

Gravitational form factors with Generalized Parton Distributions via near-threshold heavy quarkonium photo-production

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Based on works in collaboration with X. Ji, Y. Liu, J. Yang, F. Yuan and W. Zhao

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[2308.13006]

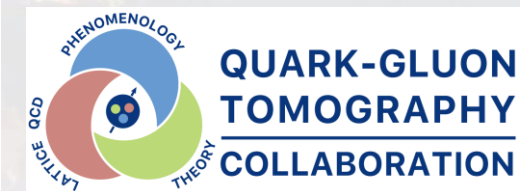
[2501.10532]

APS GHP Workshop, Anaheim, CA

March. 14 – 16th, 2025

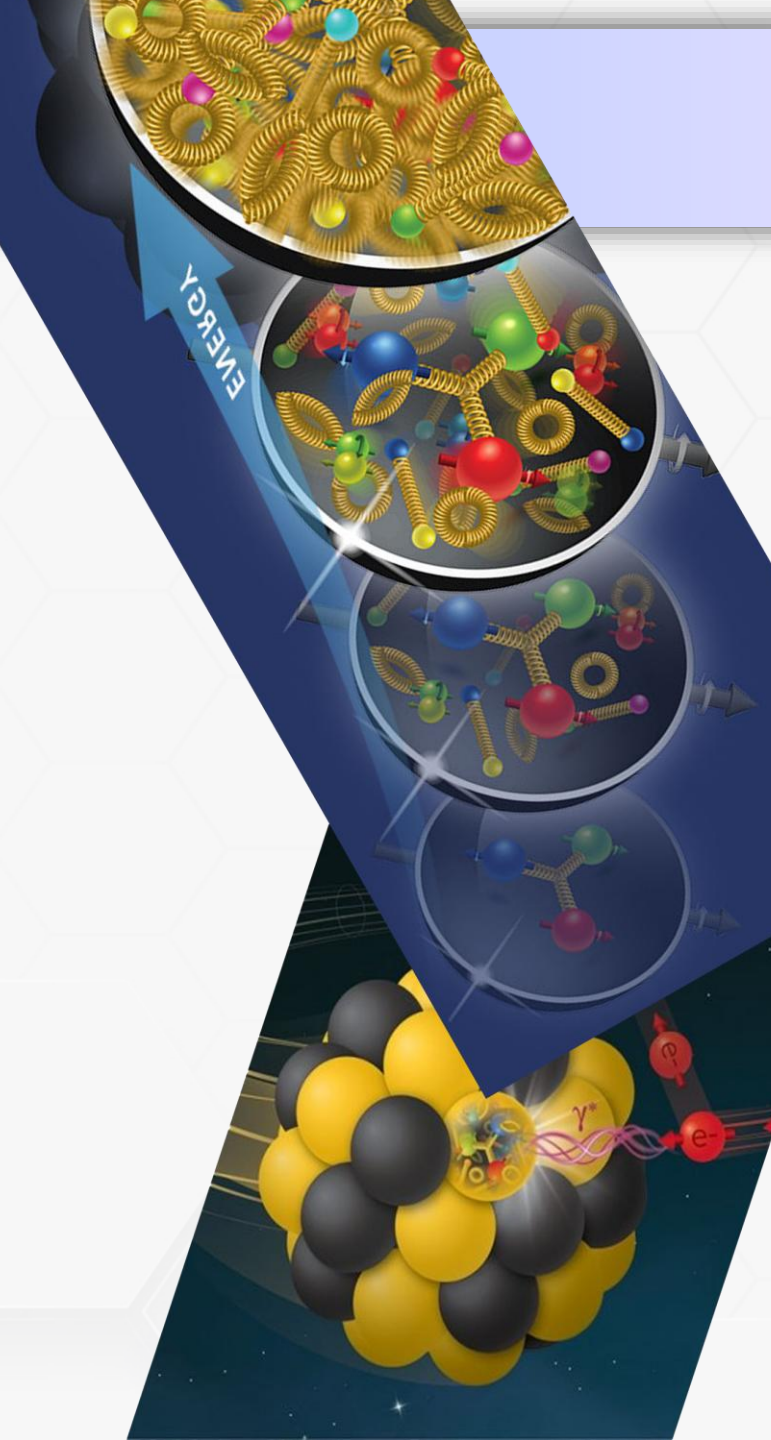


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Hadronic Physics
GHP



Outline

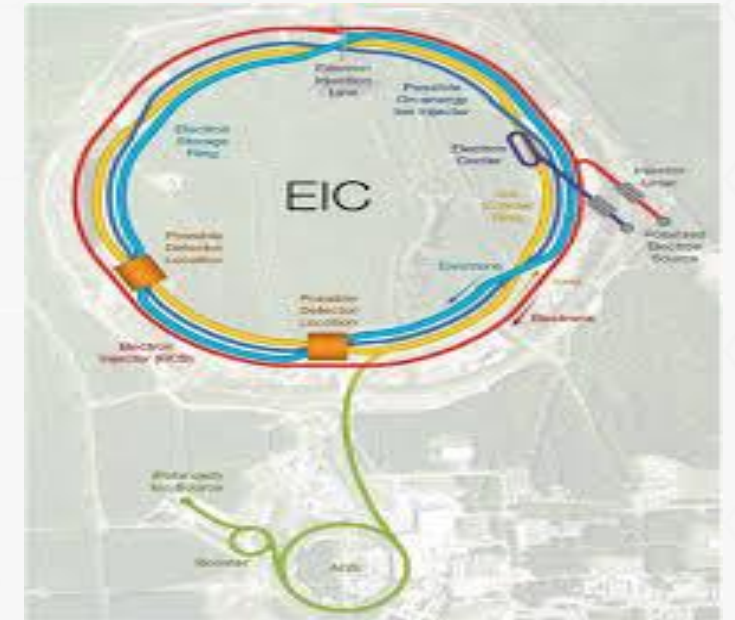
- » Gravitational form factors and Generalized Parton Dist.
- » Exclusive threshold productions of heavy quarkonium
- » Next-to-leading corrections and Bayesian Inference
- » Summary and outlook



Internal structure of the nucleon

Ever since we realized that nucleons are not fundamental particles, understanding their internal structures had become one of the most important topics.

However, finding a probe is quite hard.



