EXPLORING THE ROLE OF THE PRE-EQUILIBRIUM PHASE IN DETERMINING NUCLEAR GEOMETRY IN HIGH-ENERGY COLLISIONS

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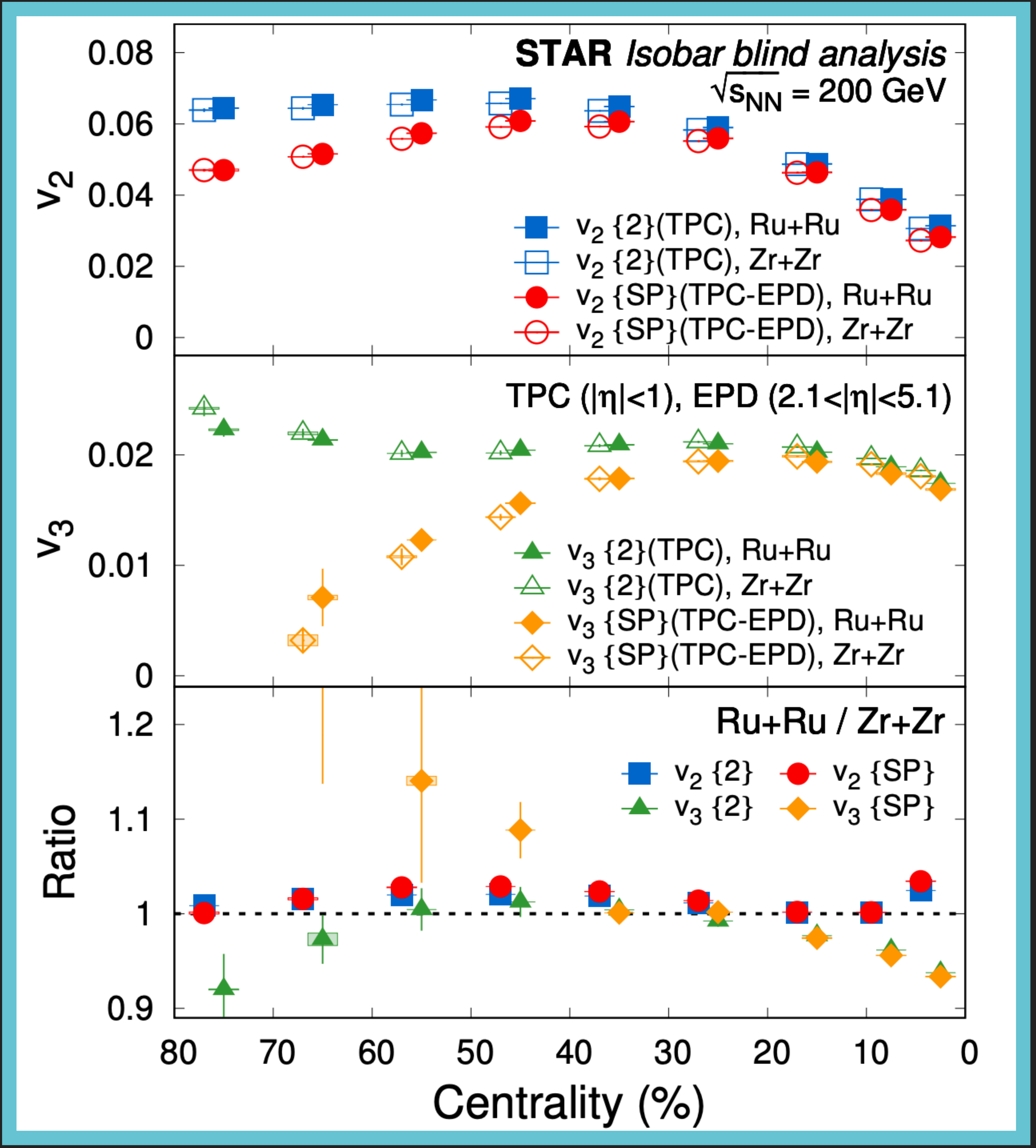
Project 2024/08903-4
FAPESP
SÃO PAULO RESEARCH FOUNDATION

STAR RESULTS

- ${}^{96}_{44}\text{Ru} + {}^{96}_{44}\text{Ru}$ and ${}^{96}_{40}\text{Zr} + {}^{96}_{40}\text{Zr}$ at $\sqrt{s_{NN}} = 200\ GeV$

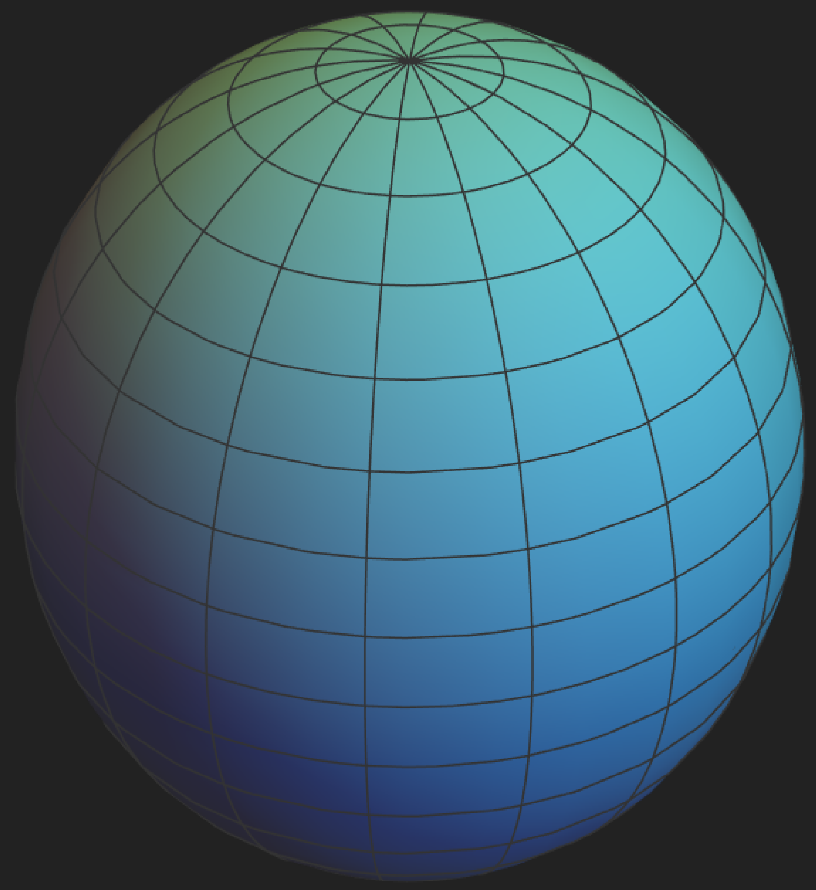
Search for the chiral magnetic effect with isobar collisions at $\sqrt{s_{NN}} = 200$ GeV by the STAR Collaboration at the BNL Relativistic Heavy Ion Collider

M. S. Abdallah *et al.* (STAR Collaboration)
Phys. Rev. C **105**, 014901 – Published 3 January 2022



PRC 105, 014901

${}^{96}_{44}\text{Ru}$



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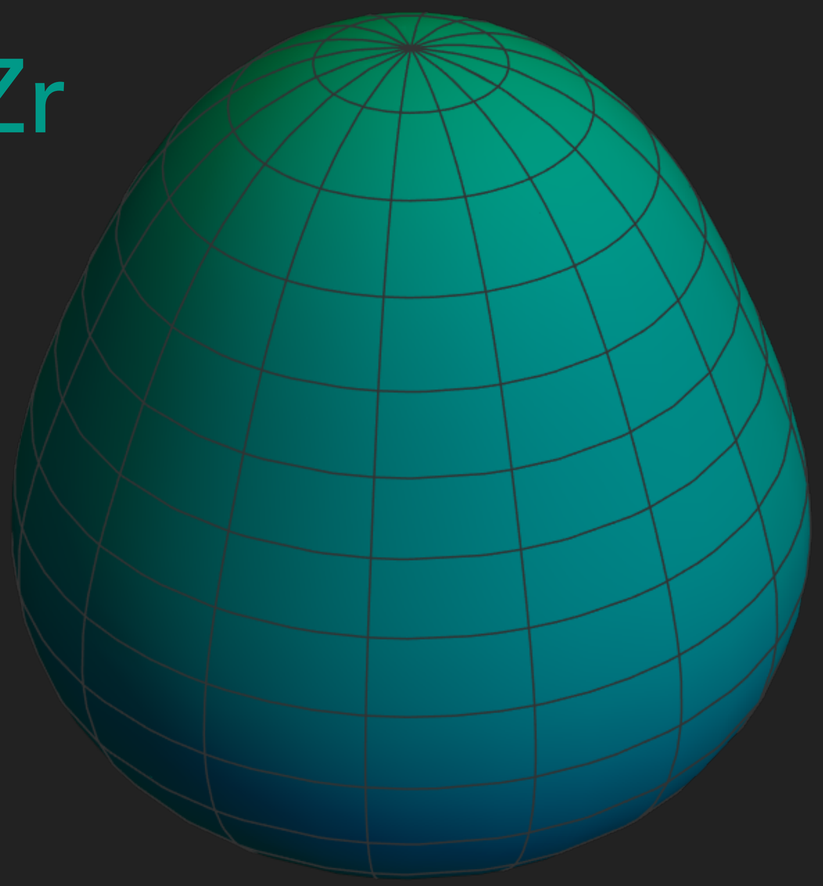


HOW DOES THE INITIAL STATE GEOMETRY AFFECTS THE FINAL STATE OBSERVABLES?

OBJECTIVES

- Numerical simulations
- To perform a systematic analysis of how differences in initial state geometry are carried out to the final state
- To study how observables sensitive to nuclear geometry are dependent on pre-equilibrium, hydrodynamics and hadronic transport

SIMULATION CHAIN

Thermalized QGP evolution

Decays, resonances and scatterings

$\tau = \sqrt{t^2 - z^2} \sim 0\, fm/c$

Pre-equilibrium until thermalization

Particlize the freeze-out surface

ISOBAR: SIMULATION CHAIN

- (2+1)D boost invariant at $\mu_b = 0$
- XSCAPE framework Putschke et al, arXiv:1903.07706, 2019

Nucleons position as input

- JETSCAPE parametrization PRC 103, 054904 (2021)

ISOBAR: SIMULATION CHAIN

- Nucleons are sampled from a deformed Woods-Saxon distribution

$$P(r,\theta,\varphi) = \frac{\rho_0}{1 + \exp\left\{[r - \mathscr{R}(\theta,\varphi)]/a\right\}}$$

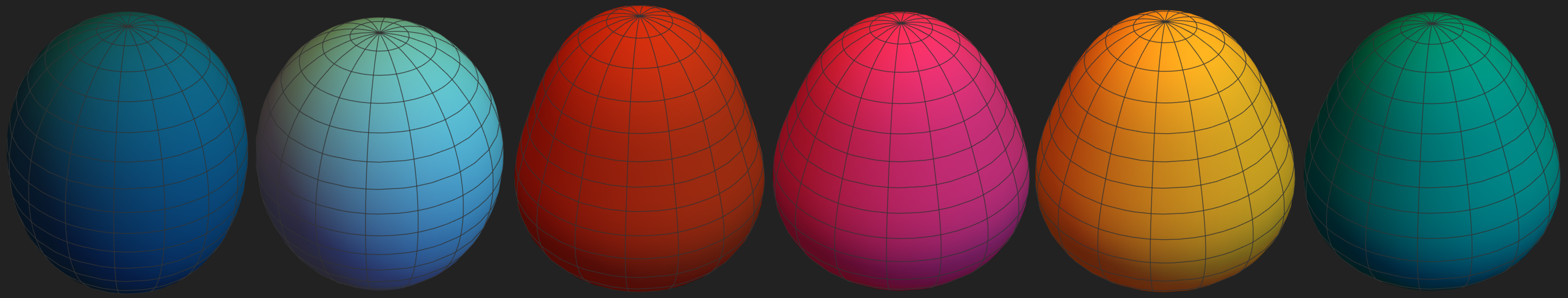
$$\mathscr{R}(\theta,\varphi) = R_0 \left\{ 1 + \beta_2 \left[Y_2^0\left(\theta,\varphi\right) \cos\gamma + \frac{2}{\sqrt{2}} \sin\gamma \, \Re Y_2^2\left(\theta,\varphi\right) \right] + \beta_3 Y_3^0(\theta,\phi) \right\}$$

No neutron-skin included

- T_RENTo input

ISOBAR: SIMULATION CHAIN

	R_0 (fm)	a (fm)	β_2	β_3	γ
Case 1	5.09	0.46	0.16	0	$\pi/6$
Case 2	5.09	0.46	0.16	0	0
Case 3	5.09	0.46	0.16	0.20	0
Case 4	5.09	0.46	0.06	0.20	0
Case 5	5.09	0.52	0.06	0.20	0
Case 6	5.02	0.52	0.06	0.20	0



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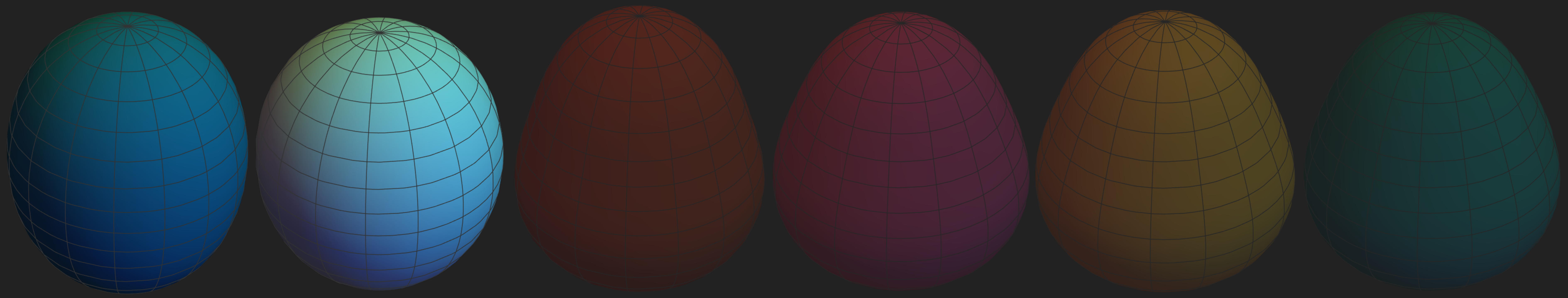

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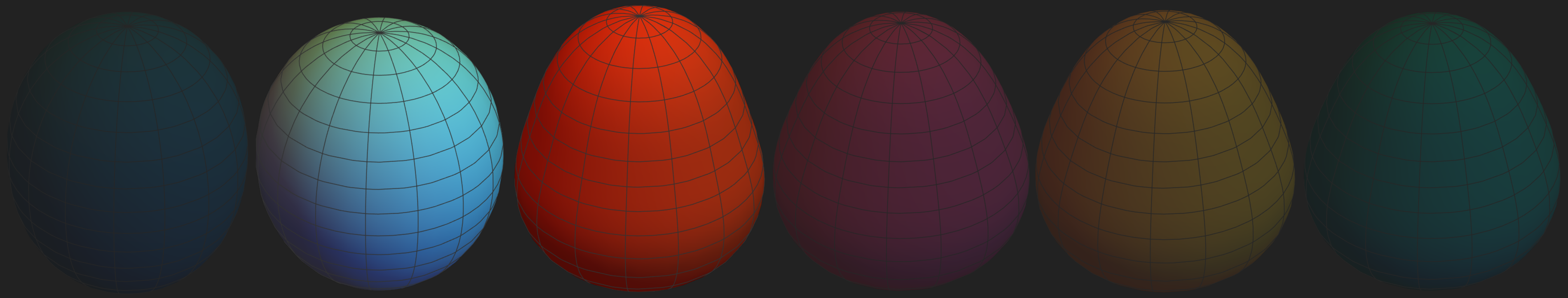

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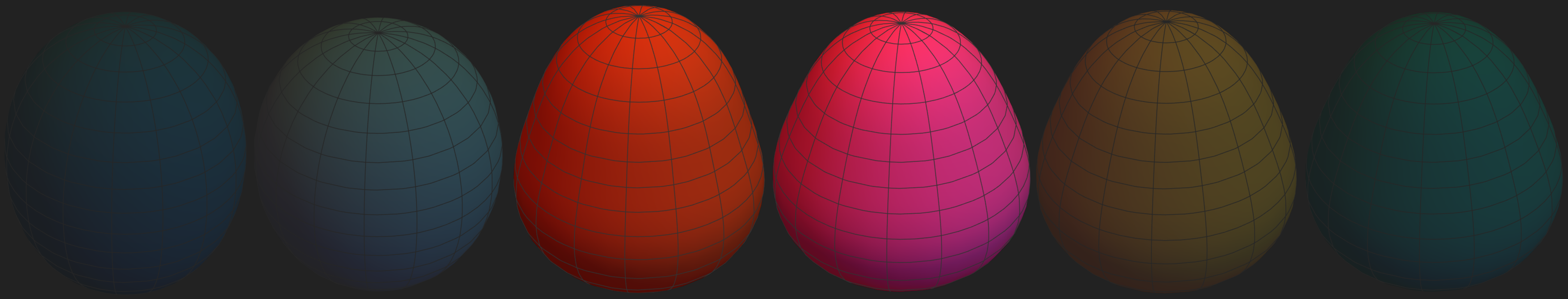

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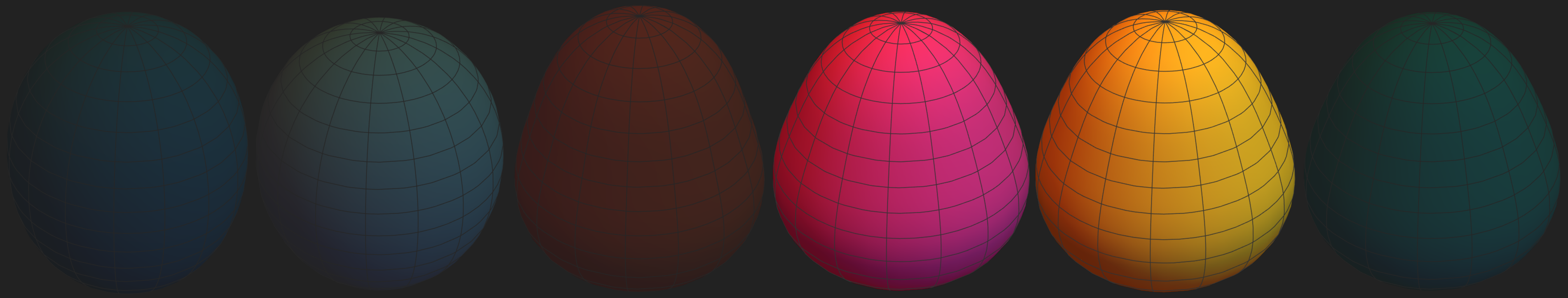

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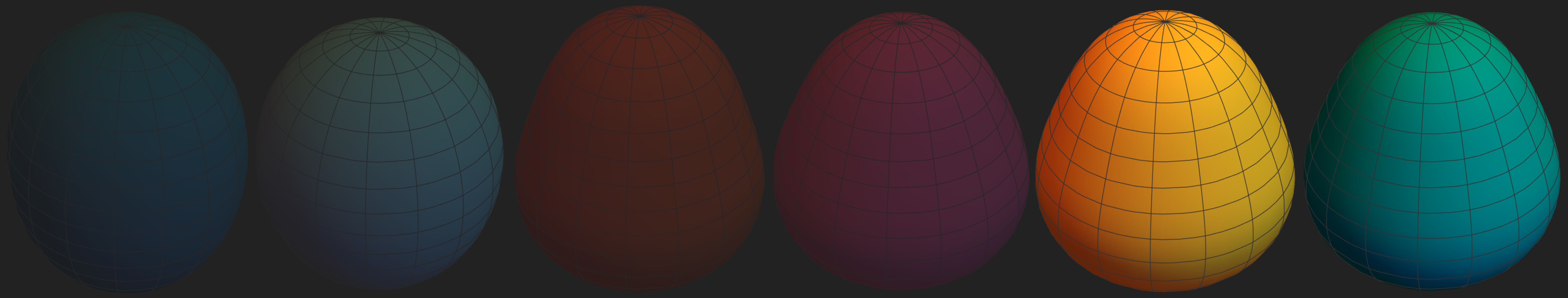
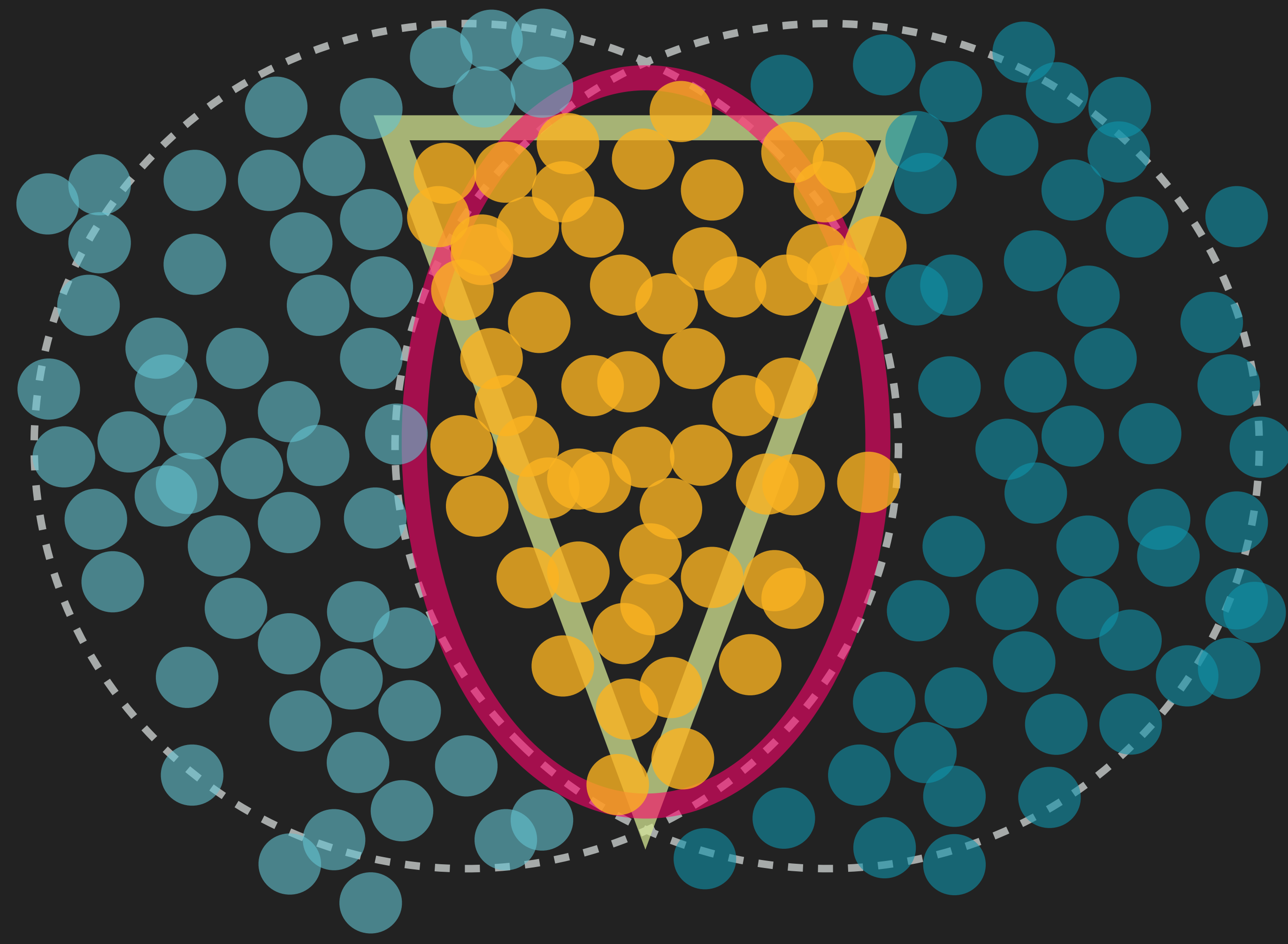


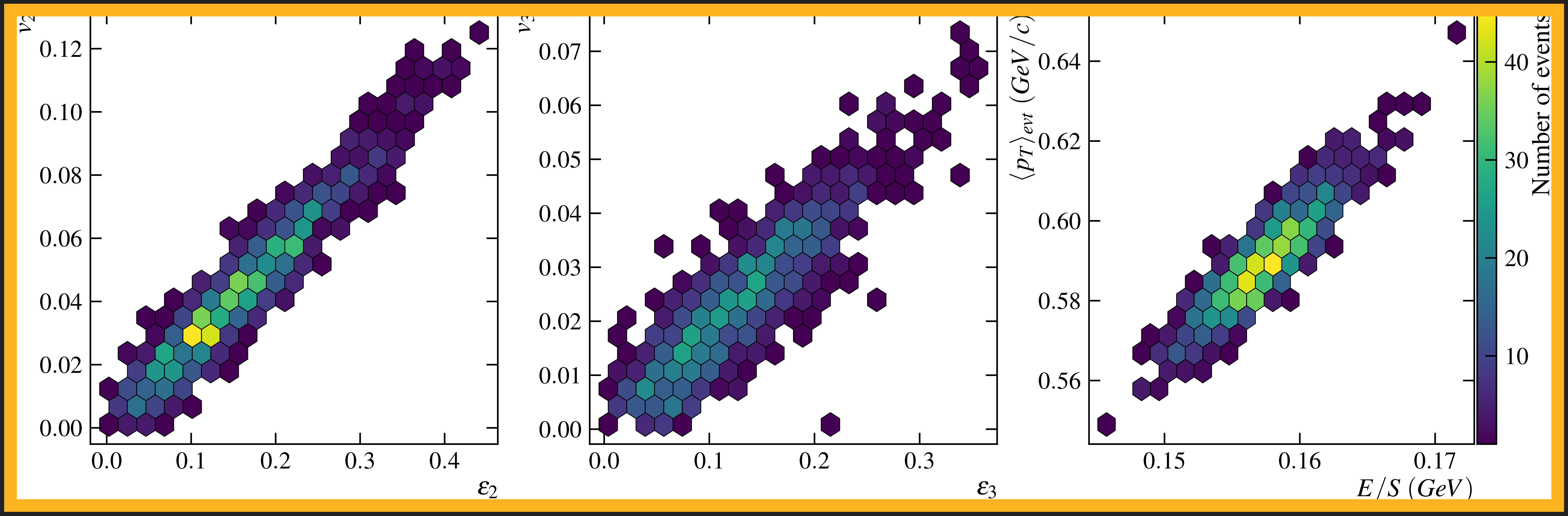
INITIAL CONDITION MAPPING

$$\varepsilon_n = \frac{|\{r^n e^{in\phi}\}|}{\{r^n\}}$$

- Relations between initial and final state observables

$$\langle p_T \rangle_{event} = \kappa_{p_T} \frac{E}{S} \qquad v_n = \kappa_n \varepsilon_n$$





