Recent Progress in Gluon Parton Distributions from Lattice QCD

Bill Good (goodwil9@msu.edu)
03.15.2025
The 11th Workshop of the APS Topical Group on Hadron Physics

In collaboration with:
Alex NieMiera, Huey-Wen Lin, Kinza Hasan, Nobuo Sato, Patrick Barry, Wally Melnitchouk
(affiliations)

MICHIGAN STATE UNIVERSITY

APS Topical Group on Hadronic Physics GHP

Outline

I. Introduction
II. Continuum Physical Studies of the Pion and Nucleon Gluon PDFs
III. Improving the JAM Pion PDF Global fit with Lattice Gluon Data
IV. LaMET for the Gluon PDF and Future Signal Improvement
V. Conclusion

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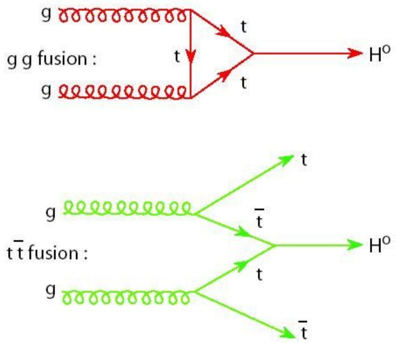
Introduction and Background

- The gluon parton distribution function (PDF) provides important input to high energy experiments, such as Higgs production and J/ψ photo-production



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Amaroso, *et al*. (Snowmass) Acta Physica Polonica. B, 53, 12 (2023)
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Achenbach, *et al.* (LRP) Nucl. Phys. A 1047:122874 (2024)

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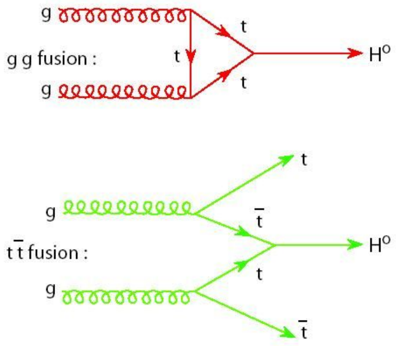

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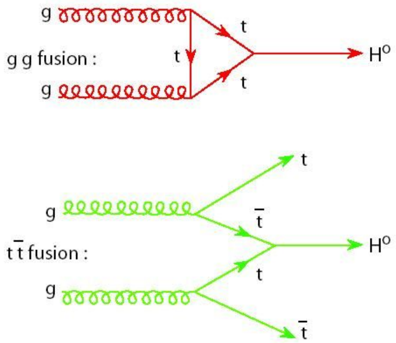

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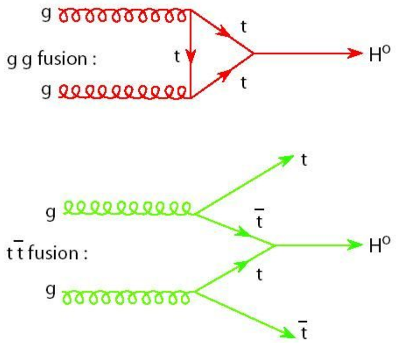

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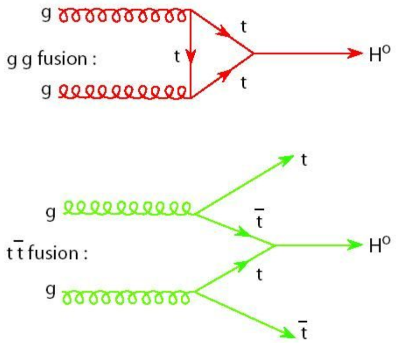

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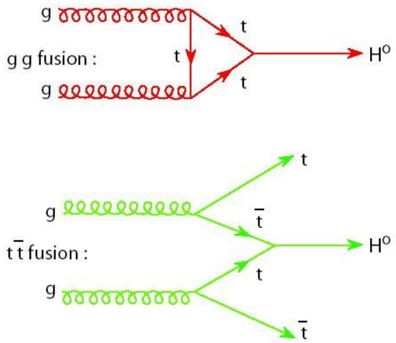
- I will present MSULat's recent work in illuminating the gluonic structure of the nucleon and pion



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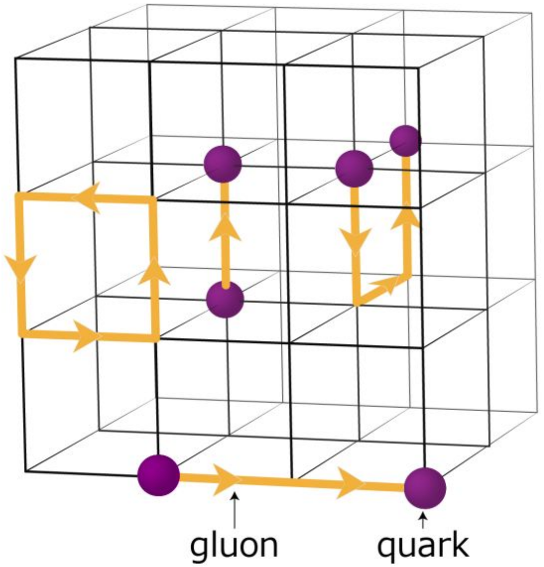

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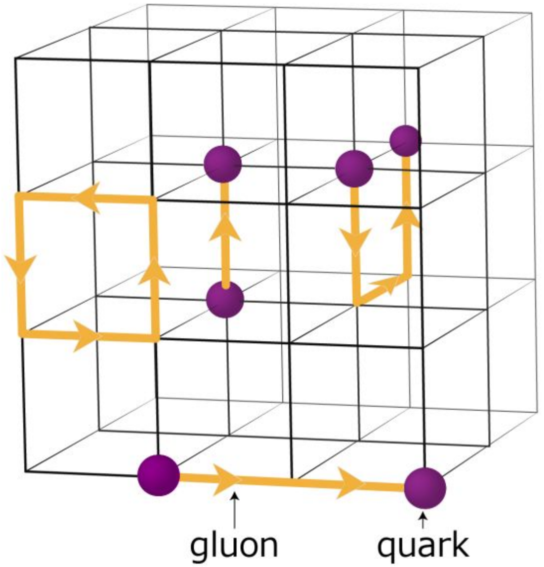

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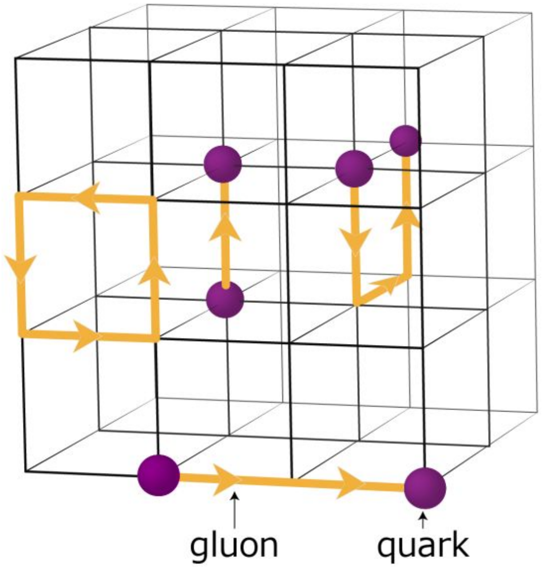

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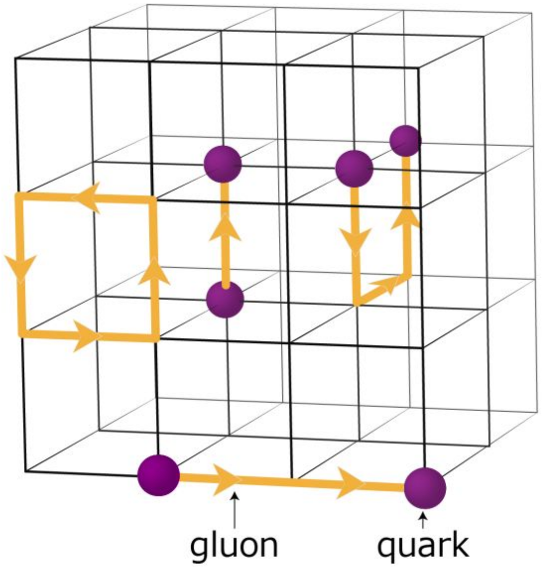

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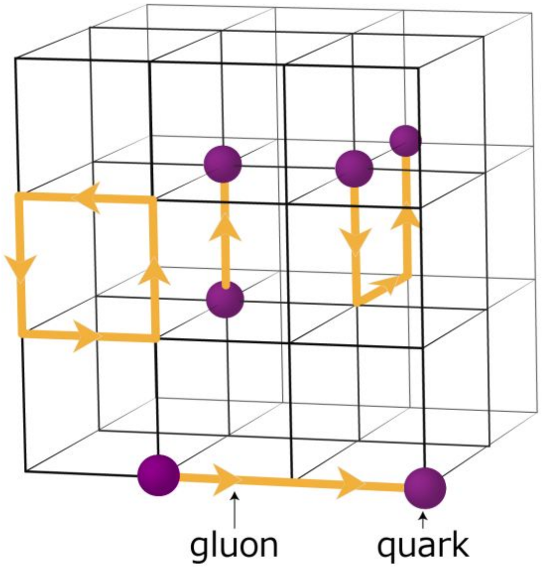

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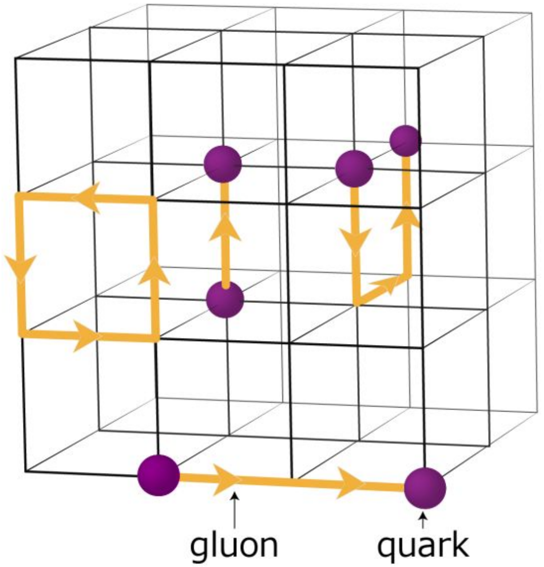

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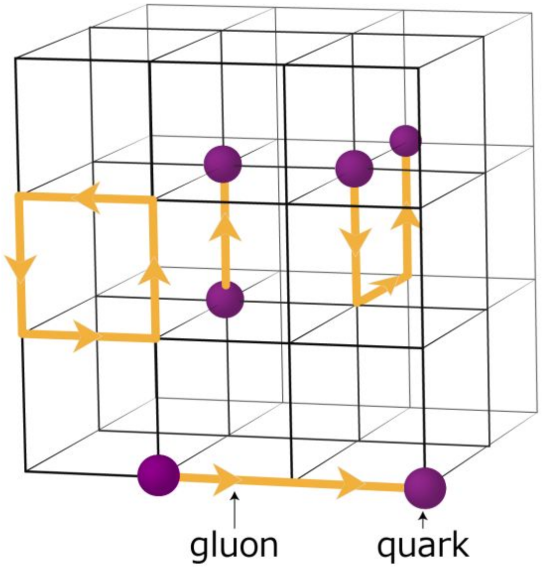

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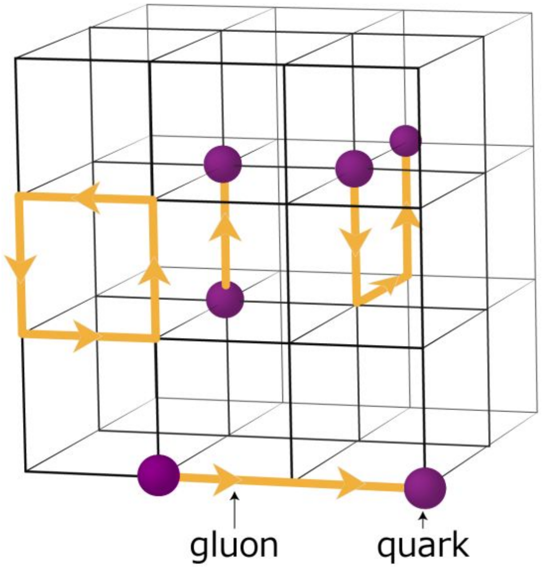

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  - Large momentum Effective Theory (LaMET or quasi-PDF) and the pseudo-PDF approach
  - Because of limited signal at long distances, the pseudo-PDF approach has been much more successful for gluon PDFs than LaMET, but we're interested in pushing towards longer distances to apply LaMET and compare the two methods

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Pseudo- and Quasi-PDF Methods

Spatially separated matrix elements (MEs)

$$\langle h(P_z)|O_g(z)|h(P_z)\rangle$$

from the lattice

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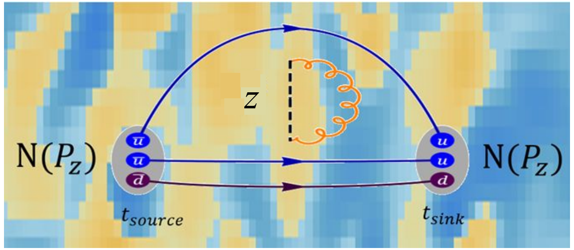

