The QCD static potential

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Introduction

static potential

$\rightarrow$ dominant interaction between heavy q-$\bar{q}$ at low energy

$T = 0$

- attractive
- coulomb-like at small r (linearly rising at large r)

$T \neq 0$

- the short-distance potential screened
- yukawa-like with screening mass $\propto T$

proposed signal for QGP formation:

suppression of heavy q-$\bar{q}$ bound state production at high T

T. Matsui and H. Satz, Phys. Lett. B **178**, *416-422 (1986).*

at LO and in thermal equilibrium:

$V(r)$ from f-transform of 00 thermal gluon propagator in zero frequency limit

$$G_{\rm lo}(0,p) = -\frac{1}{m_D^2+p^2} + \frac{i\pi T m_D^2}{p(p^2+m_D^2)^2}$$

$$\mathrm{Re}\,V_{1\rm lo}(\vec{r}) = -g^2 C_F \int \frac{d^3p}{(2\pi)^3}\,\frac{e^{i\vec{p}\cdot\vec{r}}}{p^2+m_D^2} = -\frac{\alpha C_F}{r}e^{-m_D r}$$

** important idea

if Im[V] $\sim$Re[V] where screening becomes important

$\rightarrow$ bound states disappear because decay *(become wide resonances)*

- not because V is screened *(too shallow to support them)*

M. Laine, O. Philipsen, P. Romatschke and M. Tassler, JHEP **03**, *054 (2007).*

Q: what is the scale where we expect this to happen?

propagator: $G_{\rm lo}(0,p) = -\frac{1}{m_D^2+p^2} + \frac{i\pi T m_D^2}{p(p^2+m_D^2)^2}$

if $p \sim g^a T$ then ${\rm Re}[\tilde{V}_{1LO}(p)] \sim \frac{g^{2-2a}}{T^2}$ and ${\rm Im}[\tilde{V}_{1LO}(p)] \sim \frac{g^{4-5a}}{T^2}$

for a resonance to exist need ${\rm Im}[\tilde{V}] < {\rm Re}[\tilde{V}] \Rightarrow 0 < a < 2/3$

$a = 2/3$ parametrically scale we expect quarkonium to dissociate

real time static potential beyond LO in equilibrium

MEC, C. Manuel and J. Soto, Phys. Rev. Lett. 134, 011905 (2025)

we consider $p \sim g^a T$ with $1/2 < a < 2/3 \Rightarrow m_D \ll p \ll T$

• upper bound on p: from condition $\mathrm{Re}\,\tilde{V}_{\mathrm{lo}}(p) \sim \mathrm{Im}\,\tilde{V}_{\mathrm{lo}}(p)$

$\rightarrow$ bound state decays

• lower bound on p: require p "semi-hard"

- calculation of next-to-leading order potential is simplified

consequences: $V(r)$ valid for $r\, m_D \ll 1 \ll r\, T$

<u>motivation</u>

■ a check of the idea of quarkonium dissociation

■ provides wider set of physically motivated forms of the potential

- *to use as input for methods to extract* V *from lattice correlators*

calculational method

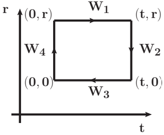
- thermalized plasma
- $M_q \gg$ all other physical scales

$\rightarrow$ static $q\bar{q}$ are (unthermalised) probe particles

- coulomb gauge
- dimensional regularization

potential obtained from real time QCD (rectangular) wilson loop

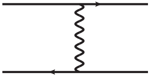
$W(t,r) = \frac{1}{N_c}\left\langle \mathcal{P}\mathrm{exp}\left(ig\int A_\mu(z)z^\mu\right)\right\rangle$



