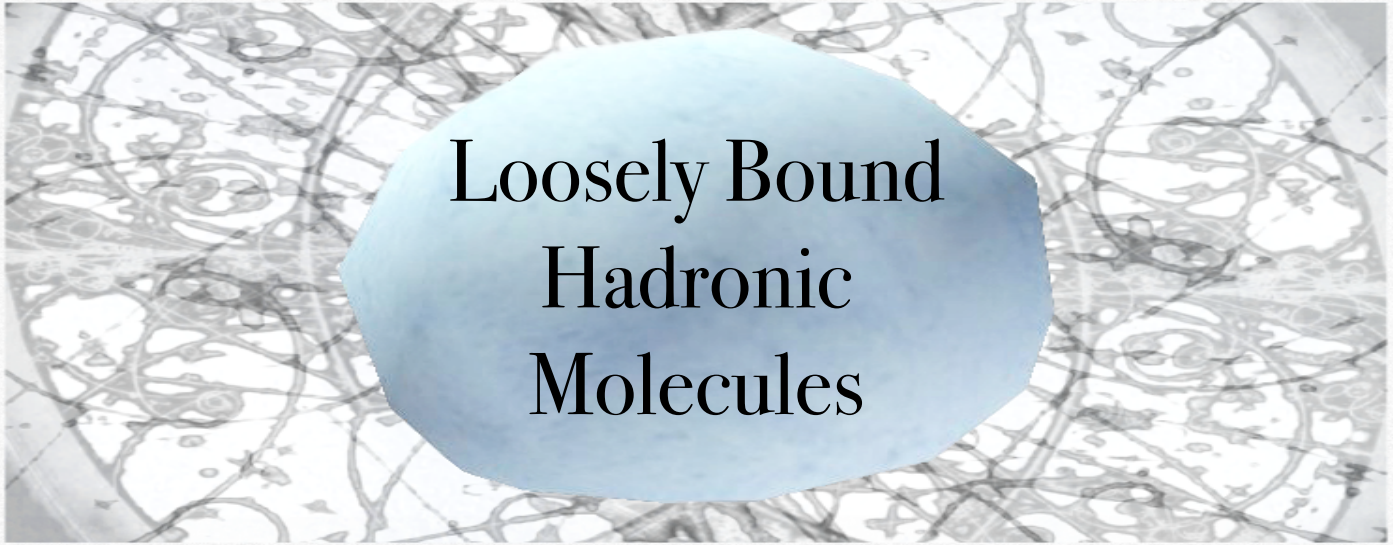
How the Contact can Produce



in Heavy-ion Collisions

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EXOHAD
EXOTIC HADRONS TOPICAL COLLABORATION

What are we Studying?

- **Loosely bound hadronic molecules** are composite particles consisting of mesons or baryons (or nuclei) bound together by the strong interaction.
- Their binding energies are tiny compared to their reduced mass.
- The $\chi_{c1}(3872)$ is a loosely bound charm-meson molecule composed of $D^{*0}\bar{D}^{0}$ or $D^{0}\bar{D}^{*0}$.

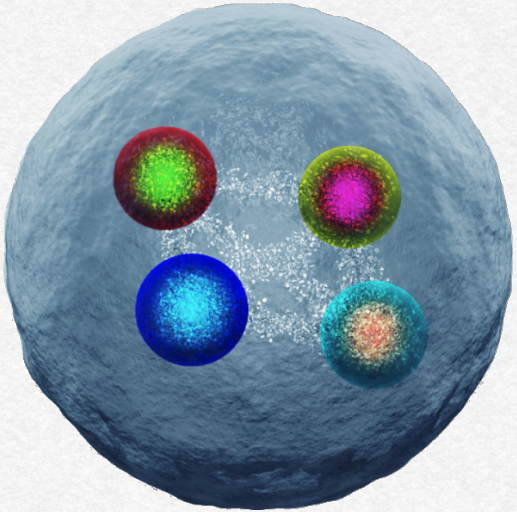
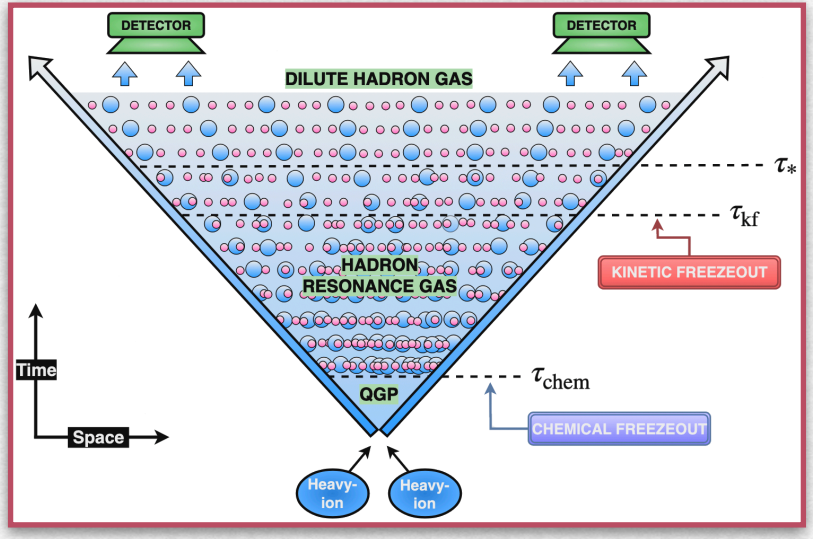