Energy-momentum tensor form factors

The energy-momentum tensor (EMT) is the tool to study the mechanical properties of the nucleon. Its nucleon matrix element can be written as:

$$\langle P' | T_{q,g}^{\mu\nu} | P \rangle = \bar{u}(P') \left[A_{q,g}(t) \gamma^{(\mu} \bar{P}^{\nu)} + B_{q,g}(t) \frac{\bar{P}^{(\mu} i \sigma^{\nu)\alpha} \Delta_\alpha}{2M_N} + C_{q,g}(t) \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2}{M_N} + \bar{C}_{q,g}(t) M_N g^{\mu\nu} \right] u(P)$$

X. Ji Phys. Rev. Lett. 78, 610 (1997)

Momentum form factors:

$$A_{q,g}(t)$$

Angular momentum form factors:

$$J_{q,g}(t) = \frac{1}{2} (A_{q,g}(t) + B_{q,g}(t))$$

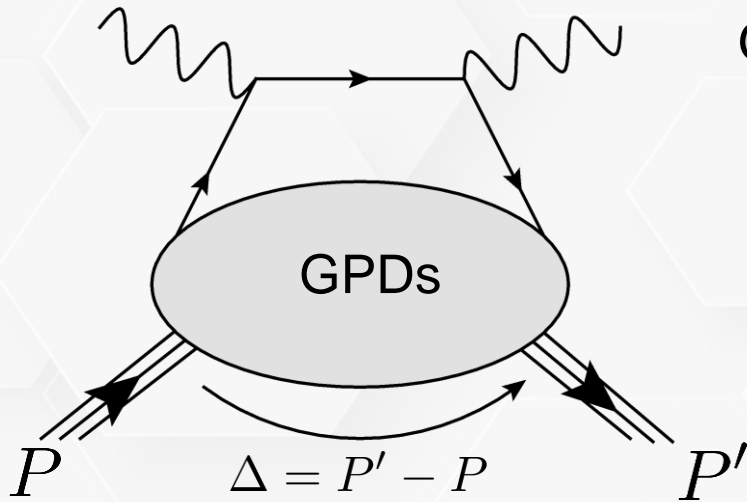
Stress tensor form factors:

$$C_{q,g}(t) \quad \text{Adam's talk}$$

EMT is coupled to gravity whereas gravitational scattering with nucleon is impossible

Generalized parton distributions (GPDs)

Instead of measuring the whole nucleon, we knock out one parton at a time.



D. Muller et. al. Fortsch.Phys. 42 101 (1994)

X. Ji Phys. Rev. Lett. 78, 610 (1997)

GPDs are distributions unifying parton distributions and form factors

$$F(x, \Delta^\mu) = F(x, \xi, t)$$

x : average parton momentum fraction

ξ : skewness – longitudinal momentum transfer $\xi \equiv -n \cdot \Delta / 2$

t : total momentum transfer squared $t \equiv \Delta^2$

Nucleon Tomography: dissect the nucleon into its partonic (quark/gluon) degrees of freedom

$$\int dx H(x, \xi, t) = F_1(t)$$

$$\int dx x H(x, \xi, t) = A(t) + (2\xi)^2 C(t)$$

Threshold heavy quarkonium productions

Color dipole probing the gluonic structure

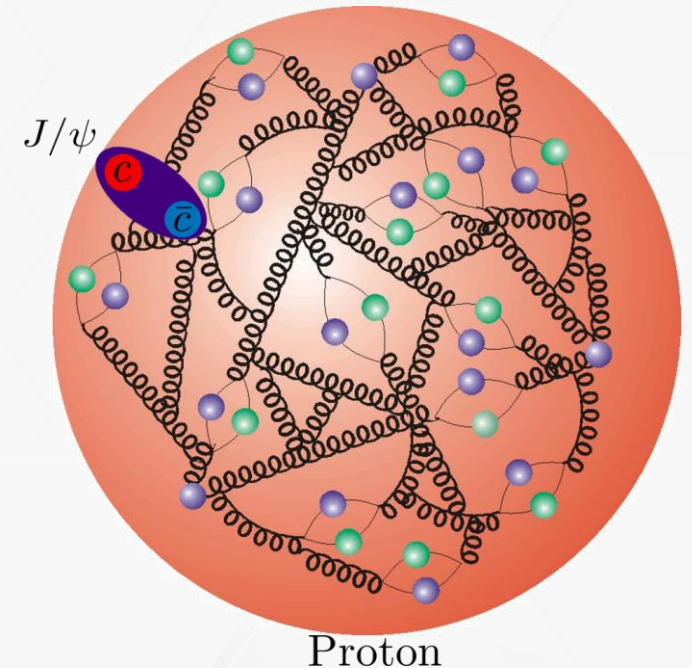
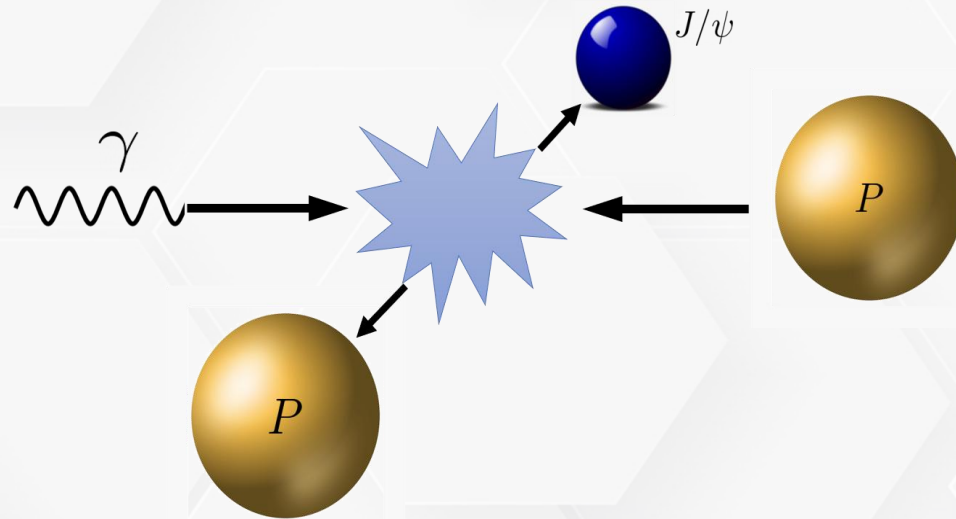
Quadratic Stark effect in QCD – measures the color electric field:

$$\mathcal{H}_{\text{Int}} = \mathbf{E}^a \cdot (\mathbf{r}_c t_c^a - \mathbf{r}_{\bar{c}} t_{\bar{c}}^a)$$

M. B. Voloshin Nucl. Phys. B 154 365-380 (1979)
M. Luke et al. Phys. Lett. B 288 355-359 (1992)
D. Kharzeev et al., Eur. Phys. J. C 9 459-462 (1999)

Elastic scattering of J/psi off the nucleon would be ideal.