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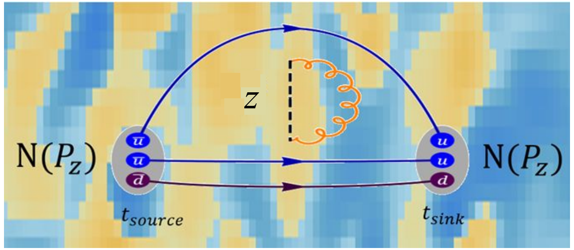
- Both methods give ways to connect spatially separated correlators to the light-cone PDFs



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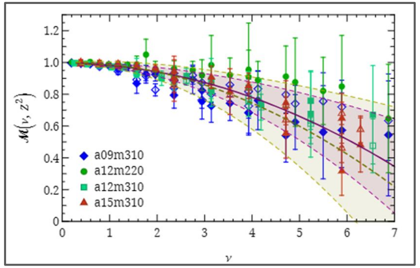

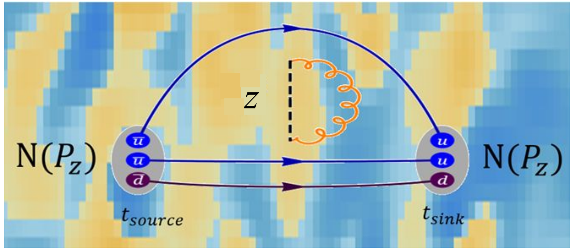

Pseudo- and Quasi-PDF Methods

Balitsky *et al.*, PLB 808:135621, 2020.

$$\mathscr{M}(\nu, z^2) = \int_0^1 dx \frac{xg(x,\mu^2)}{\langle x \rangle_g} R_{gg}(x\nu, z^2\mu^2) + \mathcal{O}(z^2m^2, z^2\Lambda_{\rm QCD})$$

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Short distance (small-z^2) matching

Fit PDF model $\frac{xg(x)}{\langle x \rangle_g} = Nx^\alpha(1-x)^\beta$

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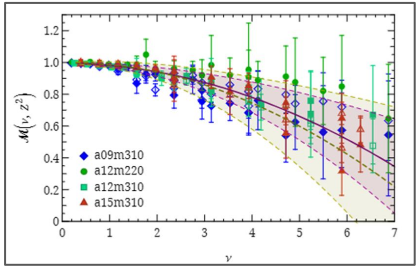

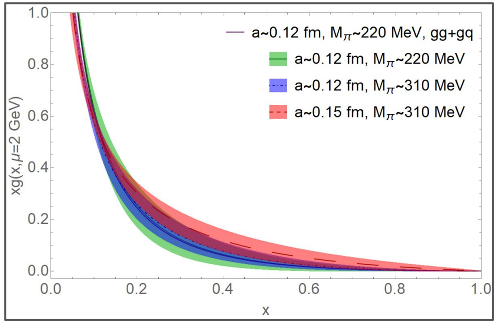

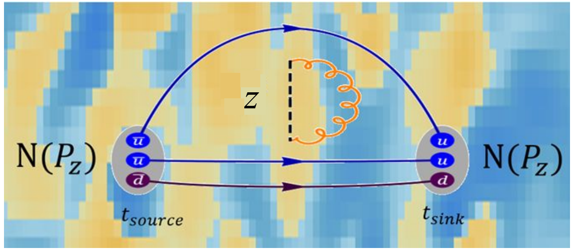

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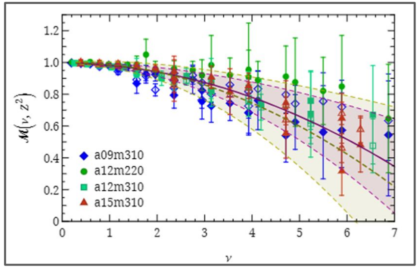

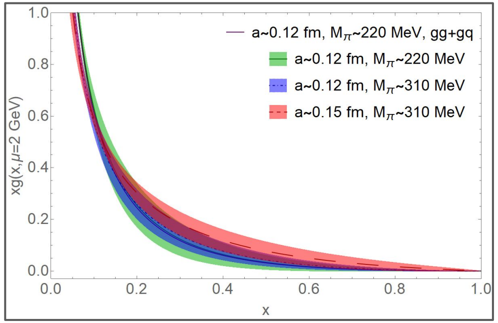

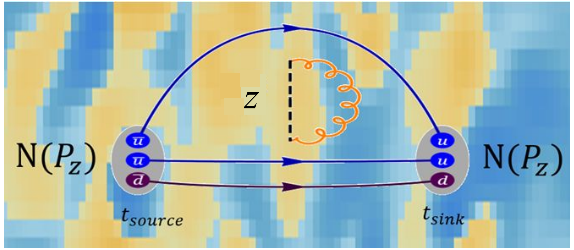

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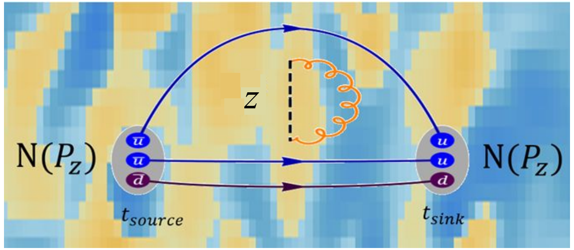

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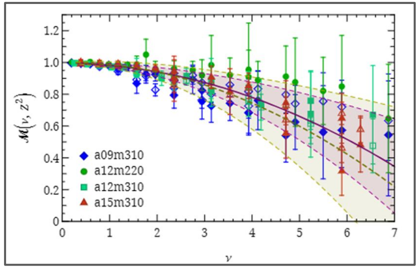

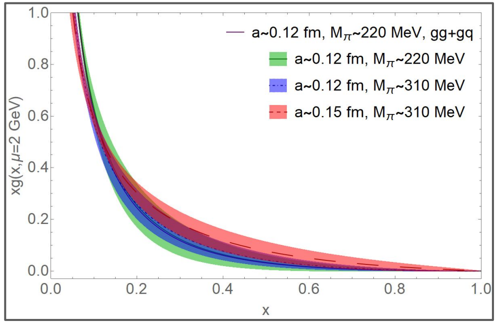

Fourier transform

Quasi-PDF $\widetilde{g}(x, P_z)$

Quasi-PDF Method

Large momentum matching

$xg(x)/\langle x\rangle_g$
Typically reliable around $x \in [0.2, 0.8]$

$$x\tilde{g}(x, P^z, \mu) = \int_{-1}^{1} dy F_{gg}(x, y, \mu) y g(y, \mu) + \mathcal{O}\left(\frac{\Lambda^2_{\mathrm{QCD}}}{(xP_z)^2}, \frac{\Lambda^2_{\mathrm{QCD}}}{((1-x)P_z)^2}\right)$$

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- The 2pt and 3pt correlators are used to extract the bare matrix elements of the gluon operator, and we apply ratio renormalization, defining the so-called "reduced pseudo-Ioffe time distribution" or RpITD

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- We measured over O(10^5-10^6) two-point (2pt) correlators on each ensemble and contracted with the gluon pseudo-PDF operator to get the three-point (3pt) correlators. We used 5 steps of hypercubic smearing

Balitsky, *et al*., PLB 808:135621 (2020)

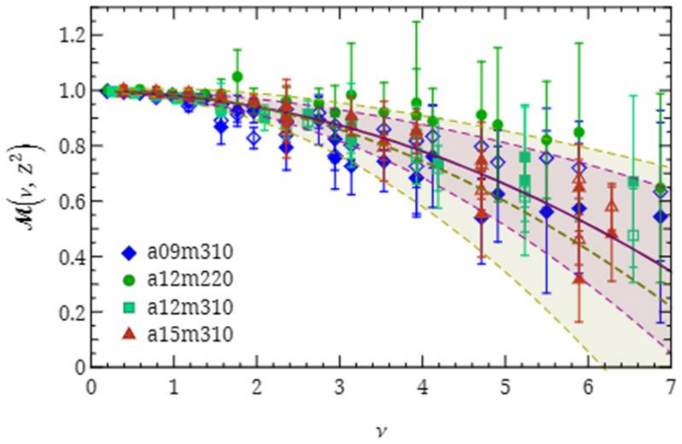
- The 2pt and 3pt correlators are used to extract the bare matrix elements of the gluon operator, and we apply ratio renormalization, defining the so-called "reduced pseudo-Ioffe time distribution" or RpITD
- I will use the following labels for the ensembles throughout the presentation:

Ensemble	a09m310	a12m220	a12m310	a15m310

Nucleon Gluon PDF

Fan, WG, Lin. PRD 108:014508 (2023)

- We took the RpITD matrix elements from each ensemble to produce the first continuum- physical extrapolated RpITD



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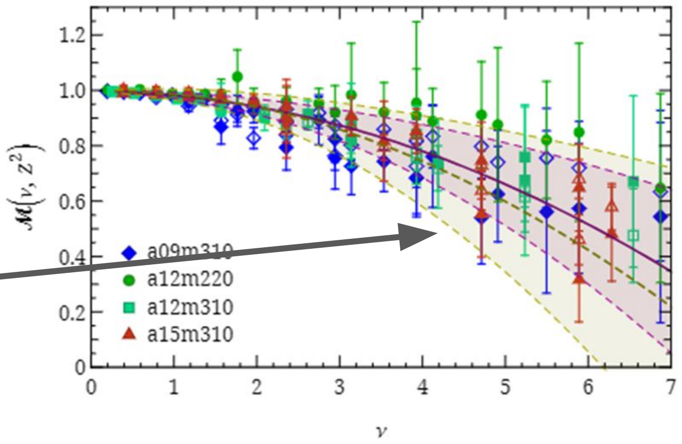

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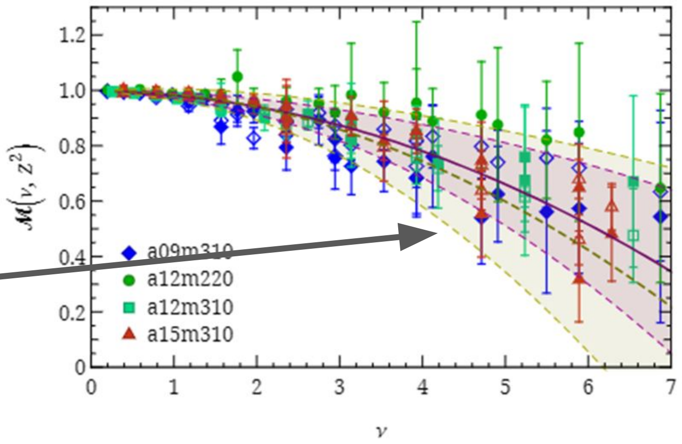

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$$\chi^2(\mu, a, M_\pi) = \sum_{\nu, z} \frac{(\mathscr{M}^{\text{fit}}(\nu, \mu, z^2, a, M_\pi) - \mathscr{M}^{\text{lat}}(\nu, z^2, a, M_\pi))^2}{\sigma^2_{\mathscr{M}}(\nu, z^2, a, M_\pi)}$$

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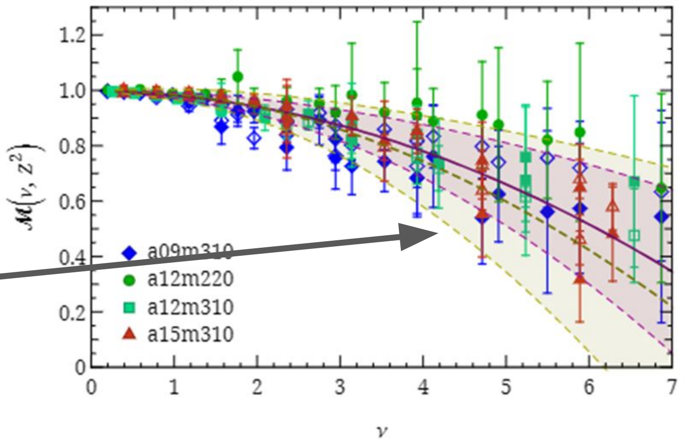

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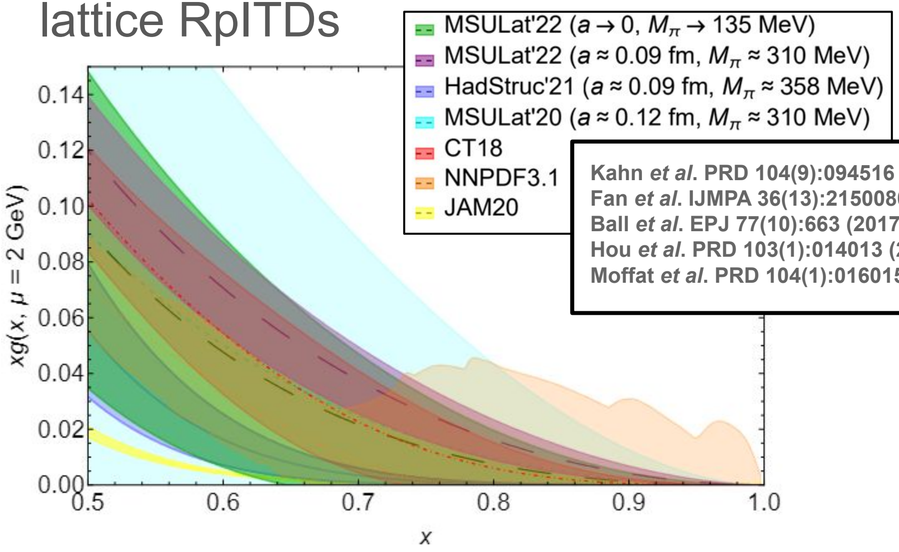