Image credit: Daniel Dominguez/Cern

Molecules from Heavy-ion Collisions

- A relativistic heavy-ion collision can create a quark-gluon plasma (**QGP**) which then cools and expands into a hadron resonance gas (**HRG**).
- Despite the temperature of the **HRG** being much higher than their binding energies, loosely bound molecules are still observed in particle detectors.
- The fact that these molecules emerge from this process has been referred to as the **"snowball in hell"** puzzle.



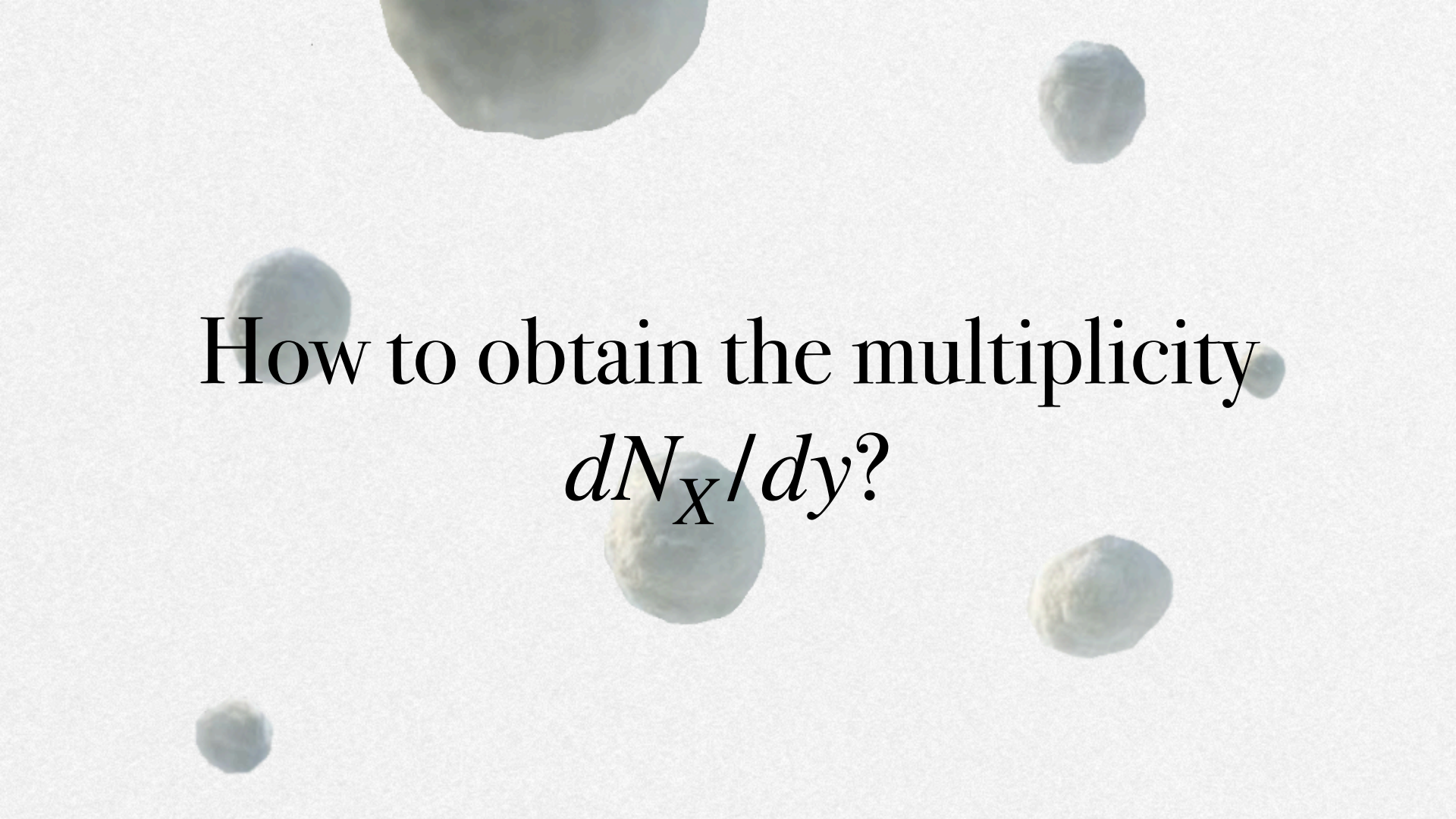
The Snowball in Hell Puzzle

- The **multiplicity** dN/dy refers to the number of particles produced in a collision event per unit rapidity.
- The "snowball in hell" puzzle is the unexpectedly large multiplicity dN_X/dy of a loosely bound hadronic molecule X.
- The path to answering the “snowball in hell” puzzle will be to derive an expression which predicts dN_X/dy in heavy-ion collisions.

