Subeikonal corrections to dijet production in DIS

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Dijet Production in DIS

- Saturation at EIC: What are the emergent properties of dense system of the gluons?
- Dijet production will be the golden channel to detect saturation at EIC.

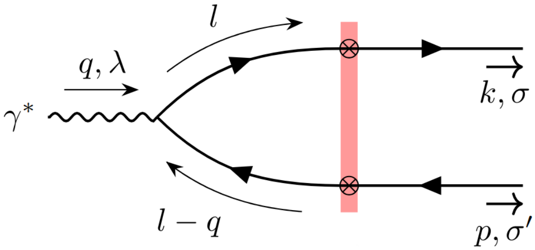


Figure: Caucal, Salazar, Venugopalan, 2108.06347

- Small $x \Rightarrow$ the dipole picture of DIS.

Motivation: Precision Calculation

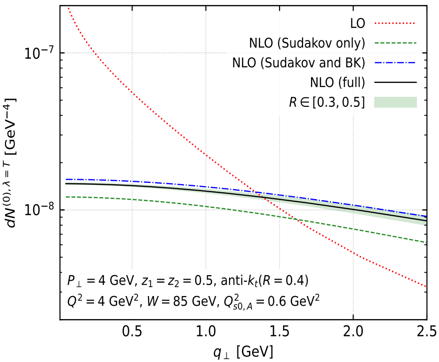


Figure: Caucal, Salazar, Schenke, Stebel, Venugopalan, 2308.00022

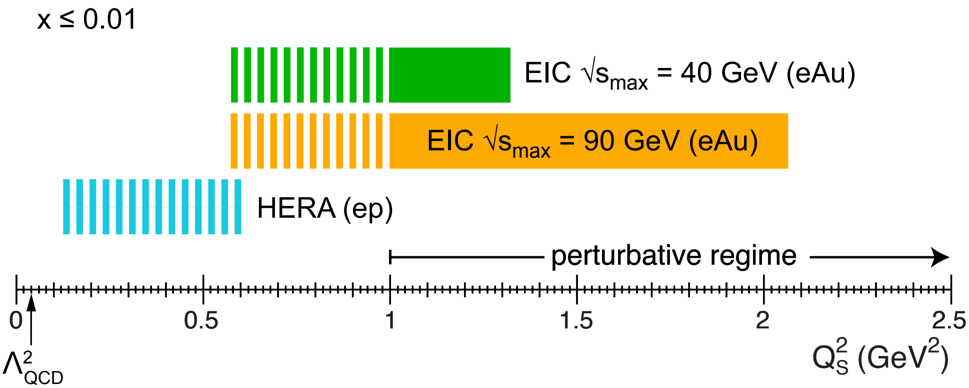
$$d\sigma^{\gamma^* A\to q\bar{q}x} = \underbrace{\text{LO}}_{\alpha_s^0 s^0} + \underbrace{\text{NLO}}_{\alpha_s^1 s^0} + \underbrace{\text{SubEik}}_{\alpha_s^0 s^{-1}} + \dots \quad (1)$$

NLO calculations to dijet:

- Caucal, Salazar, Schenke, Stebel, Venugopalan, 2308.00022.
- Caucal, Salazar, Venugopalan, 2108.06347.
- Boussarie, Grabovsky, Szymanowski, Wallon, 1606.00419.
- Beuf, 1708.06557.

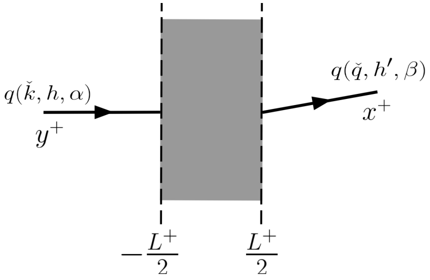
⋮

Motivation: EIC



Motivation: The Precursors

- Previous calulation of dijet production in small x:
  [Agostini, Altinoluk, Armesto, 2403.04603, Altinoluk, Beuf, Czayka, Tymowska, 2012.03886]
- Drawback: Final result depends explicitly on the finite width L^+.