Kahn *et al*. PRD 104(9):094516 (2021)
Fan *et al*. IJMPA 36(13):2150080 (2021)
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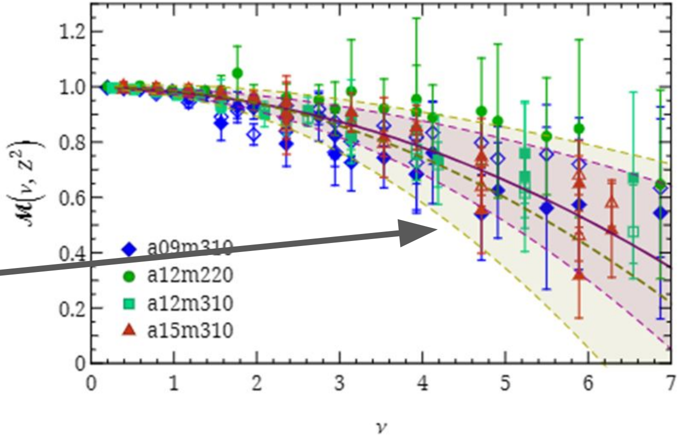

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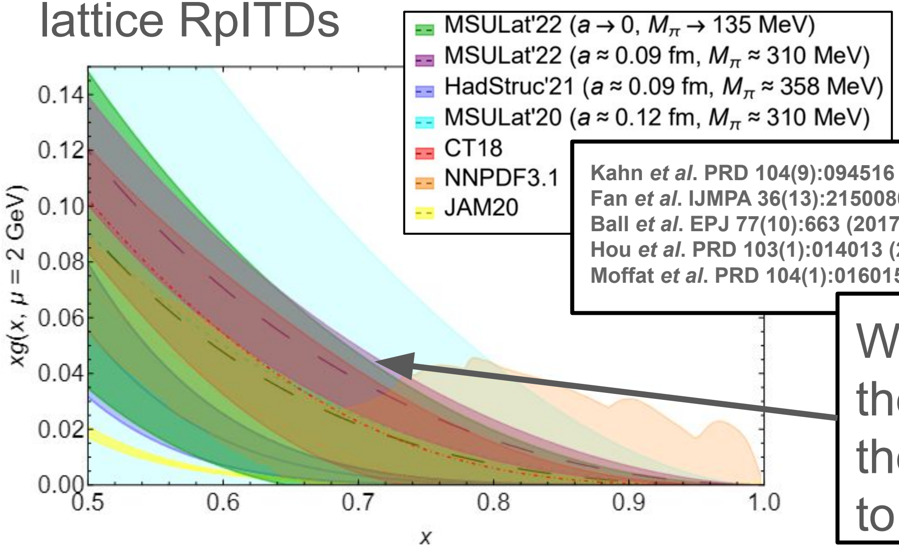

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We find good agreement between the lattice and the pheno. results in the large-x region, but we still want to reduce the errors

Lattice Spacing Dependence in Pion Gluon PDFs

WG, *et al*. PRD 109:114509(2024)

- We recently calculated pion gluon momentum fractions $\langle x \rangle_g$ and multiplied them through the normalized pion PDFs $\frac{xg(x)}{\langle x \rangle_g}$ from our previous pion gluon PDF study

Z. Fan, *et al*. PLB 823:136778 (2021)

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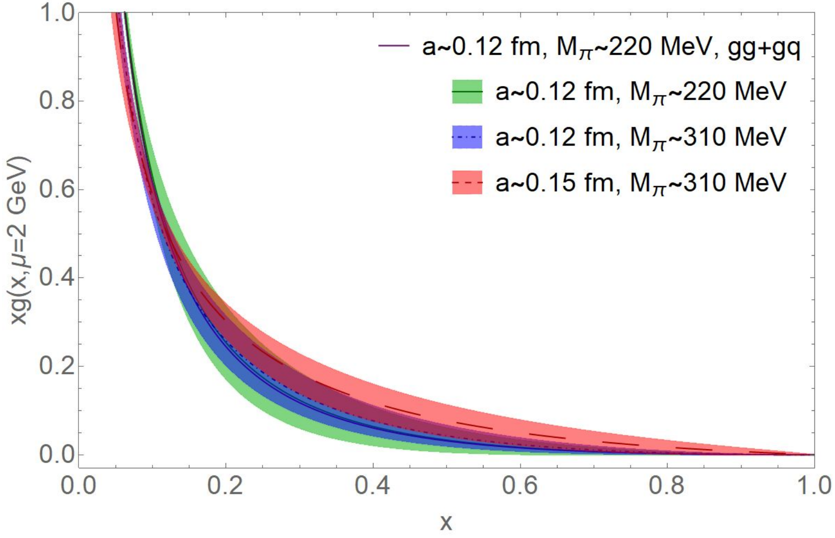

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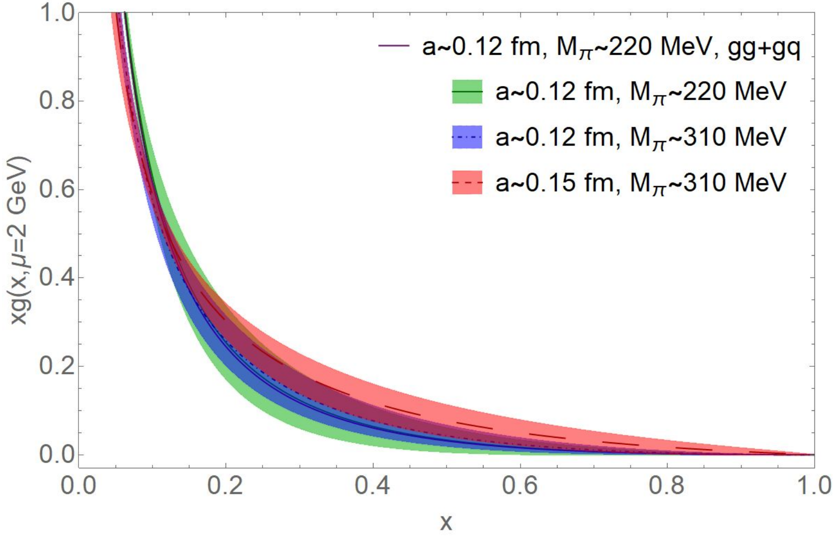
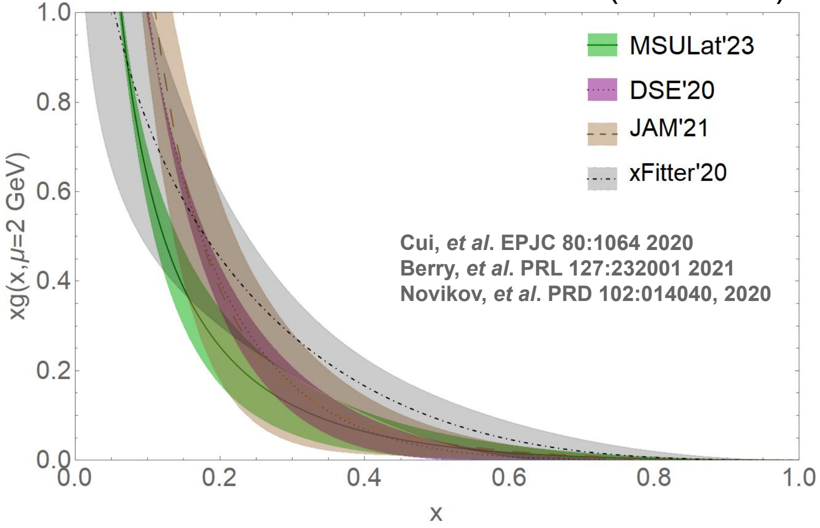


(a12m220)



Outline

I. Introduction
II. Continuum Physical Studies of the Pion and Nucleon Gluon PDFs
III. Improving the JAM Pion PDF Global fit with Lattice Gluon Data
IV. LaMET for the Gluon PDF and Future Signal Improvement
V. Conclusion

J. Conway, *et al*. PRD, 39, p. 92–122, 1989.
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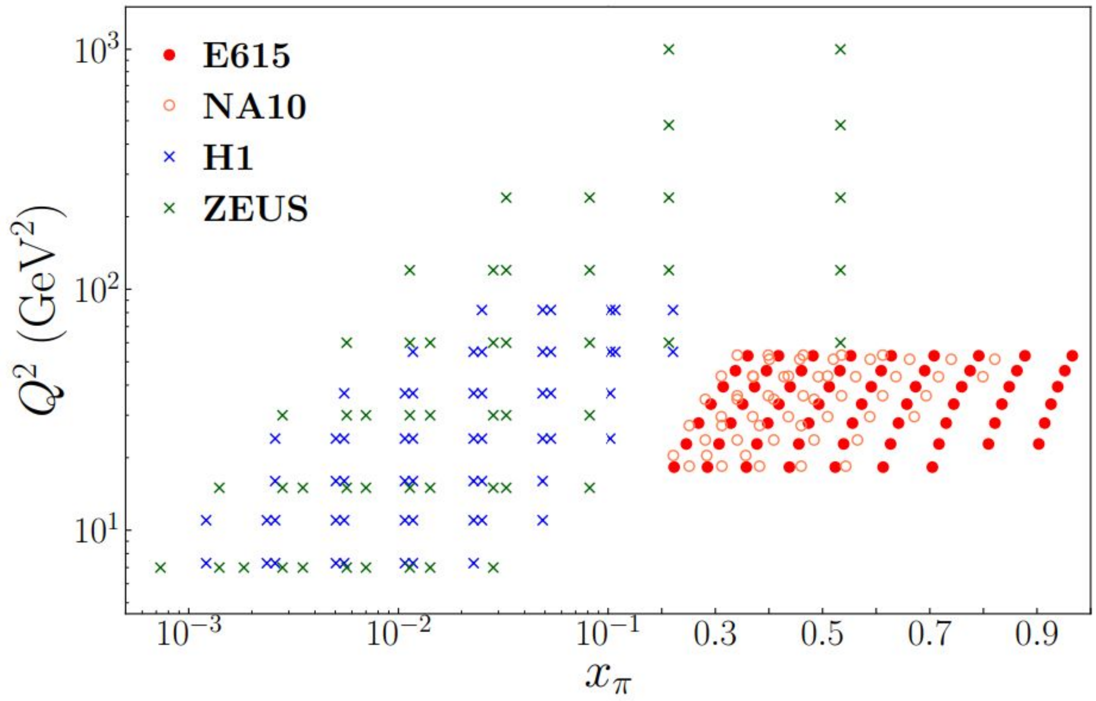

N. Y. Cao, *et al*. PRD 103:114014 2021

The experimental data in the JAM fit of pion PDFs consists of leading neutron (LN) (x's)

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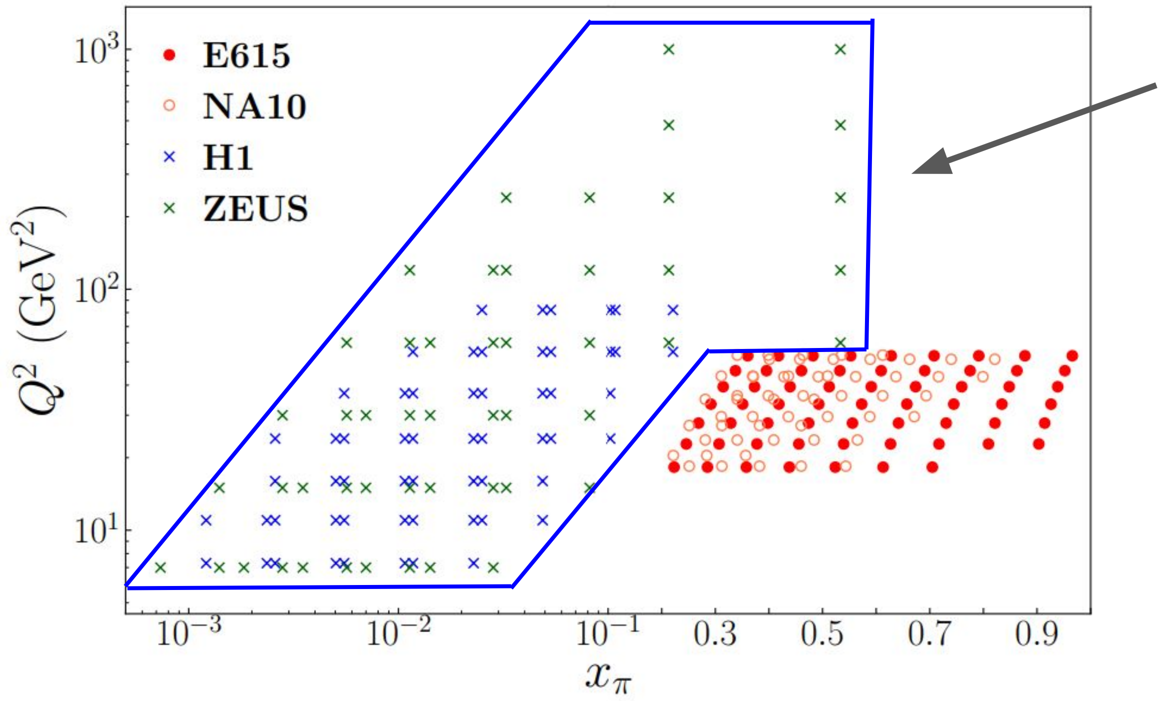

