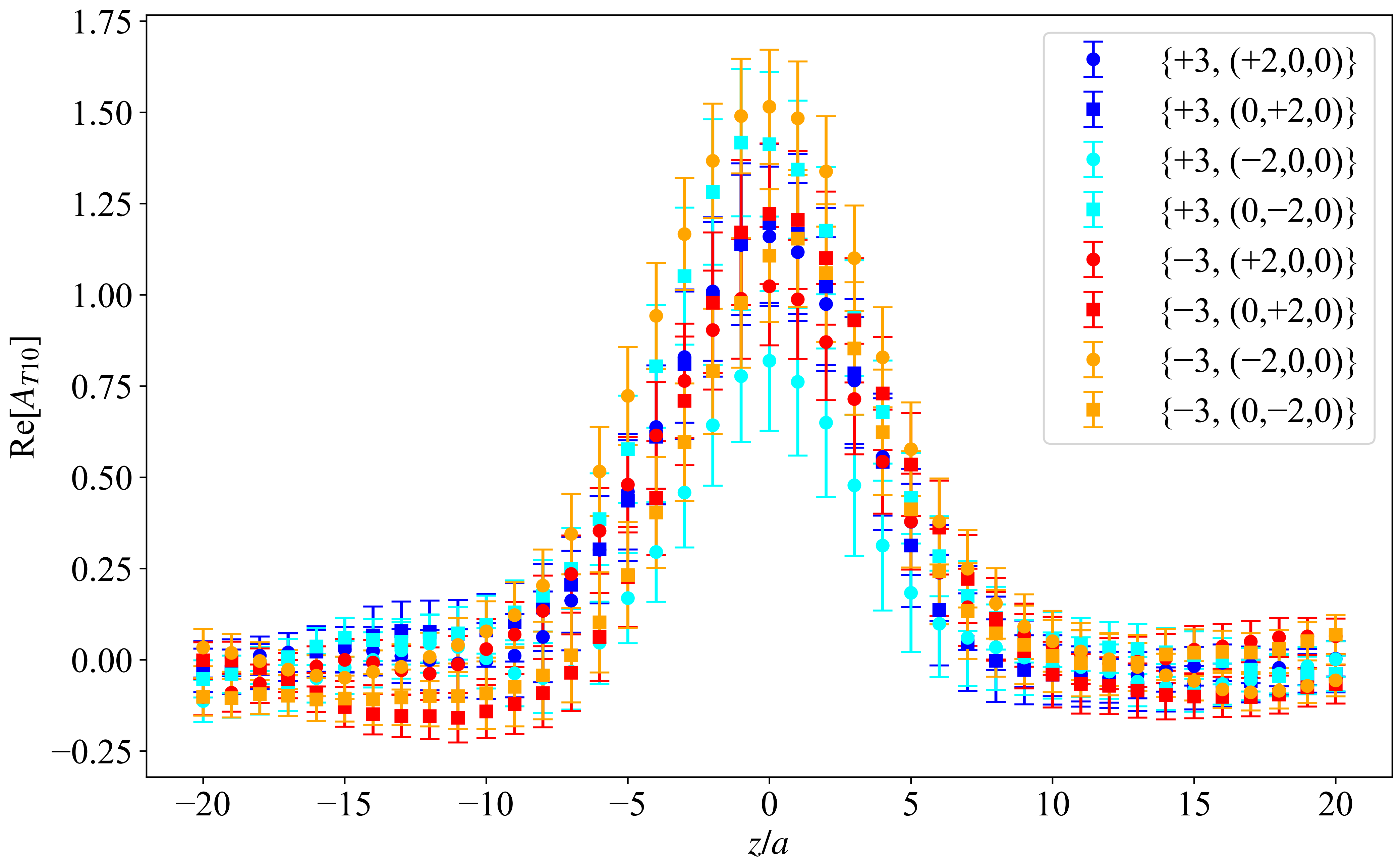

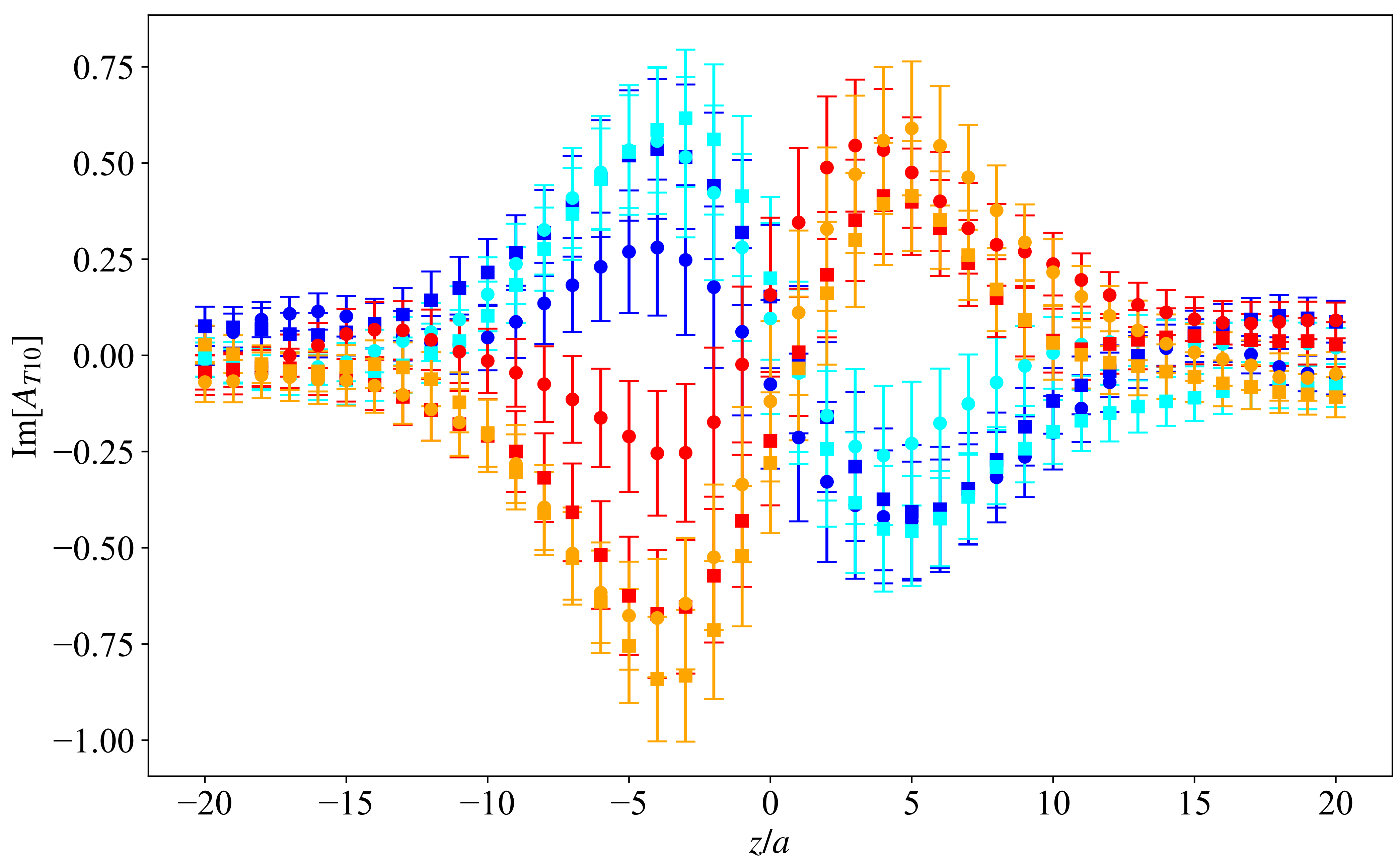

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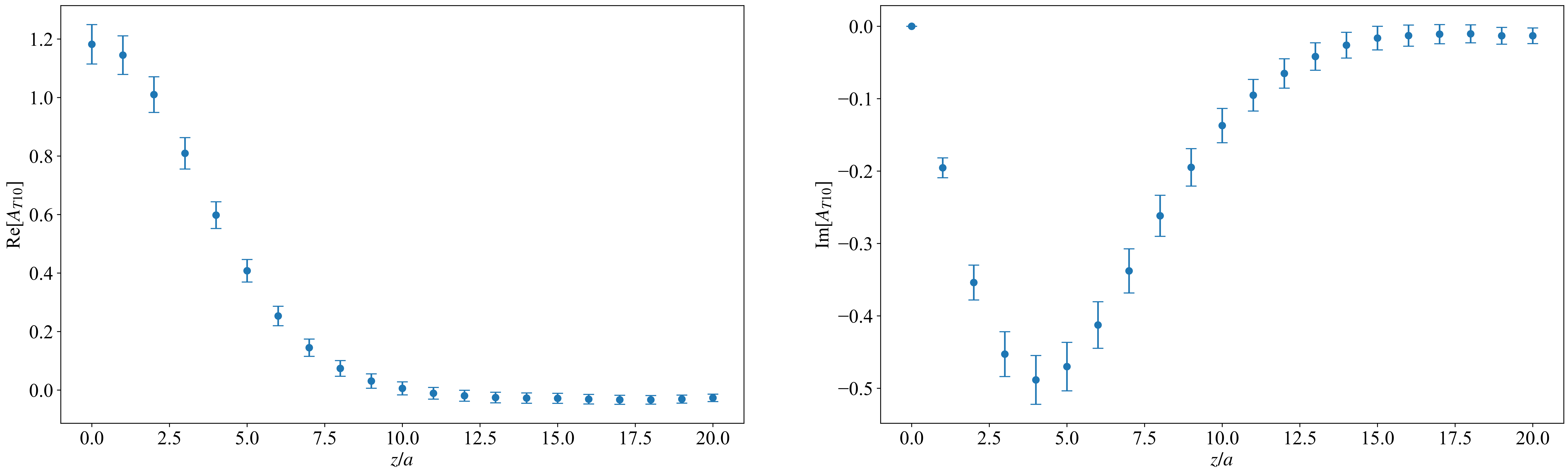
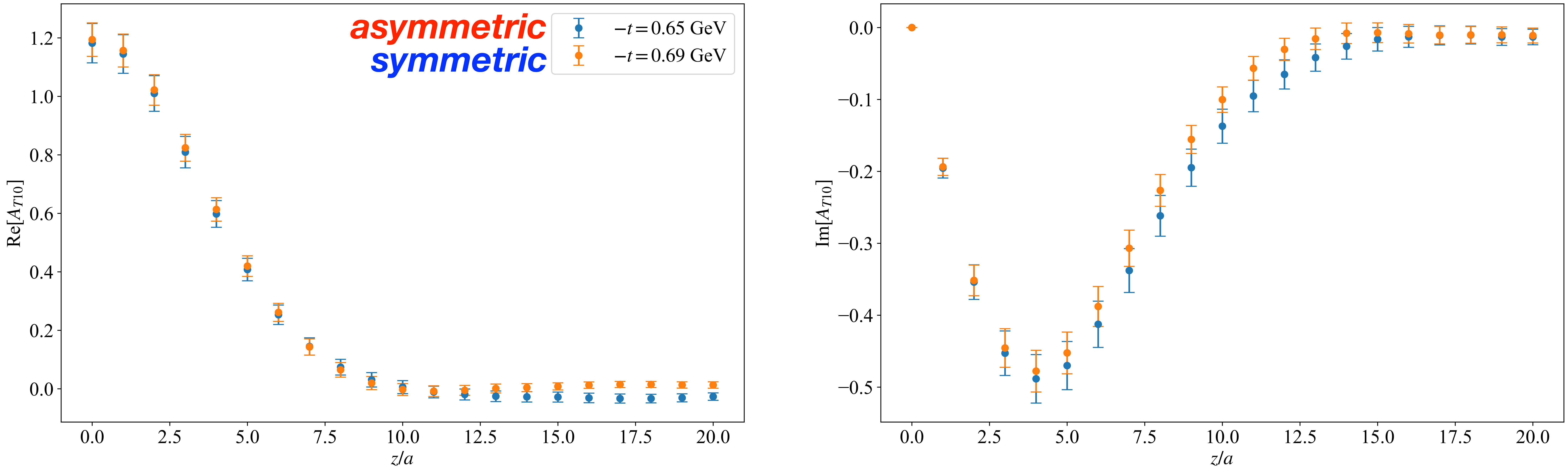


Averaging across 8 kinematic cases reduces the error by $1/\sqrt{8}$

Amplitude Comparison for different frames



- $-t = 0.65\ \text{GeV}$ corresponds to $|P_3| = 3$ and $\vec{\Delta} = (2{,}0{,}0)$+permutations in ***asymmetric*** frame
- $-t = 0.69\ \text{GeV}$ corresponds to $|P_3| = 3$ and $\vec{\Delta} = (2{,}0{,}0)$+permutations in ***symmetric*** frame
- Negligible difference between frames despite the 5% difference between $-t^s$ and $-t^a$.
  (Only $z > 15a$ for real part)

