Gravitational Form Factors in Holographic QCD

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March 16, 2025 @ GHP 2025, Anaheim, California

References

This talk is based on:

- **2106.00752** (NPB)
- **2204.08857** (PRD)
- **2103.03186** (PRD)
- **1910.04707** (PRD)

in collaboration with **Ismail Zahed**.

AdS/CFT

- The AdS/CFT correspondence can be used to compute correlation functions of local operators:

$$Z_{\rm gauge}(J\mathcal{O}, N_c, \lambda) \equiv Z_{\rm gravity}(\phi_0, g_5, \alpha'/R^2), \quad \text{where } J \equiv \phi_0.$$

- Correlation functions are evaluated via Witten diagrams in AdS.
- For non-conformal theories with a mass gap (dual to a deformed AdS background), scattering amplitudes can likewise be computed using these Witten diagrams in AdS

$$ds^2 = \frac{R^2}{z^2}\left(\eta_{\mu\nu}\, dx^\mu dx^\nu - dz^2\right), \quad \eta_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1),$$

with $0 \leq z \leq \infty$, connects the UV boundary ($z \to 0$) to the IR ($z \to \infty$), and mass gap/confinement induced by a background dilaton field $\phi(z) = \kappa^2 z^2$.

Spin-1 (Electromagnetic) Form Factors of Proton

- The Dirac and Pauli form factors of the proton are defined by

$$\langle p_2 \mid J^{\mu}_{em}(0) \mid p_1 \rangle = \overline{u}(p_2) \left(F_1(q^2)\gamma^{\mu} + F_2(q^2) \frac{i\sigma^{\mu\nu} q_{\nu}}{2m_N} \right) u(p_1).$$

- In holographic QCD, these form factors can be computed by evaluating the relevant Witten diagram in AdS.

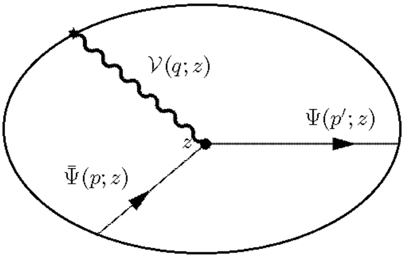


Figure: Witten diagram for the spin-1 (electromagnetic) form factors of the proton, $\langle p_2|J^{\mu}_{em}(0)|p_1\rangle$, via vector meson resonance exchange.

Spin-1 (Electromagnetic) Form Factors of Proton

- The bulk-to-boundary propagator for the virtual photon is

$$\mathcal{V}(Q,z) = g_5 \sum_n \frac{F_n\,\phi_n(z)}{Q^2+m_n^2} = \Gamma\left(1+\frac{Q^2}{4\kappa_V^2}\right)\kappa_V^2 z^2\,\mathcal{U}\left(1+\frac{Q^2}{4\kappa_V^2};2;\kappa_V^2 z^2\right),$$

  where Γ is the Gamma function and $\mathcal{U}$ is the Tricomi confluent hypergeometric function.
- The scattering amplitude in AdS is

$$S^{EM}_{Dirac}[i,f] = (2\pi)^4\,\delta^4(p'-p-q)\,\frac{1}{2g_5^2}\,(2g_5^2)\,\overline{u}_{s_f}(p')\,\epsilon_\mu(q)\gamma^\mu\,u_{s_i}(p)$$

$$\times\frac{1}{2}\int\frac{dz}{z^{2M}}\,e^{-\phi}\,\mathcal{V}(Q,z)\left(\psi_L^2(z)+\psi_R^2(z)\right).$$