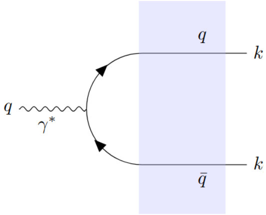


$$2\pi\delta(k_1^+ + k_2^+ - q^+)\, N_c\, \alpha_{\text{em}}\, e_f^2 \left[1 + \left(\frac{k_2^+ - k_1^+}{q^+}\right)^2\right] 2\text{Re}\,(-i)\,\frac{L^+}{2} \int_{\mathbf{z},\mathbf{v}',\mathbf{w}'} e^{i\mathbf{k}_1\cdot(\mathbf{v}'-\mathbf{z})}\, e^{i\mathbf{k}_2\cdot(\mathbf{w}'-\mathbf{z})}$$

$$\times \frac{(\mathbf{w}'^j - \mathbf{v}'^j)}{|\mathbf{w}' - \mathbf{v}'|}\, \bar{Q}\, \text{K}_1\left(\bar{Q}\,|\mathbf{w}' - \mathbf{v}'|\right) \left\langle \frac{1}{N_c} \text{Tr}\left[\mathcal{U}_F(\mathbf{w}')\mathcal{U}_F^\dagger(\mathbf{v}') - 1\right]\left[\mathcal{U}_F(\mathbf{z})\overleftrightarrow{\partial}_{\mathbf{z}^j}\mathcal{U}_F^\dagger(\mathbf{z})\right]\right\rangle$$

- Alternative aprroach: Application of background field method techniques.

Our Approach: The Background Field Method



- The amplitude of $\gamma^* \to q\bar{q}$:

$$i\mathcal{M} = ie\int d^4y\bar{u}(k_1)\gamma^\mu v(k_2)\epsilon_\mu(q)e^{ik_1y}e^{ik_2y}e^{-iqy} \tag{2}$$

- Inserting 2 complete sets of states using the Schwinger's notation,

$$i\mathcal{M} = -ie\int d^4y\int d^4x_1\int d^4x_2\bar{u}(k_1)e^{ik_1x_1}(x_1|\not{p}\frac{i}{\not{p}}|y)\gamma^\mu(y|\frac{i}{\not{p}}\not{p}|x_2)e^{ik_2x_2}v(k_2)\epsilon_\mu(q)e^{-iqy} \tag{3}$$

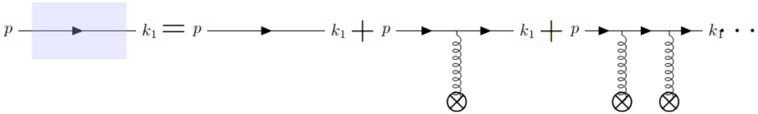
- To account for multi-gluon background interactions, we promote

$$\frac{i}{\not{p}} \to \frac{i}{\not{P}} \quad \text{where} \quad P_\mu = p_\mu + gA_\mu. \tag{4}$$

The Quark Propagator in the Background Field

- The quark propagator upon expansion in g:

$$\int d^4x_1 \bar{u}(k_1) e^{ik_1x_1} (x_1|\not{p}_1 \frac{i}{\not{p}} - \not{p}\frac{i}{\not{p}}\not{A}\not{p}\frac{1}{p^2} + \not{p}\frac{i}{\not{p}}\not{A}\not{p}\frac{1}{p^2}\not{A}\not{p}\frac{1}{p^2} + \ldots |y) \quad (5)$$



- This series can be resummed using the standard commutation relations

$$i\int d^4x_1 \bar{u}(k_1) e^{ik_1x_1} k_1^2 (x_1|\frac{1}{P^2 + \frac{1}{2}\sigma^{\mu\nu}F_{\mu\nu}}|y) \quad (6)$$

Orders of Eikonality

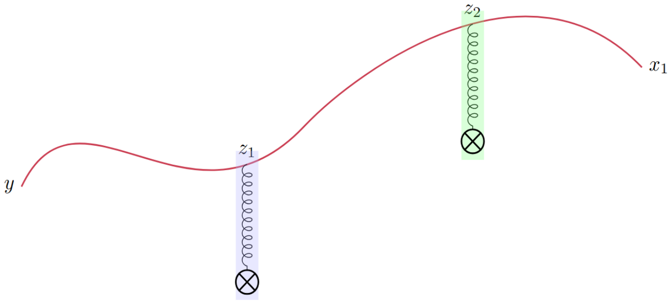
- We consider the nucleus to be boosted along the x^+ direction with the boost parameter λ.

