Quantum Entanglement Correlations in Double Parton Distributions

Eric Kolbusz
Baruch College, City University of New York

based on work done with Adrian Dumitru
arXiv:2303.07408, 2501.12312

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- Kovner et al.: gluon entanglement at small x, especially in CGC
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This talk: correlations due to quantum entanglement in two valence quarks at moderate x

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Goal: identify the presence of quantum correlations between two subsystems
(the momentum fractions of two valence quarks inside a proton)

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Given a system described by ρ_{AB}

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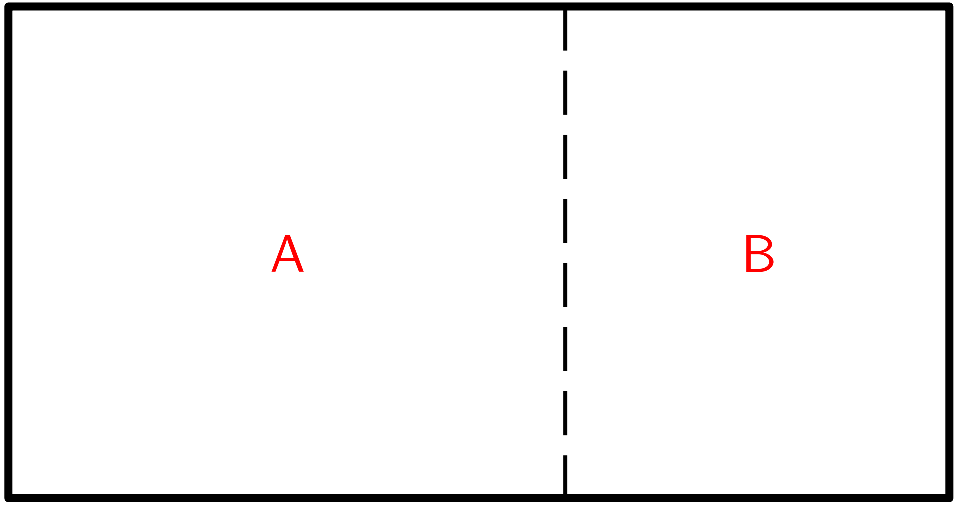

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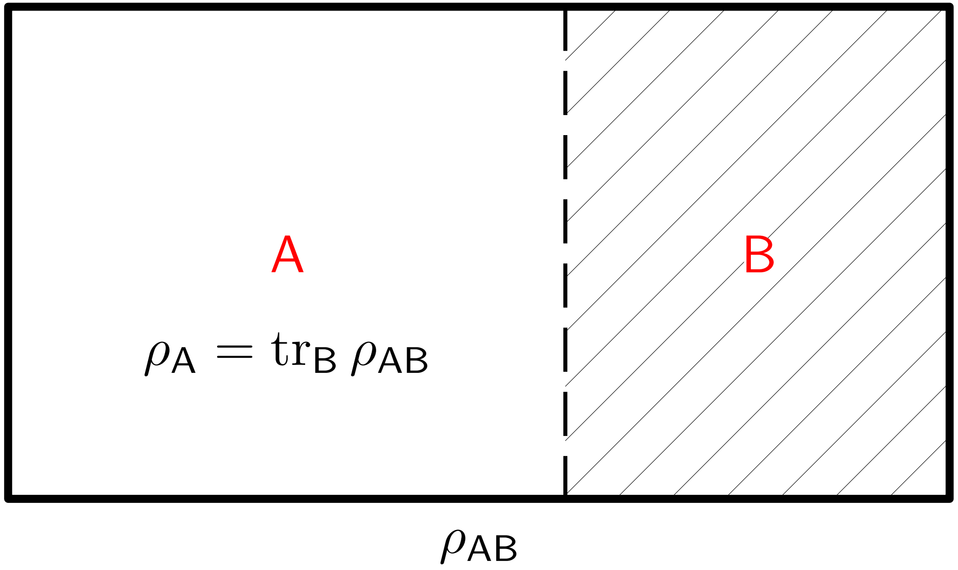