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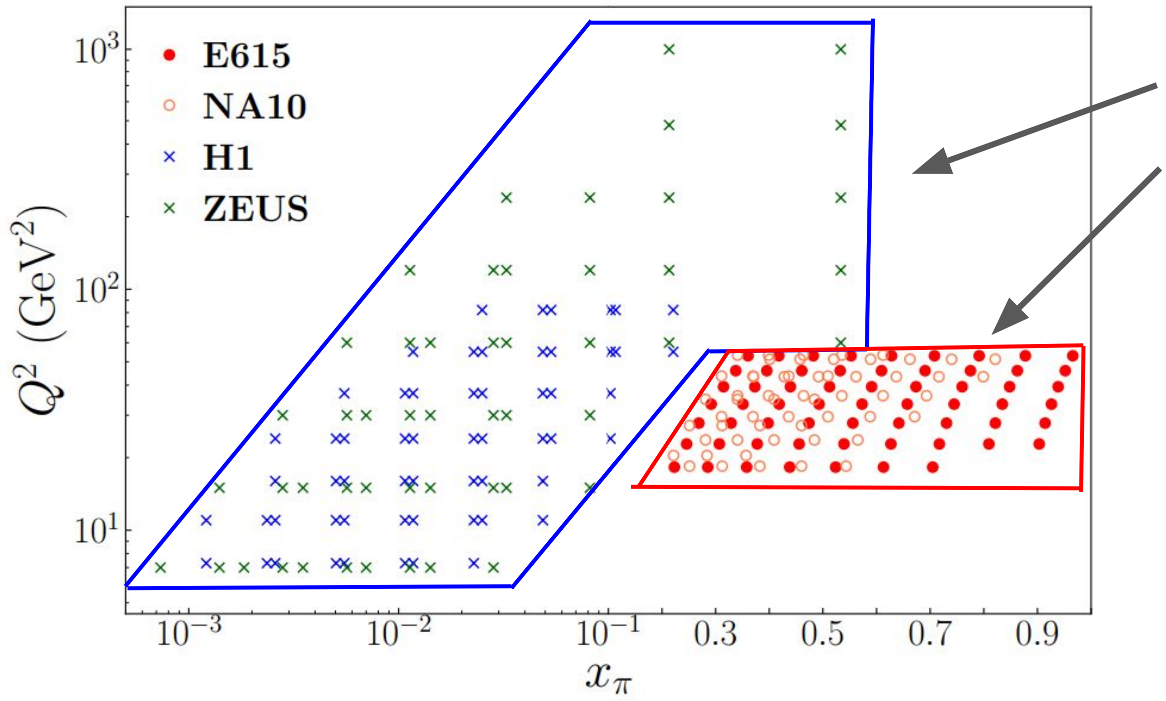

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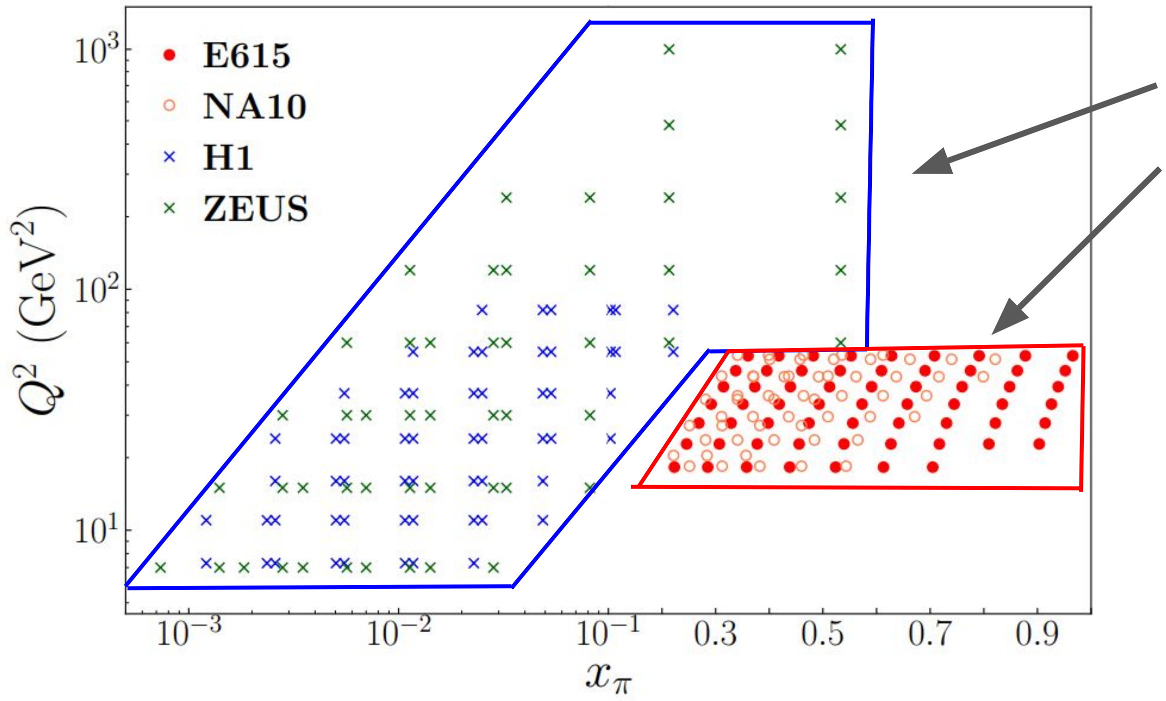

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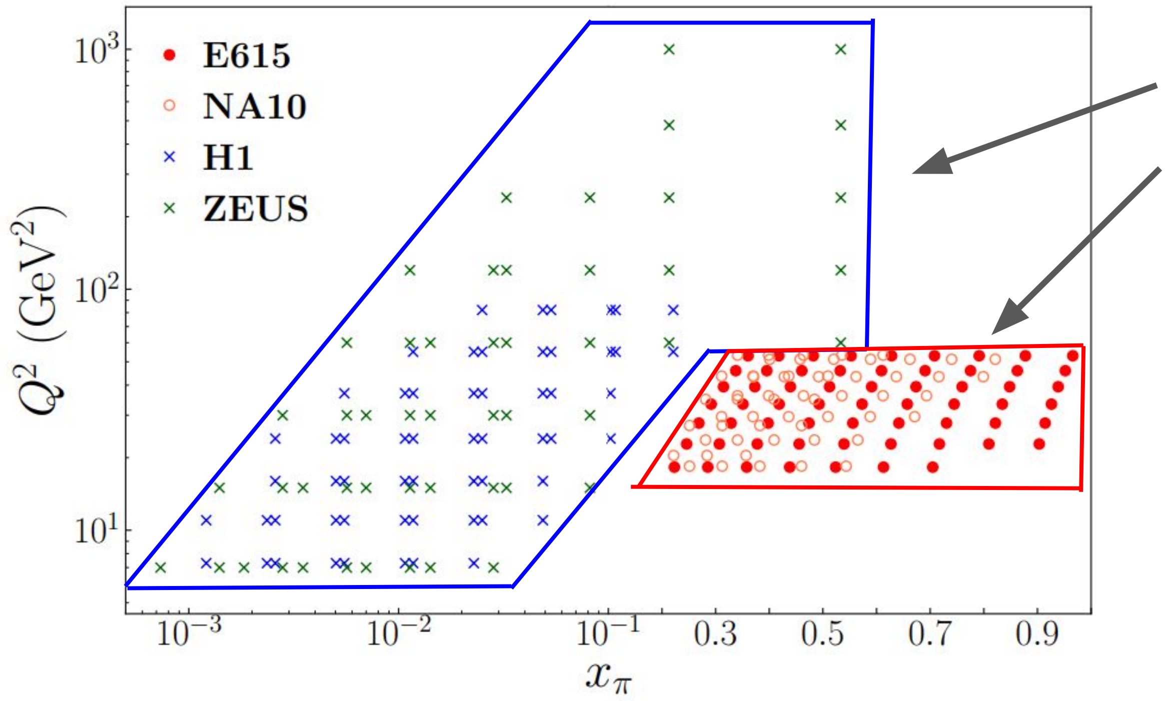

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Lattice should compliment the LN data well because it has more constraining power on the intermediate-x range!

Overall Methodology

$$f_i(x, \mu_0; \boldsymbol{a}_i) = N_i x^{\alpha_i} (1 - x)^{\beta_i} (1 + \gamma_i x^2)$$

$$\mu_0 = m_c = 1.27 \text{ GeV} \qquad \gamma_{s,g} = 0$$

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A. V. Radyushkin, PRD 96:034025, 2017.

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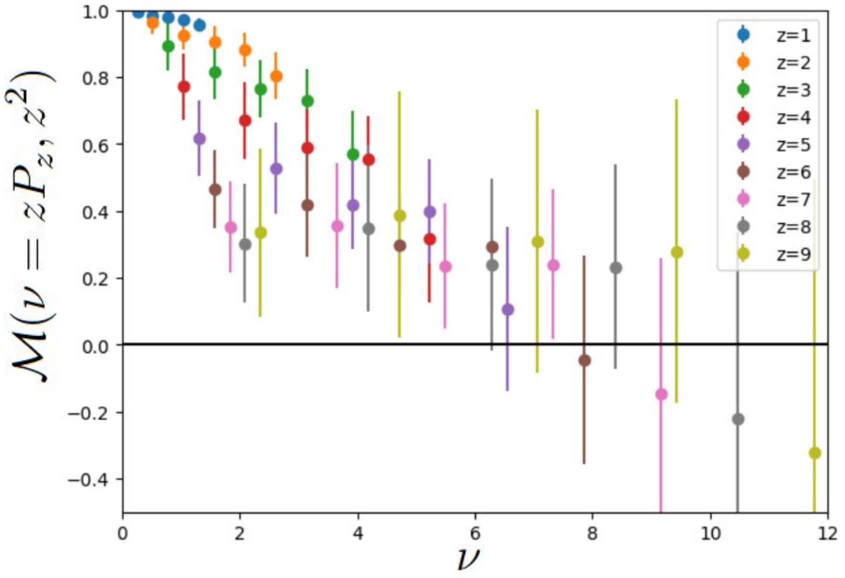
- The PDFs are refit by minimizing the new sum:

$$\chi^2_{\text{Expt}} + \chi^2_{\text{Lat}}$$

Lattice Data

- We use the a12m310 lattice ensemble with a high level of statistics



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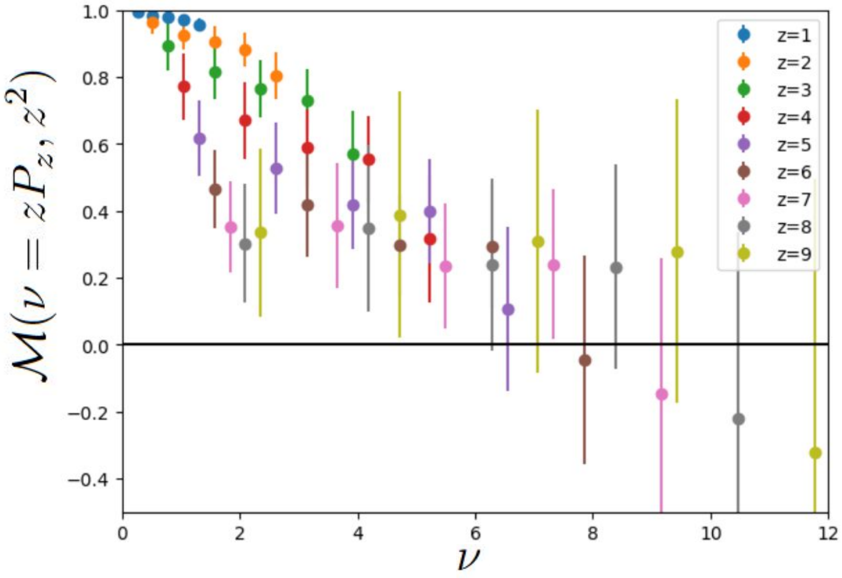

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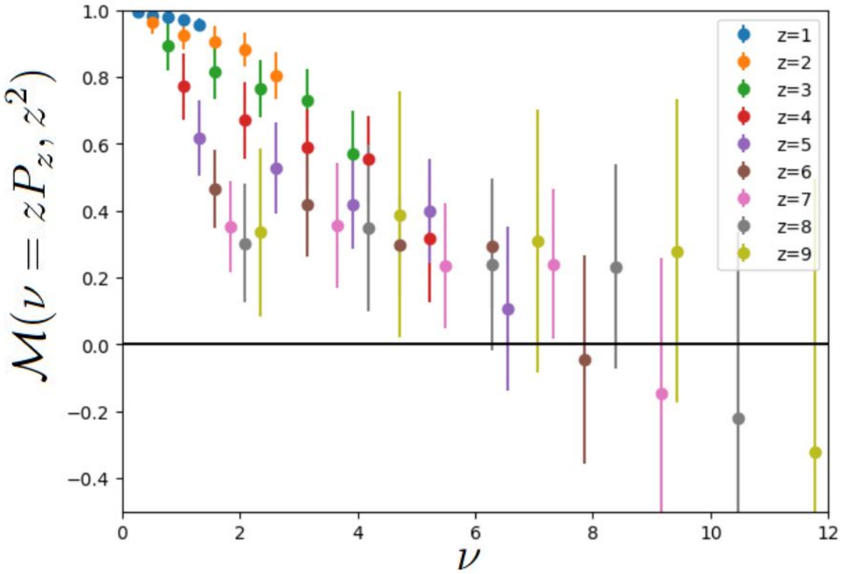

