Transverse Single-Spin Asymmetries in Inclusive DIS and SIDIS Production of Photons: Numerical Predictions using Models for Multi-Parton Correlators

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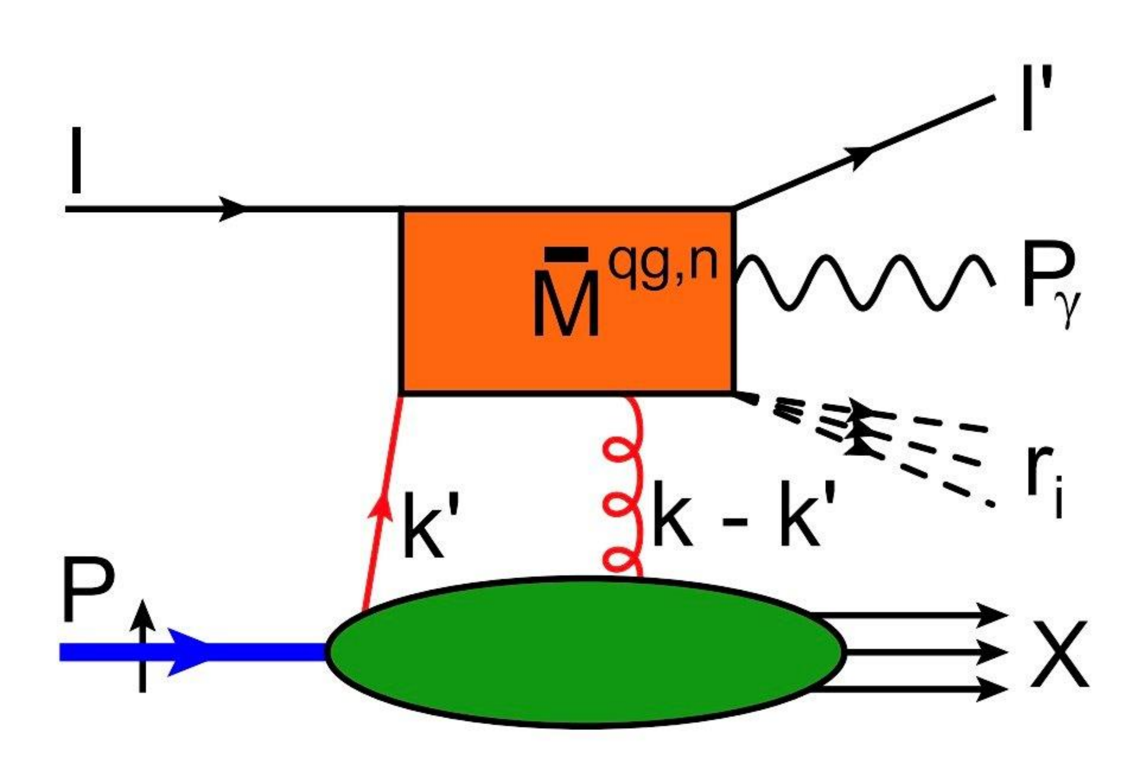
Motivation and Background

- A_{UT} – transverse single-spin asymmetry (TSSA) for the scattering of an unpolarized electron off a transversely polarized nucleon

- We focus on the process of semi-inclusive DIS (SIDIS) with a photon detected in the final state (γSIDIS)

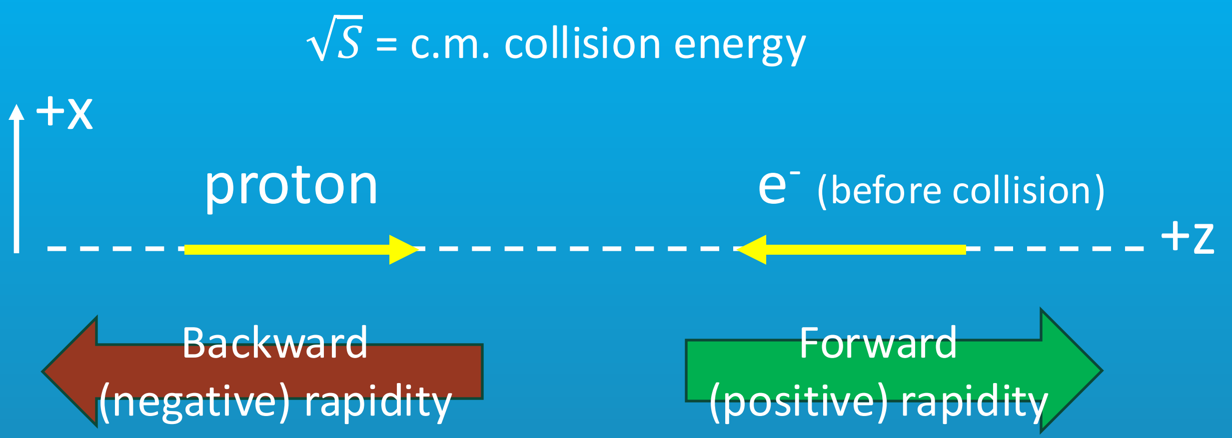
- This process is unique in that it gives us direct sensitivity to quark-gluon-quark correlators point-by-point on the full support of the momentum fractions *x*, *x'*

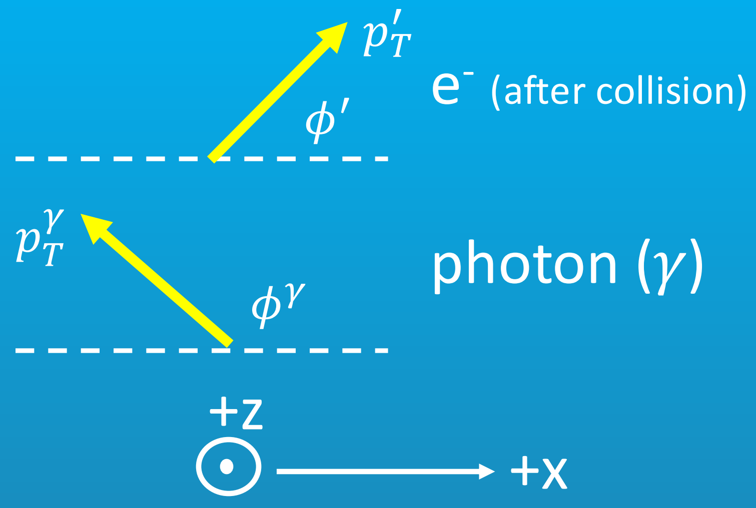
- The future EIC may be able to provide unprecedented information on these correlators, if the γSIDIS TSSA is sizeable



$$A_{UT} \equiv \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}}$$

- The γSIDIS cross section depends on the center-of-mass collision energy ($\sqrt{s}$), transverse momenta of the scattered electron and produced photon (p_T', p_T^γ), the rapidities of the scattered electron and produced photon (η', η^γ), and the azimuthal angles of the transverse momenta of the scattered electron and produced photon (ϕ', ϕ^γ)