INITIAL CONDITION MAPPING

- $\langle p_T \rangle_{event} \rightarrow \kappa_{p_T} E/S$
- $v_n \rightarrow \kappa_n \varepsilon_n$

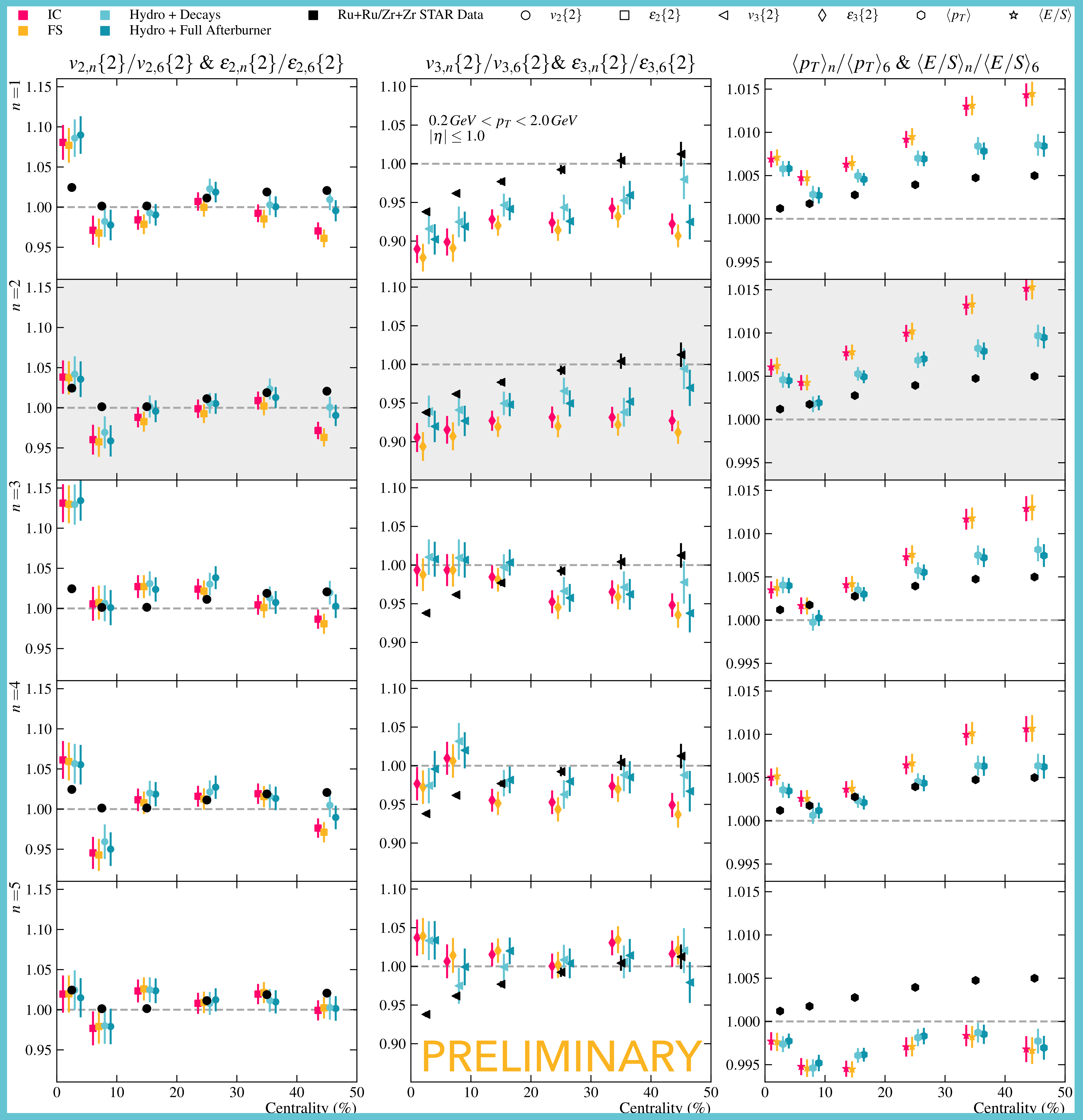
$$\rho_n^{IC} = \frac{\langle \varepsilon_n^2 E/S \rangle - \langle \varepsilon_n^2 \rangle \langle E/S \rangle}{\sigma_{\varepsilon_n^2} \sigma_{E/S}}$$

$$\varepsilon NSC(2,3) = \frac{\langle \varepsilon_2^2 \varepsilon_3^2 \rangle - \langle \varepsilon_2^2 \rangle \langle \varepsilon_3^2 \rangle}{\langle \varepsilon_2^2 \rangle \langle \varepsilon_3^2 \rangle}$$

$$\sigma_{\varepsilon_n^2} = \sqrt{\langle \varepsilon_n^4 \rangle - \langle \varepsilon_n^2 \rangle^2}$$

$$\sigma_{E/S} = \sqrt{\langle (E/S)^2 \rangle - \langle E/S \rangle^2}$$

~ 20k events
for each case



~ 20k events
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$$\frac{v_{2,n}^{Full}}{v_{2,6}^{Full}} \approx \frac{v_{2,n}^{Decays}}{v_{2,6}^{Decays}} \approx \frac{\varepsilon_{2,n}^{FS}}{\varepsilon_{2,6}^{FS}} \approx \frac{\varepsilon_{2,n}^{IC}}{\varepsilon_{2,6}^{IC}}$$

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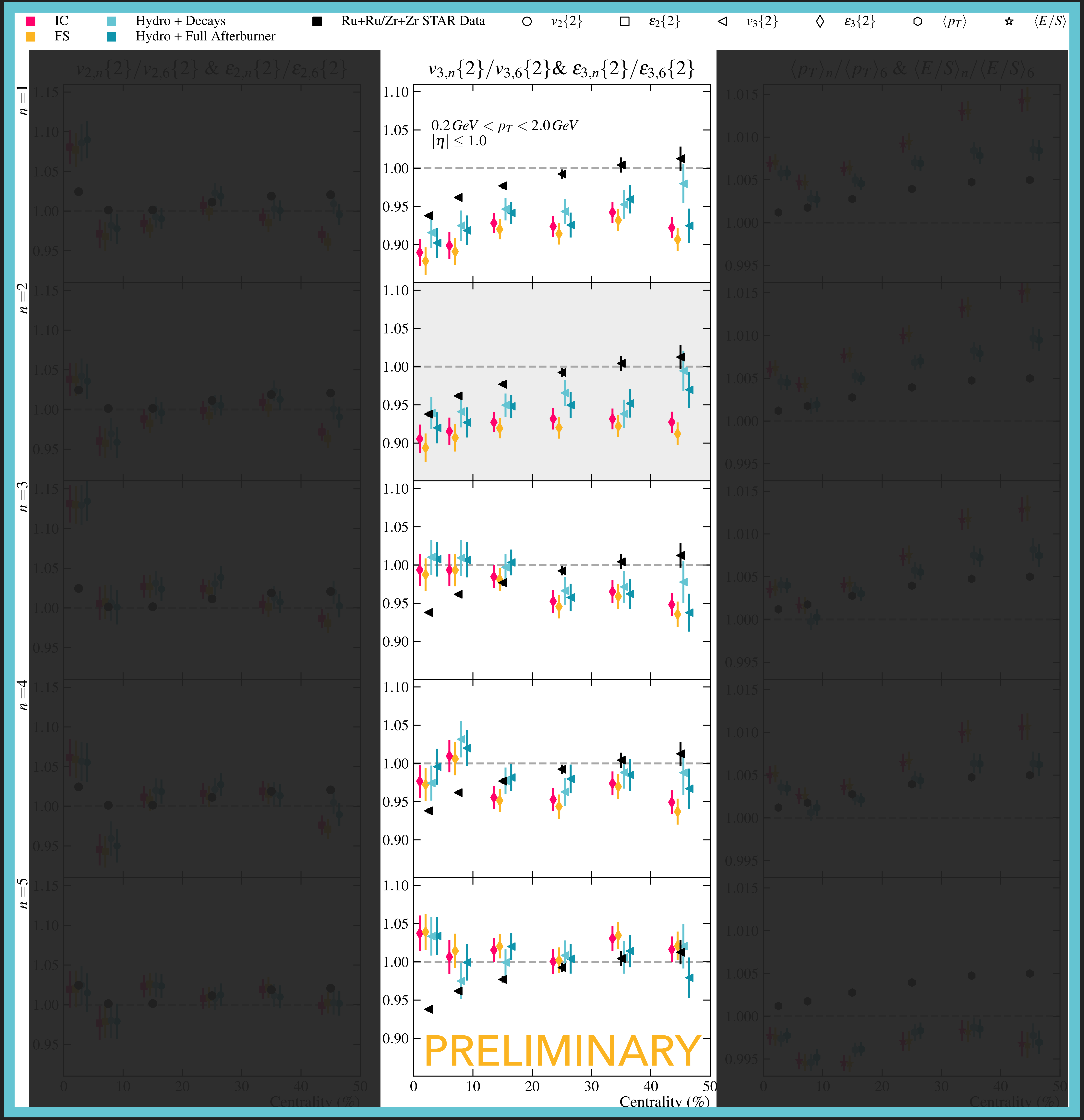

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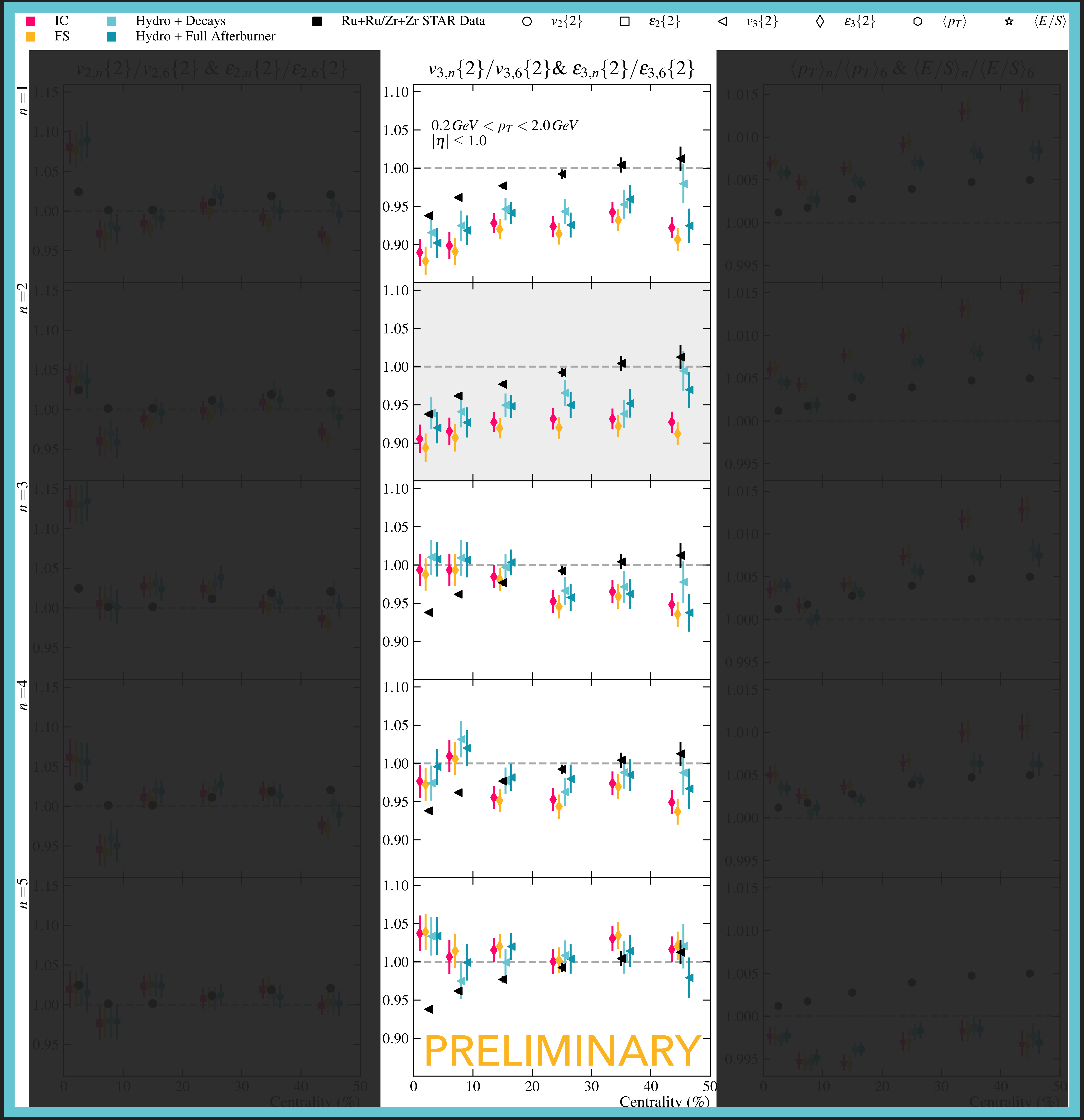
Fails for mid-central



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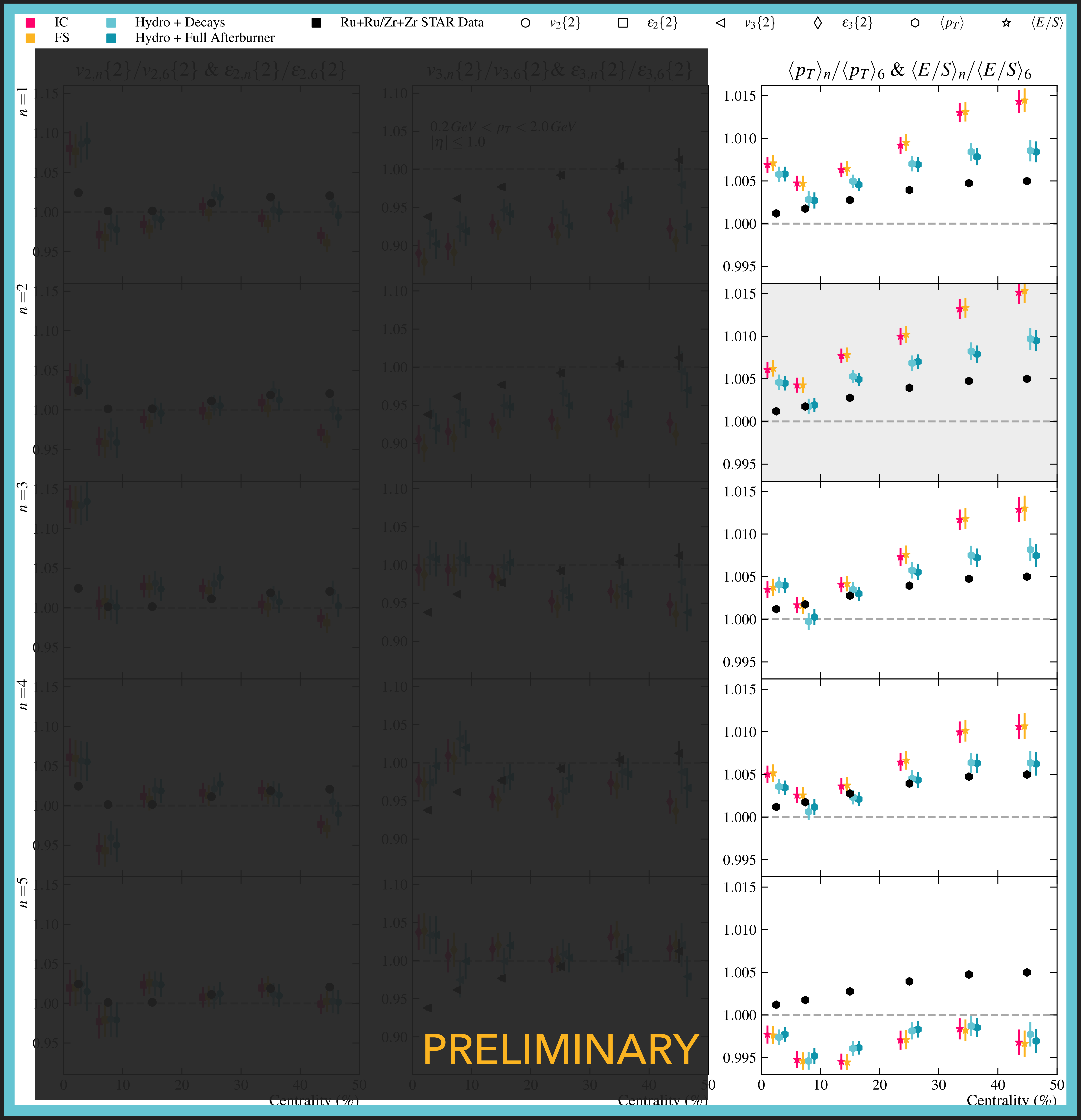

$\langle E/S \rangle$ ratios 1% greater than $\langle p_T \rangle$ ratios, which decreases for central collisions

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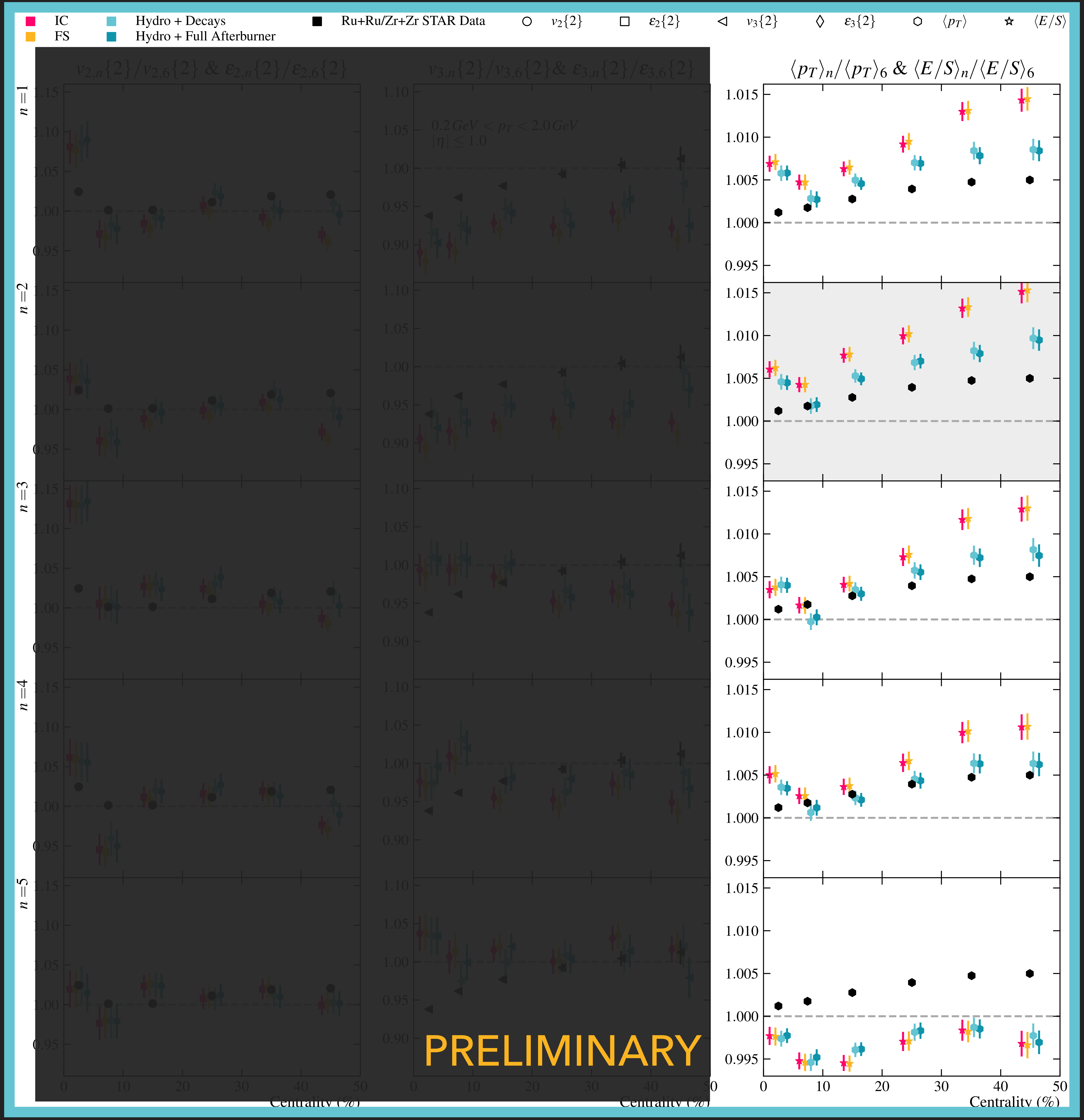

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Matches for case 5

Indicates that κ_{p_T} may depend on a

~ 20k events for each case

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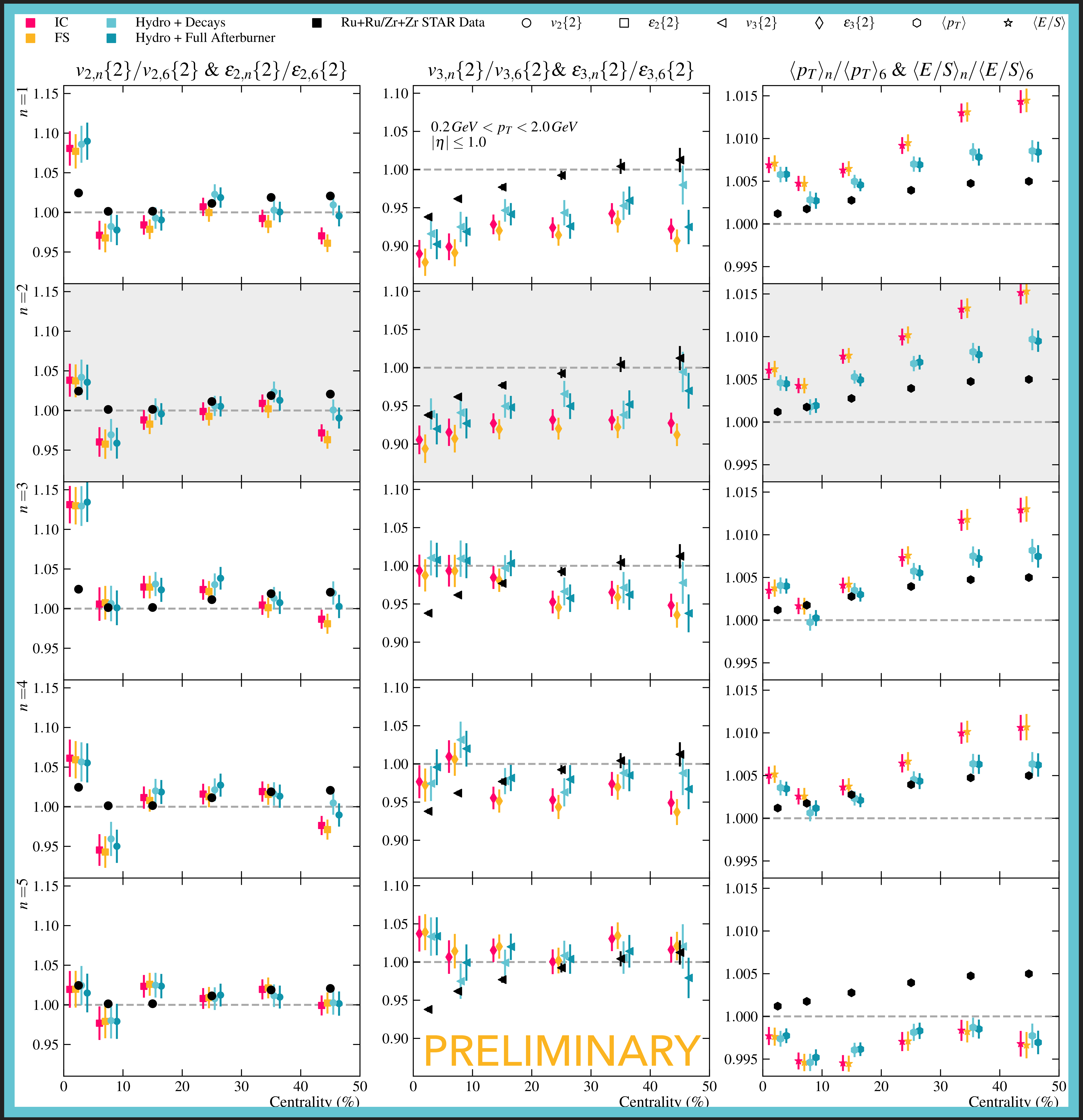


$\langle E/S \rangle$ ratios 1% greater than $\langle p_T \rangle$ ratios, which decreases for central collisions