$V(r) = \lim_{t\to\infty}\frac{i}{t}\ln[W(t,r)]$

in limit $t\to\infty$ lines on sides set to 1

$q\bar{q}$ couple to A_0 on 11 branch of CTP contour

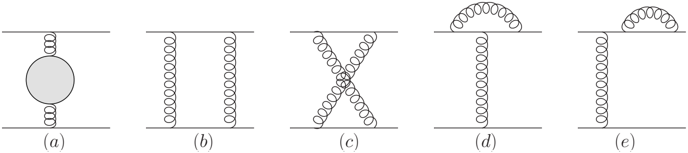


LO is the $\mathcal{O}(g^2)$ term proportional to G_{00}

HTL propagator $\to$ yukawa potential

how to calculate static potential beyond leading order

- expand W to higher order in g
- dress the propagator in the LO contribution



- iterate the LO potential (not shown)

determine how to dress lines/vertices for $p \sim g^a T$ with $\frac{1}{2} < a < \frac{2}{3}$

comment about power counting

calculation of NLO HTL n-point functions

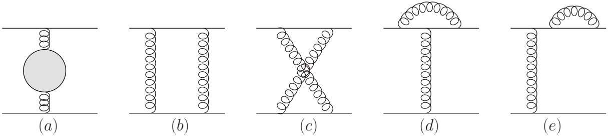
$\rightarrow$ follow prescription ...

for the static potential there are two important differences

1. fermion lines have the form $\frac{1}{p_0 \pm i\eta}$ ($M_q \gg$ all other scales)
2. external frequencies are taken to zero

$\Rightarrow$ external momenta don't flow through the diagram

** power counting is different from standard thermal field theory



in (a) we include corrections to HTL self-energy

- power correction to HTL gluon bubble

C. Manuel, J. Soto and S. Stetina, Phys. Rev. D **94**, 025017, (2016).
S. Carignano, C. Manuel and J. Soto, Phys. Lett. B, 308, **780** (2018).

- one loop gluon bubble with loop momenta semi-hard

- can be done with bare lines and vertices

- bose-einstein distributions $\sim T/p_0$ *since* $p \ll T$

- no quark loop (pauli blocking)

Re$[V]$ *in A. K. Rebhan, Phys. Rev. D* **48**, *R3967 (1993)*
Im$[V]$ *in J. Q. Zhu, Z. L. Ma, C. Y. Shi and Y. D. Li, Nucl. Phys. A* **942**, *54-64 (2015)*

diagrams (bcde) = ladder diagrams

- HTL propagators and bare vertices

also: additional contributions from static quark self-energies

$\rightarrow$ *constant contributions that we have not calculated*

- this will be explained below

• we take into account corrections to LO

real part: larger than g^2 & imag part: larger than g^{2-a}

• denominators $\sim p^2 + m_D^2$ kept unexpanded *(damped approximation)*

$\rightarrow$ extends region that coordinate space potential is valid coordinate space potential beyond leading order

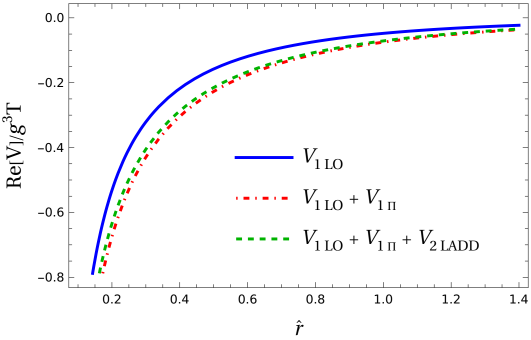
$\hat{r} = rm_D$ and $m_D = gT\hat{m}_D$

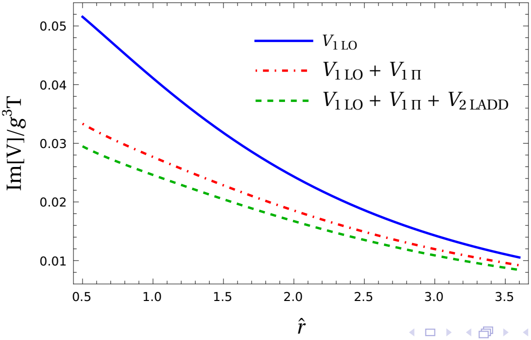
$I_j(\hat{r}) = \int_0^\infty d\hat{p}\, \sin(\hat{p}\hat{r})\,(\hat{p}^2+1)^{-j}$