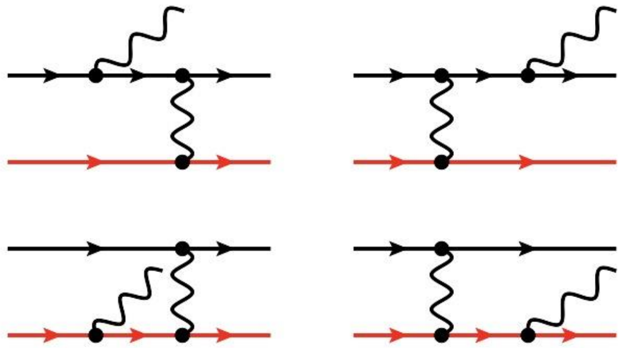




Theoretical Input

γSIDIS cross section formula for $d\sigma_{UU}$

$$\frac{d\sigma_{UU}}{dp'_T d\eta' d\phi' dp^\gamma_T d\eta^\gamma d\phi^\gamma} = \frac{\alpha^3_{em}}{4\pi^2 s Q^4} p'_T p^\gamma_T \sum_{n=\mathrm{BH,C,I}} \hat{\sigma}^n_{UU}\, f^n_1(x_B, \mu_n)$$



BH = Bethe-Heitler – square of diagrams where photon is radiated off the lepton
C = Compton – square of diagrams where photon is radiated off the quark
I = Interference – diagrams where the photon is radiated off the lepton and quark

$$f^{\mathrm{BH}} \equiv \sum_{q=u,d,s} e_q^2 \left(f^q + f^{\bar{q}}\right), \quad f^{\mathrm{C}} \equiv \sum_{q=u,d,s} e_q^4 \left(f^q + f^{\bar{q}}\right), \quad f^{\mathrm{I}} \equiv \sum_{q=u,d,s} e_q^3 \left(f^q - f^{\bar{q}}\right)$$

Relevant kinematic variables: $s = (P + l)^2, \quad Q^2 = -(l - l' - P_\gamma)^2, \quad \tilde{Q}^2 = -(l - l')^2, \quad x_B = \frac{Q^2}{2P \cdot q}, \quad \tilde{x}_B = \frac{\tilde{Q}^2}{2P \cdot q}$

Partonic description is valid if $Q^2 \gg M^2,\ \tilde{Q}^2 \gg M^2,\ Q^2 - \tilde{Q}^2 \gg M^2$ (Brodsky, Gunion, Jaffe (1972))

Scale choices in the PDFs: $\mu_{\mathrm{BH}} = Q,\ \mu_{\mathrm{C}} = \tilde{Q},\ \mu_{\mathrm{I}} = \sqrt{Q\tilde{Q}}$

W. Albaltan, A. Prokudin, M. Schlegel, Phys. Lett. B 804, 135367 (2020)

Theoretical Input

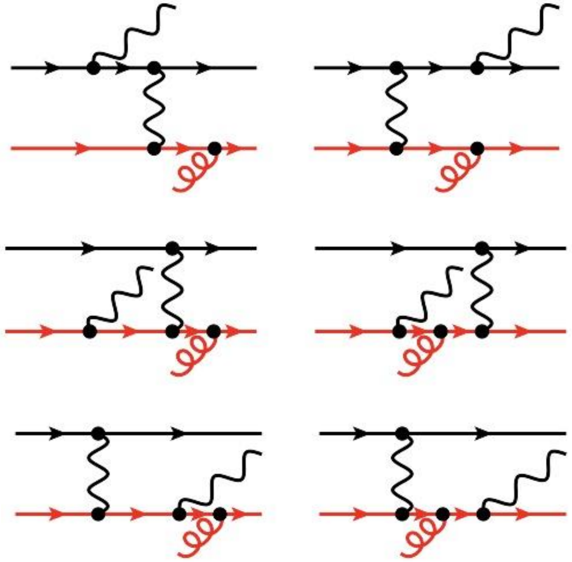
γSIDIS cross section formula for $d\sigma_{UT}$

$$\frac{d\sigma_{UT}}{dp'_T d\eta' d\phi' dp^\gamma_T d\eta^\gamma d\phi^\gamma} = \frac{\alpha^3_{em}}{4\pi^2 s Q^4} p'_T p^\gamma_T \left[\frac{\pi M \epsilon^{Pll'S}}{Q^4} \sigma^{\phi'}_{UT} + \frac{\pi M \epsilon^{PlP_\gamma S}}{Q^4} \sigma^{\phi_\gamma}_{UT} \right]$$

where

$$\sigma^{\phi'}_{UT} = \sum_{n=\mathrm{C,I}} \Big[\hat{\sigma}^{n,\phi'}_{\mathrm{HP},F} F^n_{FT}(x_B, \tilde{x}_B, \mu_n) + \hat{\sigma}^{n,\phi'}_{\mathrm{SFP},F} F^n_{FT}(x_B, 0, \mu_n)$$
$$+ \hat{\sigma}^{n,\phi'}_{\mathrm{HP},G} G^n_{FT}(x_B, \tilde{x}_B, \mu_n) + \hat{\sigma}^{n,\phi'}_{\mathrm{SFP},G} G^n_{FT}(x_B, 0, \mu_n) \Big]$$

$$\sigma^{\phi_\gamma}_{UT} = \sum_{n=\mathrm{C,I}} \Big[\hat{\sigma}^{n,\phi_\gamma}_{\mathrm{HP},F} F^n_{FT}(x_B, \tilde{x}_B, \mu_n) + \hat{\sigma}^{n,\phi_\gamma}_{\mathrm{SFP},F} F^n_{FT}(x_B, 0, \mu_n)$$
$$+ \hat{\sigma}^{n,\phi_\gamma}_{\mathrm{HP},G} G^n_{FT}(x_B, \tilde{x}_B, \mu_n) + \hat{\sigma}^{n,\phi_\gamma}_{\mathrm{SFP},G} G^n_{FT}(x_B, 0, \mu_n) \Big]$$



Note: SGP terms (where $x = x'$) cancel, as does the BH part

$\epsilon^{Pll'S} \propto \sin(\phi_S - \phi')$

$\epsilon^{PlP_\gamma S} \propto \sin(\phi_S - \phi^\gamma)$

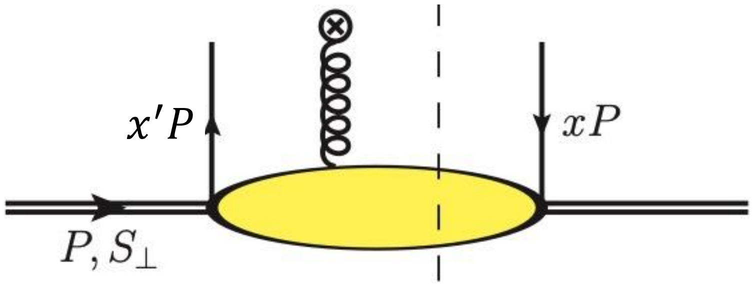
(fix $\phi_S = \pi/2$)

F_{FT} and G_{FT} are quark-gluon-quark correlators that can be probed point-by-point on the full support *x, x'*, which is an unprecedented feature of this observable

W. Albaltan, A. Prokudin, M. Schlegel, Phys. Lett. B 804, 135367 (2020)

Theoretical Input

Properties of quark-gluon-quark correlators F_{FT} and G_{FT}



x and x' are the momentum fractions of the nucleon momentum carried by the quarks

Support for the functions is on

$$|x| \leq 1,\ |x'| \leq 1,\ |x - x'| \leq 1$$

$$F_{FT}(x, x') = F_{FT}(x', x) \qquad F_{FT}^{\bar{q}}(x, x') = F_{FT}^{q}(-x', -x)$$

$$G_{FT}(x, x') = -G_{FT}(x', x) \qquad G_{FT}^{\bar{q}}(x, x') = -G_{FT}^{q}(-x', -x)$$

$$\pi\, F_{FT}^{q}(x, x) = f_{1T}^{\perp(1),q}(x)$$

Note: $f_{1T}^{\perp}$ is the Sivers TMD PDF (use JAM3D-22), which is the probability of finding an unpolarized quark inside a transversely polarized nucleon

