

# Small- $x$ Helicity Global Analysis: Now With Polarized $pp$ Data

NICHOLAS BALDONADO

NEW MEXICO STATE UNIVERSITY



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# Proton Spin Puzzle and Small-x

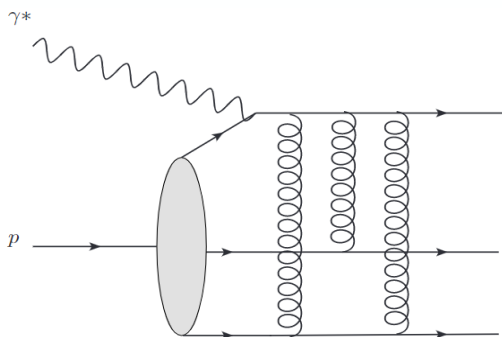
- Proton Spin  $\equiv \frac{1}{2}$ , Experimental Measurement of spin coming from partons  $\neq \frac{1}{2}$

- Jaffe-Manohar sum rule [1]:  $S_q + L_q + S_G + L_G = \frac{1}{2}$

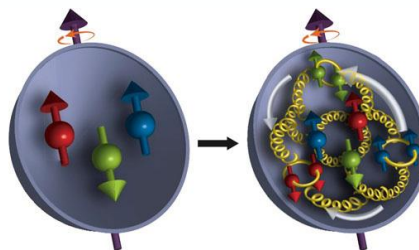
$$S_q(Q^2) = \frac{1}{2} \int_0^1 dx \Delta\Sigma(x, Q^2), \quad S_G(Q^2) = \int_0^1 dx \Delta G(x, Q^2)$$

- Experimental data only exists for a range of values  $x \in [x_{min}, 1]$

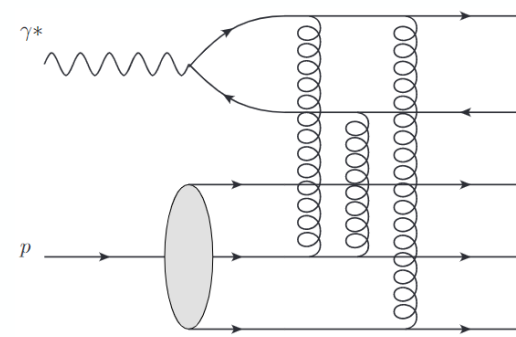
“Knockout” DIS at Large- $x$



$$x = \frac{Q^2}{s + Q^2} \sim \frac{Q^2}{E_{CM}^2} \sim \frac{1}{m \Delta t_{Ioffe}}$$



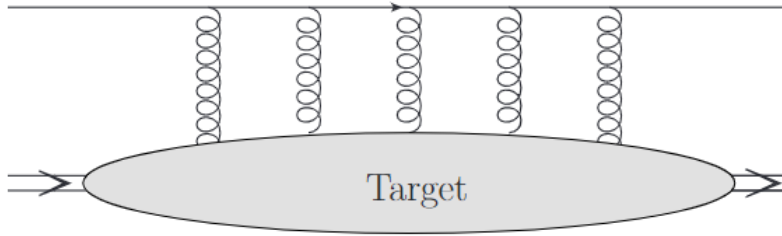
“Dipole” DIS at Small- $x$



At small- $x$  and with large Ioffe time our degrees of freedom change from quarks and gluons to eikonal Wilson Lines, and the proton appears very dense and frozen.

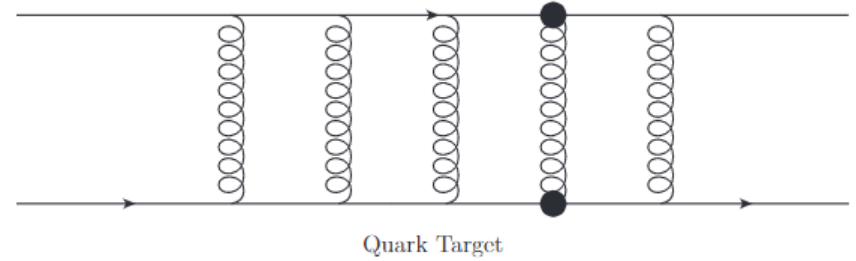
# Sub-Eikonal Corrections and Double Logs