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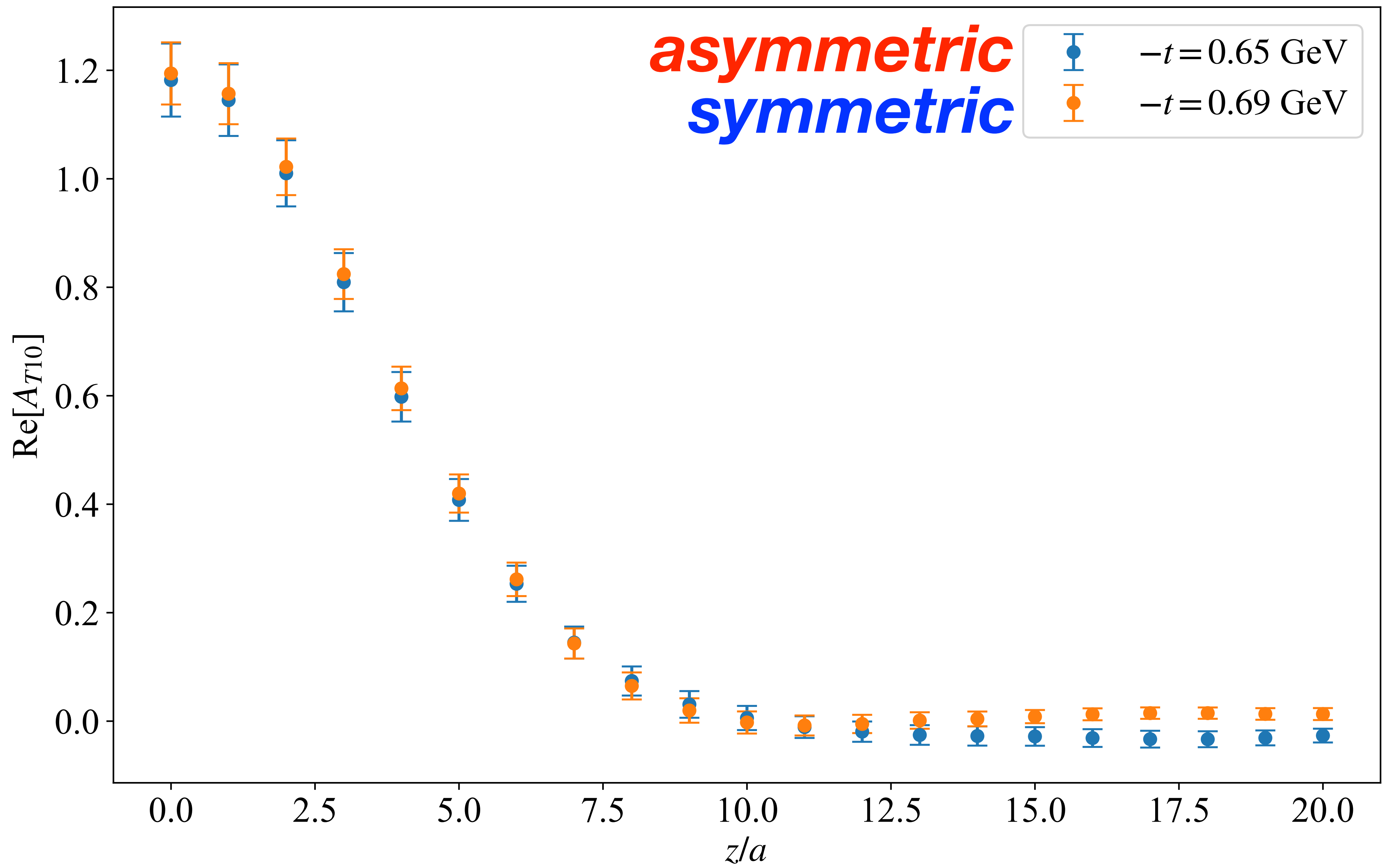

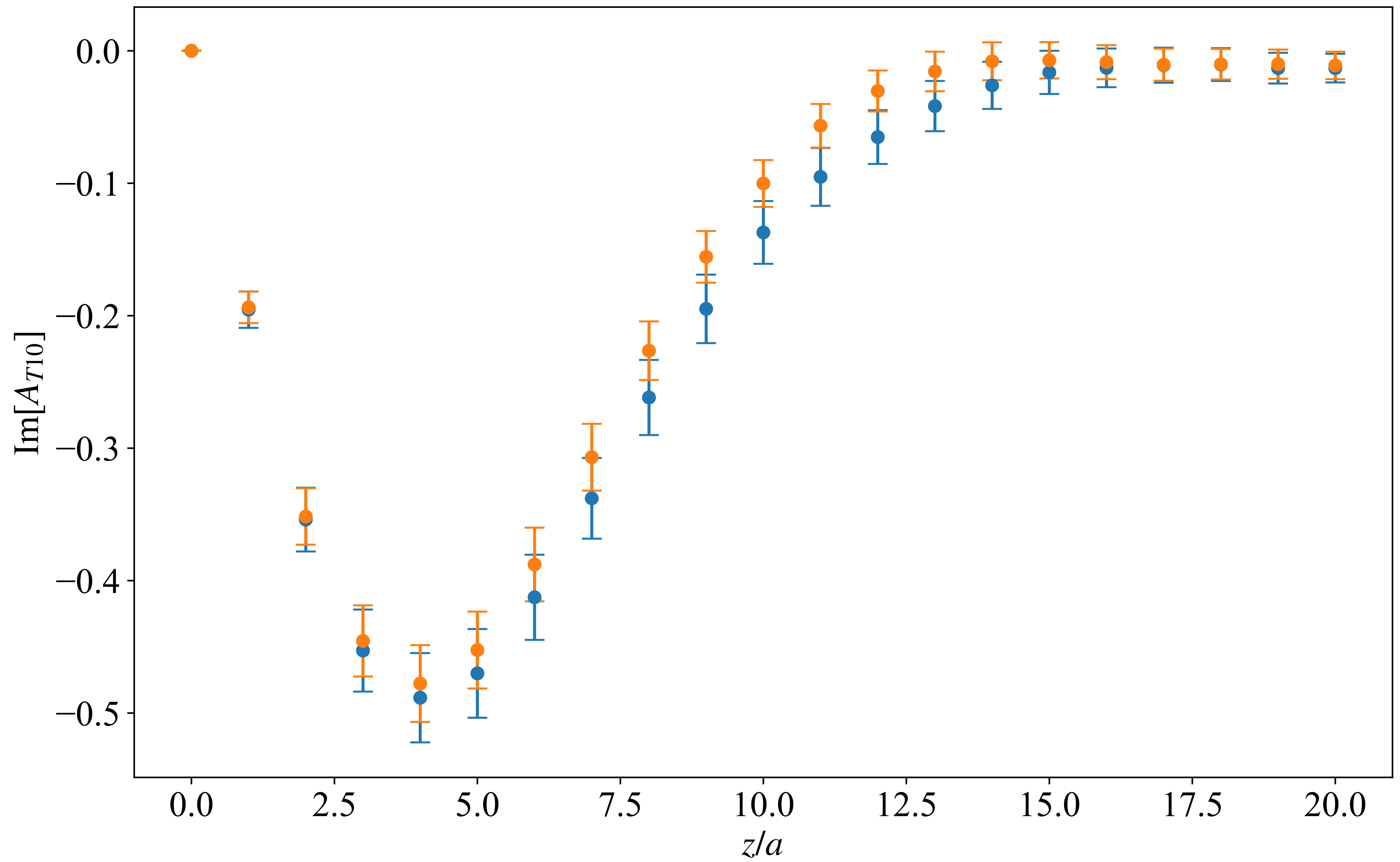

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  (Only $z > 15a$ for real part)

Amplitudes are frame independent!

Quasi-GPDs

$$F_{\lambda,\lambda'}^{[i\sigma^{j+}\gamma_5]}(z,\Delta,P) = -i\epsilon^{-+ij}\bar{u}(p',\lambda')\left[i\sigma^{+i}H_T + \frac{\gamma^+\Delta_\perp^i - \Delta^+\gamma_\perp^i}{2M}E_T + \frac{P^+\Delta_\perp^i - P_\perp^i\Delta^+}{M^2}\widetilde{H}_T + \frac{\gamma^+P_\perp^i - P^+\gamma_\perp^i}{M}\widetilde{E}_T\right]u(p,\lambda)$$

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↕

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$$\mathscr{H}^{s/a}_T = -2A_{T2}\left(1 - \frac{\bar{P}^2}{m^2}\right) + A_{T4} - zA_{T8}\left(\frac{E_f^2 - E_i^2}{2P_3}\right) + A_{T10}$$

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Only use $\bar{P}^a$ and Δ^a

Amplitude decomposition matches results from previous results **[ETMC, PRD 105, 034501 (2022)]**

Quasi-GPDs

- Definition comparison

$$\Delta^s = \left(0, \Delta_x\sqrt{\frac{2E_f}{(E_f+E_i)}}, \Delta_y\sqrt{\frac{2E_f}{(E_f+E_i)}}, 0\right)$$

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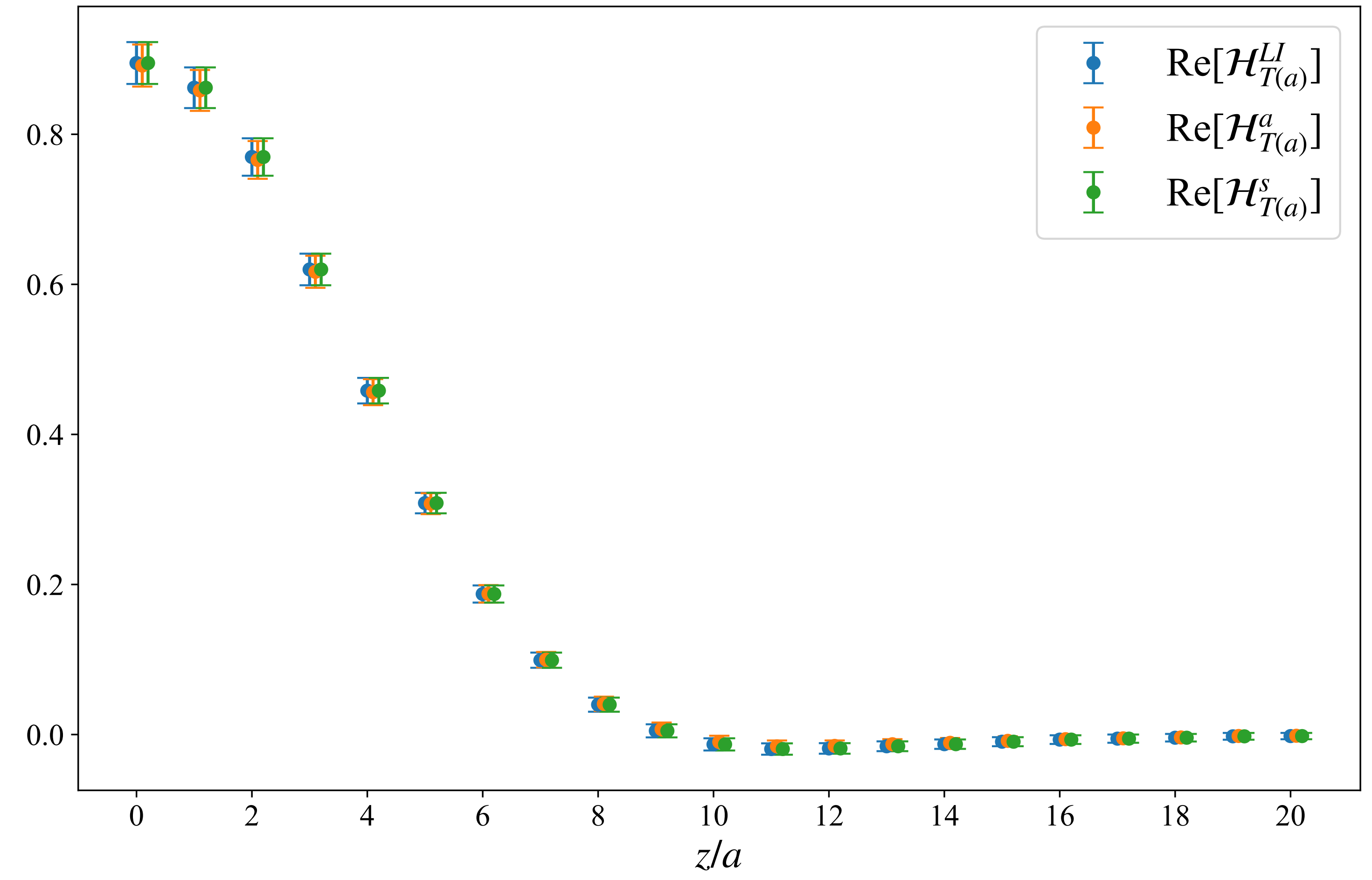

