Transitions of few-hadron systems from QCD

Felipe G. Ortega-Gama

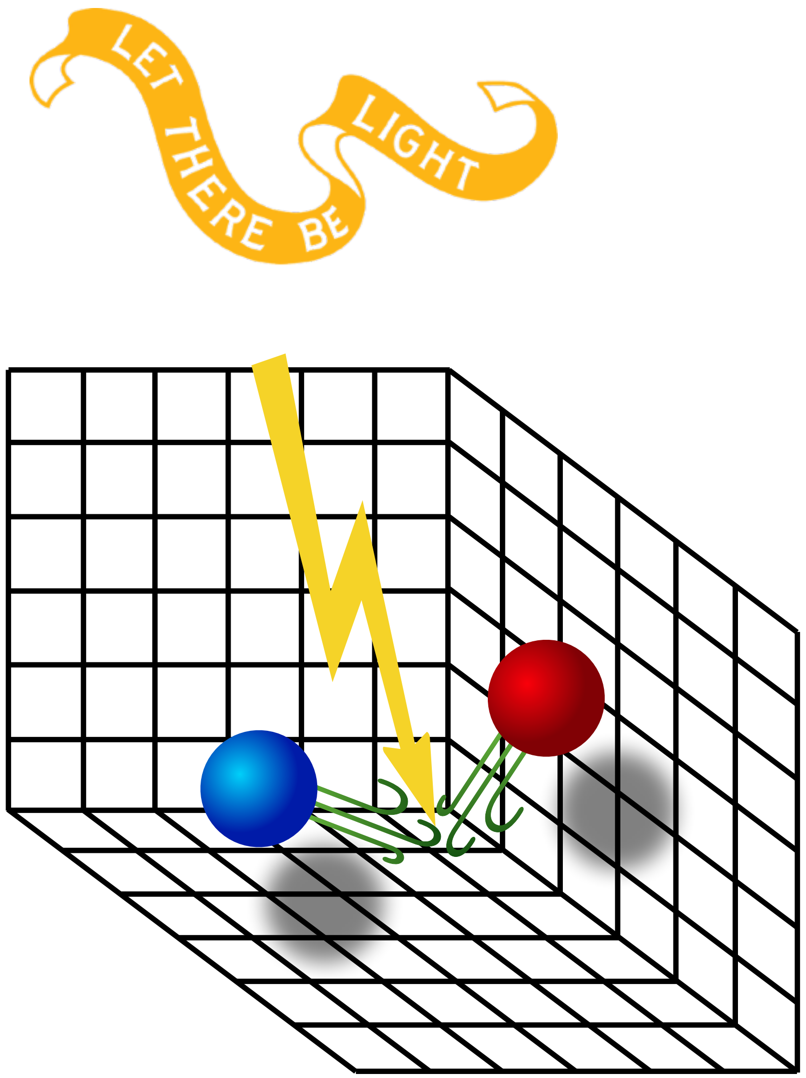
UC Berkeley

fgortegagama@berkeley.edu

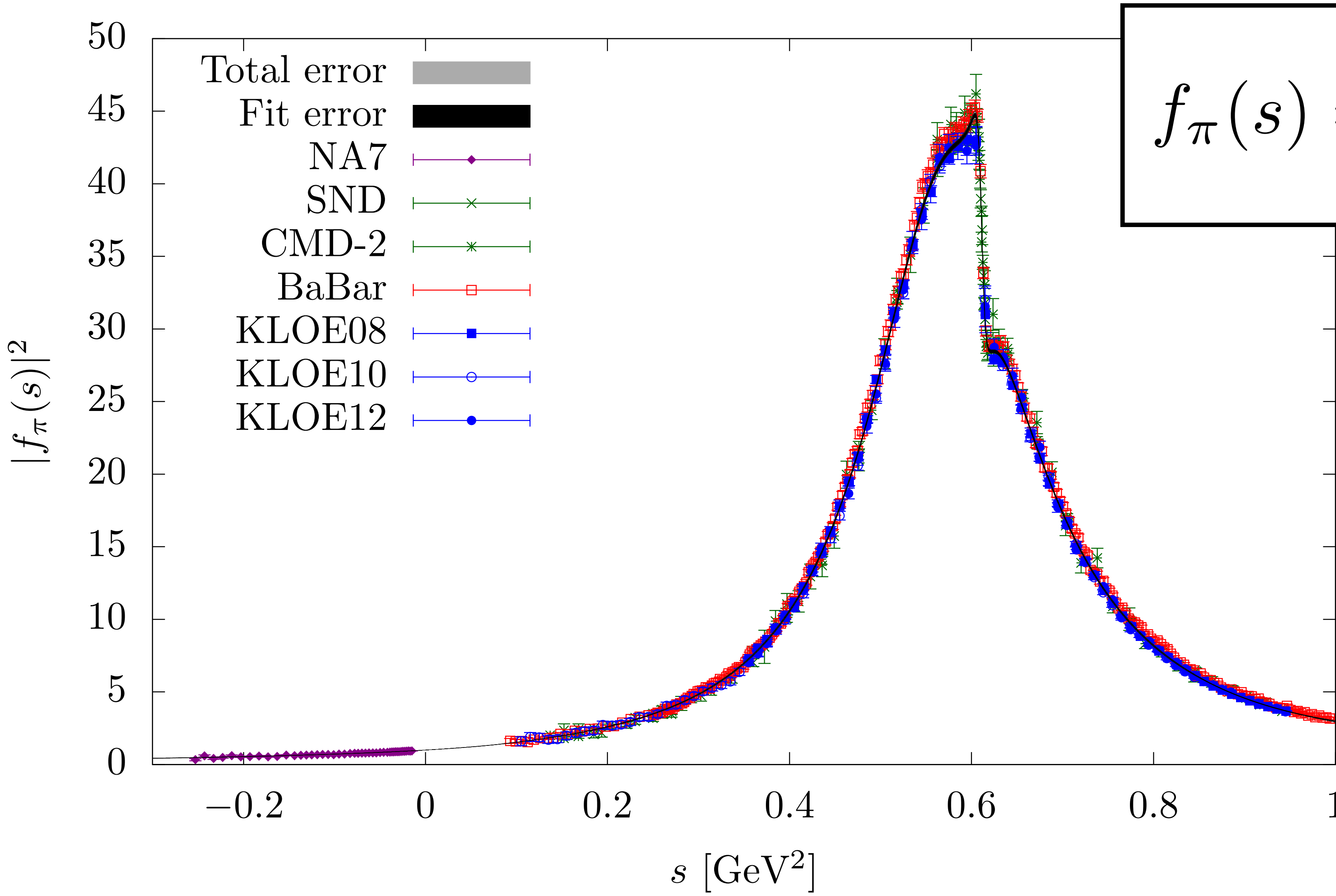
2025 March 15 - GHP

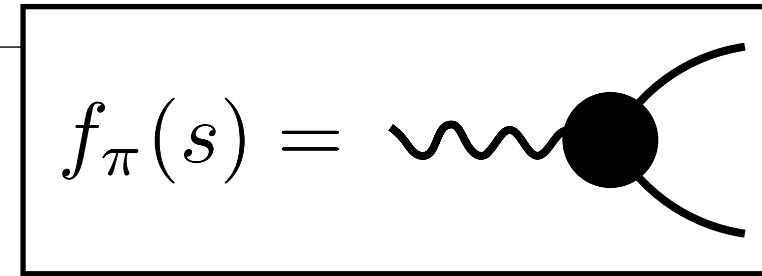
Phys. Rev. D **110**, 094505

In collaboration with
J. Dudek (William & Mary)
R. Edwards (Jefferson Lab)