Recap pt. 1

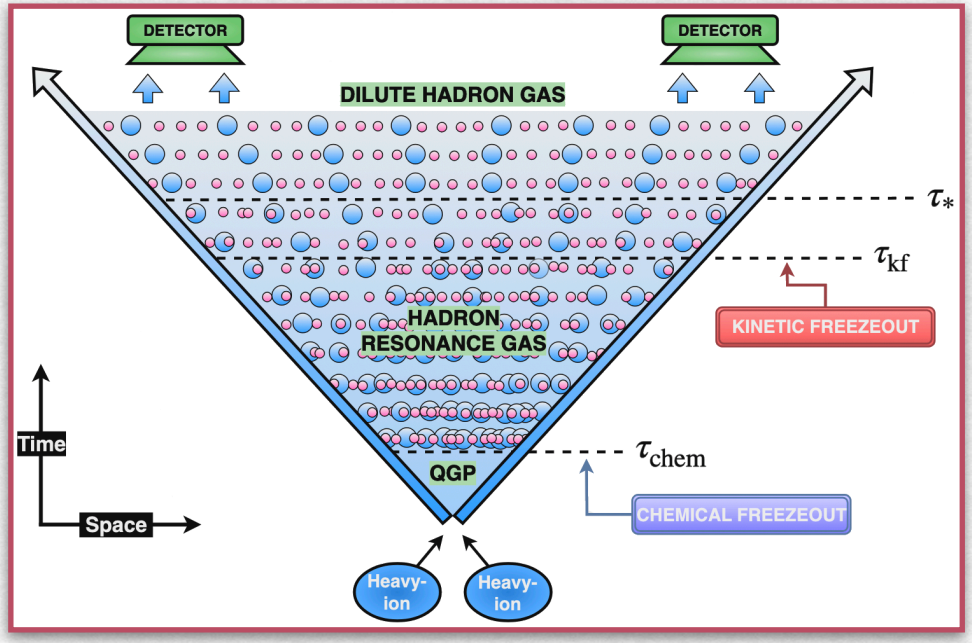
- Molecules emerging from heavy-ion collisions have been referred to as “snowballs in hell” since the energy that binds them is much weaker than the temperature of medium they’re in.

- We want to predict their multiplicity (or more simply—answer how many snowballs are formed in a given region of space), as well as provide more insight into the physics at play here.


How to obtain the multiplicity
dN_X/dy?

Toy Model for the System

- At a proper time τ after the collision, the system is locally homogeneous with temperature $T(\tau)$ and volume $V(\tau)$.
- After the transition to a HRG from a QGP at **chemical freezeout** hadron abundances are fixed.
- The HRG then expands and cools until **kinetic freezeout**, when scattering stops due to the diluteness of the gas.