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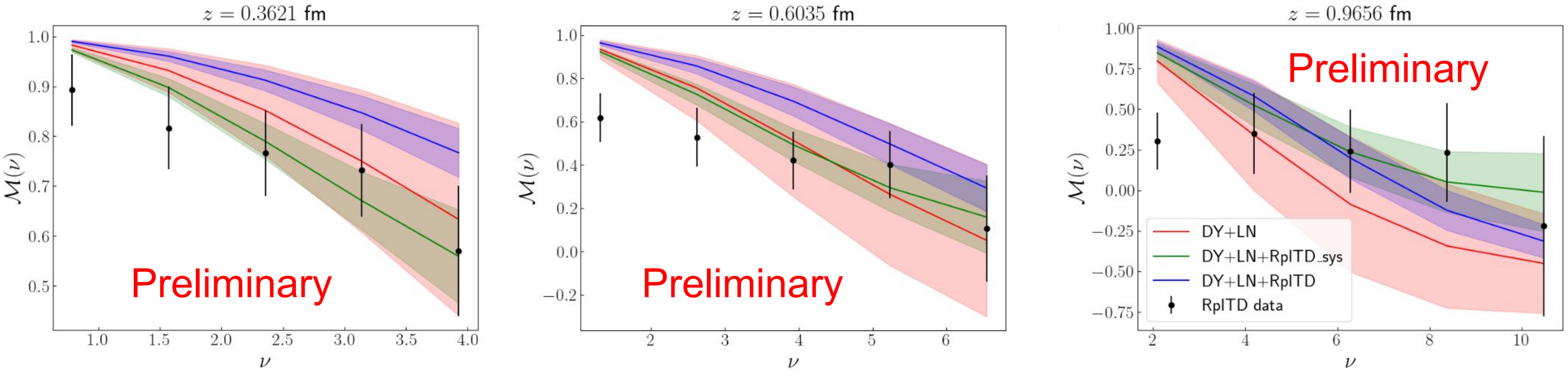
- We fit systematic corrections for higher-twist and discretization effects using the same parameterization as a previous study of the valence quark PDFs

$$z^2 B_1(\nu) + \frac{a}{|z|} P_1(\nu)$$

P. C. Barry, et al., PRD 105:114051 (2022)

Fits to the Lattice Data

- We match the lattice RpITD data to the RpITDs reconstructed from the fitted PDFs. We use the scale $\mu^2 = m_c^2$



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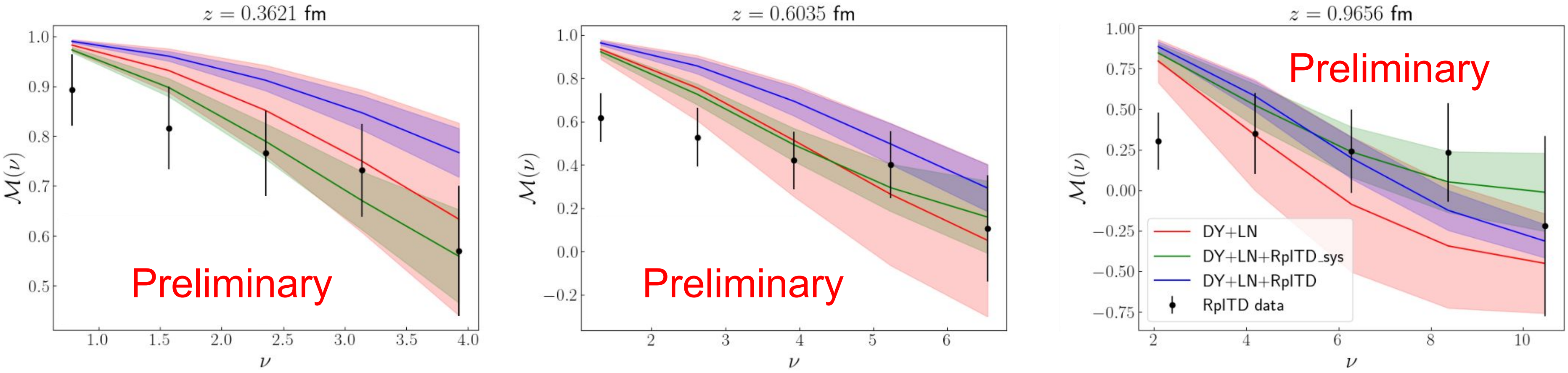
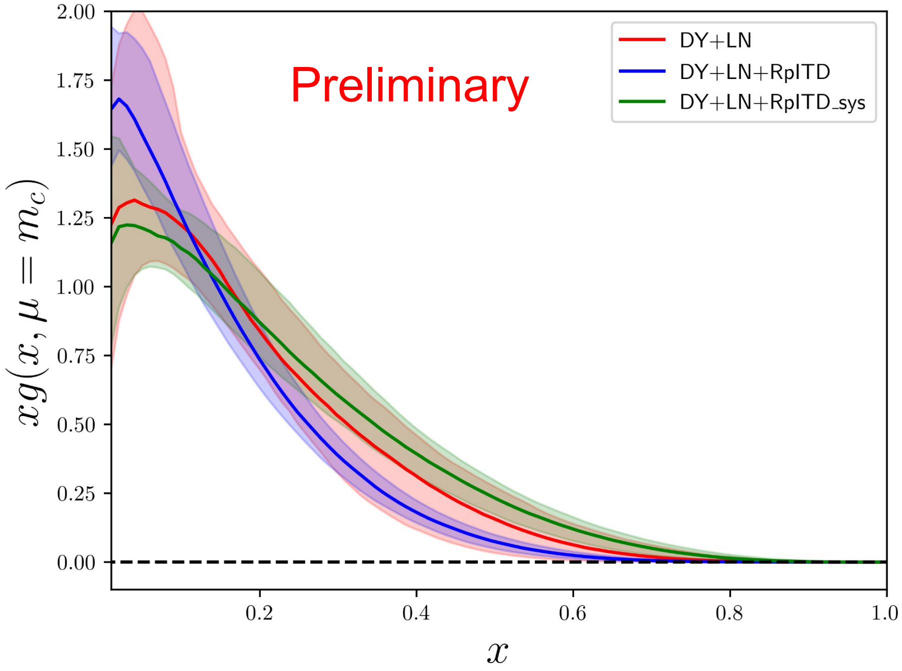


Fit	$\chi^2_{\mathrm{Expt}}/N_{\mathrm{pts}}$	Z-Score Expt.	$\chi^2_{\mathrm{Lat}}/N_{\mathrm{pts}}$	Z-Score Lat
DY+LN	0.847(37)	1.69(43)	2.8(47)	N/A
DY+LN+RpITD	0.840(39)	1.78(45)	1.854(69)	2.95(19)
DY+LN+RpITD_sys	0.845(38)	1.71(43)	1.24(13)	1.02(46)

New Pion PDFs



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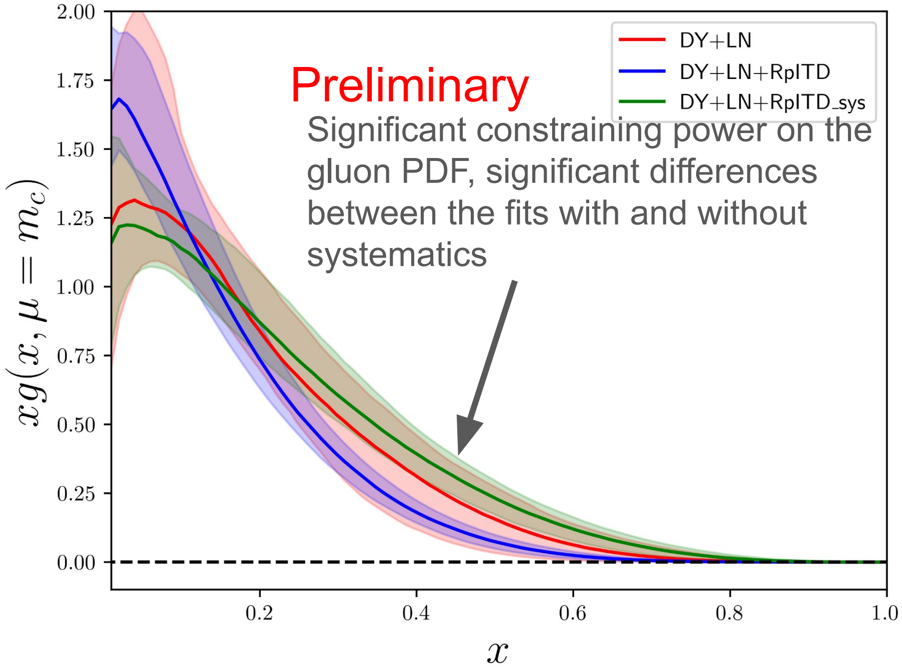

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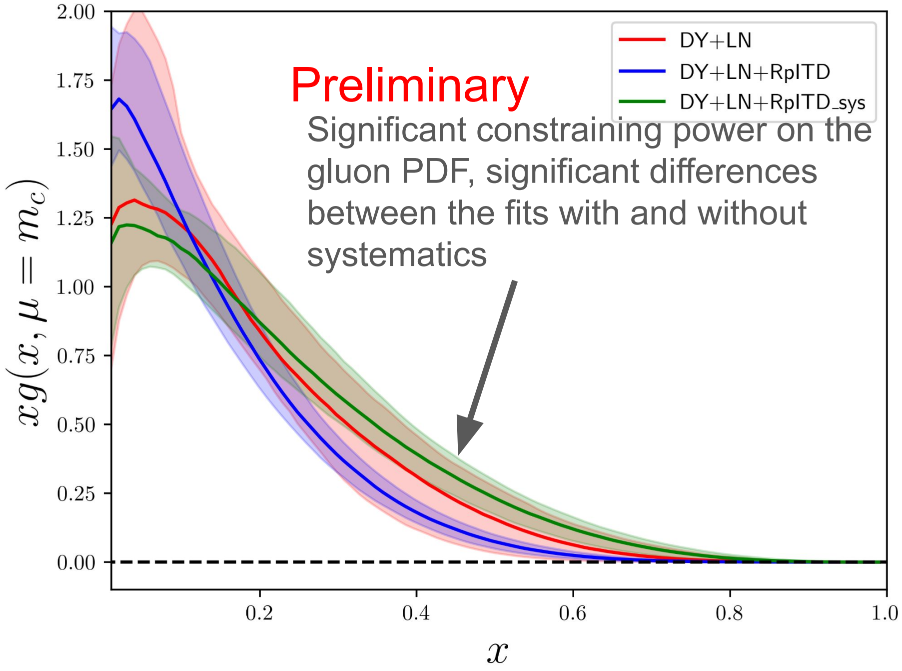
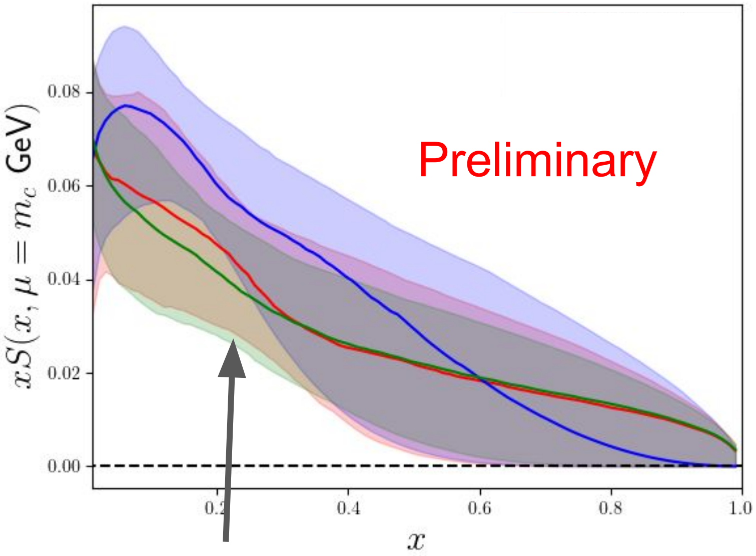


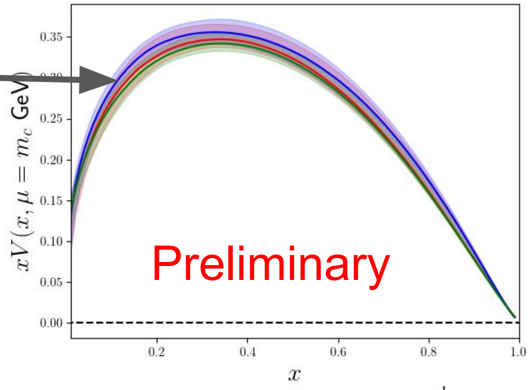
Significant constraining power on the gluon PDF, significant differences between the fits with and without systematics