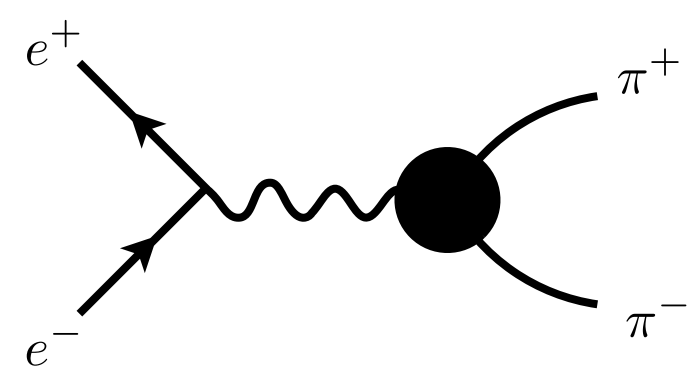


Pion form factor experimental data

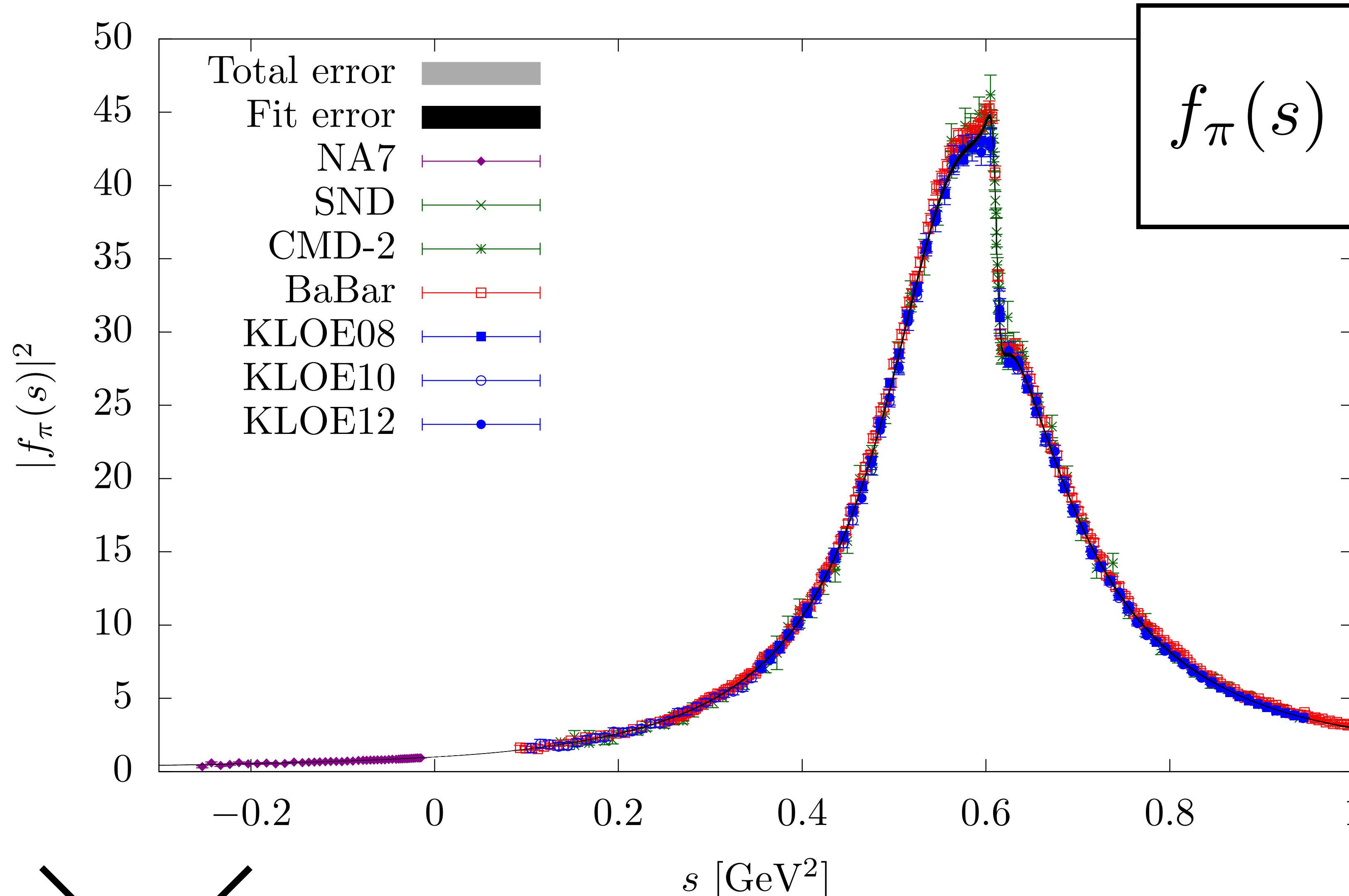


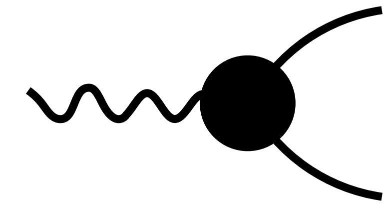


s : Energy squared of the hadron pair

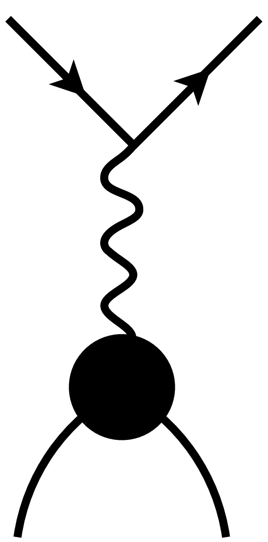


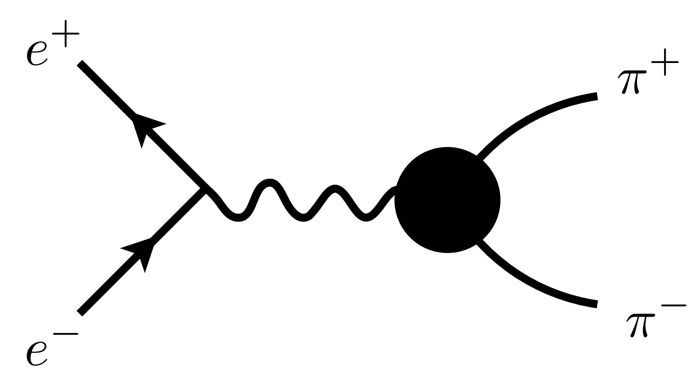
Pion form factor experimental data



$f_\pi(s) =$ 

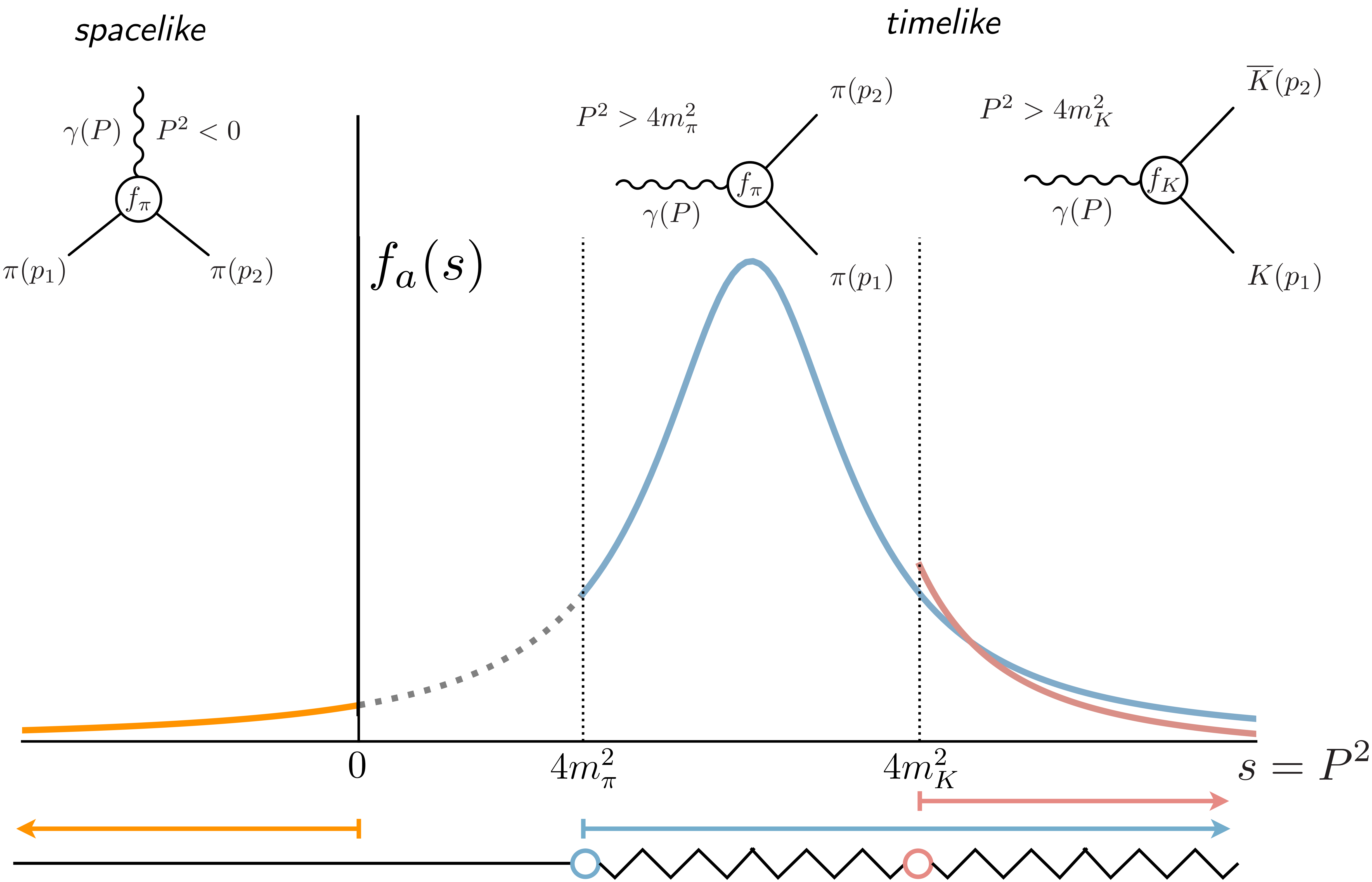
s : Energy squared of the hadron pair



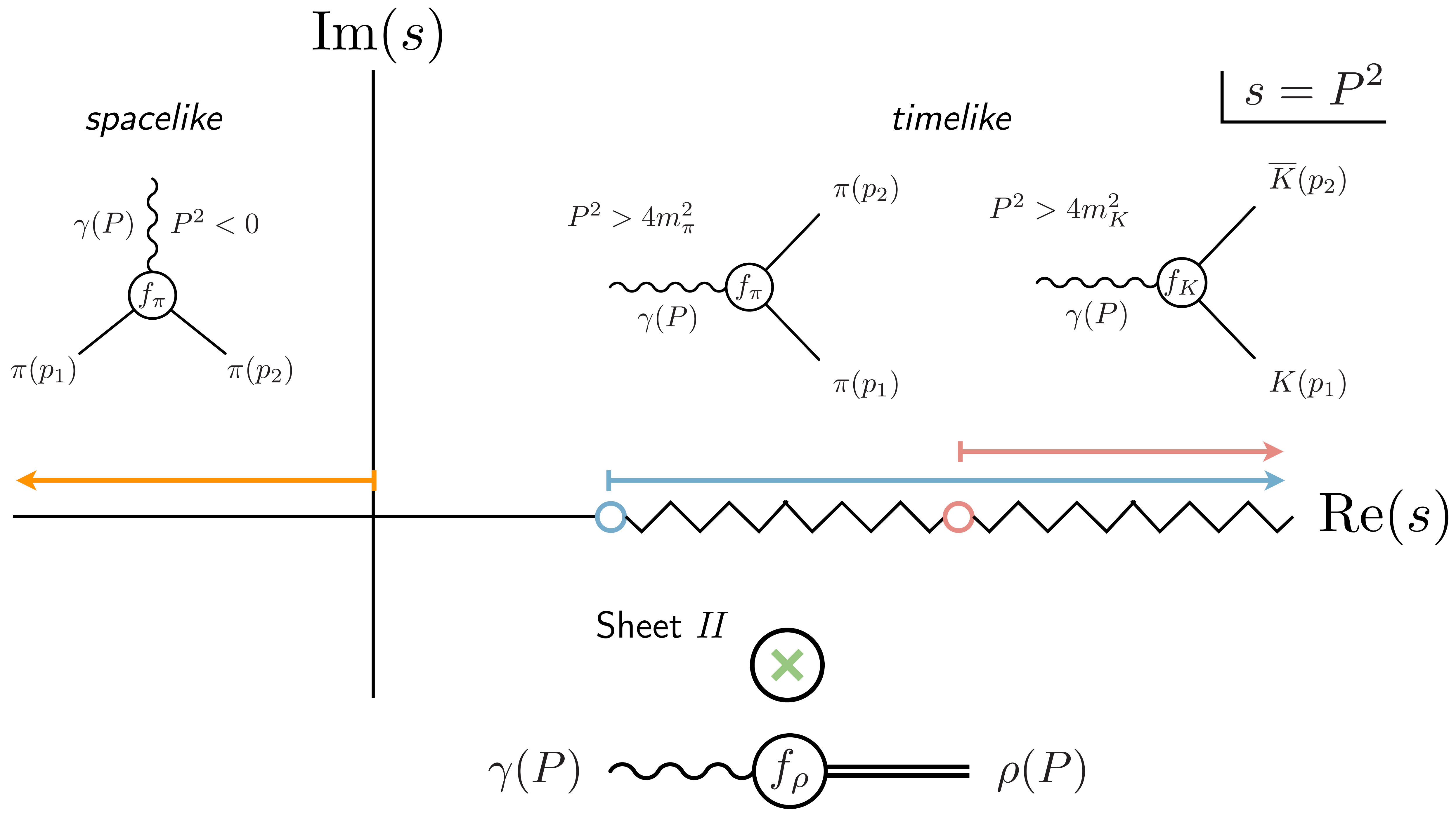


Crossing symmetry

Vector form factor of pseudoscalars

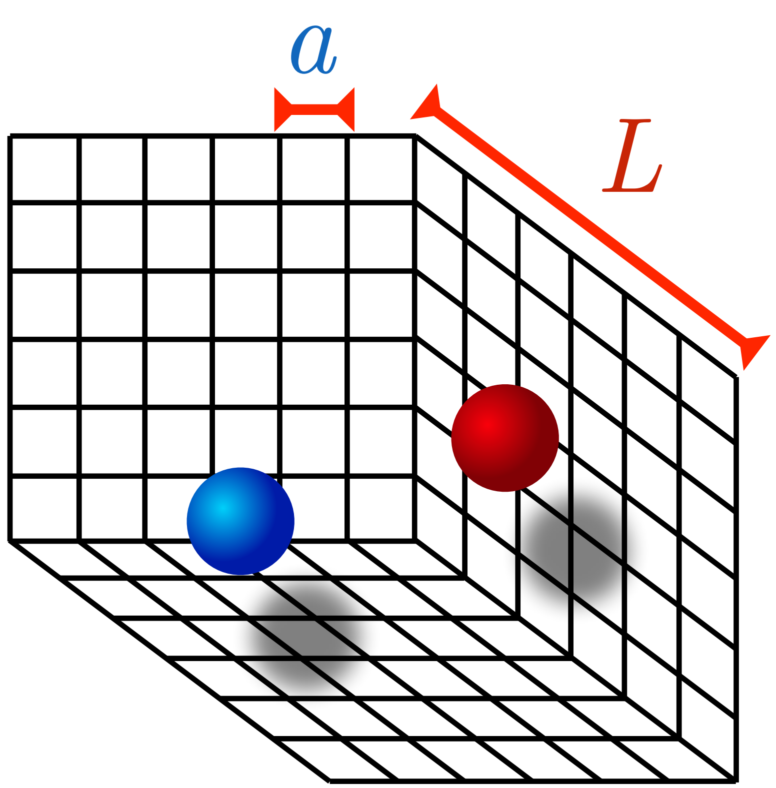


Analytic structure



Physics in a Finite Volume

✦LQCD Path Integral:



✦ Finite Lattice spacing

✦ Finite Volume

✦ Wick rotated: Euclidean spacetime

$$e^{-S_{\mathrm{QCD}}} \implies \langle \mathcal{O}_1(\tau_1) \cdots \mathcal{O}_n(\tau_n) \rangle$$

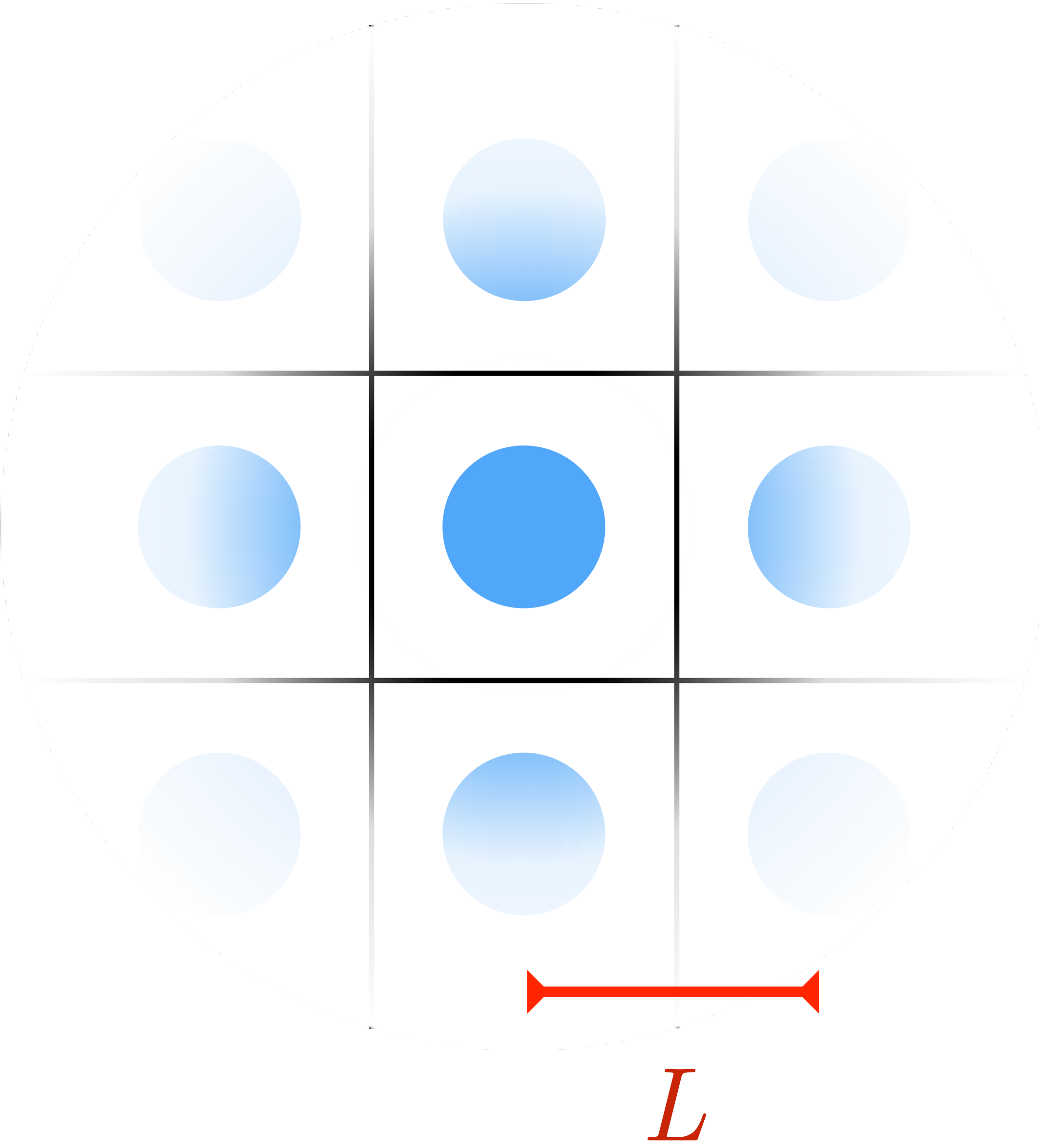
- Discrete FV Spectrum:

$$\left\langle O_i(t) O_j^\dagger(0) \right\rangle = \sum_{n=0} Z_i^{n*} Z_j^n e^{-E_n t}$$

- Local matrix elements

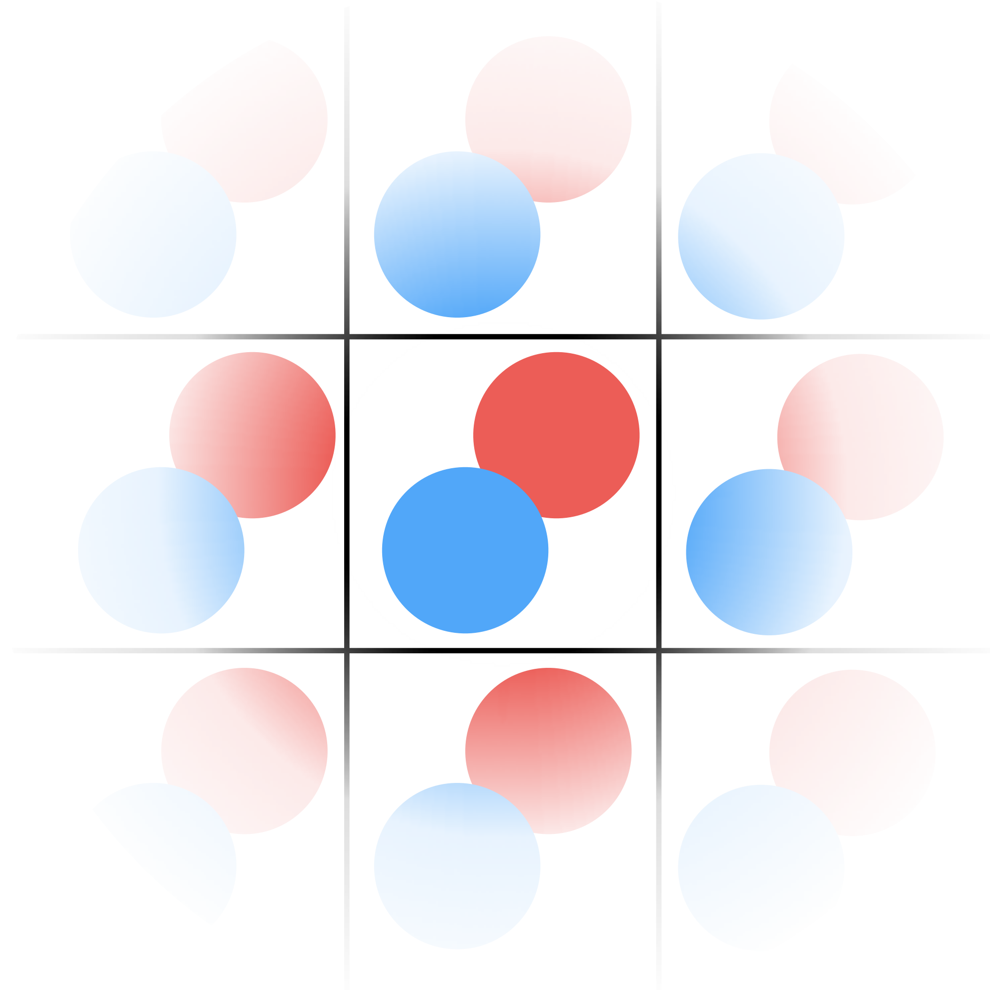
$$\left\langle O_i(\Delta t) \mathcal{J}^\mu(t) O_j^\dagger(0) \right\rangle = \sum_{n,m=0} \langle n | \mathcal{J}^\mu(0) | m \rangle \, Z_j^m Z_i^{n*} e^{-E_n(\Delta t - t)} e^{-E_m t}$$

Form factors in finite volume



Strong interaction $(r \gg 1) \sim e^{-mr}$

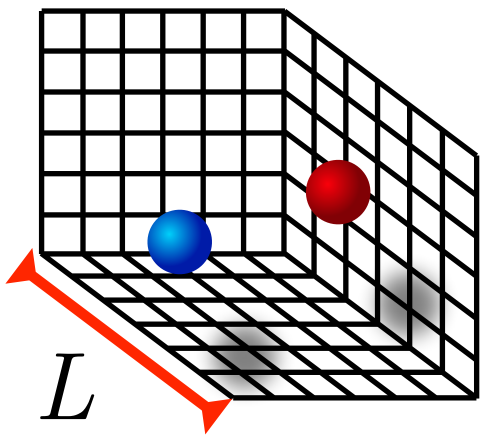
$$\langle \pi, L | \mathcal{J} | \pi, L \rangle = \langle \pi | \mathcal{J} | \pi \rangle + \mathcal{O}(e^{-mL})$$



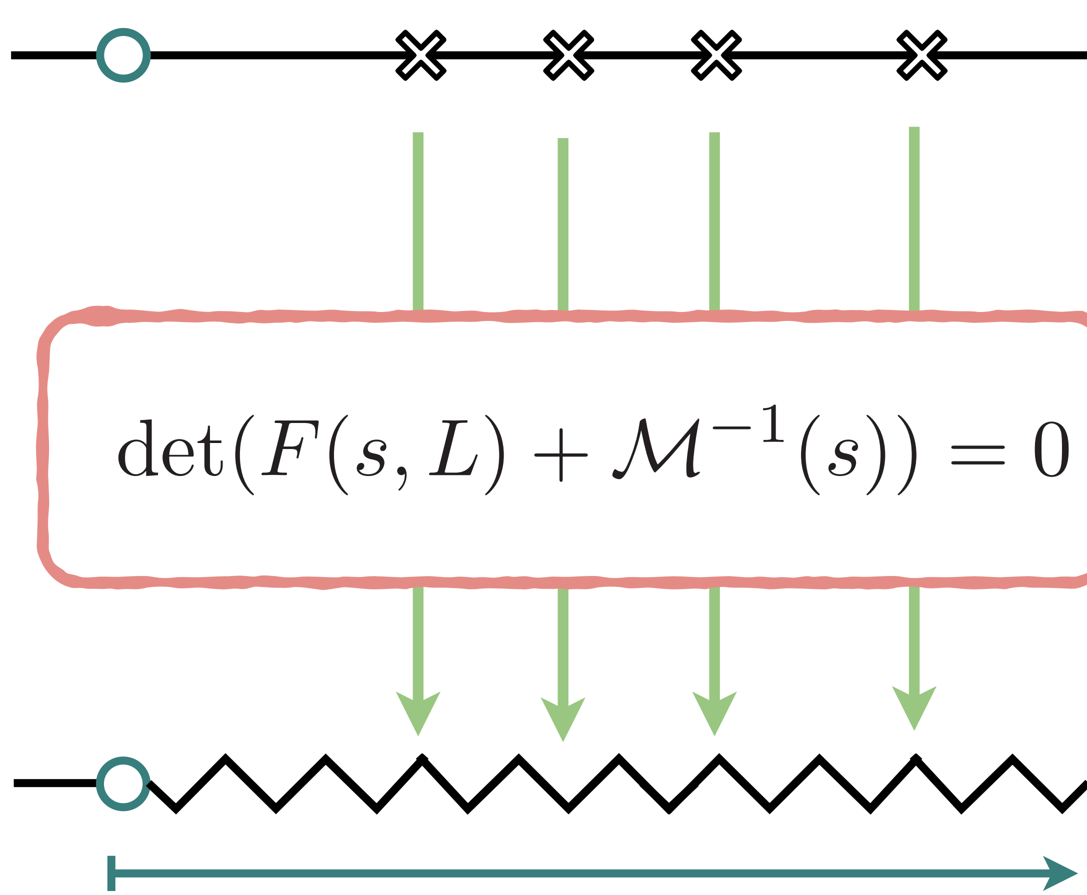
Final state interactions are modified by FV

$$\langle \pi\pi, L | \mathcal{J} | 0 \rangle \not\approx \langle \pi\pi | \mathcal{J} | 0 \rangle$$