After Kinetic Freeze-out

- After kinetic freeze-out, when the gas becomes sufficiently dilute, both the number density $\mathfrak{n}_h$ of a hadron h and the number density $\mathfrak{n}_\pi$ of pions decrease in proportion to $1/V(\tau)$.

- Since interactions (other than those within the molecule) have ceased, the ratio of the number densities of a molecule X and π remain fixed and must be equal to the ratio of the multiplicities observed in the detector:

$$\Longrightarrow \quad \frac{\mathfrak{n}_X}{\mathfrak{n}_\pi} = \frac{dN_X/dy}{dN_\pi/dy} \quad \text{remains fixed.}$$

After Kinetic Freeze-out (cont.)

$$\frac{\mathfrak{n}_X}{\mathfrak{n}_\pi} = \frac{dN_X/dy}{dN_\pi/dy}$$

- The ratio of multiplicities can be predicted by the ratio of densities.
- The pion multiplicity is readily available from experimental data. We will show later how to deal with $\mathfrak{n}_\pi$.
- To predict the multiplicity dN_X/dy, we must find a way to obtain $\mathfrak{n}_X$.

So to obtain dN_X/dy we need $\mathfrak{n}_X$ after kinetic freeze-out.

But now how to find $\mathfrak{n}_X$?

The Contact

- Molecules X (with reduced mass μ) have tiny binding energies $|\varepsilon_X| = \gamma^2/2\mu$ with binding momentum $\gamma = 1/a$, where a is the large s-wave scattering length.
- For systems with a large s-wave scattering length, the thermodynamic variable conjugate to γ is called the **contact** C.
- The contact can be thought of as a measure of the probability for two particles to be short distance from each other.

The Contact Density

The contact C was originally introduced by Shina Tan in 2005 in the context of a strongly interacting fermi gas. Tan derived many universal relations (all involving C) between short-distance properties of the many-body system.

Tan's adiabatic relation implies that the contact for a molecule X is given by

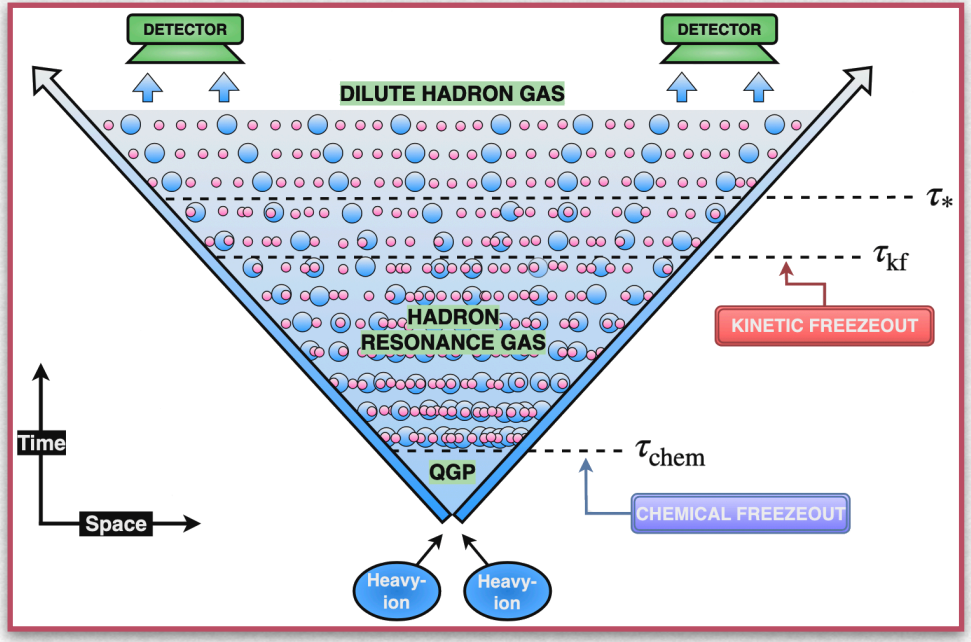
$$C_X = 8\pi\gamma.$$

For a dilute gas of volume V, the **contact density** $\mathscr{C}_X = C_X/V$ reduces to the contact for X multiplied by the **molecule number density** $\mathfrak{n}_X$:

$$\mathscr{C}_X = C_X\mathfrak{n}_X = 8\pi\gamma\mathfrak{n}_X\,.$$

The Crossover τ_*

- The previous slide implies that at some proper time τ_* there is a **crossover** where the HRG becomes dilute enough such that: $\mathfrak{n}_X(\tau) = (1/8\pi\gamma)\mathscr{C}_X(\tau)$
- We have now replaced the problem of finding $\mathfrak{n}_X(\tau)$ with finding $\mathscr{C}_X(\tau)$.