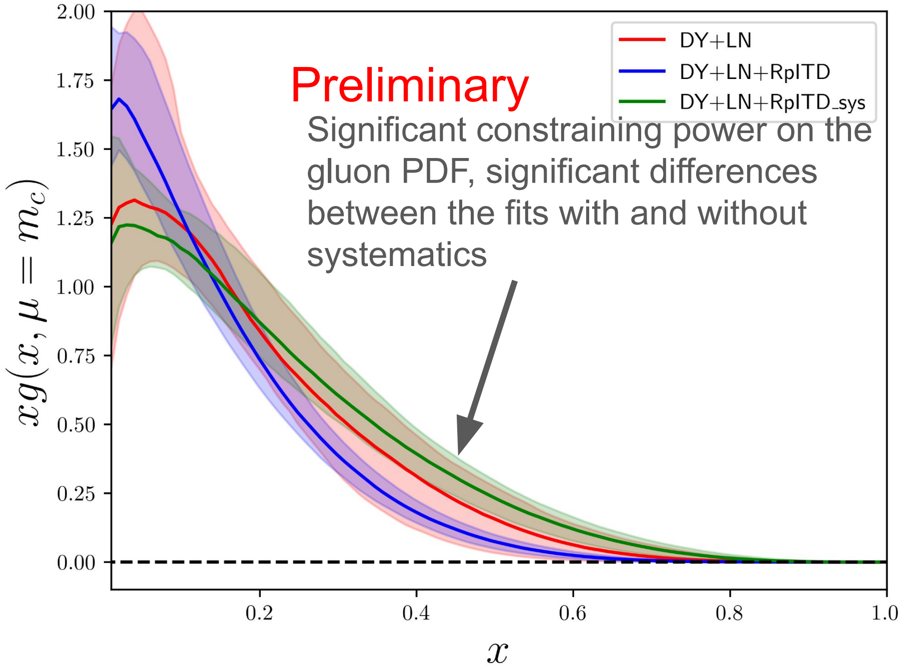


Including systematics makes the sea PDF go back into better agreement with the fits without lattice data. Interesting…

New Pion PDFs

Not much effect on the valence quarks (as expected)





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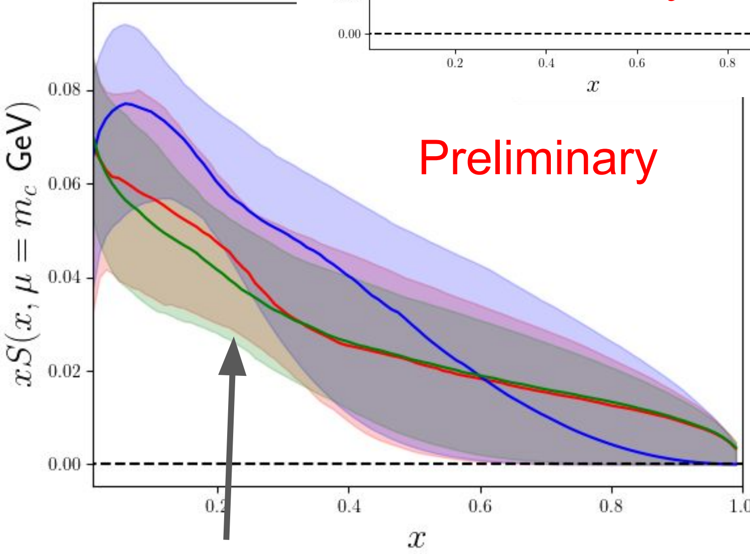

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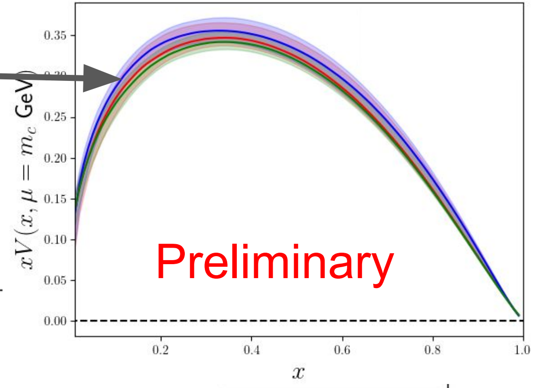

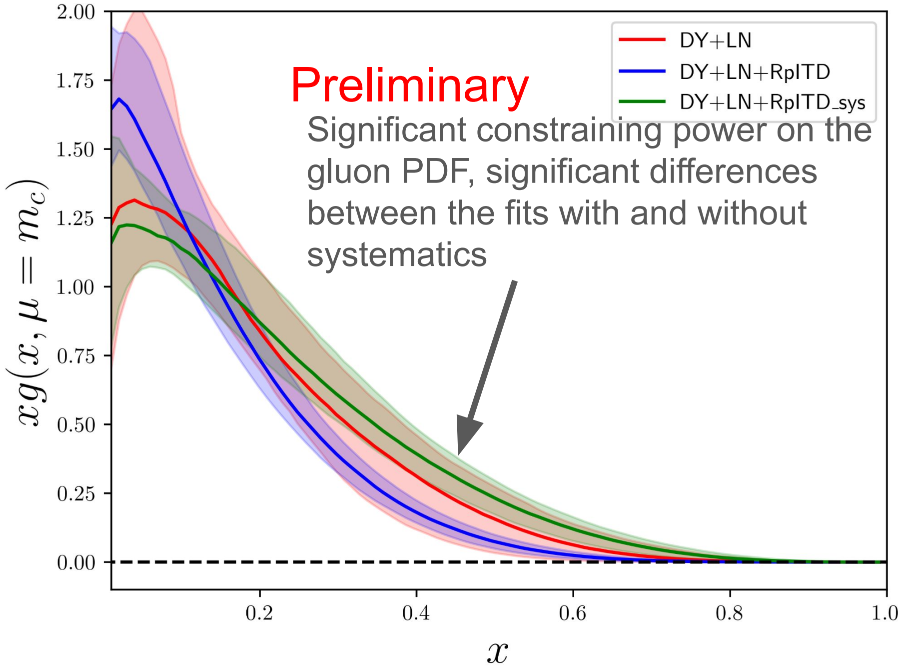

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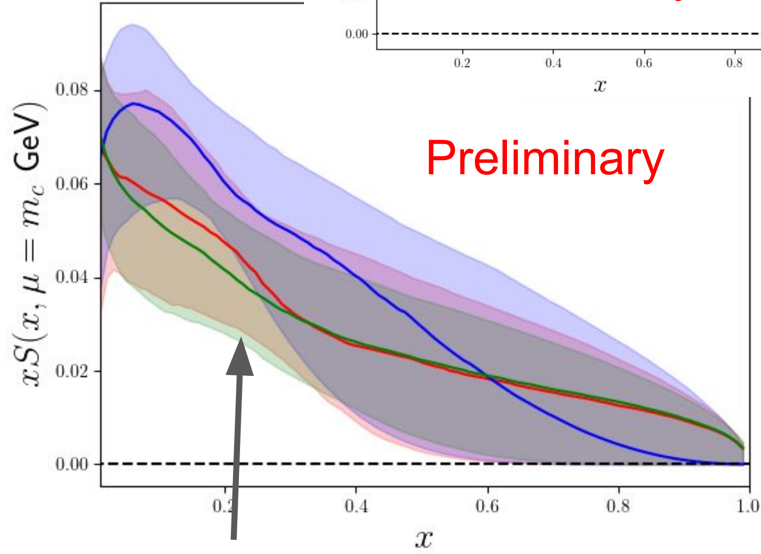
Overall compatibility with "before" lattice PDFs



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I. Introduction
II. Continuum Physical Studies of the Pion and Nucleon Gluon PDFs
III. Improving the JAM Pion PDF Global fit with Lattice Gluon Data
IV. LaMET for the Gluon PDF and Future Signal Improvement
V. Conclusion

Gluon PDF Operator Study

- There are several operators that can be measured on the lattice which all have the same light-cone behavior

Wang, *et al*. PRD 100:074509 (2019)

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$$O^{(3)}(z) = F^{ti}(z)W(z,0)F^{t}_{\ i}(0) - F^{ij}(z)W(z,0)F_{ij}(0)$$

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- We wanted to determine which operator performs best on the lattice

Gluon PDF Operator Study

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- Took the CT18 nucleon gluon PDF at $\overline{\mathrm{MS}}$ scale μ = 2 GeV and used LaMET matching and a Fourier transform to compare to the operators in the ratio renormalization scheme from the a12m310 ensemble

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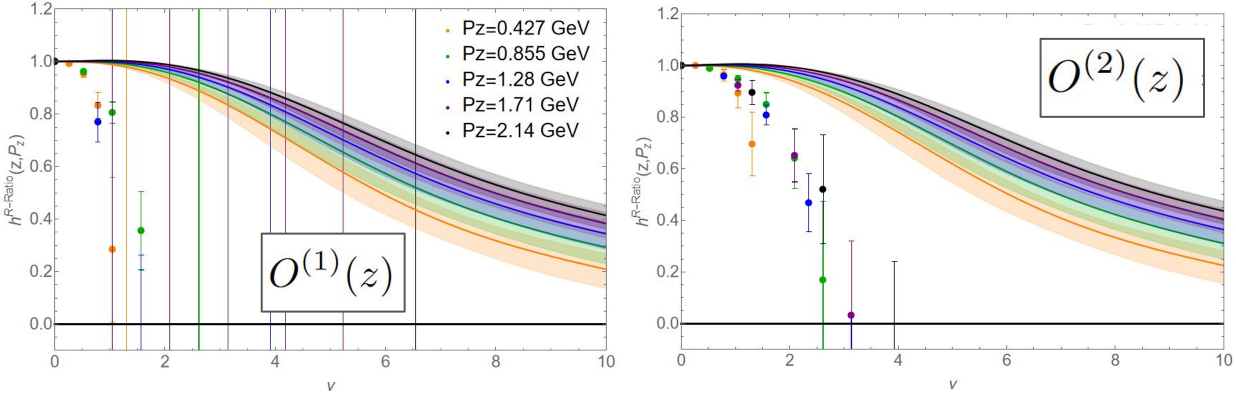

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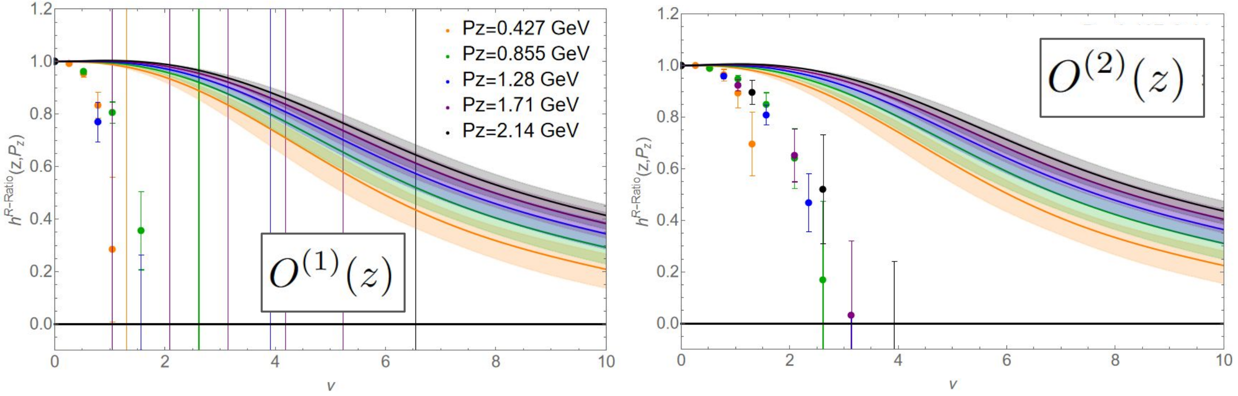

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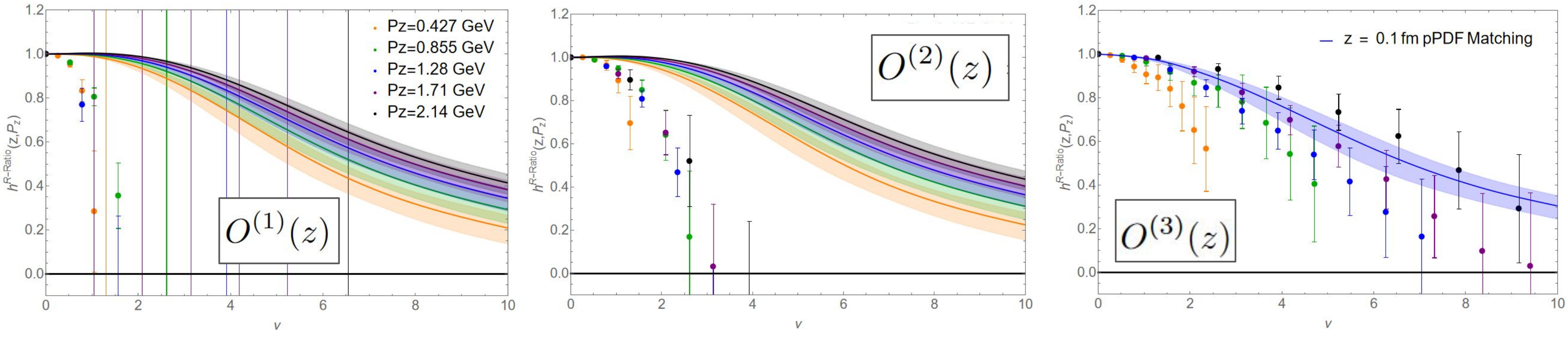