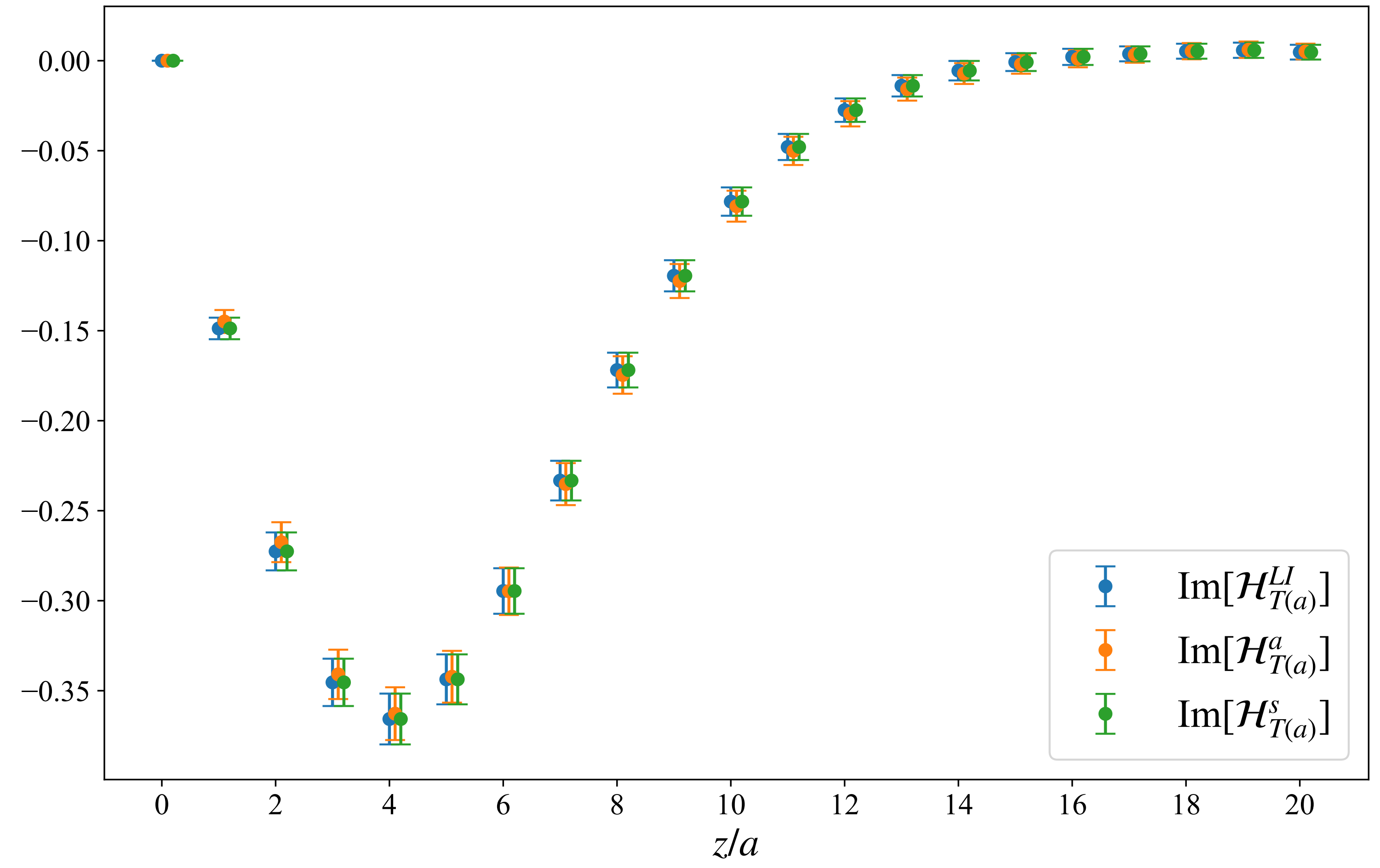

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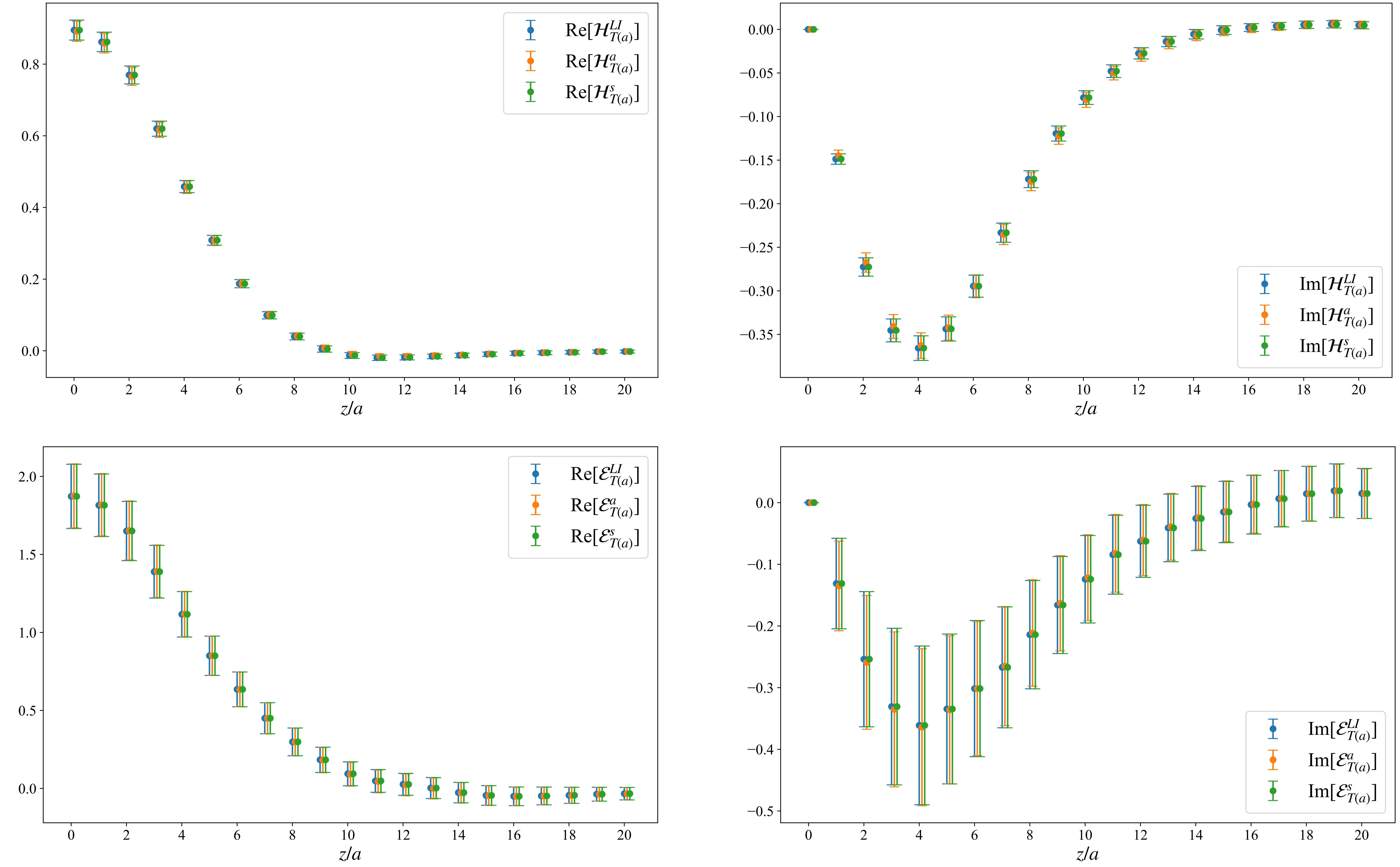

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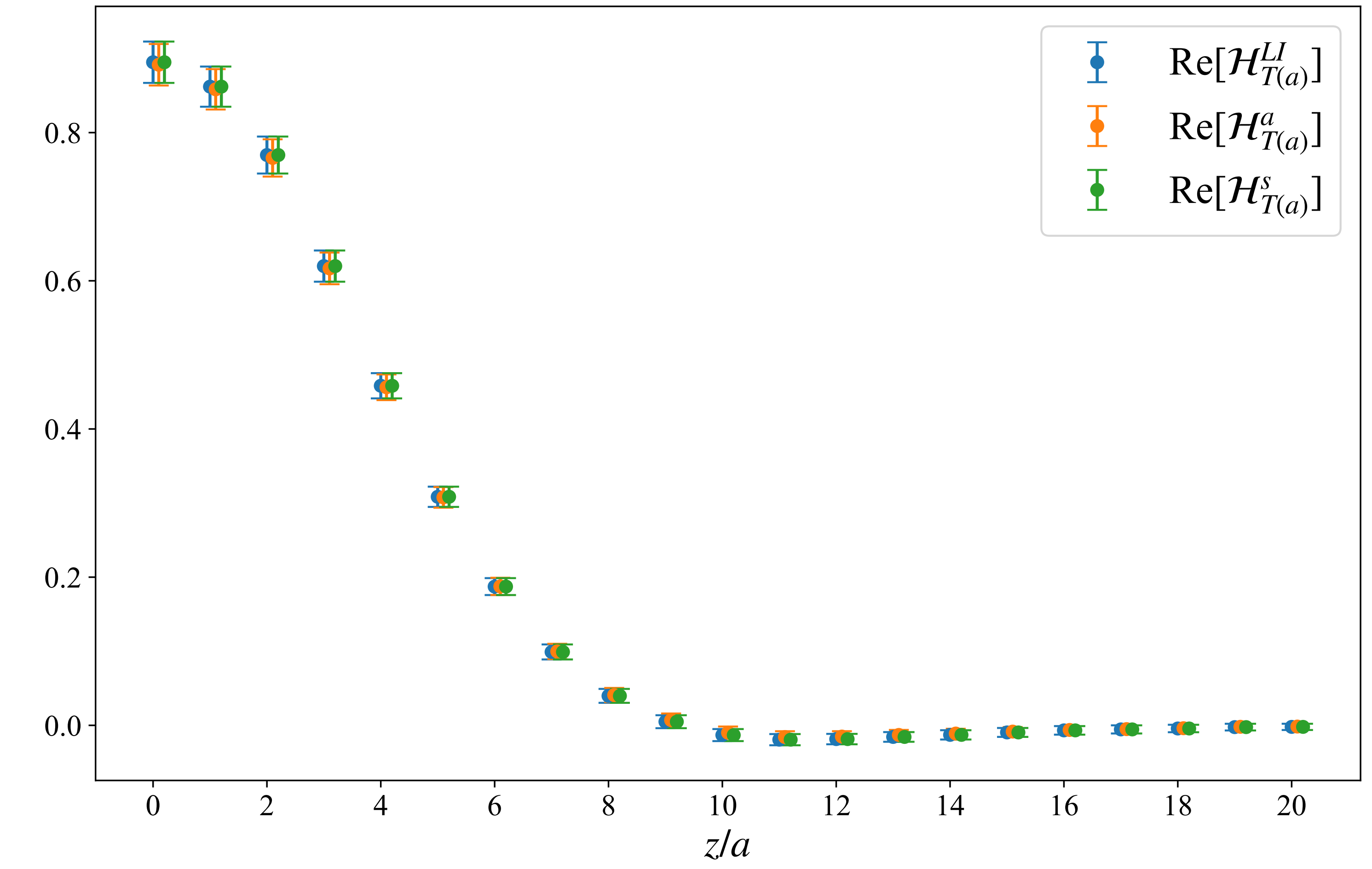