Recap pt. 2

- To understand the production of loosely bound hadronic molecules in heavy-ion collisions, we set out to derive an expression for the multiplicity dN_X/dy.
- After a crossover at τ_*, the HRG is sufficiently dilute such that $\dfrac{\mathfrak{n}_X}{\mathfrak{n}_\pi} = \dfrac{dN_X/dy}{dN_\pi/dy}$, where $\mathfrak{n}_X(\tau) = (1/8\pi\gamma)\mathscr{C}_X(\tau)$ for $\tau \gtrsim \tau_*$.
- The last remaining piece to the puzzle will be to derive an expression for $\mathscr{C}_X(\tau)$. Once we have done that, everything can be put together to solve for dN_X/dy.


Time for a little bit of algebra...

Evolution of $\mathscr{C}_X(\tau)$

Quantum field theory tells us that the contact density is the expectation value of an operator with *scaling dimension* 4.

Before Crossover

$(\tau_{\mathrm{kf}} < \tau \lesssim \tau_*)$

The contact density of a molecule X has *scaling dimension* 4. It decreases as

$$\mathscr{C}_X(\tau) \propto 1/V(\tau)^{4/3}$$

After Crossover

$(\tau \gtrsim \tau_*)$

After some time τ_*, the contact density decreases as

$$\mathscr{C}_X(\tau) \propto 1/V(\tau)$$

Evolution of $\mathscr{C}_X(\tau)$ (cont.)

Using the fact $\mathfrak{n}_\pi(\tau) \propto 1/V(\tau)$, we find:

Before Crossover

$(\tau_{\rm kf} < \tau \lesssim \tau_*)$

$$\frac{\mathscr{C}_X(\tau)}{\mathscr{C}_X(\tau_{\rm kf})} = \left[\frac{V(\tau_{\rm kf})}{V(\tau)}\right]^{4/3} = \left[\frac{\mathfrak{n}_\pi(\tau)}{\mathfrak{n}_\pi(\tau_{\rm kf})}\right]^{4/3}$$

After Crossover

$(\tau \gtrsim \tau_*)$

$$\frac{\mathscr{C}_X(\tau)}{\mathscr{C}_X(\tau_*)} = \frac{V(\tau_*)}{V(\tau)} = \frac{\mathfrak{n}_\pi(\tau)}{\mathfrak{n}_\pi(\tau_*)}$$

Evolution of $\mathscr{C}_X(\tau)$ (cont.)

Before Crossover

$(\tau_{\rm kf} < \tau \lesssim \tau_*)$

$$\mathscr{C}_X(\tau) = \mathscr{C}_X\left(\tau_{\rm kf}\right)\left[\mathfrak{n}_\pi(\tau)/\mathfrak{n}_\pi\left(\tau_{\rm kf}\right)\right]^{4/3}$$

Use to obtain $\mathscr{C}_X(\tau_*)$

After Crossover

$(\tau \gtrsim \tau_*)$

$$\mathscr{C}_X(\tau) = \mathscr{C}_X(\tau_*)\left[\mathfrak{n}_\pi(\tau)/\mathfrak{n}_\pi(\tau_*)\right]$$

$$\mathscr{C}_X(\tau) = \mathscr{C}_X(\tau_{\rm kf})\left[\frac{\mathfrak{n}_\pi(\tau)}{\mathfrak{n}_\pi(\tau_*)}\right]\left[\frac{\mathfrak{n}_\pi(\tau_*)}{\mathfrak{n}_\pi(\tau_{\rm kf})}\right]^{4/3}$$