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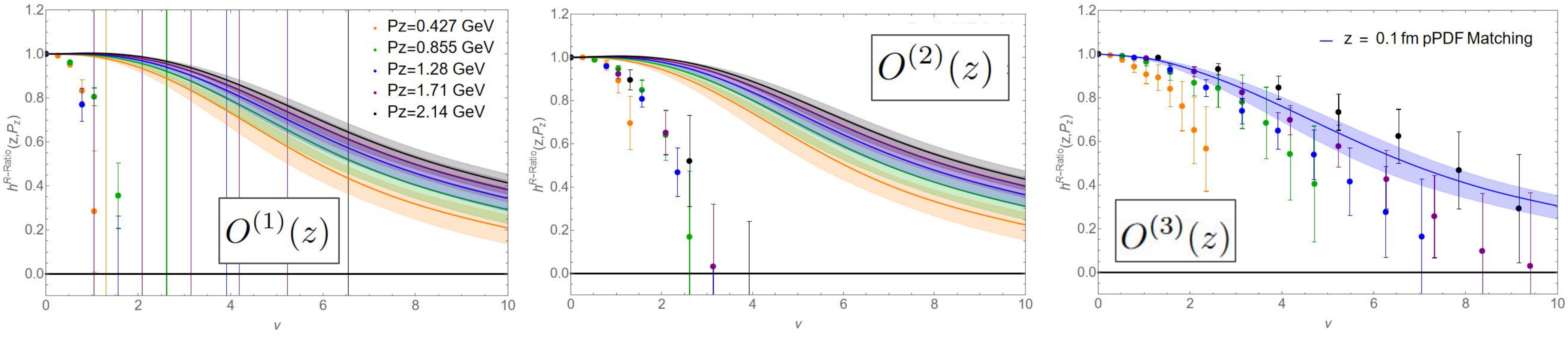

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We used this study to urge the perturbative community to study the matching for the $O^{(3)}$ operator as well

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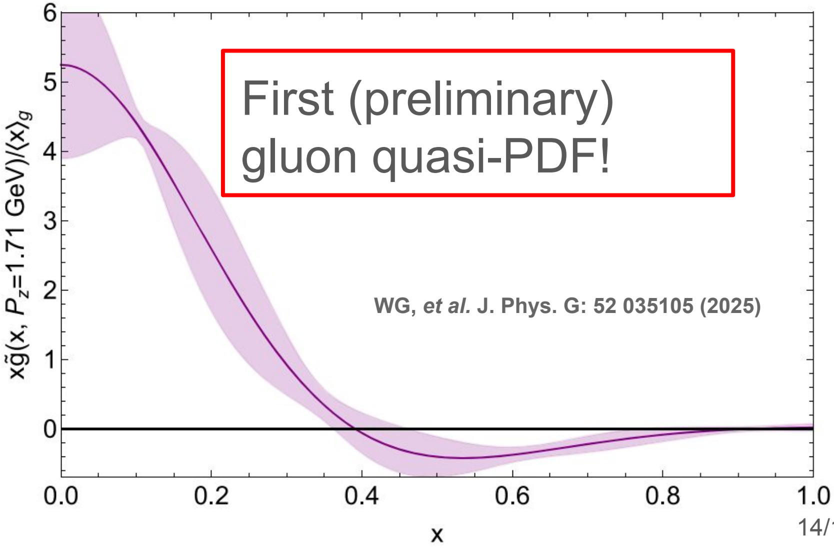

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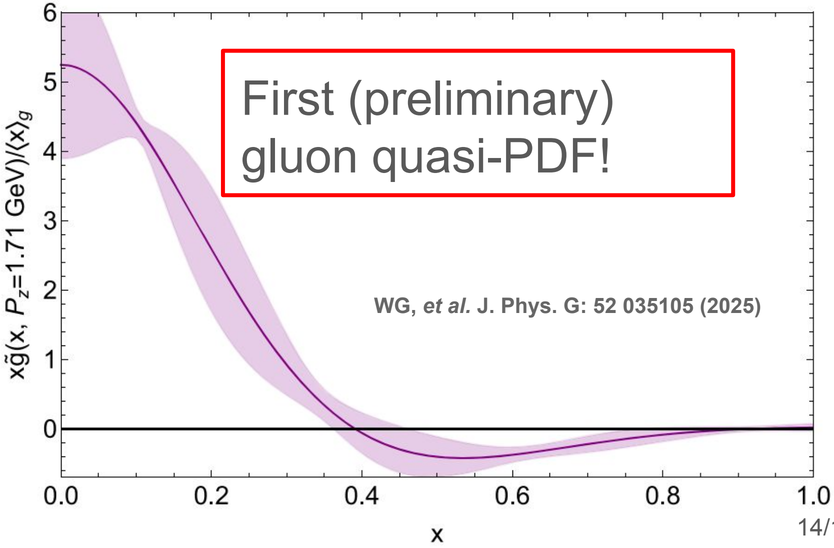
- We were able to obtain a quasi-PDF with reasonable signal, which could be matched to the light-cone PDF, if the matching kernels are calculated, as well
- Note: we used Wilson smearing with T = 3a^2 to obtain these results as a preliminary study. This amount of smearing likely has some effect on the physics



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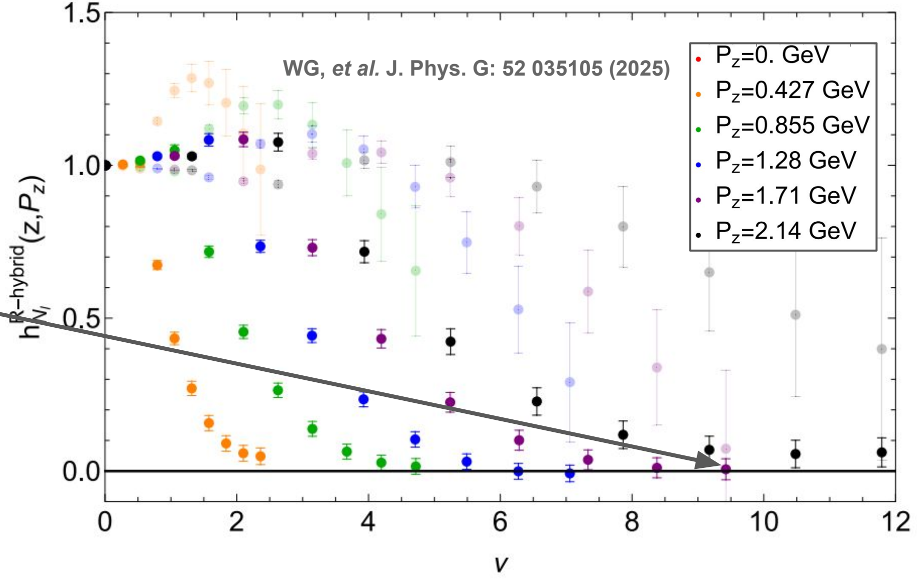

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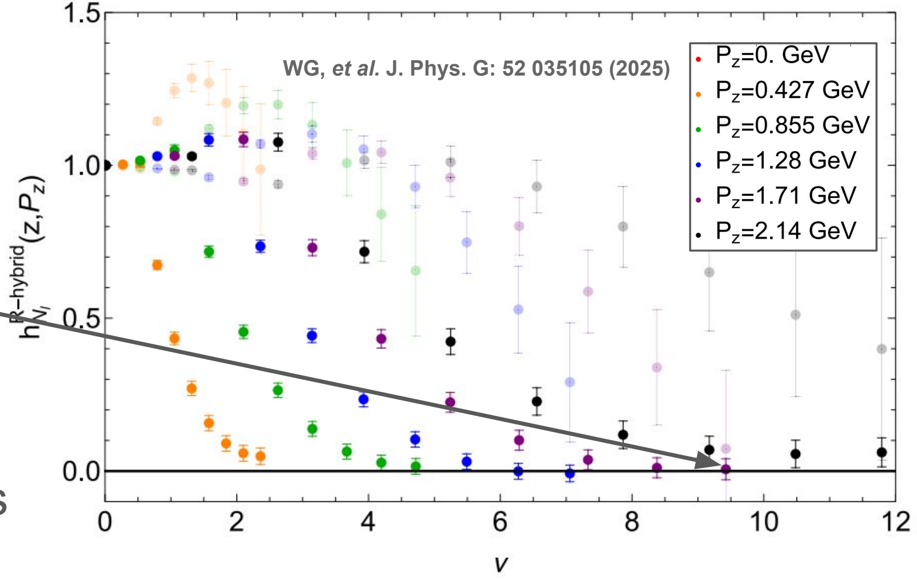

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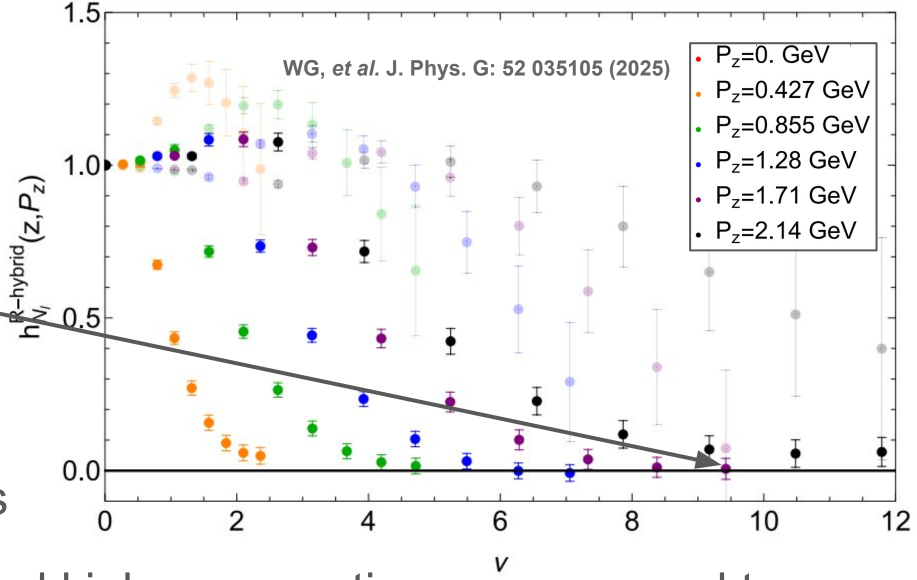


- We also see some interesting bumps and kinks, suggesting we may need to understand some numerical aspects better, as well

Summary and Outlook

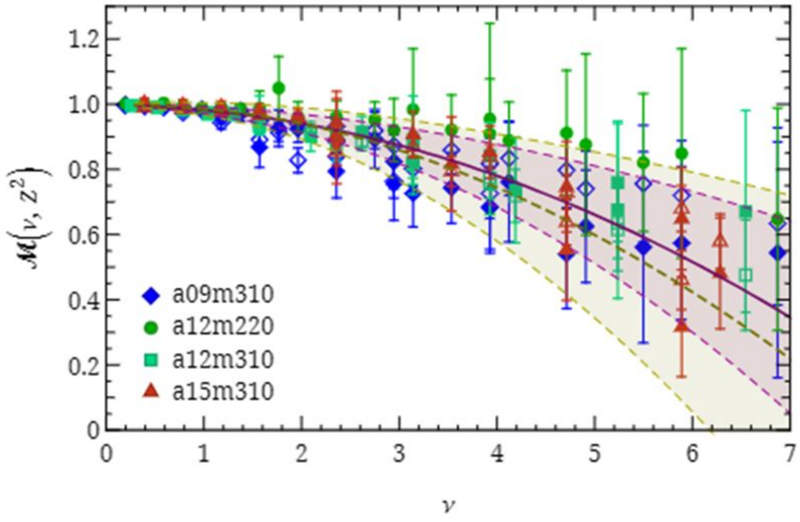
- There has been steady progress in elucidating the gluonic structure of the nucleon and pion through lattice PDFs
- We find that lattice data can have a significant impact on the JAM global fit of the pion PDF, but the lattice systematics must be taken into account
- We still need to make improvements in large momentum and larger distance signal to improve gluon PDFs in both the pseudo- and quasi-PDF methods
- We're exploring different signal improvement methods and constantly increasing our statistics to push the limits on the gluon PDF

Backup Slides

Continuum-Physical Extrapolation

- We use the following fit form for the physical continuum extrapolation of the nucleon gluon RpITD:

$$\mathscr{M}(\nu, z^2, a, M_\pi) = \left(\sum_{k=0}^{k_{\max}} \lambda_k(a, M_\pi)\nu^k + c_z(a, M_\pi)z^2\right) \times (1 + c_a a^2 + c_M(M_\pi^2 - (M_\pi^{\text{phys}})^2),$$



- The gold band, is a similar form with a instead of a^2. The results are similar, but the error is larger, we moved forward with the a^2 extrapolation