Theoretical Input

Models for quark-gluon-quark correlators F_{FT} and G_{FT} - Fourier expansion

$$F^q_{FT}(x,x') = \Bigg\{ \frac{1}{2\pi} f_{1T}^{\perp(1),q+\bar{q}}(r/\sqrt{2}) \left[1 + \sum_{n=1}^{3} [a^q_{2n}(\cos(2n\phi) - 1)] \right]$$

$$+ \frac{1}{2\pi} f_{1T}^{\perp(1),q-\bar{q}}(r/\sqrt{2}) \left[\cos(\phi) + \sum_{n=1}^{3} [a^q_{2n+1}(\cos((2n+1)\phi) - \cos(\phi))] \right] \Bigg\} e(x,x')$$

$$G^q_{FT}(x,x') = \left\{ -\frac{1}{\pi} f_{1T}^{\perp(1),q+\bar{q}}(r/\sqrt{2}) \sum_{n=1}^{6} [b^q_n \sin(n\phi)] \right\} e(x,x')$$

where $$e(x,x') = \left(\frac{2}{1+e^{-50(1-x^2)^3}} - 1 \right) \left(\frac{2}{1+e^{-50(1-(x')^2)^3}} - 1 \right)$$

$$\times \left(\frac{2}{1+e^{-50(1-(x-x')^2)^3}} - 1 \right) \theta(1-|x|)\theta(1-|x'|)\theta(1-|x-x'|)$$

$$r = \sqrt{x^2 + x'^2}$$

$$\phi = \begin{cases} -\frac{\pi}{4} + \arctan(x'/x) \text{ if } x' \geq |x| \geq 0 \\ \frac{3\pi}{4} + \arctan(x'/x) \text{ if } -x \geq |x'| \geq 0 \\ \frac{3\pi}{4} + \arctan(x'/x) \text{ if } -x' \geq |x| \geq 0 \\ \frac{7\pi}{4} + \arctan(x'/x) \text{ if } x \geq |x'| \geq 0 \end{cases}$$

NB: These expressions are consistent with the aforementioned properties of F_{FT} and G_{FT}

Assumptions:

- all Fourier coefficients a_n and b_n are constants (independent of r)
- all Fourier coefficients a_n for $n \geq 8$ and b_n for $n \geq 7$ vanish
- $f_{1T}^{\perp}$ sets the size of F_{FT} and G_{FT}

Theoretical Input

Models for quark-gluon-quark correlators F_{FT} and G_{FT} - Lattice QCD constraint

$$\boldsymbol{a^q} = (a_2^q, a_4^q, a_6^q; a_3^q, a_5^q, a_7^q) \qquad \boldsymbol{b^q} = (b_1^q, b_2^q, b_3^q, b_4^q, b_5^q, b_6^q)$$

There is no experimental data to constrain these parameters, but lattice QCD has calculated the d_2 matrix element for u and d quarks:

$$d_2^q = -\int_{-1}^{1} dx \int_{-1}^{1} dx' \, F_{FT}^q(x, x')$$

$$d_2^u = 0.026(4)(13)$$
$$d_2^d = -0.0086(26)(146)$$

Lattice QCD results at μ = 2 GeV from RQCD Collaboration (2022)

These values can be used to constrain a_2 once values are chosen for $a_{4,6,3,5,7}$:

$$a_2^q = (d_2^q - (A_0^q + A_4^q a_4^q + A_6^q a_6^q + A_3^q a_3^q + A_5^q a_5^q + A_7^q a_7^q))/A_2^q$$

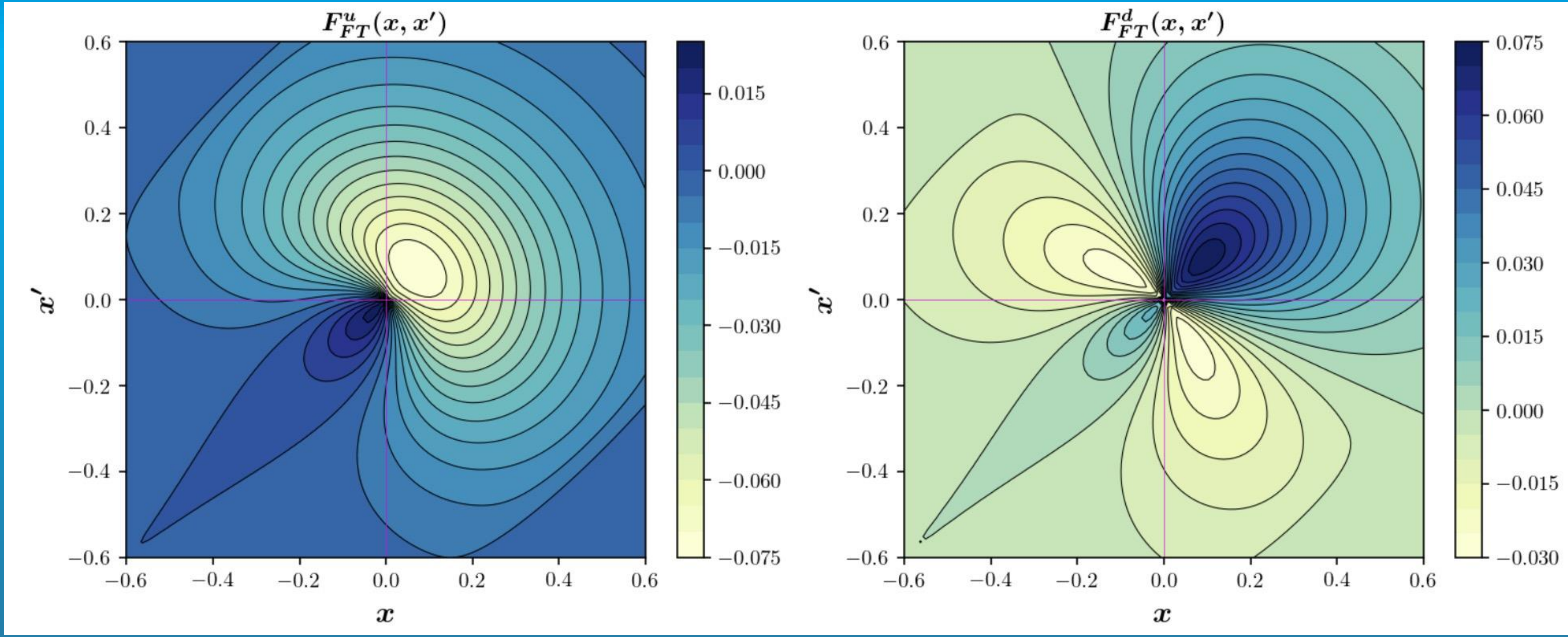
where $A_0^q \equiv -\int_{-1}^{1} dx \int_{-1}^{1} dx' \, F_{FT}^q(x, x')\Big|_{\boldsymbol{a^q}=0}, \qquad A_i^q \equiv -A_0^q - \int_{-1}^{1} dx \int_{-1}^{1} dx' \, F_{FT}^q(x, x')\Big|_{a_i^q=1, a_{j\neq i}^q=0}$

Note that we also fix $b_1 = -a_2$.