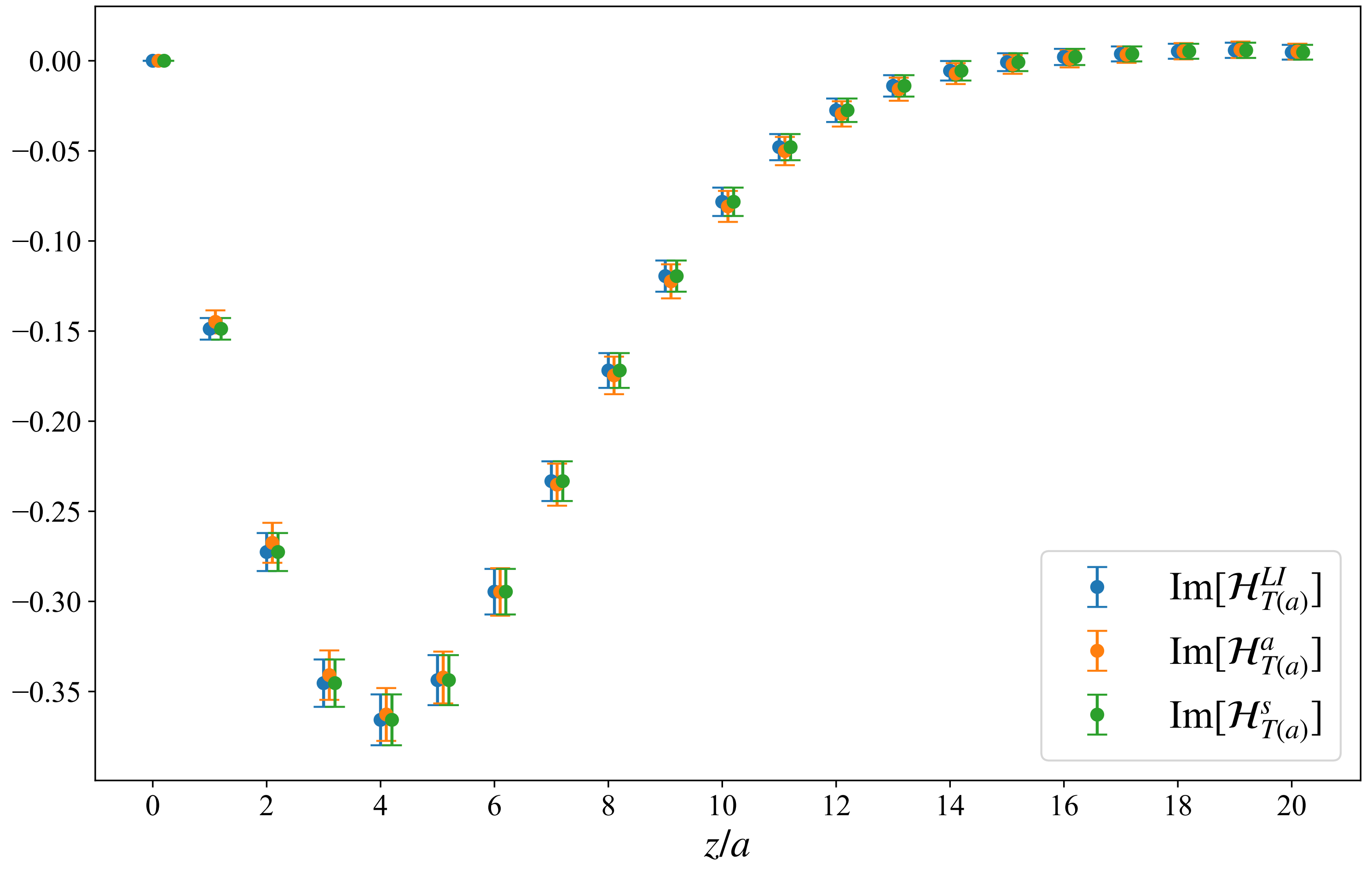

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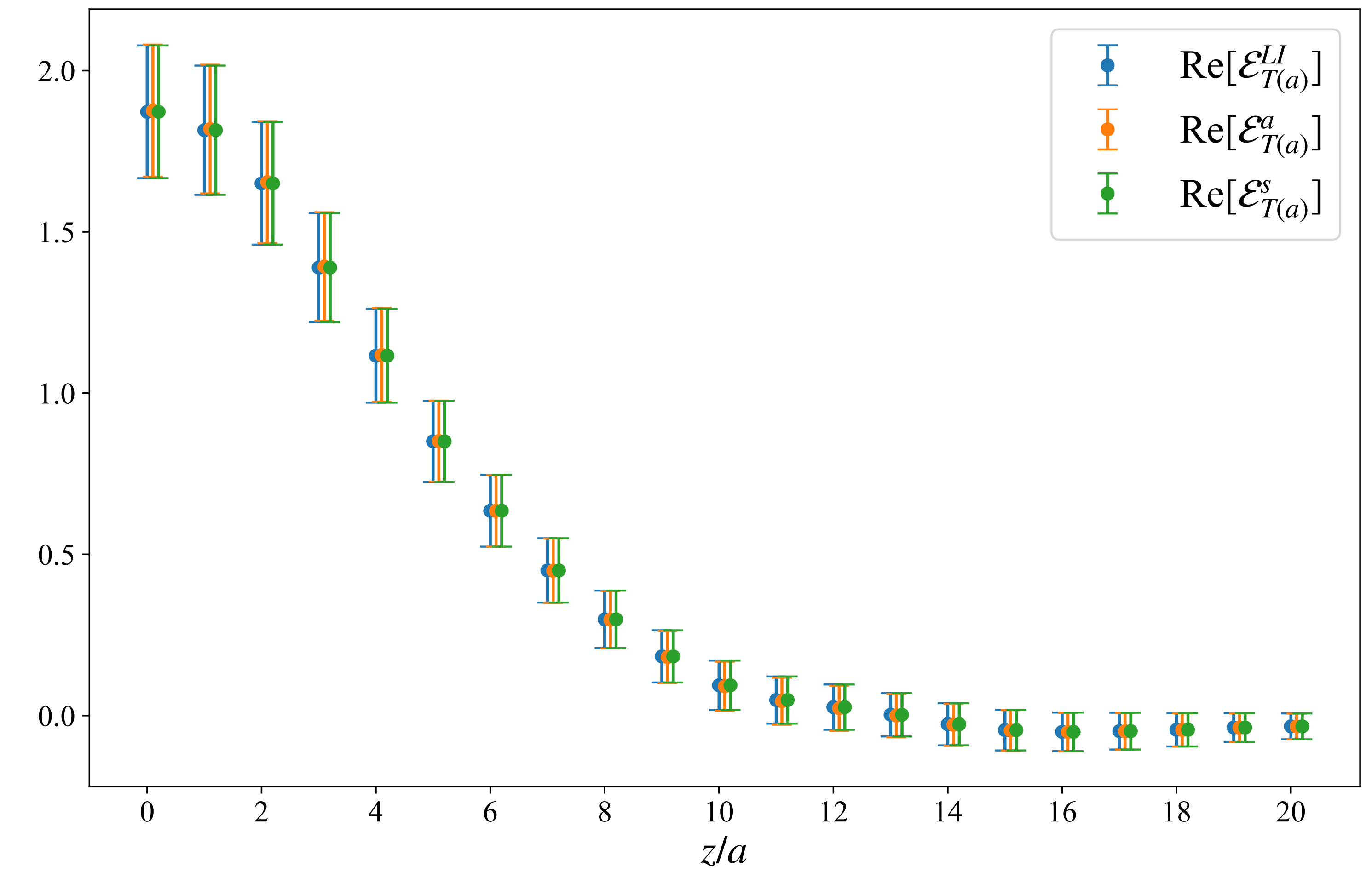
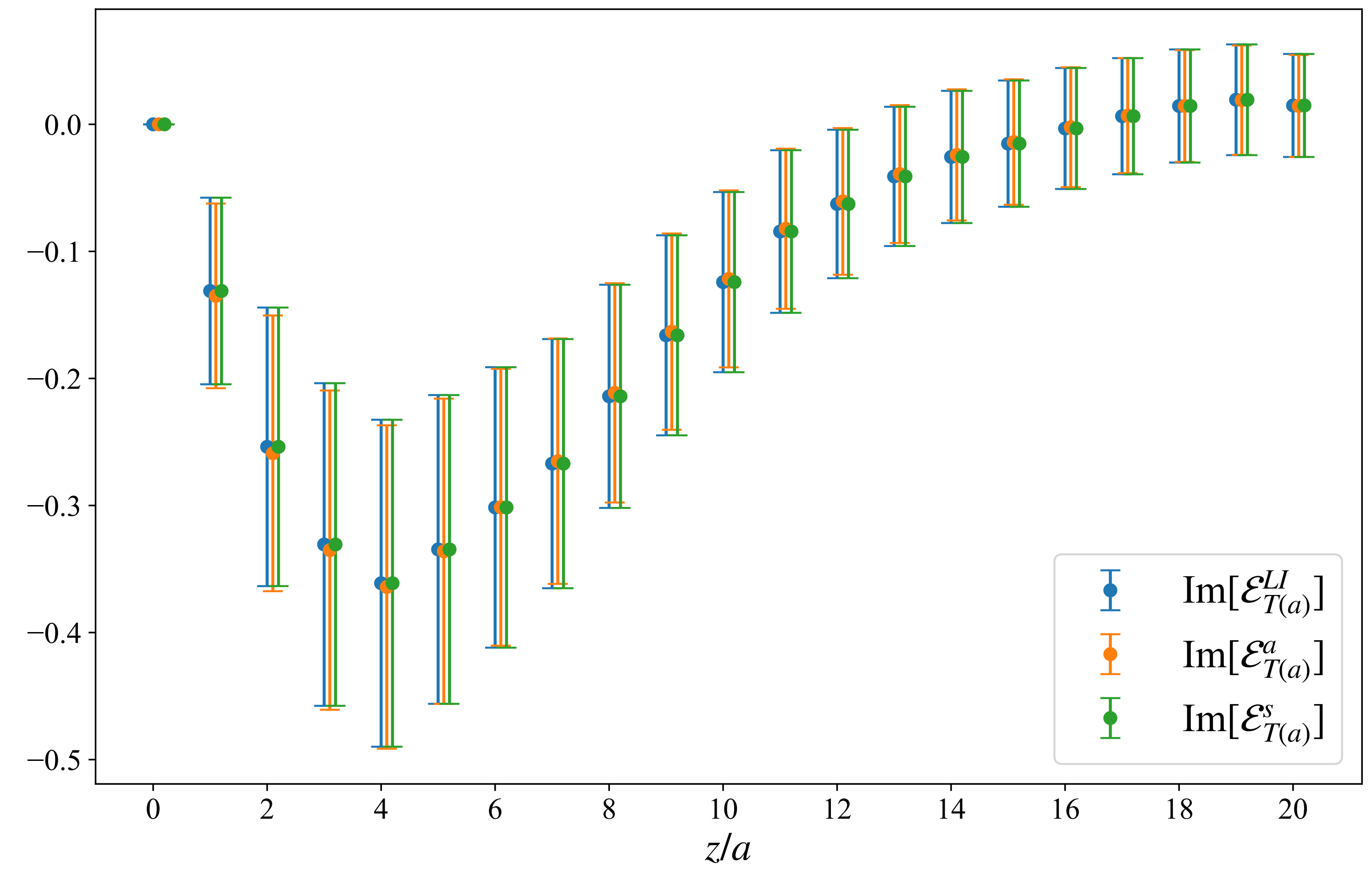
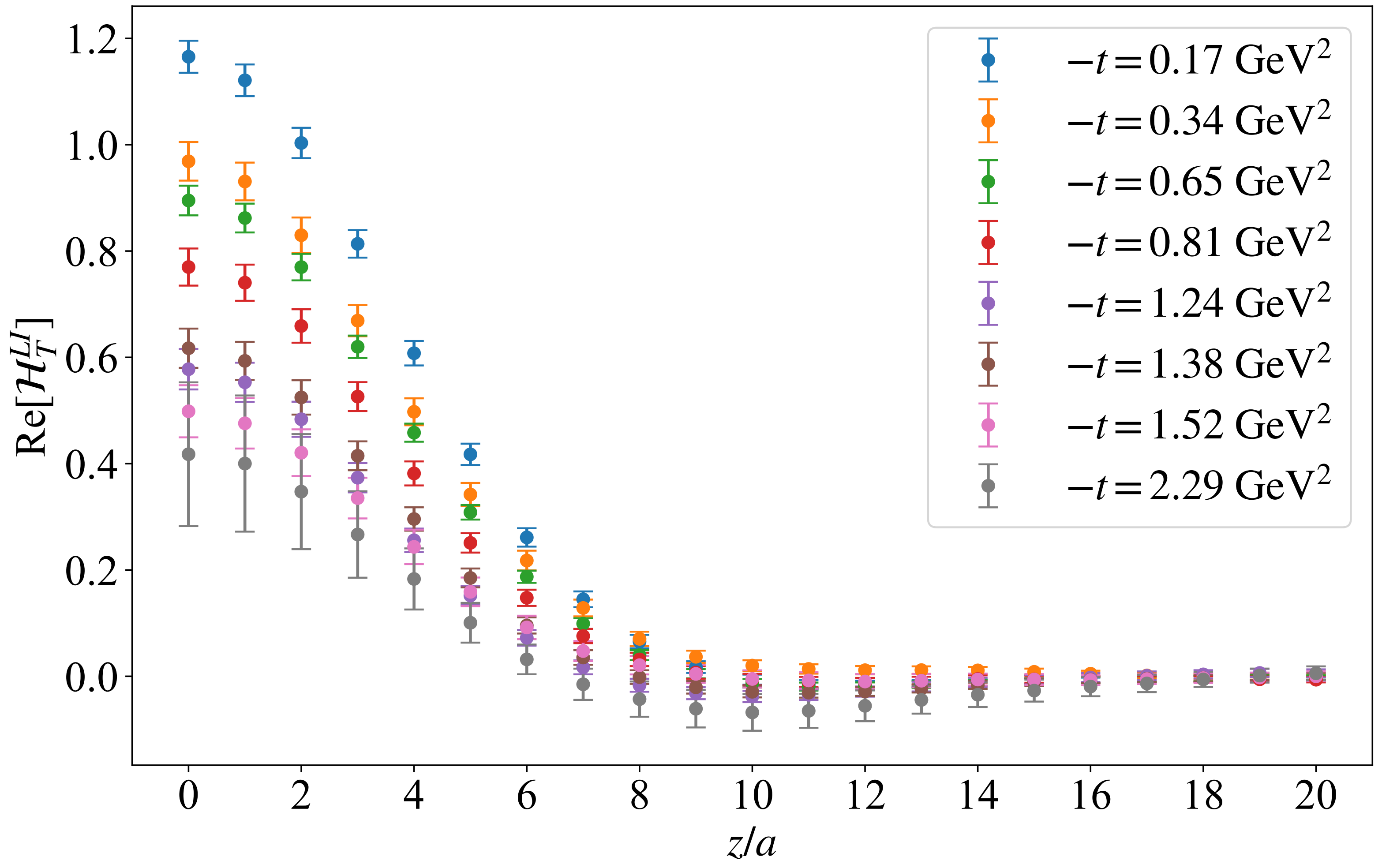
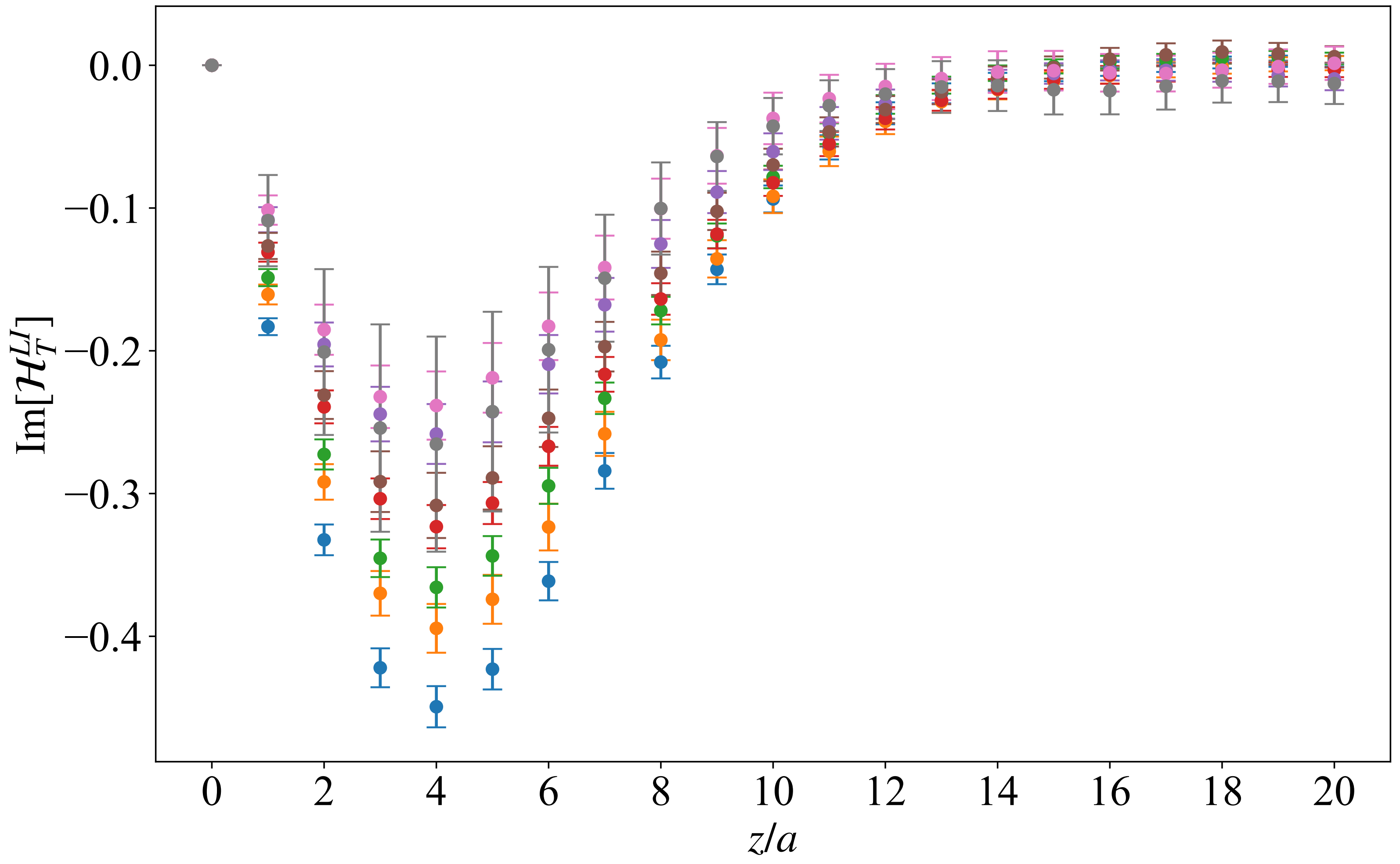