$$
\begin{aligned}
A_-(x^+, x^-, x_\perp) &\sim \lambda \tilde{A}_-(\frac{1}{\lambda}x^+, \lambda x^-, x_\perp) \\
A_\perp(x^+, x^-, x_\perp) &\sim \tilde{A}_\perp(\frac{1}{\lambda}x^+, \lambda x^-, x_\perp) \\
A_+(x^+, x^-, x_\perp) &\sim \frac{1}{\lambda}\tilde{A}_+(\frac{1}{\lambda}x^+, \lambda x^-, x_\perp)
\end{aligned}
\qquad
\begin{aligned}
F_{-\perp} &\sim \lambda \tilde{F}_{-\perp} \\
F_{-+} &\sim \tilde{F}_{-+} \\
F_{ij} &\sim \tilde{F}_{ij} \\
F_{+\perp} &\sim \frac{1}{\lambda}\tilde{F}_{+\perp}
\end{aligned}
\tag{7}
$$

Expansion of the Resummed Propagator

- Expanding the propagator,

$$k_1^2(k_1|\frac{1}{P^2+\frac{g}{2}\sigma^{\mu\nu}F_{\mu\nu}}|k+q) = k_1^2(k_1|\frac{1}{p^2} - \frac{1}{p^2}\left(g\{p^\mu, A_\mu\} + g^2A^\mu A_\mu + \frac{g}{2}\sigma^{\mu\nu}F_{\mu\nu}\right)\frac{1}{p^2}$$
$$+\frac{1}{p^2}\left(g\{p^\mu, A_\mu\} + g^2A^\mu A_\mu + \frac{g}{2}\sigma^{\mu\nu}F_{\mu\nu}\right)\frac{1}{p^2}\left(g\{p^\mu, A_\mu\} + g^2A^\mu A_\mu + \frac{g}{2}\sigma^{\mu\nu}F_{\mu\nu}\right)\frac{1}{p^2} - \ldots|k+q) \quad (8)$$



Example

- Keeping the dominant component of the background field A_-:

$$k_1^2(k_1|\frac{1}{p^2+2p^-A_-+i\epsilon}|k+q) = k_1^2(k_1|\frac{1}{p^2+i\epsilon} - \frac{1}{p^2+i\epsilon}2p^-A_-\frac{1}{p^2+i\epsilon} + \frac{1}{p^2+i\epsilon}2p^-A_-\frac{1}{p^2+i\epsilon}2p^-A_-\frac{1}{p^2+i\epsilon} - \ldots|k+q) \tag{9}$$

- Taking the poles p^+ of the intermediate propagator,

$$\Big\{(k_{1\perp}|k_\perp+q_\perp)2\pi\delta(k_1^+-k^+-q^+) + \int_{-\infty}^{\infty} dz^- e^{-i(k^++q^+)z^-}(k_{1\perp}|i\int_{z^-}^{\infty} dz_1^- e^{i\frac{p_\perp^2}{2k_1^-}z_1^-} A_-(z_1^-)e^{-i\frac{p_\perp^2}{2k_1^-}z_1^-} e^{i\frac{p_\perp^2}{2k_1^-}z^-} + (i)^2\int_{z^-}^{\infty} dz_2^- \int_{z_2^-}^{\infty} dz_1^- e^{i\frac{p_\perp^2}{2k_1^-}z_1^-} A_-(z_1^-)e^{-i\frac{p_\perp^2}{2k_1^-}z_1^-} e^{i\frac{p_\perp^2}{2k_1^-}z_2^-} A_-(z_2^-)e^{-i\frac{p_\perp^2}{2k_1^-}z_2^-} e^{i\frac{p_\perp^2}{2k_1^-}z^-} - \ldots|k_\perp+q_\perp)\Big\}2\pi\delta(k_1^- - k^- - q^-) \tag{10}$$

- Exponential phases describe the transverse motion of quarks inside the background field.