Matches for case 5

Indicates that κ_{p_T} may depends on a

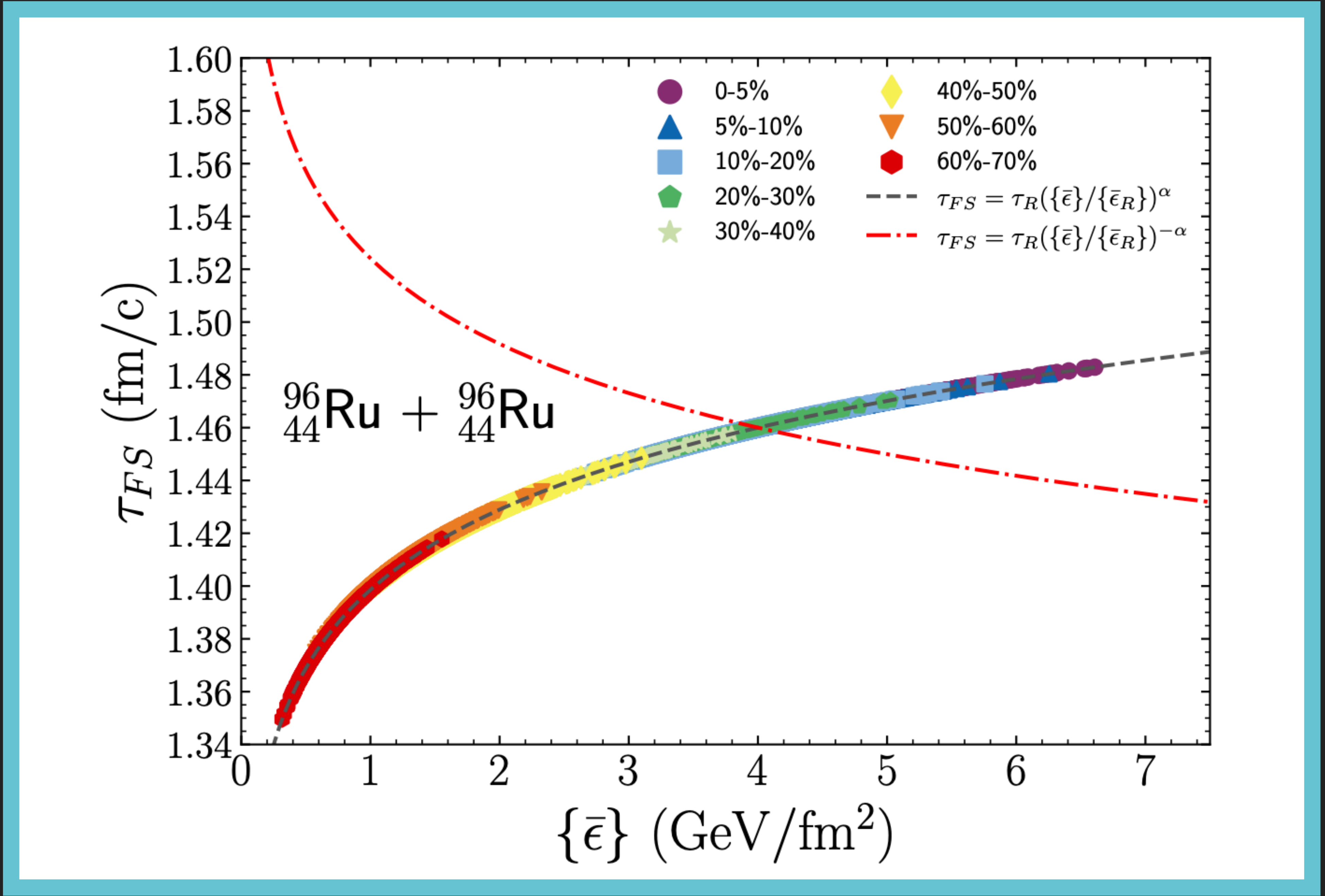
ISOBAR: RESULTS

- Results indicate that initial conditions can be used to predict observables for central collision
- Now we can focus on ICs and FS —— 10M events for each case
- Two FS time ($\tau_{switch} = 1\ fm/c$) and JETSCAPE parametrization

$$\tau_{FS} = \tau_R \left(\frac{\{\bar{\varepsilon}\}}{4\ Gev/fm^2} \right)^{\alpha}$$

τ_R: 1.46 fm/c; α: 0.031

$$\{a\} = \frac{\int dx\, dy\ \bar{\varepsilon}(x,y) a(x,y)}{\int dx\, dy\ \bar{\varepsilon}(x,y)}$$

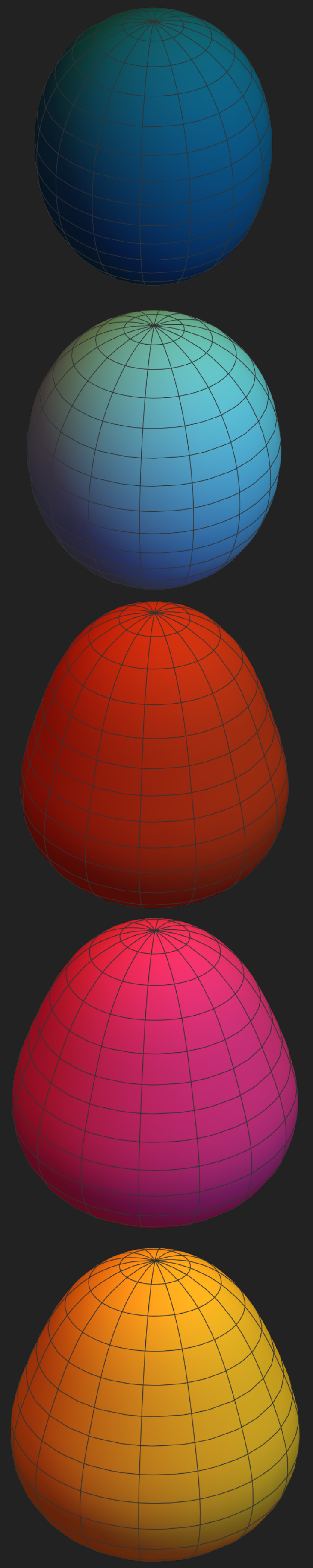


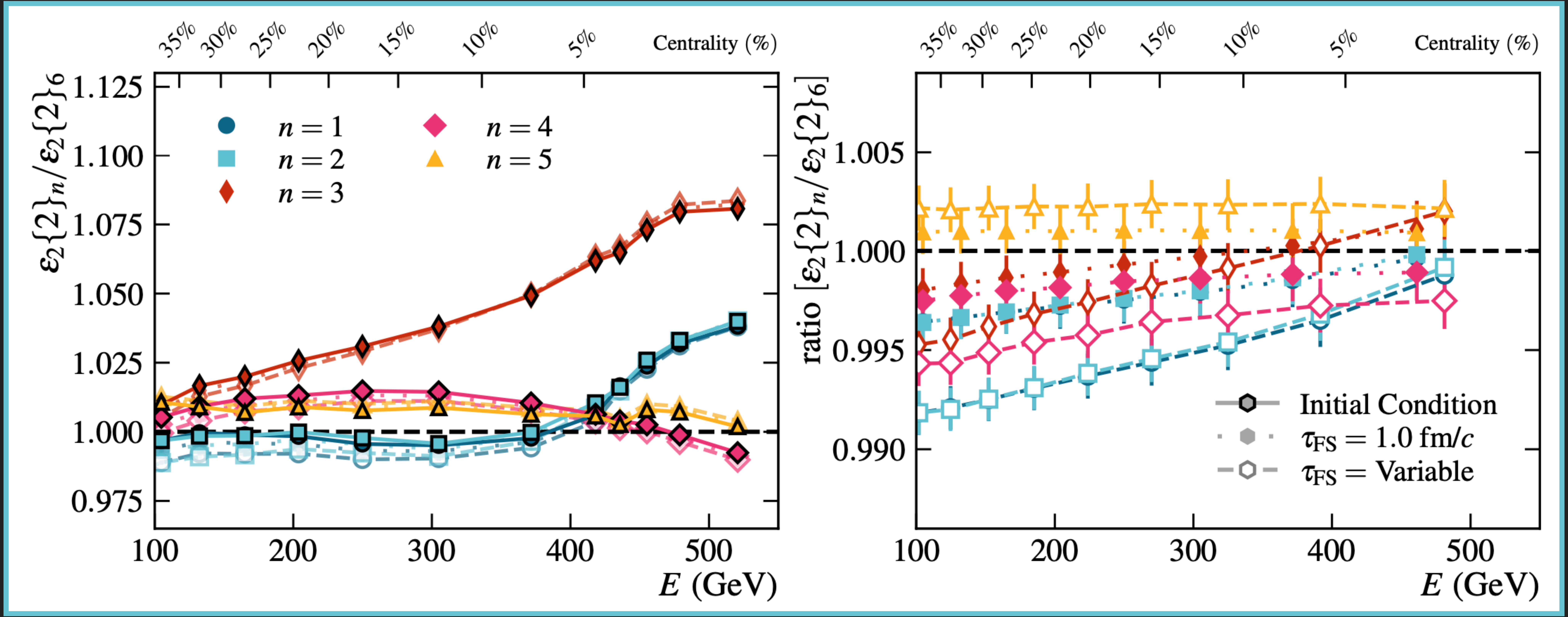
ISOBAR: RESULTS

$$\varepsilon_n\{2\} = \sqrt{\langle \varepsilon_n^2 \rangle}\,,$$

$$\varepsilon_n\{4\} = \sqrt[4]{2\langle \varepsilon_n^2 \rangle^2 - \langle \varepsilon_n^4 \rangle}\,.$$

Gardim, Giannini, Grassi, P. Pala , M. Serenone
Phys. Rev. C **110**, 064907





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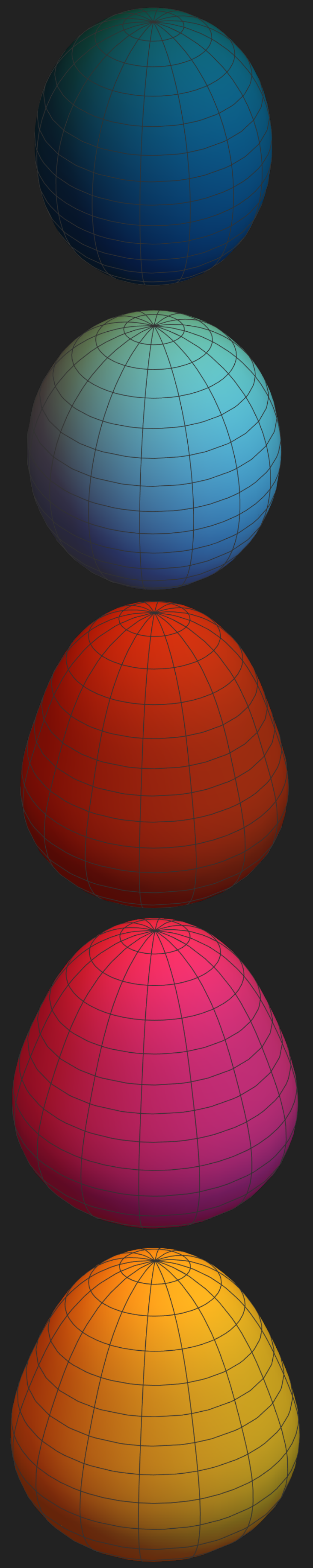

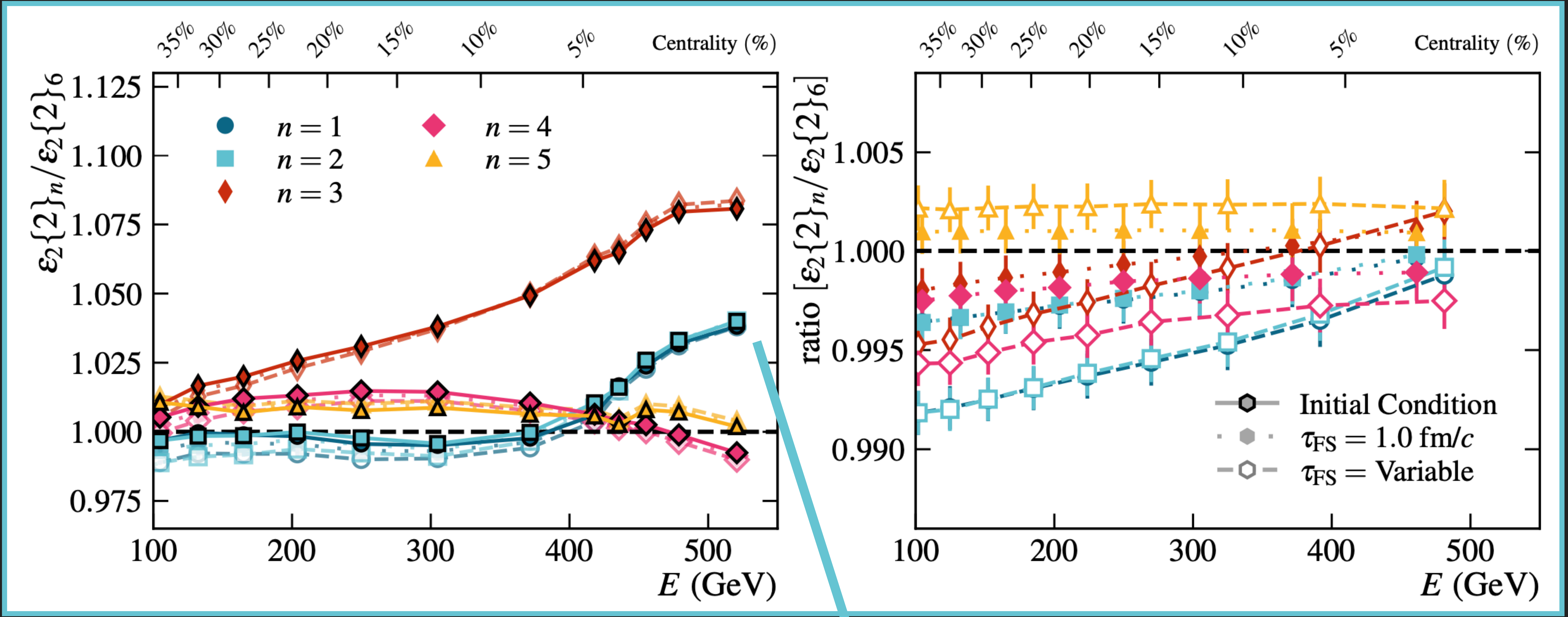

Effects of β_2

No effect of γ

ISOBAR: RESULTS

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Non trivial interplay between β_2 and β_3

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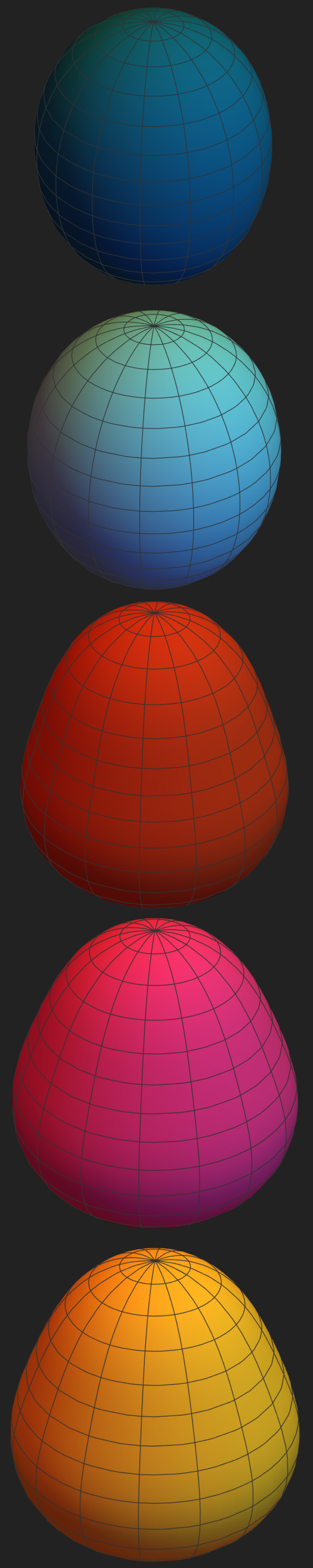

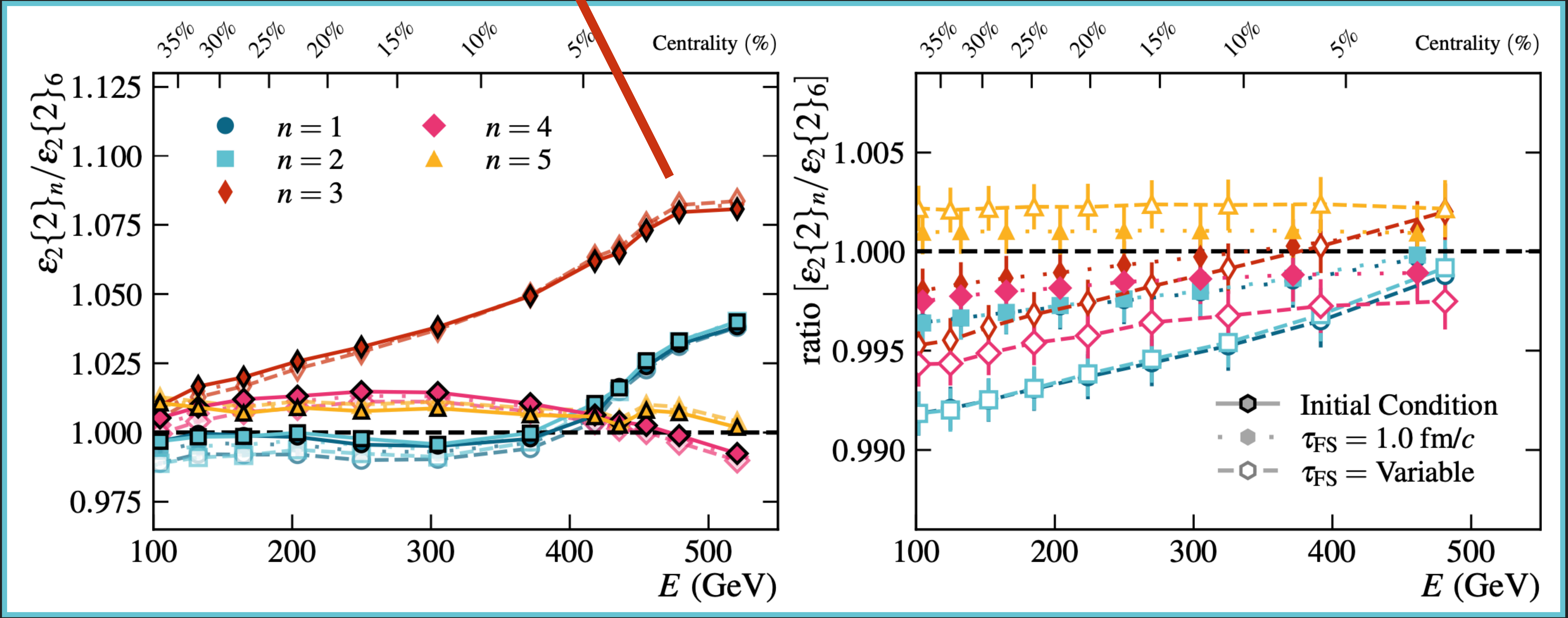

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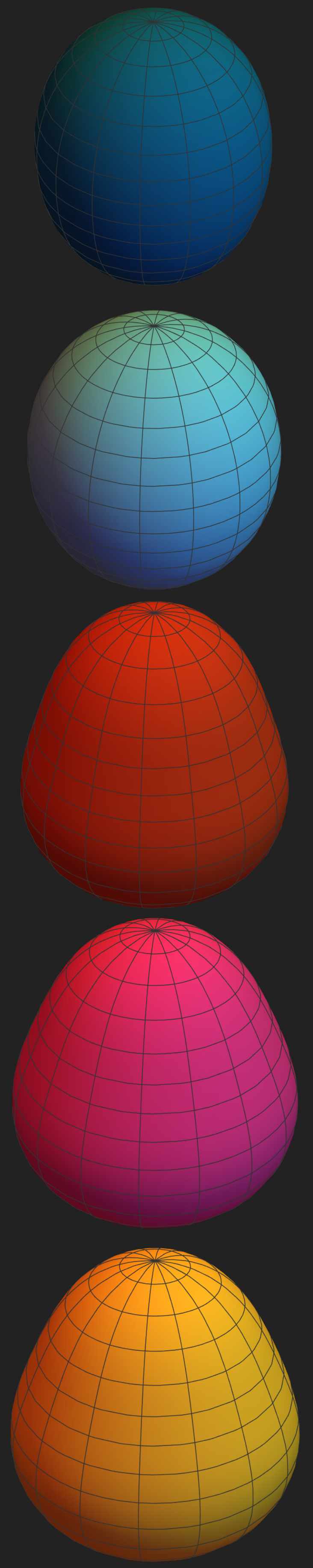

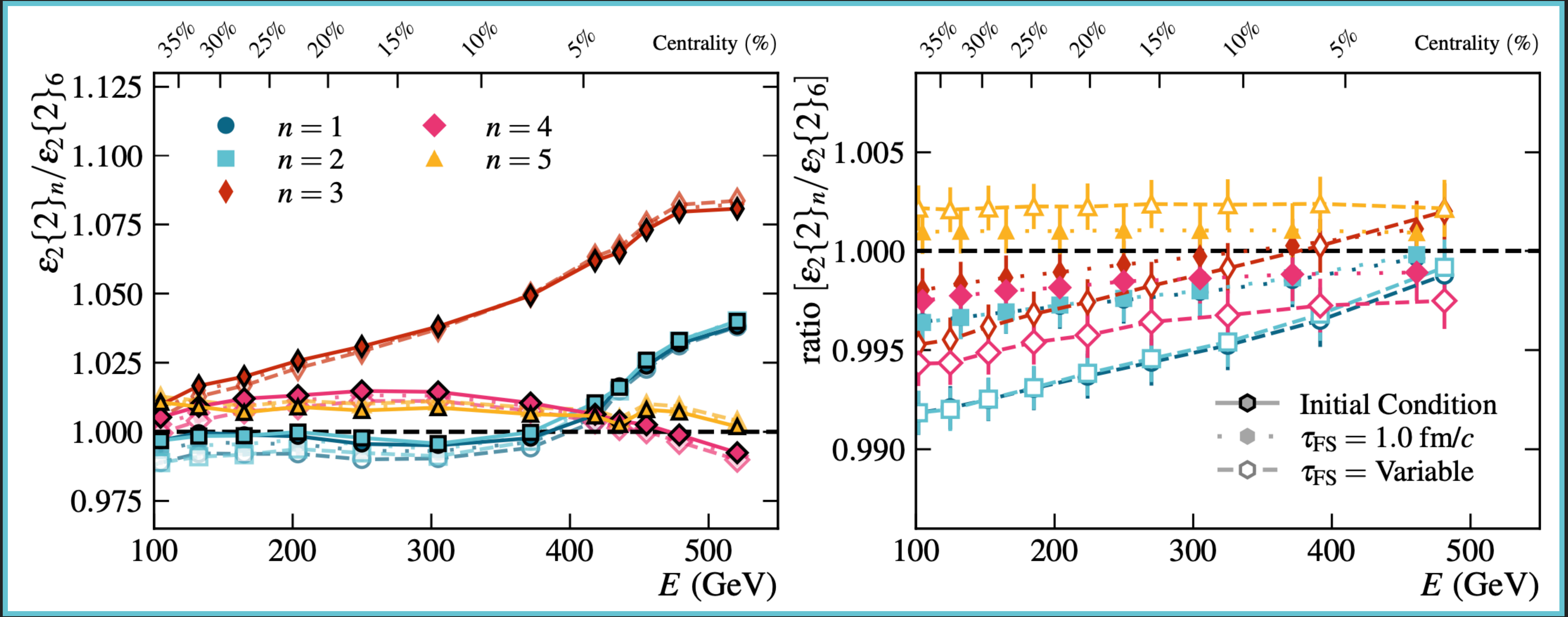

FS effects around at most 1%

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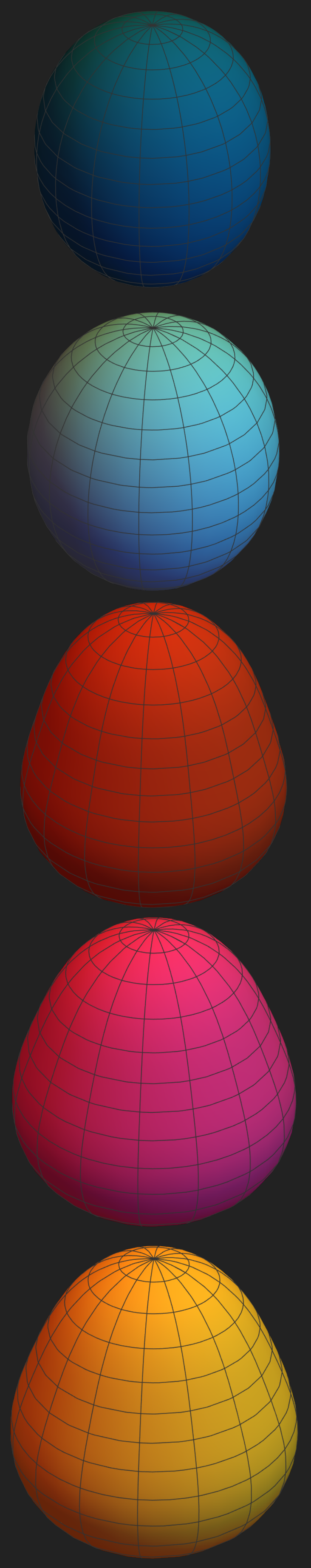