Theoretical Input

Models for quark-gluon-quark correlators F_{FT} and G_{FT} - "Scenario 0"

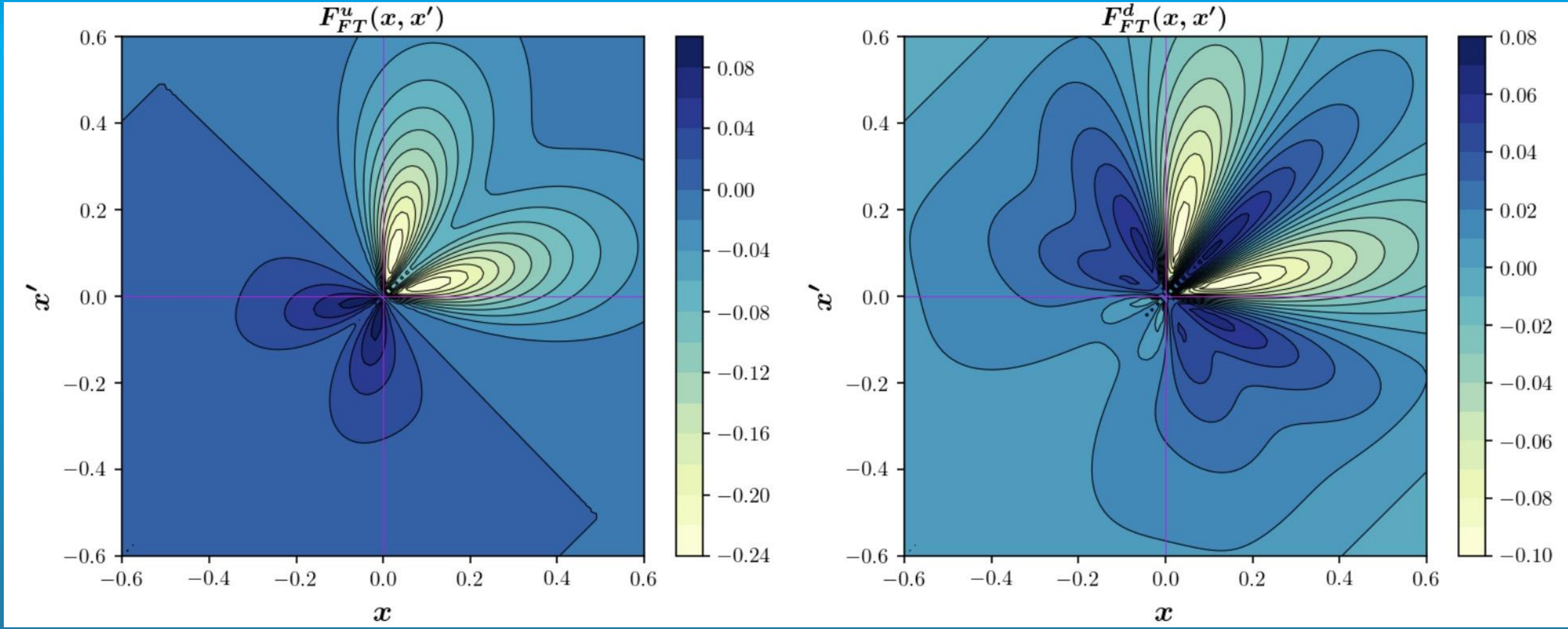
$$\boldsymbol{a}^{\boldsymbol{u}} = (-0.2691, 0, 0; 0, 0, 0)\,, \quad \boldsymbol{a}^{\boldsymbol{d}} = (0.7822, 0, 0; 0, 0, 0)\,, \quad \boldsymbol{b}^{\boldsymbol{u}} = (0, 0, 0, 0, 0, 0)\,, \quad \boldsymbol{b}^{\boldsymbol{d}} = (0, 0, 0, 0, 0, 0)$$



Theoretical Input

Models for quark-gluon-quark correlators F_{FT} and G_{FT} - "Scenario 1"

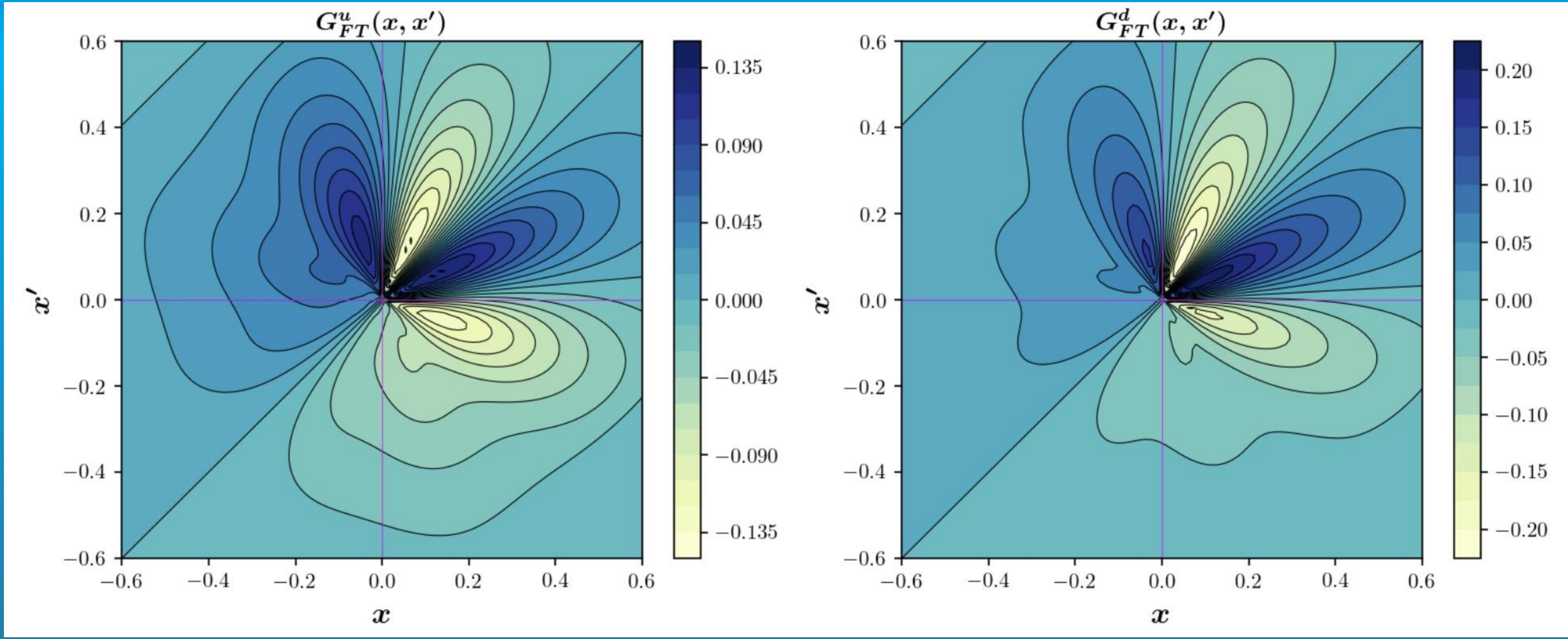
$$\boldsymbol{a}^{\boldsymbol{u}} = \left(1.1585, -\tfrac{2}{3}, -\tfrac{2}{3}; -\tfrac{1}{3}, -1, -\tfrac{1}{3}\right), \quad \boldsymbol{a}^{\boldsymbol{d}} = \left(-0.6658, \tfrac{2}{3}, \tfrac{2}{3}; \tfrac{1}{3}, 1, \tfrac{1}{3}\right)$$