Evolution of $\mathfrak{n}_X(\tau)$

Previously: $$\mathscr{C}_X(\tau) = \mathscr{C}_X(\tau_{\rm kf}) \left[\frac{\mathfrak{n}_\pi(\tau)}{\mathfrak{n}_\pi(\tau_*)}\right] \left[\frac{\mathfrak{n}_\pi(\tau_*)}{\mathfrak{n}_\pi(\tau_{\rm kf})}\right]^{4/3}, \quad \tau \gtrsim \tau_*$$

Dilute limit: $$\mathscr{C}_X(\tau) = 8\pi\gamma \mathfrak{n}_X(\tau), \quad \tau \gtrsim \tau_*$$

Equation the two expressions for $\mathscr{C}_X(\tau)$, the molecule number density is

$$\mathfrak{n}_X(\tau) = \frac{1}{8\pi\gamma} \mathscr{C}_X(\tau_{\rm kf}) \left[\frac{\mathfrak{n}_\pi(\tau_*)}{\mathfrak{n}_\pi(\tau_{\rm kf})}\right]^{1/3} \frac{\mathfrak{n}_\pi(\tau)}{\mathfrak{n}_\pi(\tau_{\rm kf})}, \quad \tau \gtrsim \tau_*$$

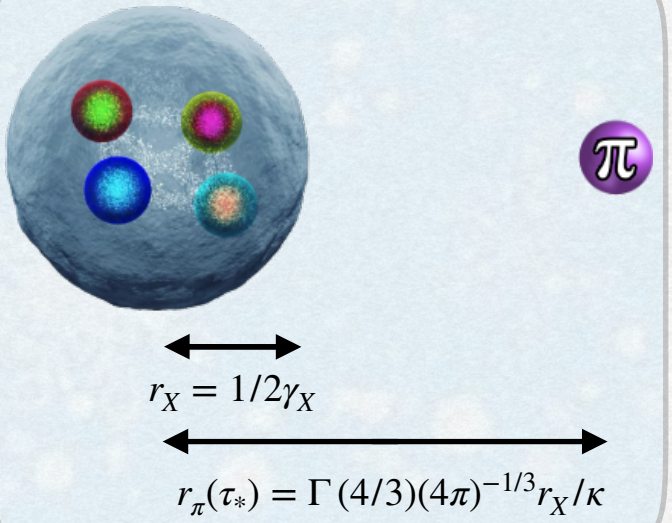
Estimating $\mathfrak{n}_\pi(\tau_*)$

- Assuming $\mathfrak{n}_\pi(\tau)$ is uniform, the mean nearest-pion distance is given by

$$r_\pi(\tau) = \Gamma(4/3)\left[4\pi\mathfrak{n}_\pi(\tau)\right]^{-1/3}.$$

- To estimate τ_* we use the time when $r_\pi(\tau)$ exceeds r_X by a small numerical factor parameterized by κ:

We estimate $\mathfrak{n}_\pi(\tau_*) = (2\kappa\gamma)^3$ $\leftarrow \kappa$ is treated as a phenomenological parameter.



Now let’s put the pieces together.

Molecule Multiplicity

Previously: $\mathfrak{n}_X(\tau) = \dfrac{1}{8\pi\gamma}\mathscr{C}_X(\tau_{\rm kf})\left[\dfrac{\mathfrak{n}_\pi(\tau_*)}{\mathfrak{n}_\pi(\tau_{\rm kf})}\right]^{1/3}\dfrac{\mathfrak{n}_\pi(\tau)}{\mathfrak{n}_\pi(\tau_{\rm kf})}, \qquad \mathfrak{n}_\pi(\tau_*) = (2\kappa\gamma)^3$

After τ_* the ratio $\mathfrak{n}_X(\tau)/\mathfrak{n}_\pi(\tau)$ remains fixed and is equal to the ratio $(dN_X/dy)/(dN_\pi/dy)$ of the multiplicities observed at the detector:

$$\Longrightarrow \quad dN_X/dy = \frac{\kappa}{4\pi}\left[\mathscr{C}_X(\tau_{\rm kf})/\mathfrak{n}_\pi(\tau_{\rm kf})^{4/3}\right] dN_\pi/dy\,.$$

Molecule Multiplicity (cont.)