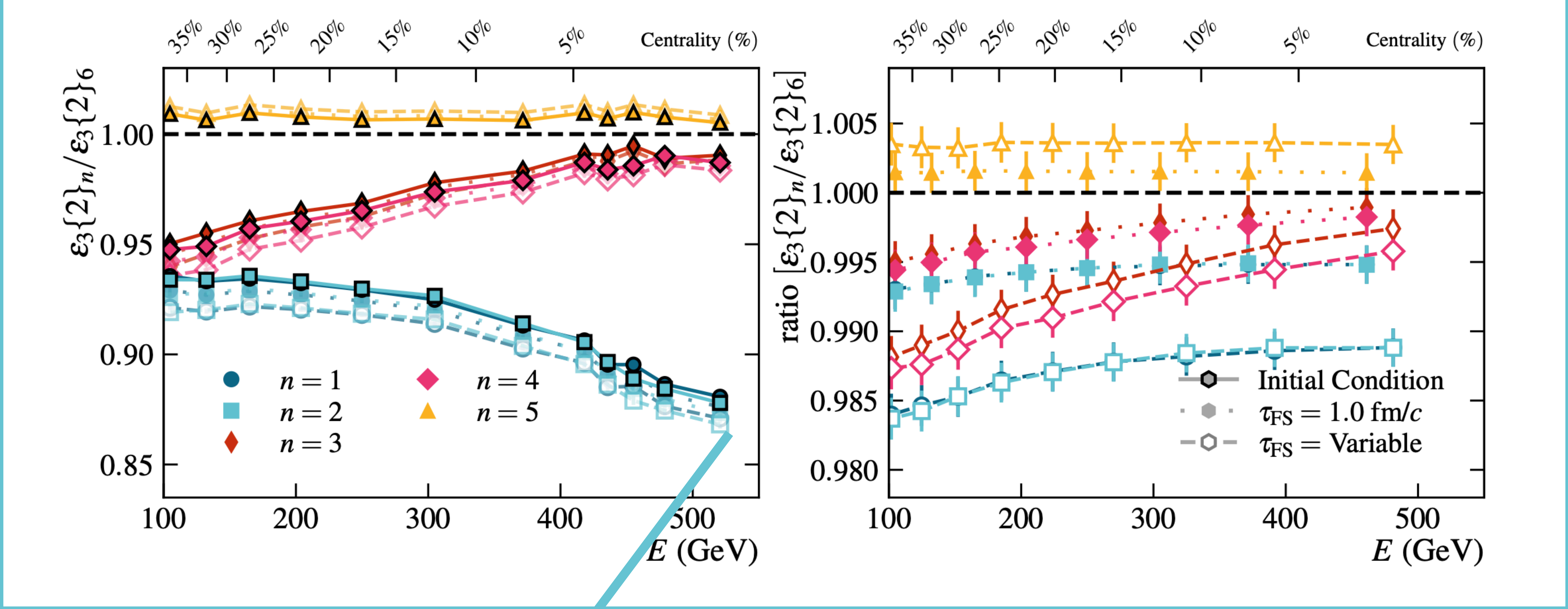

Great effect of β_3

ISOBAR: RESULTS

Sensitive to a

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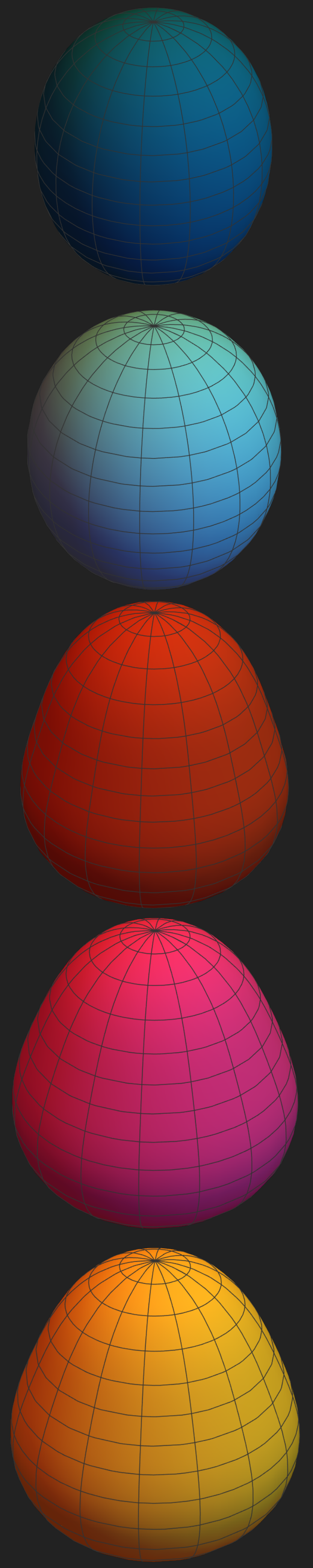

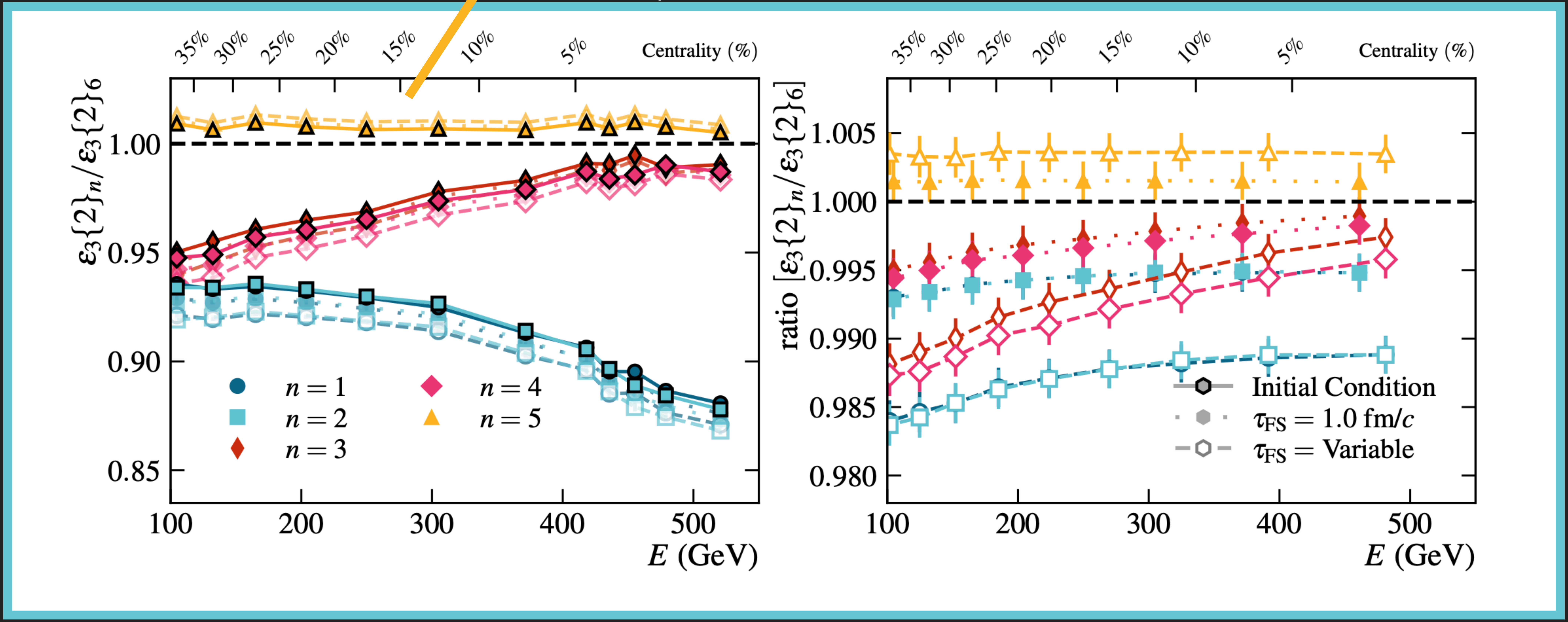

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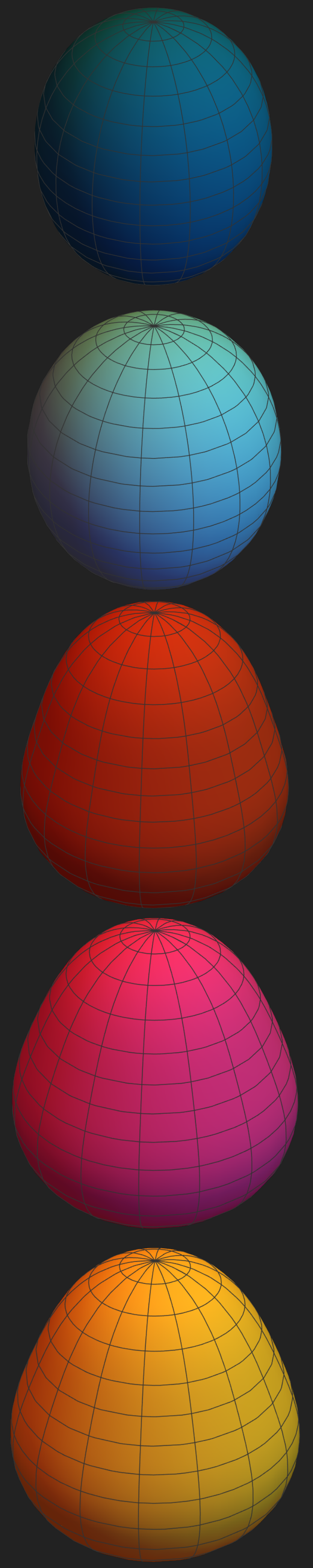

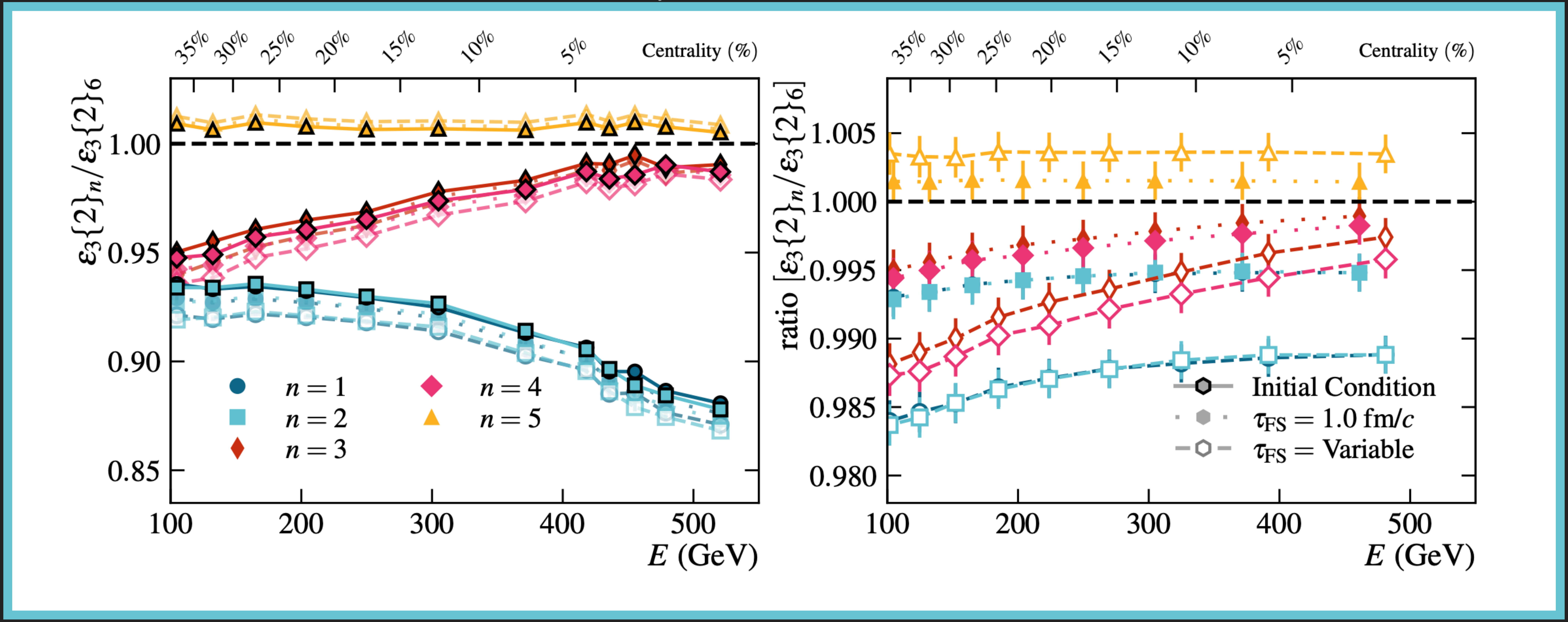

Increase in FS effects

ISOBAR: RESULTS

Sensitivity to a

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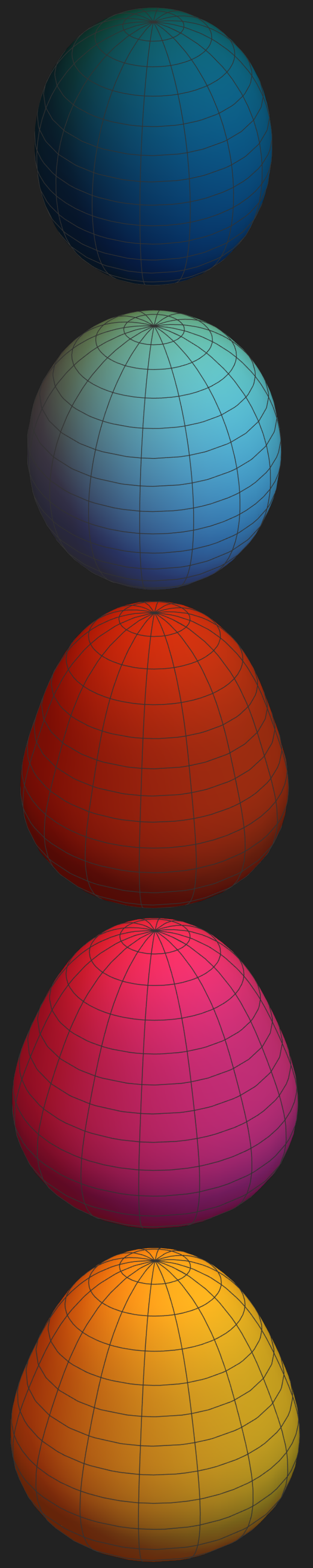

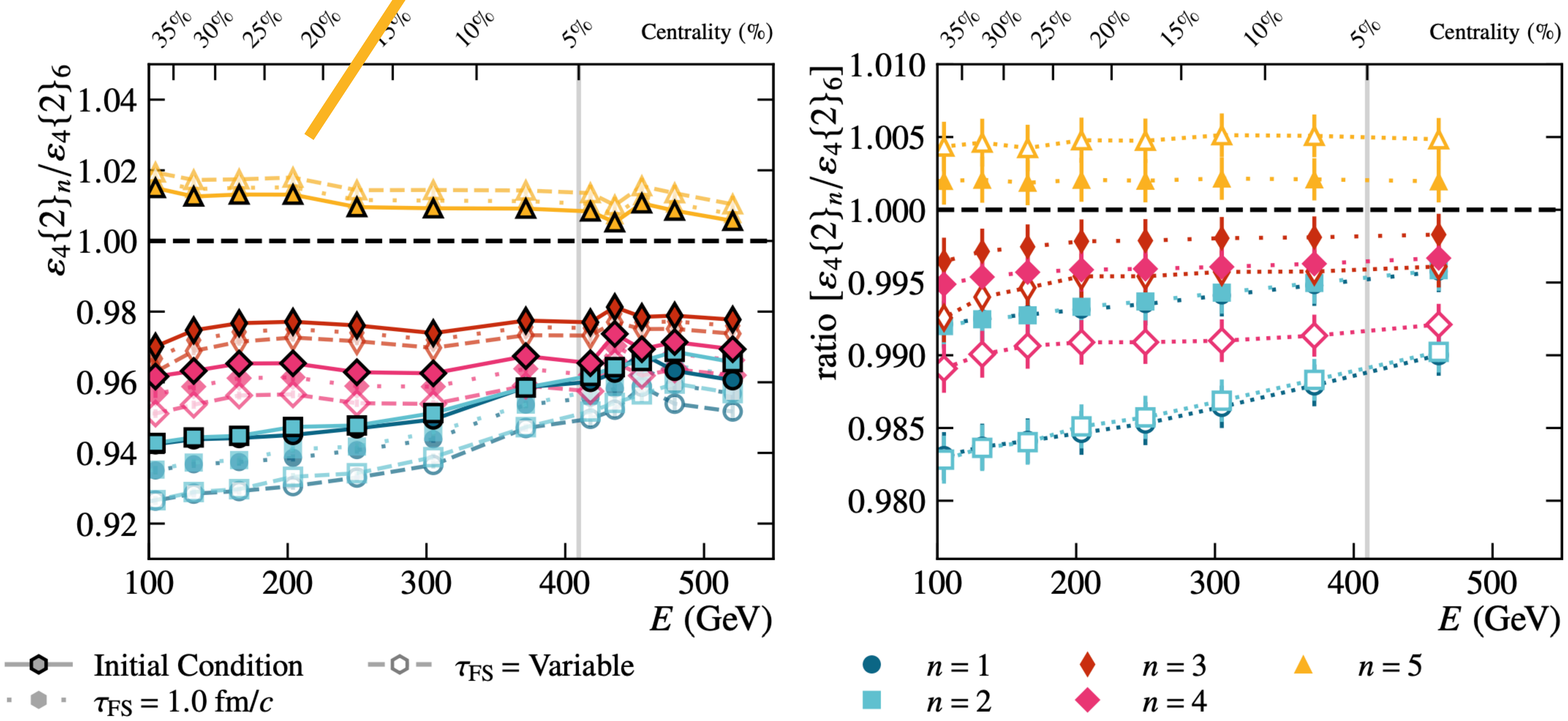

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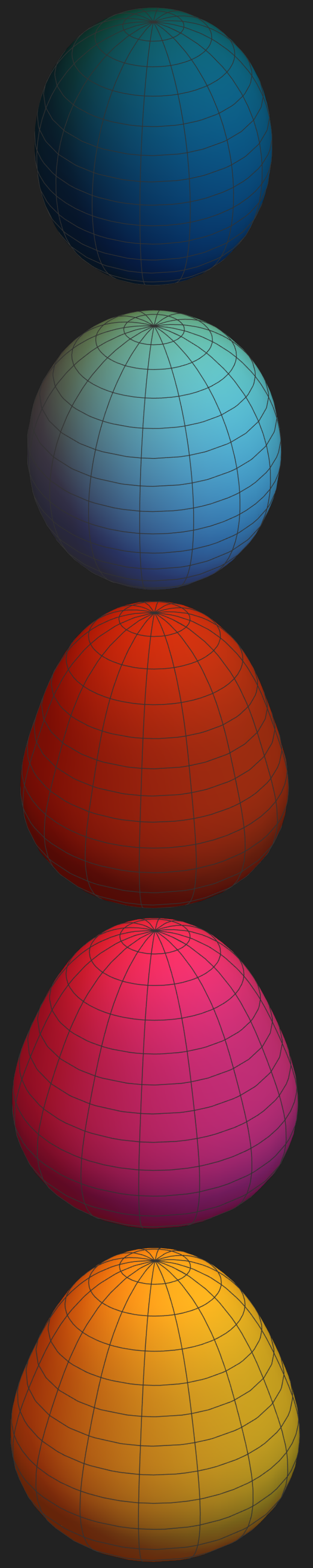

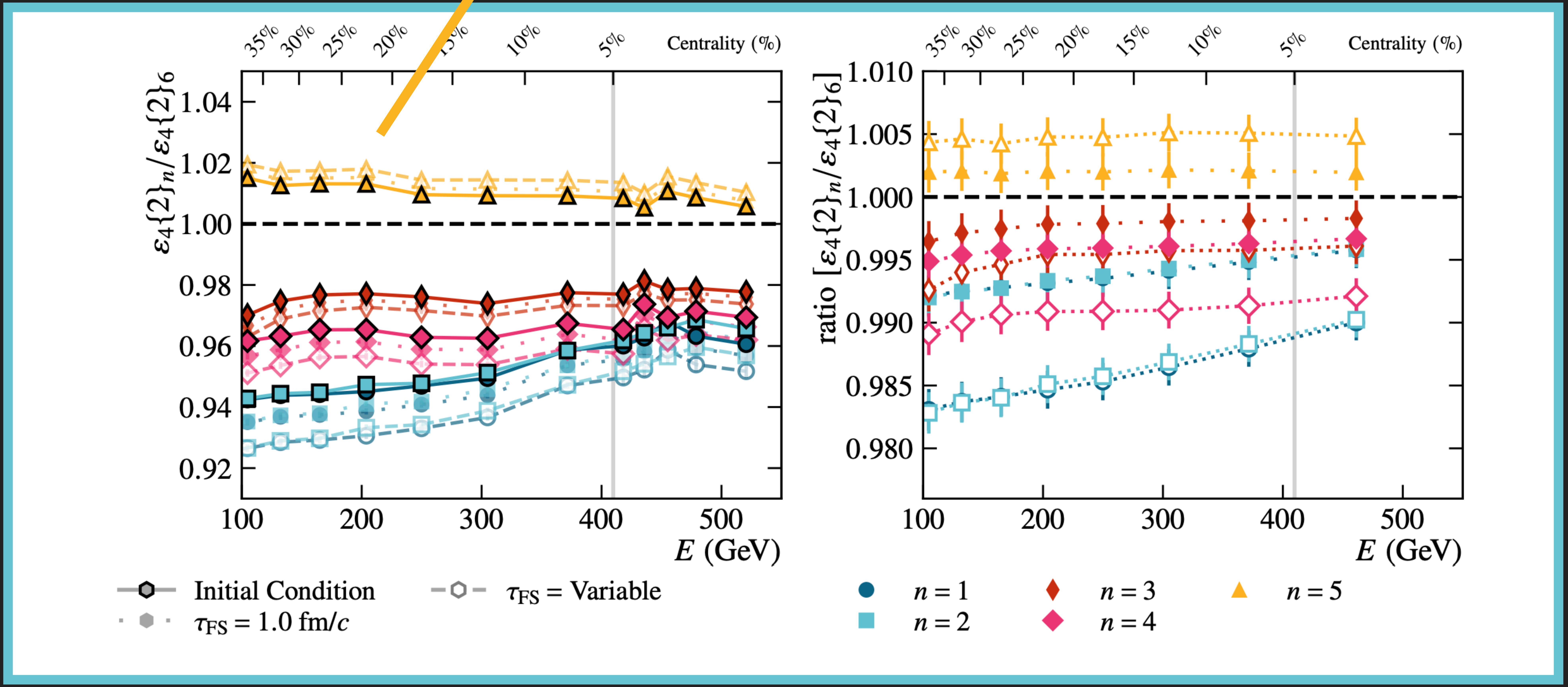

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Even stronger FS effect

FS effect increases with n

ISOBAR: RESULTS

$$\varepsilon NSC(2,3) = \frac{\langle \varepsilon_2^2 \varepsilon_3^2 \rangle - \langle \varepsilon_2^2 \rangle \langle \varepsilon_3^2 \rangle}{\langle \varepsilon_2^2 \rangle \langle \varepsilon_3^2 \rangle}$$

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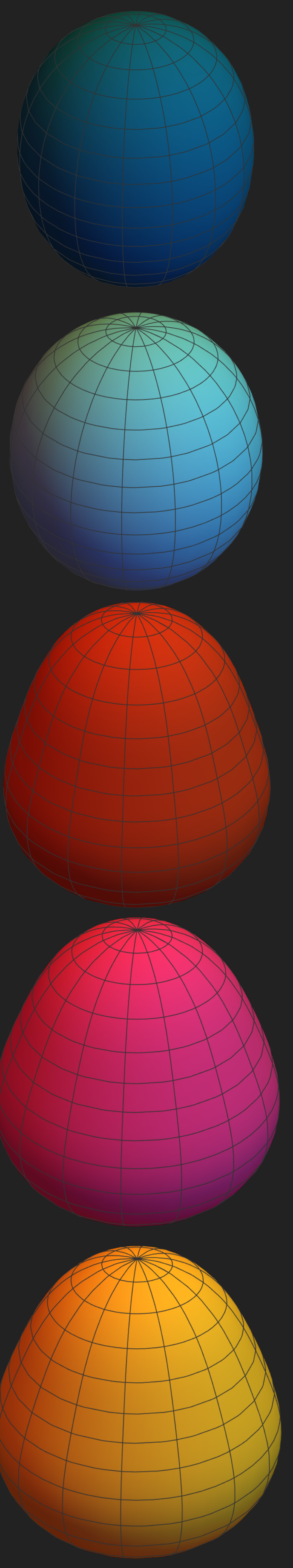