We stick with LI definitions

Quasi-GPDs



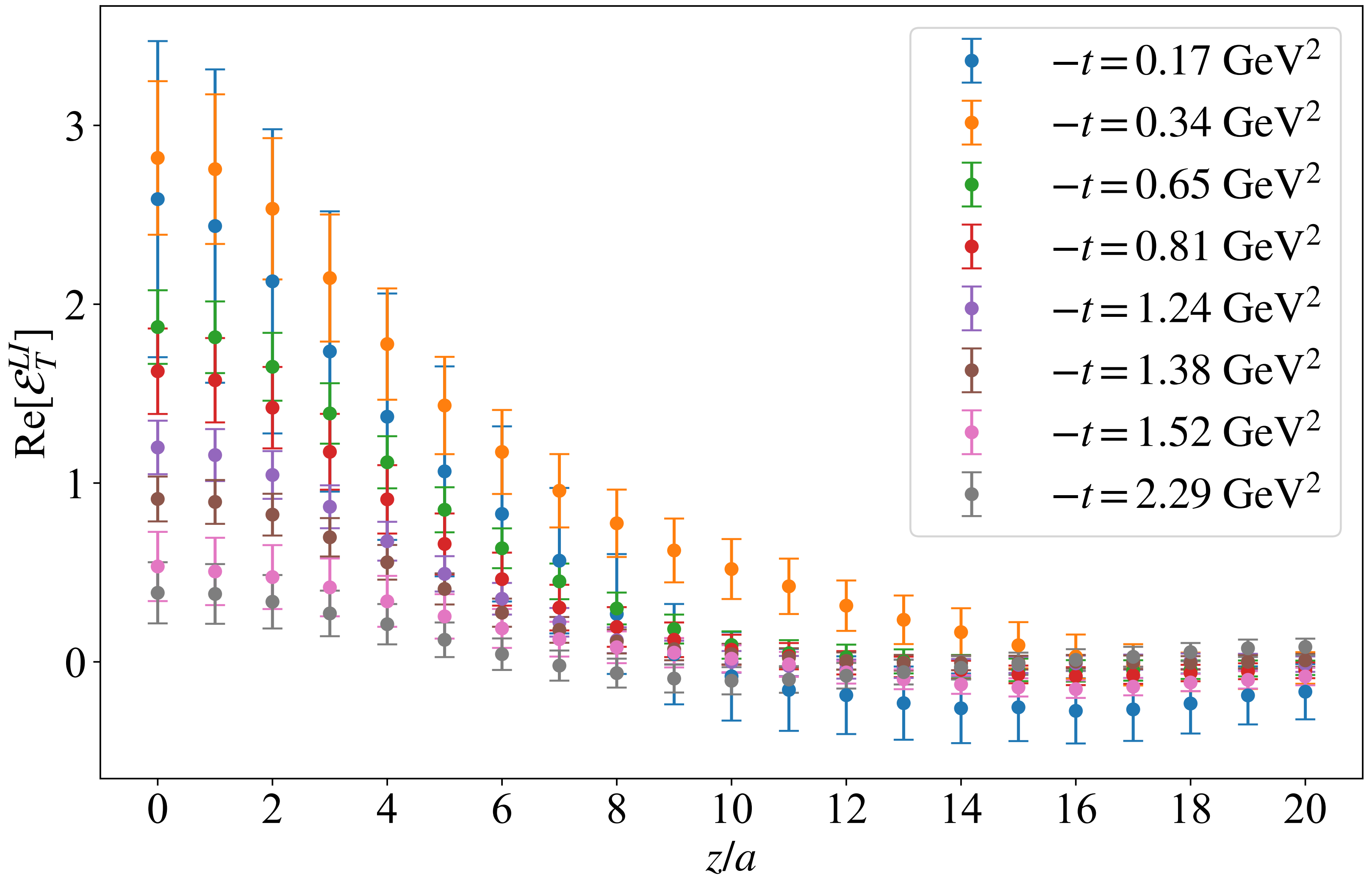


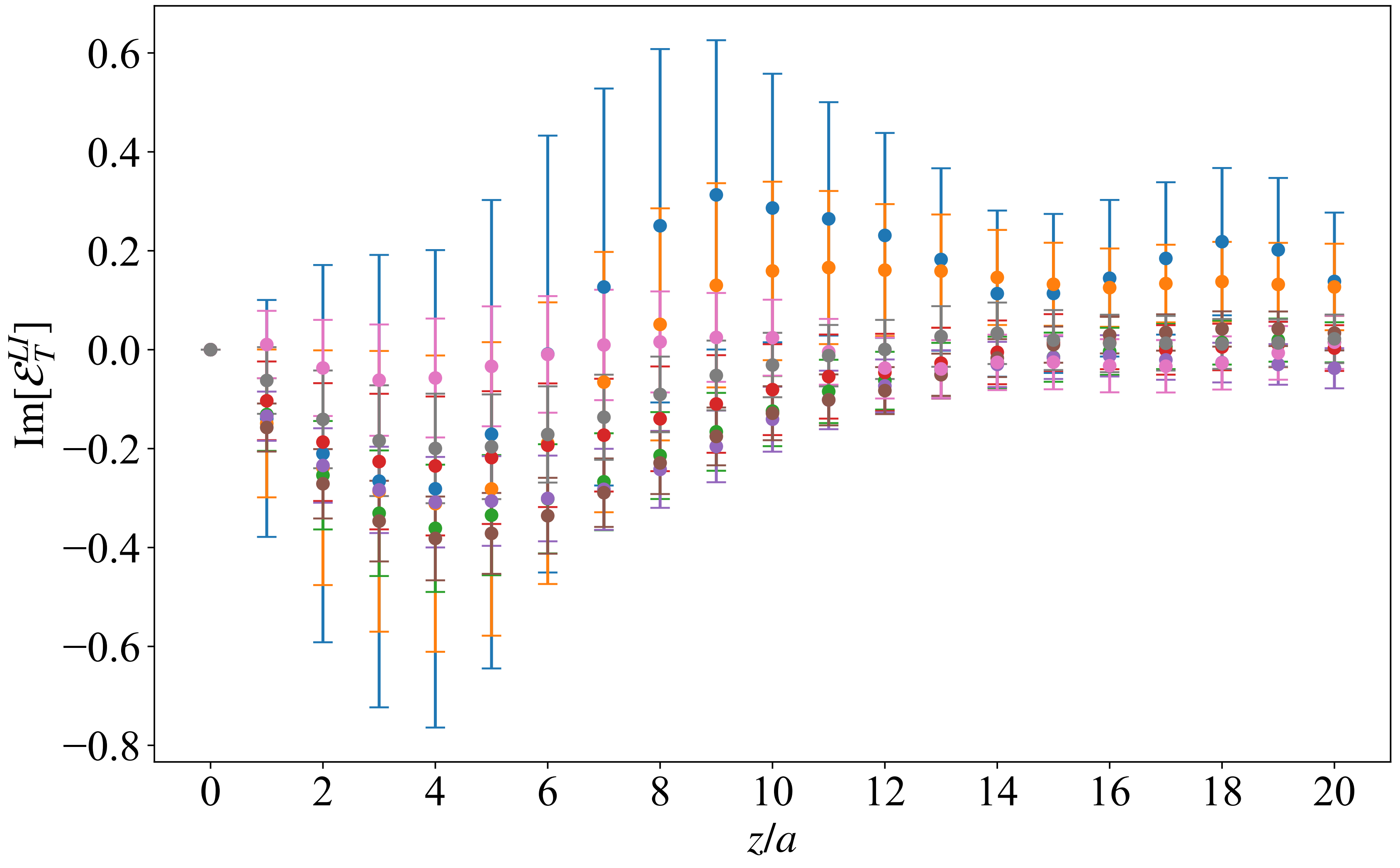
$$\mathscr{H}_T^{LI} = -2A_{T2}\left(1 - \frac{\bar{P}^2}{m^2}\right) + A_{T4} + A_{T10}$$

- ❖ Good signal to noise ratio
- ❖ Magnitude decreases as $-t$ increases

$$\mathscr{E}_T^{LI} = 2A_{T2} - A_{T4}$$

- ❖ More noise, but still a good signal





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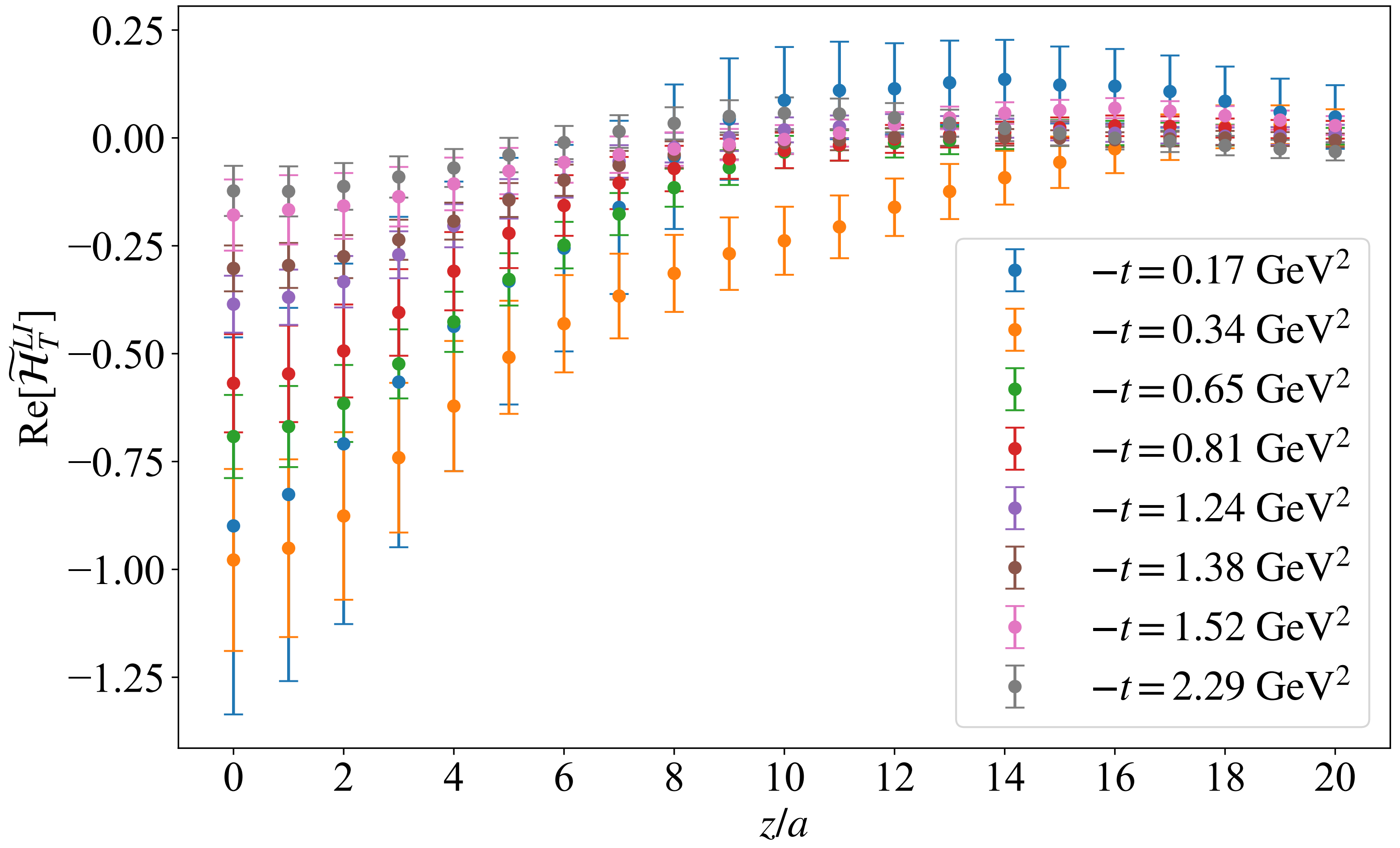