Deuteron Multiplicity from ALICE

Centrality	dN_d/dy
$(0-10)\,\%$	$(9.82 \pm 1.58) \times 10^{-2}$
$(10-20)\,\%$	$(7.6 \pm 1.25) \times 10^{-2}$
$(20-40)\,\%$	$(4.76 \pm 0.82) \times 10^{-2}$
$(40-60)\,\%$	$(1.90 \pm 0.41) \times 10^{-2}$
$(60-80)\,\%$	$(0.51 \pm 0.14) \times 10^{-2}$

Multiplicity is measured in ranges of **centrality**: It indicates how *head-on* (small %) or *peripheral* (large %) the collision was.

Previously: $dN_X/dy = \frac{\kappa}{4\pi}\left[\mathscr{C}_X(\tau_{\rm kf})/\mathfrak{n}_\pi(\tau_{\rm kf})^{4/3}\right] dN_\pi/dy$

- Dividing dN_X/dy by dN_d/dy, we find:

$$dN_X/dy = \left[\mathscr{C}_X(\tau_{\rm kf})/\mathscr{C}_d(\tau_{\rm kf})\right] dN_d/dy$$

- The ratio of molecule multiplicities equals the ratio of their contact densities at kinetic freeze-out.

Evaluating the Contact Density

- Thermodynamic variables can be calculated using the **virial expansion**.
- Labeling the constituents of the molecule by σ, the virial expansion is a power series around small **fugacities** z_σ:

$$z_\sigma = \mathfrak{n}_\sigma (2\pi/m_\sigma T)^{3/2} \quad \leftarrow \text{Boltzmann approximation}$$

number density: $\mathfrak{n}_\sigma$
mass: m_σ

- For sufficiently small $\mathfrak{n}_\sigma$, to leading order in z_σ the virial expansion of the contact density $\mathscr{C}_{X\mathrm{kf}}$ is given by:

$$\mathscr{C}_X(\tau_{\mathrm{kf}}) = \frac{16}{\pi} (\mu T)^2 z_1 z_2 F\left(\gamma/\sqrt{2\mu T}\right)$$

$$F(w) = 1 + \sqrt{\pi} w + \ldots, \quad (w > 1)$$


How many snowballs?

Hypertriton (${}^{3}_{\Lambda}H$) Multiplicity

- ${}^{\Lambda}_{3}H$ is a bound state of a deuteron and strange baryon Λ with a Λ separation energy of 148 ± 40 keV.
- In Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV, the ALICE collaboration also measured the multiplicity of ${}^{\Lambda}_{3}H$ (and ${}^{\Lambda}_{3}\bar{H}$).
- Using the measured dN_d/dy, a prediction for the mean hypertriton multiplicity $dN_{{}^{3}_{\Lambda}H}/dy$ can be made in the (0-10)% centrality bin.

Our Formula:

$$dN_{{}^{3}_{\Lambda}H}/dy = \left(\mathscr{C}_{{}^{3}_{\Lambda}H\mathrm{kf}}/\mathscr{C}_{d\mathrm{kf}}\right) dN_d/dy$$

Centrality	Observed $dN_{{}^{3}_{\Lambda}H}/dy$	Predicted $dN_{{}^{3}_{\Lambda}H}/dy$
$(0-10)\,\%$	$(14.7 \pm 3.6) \times 10^{-5}$	$(10.4 \pm 3.9) \times 10^{-5}$

The prediction for $dN_{{}^{3}_{\Lambda}H}/dy$ is consistent with the ALICE result to within the errors.

$\chi_{c1}(3872)$ Multiplicity

- The $\chi_{c1}(3872)$ is a loosely bound charm-meson molecule discovered in 2003. The difference between its mass and the threshold for the charm meson pair $D^{*0}\bar{D}^{0}$ is -50 ± 93 keV.
- The CMS collaboration has presented evidence for the production of $\chi_{c1}(3872)$ in Pb-Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV.
- Using the measured dN_d/dy, we can make a prediction for the $\chi_{c1}(3872)$ multiplicity $dN_{\chi_{c1}}/dy$ in the (0-10)% bin.

Our Formula:

$$dN_{^3_\Lambda H}/dy = \left(\mathscr{C}_{^3_\Lambda H\mathrm{kf}}/\mathscr{C}_{d\mathrm{kf}}\right) dN_d/dy$$

Centrality	Predicted $dN_{\chi_{c1}}/dy$
$(0-10)\,\%$	$(23.4 \pm 7.8) \times 10^{-5}$

Future Outlook

- The contact provides a simple explanation for a 30+ year old puzzle.
- Using an approximation for the contact density, we calculated the multiplicities of loosely bound molecules.
- Our approach can also be extended to obtaining the transverse momentum distributions for loosely bound hadronic molecules.