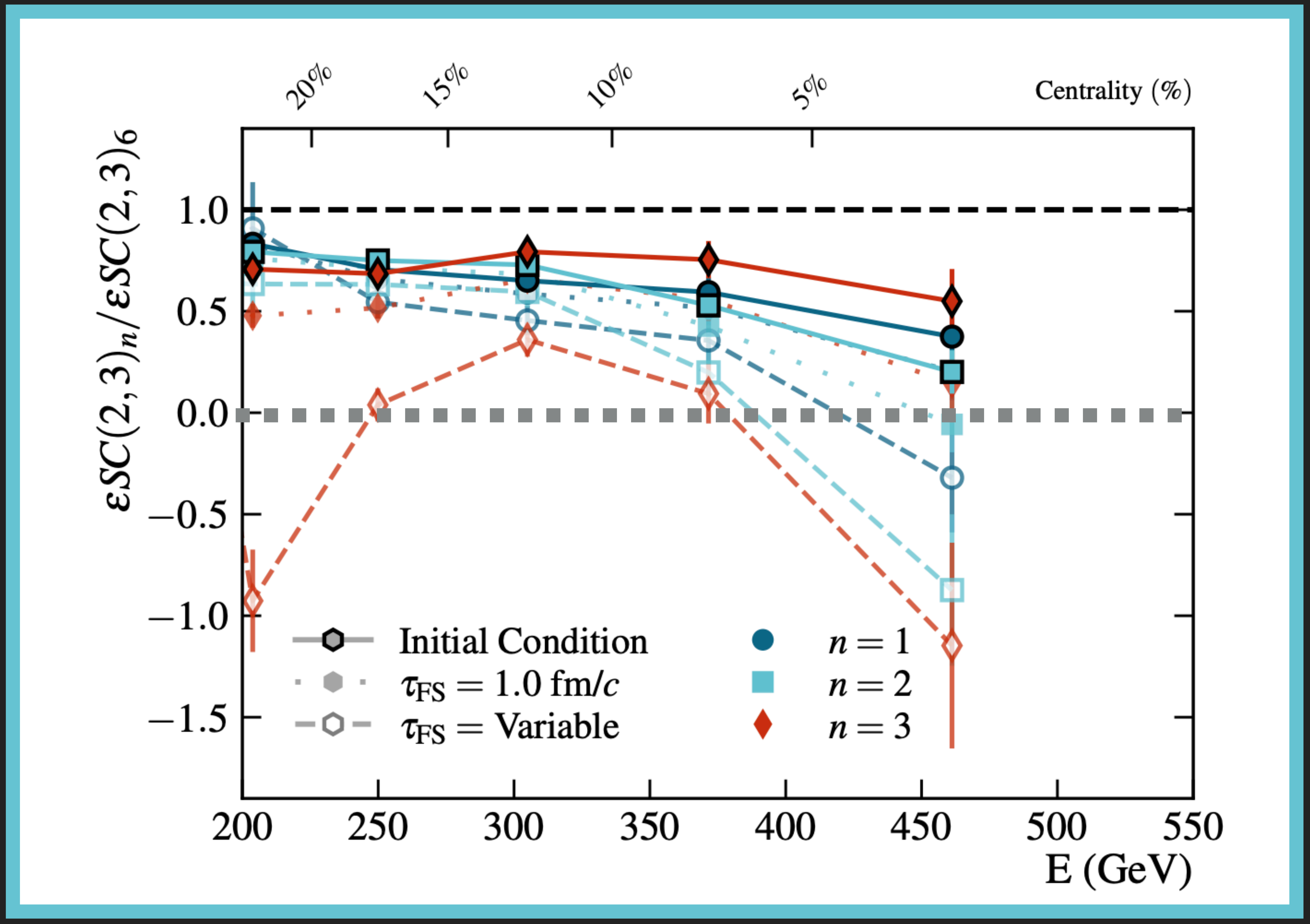

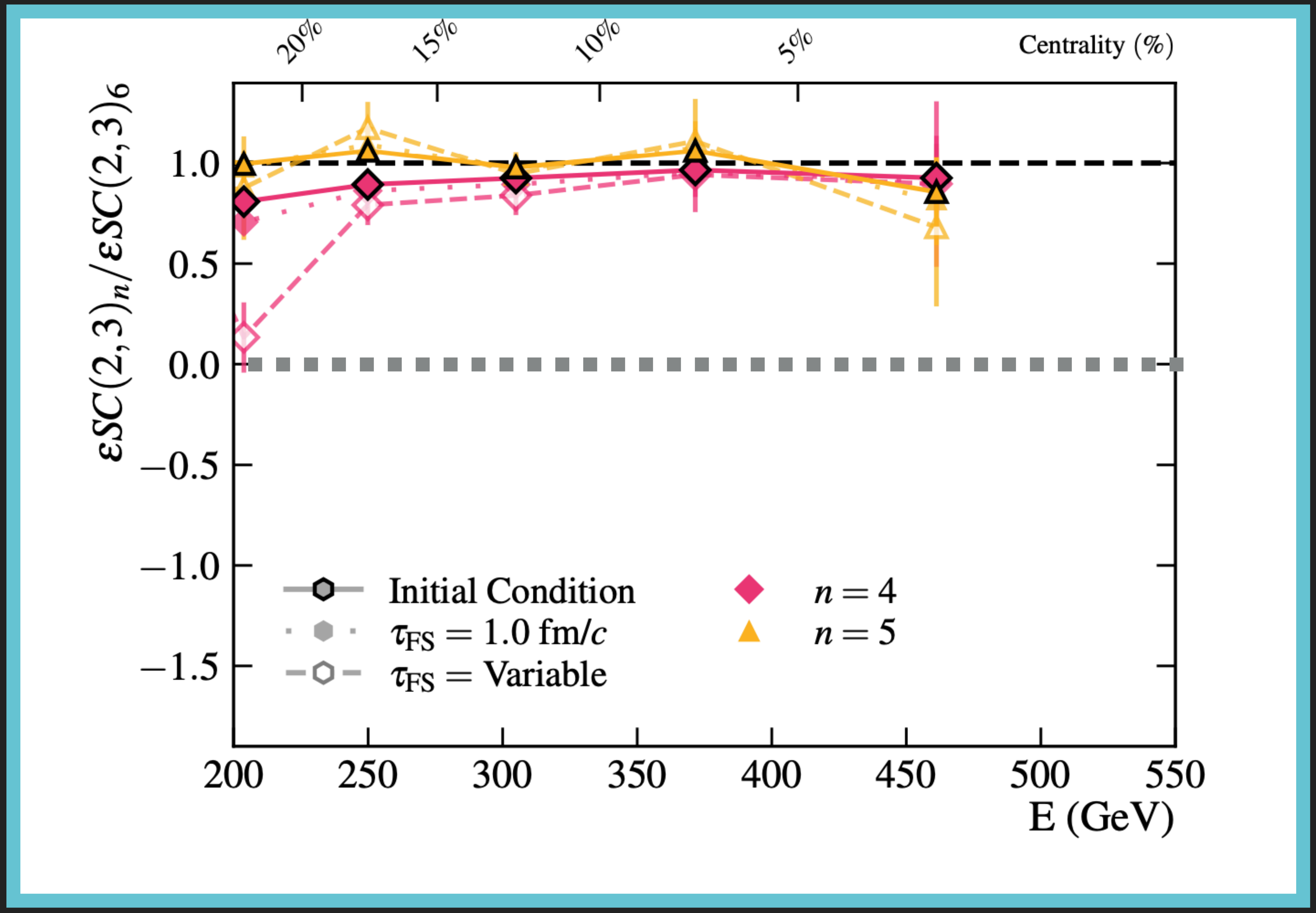

ISOBAR: RESULTS

Large FS effect

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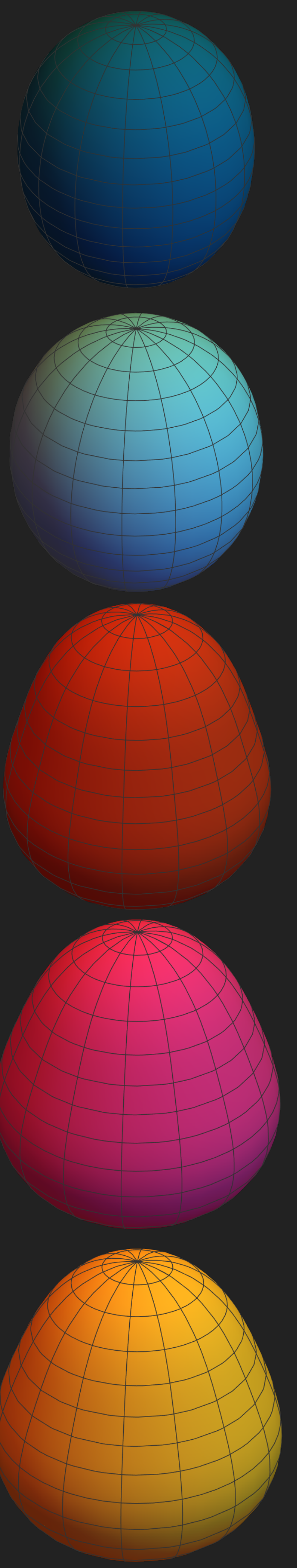
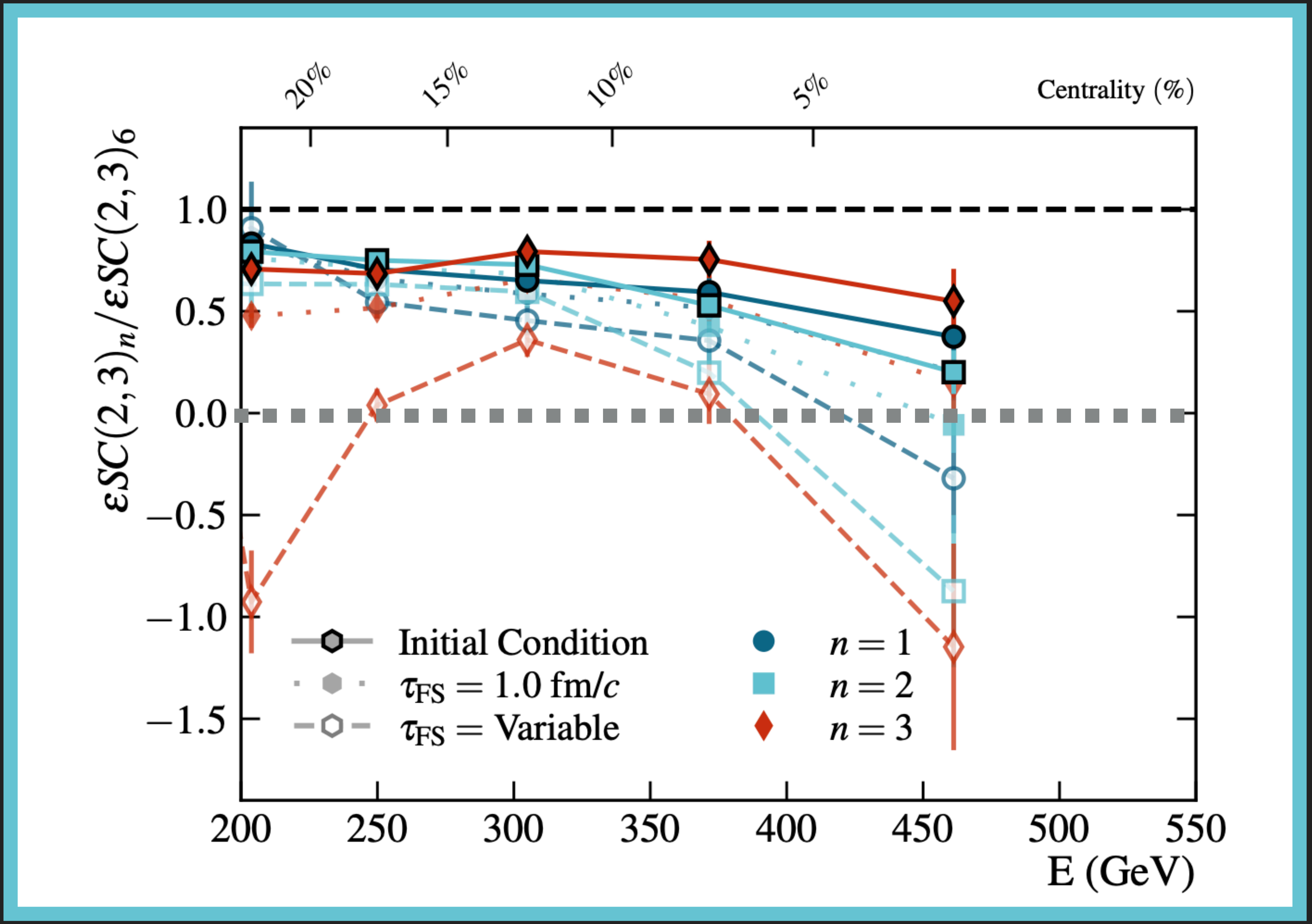
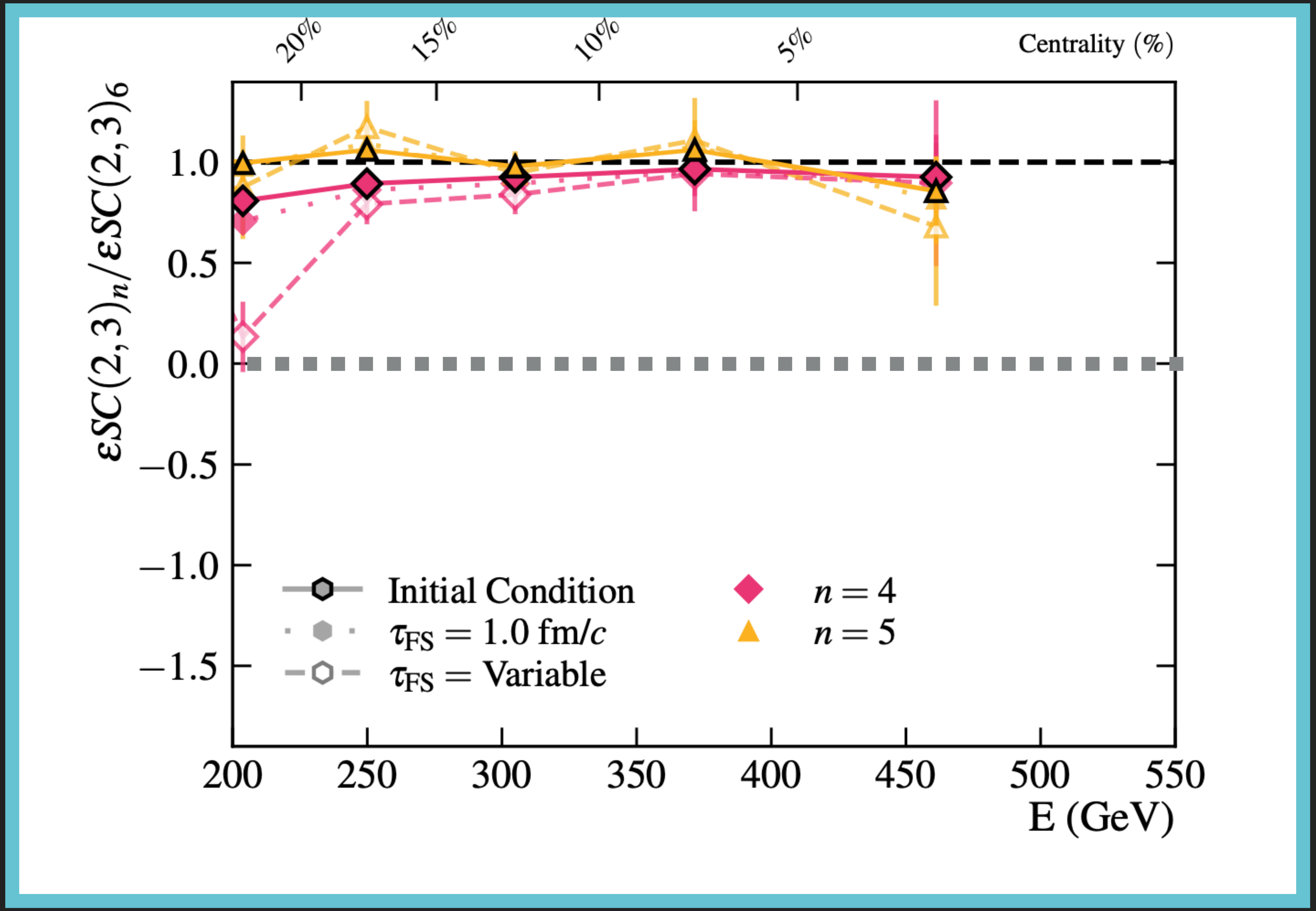
$$\varepsilon NSC(2,3) = \frac{\langle \varepsilon_2^2 \varepsilon_3^2 \rangle - \langle \varepsilon_2^2 \rangle \langle \varepsilon_3^2 \rangle}{\langle \varepsilon_2^2 \rangle \langle \varepsilon_3^2 \rangle}$$







ISOBAR: RESULTS

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Sign change

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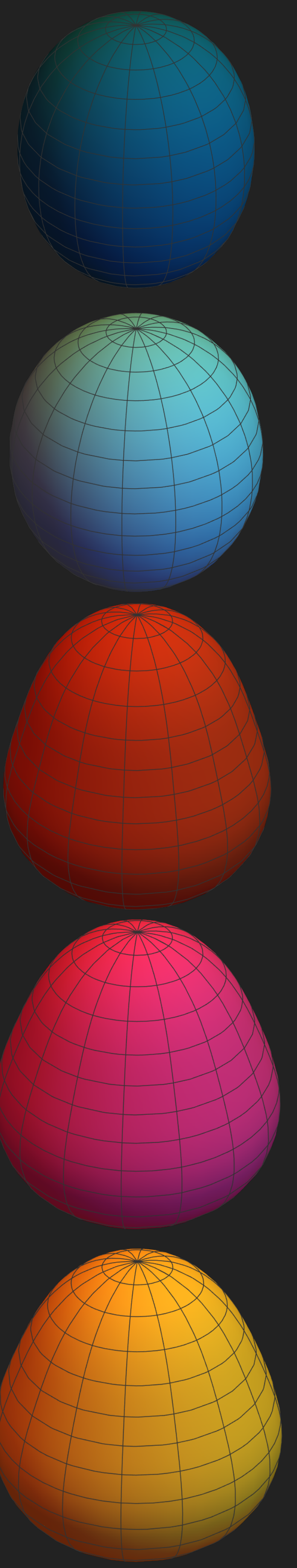

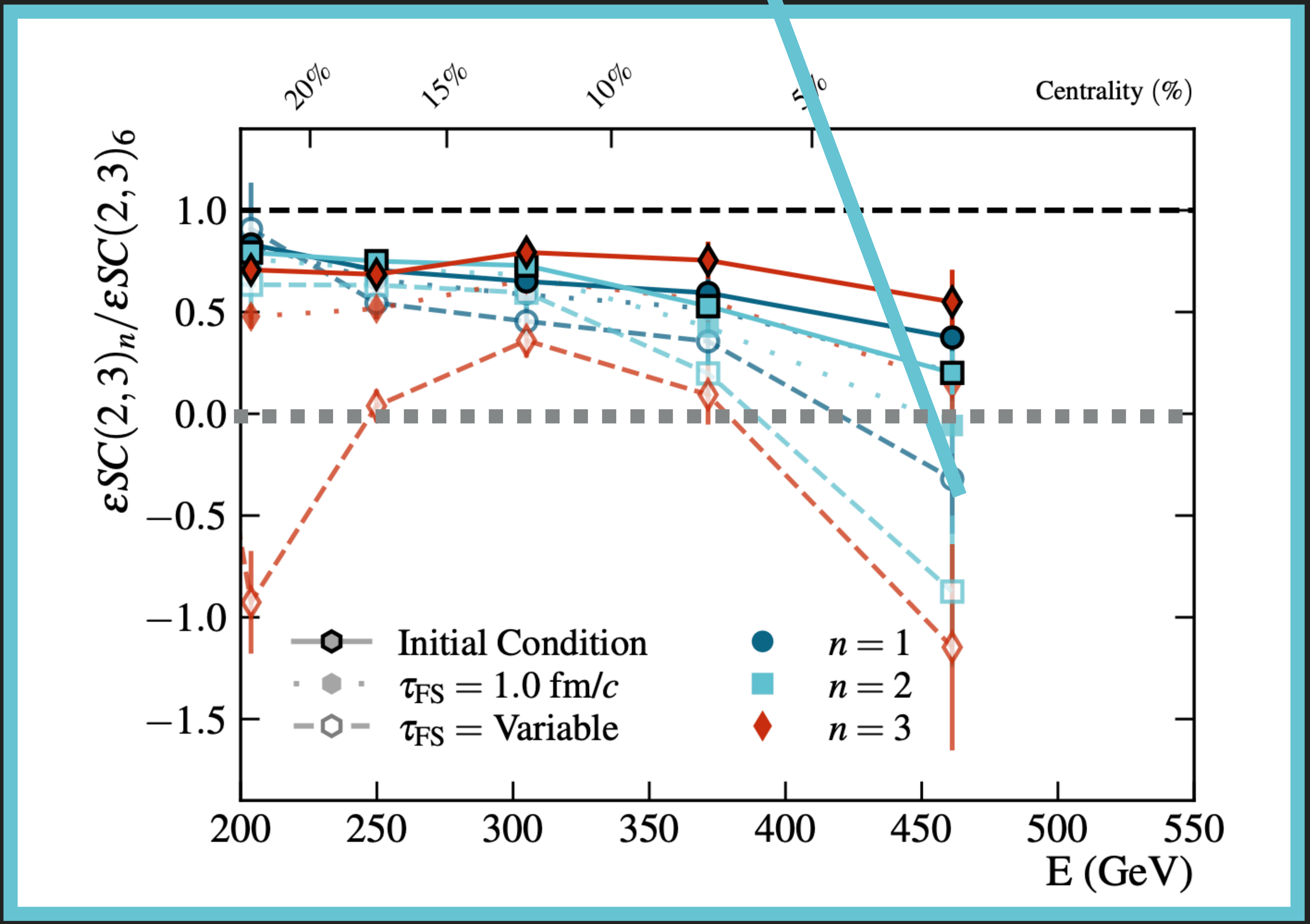

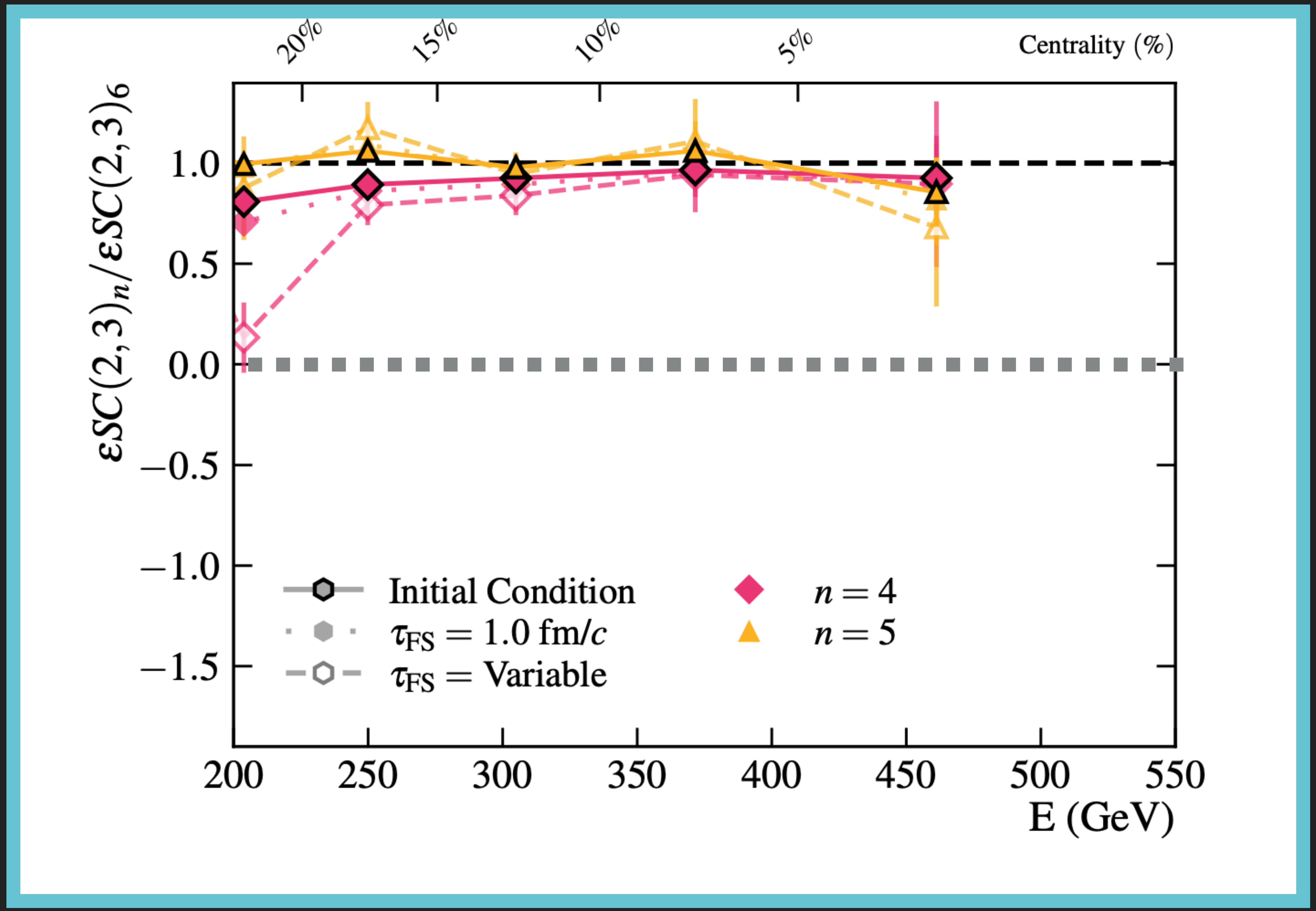

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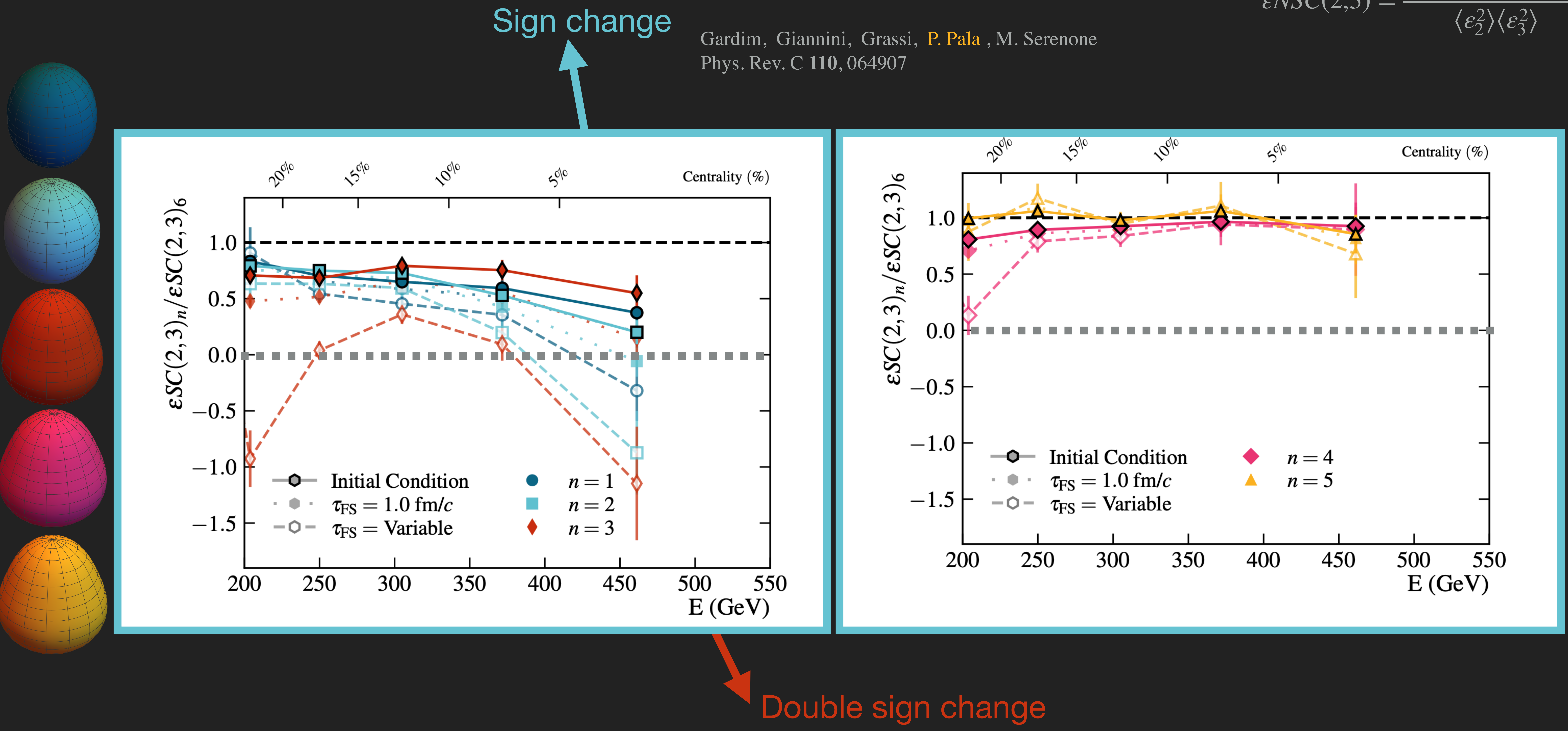