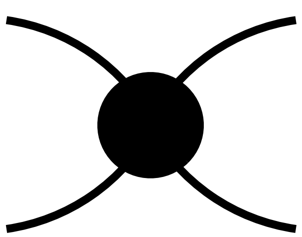
Lüscher formalism

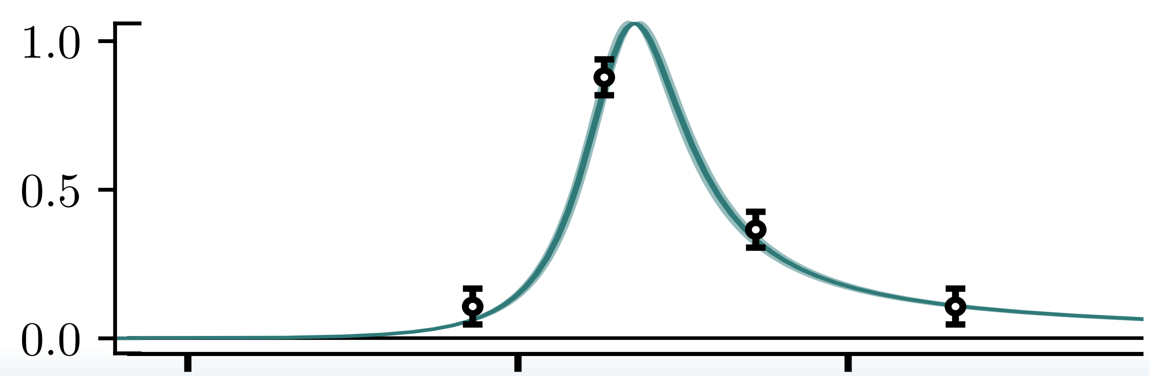


$$\left\langle O_i(t) O_j^\dagger(0) \right\rangle = \sum_{n=0} Z_i^{n*} Z_j^n e^{-E_n t}$$

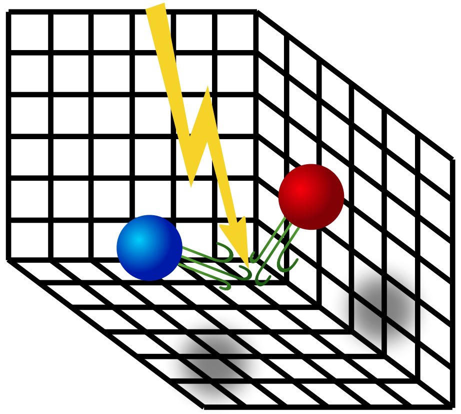


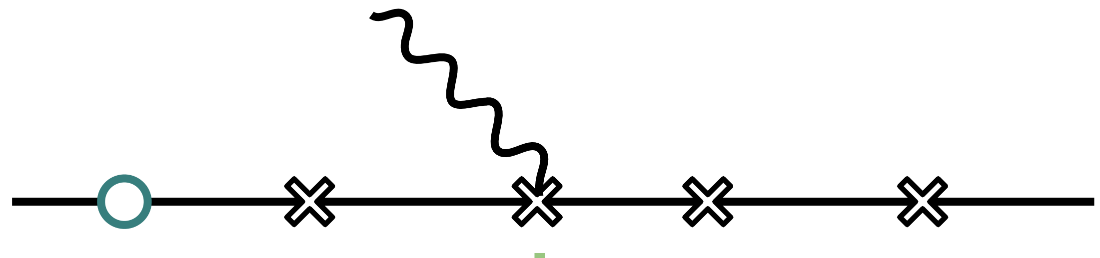
$$\det(F(s,L) + \mathcal{M}^{-1}(s)) = 0 \quad +\mathcal{O}(e^{-m_\pi L})$$

$\mathcal{M}(s) =$ 

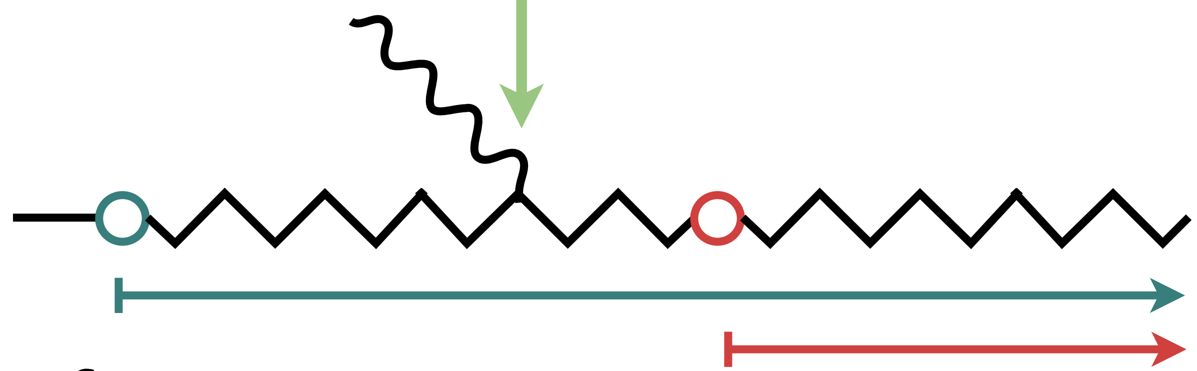


Two-hadron production

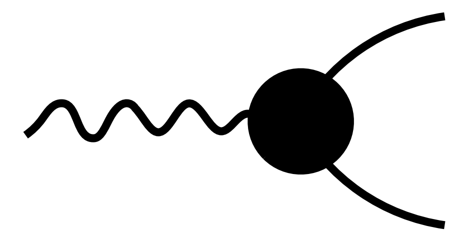




$$|\langle n|\mathcal{J}(0)|0\rangle|^2 = \frac{1}{L^3} f(s) \cdot \mathcal{R}_n \cdot f(s)$$

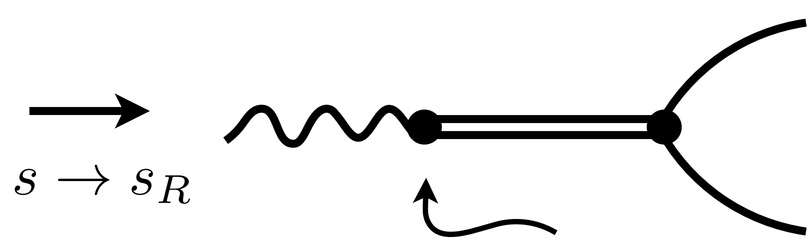


- Lellouch-Lüscher factor



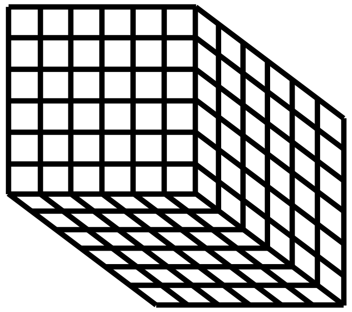
Normalization of dynamic FV state & rescattering effects

$$\mathcal{R}_n \equiv \lim_{E \to E_n} \frac{E - E_n}{F^{-1} + \mathcal{M}}$$

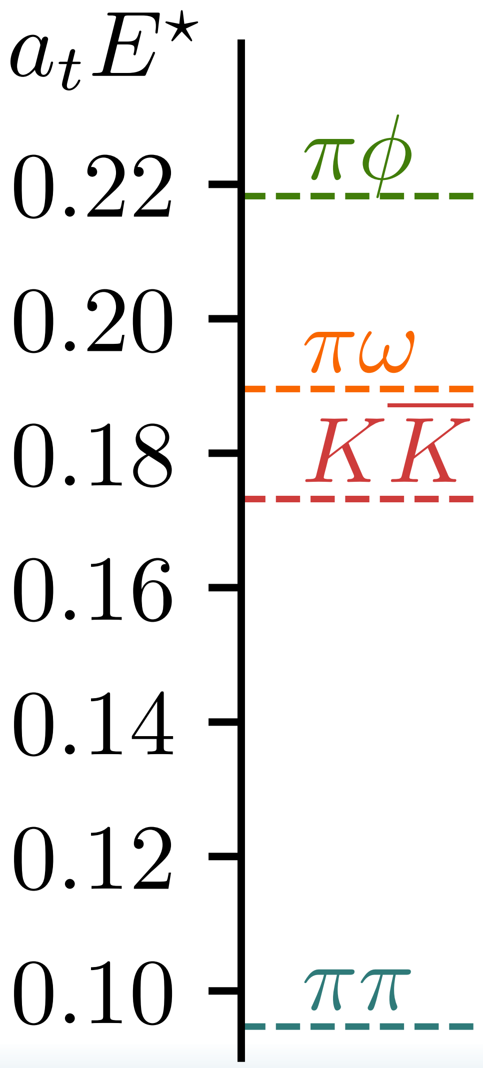


Lattice QCD setup

	$a_t m$	m/MeV
π	0.0474	284
K	0.0866	519



$J^P(I^G)$=1⁻(1⁺)