Spin-1 (Electromagnetic) Form Factors of Proton

- The electric and magnetic Sachs form factors are given by

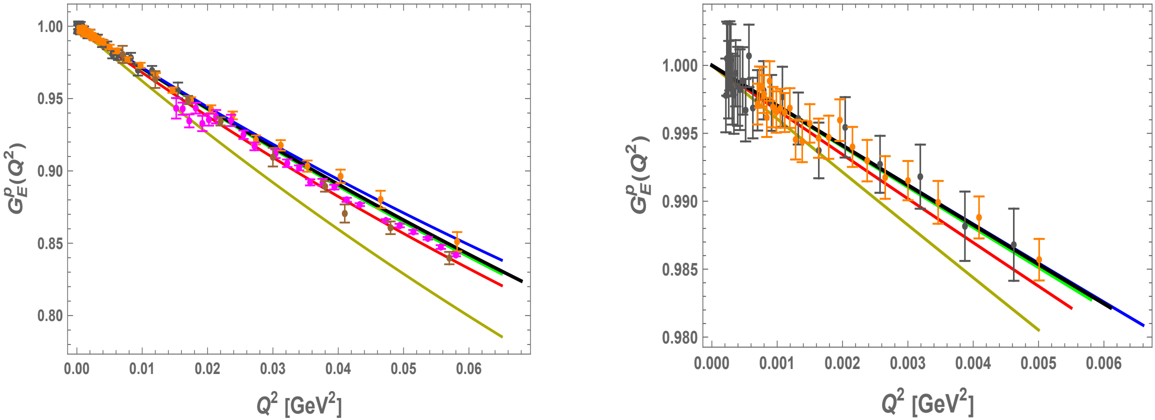
$$G_E^{p,n}(Q) = F_1^{p,n}(Q) - \frac{Q^2}{4m_N^2} F_2^{p,n}(Q), \quad G_M^{p,n}(Q) = F_1^{p,n}(Q) + F_2^{p,n}(Q).$$

- In AdS/QCD, they can be explicitly expressed via Beta functions [Abidin & Carlson (2009), Vega (2010)], for example

$$F_1^P(Q,\kappa,\tau) = \left(\frac{a}{2} + \tau\right) B(\tau, a+1) + \eta_P\,(\tau - 1)\,\frac{a(a(\tau-1)-1)}{(a+\tau)(a+\tau+1)}\, B(\tau-1, a+1)$$

with $a = \frac{Q^2}{4\kappa^2}$ where κ sets the mass scale, and the twist τ is related to the mass dimension of nucleon operator. The explicit expressions involve Beta functions $B(x,y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$.

Comparison to Data



Sample comparisons with world data (brown), Arrington (red), Mainz A1 (magenta), and PRad (orange/gray with blue fit). See [Bernauer:2013], [Xiong:2019]. Our prediction (black).

Charge Radius of the Proton

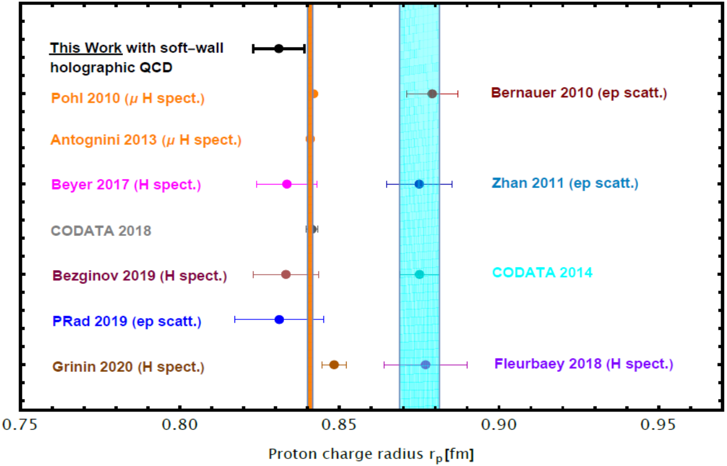


Figure: Various experimental determinations of the proton charge radius.

Spin-2/0 (Gravitational) Form Factors of Proton

- The gravitational form factors (GFFs) of the proton are defined via the energy-momentum tensor (EMT):