In real life we have photo-/electro- production instead



Heavy meson corresponds to small color dipole --- local gluonic distribution.

Near-threshold J/ψ photoproduction

First measurement of near-threshold exclusive J/ψ photoproduction off the proton by GlueX

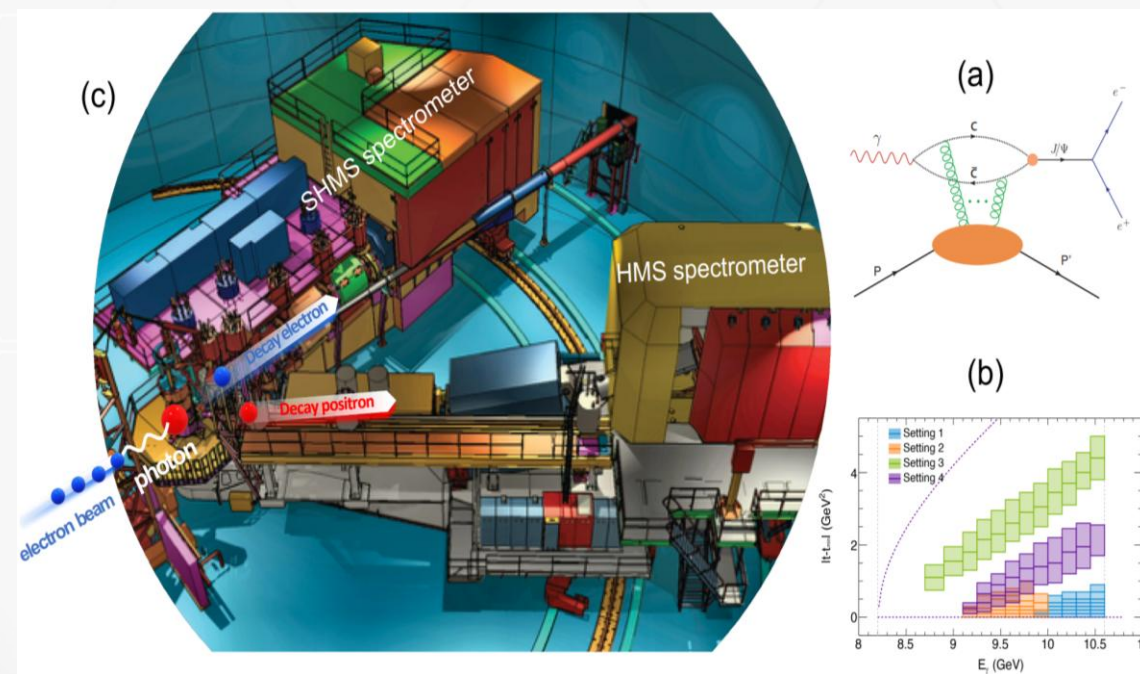


GlueX Collaboration Phys. Rev. Lett. 123, 072001 (2019)
GlueX Collaboration Phys. Rev. C 108 2, 025201 (2023)

Sean's and Sylvester's talk

Yuxun Guo @ APS GHP 2025

Another recent measurement by the J/ψ 007 experiment at JLab Hall C



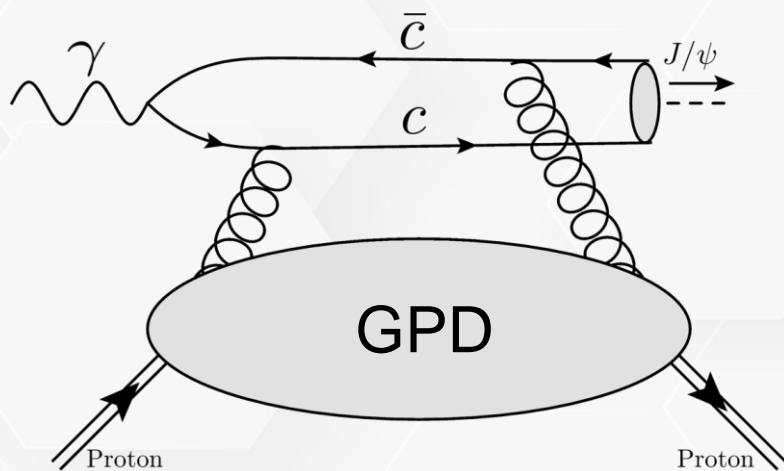
B. Duran et. al. Nature 615 7954, 813-816 (2023)

Also, CLAS12 at Hall B (Pita's and Tyson's talk)

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GPD framework for threshold production

Near-threshold exclusive heavy vector meson production in the GPD framework.



Y. Guo et. al. Phys. Rev. D 103 9, 096010 (2021)

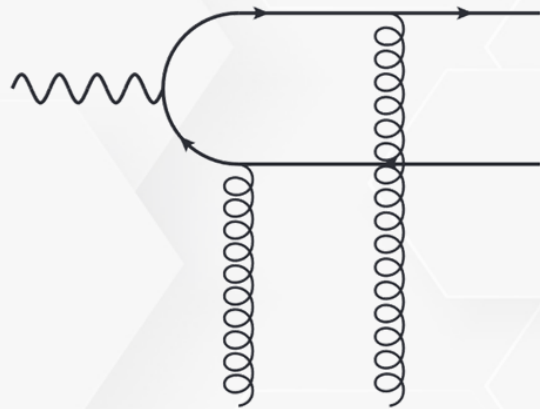
- ❑ Leading order factorization with GPDs
- ❑ The same amplitude as the collinear case but with different kinematics;
- ❑ Large momentum transfer/skewness in the heavy quark limit;

$$\frac{d\sigma}{dt} \propto \left[(1 - \xi^2) |\mathcal{H}_C|^2 - 2\xi^2 \text{Re} [\mathcal{H}_C^* \mathcal{E}_C] - \left(\xi^2 + \frac{t}{4M_p^2} \right) |\mathcal{E}_C|^2 \right],$$