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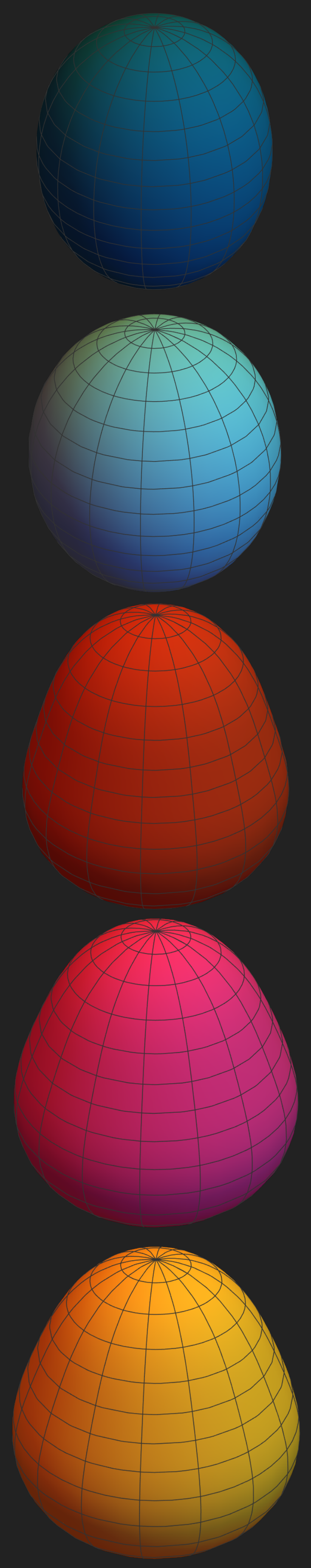

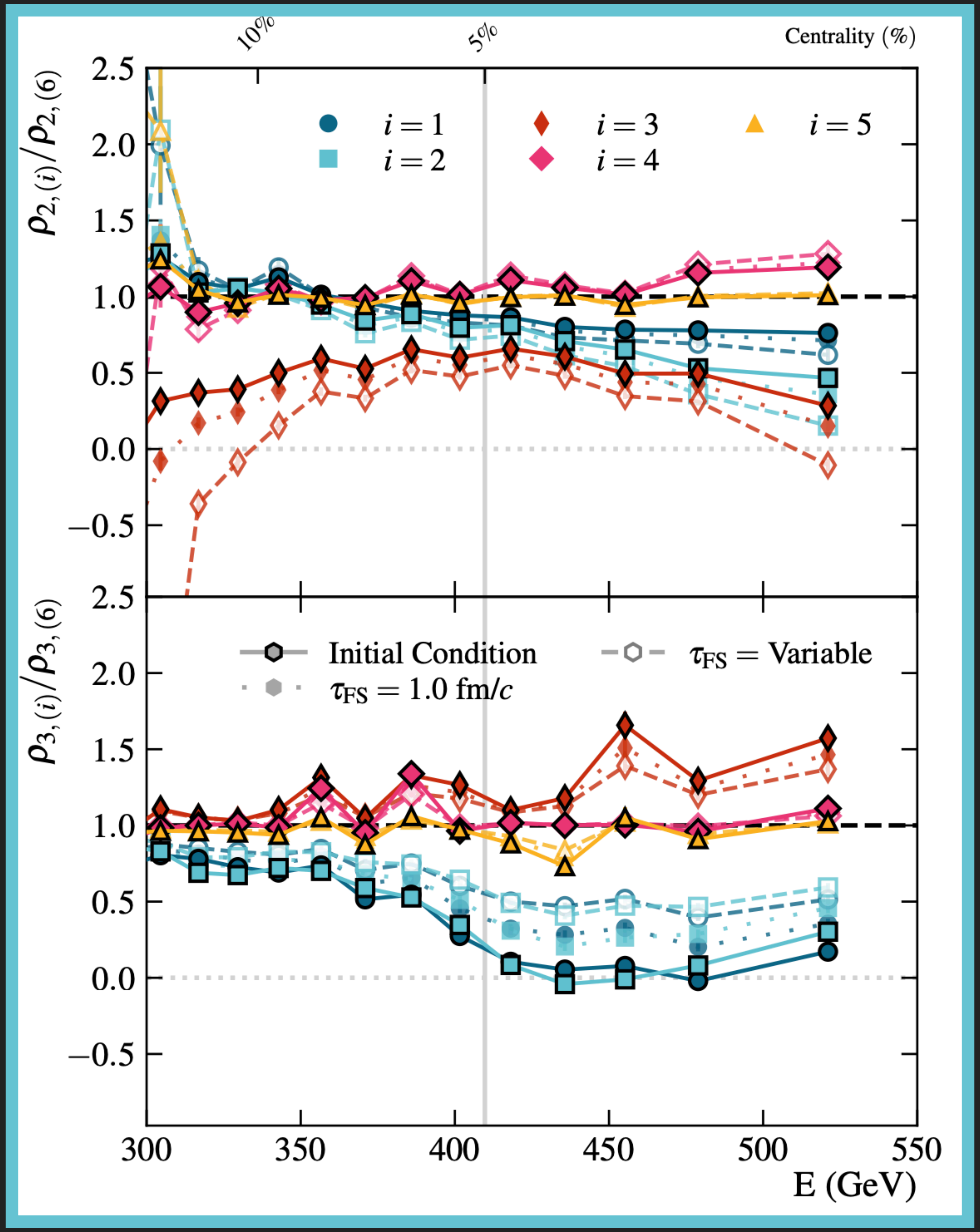

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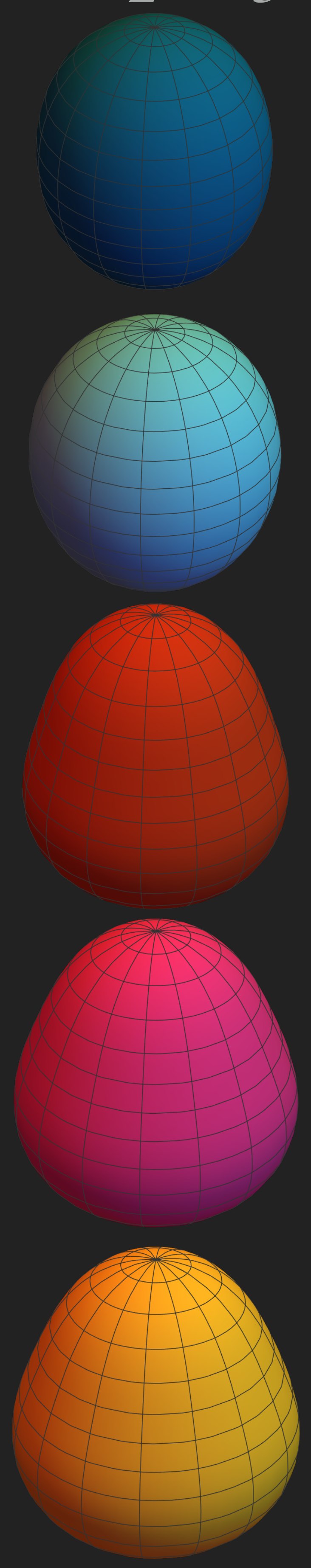
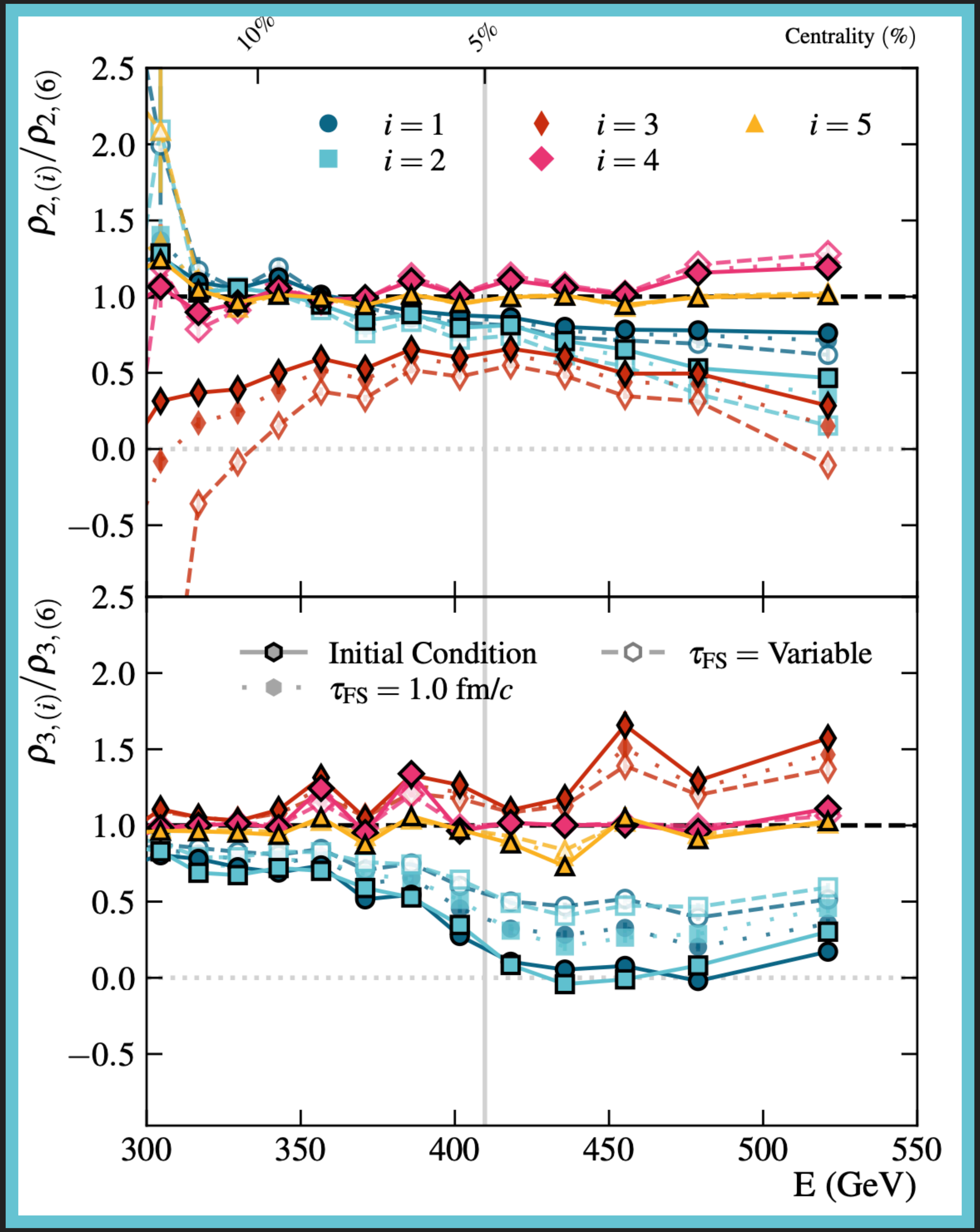
ISOBAR: RESULTS

ρ_2 , ρ_3 are sensitive to free-streaming





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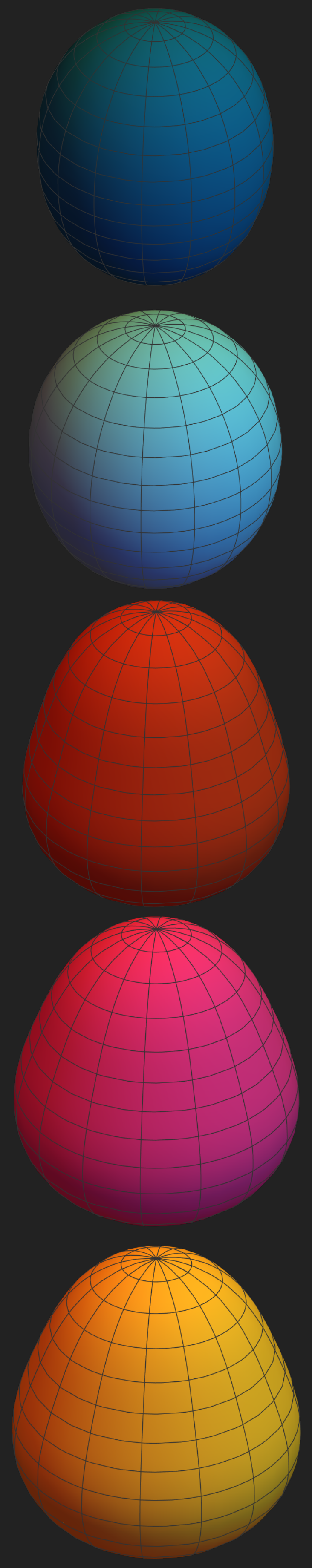

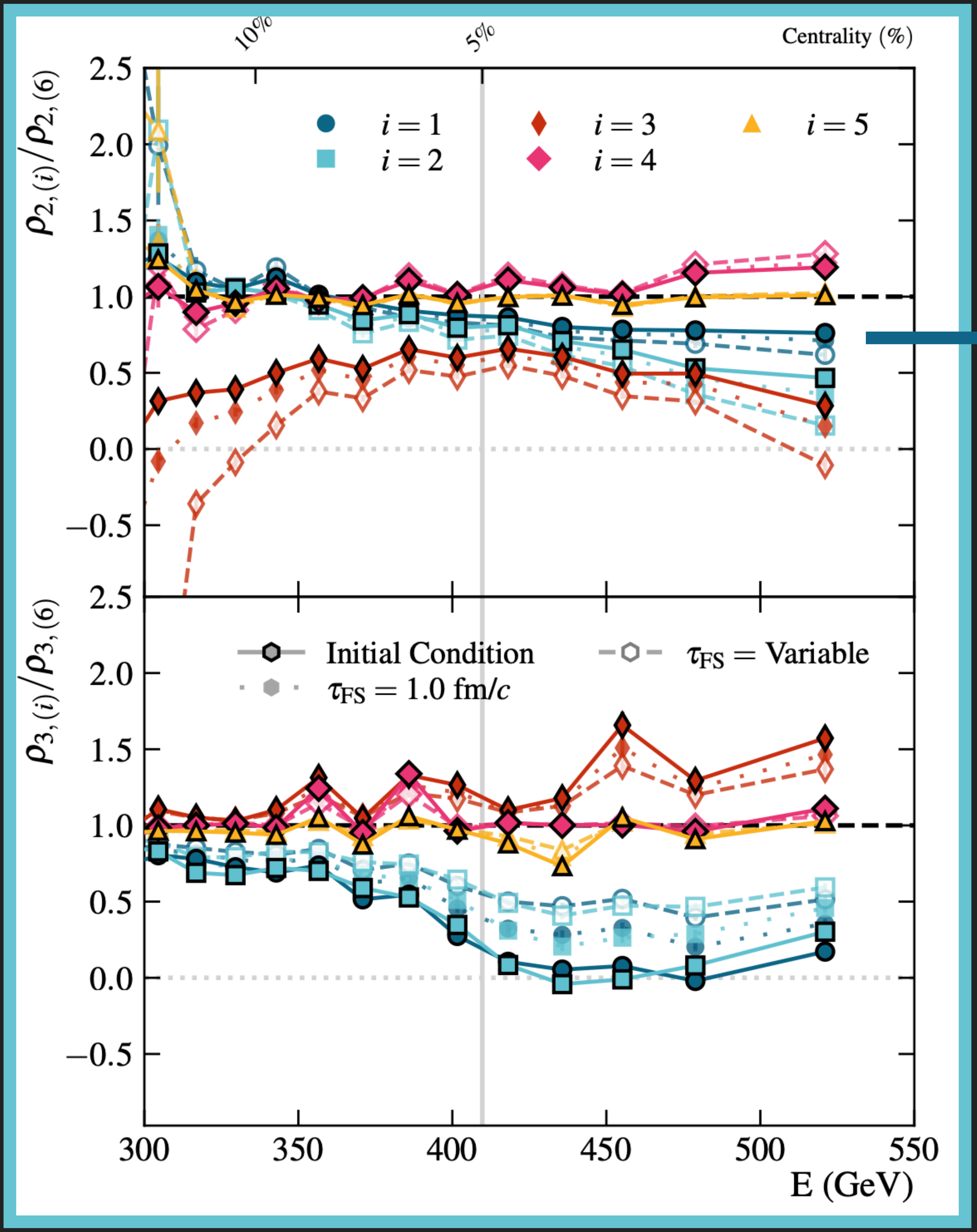

Sensitive to γ

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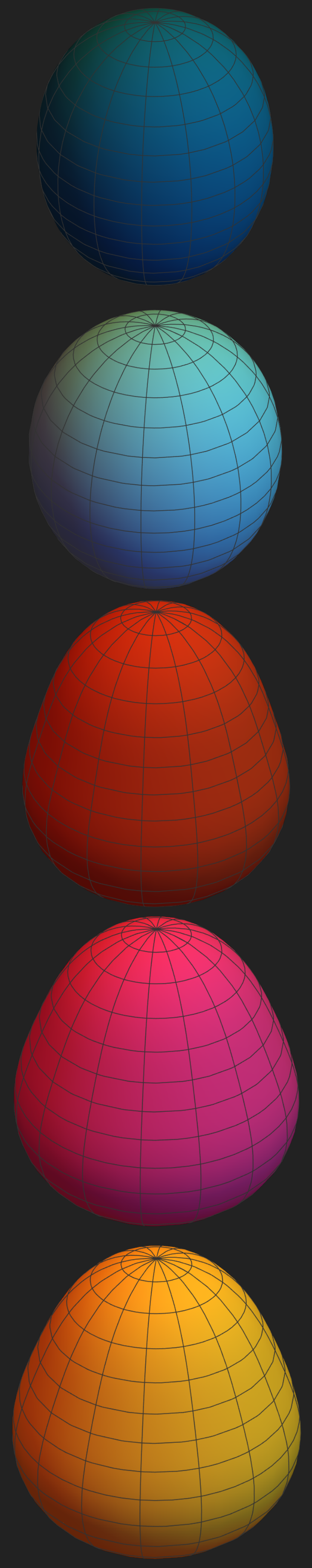

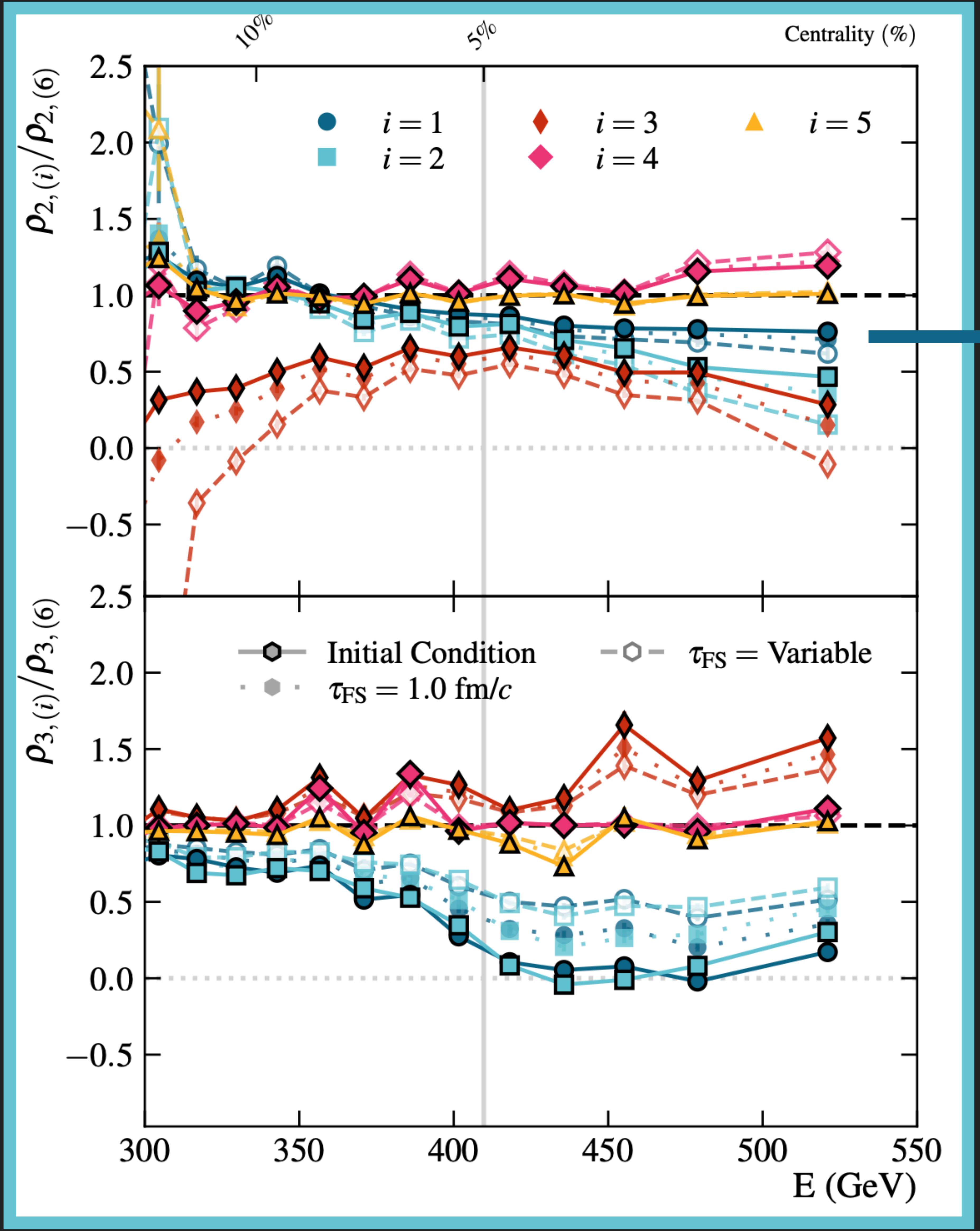

Increases with FS time

Sensitive to γ

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ISOBAR: RESULTS

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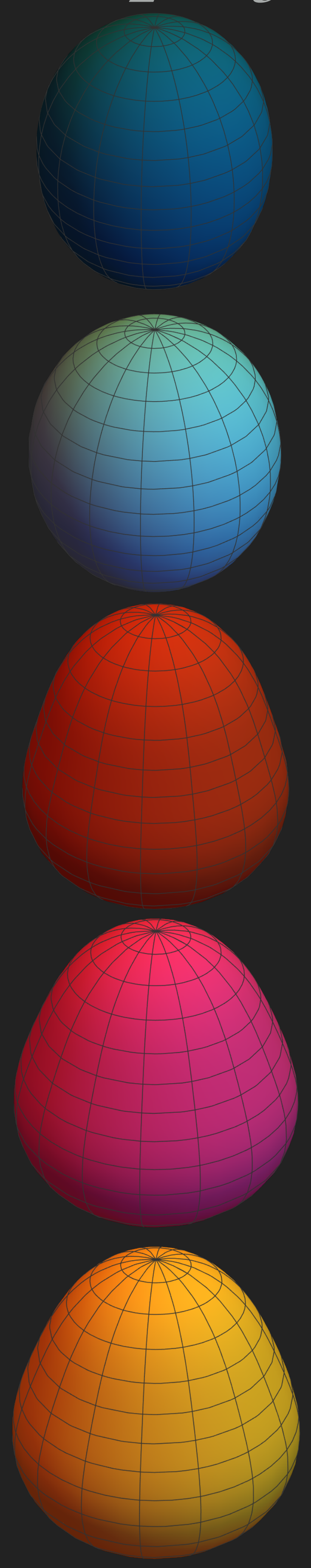
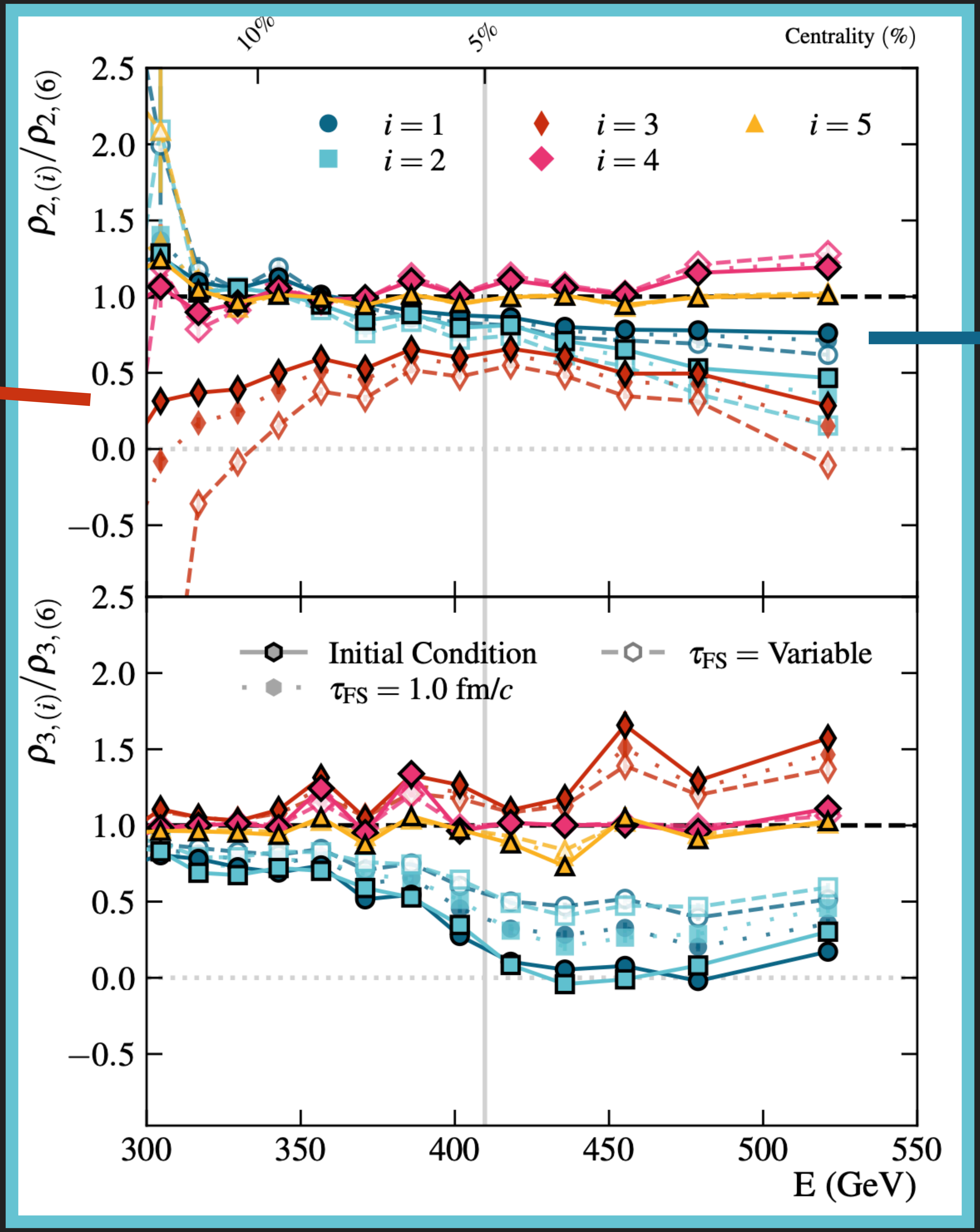