$$L^3 \times T = 24^3 \times 256$$

$$m_\pi L \approx 4 \qquad 1 \gg e^{-m_\pi L}$$

$$a_t^{-1} \approx 6\ \mathrm{GeV}$$

400 gauge configurations

$$\xi = a_s/a_t = 3.455(6)$$

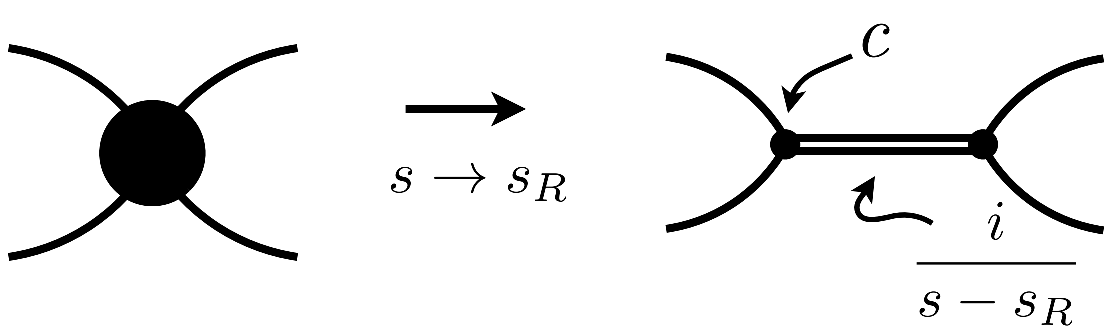
Elastic Lüscher formalism

	m/MeV
π	284
K	519

$$\mathcal{M}(s) = F^{-1}(s, L)$$

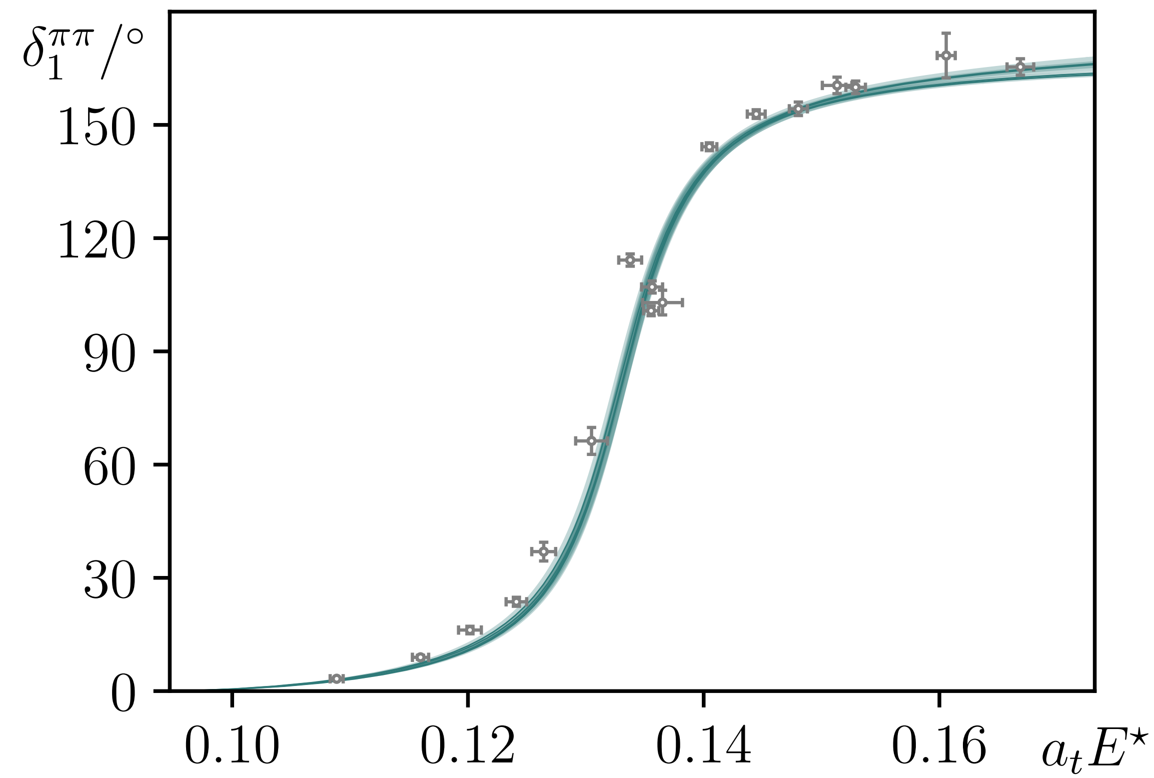
Neglecting higher spin

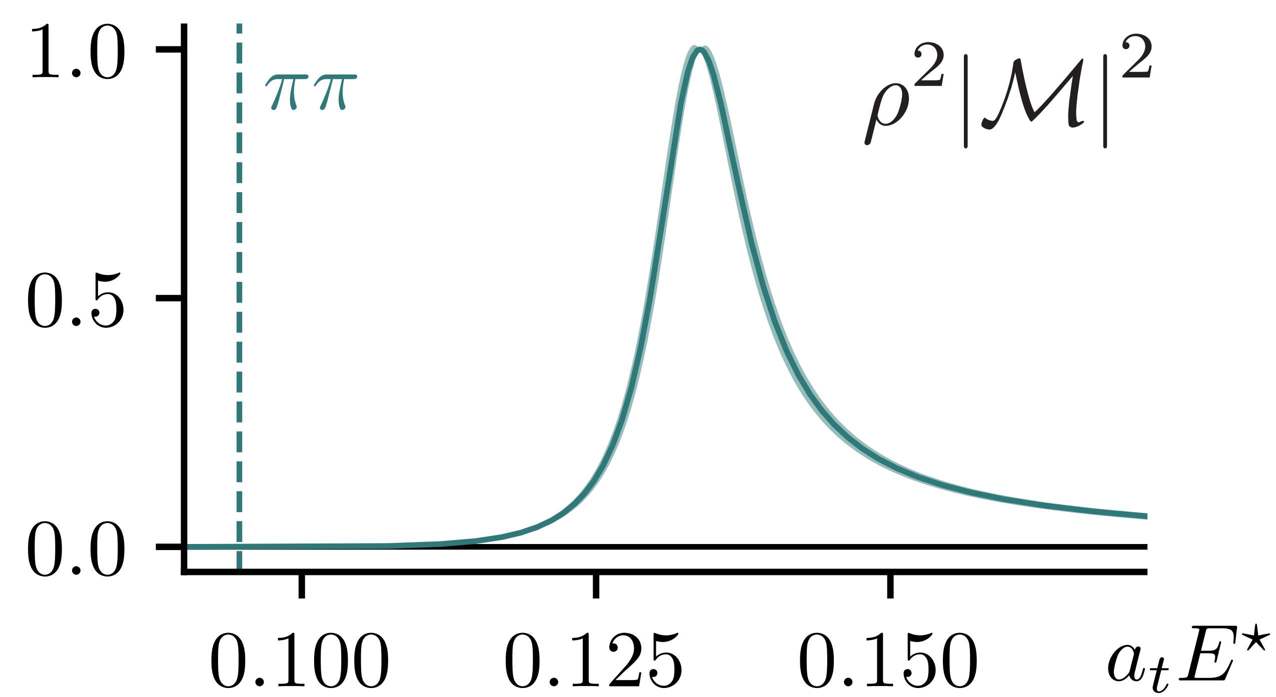
$$\mathcal{M}_{\pi\pi\to\pi\pi} = \frac{1}{\rho} \sin(\delta_1^{\pi\pi})\, e^{i\delta_1^{\pi\pi}}$$



$\rho(1^{--})$:

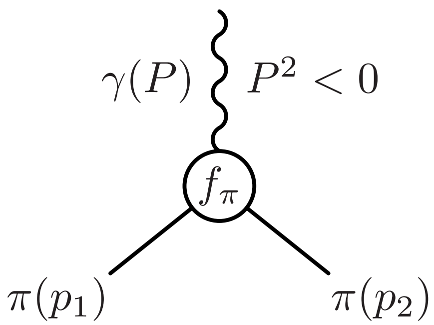
$$\mathrm{Re}(\sqrt{s_R}) = 797 \pm 2.6\ \mathrm{MeV}$$
$$\mathrm{Im}(\sqrt{s_R})/2 = 28.5 \pm 1\ \mathrm{MeV}$$

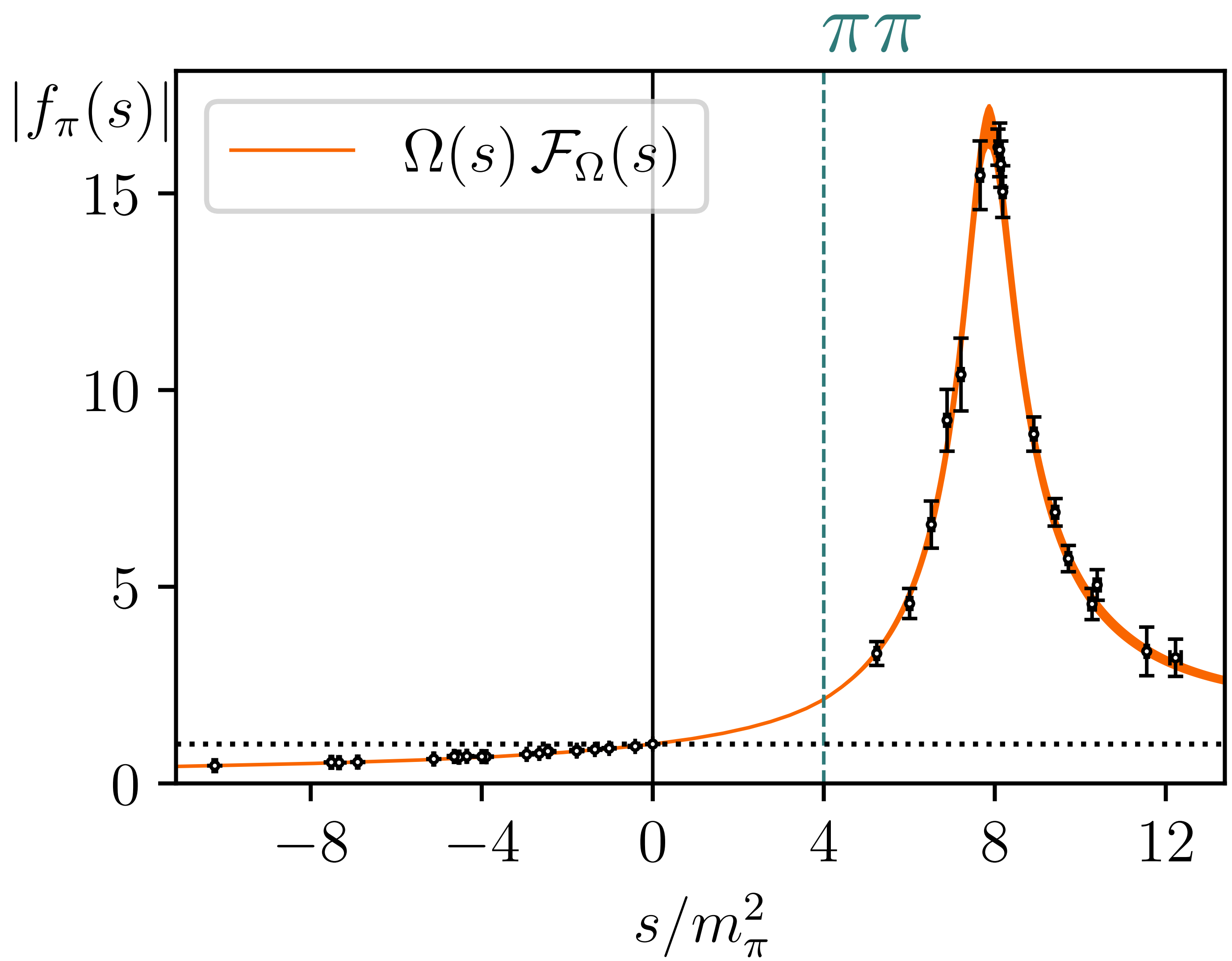


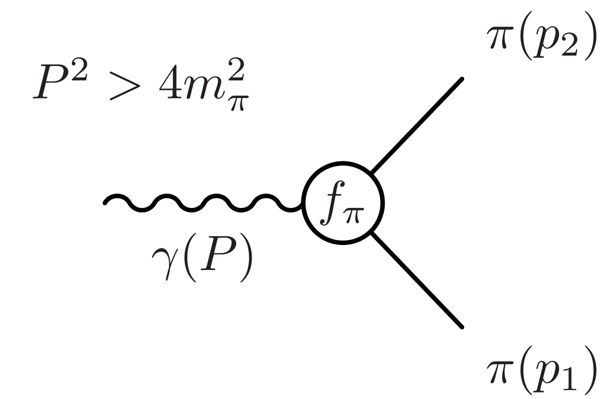


Pion vector form factor

$$|\langle n|\mathcal{J}(0)|0\rangle|^2 = \frac{1}{L^3} f(s) \cdot \mathcal{R}_n \cdot f(s)$$