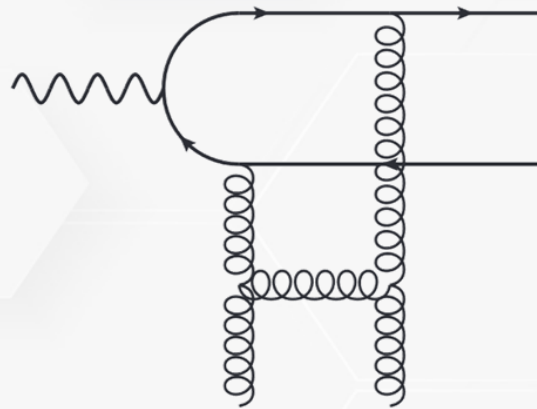
Will be sensitive to the so-called quark/gluonic Compton-like form factors (q/gCFFs)

Next-to-leading corrections in strong coupling

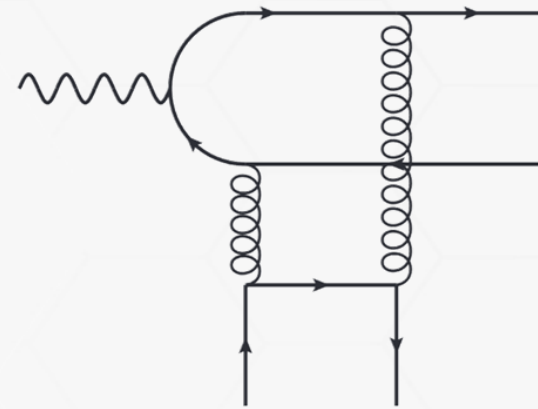
Next-to-leading order corrections in the strong coupling are also very interesting.



(a)



(b)



(c)

Z-Q Chen and C-F Qiao Phys. Lett. B 797 134816 (2019)

- ❑ Important to justify the perturbative expansion, particularly for J/psi
- ❑ Additional sensitivity to the quark GPDs and eventually GFFs!

GFFs and CFFs at near-threshold kinematics

The amplitudes (q/gCFFs) are factorizable in terms of GPDs, but their relations to the GFFs we are interested in are unclear.

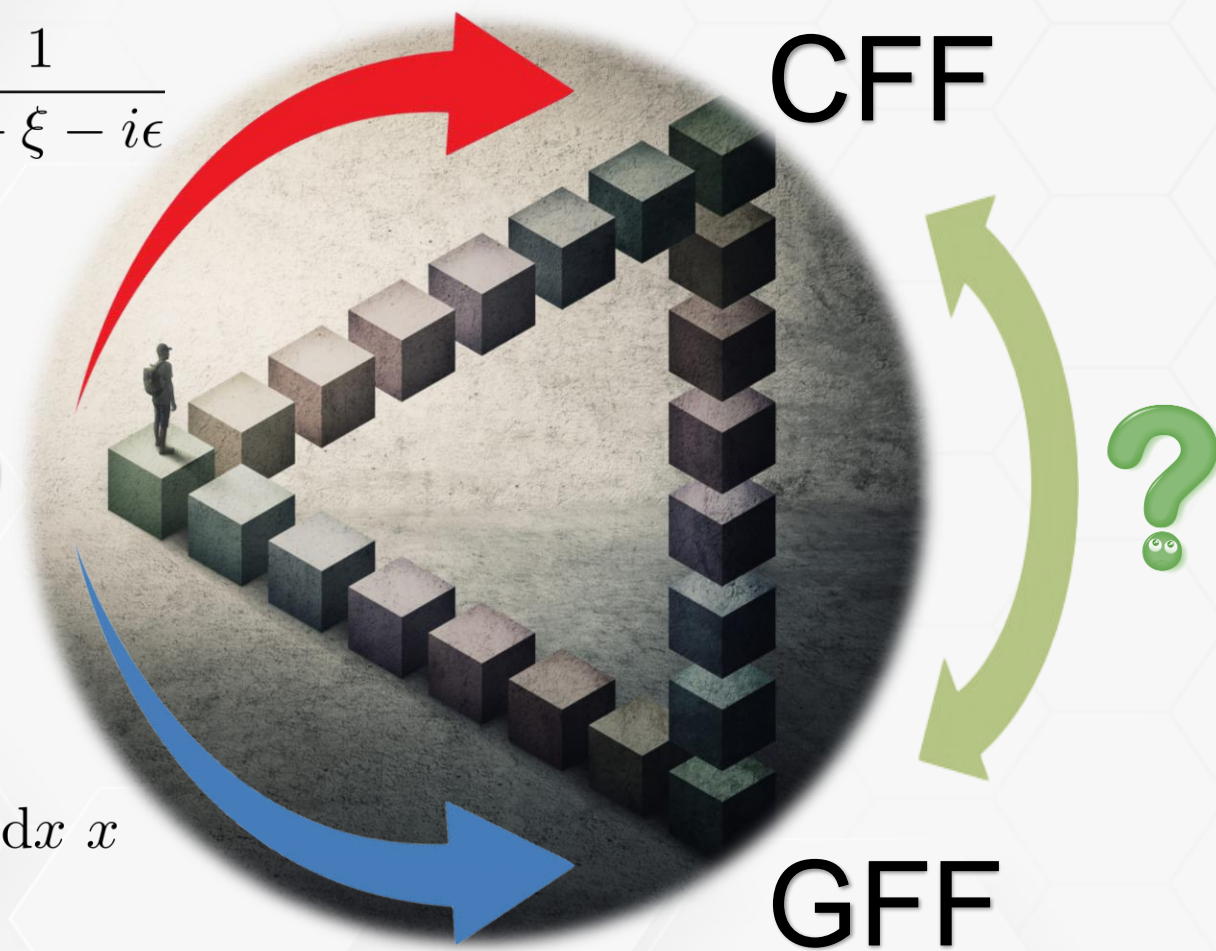
$$\int_{-1}^1 dx \frac{1}{x + \xi - i\epsilon}$$

GPD

$$\int_{-1}^1 dx x$$

CFF

GFF



Global analysis with GPDs (e.g. GUMP)