Unpolarized Wilson Line

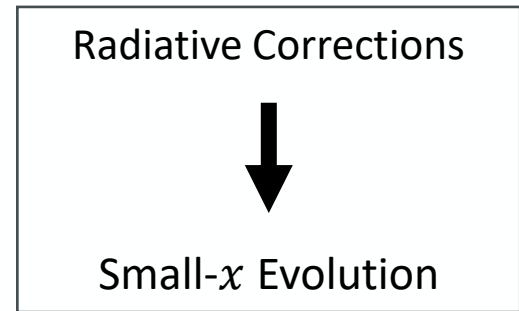
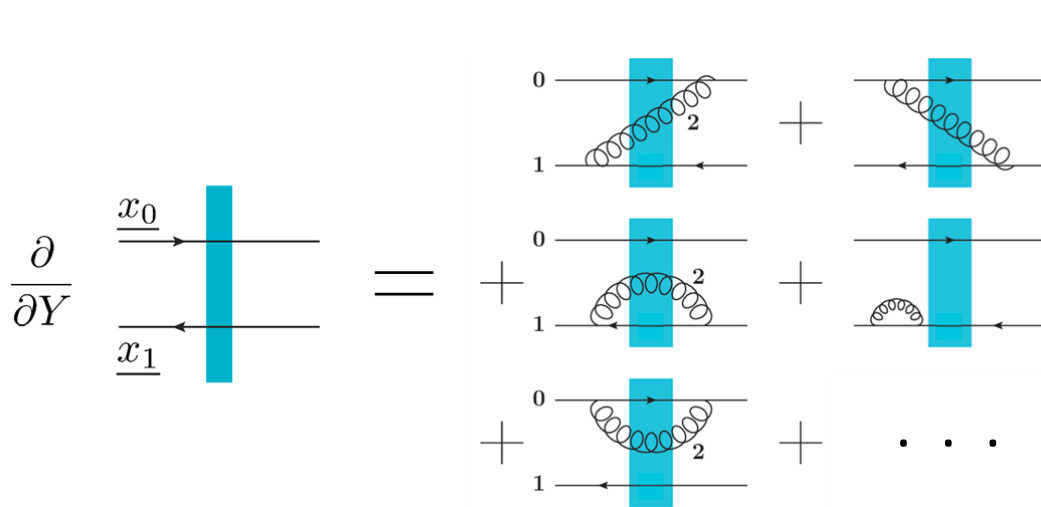


$$V_{\underline{0}} = \mathcal{P} \exp \left[ ig \int_{-\infty}^{+\infty} dx^- A^+(x^+ = 0, x^-, \underline{x}_0) \right]$$

Polarized Wilson Line



$$(V_{\underline{x}}^{pol})^g \propto \int_{-\infty}^{\infty} dx^- V_{\underline{x}}[+\infty, x^-] F^{12}(x^-, \underline{x}) V_{\underline{x}}[x^-, -\infty]$$



Unpolarized evolution:  $\alpha_s \ln \frac{1}{x} \sim 1$

**Polarized evolution:  $\alpha_s \ln^2 \frac{1}{x} \sim 1$**

# KPS-CTT Evolution and Helicity Distribution

KPS-CTT = Kovchegov-Pitonyak-Sievert-Cougoulic-Tarasov-Tawabutr

$$Q_{10}(zs) \equiv \frac{1}{2N_c} \text{Re} \left\langle \left\langle \text{Tr tr} \left[ V_{\underline{0}} V_{\underline{1}}^{\text{pol}[1]\dagger} \right] + \text{Tr tr} \left[ V_{\underline{1}}^{\text{pol}[1]} V_{\underline{0}}^\dagger \right] \right\rangle \right\rangle (zs)$$

Polarized dipole amplitudes are various combinations of polarized and unpolarized Wilson lines [2-4].

$$G_{10}^i(zs) \equiv \frac{1}{2N_c} \left\langle \left\langle \text{tr} \left[ V_{\underline{0}}^\dagger V_{\underline{1}}^{iG[2]} + \left( V_{\underline{1}}^{iG[2]} \right)^\dagger V_{\underline{0}} \right] \right\rangle \right\rangle (zs)$$

...

$$\begin{aligned}
 Q_f(x_{10}^2, zs) &\sim Q_f^{(0)}(x_{10}^2, zs) + \iint [2\tilde{G} + \tilde{\Gamma} + Q_f - \bar{\Gamma}_f + 2\Gamma_2 + 2G_2] + \iint [Q_f + 2G_2] \\
 \bar{\Gamma}_f(x_{10}^2, x_{21}^2, z's) &\sim Q_f^{(0)}(x_{10}^2, z's) + \iint [2\tilde{G} + 2\tilde{\Gamma} + Q_f - \bar{\Gamma}_f + 2\Gamma_2 + 2G_2] + \iint [Q_f + 2G_2] \\
 \tilde{G}(x_{10}^2, zs) &\sim \tilde{G}^{(0)}(x_{10}^2, zs) + \iint [3\tilde{G} + \tilde{\Gamma} + 2G_2 + \Gamma_2 - \bar{\Gamma}_f] - \iint [Q_f + 2G_2] \\
 \tilde{\Gamma}(x_{10}^2, x_{21}^2, z's) &\sim \tilde{\Gamma}^{(0)}(x_{10}^2, z's) + \iint [3\tilde{G} + \tilde{\Gamma} + 2G_2 + \Gamma_2 - \bar{\Gamma}_f] - \iint [Q_f + 2G_2] \\
 G_2(x_{10}^2, zs) &\sim G_2^{(0)}(x_{10}^2, zs) + \iint [\tilde{G} + 2G_2] \\
 \Gamma_2(x_{10}^2, x_{21}^2, z's) &\sim G_2^{(0)}(x_{10}^2, z's) + \iint [\tilde{G} + 2G_2]
 \end{aligned}$$