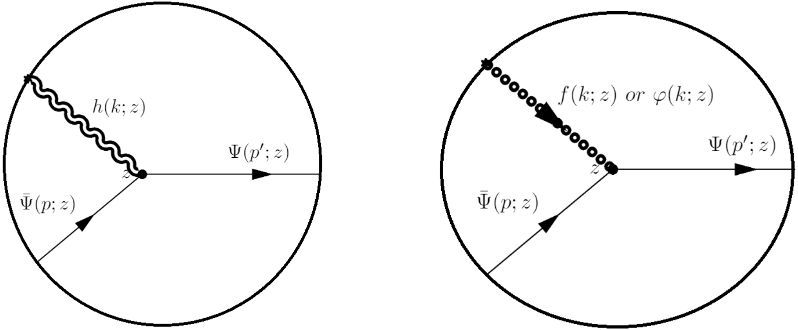
$$\langle p_2 \mid T^{\mu\nu}(0) \mid p_1 \rangle = \overline{u}(p_2)\left(A(k)\gamma^{(\mu}p^{\nu)} + B(k)\,\frac{i\,p^{(\mu}\sigma^{\nu)\alpha}k_\alpha}{2m_N} + C(k)\,\frac{k^\mu k^\nu - \eta^{\mu\nu}k^2}{m_N}\right)u(p_1),$$

  with $k = p_2 - p_1$. Often one writes $D(k) \equiv 4\,C(k)$.
- In holographic QCD, one can compute these via Witten diagrams coupling AdS metric fluctuations $h_{\mu\nu}$ to the bulk Dirac fermion representing the proton.
- The bulk metric fluctuations decompose into spin-2 (transvers-traceless part h) and spin-0 (traceful part f) [Kanitscheider (2008)]:

$$h_{\mu\nu}(k,z) \;\supset\; \left[\epsilon_{\mu\nu}^{TT}\,h(k,z)\right] \;+\; \left[\tfrac{1}{3}\,\eta_{\mu\nu}\,f(k,z)\right].$$

- For non-degenerate 2^{++} and 0^{++} glueball spectra, the holographic coupling includes both transverse-traceless (spin-2) and scalar (spin-0) fluctuations, respectively.

Spin-2/0 (Gravitational) Form Factors of Proton



Witten diagrams for the spin-2 (left) and spin-0 (right) contributions to proton gravitational form factors.

Holographic Gravitational Form Factors

- The holographic spin-2 form factor $A(K, \kappa_T)$ can be written [Abidin & Carlson (2009)]:

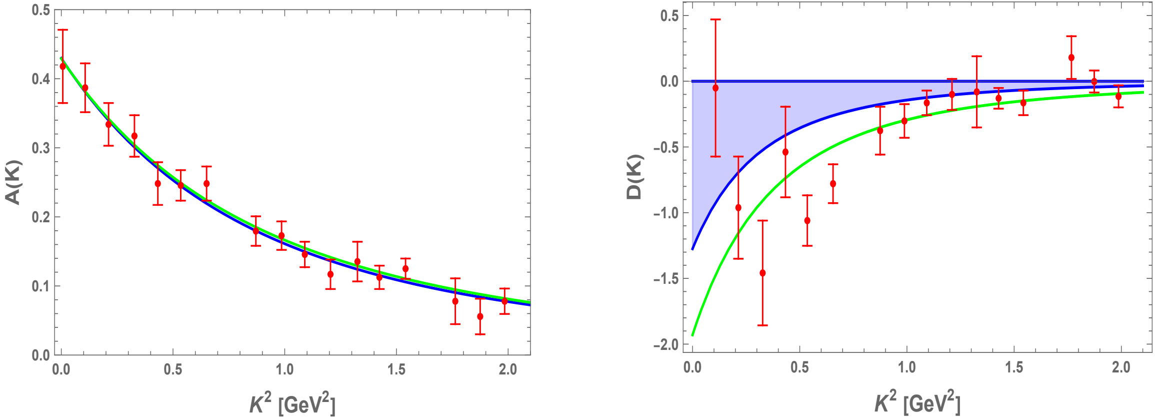
$$A(K, \kappa_T) \ = \ \frac{1}{2} \int dz \, \sqrt{g} \, e^{-\phi} \, z \, \left(\psi_R^2(z) + \psi_L^2(z)\right) \sum_{n=0}^{\infty} \frac{\sqrt{2}\, \kappa \, F_n \, \psi_n(z)}{K^2 + m_n^2}.$$

  It often behaves like a tripole in K^2. Numerically, $A(0) = 0.430$, $m_{TT} \approx 1.612\,\mathrm{GeV}$.
- For the scalar (spin-0) form factor $D(K)$, if the glueball masses are non-degenerate:

$$D(K, \kappa_T, \kappa_S) = -\frac{4\, m_N^2}{3\, K^2} \Big[A(K, \kappa_T) \ - \ A_S(K, \kappa_S) \Big],$$

  giving $D(0) \approx -1.275$ and $m_{SS} \approx 0.963\,\mathrm{GeV}$ for a corresponding tripole fit.