PDF Forms, Assumptions, Constraints, and Theory

- We use the PDF forms: $f_i(x, \mu_0; \boldsymbol{a}_i) = N_i x^{\alpha_i}(1-x)^{\beta_i}(1+\gamma_i x^2)$
  with $\mu_0 = m_c = 1.27$ GeV and $\gamma_{s,g} = 0$
- We assume charge symmetry and a flavor symmetric sea:

$$q_v \equiv \bar{u}_v^{\pi^-} = \bar{u}^{\pi^-} - u^{\pi^-} = d_v^{\pi^-} \qquad q_s \equiv u^{\pi^-} = d^{\pi^-} = s^{\pi} = \bar{s}^{\pi}$$

- We constrain the normalizations using valence quark number conservation and the momentum sum rule:

$$\int_0^1 dx q_v(x,\mu) = 1 \qquad \int_0^1 dx x[2q_v(x,\mu) + 6q_s(x,\mu) + g(x,\mu)] = 1$$

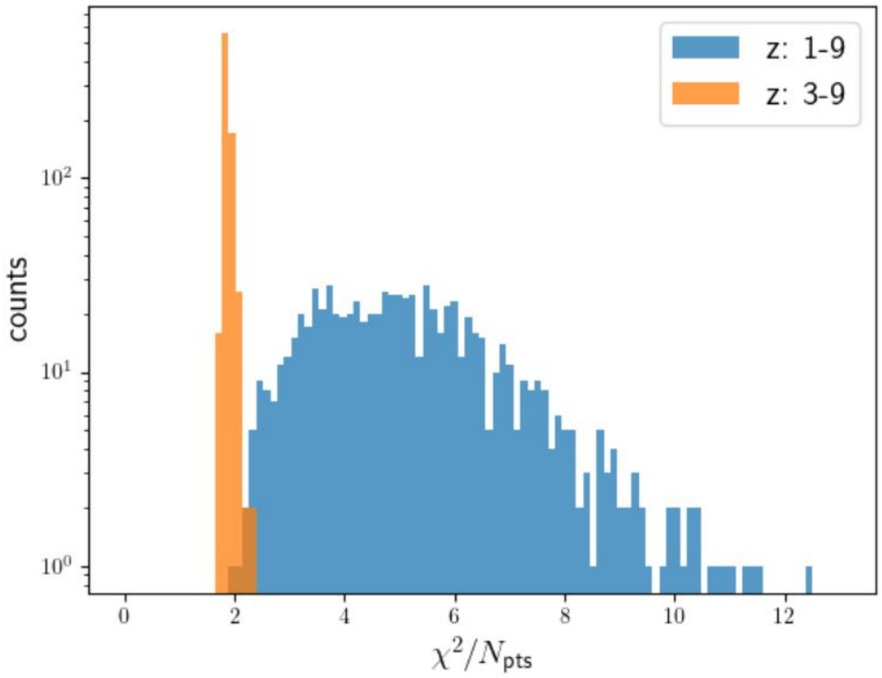
- Perturbative theory at NLO with threshold resummation for the DY with a double Mellin transform

P. C. Barry, et al., PRL 127:232001 (2021)

z-Cut Justification

- Attempting a fit with z = 1a and 2a results in a large χ^2/N_{pts} with a very large spread: 5.2(17)

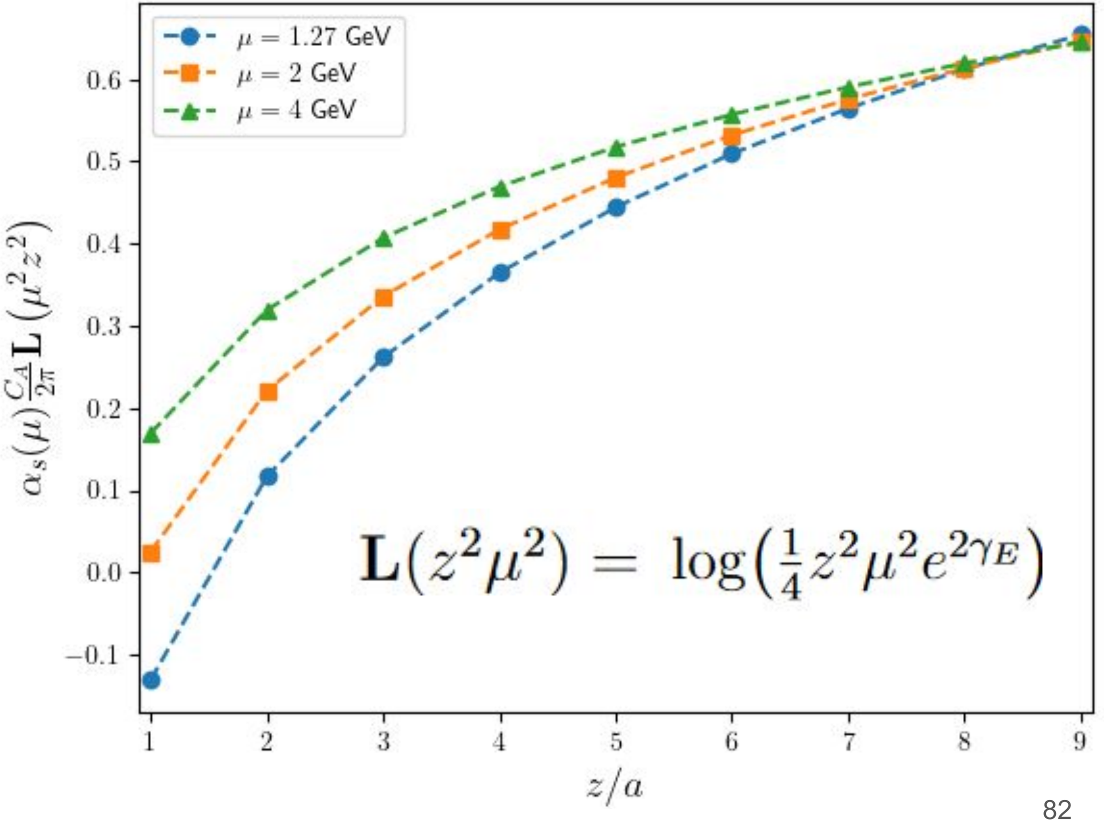
- The small-z have large discretization effects!



High-z Validity

$$R_{gg}(y, z^2\mu^2) = \cos y - \frac{\alpha_s}{2\pi} C_A \left\{ \left[\ln\left(z^2\mu^2 \frac{e^{2\gamma_E}}{4} \right) \right] R_B(y) + R_L(y) + R_C(y) \right\}$$

- We can plot the leading coefficient in the matching kernel expansion, which grows with z
- The term that multiplies this coefficient is O(1)
- This supports that we need the higher twist effects, but we aren't breaking our perturbative expansion



Systematic Parameterization

- Used the form from **P. C. Barry, et al., PRD 105:114051 (2022)**

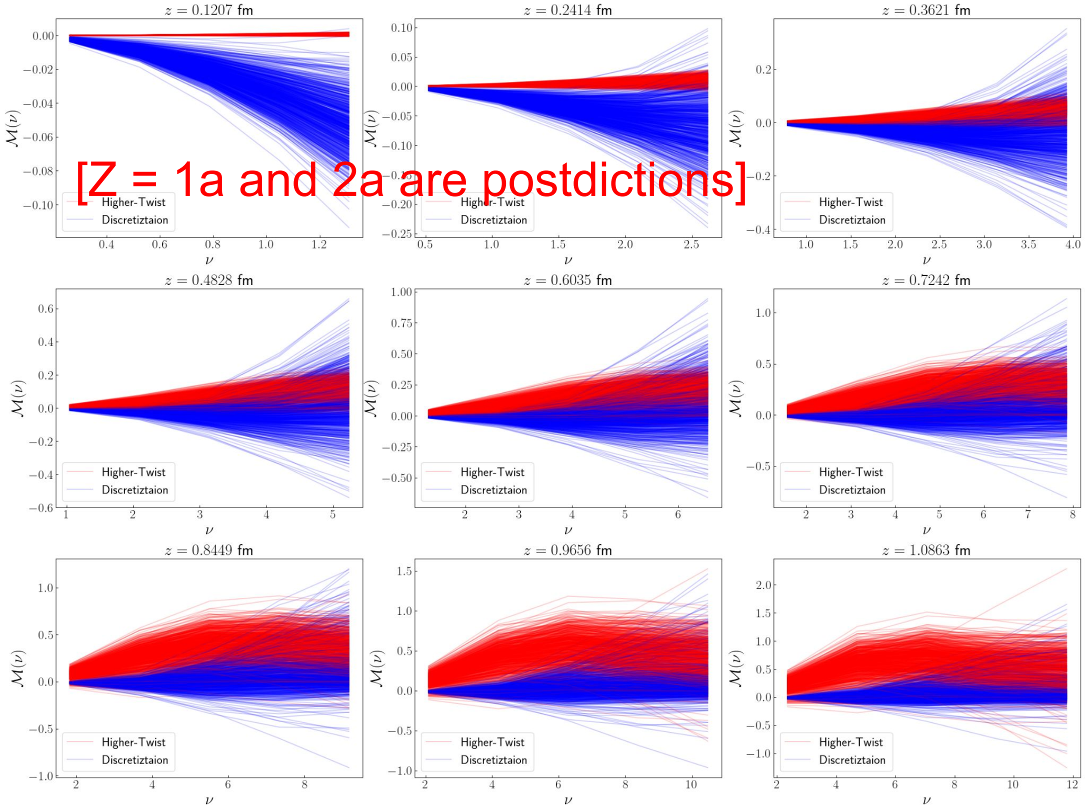
$$\mathrm{Re}\,\mathfrak{M}(\nu, z^2) = \int_0^1 \mathrm{d}x\ q_v(x, \mu_{\mathrm{lat}})\,\mathcal{C}^{\mathrm{Rp\text{-}ITD}}\left(x\nu, z^2, \mu_{\mathrm{lat}}\right) + z^2 B_1(\nu) + \frac{a}{|z|} P_1(\nu)$$

$$B_1(\nu) = \sum_n \sigma_{0,n}(\nu)\, b_n,$$

$$P_1(\nu) = \sum_n \sigma_{0,n}(\nu)\, p_n,$$

$$\sigma_{0,n}(\nu) = \int_0^1 \mathrm{d}x \cos(\nu x)\, x^a (1-x)^b\, J_n^{(a,b)}(x)$$

Systematic Effects

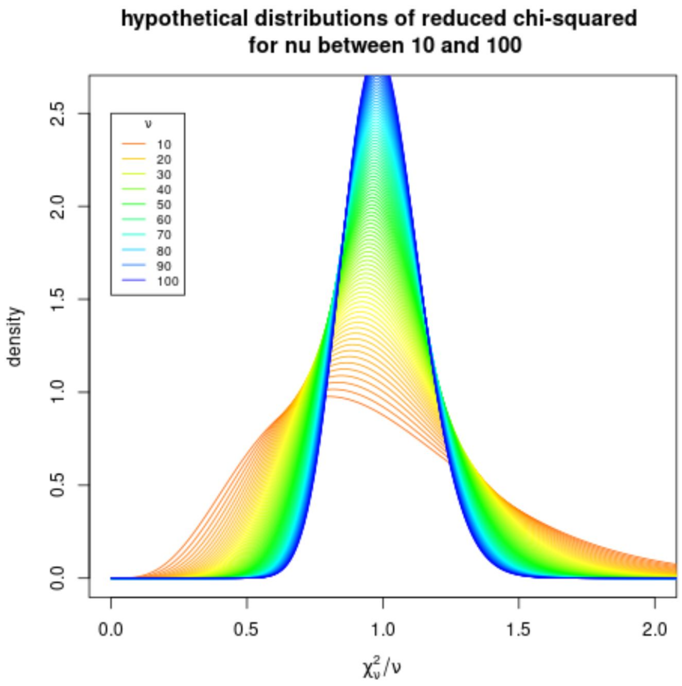


[Z = 1a and 2a are postdictions]

- Discretization effects pull the RpITD down at small z (and don’t do much at high-z)
- Higher-twist effects push the RpITD upwards at intermediate to larger-z (and don’t do much at the smallest-z)

Z-Score

- The chi^2/dof distribution changes width as the dof changes, so chi^2/dof is not necessarily the full picture
- The Z-score gives the number of standard deviations, your value of chi^2/dof is away from 1
- Ex. A chi^2/dof of 1.5 is reasonable with 10 points, but not so much 100 points



Hybrid-Ratio Scheme

Ji, *et al*. Nucl. Phys. B. 964:115311 (2021)

- The hybrid-ratio scheme is a renormalization scheme which handles the linear divergence from the Wilson line self energy at long distances
- We renormalize the quasi-PDF matrix elements as:

$$h^R(z,P_z) = \begin{cases} \frac{h^{\mathrm{B}}(0,0)}{h^{\mathrm{B}}(0,P_z)} \frac{h^{\mathrm{B}}(z,P_z)}{h^{\mathrm{B}}(z,0)} & z \le z_s \\ \frac{h^{\mathrm{B}}(0,0)}{h^{\mathrm{B}}(0,P_z)} \frac{h^{\mathrm{B}}(z,P_z)}{h^{\mathrm{B}}(z_s,0)} \times e^{(\delta m + m_0)(z - z_s)} & z > z_s \end{cases}$$

- z_s is a distance scale, before which the divergence is mostly ignorable
  - Should not be much more than ~0.3 fm
- $\delta m + m_0$ can be fit by matching to the Wilson coefficients for the given operator
- The hybrid-ratio scheme agrees with the standard ratio scheme for $z_s \to \infty$