(Large- $N_c$  &  $N_f$ )

$$\begin{aligned}
 \Delta G &\propto G_2 \\
 \Delta \Sigma &\propto -\iint [Q_f + 2G_2] \\
 \Delta q_f^{NS} &\propto \iint [G_f^{NS}] \\
 A_{\parallel}^{DIS} &\propto A_1 \propto g_1^p \propto -\iint [Q_f + 2G_2]
 \end{aligned}$$

$$Q_f^{(0)}, \tilde{G}^{(0)}, G_2^{(0)}, G_f^{NS(0)} \\
 (x \approx 0.1)$$



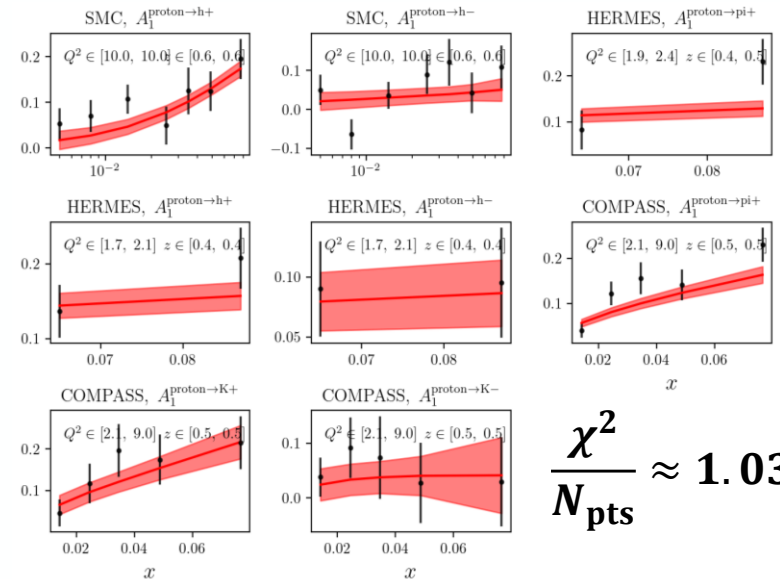
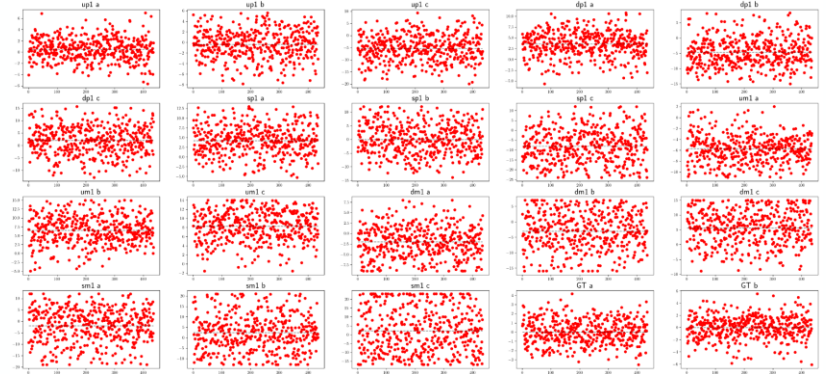
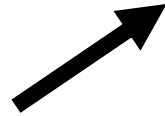
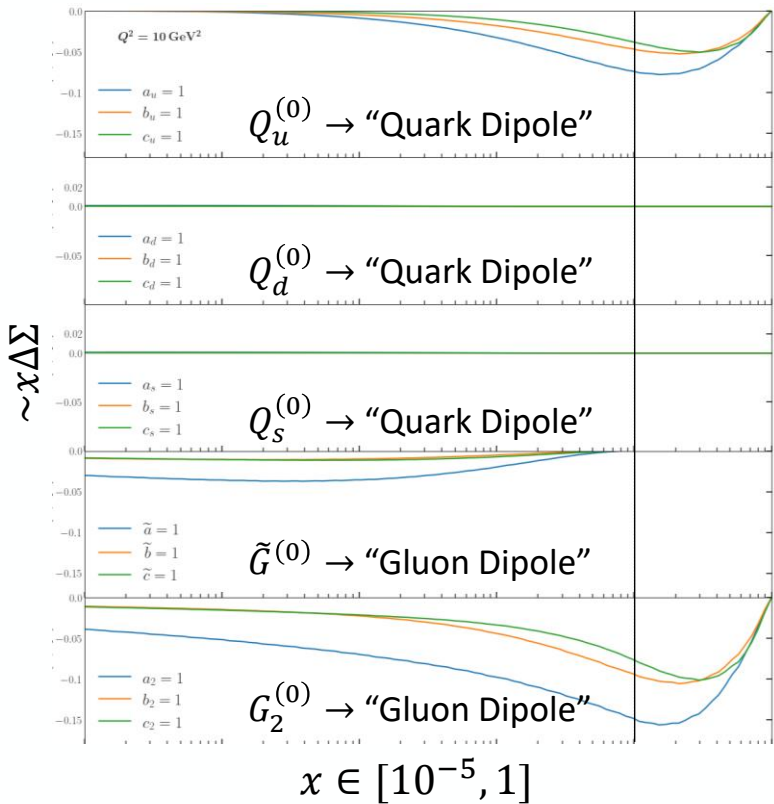
KPS-CTT [5]