Strong effect of β_3 in ρ_2 for non-central collisions



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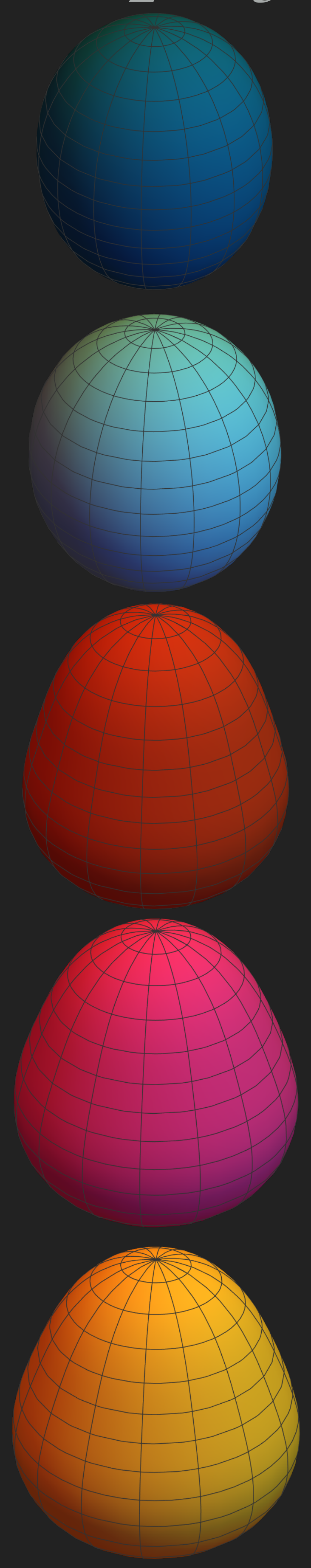
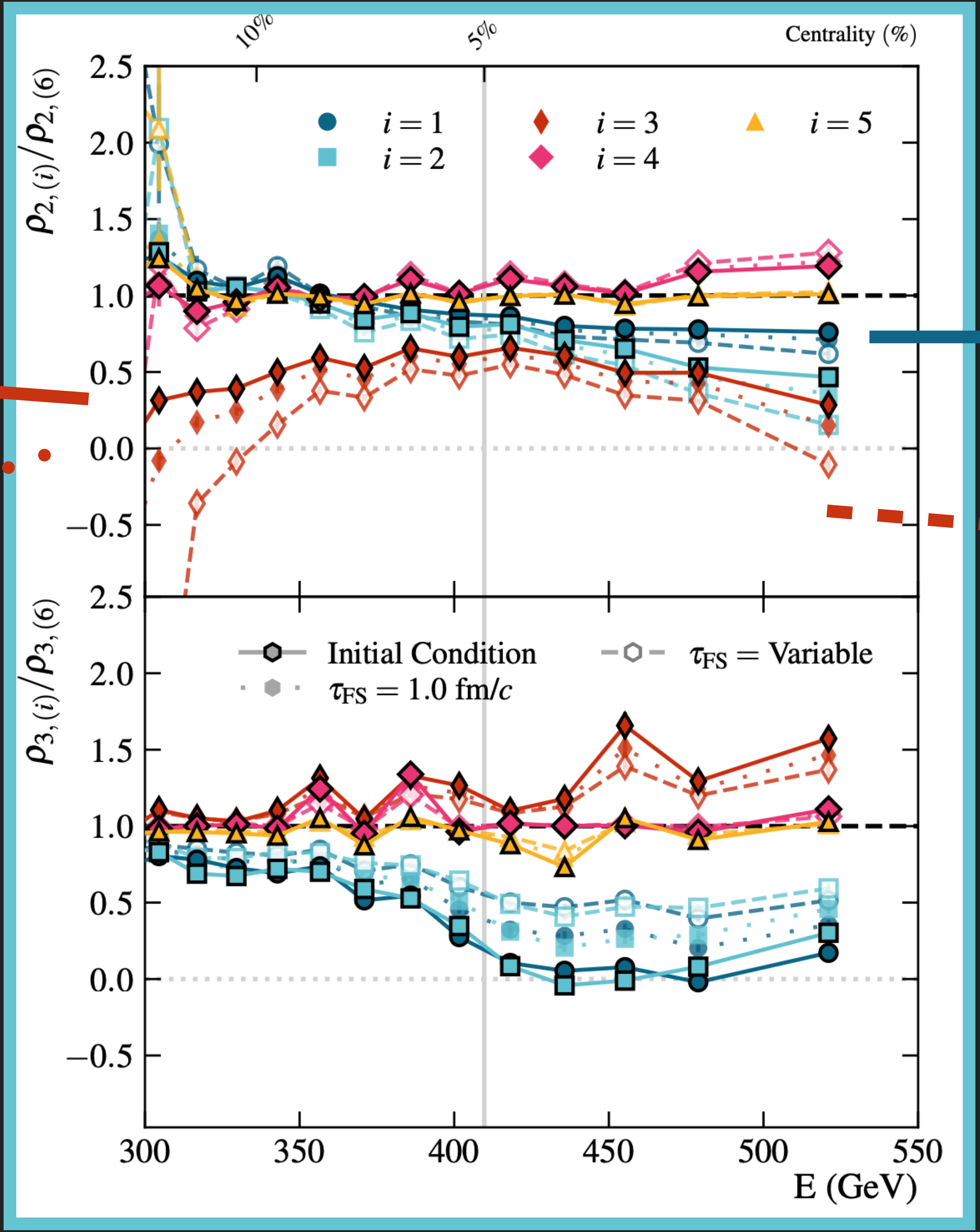


Strong effect of β_3 in ρ_2 for non-central collisions

Sign change



Increases with FS time

Sensitive to γ

Double sign change

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ISOBAR: RESULTS

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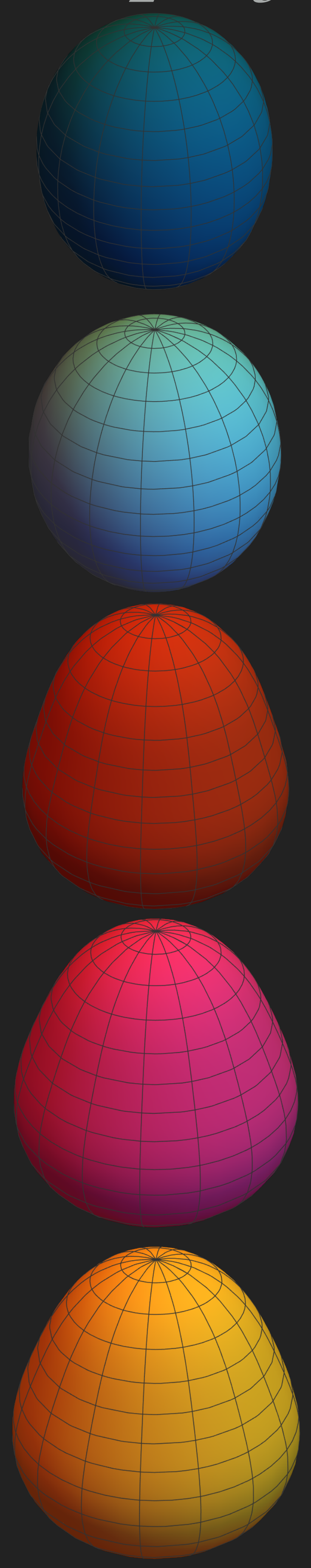

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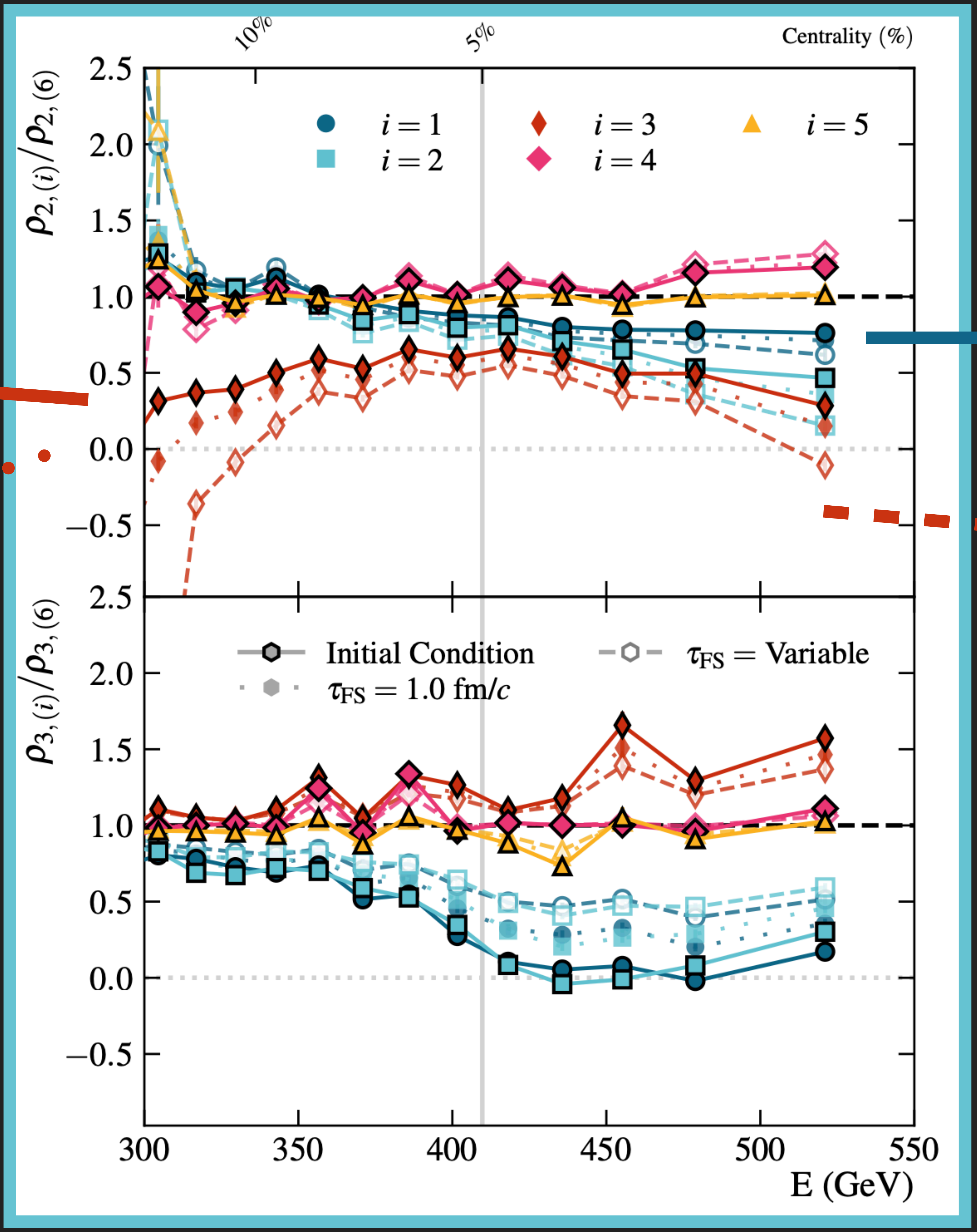

Increases with FS time

Sensitive to γ

Double sign change

May help to constrain the duration of the pre-hydrodynamic phase in isobar collisions

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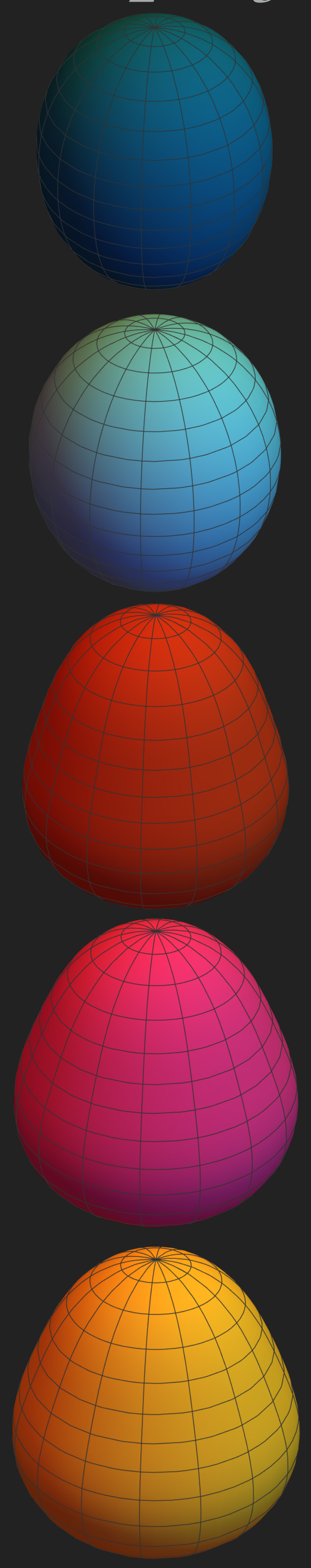
ISOBAR: RESULTS

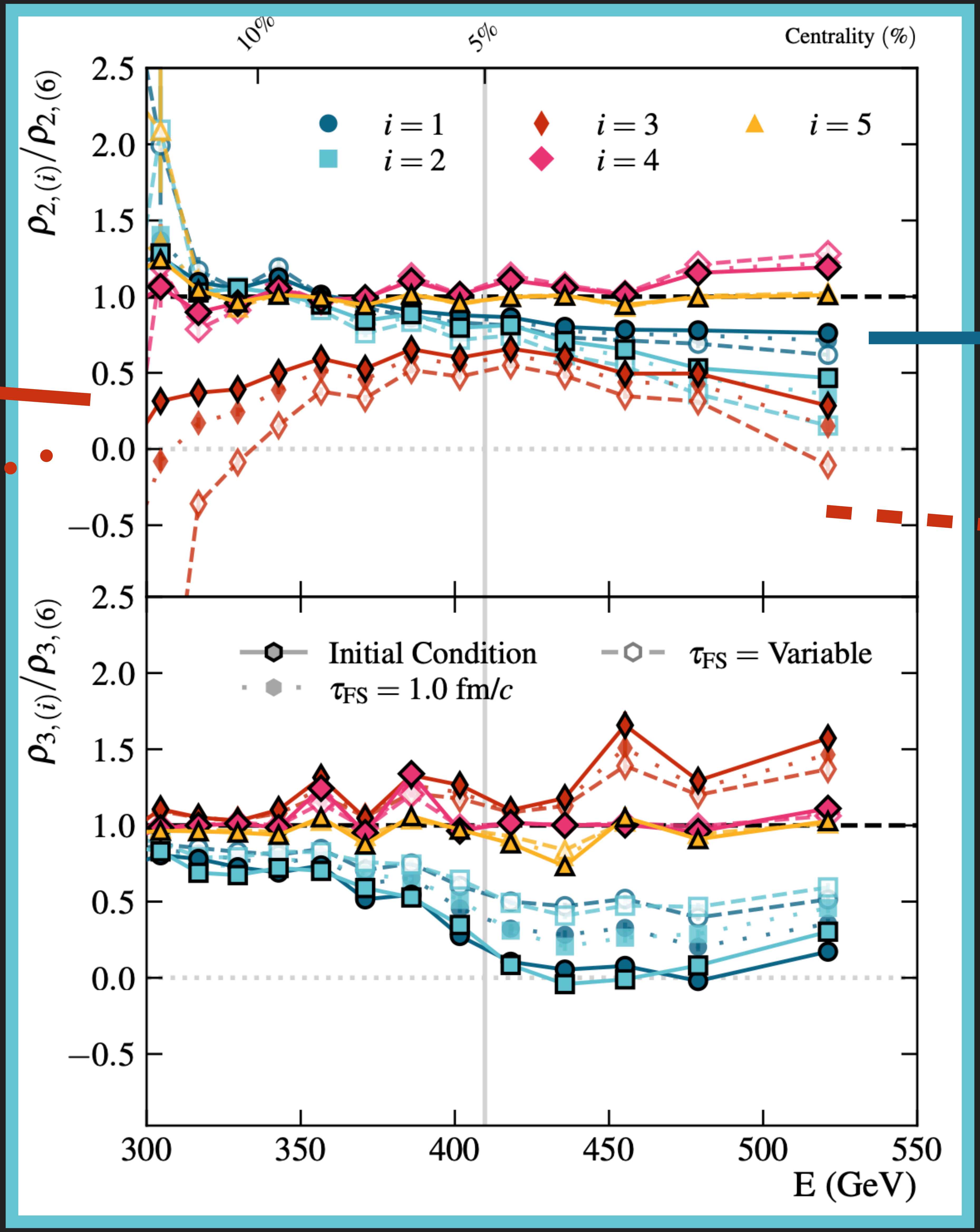
$\rho_2\,,\rho_3$ are sensitive to free-streaming



Strong effect of β_3 in ρ_2 for non-central collisions

Sign change

Overall extreme sensitive to β_3



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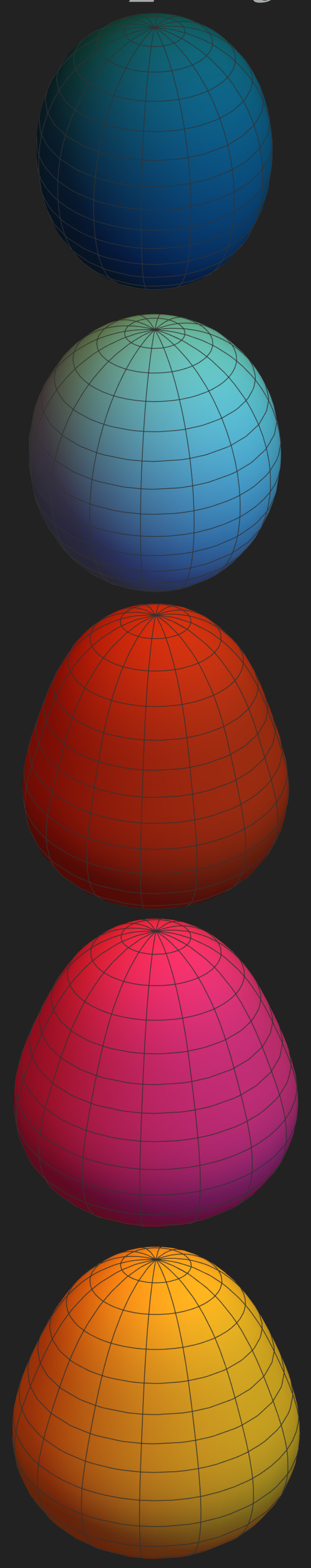
ISOBAR: RESULTS

ρ_2 , ρ_3 are sensitive to free-streaming

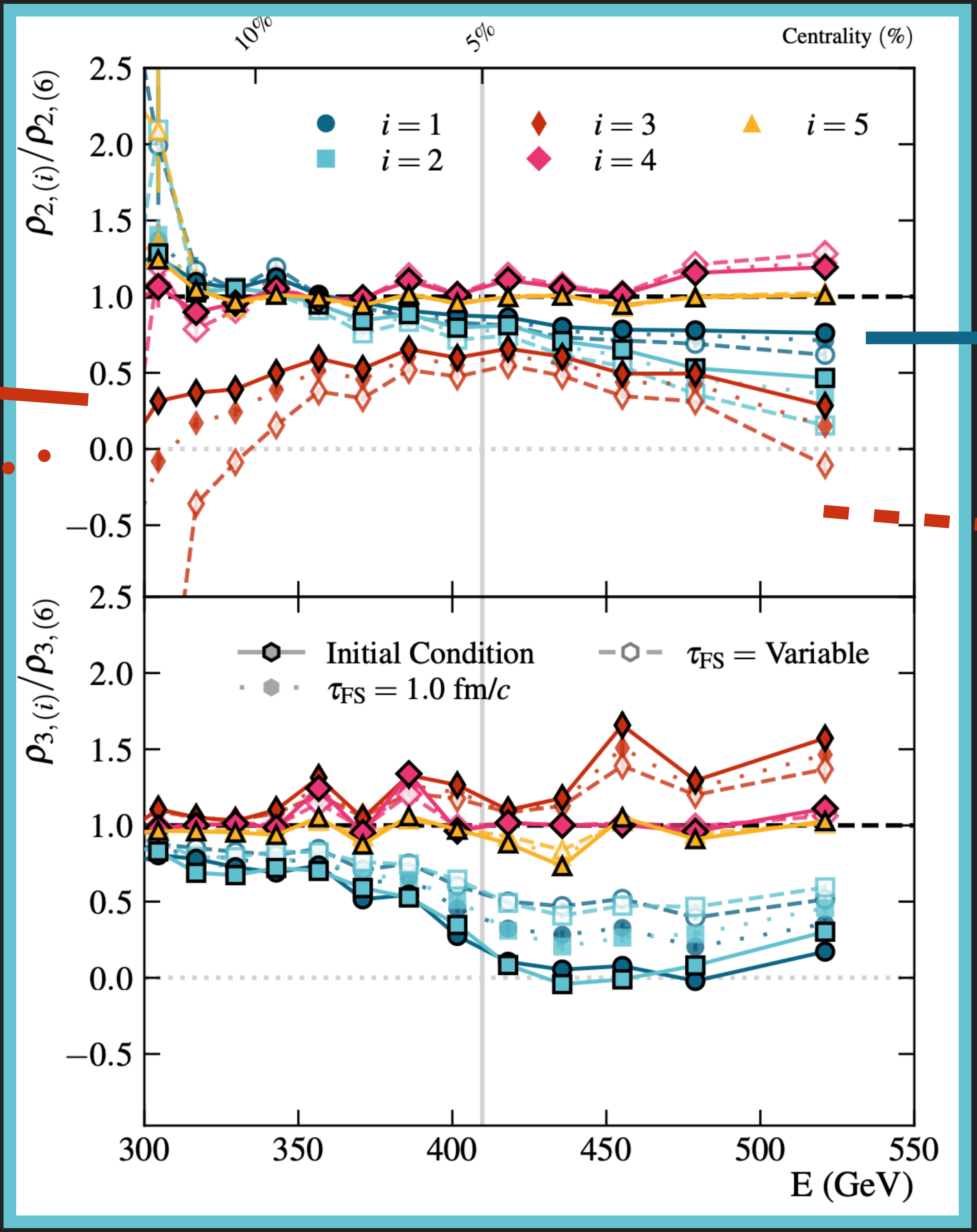


Strong effect of β_3 in ρ_2 for non-central collisions

Sign change

Overall extreme sensitive to β_3

Decreases with FS time



Increases with FS time

Sensitive to γ

Double sign change

May help to constrain the duration of the pre-hydrodynamic phase in isobar collisions

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SUMMARY

- ε_2 ratios predicts v_2 ratios, ε_3 ratios predicts v_3 ratios
- $\langle E/S \rangle$ ratios do not follow $\langle p_T \rangle$ ratios (except for central collisions)
- Effects of hadronic transport is minimal in these ratios
- $\rho_{2,3}$ can be used together with $\varepsilon_{2,3}$ to better constraint the nuclear structure parameters, but more statistics to calculate $\rho_{2,3}^{Hydro}$ with full simulations is necessary
- In the results, a strong effect of the free-streaming time on $\rho_{2,3}$ and $\varepsilon NSC(2,3)$ is observed, which may be used to constrain the duration of the preequilibrium phase.