- Expansion of the phase:

$$e^{i\frac{p_\perp^2}{2k_1^-}z_1^-}A_-(z_1^-)e^{-i\frac{p_\perp^2}{2k_1^-}z_1^-} = A_-(z_1^-) + \frac{iz_1^-}{2k_1^-}[p_\perp^2, A_-(z_1^-)] + \dots$$
$$= A_-(z_1^-) - \frac{iz_1^-}{2k_1^-}\{p^k, i\partial_k A_-(z_1^-)\} + \dots \quad (11)$$

- Power counting scheme $\implies$ Each subsequent term is suppressed by powers of z^-.

$$A_-(z^-) \sim \lambda\tilde{A}_-(\lambda x^-)$$
$$\implies \int dz^- A_-(z^-) \sim \int d\tilde{z}^- \tilde{A}_-(\tilde{z}^-)$$
$$\implies \int dz^- z^- A_-(z^-) \sim \frac{1}{\lambda}\int d\tilde{z}^- \tilde{z}^- \tilde{A}_-(\tilde{z}^-) \quad (12)$$

The Eikonal Propagator

- Keeping the leading term,

$$e^{i\frac{p_\perp^2}{2k_1^-}z_1^-}A_-(z_1^-)e^{-i\frac{p_\perp^2}{2k_1^-}z_1^-} \to A_-(z_1^-) \tag{13}$$

- We reproduce the eikonal result:

$$\begin{aligned} k_1^2(k_1|\frac{1}{p^2+2p^-A_-+i\epsilon}|k+q) =& 2\pi\delta(k_1^- - k^- - q^-)(k_{1\perp}|2\pi\delta(k^+ + q^+ - \frac{p_\perp^2}{2k_1^-}) \\ &+ (U-1)\frac{i}{k^+ + q^+ - \frac{p_\perp^2}{2k_1^-}}|k_\perp + q_\perp) \end{aligned} \tag{14}$$

Subeikonal Propagator

- Considering the first 2 terms of the expansion,

$$e^{i\frac{p_\perp^2}{2k_1^-}z_1^-}A_-(z_1^-)e^{-i\frac{p_\perp^2}{2k_1^-}z_1^-} \to A_-(z_1^-) - \frac{z_1^-}{2k_1^-}\{p^k, F_{-k}(z_1^-)\} \tag{15}$$

- With the subeikonal correction,

$$k_1^2(k_1|\frac{1}{p^2+2p^-A_-+i\epsilon}|k+q) = 2\pi\delta(k_1^- - k^- - q^-) \tag{16}$$

$$\times\ (k_{1\perp}|\int_{-\infty}^{\infty}dz^-\Big\{[\infty,z^-] - i\int_{z^-}^{\infty}dz_1^-\frac{z_1^-}{2k_1^-}[\infty,z_1^-]\{p^k,F_{-k}(z_1^-)\}[z_1^-,z^-]+\dots\Big\}$$

$$e^{-i(k^+ + q^+ - \frac{p_\perp^2}{2k_1^-})z^-}|k_\perp + q_\perp) \tag{17}$$

- This is not the complete subeikonal propagator.
- The general full result with all the components of the background field can be obtained using similar techniques.

The General Subeikonal Propagator

- Taking all the terms,

$$k_1^2(k_1|\frac{1}{P^2+\frac{1}{2}\sigma^{\mu\nu}F_{\mu\nu}+i\epsilon}|k+q) = 2\pi\delta(k_1^- - k^- - q^-)\Big[(k_{1\perp}|\int_{-\infty}^{\infty}dz^-[\infty,z^-]$$

$$\times\, e^{-i(k^+ + q^+ - \frac{p_\perp^2}{2k_1^-})z^-} + \int_{-\infty}^{\infty}dz^-\int_{z^-}^{\infty}dz_1^-[\infty,z_1^-]\Big\{\frac{1}{2k_1^-}\sigma^{-k}F_{-k}(z_1^-) - \frac{iz_1^-}{2k_1^-}\{P^k, F_{-k}(z_1^-)\}$$

$$-\frac{z_1^-}{(2k_1^-)^2}[P_\perp^2,\sigma^{-k}F_{-k}(z_1^-)] + \frac{i}{2k_1^-}\sigma^{+-}F_{+-}(z_1^-) + \frac{i}{4k_1^-}\sigma^{ij}F_{ij}(z_1^-)\Big\}[z_1^-,z^-]e^{-i(k^+ + q^+ - \frac{p_\perp^2}{2k_1^-})z^-}$$

$$|k_\perp + q_\perp)\Big] \tag{18}$$

- Different terms in this equation have different *orders of eikonality*.