$$\Delta G, \Delta \Sigma, \Delta q_f^{NS}, g_1 \\
 (x \ll 0.1)$$

# Global Analysis: Bayesian-MC Analysis

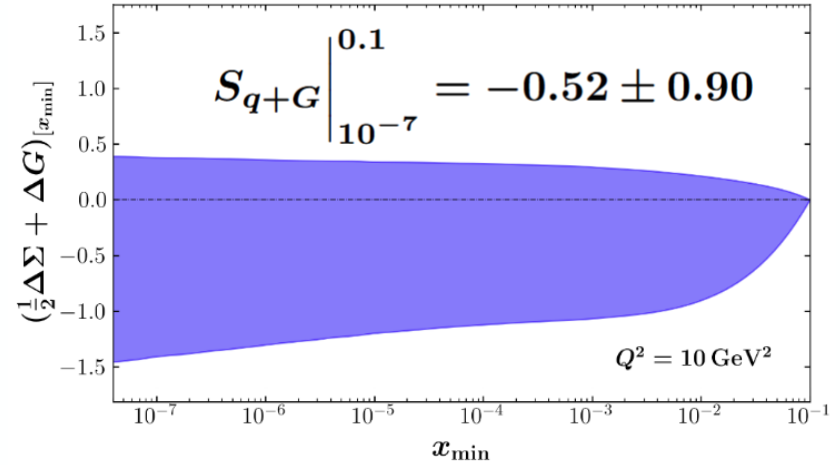
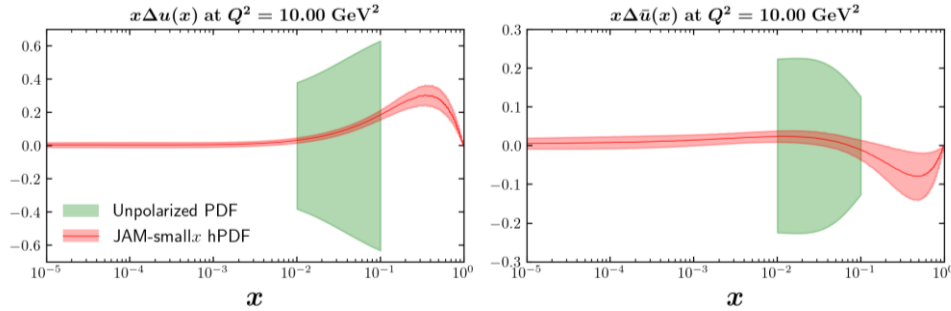
$$G^{(0)} \approx a\eta + b s_{10} + c$$