Y. Guo et. al. arxiv: 2409.17231
Y. Guo et. al. JHEP 05 150 (2023)
Y. Guo et. al. JHEP 09 215 (2022)

Fitting with Bayesian inference

General framework

1. Write down the factorized cross-section formulae:

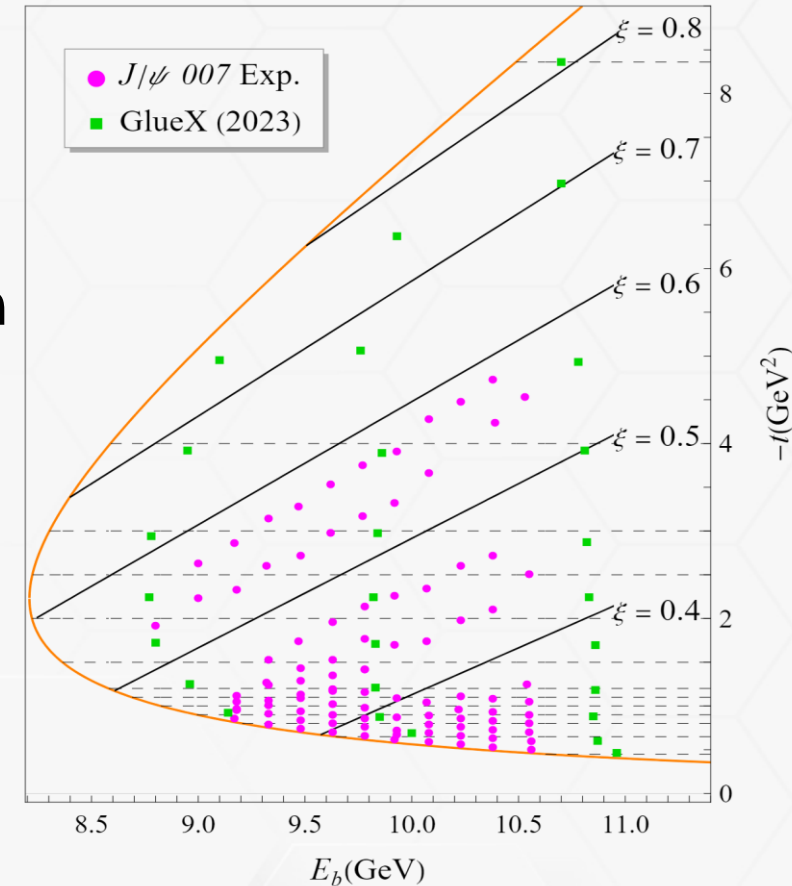
$$\frac{d\sigma}{dt} \propto \left[(1 - \xi^2) |\mathcal{H}_C|^2 - 2\xi^2 \text{Re} [\mathcal{H}_C^* \mathcal{E}_C] - \left(\xi^2 + \frac{t}{4M_p^2} \right) |\mathcal{E}_C|^2 \right],$$

2. Approximate the q/gCFFs with **large skewness expansion** AND assume **higher-moment suppression**. At LO they are

$$\mathcal{H}_{gC}(\xi, t) \approx C_g(t) + \xi^{-2} \mathcal{A}_g^{(2)}(t)$$

$$\mathcal{E}_{gC}(\xi, t) \approx -C_g(t) + \xi^{-2} \mathcal{B}_g^{(2)}(t)$$

3. Incorporate necessary theoretical corrections and extract the GFFs from the q/gCFFs (with Bayesian inference/ML).



Y. Guo et. al. Phys. Rev. D 108, 034003 (2023)

Next-to-leading order effects

The Next-to-leading corrections appear to be sizable:

1. The evolved Wilson coefficients are plotted:

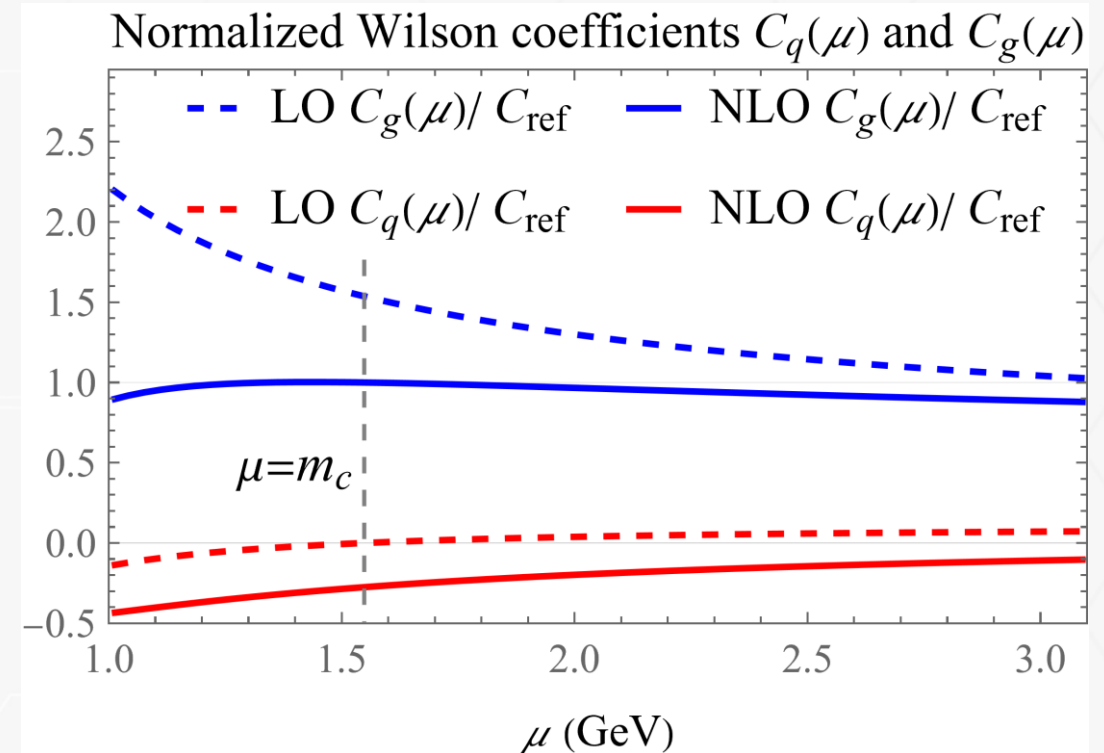
$$\bar{C}^{\text{evo}} \equiv \alpha_S(\mu_F) \bar{C}^{(1)}(\mu_F, m_c) \mathcal{E}^{(1)}(\mu_F, m_c)$$