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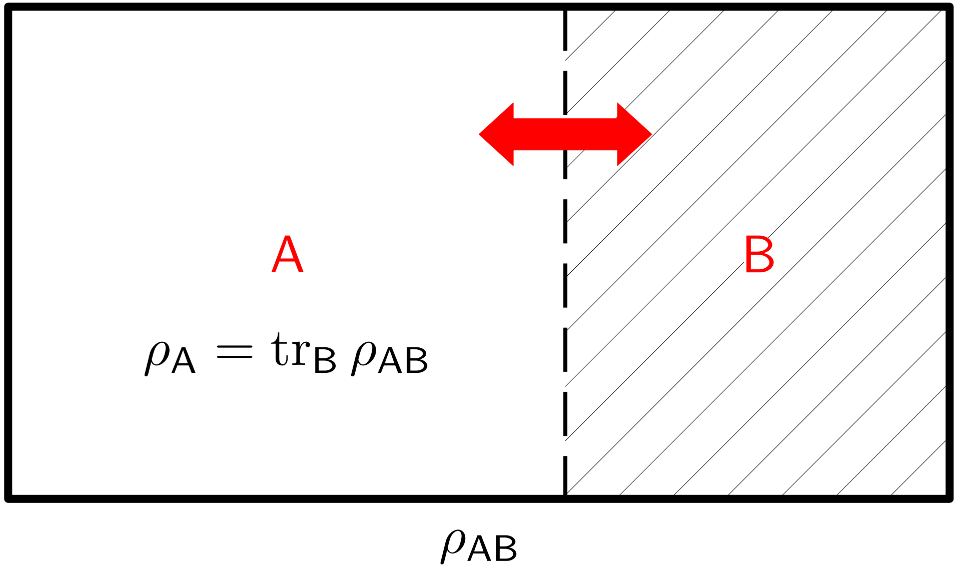

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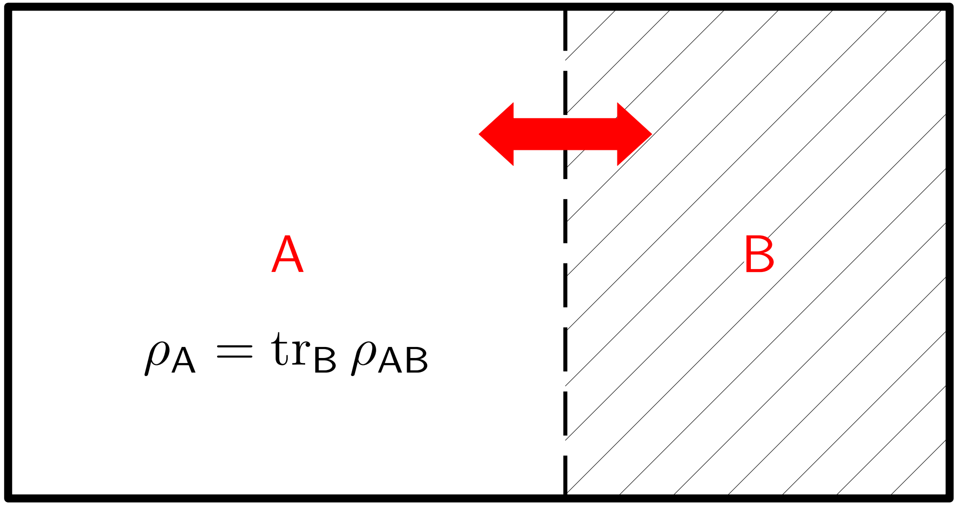
If ρ_{AB} is a pure state:

- $\mathsf{S}(\rho_{\mathsf{AB}}) = 0$
- $\mathsf{S}(\rho_{\mathsf{A}}) > 0$ means A and B are entangled



$\rho_{\mathsf{AB}} = |\psi\rangle\langle\psi|$

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Cannot use $S(\rho^A) > 0$ to draw any conclusions about the presence of quantum correlations between A and B.

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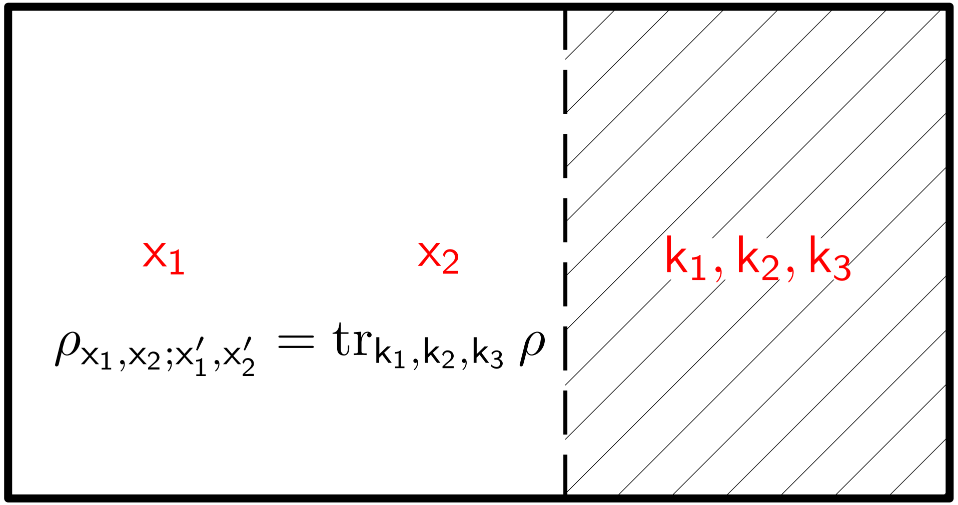