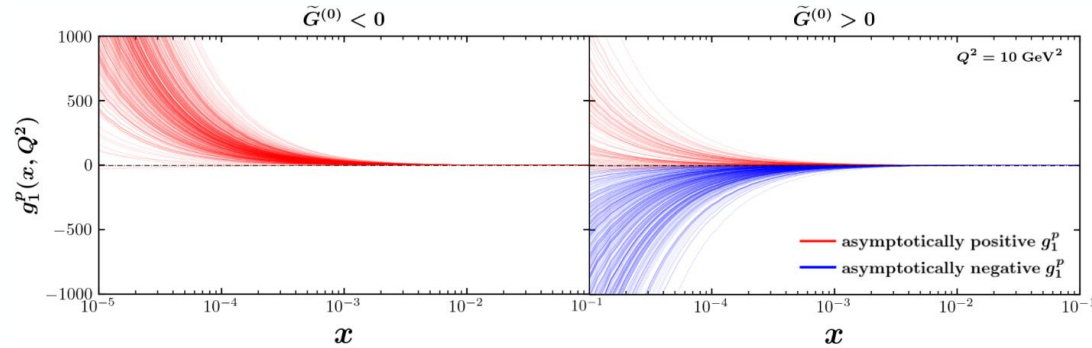
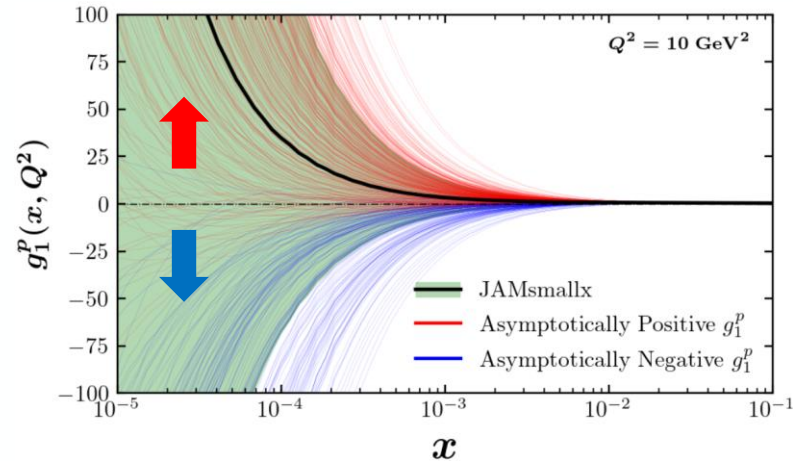
- Compute **Full Evolution** from any parameter.
- Chi-squared optimization for **DIS and SIDIS data**.

# Global Analysis: Success and Setback with SIDIS

SIDIS data improves the first analysis [6] and allows us to separate quark flavor hPDFs,  $\Delta q_f$ , from  $\Delta\Sigma$ .



$$\lim_{x \rightarrow 0} g_1^p \sim \lim_{x \rightarrow 0} \Delta\Sigma \sim \lim_{x \rightarrow 0} \Delta G \sim \left(\frac{1}{x}\right)^{\alpha_h}$$

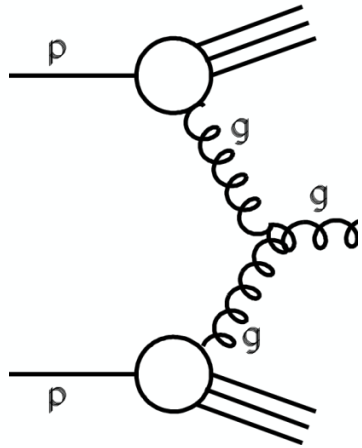


Asymptotic bimodality is caused by the insensitivity of DIS and SIDIS data to **gluon dominated dipole amplitudes** [7].



# Global Analysis with $pp$ Data

Polarized proton-proton collisions allow for leading order gluon-gluon interactions and thus can be more sensitive to the gluon dominated polarized dipole amplitudes.



(Picture from CERN)

The polarized  $pp$  data available is  $A_{LL}^{\text{jet}}$ , with jet cross-section [8]:

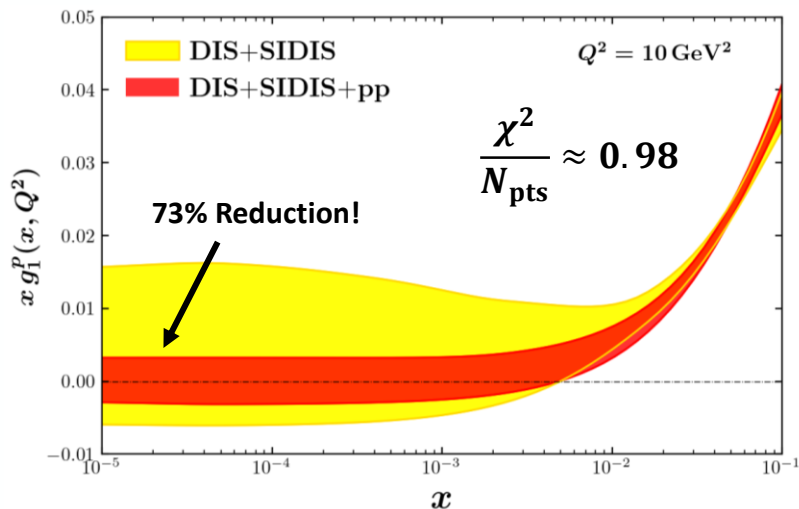
$$\frac{d\Delta\sigma^{pp\rightarrow\text{jet}X}}{d^2p_T^{\text{jet}} d\eta^{\text{jet}}} = \frac{1}{\pi S} \int_{x_g^{P,\min}}^1 \frac{dx_g^P}{x_g^P} \Delta f_g^P(x_g^P, \mu_F) \int_{x_g^{T,\min}}^1 \frac{dx_g^T}{x_g^T} \Delta f_g^T(x_g^T, \mu_F) \\ \times \int_{z_g^{\min}}^1 \frac{dz_g}{z_g} \frac{d\Delta\hat{\sigma}_{gg}^g(\lambda, \hat{s}, \hat{p}_T, \hat{\eta}, \mu_F, \mu'_F, \mu_R)}{vdvdw} \mathcal{J}_g\left(z_g, \frac{R p_T^{\text{jet}}}{\mu'_F}, \mu_R\right)$$