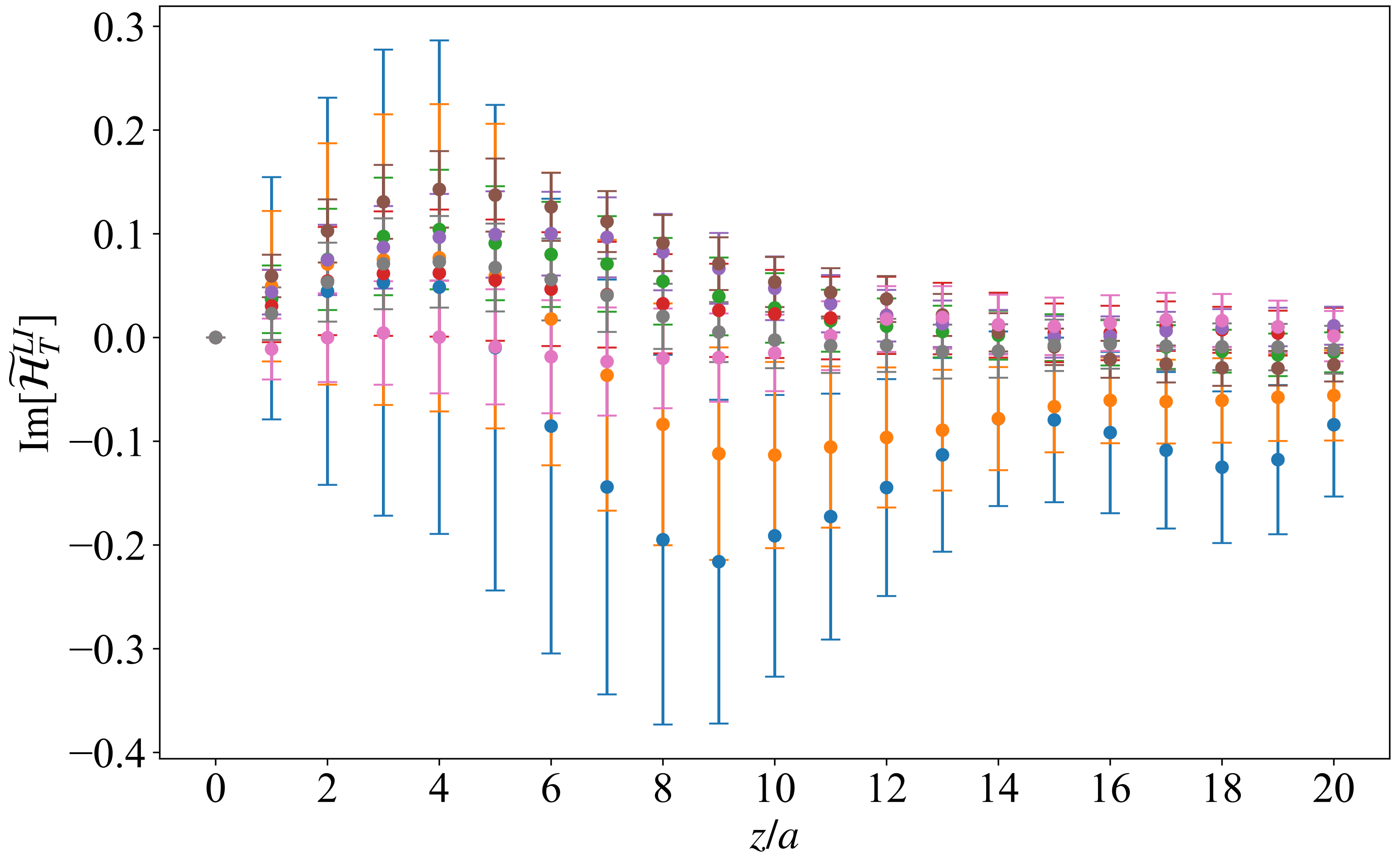

$$\tilde{\mathcal{H}}_T^{LI} = -A_{T2}$$

❖ Similar to $\mathcal{E}_T^{LI}$

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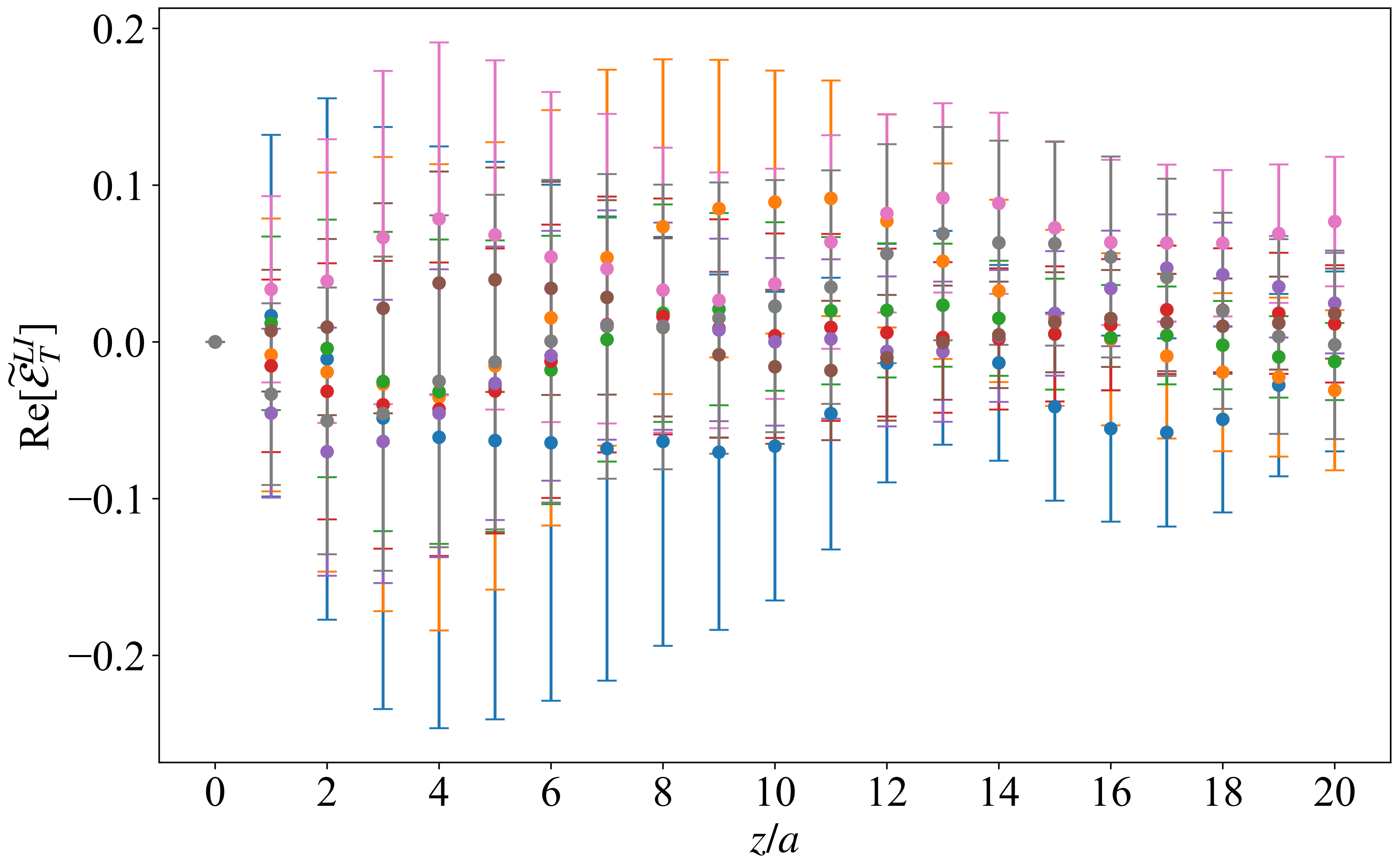
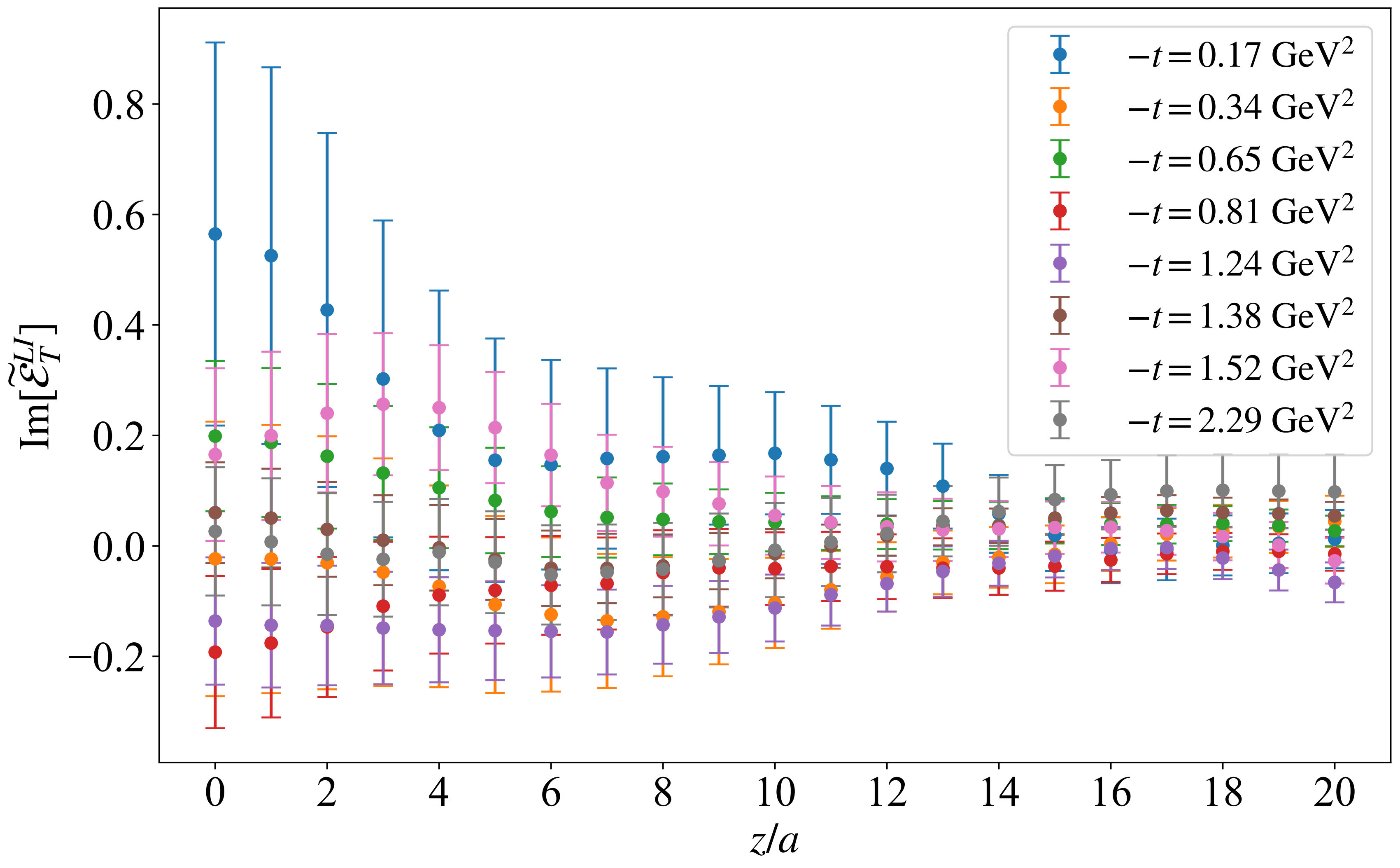
❖ Expected to be zero when $\xi = 0$





Quasi-GPDs: Momentum Space

- Multiple methods to consider
    - Standard Fourier Transform

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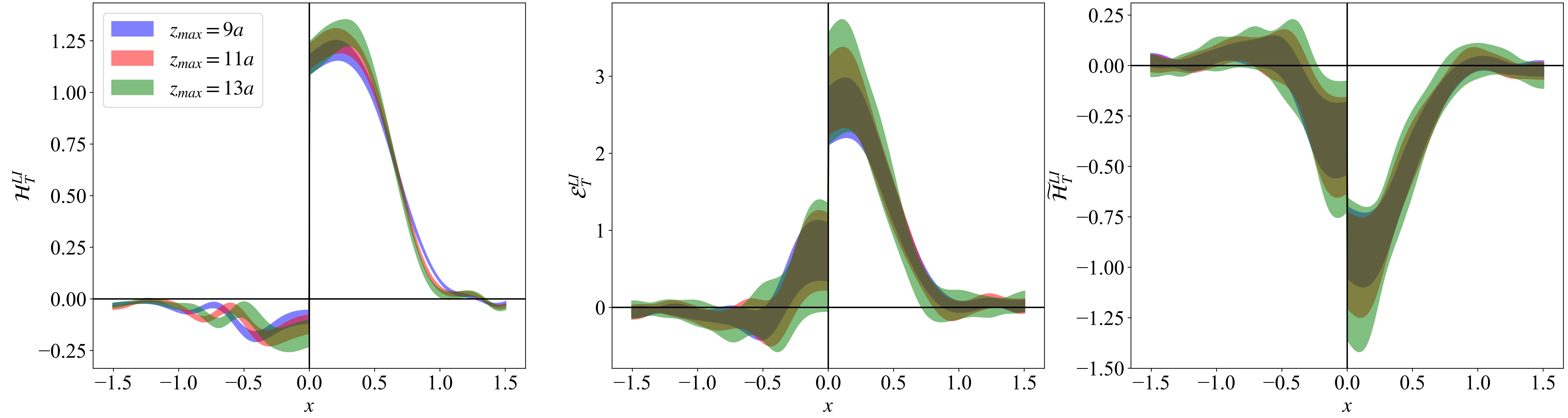
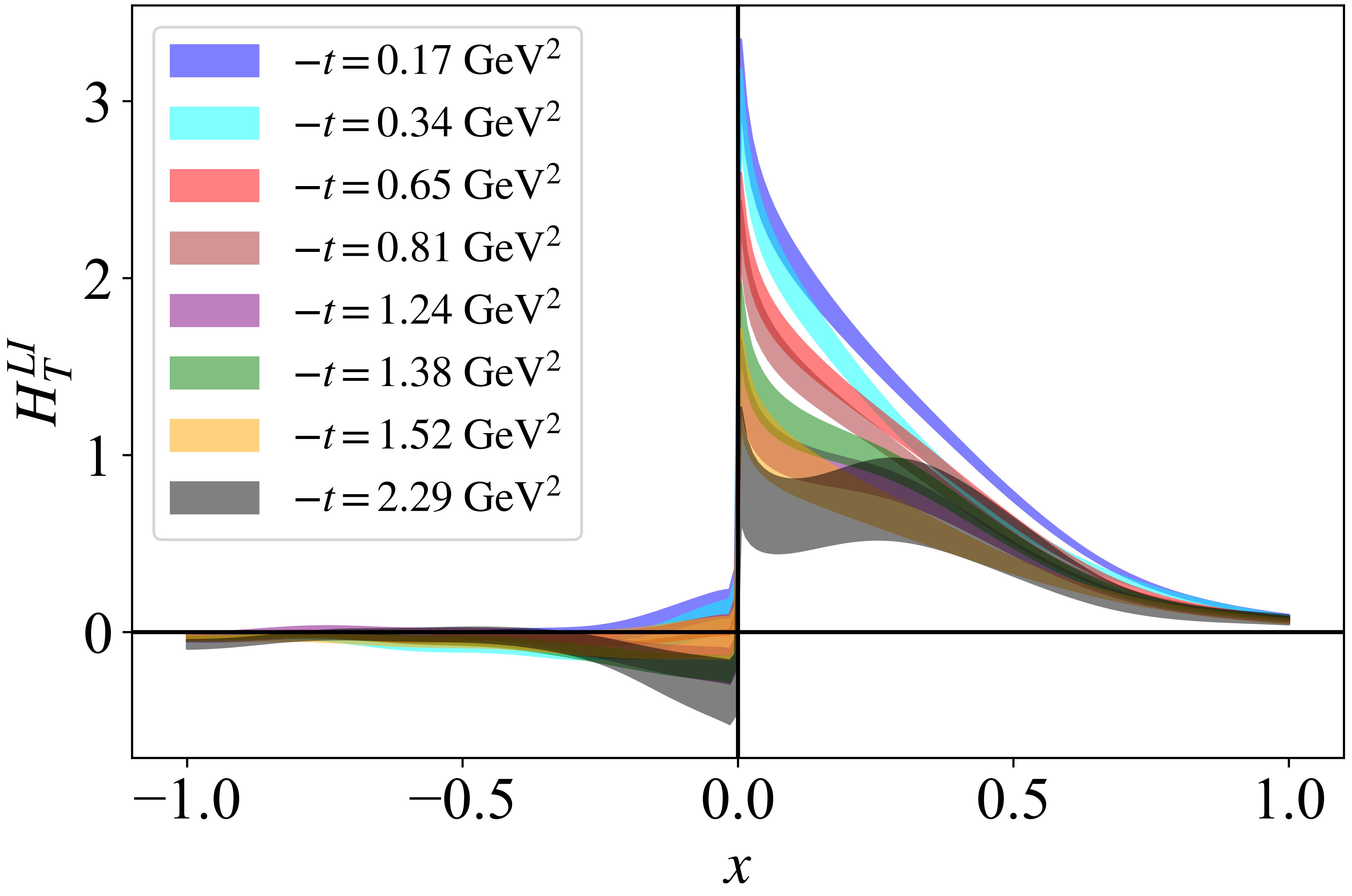
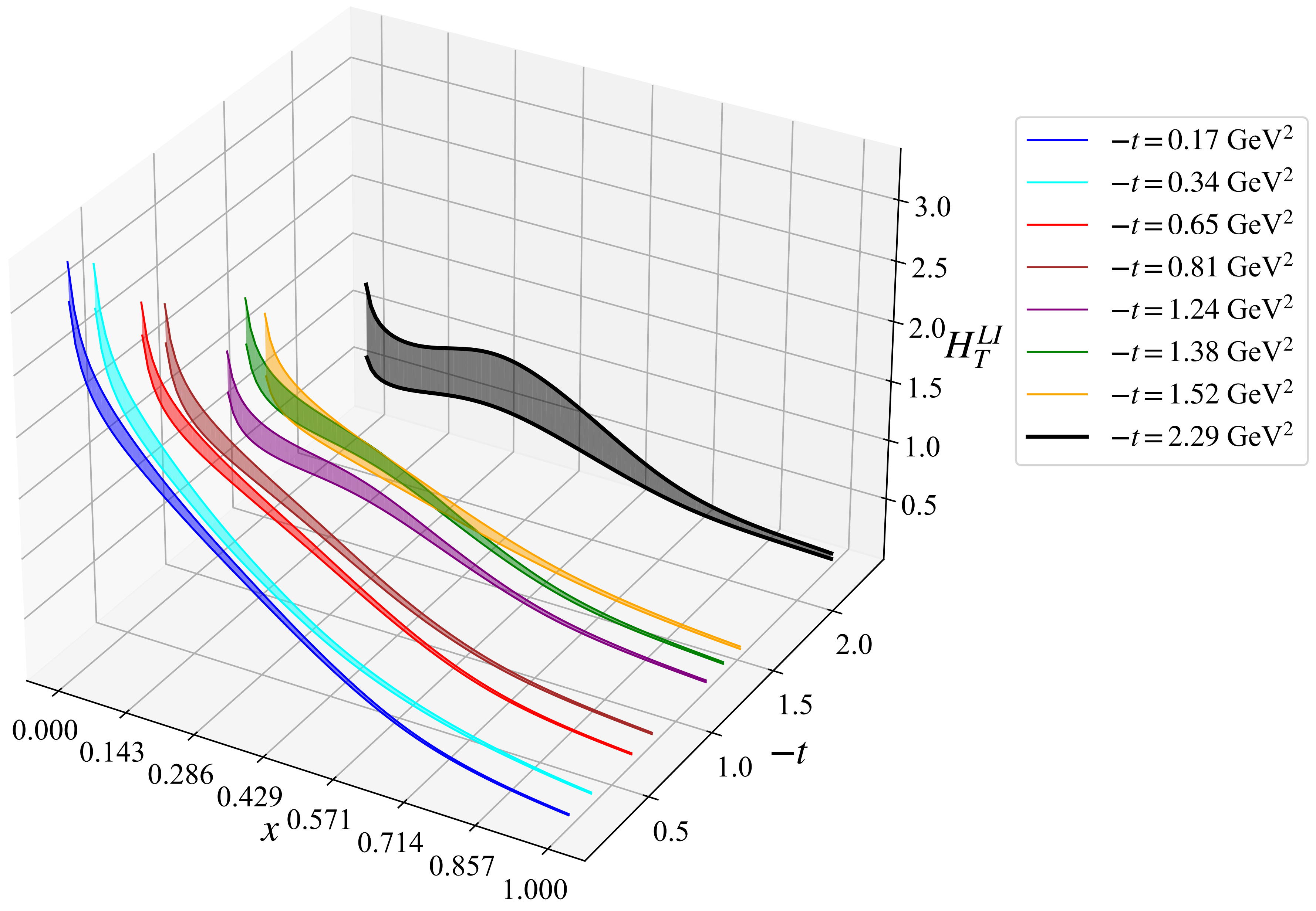
- ❖ Test the dependence on z_{max} in reconstruction