$$V_{\rm 1lo} = -\frac{g^2 C_F}{4\pi\hat{r}}\left(m_D e^{-\hat{r}} - 2iT\, I_2(\hat{r})\right)$$

$$\mathrm{Re}[V_{\rm nlo}] = \frac{g^4 N_c C_F T}{64\pi^2\hat{r}}\left\{8\left(I_2(\hat{r}) - I_1(\hat{r})\right) + \frac{e^{-\hat{r}}}{16}\left(3\pi^2 - 16 + \frac{\hat{r}}{6}\left(16-\pi^2\right)\right)\right\}$$

$$i\mathrm{Im}[V_{\rm nlo}] = -i\frac{g^3 C_F\, T}{16\pi^2\hat{m}_D}\left\{\frac{3\pi^2-16}{32\,\hat{r}} I_2(\hat{r}) + \frac{7}{3}\,N_c e^{-\hat{r}} - \frac{2g\hat{m}_D}{\pi\hat{r}}\left(N_c - \frac{N_f}{2}\right)\left(I_1(\hat{r}) - I_2(\hat{r})\right)\right\}$$


Re[V]/g³T
r̂
0.0
−0.2
−0.4
−0.6
−0.8
0.2
0.4
0.6
0.8
1.0
1.2
1.4
$V_{1\,\mathrm{LO}}$
$V_{1\,\mathrm{LO}} + V_{1\,\Pi}$
$V_{1\,\mathrm{LO}} + V_{1\,\Pi} + V_{2\,\mathrm{LADD}}$


Im[V]/g³T
r̂
0.05
0.04
0.03
0.02
0.01
0.5
1.0
1.5
2.0
2.5
3.0
3.5
$V_{1\,\mathrm{LO}}$
$V_{1\,\mathrm{LO}} + V_{1\,\Pi}$
$V_{1\,\mathrm{LO}} + V_{1\,\Pi} + V_{2\,\mathrm{LADD}}$

soft contributions

want to include contributions from $p \sim m_D$ *that we haven't calculated*

- since $pr \sim m_D r \sim g^{1-a} < 1 \to$ can expand the exponential

$$V_{\rm soft}(r) = \int \frac{d^3p}{(2\pi)^3} \left(1 + i\vec{p}\cdot\vec{r} - \frac{1}{2}(\vec{p}\cdot\vec{r})^2 + \cdots\right) \tilde{V}(p)$$

keep terms that are $\geq$ smallest contributions in analytic result

odd powers zero by symmetry in an isotropic system

- add contributions:

$\text{Re}[V] = C + g^3 q_0 T$

$\text{Im}[V] = g^3 i_0 T + g^5 i_2 r^2 T^3$

coefficients obtained by fitting to lattice results

C is a global constant that adjusts the origins of the energies

NOTES

1. can verify constants have correct form to absorb infrared poles in the fourier transform of the expanded potential

   idea: $\frac{1}{p^2+m_D^2} \to \frac{1}{p^2} - \frac{m_D^2}{p^4} \dots$

2. recall: heavy q self-energy contributions not calculated
   - they are absorbed into the fitted constants

3. we considered bottomonium $M = 4676$ MeV

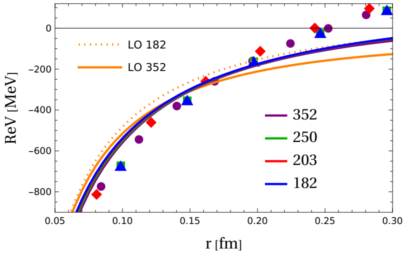
lattice calculation: A. Bazavov, D. Hoying, O. Kaczmarek, R. N. Larsen, S. Mukherjee, P. Petreczky, A. Rothkopf and J. H. Weber, [arXiv:2308.16587 [hep-lat]].