Theoretical Input

Models for quark-gluon-quark correlators F_{FT} and G_{FT} - "Scenario 1"

$$\boldsymbol{b^u} = \left(-1.1585, \tfrac{1}{3}, \tfrac{2}{3}, 1, \tfrac{2}{3}, \tfrac{1}{3}\right), \quad \boldsymbol{b^d} = \left(0.6658, -\tfrac{1}{3}, -\tfrac{2}{3}, -1, -\tfrac{2}{3}, -\tfrac{1}{3}\right)$$



Results

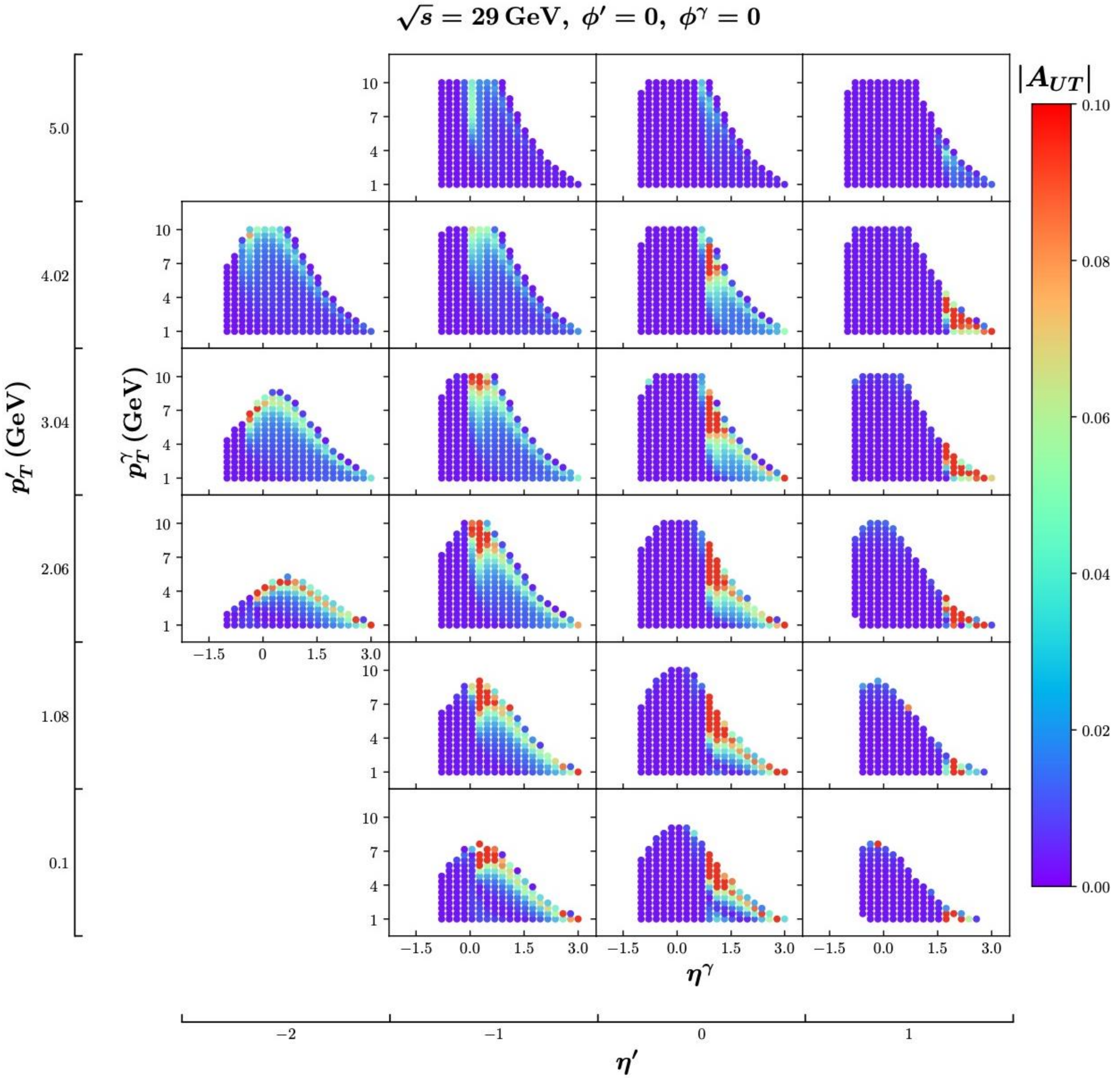
A_{UT} vs. $(p_T', \eta', p_T^\gamma, \eta^\gamma)$

with $\phi' = \phi^\gamma = 0$

for Electron-Ion Collider

$(\sqrt{S} = 29\ \mathrm{GeV})$

F_{FT}/G_{FT} Model Scenario 0



Results

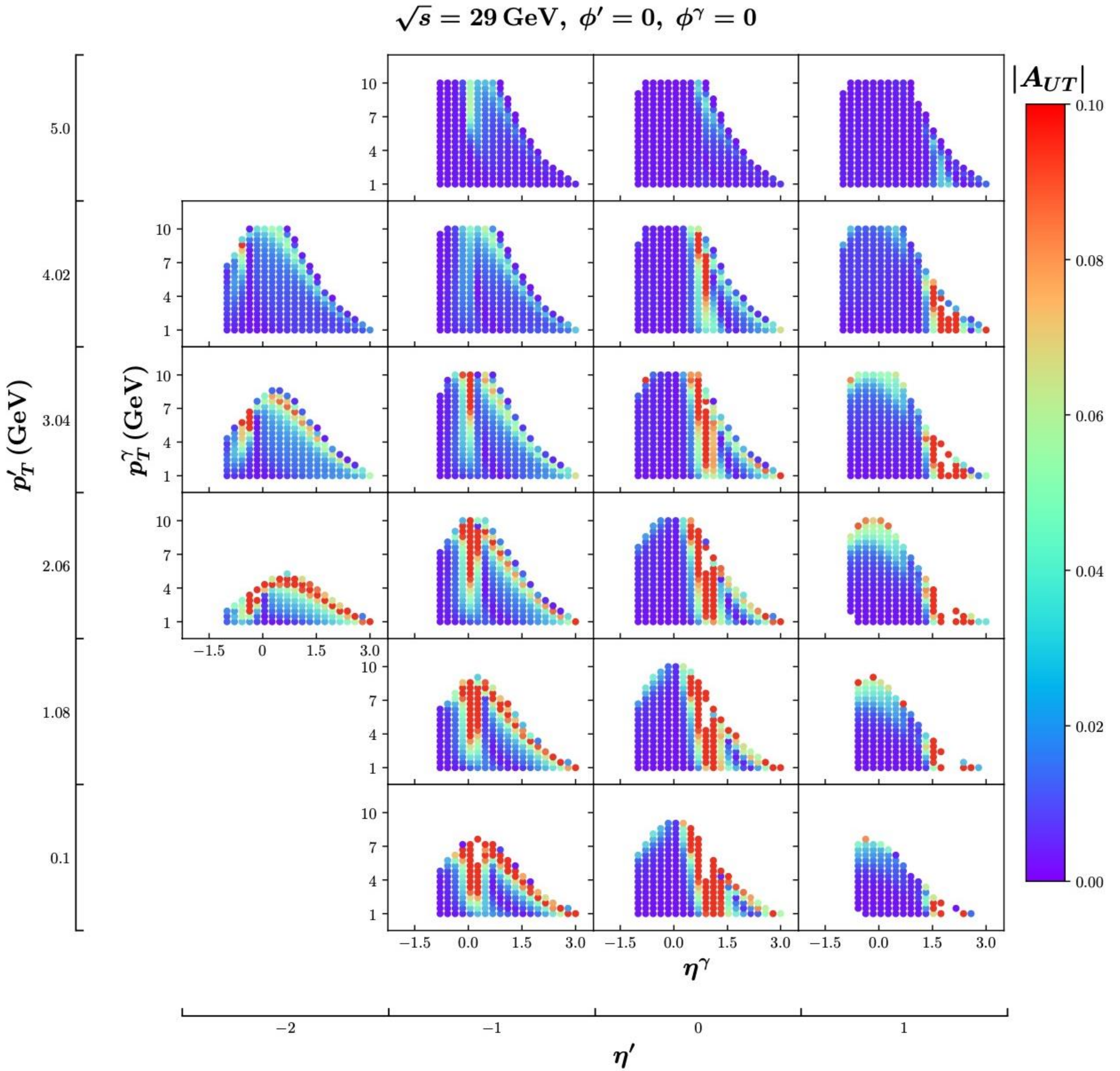
A_{UT} vs. $(p'_T, \eta', p^\gamma_T, \eta^\gamma)$