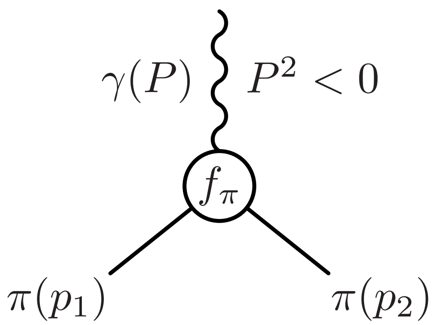


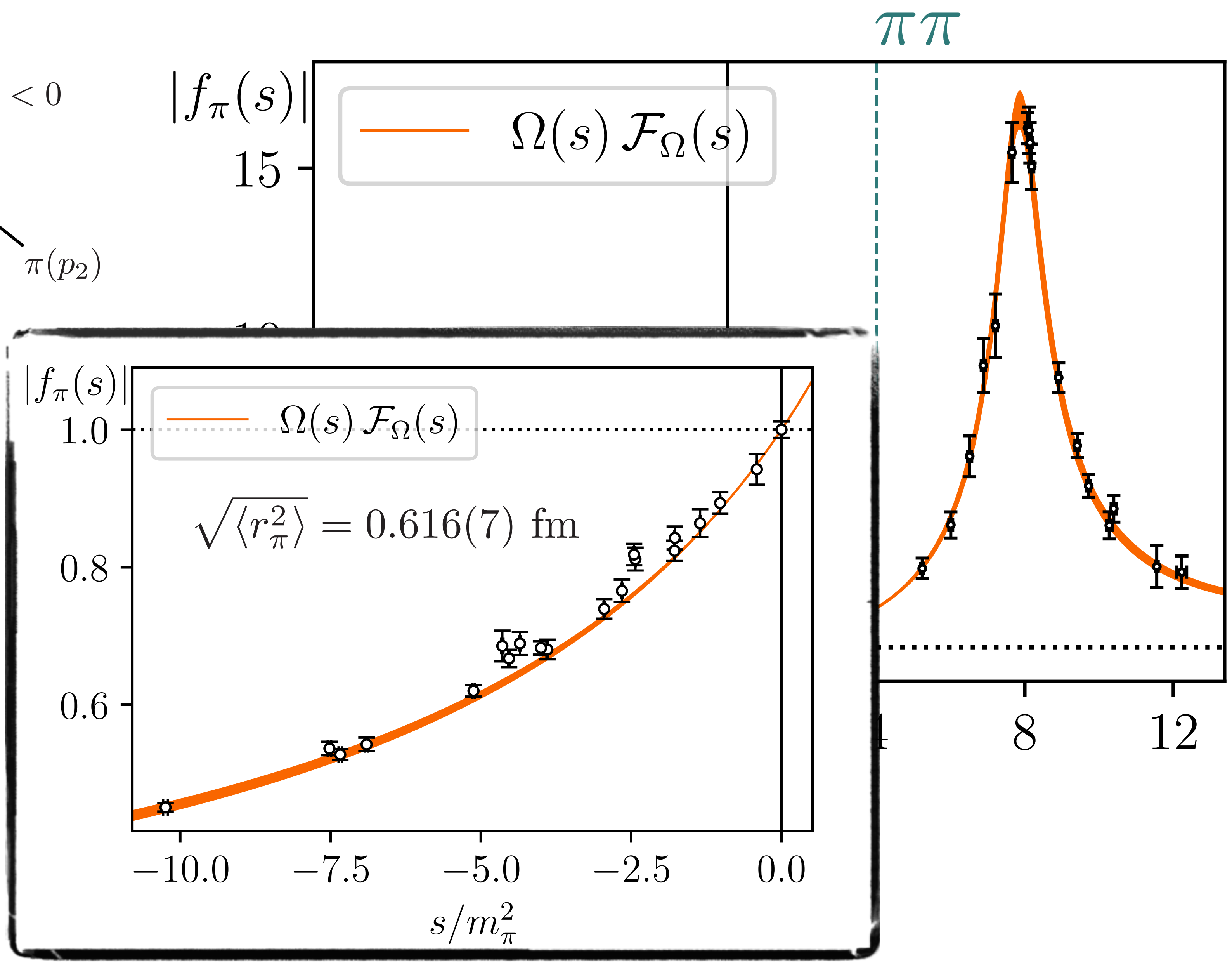
$$\Omega(s) = \exp\left(\frac{s}{\pi}\int_{4m_\pi^2}^{\infty} ds' \, \frac{\delta_1^{\pi\pi}(s')}{s'(s'-s)}\right)$$

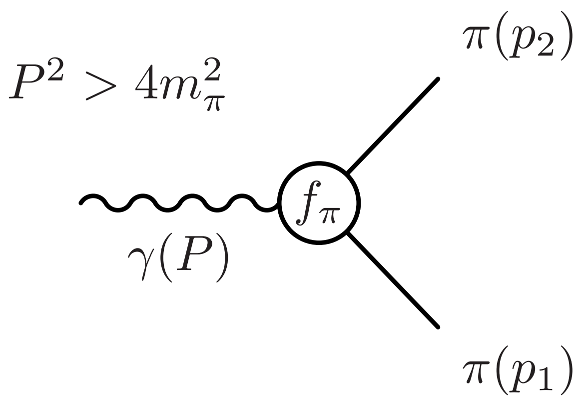
$$\mathcal{F}_\Omega(s) = Q + \sum_{n=1}^{N} c_n (z_c(s)^n - z_c(0)^n)$$

Pion vector form factor

$$|\langle n|\mathcal{J}(0)|0\rangle|^2 = \frac{1}{L^3} f(s) \cdot \mathcal{R}_n \cdot f(s)$$







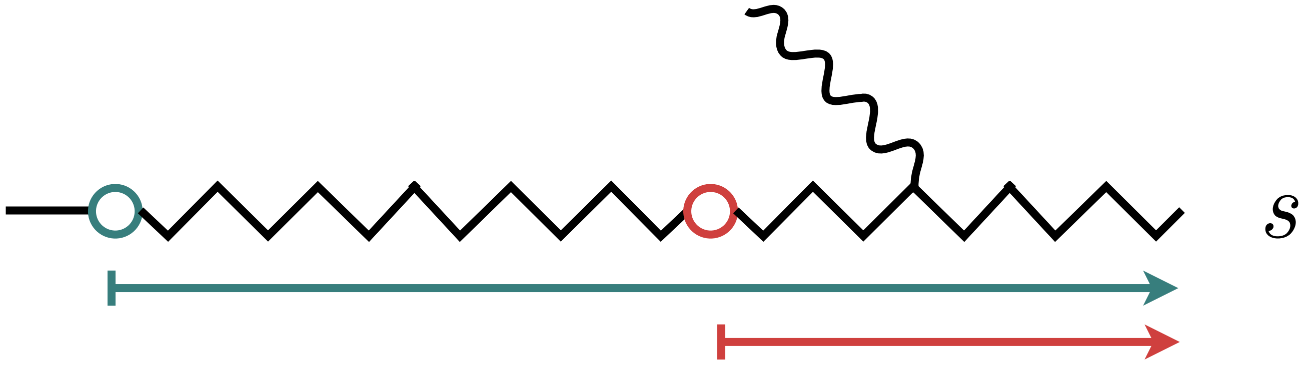
$$\Omega(s) = \exp\left(\frac{s}{\pi}\int_{4m_\pi^2}^{\infty} ds' \, \frac{\delta_1^{\pi\pi}(s')}{s'(s'-s)}\right)$$

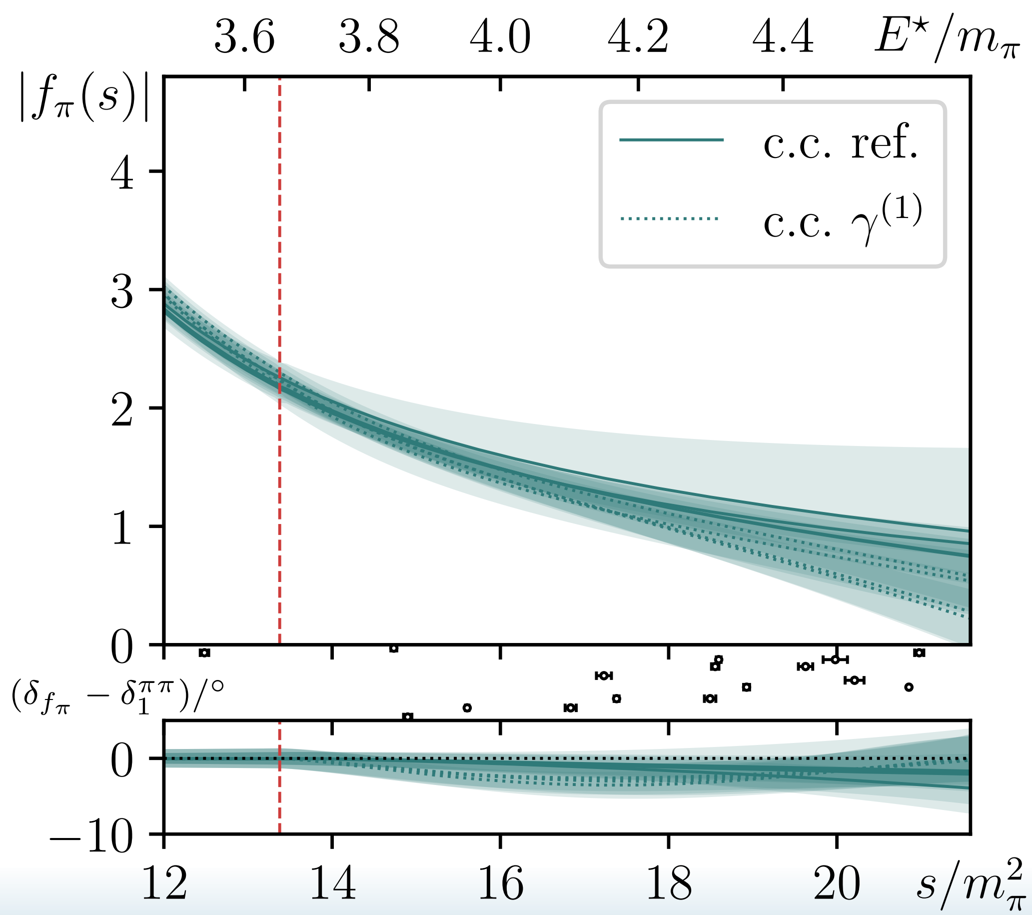
$$\mathcal{F}_\Omega(s) = Q + \sum_{n=1}^{N} c_n(z_c(s)^n - z_c(0)^n)$$

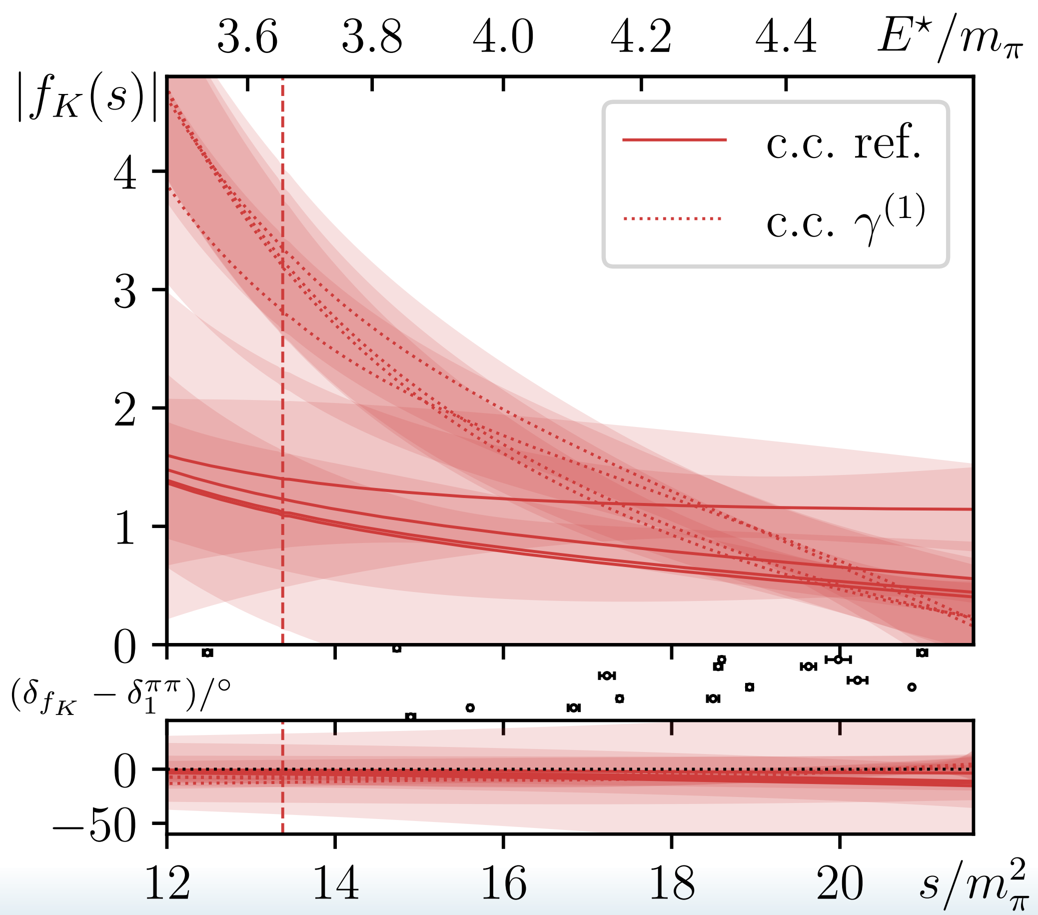
Timelike form factor above KK thr

	m/MeV
π	284
K	519

$$|\langle n|\mathcal{J}(0)|0\rangle|^2 = \frac{1}{L^3} f(s) \cdot \mathcal{R}_n \cdot f(s)$$