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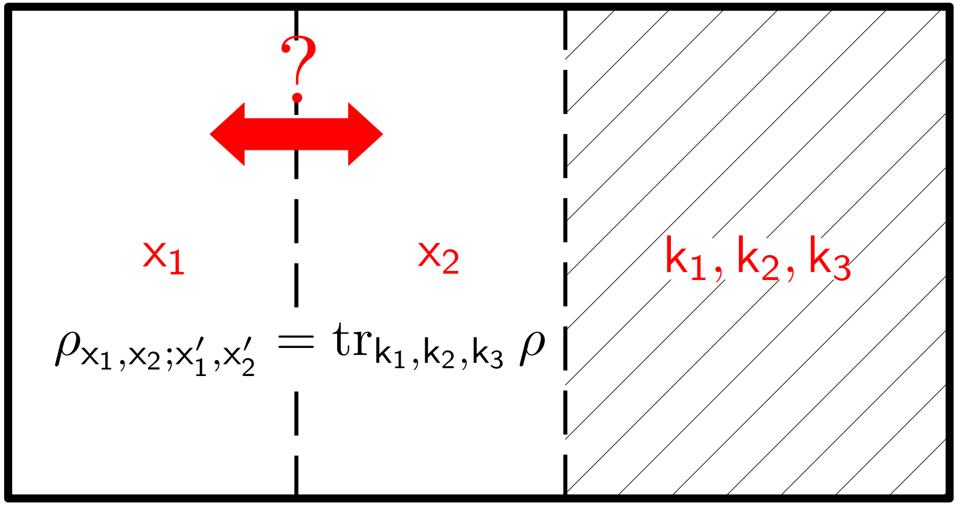
5. $S(\rho_{x_1,x_2;x_1',x_2'}) > 0$



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If ρ is separable, then $\rho = \sum_i p_i \, \rho_A^{(i)} \otimes \rho_B^{(i)} \implies \rho^{T_B} = \sum_i p_i \, \rho_A^{(i)} \otimes \rho_B^{(i)T} \implies \mathcal{N}(\rho) = 0$

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$\implies \mathcal{N}(\rho) \neq 0$ guarantees the presence of quantum correlations between A and B!

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5. untranspose, enforce $\mathrm{tr} = 1$ with $N = \dfrac{1}{1 + \mathcal{N}(\rho)}$

$$\rho' = N \left[U^{\dagger} \Theta U \rho^{T_B} \right]^{T_B}$$

SU(N) Toy Problem

Consider a totally antisymmetric state in the fundamental rep. of $SU(N_c)$ and trace over $N_c - 2$ colors

$$\rho_{i_1 \dots i_{N_c}, i'_1 \dots i'_{N_c}} = \frac{1}{N_c!} \epsilon^{i_1 \dots i_{N_c}} \epsilon^{i'_1 \dots i'_{N_c}} \implies \rho_{ij,i'j'} = \frac{1}{N_c(N_c - 1)} \left(\delta_{ii'} \delta_{jj'} - \delta_{ij'} \delta_{i'j} \right)$$

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Use PEN to remove negativity:

$$S(\rho_{ij,i'j'}) = 2\log N_c - \log 2 - \frac{1}{N_c} - \frac{1}{2{N_c}^2} + \mathcal{O}({N_c}^{-3})$$

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By construction, the differences are entirely explained by the removed quantum correlations!

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- interpolates between a Bell ρ and the totally mixed state: $\lambda\frac{\mathrm{I}_4}{4} + (1-\lambda)\rho$
- separable exactly when $\lambda \geq \frac{2}{3}$
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In this case, PEN gives the closest separable state to ρ!