with $\phi' = \phi^\gamma = 0$

for Electron-Ion Collider

$(\sqrt{S} = 29\ \mathrm{GeV})$

F_{FT}/G_{FT} Model Scenario 1



Results

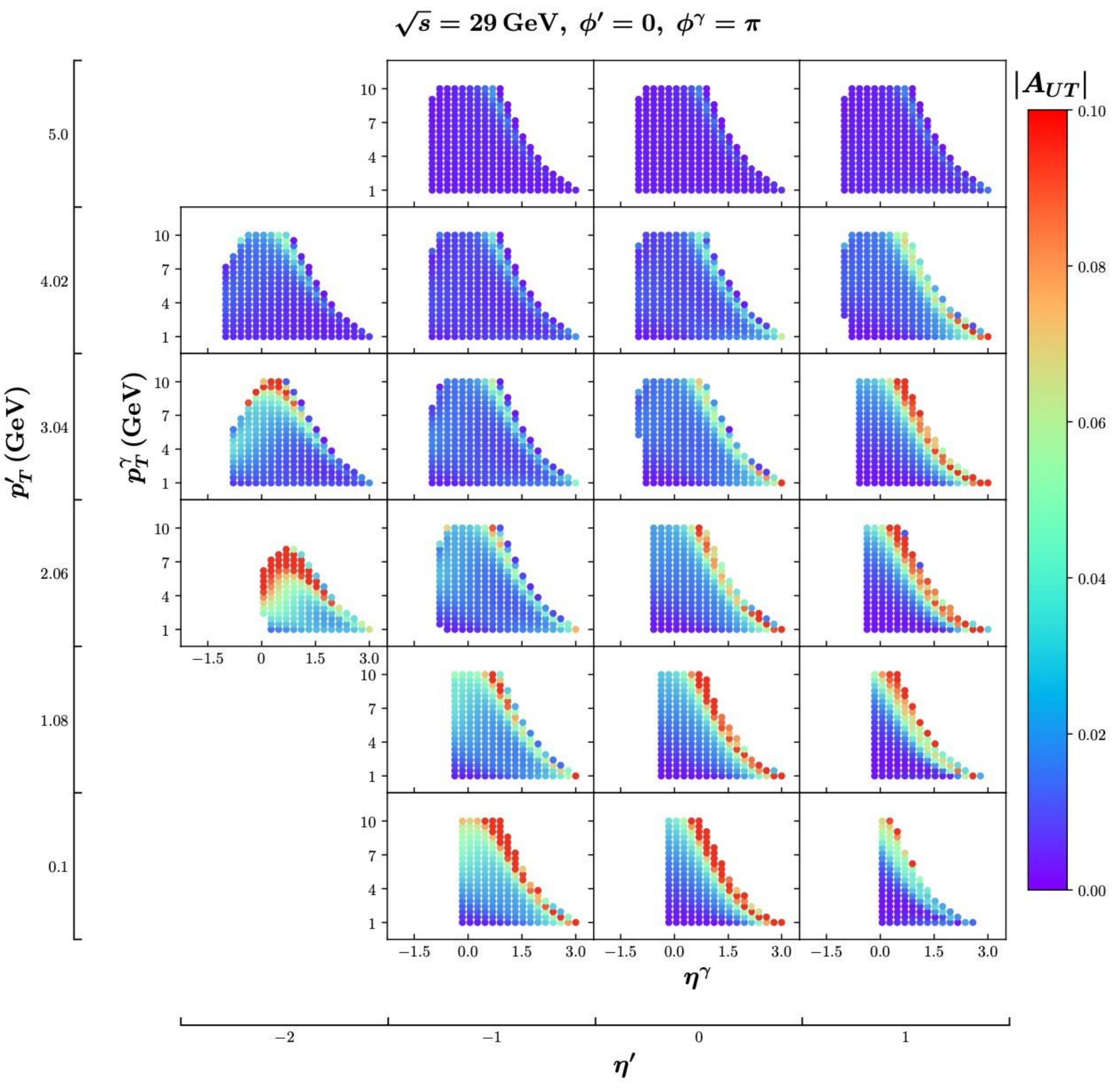
A_{UT} vs. $(p_T', \eta', p_T^\gamma, \eta^\gamma)$