We use $z_{max} = 11a$

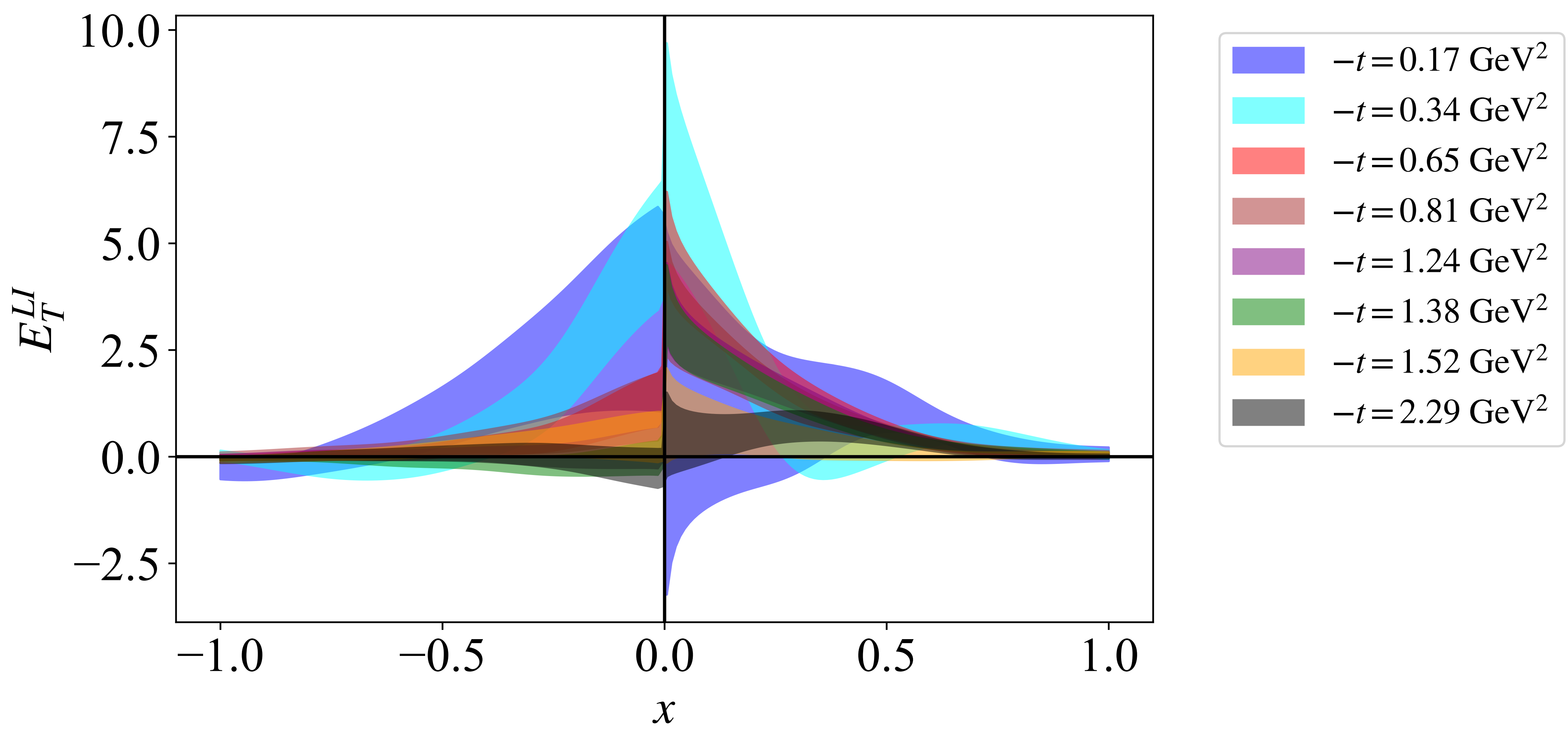
Light-Cone Results

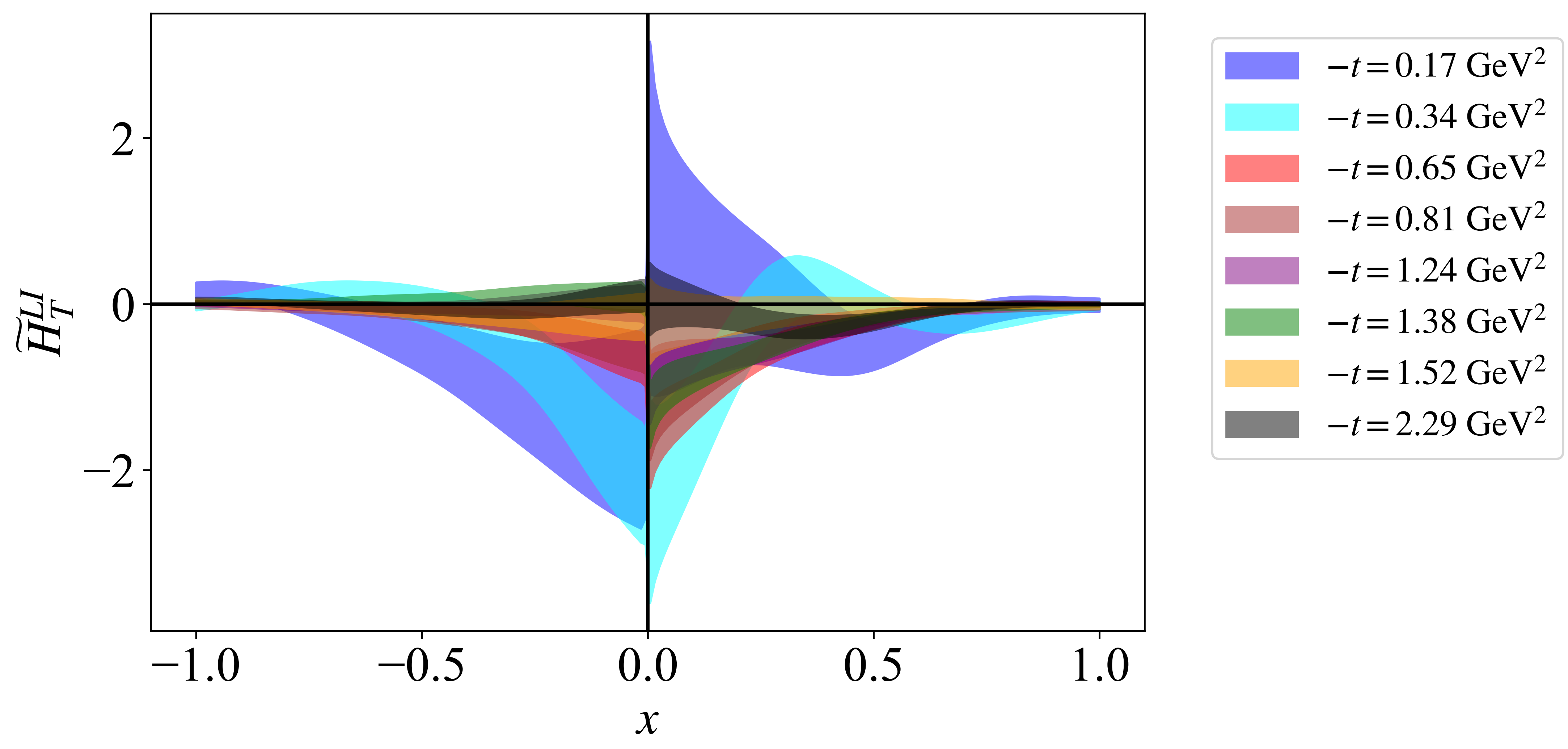




- ❖ Looks very similar to past results when extracting from symmetric frame!
- ❖ Very good signal!
- ❖ Asymmetric frame passes sanity checks!