Comparison with Lattice Data



Recent lattice QCD results [Pefkou:2021] (red points) compared to holographic fits (blue curves) with $\kappa_T = 0.388\,\mathrm{GeV}$, $\kappa_S = 0.217\,\mathrm{GeV}$. The green line is a tripole fit to the same lattice data.

Scalar and Mass Radii of Proton

- Define the scalar radius r_{GS} from the trace part of the EMT [Ji:2021]:

$$\langle r_{GS}^2 \rangle = -\frac{6}{A_S(0)} \left.\frac{dA_S(K)}{dK^2}\right|_{K=0}, \quad A_S(K) = A(K) - \frac{K^2}{4m_N^2} B(K) + \frac{3\,K^2}{4m_N^2} D(K).$$

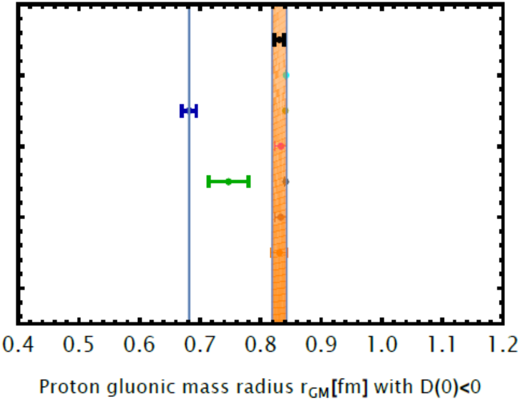
- Similarly, the mass radius r_{GM} of the proton is:

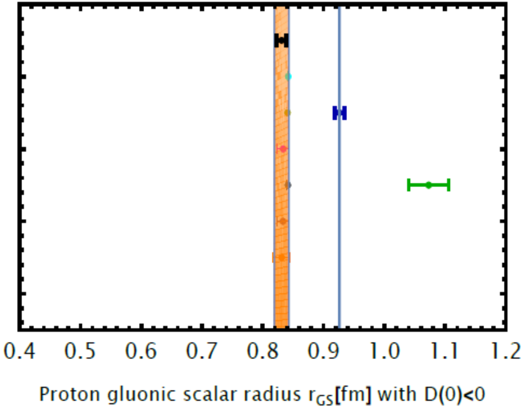
$$\langle r_{GM}^2 \rangle = -\frac{6}{A_M(0)} \left.\frac{dA_M(K)}{dK^2}\right|_{K=0},$$

where

$$A_M(K) = A(K) - \frac{K^2}{4m_N^2} B(K) + \frac{K^2}{4m_N^2} D(K).$$

Numerical Results for Radii





Our holographic predictions for (r_{GM}, r_{GS}) shown in blue. The green point is the lattice fit from [Pefkou:2021]. Also shown the charge radius in orange for comparison.

Photoproduction of Heavy Mesons Near Threshold

- The near-threshold photoproduction of heavy quarkonia (e.g. J/ψ, Υ) can probe the gluonic gravitational form factors.

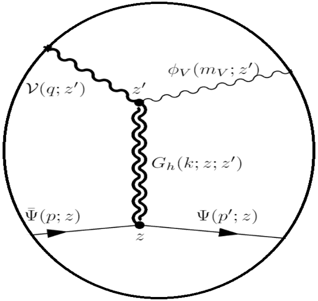


Figure: Witten diagram for diffractive photoproduction of a heavy vector meson with bulk wavefunction ϕ_V.