2. Large scale dependence are found at LO

Indicates non-negligible perturbative corrections

3. Scale-dependence reduced at NLO for the gluon

4. NLO quark contributions are suppressed by the strong coupling but are generally not small.



Y. Guo et. al. arxiv: 2501.10532

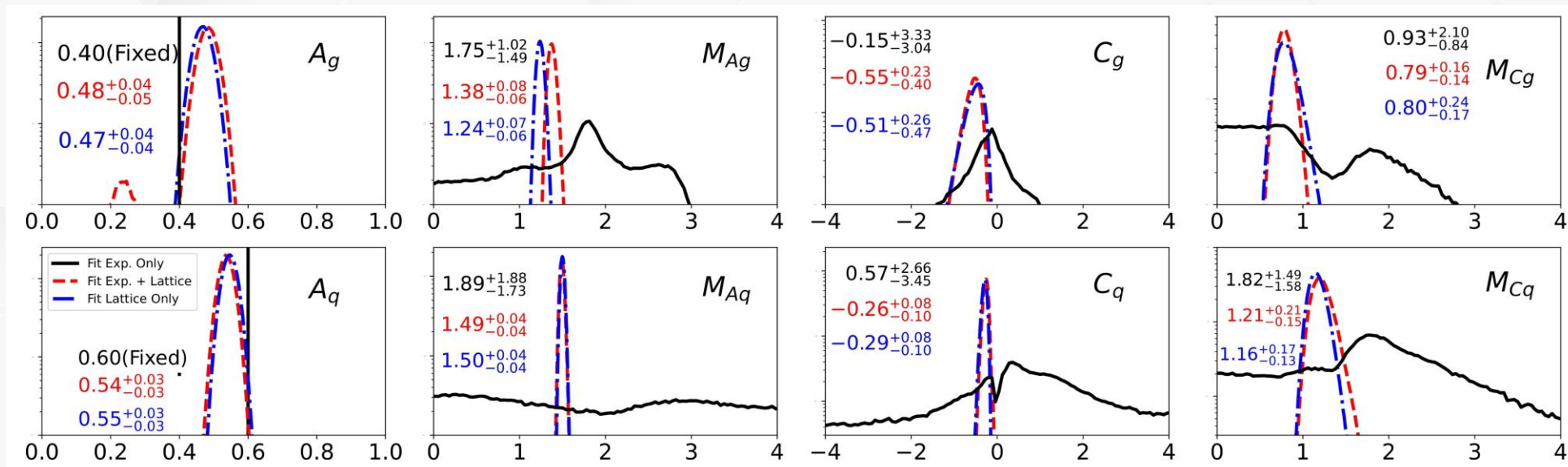
The NLO analysis may not be a simple refinement of the LO one (what about NNLO?)

Bayesian inference of proton GFFs

We consider the Bayesian inference to obtain a statistically rigorous and interpretable extraction

$$F_i(t) = F_i(0) \left(1 - t/M_{F_i}^2\right)^{-p} \quad \text{for } F_i = \{A_q, A_g, C_q, C_g\}$$

The posterior distributions of the 8 parameters are



Y. Guo et. al. arxiv: 2501.10532

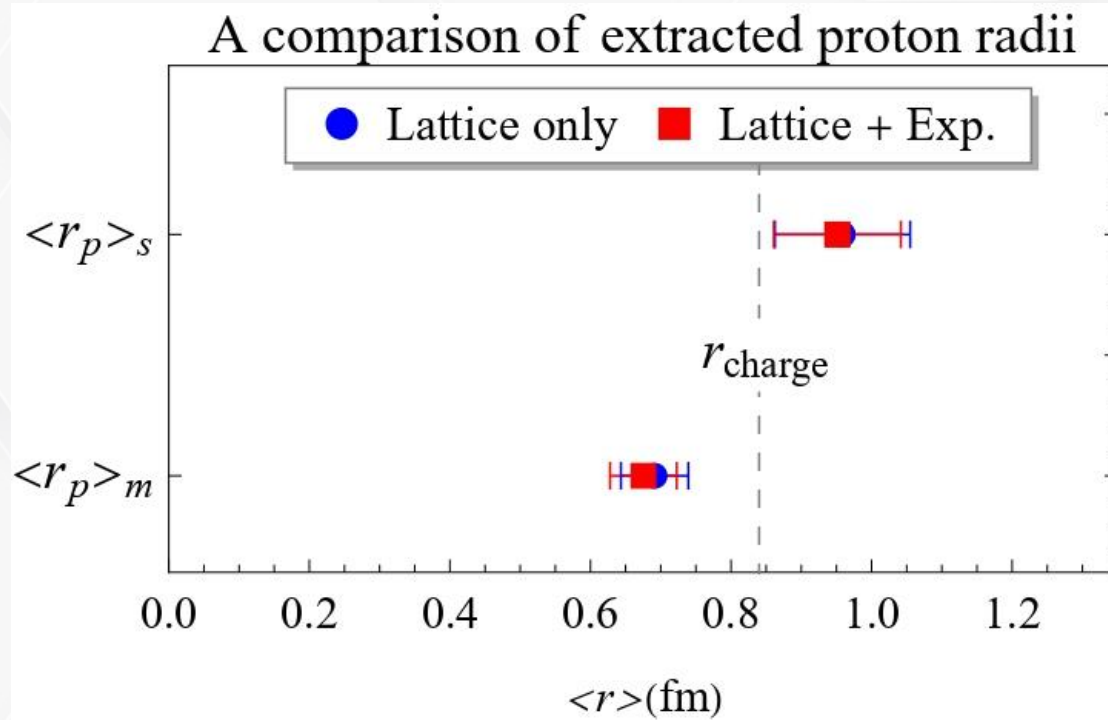
Lattice data from: D. Hackett, et. al. Phys. Rev. Lett. 132, 251904 (2024)

GlueX data from GlueX collab. Phys. Rev. C 108, 025201 (2023)

J/psi 007 data from B. Duran et. al, Nature 615, 813-816 (2023)