Analyzing the Eikonality of each Insertion

The eikonal entities:

- $[x^-, y^-]$
- $\int dz^- \frac{1}{2k_1^-} \sigma^{-i} F_{-i}$

The subeikonal entities:

- $\int dz^- \frac{z^-}{2k_1^-} \{P^k, F_{-k}(z^-)\}$
- $\int dz^- \frac{1}{2k_1^-} \sigma^{-+} F_{-+}(z^-)$
- $\int dz^- \frac{1}{4k_1^-} \sigma^{ij} F_{ij}(z^-)$

Example

$$[x^-, y^-] = 1 + i \int_{y^-}^{x^-} dz^- A_-(z^-) + i^2 \int_{y^-}^{x^-} dz_1^- A_-(z_1^-) \int_{y^-}^{z_1^-} dz_2^- A_-(z_2^-) + \dots$$

$$\sim 1 + i \int_{y^-}^{x^-} dz^- \lambda \tilde{A}_-(\lambda z^-) + i^2 \int_{y^-}^{x^-} dz_1^- \lambda \tilde{A}_-(\lambda z_1^-) \int_{y^-}^{z_1^-} dz_2^- \lambda A_-(\lambda z_2^-) + \dots$$

$$\sim 1 + i \int_{\tilde{y}^-}^{\tilde{x}^-} d\tilde{z}^- \tilde{A}_-(\tilde{z}^-) + i^2 \int_{\tilde{y}^-}^{\tilde{x}^-} d\tilde{z}_1^- \tilde{A}_-(\tilde{z}_1^-) \int_{\tilde{y}^-}^{\tilde{z}_1^-} d\tilde{z}_2^- \tilde{A}_-(\tilde{z}_2^-) + \dots$$

Subeikonal Term from the Expansion of Phase

- In our technique we observe the appearance of terms like the following.

$$\int_{-\infty}^{\infty} dz^{-} [\infty, z^{-}] e^{-i(k^{+}+q^{+}-\frac{p_{\perp}^{2}}{2k_{1}^{-}})z^{-}} \tag{19}$$

- No explicit dependence on the finite size of the shockwave!
- This term contains both contribution of eikonal and subeikonal order.

The full propagator

- Separating eikonal and subeikonal terms in our approach we obtain

$$
\begin{aligned}
&2\pi\delta(k_1^- - k^- - q^-)\Big[(k_{1\perp}|2\pi\delta(k^+ + q^+ - \frac{p_\perp^2}{2k_1^-}) + ([\infty,-\infty]-1)\frac{1}{k^+ + q^+ - \frac{p_\perp^2}{2k_1^-}} \\
&+\int_{-\infty}^{\infty} dz_1^- [\infty, z_1^-]\Big\{\frac{1}{2k_1^-}\sigma^{-k}F_{-k}(z_1^-)\Big\}[z_1^-,-\infty]\frac{i}{k^+ + q^+ - \frac{p_\perp^2}{2k_1^-}} + \int_{-\infty}^{\infty} dz_1^- [\infty, z_1^-] \\
&\Big\{-\frac{iz_1^-}{2k_1^-}\{P^k, F_{-k}(z_1^-)\} - \frac{z_1^-}{(2k_1^-)^2}[P_\perp^2, \sigma^{-k}F_{-k}(z_1^-)] + \frac{i}{2k_1^-}\sigma^{+-}F_{+-}(z_1^-) + \frac{i}{4k_1^-}\sigma^{ij}F_{ij}(z_1^-)\Big\} \\
&[z_1^-,-\infty]\frac{i}{k^+ + q^+ - \frac{p_\perp^2}{2k_1^-}} - ([\infty,-\infty]-1)\frac{1}{q^+ - k_1^+ - k_2^+} + \int_{-\infty}^{\infty} dz^-([\infty,z^-]-1)e^{-i(q^+ - k_1^+ - k_2^+)z^-} \\
&+\int_{-\infty}^{\infty} dz_1^- [\infty, z_1^-]\Big\{\frac{1}{2k_1^-}\sigma^{-k}F_{-k}(z_1^-)\Big\}\Big(\int_{-\infty}^{z_1^-} dz^-[z_1^-, z^-]e^{-i(q^+ - k_1^+ - k_2^+)z^-} \\
&-[z_1^-,-\infty]\frac{i}{q^+ - k_1^+ - k_2^+}\Big)|k_\perp + q_\perp)\Big]
\end{aligned}
\tag{20}
$$