Light-Cone Results





- E_T describes transverse nucleon deformation and spin-orbit correlations
- $\tilde{H}_T$ describes transversely polarized quark distributions and pion pole effects
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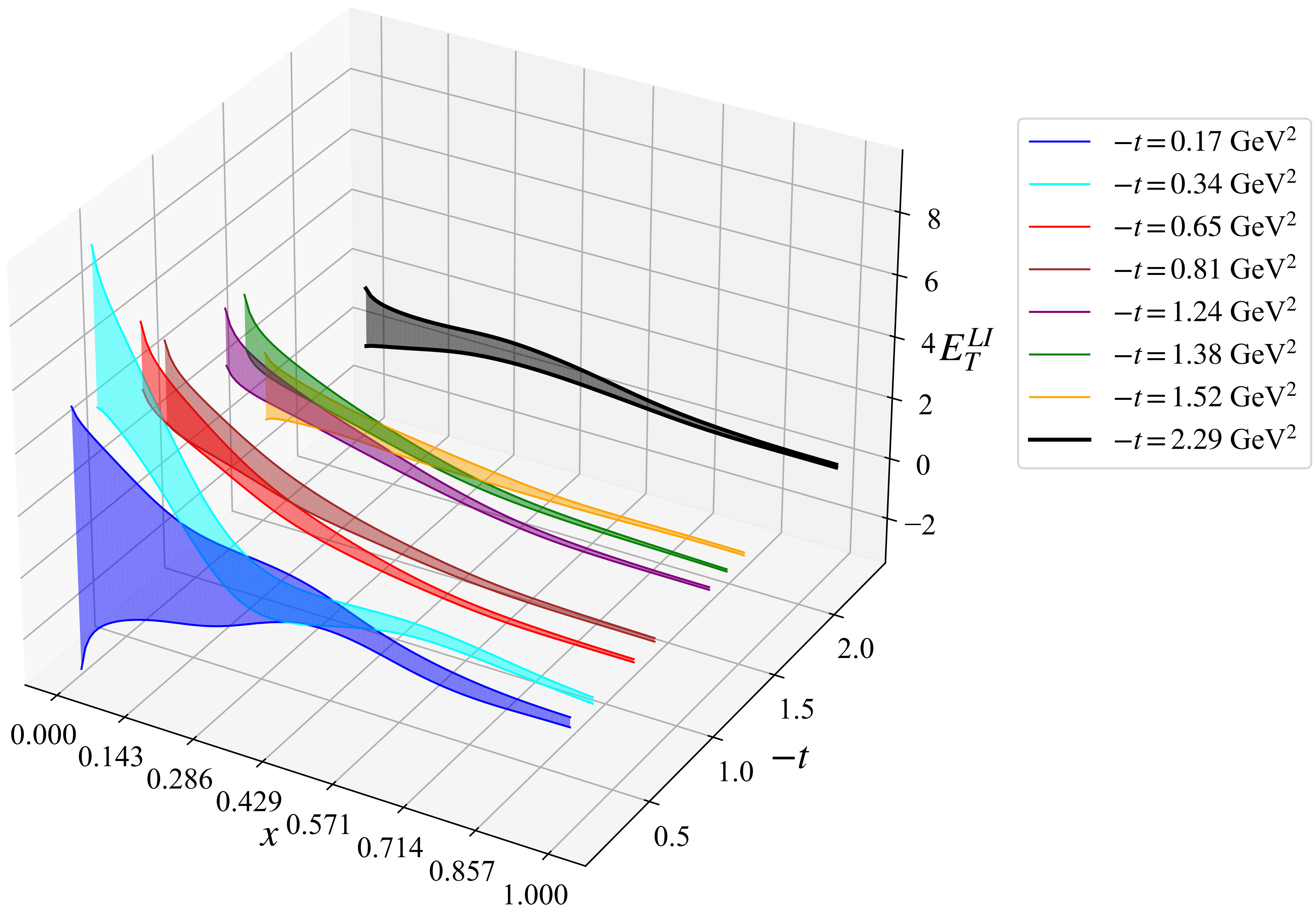

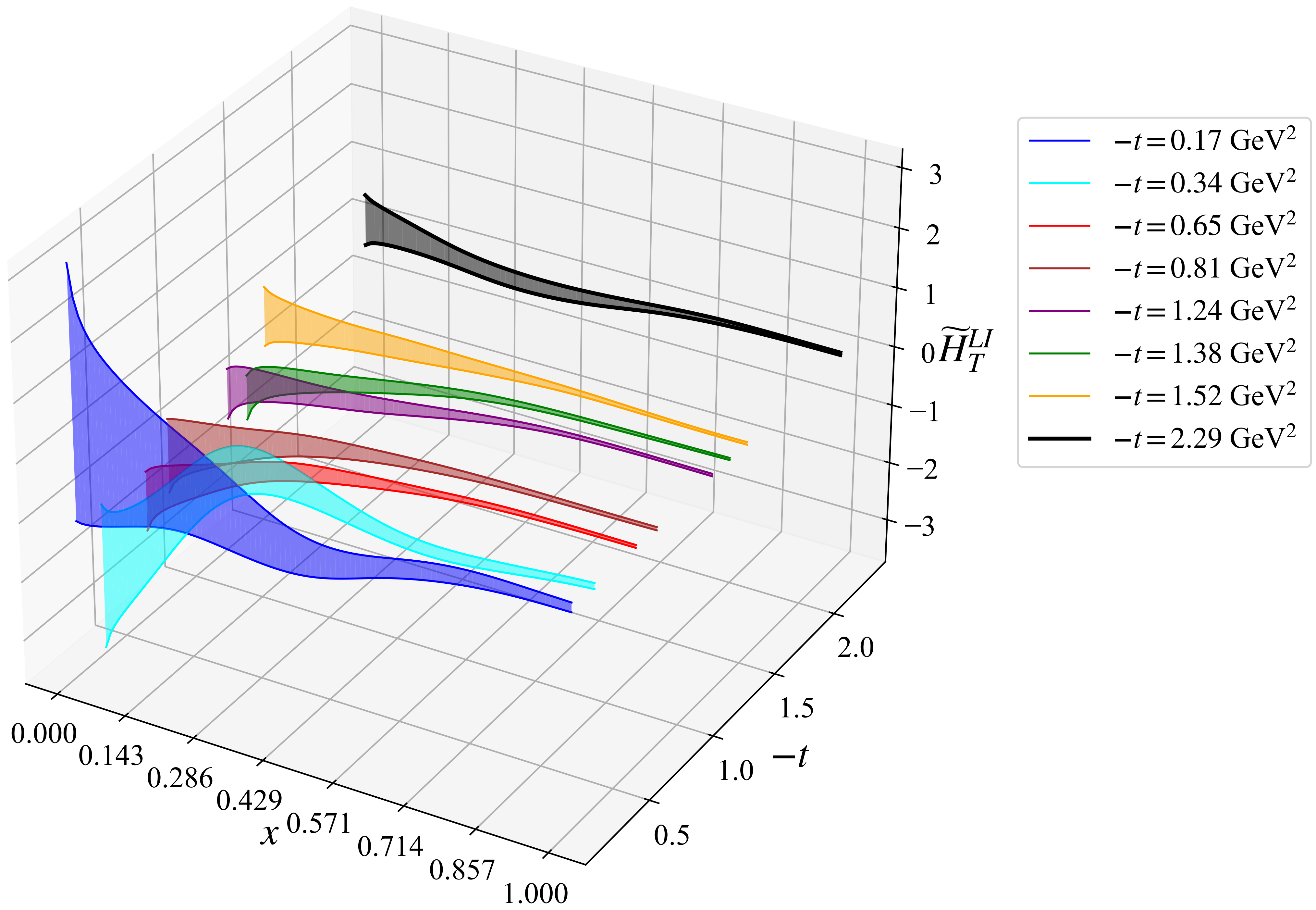

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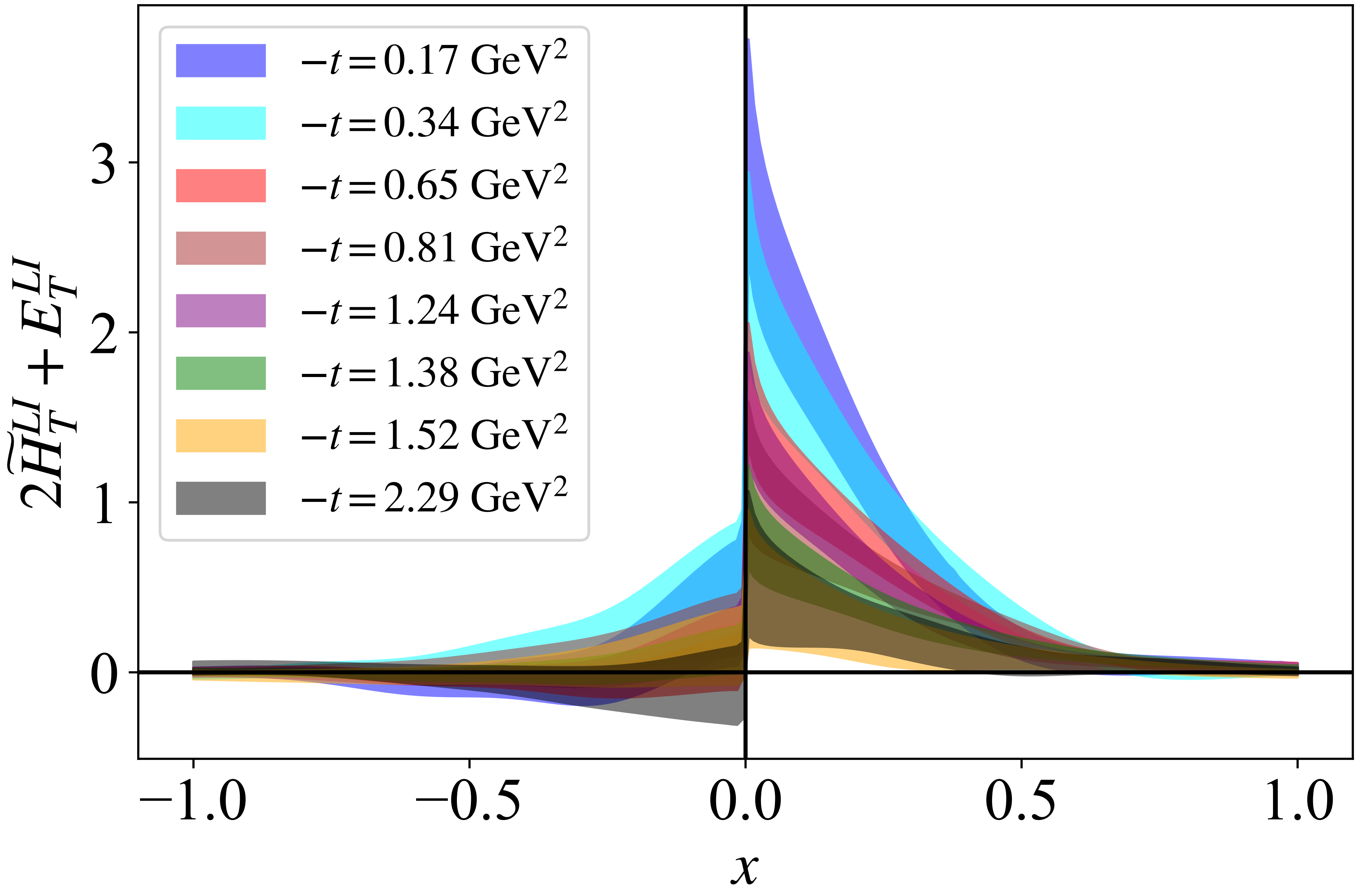

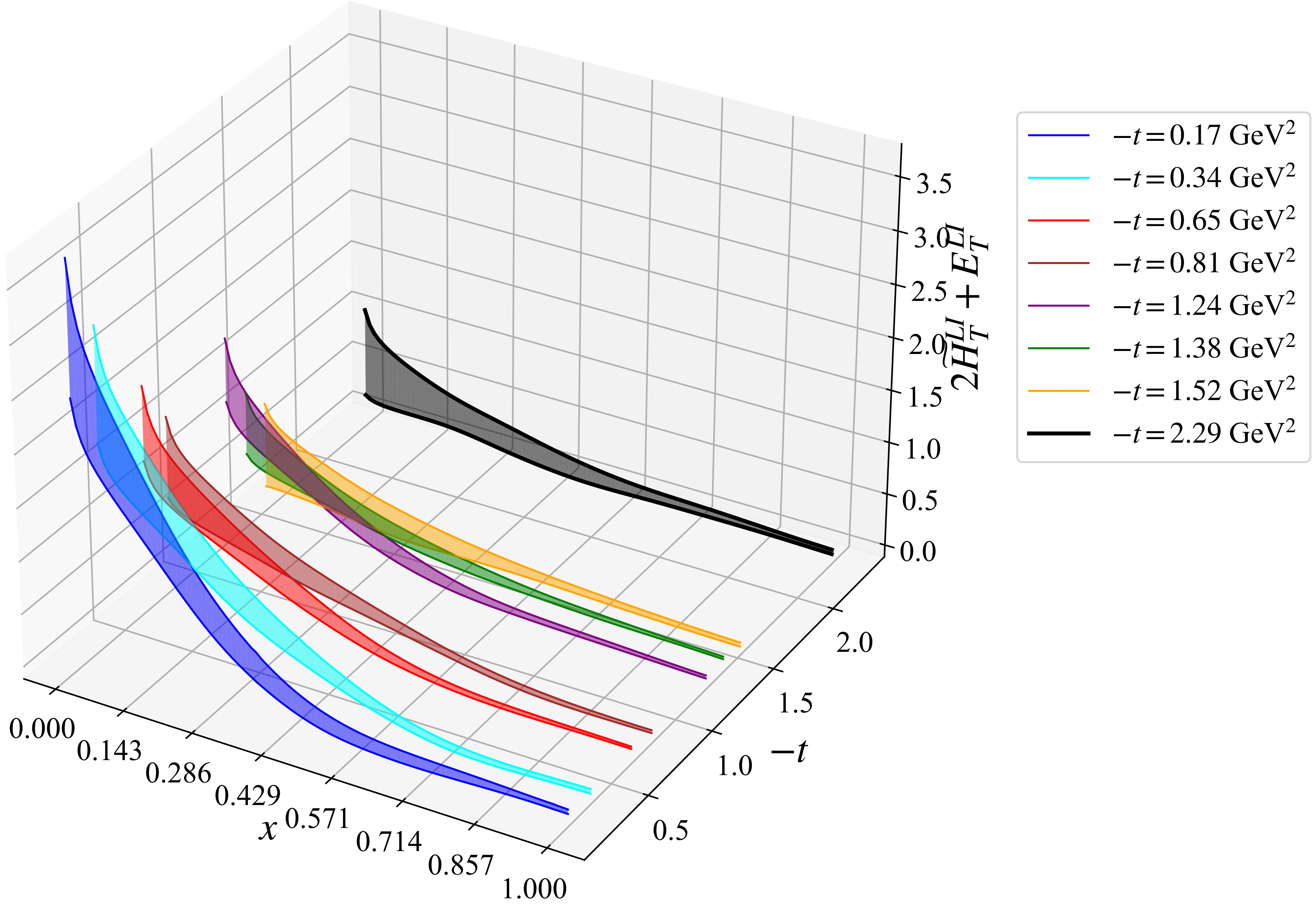

❖ Important for:

- ❖ ***Transverse Nucleon Deformation***: describes how transversely polarized nucleon is distorted in b-space
- ❖ ***Spin-Orbit Correlations***: how quark transverse spin affects transverse motion
- ❖ ***Orbital Angular Momentum***: contributes to the decomposition of nucleon spin and angular momentum structure

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Thank You!!!

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❖ Academic Computer Center in Gdańsk by Tryton