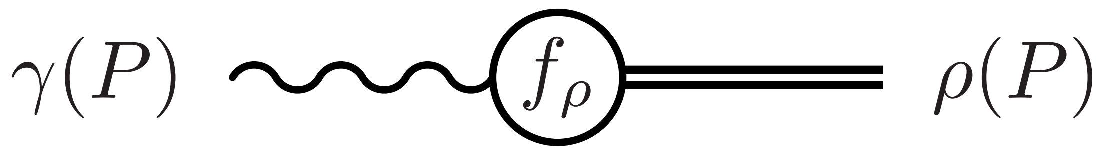


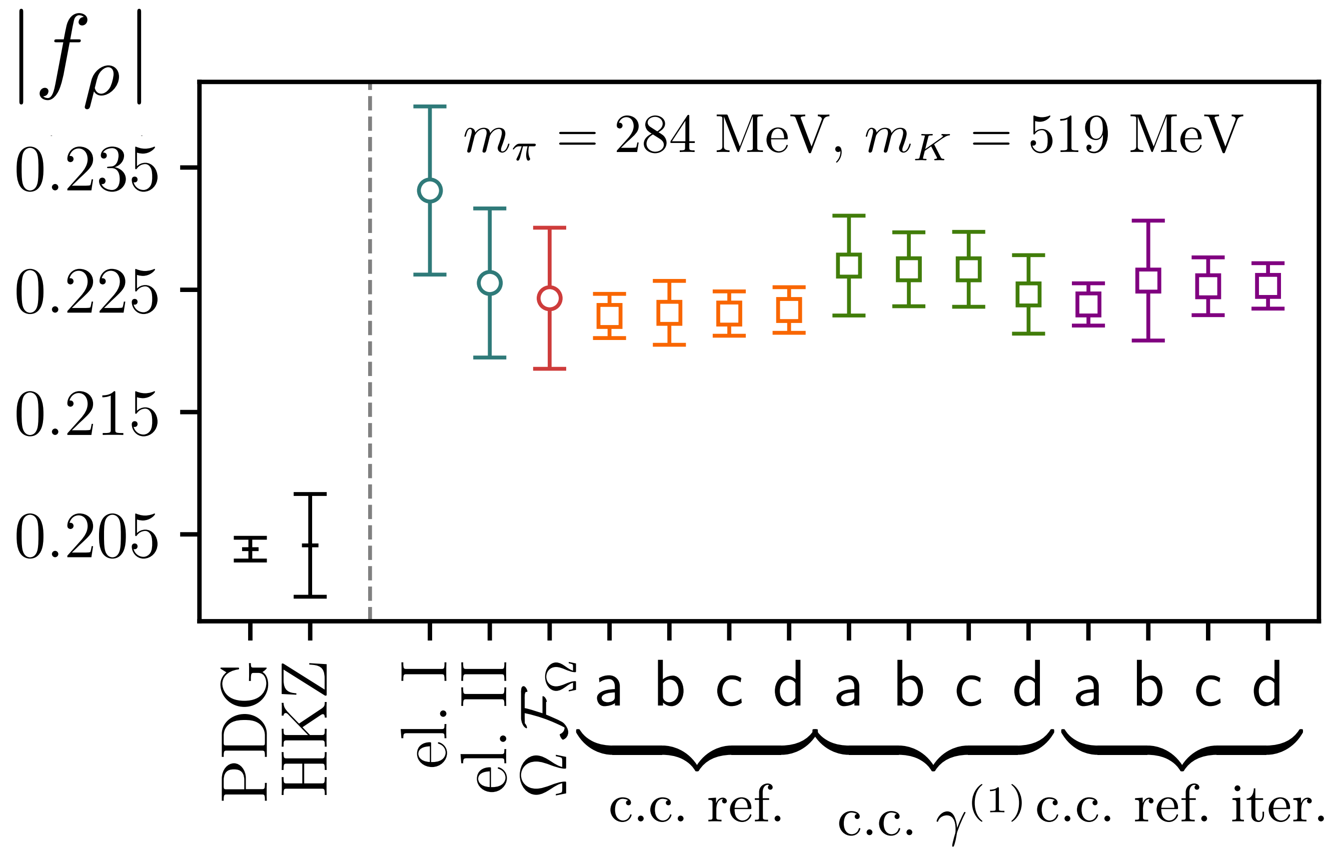


ρ meson decay constant

$$f_a(s \sim s_R) \propto \sqrt{16\pi}\,\hat{c}_a \frac{1}{s - s_R} f_\rho$$

$$\langle \rho(\vec{P}, m) | \mathcal{J}^\mu | 0 \rangle = \epsilon^{\mu *}(P, m)\, f_\rho\, m_R^2$$

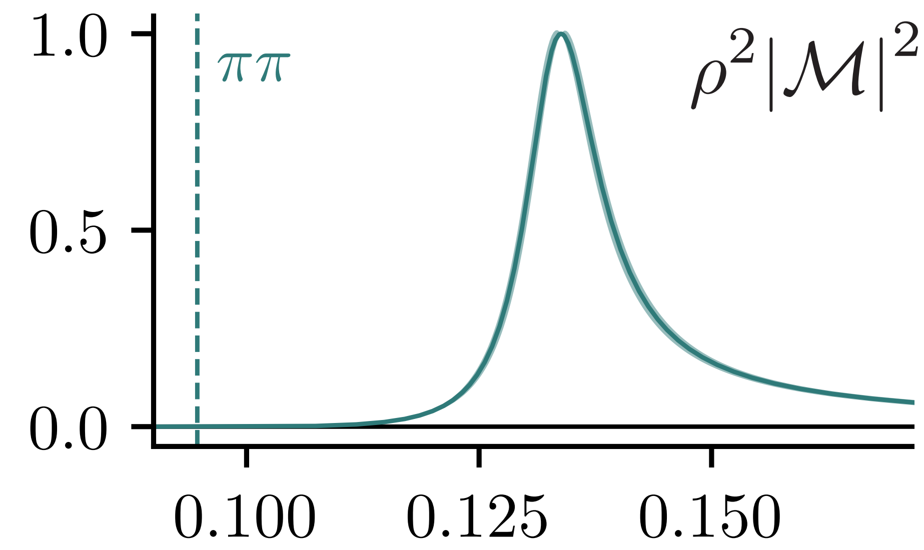




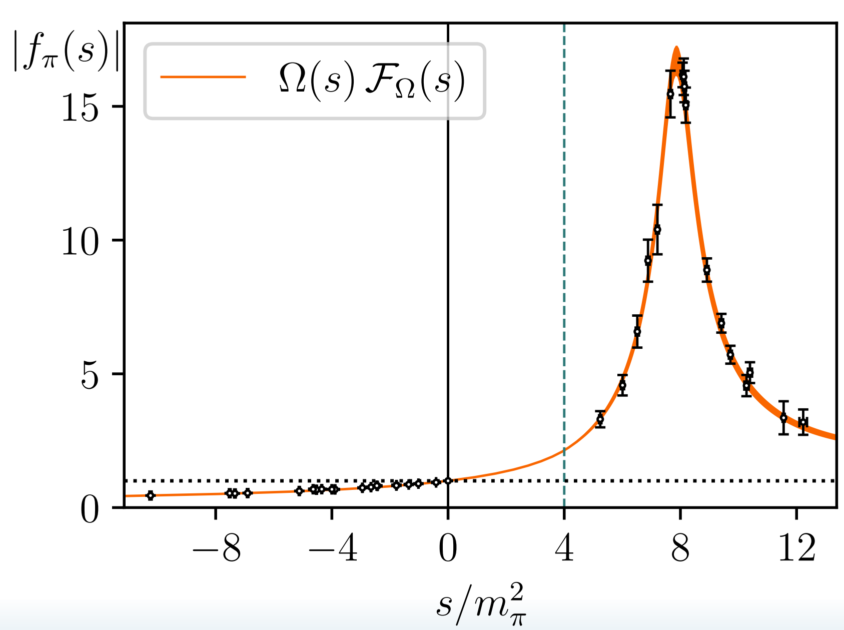
Including inelastic *KK* (cc) effects

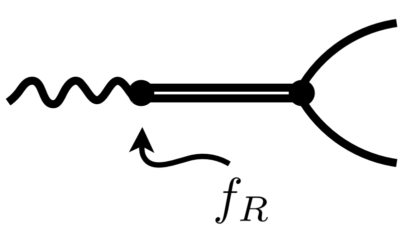
Summary and outlook

- ☑ Vector form factor
  - ☑ spacelike + timelike
  - ☑ Coupled channel
- ☐ Form factors of (*exotic*) resonances.
- ☐ CP violation: e.g. D to $\pi\pi/KK$.
  [Phys.Rev.Lett. 122 211803 (2019)]
- ☐ 2-body amplitudes from QCD.