BACKUP

TRENTO

$$T_N(x,y) = \sum_{i=1}^{N_{part}} w_i \int \mathrm{d}z \, \frac{1}{(2\pi w^2)^{3/2}} e^{-\frac{(x-x_i)^2 + (y-y_i)^2 + z^2}{2w^2}}$$

$$T_R(x,y) = T_R(p; T_A, T_B) = \left(\frac{T_A^p(x,y) + T_B^p(x,y)}{2} \right)^{\frac{1}{p}}$$

$$\bar{\varepsilon}(\overrightarrow{x_T}) = \bar{\varepsilon}(x,y) = \lim_{\tau \to 0^+} \tau \varepsilon(\tau, x, y, \eta = 0) = N T_R(x,y)$$

FREE-STREAMING

- $\bar{\varepsilon} \to$ zero mass Partons with a locally isotropic momentum distribution

$$p^{\mu}\partial_{\mu}f = 0$$

- Milne coordinates solution

$$f(\tau, \overrightarrow{x_T}, \eta_s; \overrightarrow{p_T}, y) = f(\tau_0, \overrightarrow{x_T} - (\tau_{switch} - \tau_0)\hat{p}_T, \eta_s; \overrightarrow{p_T}, y)$$

$$T^{\mu\nu}(\tau, \overrightarrow{x_T}, \eta_s) = \frac{g}{(2\pi)^3}\int \frac{\mathrm{d}^3 p}{p^0}\, p^{\mu}p^{\nu} f(\tau, \overrightarrow{x_T}, \eta_s; \overrightarrow{p_T}, y)$$

FREE-STREAMING

$$\hat{p}^\mu = \frac{p^\mu}{p_T}\Big|_{y=0} = (1, \cos\phi_p, \sin\phi_p, 0)$$

- Boost invariant

$$T^{\mu\nu}(\tau, \overrightarrow{x_T}, \eta_s = 0) = \frac{1}{\tau}\int_{-\pi}^{\pi} \mathrm{d}\phi_p\, \hat{p}^\mu \hat{p}^\nu F(\tau, \overrightarrow{x_T}; \phi_P)$$

- Where

$$F(\tau, \overrightarrow{x_T}; \phi_P) = F_0(\overrightarrow{x_T} - (\tau - \tau_0)\hat{p}_T)$$

- Using initial isotropy

$$\bar{\varepsilon}(\overrightarrow{x_T}) = \tau_0 T^{\tau\tau}(\tau_0, \overrightarrow{x_T}) = 2\pi F_0(\tau_0, \overrightarrow{x_T})$$

FREE-STREAMING

$$\hat{p}^\mu = \left.\frac{p^\mu}{p_T}\right|_{y=0} = (1, \cos\phi_p, \sin\phi_p, 0)$$

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FREE-STREAMING

$$\hat{p}^\mu = \left.\frac{p^\mu}{p_T}\right|_{y=0} = (1, \cos\phi_p, \sin\phi_p, 0)$$

$T^{\mu\nu}$ for all points

$\forall \tau < \tau_{switch}$

- Boost invariant

$$T^{\mu\nu}(\tau, \vec{x_T}, \eta_s = 0) = \frac{1}{\tau}\int_{-\pi}^{\pi} \mathrm{d}\phi_p \, \hat{p}^\mu \hat{p}^\nu F(\tau, \vec{x_T}; \phi_P)$$

- Where

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- Using initial isotropy

$$\bar{\varepsilon}(\vec{x_T}) = \tau_0 T^{\tau\tau}(\tau_0, \vec{x_T}) = 2\pi F_0(\tau_0, \vec{x_T})$$

MUSIC: EQUATIONS

- d_μ is the covariant derivative

$$\mathrm{d}_\mu T^{\mu\nu} = 0$$

$$\mathrm{d}_\mu N^{\mu} = 0$$

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- d_μ is the covariant derivative

$$\mathrm{d}_\mu T^{\mu\nu} = 0$$

$$\mathrm{d}_\mu N^{\mu} = 0$$

$$T^{\mu\nu} = \varepsilon u^\mu u^\nu - \Delta^{\mu\nu}(P + \Pi) + \pi^{\mu\nu}$$

$$N^\mu = \rho_b u^\mu$$

- $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$

MUSIC: EQUATIONS

- DNMR

$$\tau_\pi \mathrm{D}\pi^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + 2\tau_\pi \pi_a^{\langle\mu}\omega^{\nu\rangle a} - \delta_{\pi\pi}\pi^{\mu\nu}\theta - \tau_{\pi\pi}\pi_a^{\langle\mu}\sigma^{\nu\rangle a} + \lambda_{\pi\Pi}\Pi\sigma^{\mu\nu} + \phi_7 \pi_a^{\langle\mu}\pi^{\nu\rangle a}$$

$$\tau_\Pi \mathrm{D}\Pi + \Pi = -\zeta\theta - \delta_{\Pi\Pi}\Pi\theta + \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu}$$

MUSIC: EQUATIONS

- DNMR

$$\tau_\pi D\pi^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + 2\tau_\pi \pi_a^{\langle\mu}\omega^{\nu\rangle a} - \delta_{\pi\pi}\pi^{\mu\nu}\theta - \tau_{\pi\pi}\pi_a^{\langle\mu}\sigma^{\nu\rangle a} + \lambda_{\pi\Pi}\Pi\sigma^{\mu\nu} + \phi_7 \pi_a^{\langle\mu}\pi^{\nu\rangle a}$$

$$\tau_\Pi D\Pi + \Pi = -\zeta\theta - \delta_{\Pi\Pi}\Pi\theta + \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu}$$

- First order: η, ζ
- Second order : τ_π , $\delta_{\pi\pi}$, $\tau_{\pi\pi}$, $\lambda_{\pi\Pi}$, ϕ_7 , τ_Π , $\delta_{\Pi\Pi}$, $\lambda_{\Pi\pi}$.

MUSIC: EQUATIONS

$$u^\mu \mathrm{d}\mu = \mathrm{D},$$
$$\Delta^{\mu\nu\alpha\beta} = \frac{1}{2}(\Delta^{\mu\alpha}\Delta^{\nu\beta} + \Delta^{\mu\beta}\Delta^{\nu\alpha}) - \frac{1}{\Delta^\lambda_\lambda}\Delta^{\mu\nu}\Delta^{\alpha\beta},$$
$$\theta = \mathrm{d}_\mu u^\mu,$$
$$A^{<\mu\nu>} = \Delta^{\mu\nu}_{\alpha\beta}A^{\alpha\beta},$$
$$\sigma^{\mu\nu} = \nabla^{<\mu}u^{\nu>} = \Delta^{\mu\nu}_{\alpha\beta}d^\alpha u^\beta,$$
$$\omega^{\mu\nu} = \frac{1}{2}\left(\nabla^\mu u^\nu + \nabla^\mu u^\nu\right).$$

- DNMR

$$\tau_\pi \mathrm{D}\pi^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + 2\tau_\pi \pi_a^{\langle\mu}\omega^{\nu\rangle a} - \delta_{\pi\pi}\pi^{\mu\nu}\theta - \tau_{\pi\pi}\pi_a^{\langle\mu}\sigma^{\nu\rangle a} + \lambda_{\pi\Pi}\Pi\sigma^{\mu\nu} + \phi_7\pi_a^{\langle\mu}\pi^{\nu\rangle a}$$

$$\tau_\Pi \mathrm{D}\Pi + \Pi = -\zeta\theta - \delta_{\Pi\Pi}\Pi\theta + \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu}$$

- First order: η, ζ — JETSCAPE Parametrization
- Second order : τ_π , $\delta_{\pi\pi}$, $\tau_{\pi\pi}$, $\lambda_{\pi\Pi}$, ϕ_7 , τ_Π , $\delta_{\Pi\Pi}$, $\lambda_{\Pi\pi}$. — Function of ζ , η

MUSIC: EQUATIONS

- JETSCAPE parametrization for η and ζ

$$\frac{\eta}{s} = max\left[\frac{\eta}{s}\Big|_{lin}(T), 0\right]$$

$$\frac{\eta}{s}\Big|_{lin}(T) = a_{low}(T - T_\eta)\Theta(T_\eta - T) + \left(\frac{\eta}{s}\right)_{kink} + a_{high}(T - T_\eta)\Theta(T - T_\eta)$$

$$\frac{\zeta}{s}(T) = \frac{(\zeta/s)_{max}\Lambda^2}{\Lambda^2 + (T - T_\zeta)^2}$$

$$\Lambda = w_\zeta[1 + \lambda_\zeta \, \mathrm{sign}(T - T_\zeta)]$$

MUSIC: EQUATIONS

- Second order transport coefficients

$$\tau_\pi = \frac{b_\pi}{T}\frac{\eta}{s}, \qquad \delta_{\pi\pi} = \frac{4}{3}\tau_\pi,$$

$$\phi_7 = \frac{9}{70P}, \qquad \tau_{\pi\pi} = \frac{10}{7}\tau_\pi,$$

$$\lambda_{\pi\Pi} = \frac{6}{5}, \qquad \tau_\Pi = \frac{\zeta}{15\left(\frac{1}{3} - c_s^2\right)^2(\varepsilon + P)},$$

$$\delta_{\Pi\Pi} = \frac{2}{3}\tau_\Pi, \qquad \lambda_{\Pi\pi} = \frac{8}{5}\left(\frac{1}{3} - c_s^2\right)\tau_\Pi,$$

MUSIC: LANDAU MATCHING

- $T^{\mu\nu} = (\varepsilon + P + \Pi)u^\mu u^\nu - (P + \Pi)g^{\mu\nu} + \pi^{\mu\nu}$
- $\pi^\mu_\mu = 0$ e $u_\mu \pi^{\mu\nu} = 0$

Landau rest frame

$$T^\mu_\nu u^\nu = \varepsilon u^\mu$$

$$\Pi = \frac{\varepsilon - T^\mu_\mu}{3} - P(\varepsilon, \rho_b)$$

$$\pi^{\mu\nu} = \frac{u^\mu u^\nu}{3}\left(T^\alpha_\alpha - 4\varepsilon\right) + \frac{1}{3}\left(\varepsilon - T^\alpha_\alpha\right) g^{\mu\nu} + T^{\mu\nu}$$

MUSIC: EVOLUTION

- MKurganov-Tadmor
- Evolve until $T < T_{sw} \forall$ cells
- Constant T surface

$$T_{sw} = T\left(\tau, x, y, \eta_s\right)$$

- Freeze-out surface calculated from the grid using Cornellius

ISS: COOPER–FRYE

- Probability of emitting particle of species i with momentum p

$$E\frac{\mathrm{d}^3N_i}{\mathrm{d}p^3}\left(x^\mu, p^\mu\right) = p^\nu \mathrm{d}^3\sigma_\nu \left(f_{0i}\left(x^\mu, p\right) + \delta f_i\left(x^\mu, p^\mu\right)\right)$$

δf

$$\delta f_i = f_{0i}(1-\Theta f_{0i})\left[\Pi(A_T m_i^2 + A_E (u\cdot p)^2) + A_\pi \pi^{\mu\nu} p_{\langle\mu} p_{\nu\rangle}\right]$$

$$A_T = \frac{\mathscr{P}}{\mathscr{A}_{21}\mathscr{P} + \mathscr{N}_{31}\mathscr{Q} + \mathscr{J}_{41}\mathscr{R}},$$
$$A_E = \frac{\mathscr{R}}{\mathscr{A}_{21}\mathscr{P} + \mathscr{N}_{31}\mathscr{Q} + \mathscr{J}_{41}\mathscr{R}},$$
$$A_\pi = \frac{1}{2(\varepsilon + P)T^2}.$$

$$\mathscr{P} = \mathscr{N}_{30}^2 - \mathscr{J}_{40}\mathscr{M}_{20},$$
$$\mathscr{Q} = \mathscr{B}_{30}\mathscr{J}_{40} - \mathscr{A}_{20}\mathscr{N}_{30}$$
$$\mathscr{R} = \mathscr{A}_{20}\mathscr{M}_{20} - \mathscr{B}_{10}\mathscr{N}_{30},$$

$$\mathscr{J}_{rq} = \frac{1}{(2q+1)!!}\sum_i \int_p (u\cdot p)^{r-2q}(-p\cdot\Delta\cdot p)^q f_{0i}(1-\Theta f_{0i})$$
$$\mathscr{N}_{rq} = \frac{1}{(2q+1)!!}\sum_i b_i \int_p (u\cdot p)^{r-2q}(-p\cdot\Delta\cdot p)^q f_{0i}(1-\Theta f_{0i})$$
$$\mathscr{M}_{rq} = \frac{1}{(2q+1)!!}\sum_i b_i^2 \int_p (u\cdot p)^{r-2q}(-p\cdot\Delta\cdot p)^q f_{0i}(1-\Theta f_{0i})$$
$$\mathscr{A}_{rq} = \frac{1}{(2q+1)!!}\sum_i m_i^2 \int_p (u\cdot p)^{r-2q}(-p\cdot\Delta\cdot p)^q f_{0i}(1-\Theta f_{0i})$$
$$\mathscr{B}_{rq} = \frac{1}{(2q+1)!!}\sum_i b_i m_i^2 \int_p (u\cdot p)^{r-2q}(-p\cdot\Delta\cdot p)^q f_{0i}(1-\Theta f_{0i}).$$

ISS: COOPER-FRYE

- Probability of emitting particle of species i with momentum p

Surface element

$$E\frac{\mathrm{d}^3N_i}{\mathrm{d}p^3}\left(x^\mu, p^\mu\right) = p^\nu \mathrm{d}^3\sigma_\nu \left(f_{0i}\left(x^\mu, p\right) + \delta f_i\left(x^\mu, p^\mu\right)\right)$$

Equilibrium distribution Maxwell-Jüttner

$$\frac{g_i}{(2\pi)^3}\frac{1}{\exp\left((u(x)\cdot p + b_i\mu_b(x))/T\right) + \Theta}$$

Out of equilibrium correction

$$\delta f_i = f_{0i}(1-\Theta f_{0i})\left[\Pi(A_T m_i^2 + A_E(u\cdot p)^2) + A_\pi \pi^{\mu\nu} p_{\langle\mu}p_{\nu\rangle}\right]$$

SMASH

- In cascade mode, solves the Boltzmann equation

$$p^{\mu}\partial_{\mu}f_i(x,p) = C^{i}_{coll}$$

  - Collision term
  - Resonances and decays
  - Binary elastic and inelastic collisions

- Propagate in a straight line until one of the process occurs
- When Kinect freeze-out is reached free-stream the particles until final time

ISOBAR: SIMULATION CHAIN

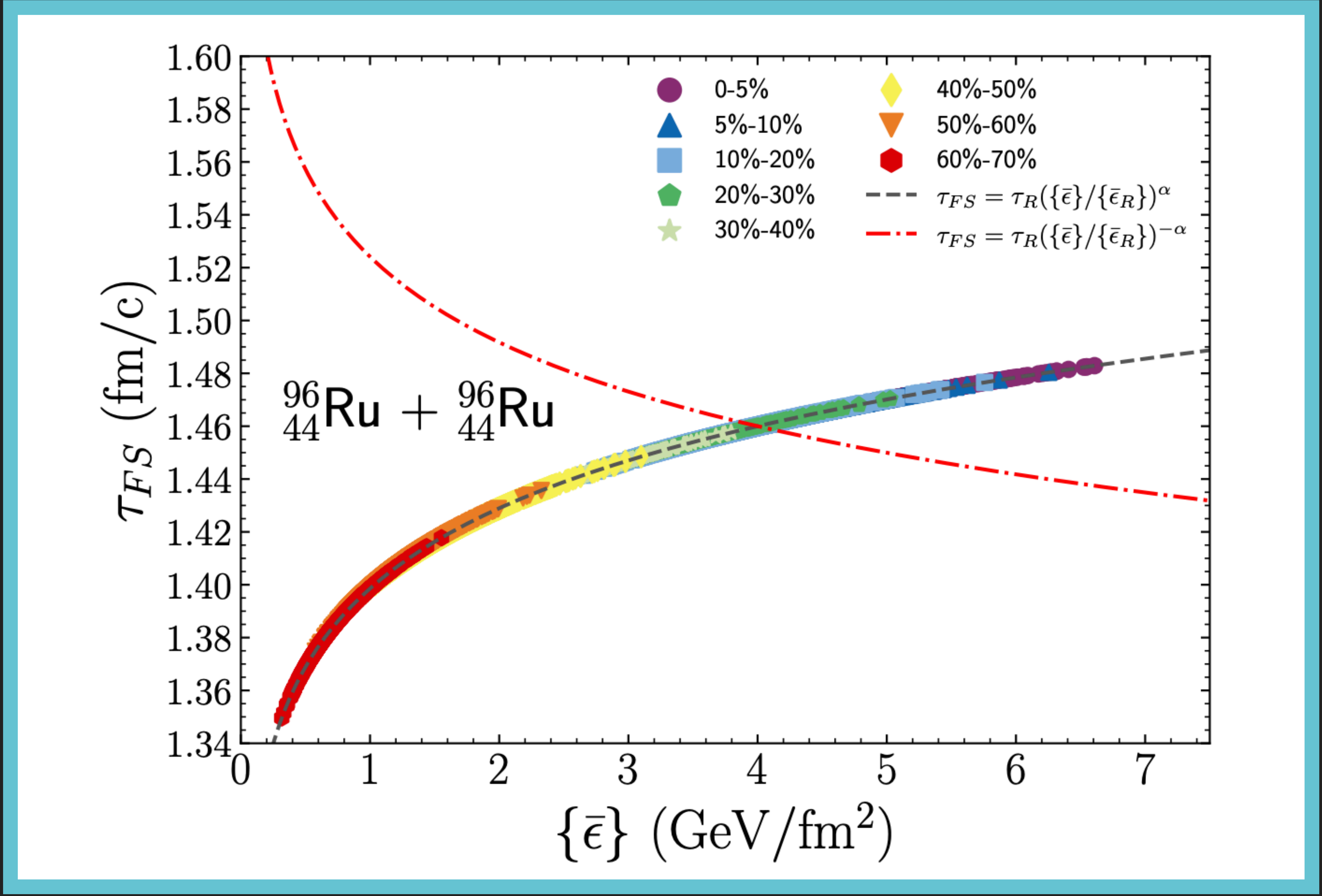
- JETSCAPE parametrization

1.46 fm/c

0.031

$$\tau_{FS} = \tau_R \left(\frac{\{\bar{\varepsilon}\}}{4\ Gev/fm^2} \right)^{\alpha}$$

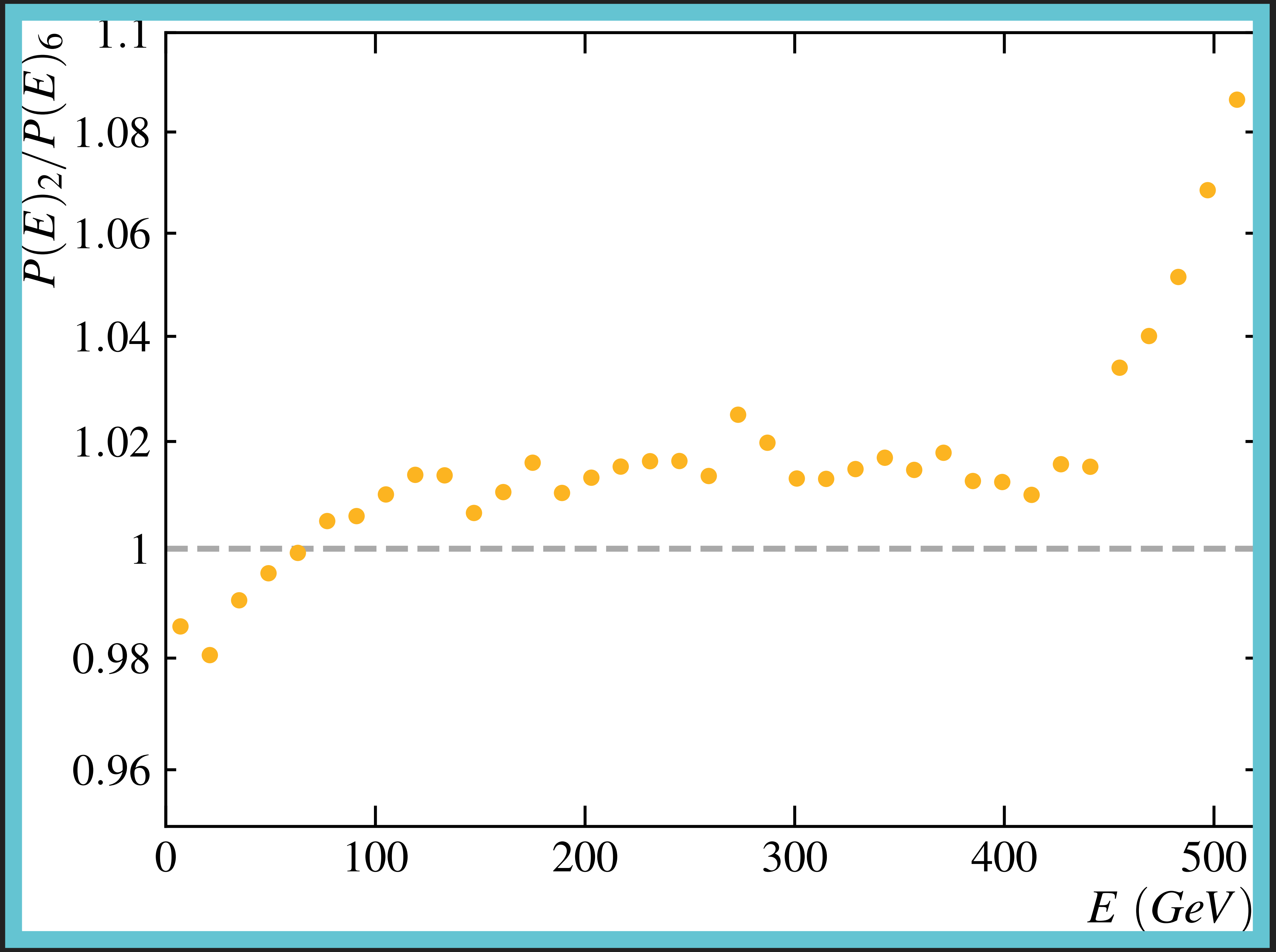
$$\{a\} = \frac{\int dx\, dy\ \bar{\varepsilon}(x,y) a(x,y)}{\int dx\, dy\ \bar{\varepsilon}(x,y)}$$

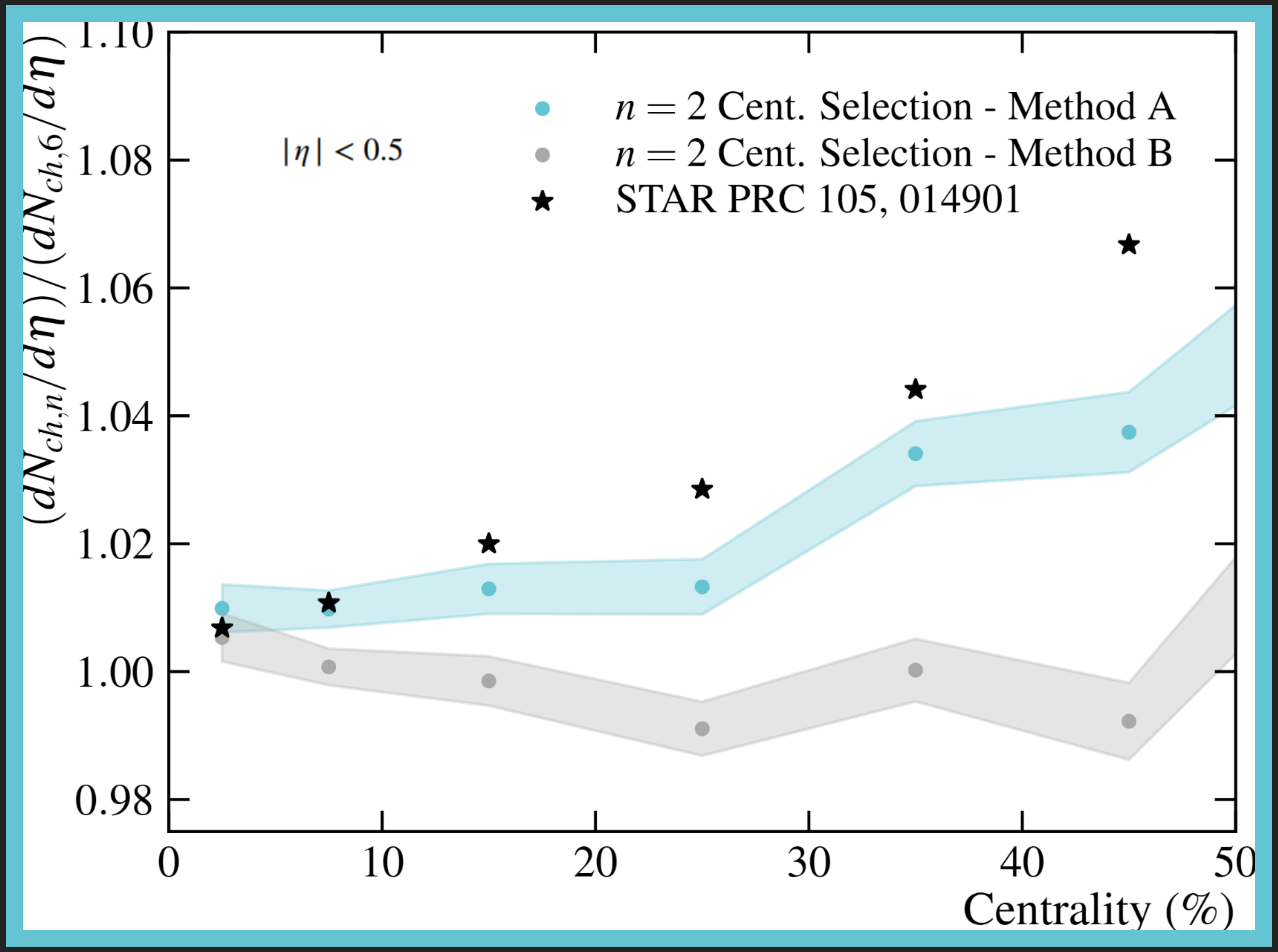


Gardim, Giannini, Grassi, P. Pala, M. Serenone
Phys. Rev. C **110**, 064907

ISOBAR: CENTRALITY SELECTION

- Two selection methods
- Effects on $dN/d\eta$ rations
- Method A - Different bins for each case
- Method B - Common bins for all cases





ISOBAR: SIMULATION CHAIN

- (2+1)D boost invariant at $\mu_b = 0$
- XSCAPE framework Putschke et al, arXiv:1903.07706, 2019

Assume that $\bar{\varepsilon}$ represents massless partons

Emit particles from the Cooper-Frye distribution

HotQCD

Freestream $T^{\mu\nu}$ untill τ_{FS}

$d_\mu T^{\mu\nu} = 0$

+ Equations for Π and $\pi^{\mu\nu}$

Responsible for collisions and decays, the system evolves until kinetic freeze-out

Simple and fast parametric model

Nucleons position as input

$$\bar{\varepsilon}(\overrightarrow{x_T}) = \bar{\varepsilon}(x, y) = \lim_{\tau \to 0^+} \tau\varepsilon(\tau, x, y, \eta = 0)$$