Implication on the proton radii

We can also calculate the proton radii with the extracted proton GFFs:



$$\langle r^2 \rangle_m = 6 \left. \frac{dA(t)}{dt} \right|_{t=0} - 6 \frac{C(0)}{M_N^2}$$
$$\langle r^2 \rangle_s = 6 \left. \frac{dA(t)}{dt} \right|_{t=0} - 18 \frac{C(0)}{M_N^2}$$

Mostly rely on the lattice constraint due to the lack of sensitivity to the dipole/tripole mass of GFFs

Hint to other processes/analyses

This analysis is not limited to just heavy vector meson photo-productions

- Light(er) meson electro-productions at large skewness

Y. Hatta et. al. arxiv: 2501.12343

- Time-like Compton scattering (TCS) at large skewness

- Deeply virtual Compton scattering (DVCS) at large skewness

JLab Hall A and F. Georges Phys. Rev. Lett. 128 25, 252002 (2022)

In lower-skewness region, the imaginary part of the amplitudes are no longer small.

Method like dispersion analysis can incorporate the imaginary parts to the extractions.

Summary and outlook

Summary

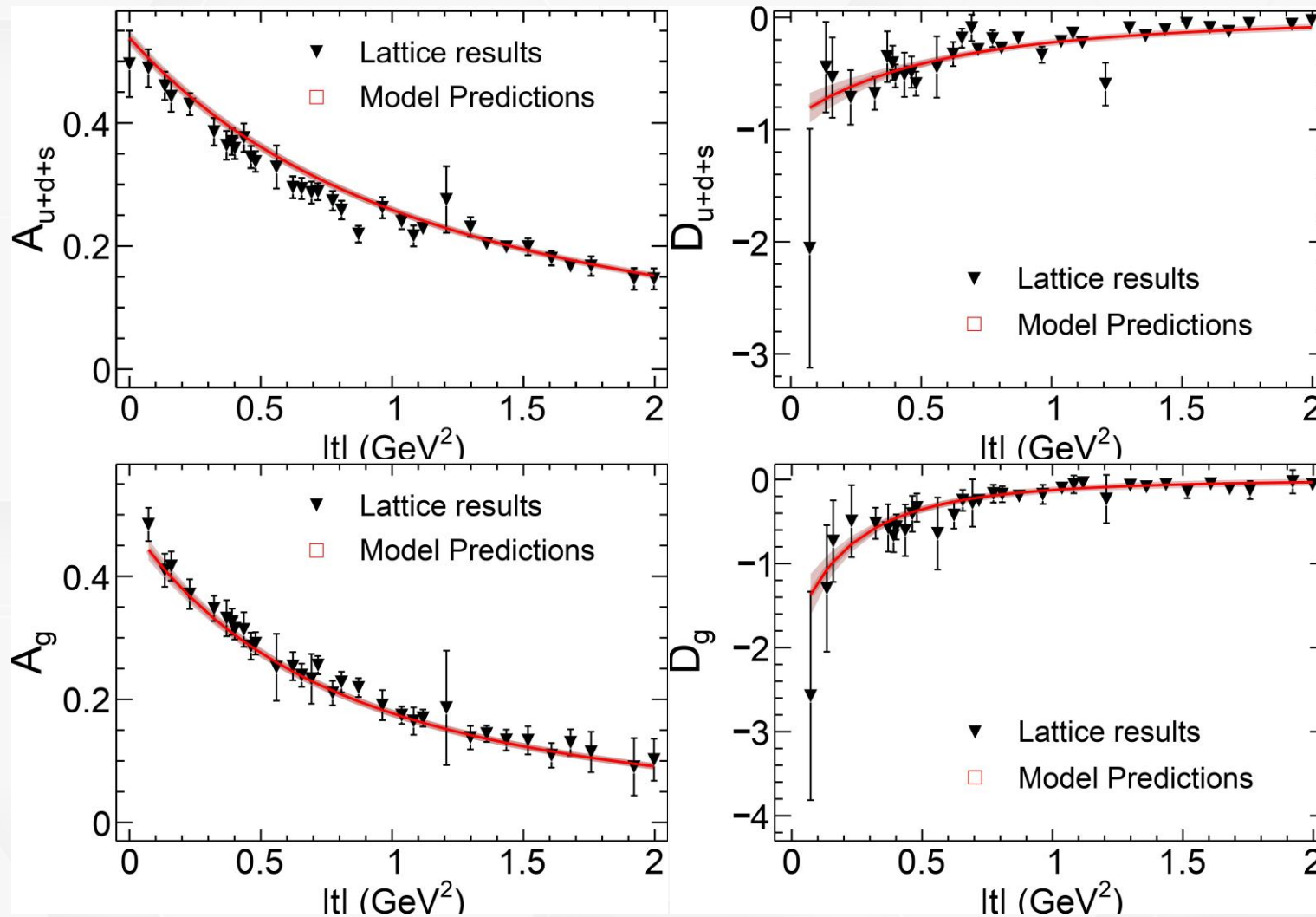
- Exclusive heavy quarkonium production as probe of gluonic structures
- Large skewness behavior of q/g CFFs in near-threshold heavy quarkonium production
- Extraction of quark and gluon GFFs in the nucleon with Bayesian inference