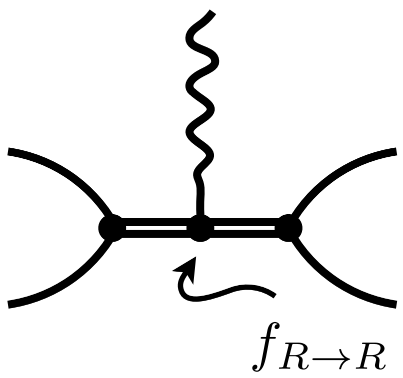


$m_\pi = 284$ MeV, $m_K = 519$ MeV



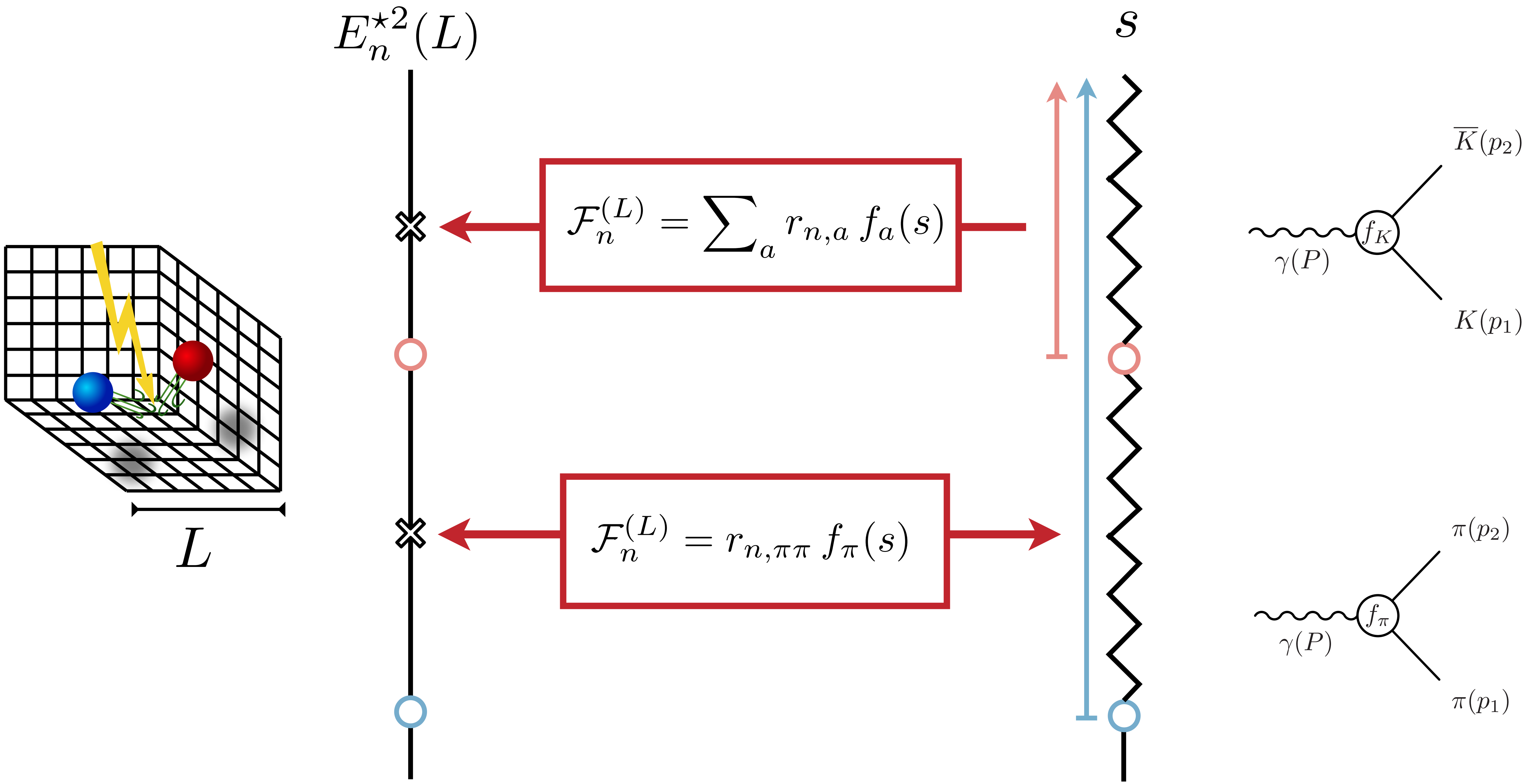


$|f_\rho| = 0.224(6)$



$\langle r_\rho^2 \rangle = ?$

FV correction: $r_{n,a}$

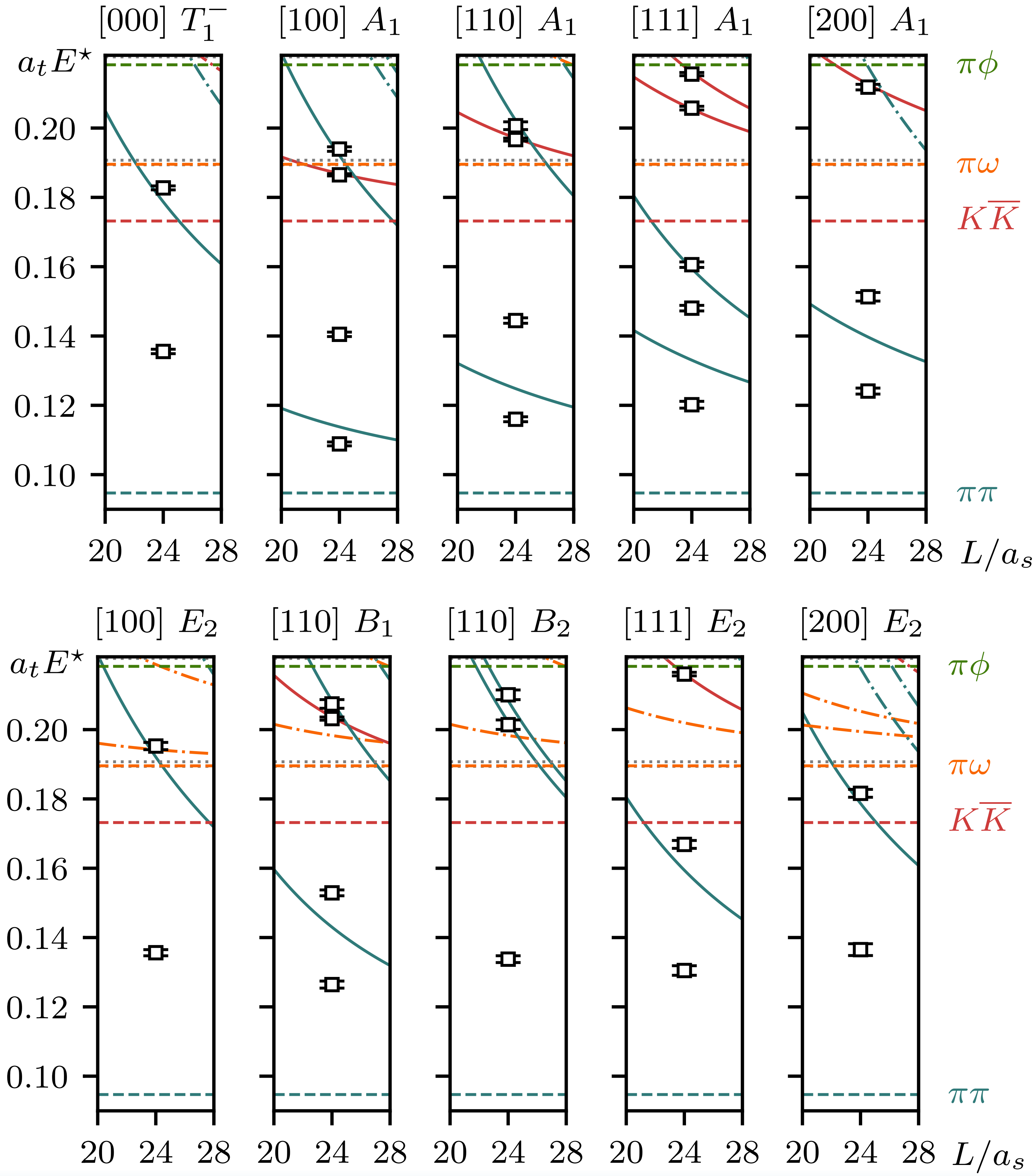


$$\langle 0|\mathcal{J}|E_n^\star\rangle_L \propto \mathcal{F}_n^{(L)} \qquad \langle 0|\mathcal{J}|P,a\rangle \propto f_a$$

Finite volume spectrum

$[\vec{d}] = \vec{P}/(2\pi/L)$

- 17 elastic levels
- 15 above KK threshold

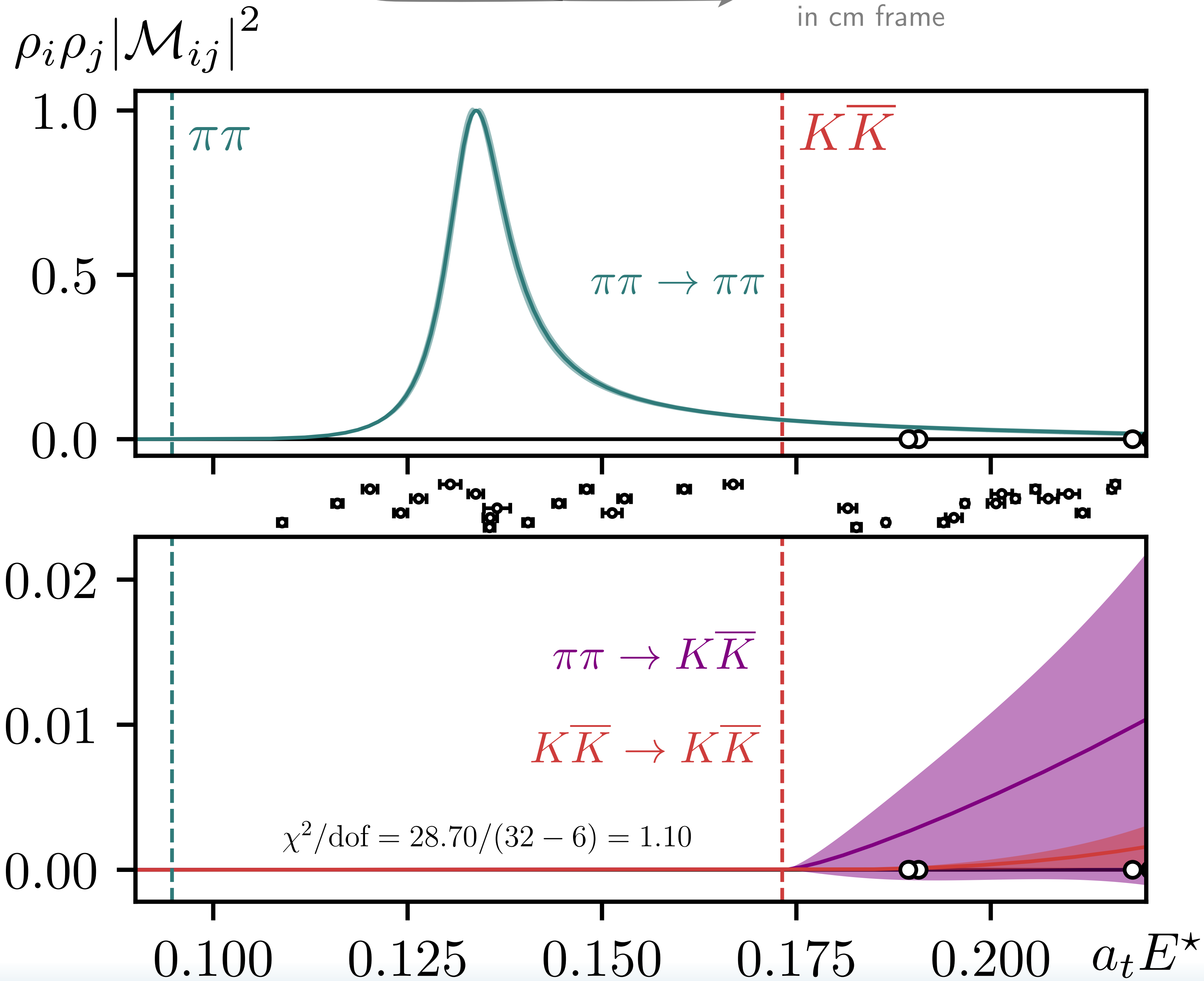


Scattering coupled channel fit

$$\mathcal{M}^{-1}_{ab} = \frac{1}{2k^\star_a} K^{-1}_{ab} \frac{1}{2k^\star_b} - i\rho_{\mathrm{CM},ab}$$

Relative momentum in cm frame

$$K_{ab} = \frac{g_a g_b}{-s + m_r^2} + \gamma_{ab}$$



ρ resonance

$$\mathcal{M}_{ab}(s \sim s_R) \sim \frac{c_a\, c_b}{s_R - s}$$

$$\mathrm{Re}(\sqrt{s_R}) = 797 \pm 2.6 \text{ MeV}$$
$$\mathrm{Im}(\sqrt{s_R})/2 = 28.5 \pm 1 \text{ MeV}$$

$$\left|\frac{c_{\pi\pi}}{k^\star_{\pi\pi}(s_R)}\right| = 6.41 \pm 0.13$$

$$\left|\frac{c_{K\overline{K}}}{k^\star_{K\overline{K}}(s_R)}\right| = 2.4 \pm 4.0$$