The x^+ dependence of the fields

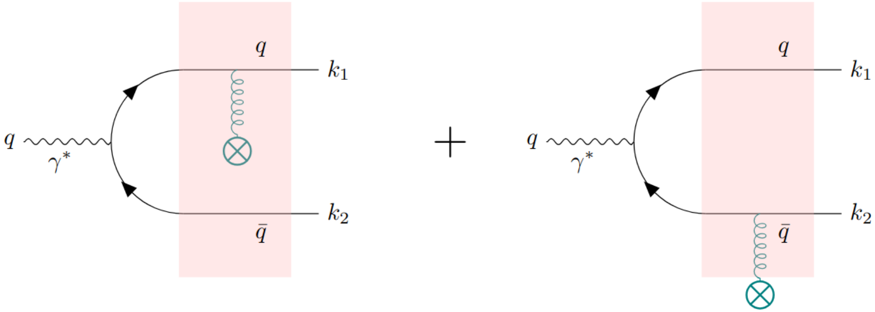
- What if we consider the dependence of the fields on x^+?
- This can effectively be of subeikonal order.
- The recoil effect on the quark.

$$2\pi\delta(k_1^- - k^- - q^-) \to \int dx^+ e^{i(k_1^- - k^- - q^-)x^+} \tag{21}$$

The Amplitude

- In terms of the resummed propagators, the amplitude in the background field is

$$i\mathcal{M} = ie\int \frac{d^4k}{(2\pi)^4}\bar{u}(k_1)k_1^2(k_1|\frac{1}{P^2+\frac{1}{2}\sigma^{\mu\nu}F_{\mu\nu}}|k+q)\gamma^\mu(k|\frac{1}{P^2+\frac{1}{2}\sigma^{\mu\nu}F_{\mu\nu}}|-k_2)k_2^2 v(k_2)\epsilon_\mu(q) \tag{22}$$



Example: The Eikonal Amplitude

- In the amplitude we replace

$$k_1^2(k_1|\frac{1}{P^2+\frac{1}{2}\sigma^{\mu\nu}F_{\mu\nu}}|k+q) \to 2\pi\delta(k_1^- - k^- - q^-)(k_{1\perp}|\Bigg(([\infty,-\infty]-1)\frac{1}{k^+ + q^+ - \frac{p_\perp^2}{2k_1^-}} + \int_{-\infty}^{\infty} dz_1^-[\infty, z_1^-]\Big\{\frac{1}{2k_1^-}\sigma^{-k}F_{-k}(z_1^-)\Big\}[z_1^-,-\infty]\frac{i}{k^+ + q^+ - \frac{p_\perp^2}{2k_1^-}}\Bigg)|k_\perp + q_\perp) \tag{23}$$

- Entire F_{-k} is eikonal.

$$F_{-k} = \partial_- A_k - \partial_k A_- - ig[A_-, A_k] \tag{24}$$

- This is mostly replaced by $\partial_i A_-$.