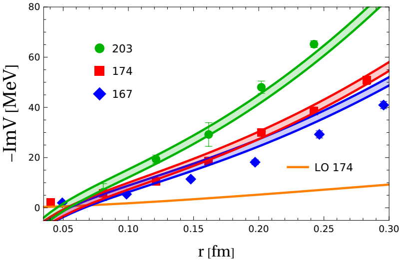
use $g = 1.8$ from fit to $T = 0$ lattice data with $r \in (0.0, 0.3)$ fm

find (C, q_0, i_0, i_2) with fit to all available T and $r \in (0.02, 0.3)$ fm

- real part of potential varies little with T (like data)
- imaginary part gets big contro from soft region

– solid bands are uncertainties in fitted coefficients inherited from lattice data





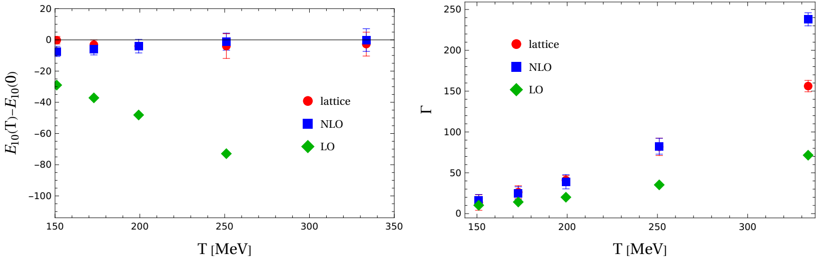
lattice calculation: R. Larsen, S. Meinel, S. Mukherjee and P. Petreczky, Phys. Lett. B **800**, *135119 (2020).*

solve the schrödinger equation using our result for Re$[V]$

$\to$ binding energies and $\Gamma = -\langle \mathrm{Im}[V] \rangle$

find soft coefficients by fitting to all available temperatures

- error bars from fitting to upper/lower values



$\Rightarrow$ reasonable description of data for both E-bind and Γ

fitted soft contribution

recall: contribution to $V(r)$ from $p \sim m_D$

$\mathrm{Re}[V] = C + g^3 q_0 T$

$\mathrm{Im}[V] = g^3 i_0 T + g^5 i_2 r^2 T^3$

find values of coefficients by fitting to 2 sets of lattice data

in our calculation all scales are explicit

$\rightarrow$ expect same size for numerical coefficients from the two fits

$$(q_0, i_0, i_2) = (0.049, \qquad -0.021 \pm 0.002, 0.205 \pm 0.001)$$
$$(q_0, i_0, i_2) = (0.078 \pm -0.004, -0.026 \pm 0.009, 0.053 \pm 0.002)$$

i_2 from the first fit is significantly larger

C=219 MeV from first calculation

in second the coulomb binding energy is subtracted (C plays no role) dissociation:

- bound states disappear because decay (become wide resonances)
- not because V is screened *too shallow to support them*

$T_{\rm diss} \approx$ temperature where ground state $E_{\rm bind} = \Gamma = -2\langle {\rm Im} V \rangle$

- define $E_{\rm bind}$ as eigenvalue of V with threshold set to 0

lo result: $T_{\rm diss} = 193.2$ MeV

nlo result: $T_{\rm diss} = 151.8 \pm 1.2$ MeV $\leftarrow$ using first fit

*** outlying result for i_2*

nlo result: $T_{\rm diss} = 225 \pm 10$ MeV $\leftarrow$ using second fit

conclusions

- calculated beyond-lo corrections to momentum space potential

- when the typical momentum transfer p satisfies $m_D \ll p \ll T$

- relevant region to obtain dissociation T for heavy quarkonium

- we include soft contributions $p \lesssim m_D$

- have universal form because we can expand exponential in f-transform

- coefficients from fitting to lattice data

- reasonable description of lattice data (LO fails)

- *identify an inconsistency between 2 different sets of lattice data*

- results provide useful inputs for the Bayesian methods required in the effort to determine the potential from euclidean lattice data