**DLA Accuracy:** Only the leading order term in the jet function  $\mathcal{J}_g$  preserves the resummation parameter  $\alpha_s \ln^2(\frac{1}{x})$ :  $\mathcal{J}_g \equiv \delta(1 - z_g)$

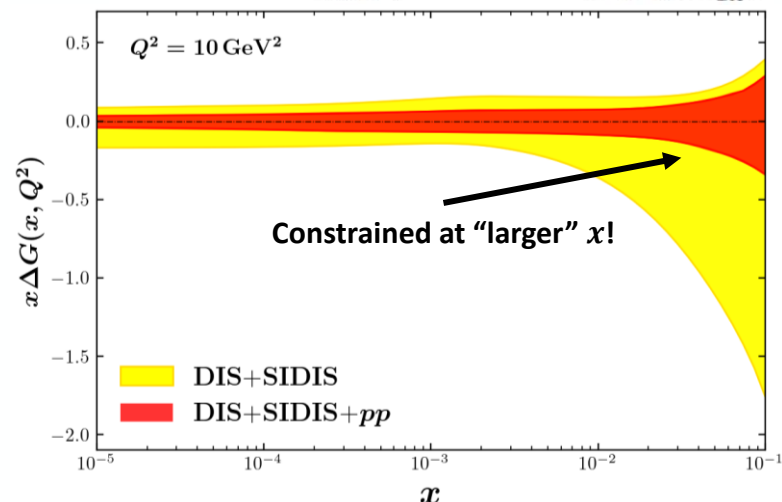
The **small- $x$**  cross-section in the pure-gluon limit:

$$\frac{d\Delta\sigma^{pp\rightarrow\text{jet}X}}{d^2p_T dy} = \frac{8C_F}{\pi^3} \frac{1}{s p_T^2} \int_0^\infty dx_\perp \frac{1}{\alpha_s(1/x_\perp^2)} x_\perp J_0(p_T x_\perp) \left( x_{T,P} \approx \frac{p_T}{\sqrt{s}} e^{\pm y} < 0.1 \right) \\ \times \left[ 2G_{2,P} \nabla_\perp^2 \tilde{G}_T + 2(\nabla_\perp^2 \tilde{G}_P) G_{2,T} + \frac{\partial}{\partial x_\perp} \tilde{G}_P \frac{\partial}{\partial x_\perp} \tilde{G}_T + 2 \frac{\partial}{\partial x_\perp} G_{2,P} \frac{\partial}{\partial x_\perp} \tilde{G}_T + 2 \frac{\partial}{\partial x_\perp} \tilde{G}_P \frac{\partial}{\partial x_\perp} G_{2,T} \right]$$

# Influence of $pp$ Data



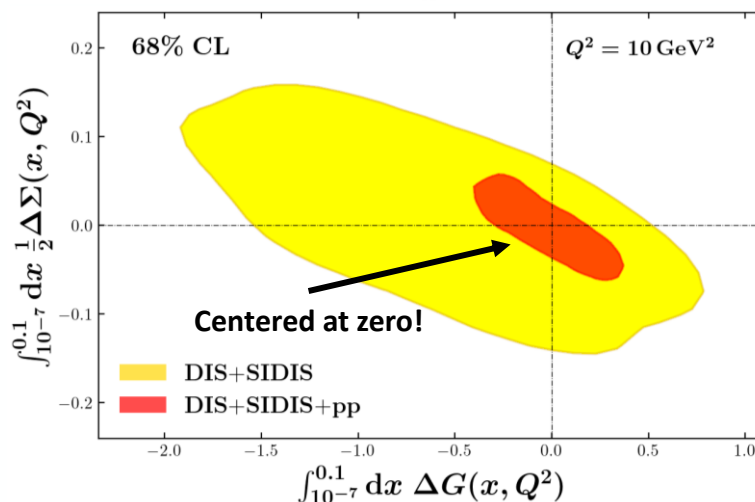
$$\Delta G(x, Q^2) = \frac{2N_c}{\alpha_s(Q^2)\pi^2} G_2(s_{10}, \eta) \Big|_{\eta=\sqrt{\frac{N_c}{2\pi}} \ln \frac{Q^2}{x\Lambda^2}}^{s_{10}=\sqrt{\frac{N_c}{2\pi}} \ln \frac{Q^2}{\Lambda^2}}$$