The General Eikonal Amplitude (γ_L^*)

- For the longitudinal polarization of the virtual photon,

$$i\mathcal{M} = -e\int \mathrm{d}^2 z_\perp \int \mathrm{d}^2 z'_\perp e^{-ik_{1\perp}.z_\perp} e^{-ik_{2\perp}.z'_\perp}([\infty,-\infty;z_\perp][-\infty,\infty;z'_\perp]-1)$$
$$\zeta\bar{\zeta}K_0(\varepsilon_f|z'_\perp - z_\perp|)\bar{u}(k_1)\gamma^- v(k_2)\epsilon_-(q)2\delta(\zeta+\bar{\zeta}-1) \tag{25}$$

The Eikonal Amplitude (γ_T^*)

For the transverse polarization of the photon:

$$-ie\int \mathrm{d}^2 z_\perp \int \mathrm{d}^2 z'_\perp e^{-ik_{1\perp}.z_\perp} e^{-ik_{2\perp}.z'_\perp} 2\pi\delta(k_1^- + k_2^- - q^-) 2q^-\zeta\bar{\zeta}\frac{-i}{2\pi}K_0(\varepsilon_f|z'_\perp - z_\perp|)$$
$$\Bigg([\infty,-\infty;z_\perp][-\infty,\infty;z'_\perp] - 1 + \int_\infty^{-\infty} \mathrm{d}z_1'^-[\infty,-\infty;z_\perp][-\infty,z_1'^-;z'_\perp]\frac{i}{2k_2^-}\sigma^{-i}F_{-i}(z_1'^-,z'_\perp)$$
$$[z_1'^-,\infty;z'_\perp] + \int \mathrm{d}z_1^-[\infty,z_1^-;z_\perp]\frac{i}{2k_1^-}\sigma^{-i}F_{-i}(z_1^-,z_\perp)[z_1^-,-\infty;z_\perp][-\infty,\infty;z'_\perp]\Bigg)$$
$$\bar{u}(k_1)\gamma^t v(k_2)\epsilon_t(q) \tag{26}$$

The General Eikonal Amplitude (γ_T^*)

$$
\begin{aligned}
-i\mathcal{M} = &- e\int \mathrm{d}^2 z_\perp \int \mathrm{d}^2 z'_\perp e^{-ik_{1\perp}.z_\perp} e^{-ik_{2\perp}.z'_\perp}([\infty, -\infty; z_\perp][-\infty, \infty; z'_\perp] - 1) \\
&\zeta\bar{\zeta}\frac{(z'-z)^i}{|z'_\perp - z_\perp|} K_1(\varepsilon_f |z'_\perp - z_\perp|)\bar{u}(k_1)\gamma^- \frac{q^-}{k_1^- k_2^-}\left(\frac{(2k_1^- - q^-)}{q^-}\delta^{it} - \frac{[\gamma^i, \gamma^t]}{2}\right) v(k_2) \\
&\epsilon_t(q)\delta(\zeta + \bar{\zeta} - 1) \qquad (27)
\end{aligned}
$$

- However, one should use F_{-k} instead of just the transverse derivative of the Wilson line.

A Subeikonal Amplitude

- We calculate all subeikonal amplitudes as well.
- An example of a term in the subeikonal order:

$$
\begin{aligned}
i\mathcal{M} = - e \int \mathrm{d}^2 z_\perp \int \mathrm{d}^2 z'_\perp \int_{\infty}^{-\infty} \mathrm{d}z_1'^{-} e^{-ik_{1\perp}.z_\perp} e^{-ik_{2\perp}.z'_\perp} \delta(\zeta + \bar{\zeta} - 1)[\infty, -\infty; z_\perp] \\
[-\infty, z_1'^{-}; z'_\perp] i\sigma^{ij} F_{ij}(z_1'^{-}, z'_\perp)[z_1'^{-}, \infty; z'_\perp] \bar{u}(k_1)\gamma^{-} v(k_2) \epsilon_{-}(q) \frac{\zeta}{2q^{-}} K_0(\varepsilon_f |z'_\perp - z_\perp|)
\end{aligned}
\tag{28}
$$

Conclusion

- We derive the general structure of operators in subeikonal order in the Regge limit using the background field method.
- We rely solely on the power counting of the background field operators without taking a finite size of the shockwave into account.
- Our basic set of operators differs from the standard eikonal calculations (MV model) and available subeikonal results.
- All terms of F_{-i} are equally important and should be understood as a generalization of $\partial_i U$.
- Our set of operators can be directly connected to the TMDPDF operators.