with $\phi' = 0, \phi^\gamma = \pi$

for Electron-Ion Collider

$(\sqrt{S} = 29 \text{ GeV})$

F_{FT}/G_{FT} Model Scenario 1



Results

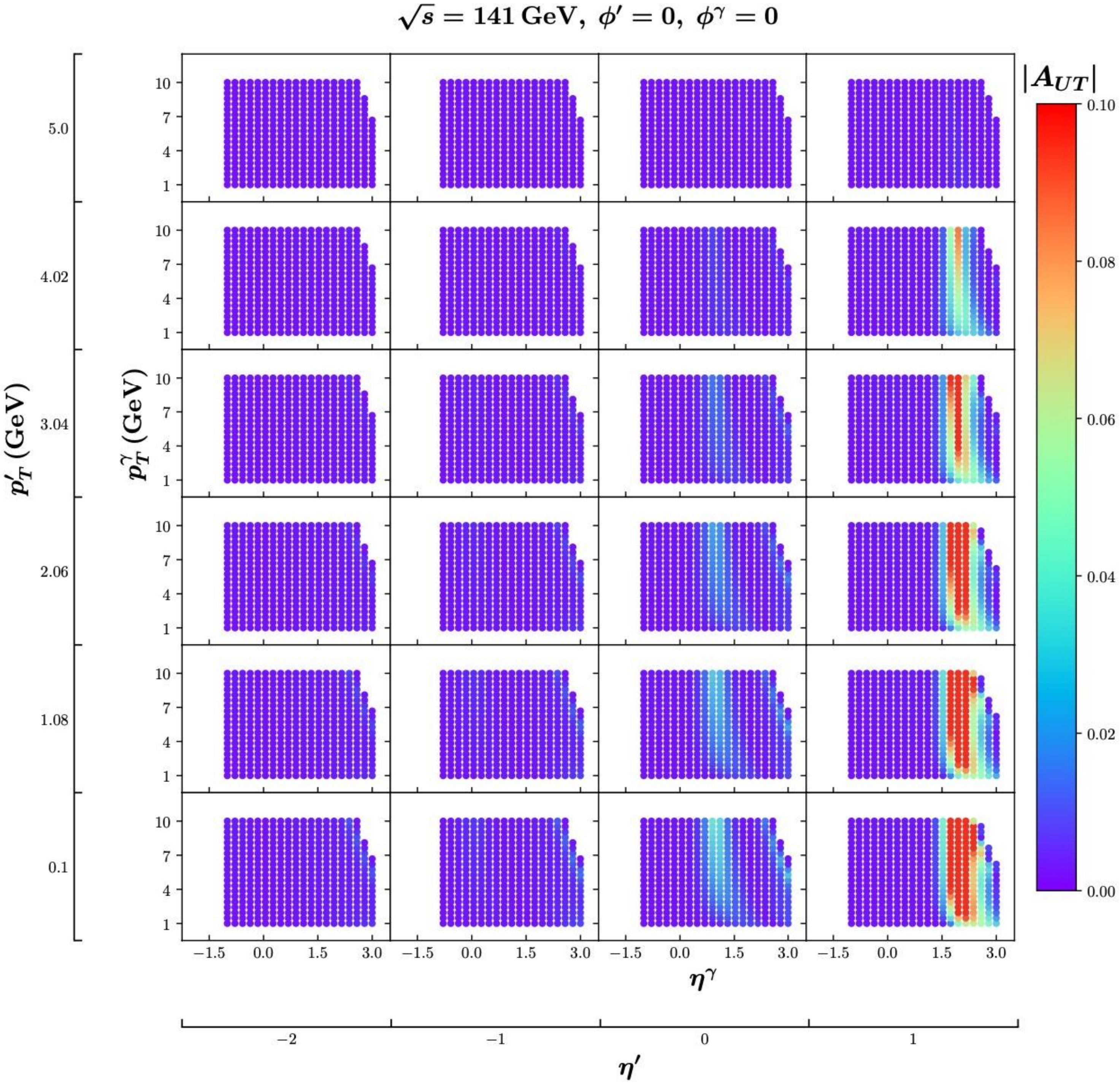
A_{UT} vs. $(p'_T, \eta', p^\gamma_T, \eta^\gamma)$

with $\phi' = \phi^\gamma = 0$

for Electron-Ion Collider

$(\sqrt{S} = 141$ GeV$)$

F_{FT}/G_{FT} Model Scenario 1

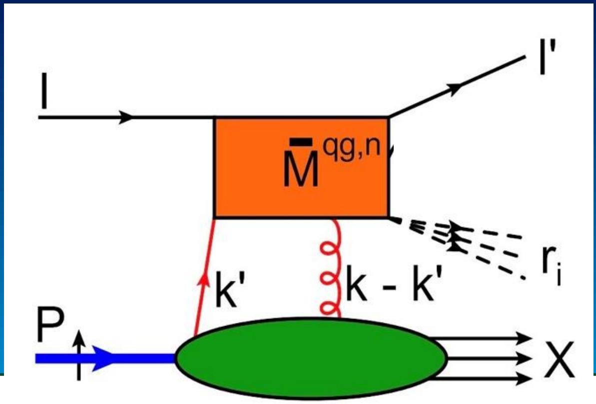


- At the low-energy EIC configuration ($\sqrt{S} = 29$ GeV), the asymmetry has the largest values (~10% or more) when the following kinematic criteria are satisfied:
  - ✓ outgoing electron is at mid or backward rapidity
  - ✓ photon is at mid to forward rapidity
  - ✓ smaller electron transverse momentum
  - ✓ higher photon transverse momentum
  - ✓ electron and photon are produced at azimuthal angles of 0 or π

- At the high-energy EIC configuration ($\sqrt{S} = 141$ GeV), the asymmetry has the largest values (~10% or more) when the following kinematic criteria are satisfied:
  - ✓ outgoing electron and photon are at forward rapidity
  - ✓ small electron transverse momentum
  - ✓ higher photon transverse momentum
  - ✓ electron and photon are produced at azimuthal angles of 0 or π

A_{UT} Inclusive DIS

DIS cross section formula for $d\sigma_{UT}$

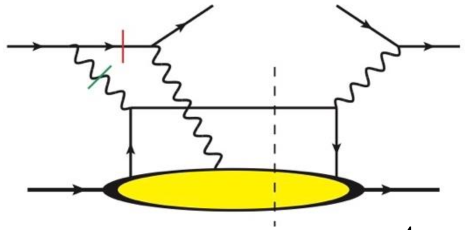


$$d\sigma_{UT}^{DIS} = -|S_T|\sin(\phi_S)\frac{4\alpha_{EM}^3}{yQ^4}\frac{M}{Q}\frac{x_B y}{\sqrt{1-y}}$$
$$\times \sum_q \left[e_q^3 \int_0^1 dx \left(\hat{C}_+ F_{FT}^q(x_B, x) + \hat{C}_- G_{FT}^q(x_B, x) \right) \right.$$
$$\left. + (1-y)\frac{e_q^3 m_q}{M} h_1^q(x_B) + \frac{2-y}{2y} e_q^2 (1 - x_B \frac{d}{dx_B}) F_{FT}^{\gamma,q}(x_B, x_B) \right]$$

qgq terms

qγq term

quark-mass term



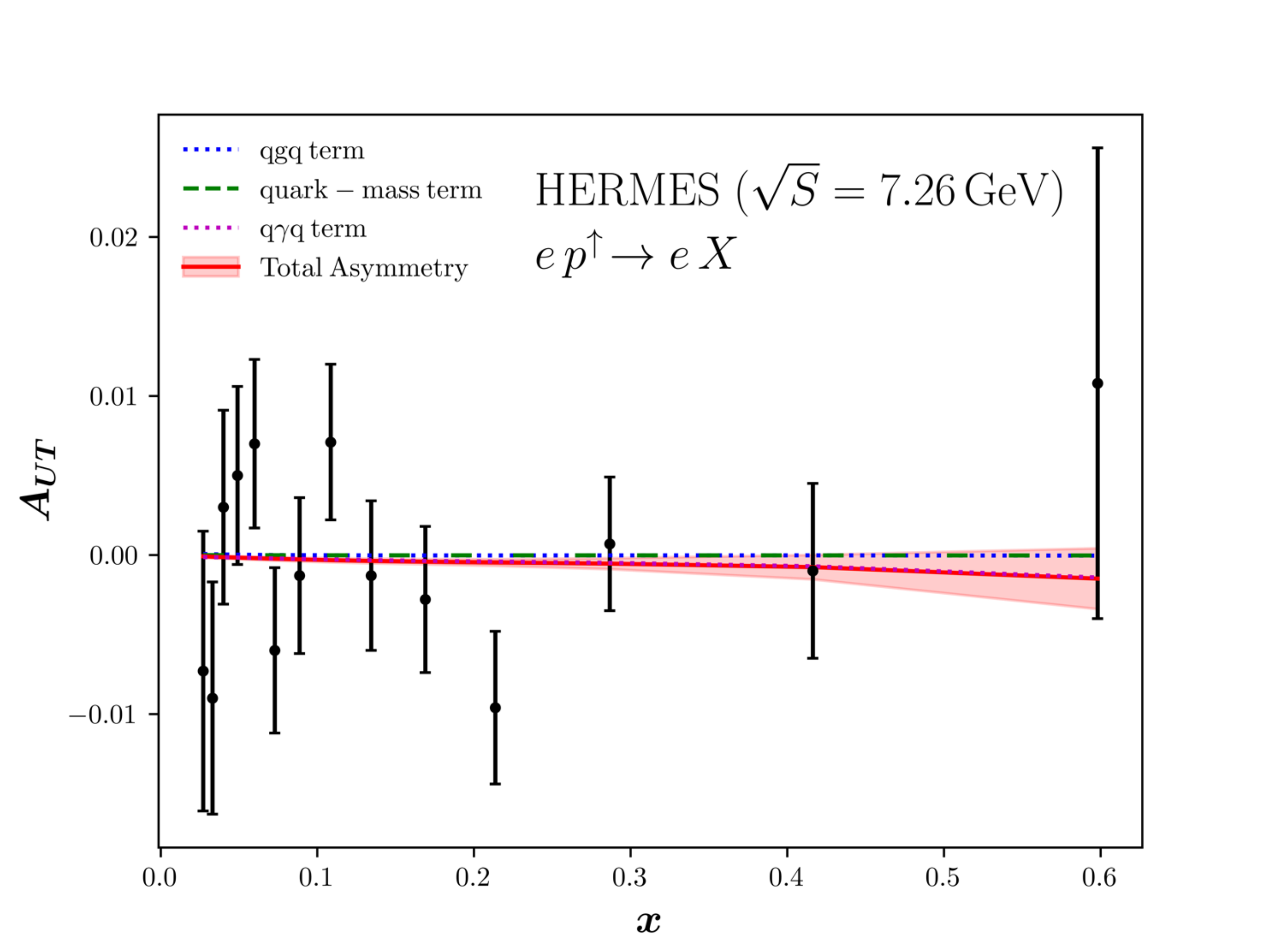
$$F_{FT}^{\gamma,u/p}(x,x) = \frac{\alpha_{em}}{3\pi C_F \alpha_s} f_{1T}^{\perp(1),u/p}(x)\,, \quad F_{FT}^{\gamma,d/p}(x,x) = \frac{4\alpha_{em}}{3\pi C_F \alpha_s} f_{1T}^{\perp(1),d/p}(x)$$
$$F_{FT}^{\gamma,u/n}(x,x) = -\frac{2\alpha_{em}}{3\pi C_F \alpha_s} f_{1T}^{\perp(1),u/n}(x)\,, \quad F_{FT}^{\gamma,d/n}(x,x) = \frac{\alpha_{em}}{3\pi C_F \alpha_s} f_{1T}^{\perp(1),d/n}(x)$$