DIS+SIDIS:  $S_{q+G} \Big|_{10^{-7}}^{0.1} = -0.52 \pm 0.90$

DIS+SIDIS+pp:  $S_{q+G} \Big|_{10^{-7}}^{0.1} = -0.04 \pm 0.23$

Uncertainty spans zero  $\rightarrow$  Bimodality persists

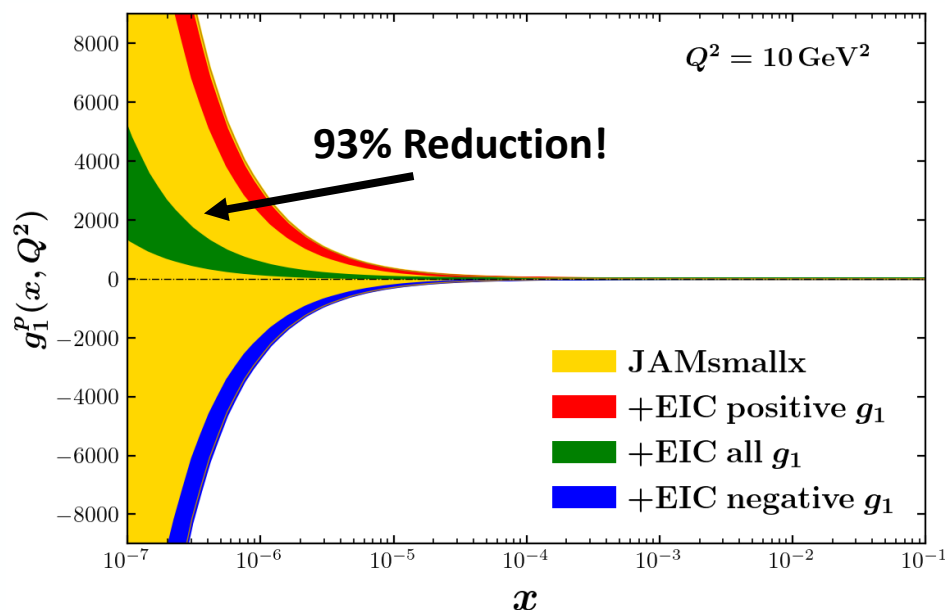




# EIC Impact Study

**EIC Pseudo data:** Create data points with EIC-consistent uncertainties/kinematics [10] using small-x theory input.

- $10^{-4} < x < 10^{-1}$ ,  $1.69 \text{ GeV}^2 < Q^2 < 50 \text{ GeV}^2$
- Proton DIS:  $\sqrt{s} = \{29, 45, 63, 141\} \text{ GeV}$ ,  $L = 100 \text{ fb}^{-1}$
- Deuteron/ $^3\text{He}$  DIS:  $\sqrt{s} = \{29, 66, 89\} \text{ GeV}$ ,  $L = 10 \text{ fb}^{-1}$
- 2% point-to-point systematic uncertainty



# Final Thoughts

- Small- $x$  helicity global analysis is successful and **not degraded** with the inclusion of polarized  $pp$  data.
- $pp$  data drastically reduces uncertainty, especially for larger- $x$  gluon hPDFs.
- Asymptotic bimodality persists → Can be broken with smaller- $x$  data ( $x \approx 10^{-4}$ ).
- Net quark and gluon spin now consistent with zero:
  - How will this be affected by re-introducing quarks?
  - Quark and gluon OAM contributions?

**Interested? Check arXiv soon!**  
**(also I'm available for a postdoc position!)**



# Bibliography

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# Extras

## Cuts on Data

- We fit to all available DIS and SIDIS data within the kinematic ranges  $5 \times 10^{-3} < x < 0.1$  and  $1.69 \text{ GeV}^2 < Q^2 < 10.4 \text{ GeV}^2$
- This leaves a total of 240 data points in the JAM database.