Outlook

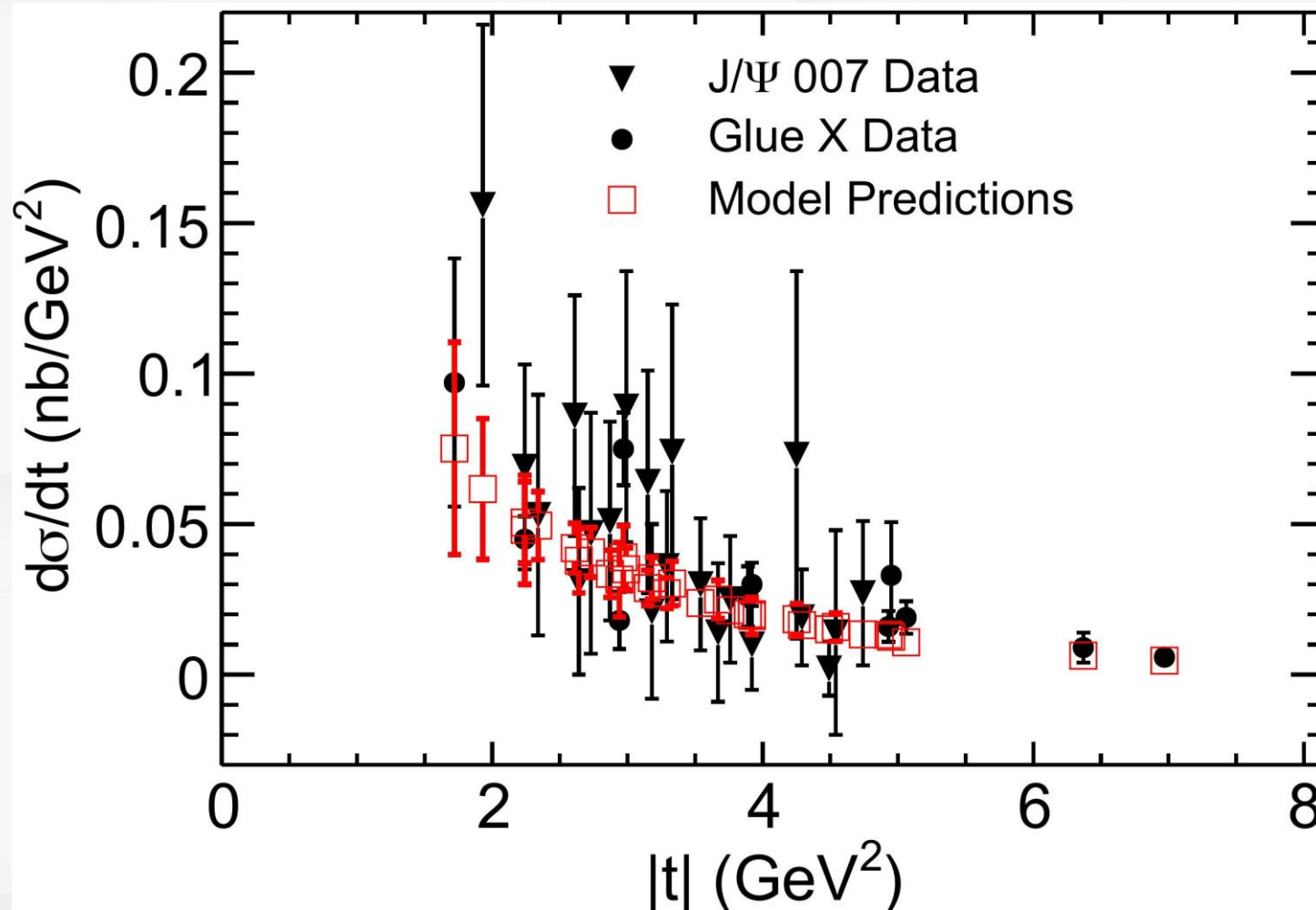
- ▽ Next-to-next-to-leading order and relativistic corrections
- ▽ Extraction with NN/ML to reduce the parameterization bias.
- ▽ Extension to other processes and combined analysis

Thank you!

Lattice form factors



Fit to the experiment data



Large skewness near the threshold

When a heavy quarkonium is produced from a massless real photon near threshold, the momentum transfer must be large. Then consider a Taylor expansion:

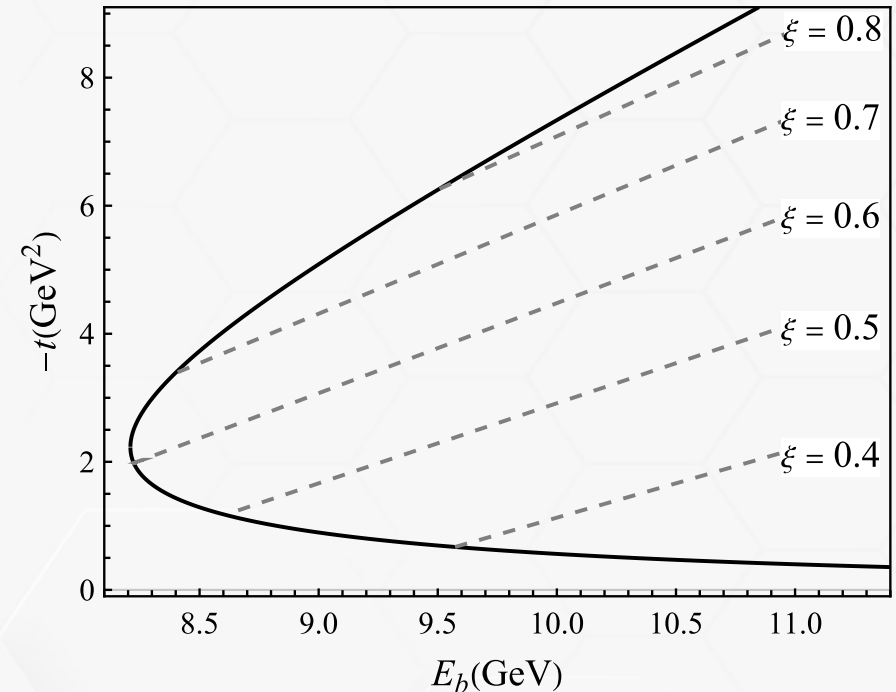
$$\text{Re} \left[\frac{1}{2\xi} \left(\frac{1}{x + \xi - i\epsilon} - \frac{1}{x - \xi + i\epsilon} \right) \right] = \sum_{n=0}^{\infty} \frac{x^{2n}}{\xi^{2+2n}}$$

The real part of the CFF can then be written as:

$$\text{Re}\mathcal{H}_{gC}(\xi, t) = \frac{2}{\xi^2} \sum_{n=0}^{\infty} \int_0^1 dx \left(\frac{x}{\xi} \right)^{2n} H_g(x, \xi, t)$$

These x-integrals lead to generalized form factors

$$\text{Re}\mathcal{H}_{gC}(\xi, t) = C_g(t) + \sum_{n=1}^{\infty} \xi^{-2n} \mathcal{A}_g^{(2n)}(t) \stackrel{?}{\approx} C_g(t) + \xi^{-2} \mathcal{A}_g^{(2)}(t)$$



Asymptotic expansion of the q/gCFFs

The q/gCFFs can be approximated by the partial sum of the contributions of each GPD moment at large skewness