Photoproduction of heavy mesons near threshold

- the differential cross section for photoproduction of heavy vector mesons (J/ψ or Υ), near threshold, is given by

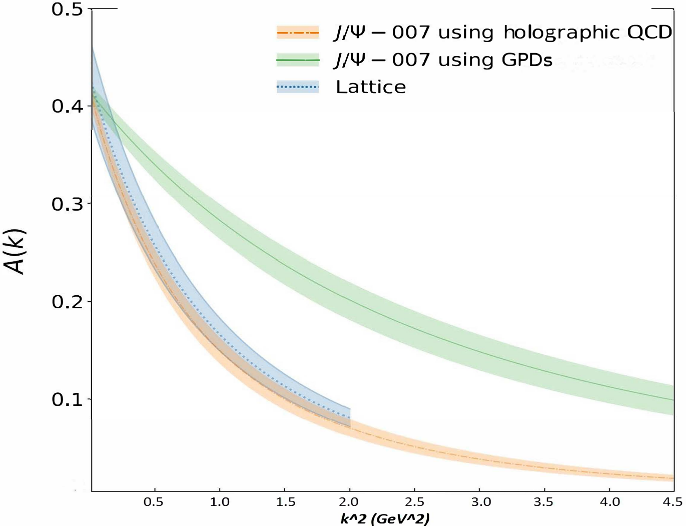
$$\frac{d\sigma}{dt} = \mathcal{N}^2 \times \left[A(t) + \eta^2 D(t)\right]^2 \times \frac{1}{A^2(0)} \times \frac{1}{32\pi(s - m_N^2)^2} \times F(s, t, M_V, m_N) \times \left(1 - \frac{t}{4m_N^2}\right)$$

with the normalization factor $\mathcal{N}$ defined as

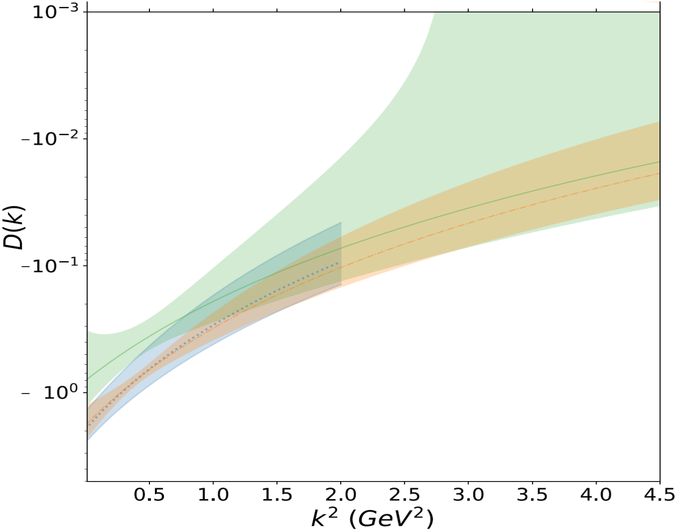
$$\mathcal{N}^2 \equiv e^2 \times \left(\frac{f_V}{M_V}\right)^2 \times \mathbb{V}_{hAA}^2 \times 2\kappa^2 \times A^2(0) = 7.768\,\mathrm{GeV}^{-4}$$

- note that $F(s,t) \sim s^4 \sim 1/\eta^4$ with the amplitude $\mathcal{A} \sim s^2 \times A(t) + s^0 \times D(t)$ as expected from 2^{++} and 0^{++} glueball t-channel exchanges

Extraction of $A(t)$ and $D(t)$



Spin-0 Contribution



Summary

- The J/ψ-007 Collaboration at JLab [Duran et al. (2022)], using our holographic amplitude, has extracted the values of the proton's **mass radius** $\approx 0.755 \pm 0.035\,\mathrm{fm}$ and scalar radius $\approx 1.069 \pm 0.056\,\mathrm{fm}$.
- These are in excellent agreement with lattice QCD results [Pefkou et al. (2021)] of $\approx 0.7464 \pm 0.025\,\mathrm{fm}$ (**mass radius**) and $\approx 1.073 \pm 0.066\,\mathrm{fm}$ (scalar radius).
- The gluonic mass distribution, therefore, is determined to lie predominantly within the proton's charge distribution. Meanwhile, the scalar (trace) distribution appears to extend further, suggesting a long-range confining component.

Thank You!