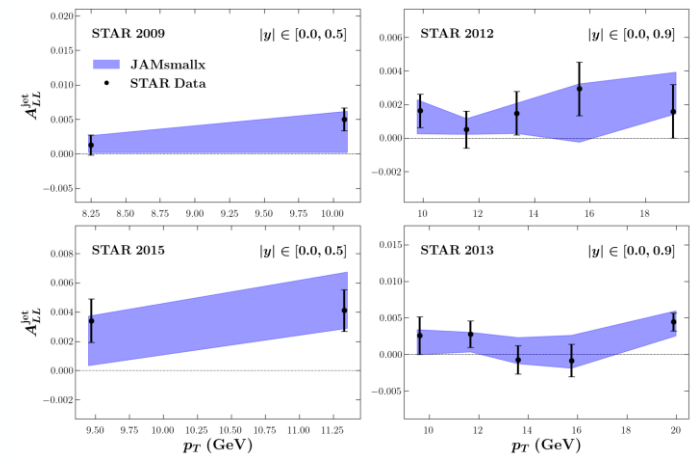


TABLE II. Summary of the polarized SIDIS data on  $A_1^1$  included in the fit, along with the  $\chi^2/N_{\text{pts}}$  for each data set.

Dataset ( $A_1^1$ )	Target	Tagged Hadron	$N_{\text{pts}}$	$\chi^2/N_{\text{pts}}$
COMPASS [132]	$d$	$\pi^+$	5	0.63
	$d$	$\pi^-$	5	0.83
	$d$	$h^+$	5	1.01
	$d$	$h^-$	5	1.02
	$d$	$K^+$	5	1.60
	$d$	$K^-$	5	0.71
COMPASS [133]	$p$	$\pi^+$	5	1.94
	$p$	$\pi^-$	5	1.18
	$p$	$K^+$	5	0.46
	$p$	$K^-$	5	0.23
HERMES [134]	$^3\text{He}$	$h^+$	2	0.55
	$^3\text{He}$	$h^-$	2	0.29
HERMES [135]	$p$	$\pi^+$	2	2.75
	$p$	$\pi^-$	2	0.00
	$p$	$h^+$	2	1.25
	$p$	$h^-$	2	0.19
	$d$	$\pi^+$	2	0.58
	$d$	$\pi^-$	2	1.23
	$d$	$h^+$	2	3.03
	$d$	$h^-$	2	1.24
	$d$	$K^+$	2	0.82
	$d$	$K^-$	2	0.25
	$d$	$K^+ + K^-$	2	0.36
SMC [136]	$p$	$h^+$	7	1.22
	$p$	$h^-$	7	1.41
	$d$	$h^+$	7	0.84
	$d$	$h^-$	7	1.52
<b>Total</b>			<b>104</b>	<b>1.04</b>

TABLE I. Summary of polarized DIS data included in the fit, separated into  $A_1$  (left) and  $A_{\parallel}$  (right), along with the  $\chi^2/N_{\text{pts}}$  for each data set.

Data set ( $A_1$ )	Target	$N_{\text{pts}}$	$\chi^2/N_{\text{pts}}$
COMPASS [120]	$p$	5	0.77
COMPASS [121]	$p$	17	0.93
COMPASS [122]	$d$	5	0.34
EMC [123]	$p$	5	0.23
HERMES [124]	$n$	2	1.11
SLAC (E142) [125]	$^3\text{He}$	1	1.47
SMC [126, 127]	$p$	6	1.26
	$p$	6	0.43
	$d$	6	0.65
	$d$	6	2.13
<b>Total</b>		<b>59</b>	<b>0.90</b>

Data set ( $A_{\parallel}$ )	Target	$N_{\text{pts}}$	$\chi^2/N_{\text{pts}}$
HERMES [128]	$p$	4	1.47
	$d$	4	1.00
SLAC (E143) [129]	$p$	9	0.55
	$d$	9	1.01
SLAC (E154) [130]	$^3\text{He}$	5	0.69
SLAC(E155) [131]	$p$	16	1.07
	$d$	16	1.57
<b>Total</b>		<b>63</b>	<b>1.10</b>

TABLE III. Summary of polarized  $pp$  data on  $A_{\text{LL}}^{\text{jet}}$  included in the fit along with the  $\chi^2/N_{\text{pts}}$  for each data set.

Data set ( $A_{\text{LL}}^{\text{jet}}$ )	$N_{\text{pts}}$	$\chi^2/N_{\text{pts}}$
STAR [137]	2	0.60
STAR [138]	5	0.30
STAR [139]	2	0.55
STAR [140]	5	0.24
<b>Total</b>	<b>14</b>	<b>0.36</b>

# Extras

## EIC impact: DGLAP extrapolation vs KPS-CTT prediction

- EIC pseudodata can break the bimodality, predicting a specific sign as  $x \rightarrow 0$ .
- Uncertainties in the extrapolation region are much smaller using small- $x$  helicity than those using DGLAP evolution.