The Contact Density

Tan's adiabatic relation states that the contact C can be expressed as a derivative of the internal energy E with entropy S held fixed:

$$C = -8\pi\mu \left(\frac{\partial E}{\partial \gamma}\right)_S .$$

If we insert the binding energy $\varepsilon_X = -\gamma_X^2/2M$, we find that the contact for a molecule X is

$$C_X = 8\pi\gamma_X .$$

For a dilute gas of volume V, the **contact density** $\mathscr{C}_X = C_X/V$ reduces to the contact for X multiplied by the **molecule number density** $\mathfrak{n}_X$:

$$\mathscr{C}_X = 8\pi\gamma\mathfrak{n}_X .$$

Estimating the Crossover Time

To estimate τ_*, we use the time when the mean nearest-pion distance $r_\pi(\tau)$ surpasses the mean molecule constituent separation r_X by a numerical factor:

$$r_\pi(\tau_*) = \Gamma(4/3)(4\pi)^{-1/3} r_X/\kappa.$$

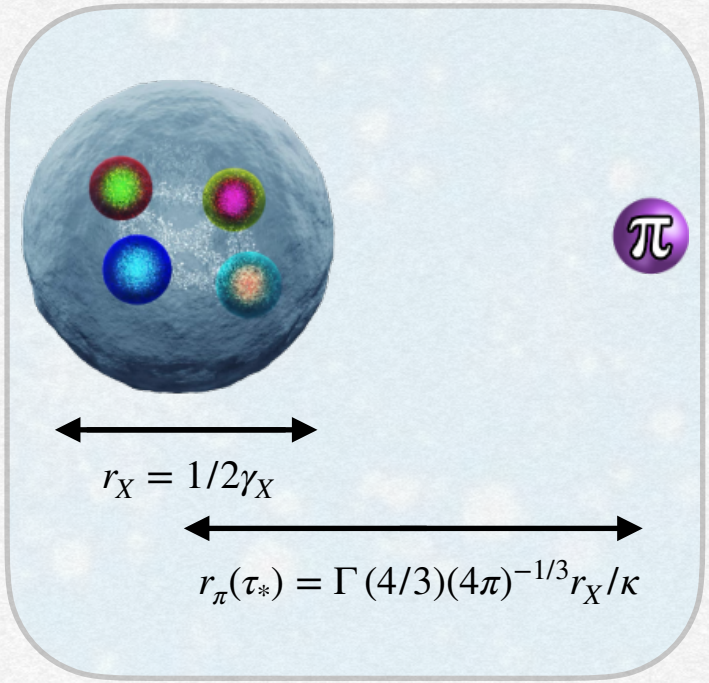
The molecule has a universal wavefunction:

$$\psi_X(r) = \sqrt{\frac{\gamma}{2\pi}}\frac{1}{r}e^{-\gamma r} \implies r_X = \langle r \rangle = 1/2\gamma.$$

The mean pion distance is $r_\pi(\tau) = \Gamma(4/3)\left[4\pi \mathrm{n}_\pi(\tau)\right]^{-1/3}$.

This implies the pion number density at τ_* reduces to:

$$\mathfrak{n}_\pi(\tau_*) = (2\kappa\gamma)^3.$$



Estimating κ

Previously: $dN_X/dy = \dfrac{\kappa}{4\pi}\left(\mathscr{C}_{X\mathrm{kf}}/\mathfrak{n}_{\pi\mathrm{kf}}^{4/3}\right) dN_\pi/dy\,.$

- The **deuteron** (d) is a proton-neutron (pn) bound state with a relatively small binding energy $\varepsilon_d = 2.225$ MeV.

- Using the measured deuteron multiplicity dN_d/dy, the parameter κ_d can be obtained from the expression for dN_X/dy:

$$\kappa_d = 4\pi\left(\mathfrak{n}_{\pi\mathrm{kf}}^{4/3}/\mathscr{C}_{d\mathrm{kf}}\right)\frac{dN_d/dy}{dN_\pi/dy}.$$

Using ALICE data we find $\kappa_d = 0.18 \pm 0.04$