A. Metz, et al., Phys. Rev. D 86, 094039 (2012)
M. Schlegel, Phys. Rev. D 87, 034006 (2013)

Note: h_1 is the transversity PDF (use JAM3D-22)

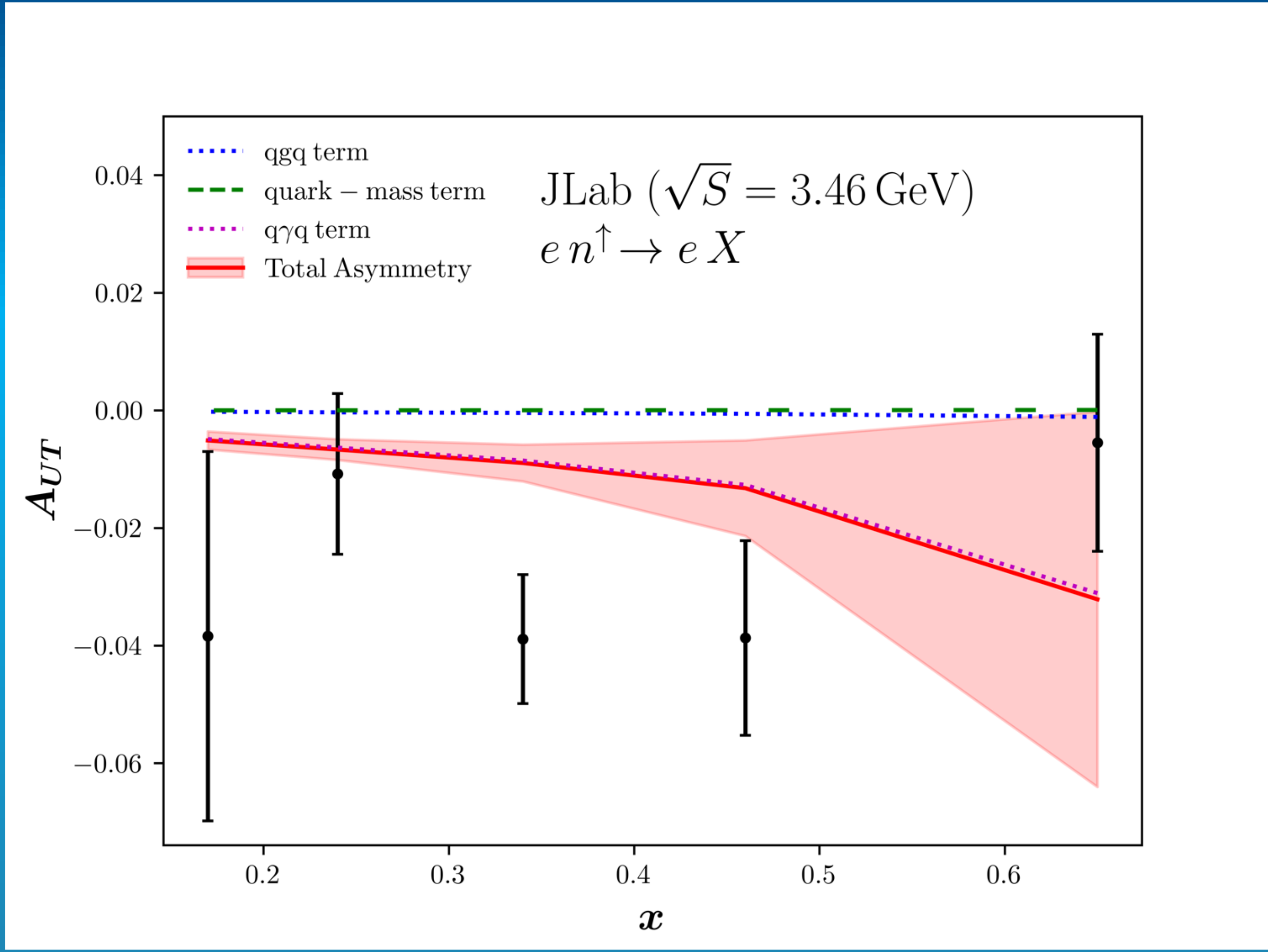
A_{UT} vs. x for HERMES

- Predicted asymmetry consistent with HERMES measurement of a zero A_{UT}
- Theoretical calculation does go negative somewhat near higher momentum fractions
- Still within range of errors of the measurement
- Quark-photon-quark term seems to be main cause of the asymmetry



A_{UT} vs. x for JLab Data

- The theoretical result when compared to JLab slightly undershoot the data at intermediate *x*
- Overall, it is in line with the magnitude and sign of the asymmetry



Conclusions and Outlook

- We have numerically computed A_{UT} for γSIDIS by modeling quark-gluon-quark correlators with reasonable assumptions, using input from the Sivers function and a lattice QCD calculation of d_2

- We have determined which kinematic regions A_{UT} for γSIDIS may be sizeable (~10% of more) at the future Electron-Ion Collider

- The theoretical calculation of A_{UT} for inclusive DIS describes relatively well the experimental data from HERMES and JLab

- Our research highlights the necessity of measuring these observables in order to extract information on quark-gluon-quark correlations in the nucleon, which remain basically unknown