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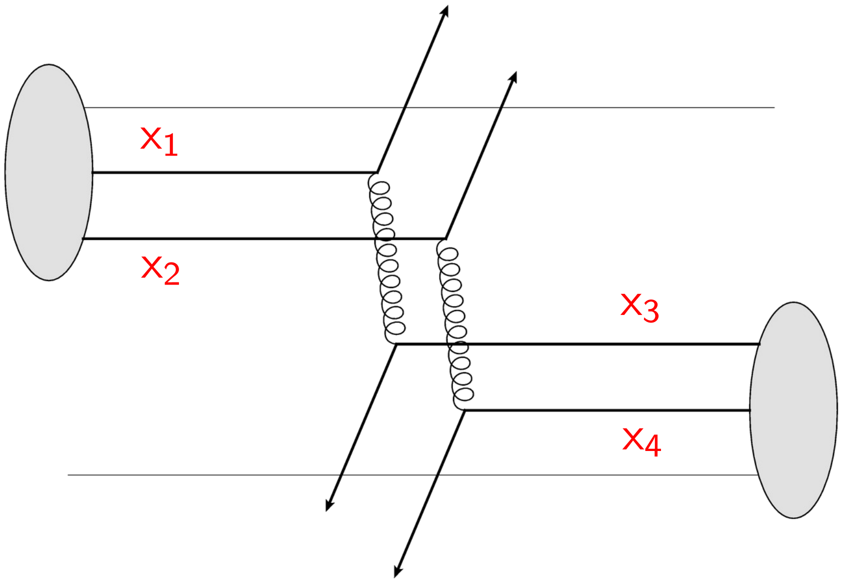
In high energy collisions two (or more) hard parton scatterings may occur

$$\sigma_{\mathrm{DPS}} \sim \int \mathrm{d}x_1 \cdots \mathrm{d}x_4 \, f_{qq}(x_1, x_2) \, f_{qq}(x_3, x_4) \, \hat{\sigma}(x_1, x_3) \, \hat{\sigma}(x_2, x_4)$$



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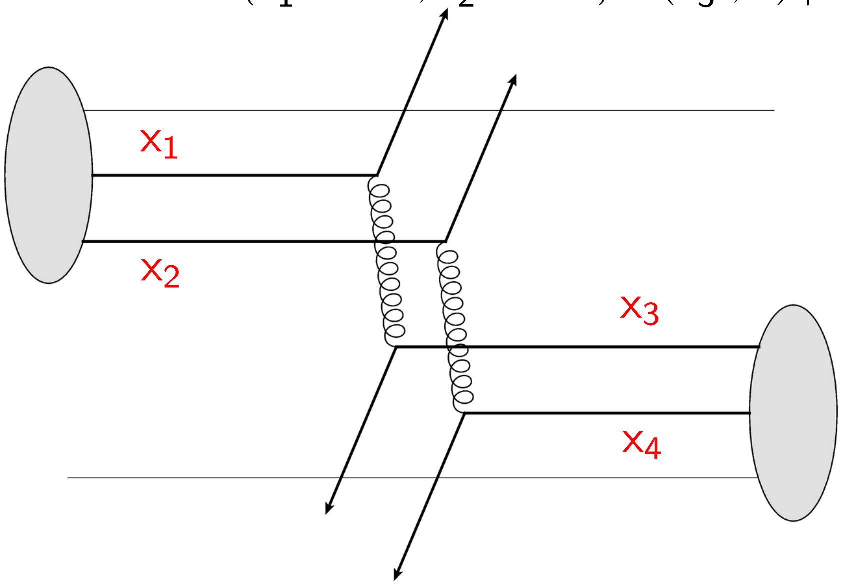
$$\sigma_{\rm DPS} \sim \int dx_1 \cdots dx_4 \, f_{qq}(x_1, x_2) \, f_{qq}(x_3, x_4) \, \hat{\sigma}(x_1, x_3) \, \hat{\sigma}(x_2, x_4)$$

Operator definition of dPDF:

$$f_{qq}(x_1, x_2) = \langle P| \, \frac{\pi P^+}{(2\pi)^3} \int d^2z \int dz_1^- dz_2^- dz_3^- \; e^{-ix_2P^+(z_1^- - z_2^-) - ix_1P^+z_3^-} \, O(z_1^- + \vec{z}, z_2^- + \vec{z}) \, O(z_3^-, 0) \, |P\rangle$$

$$= \rho_{x_1x_2, x_1x_2}$$

with $O(z, y) = \bar{q}(z)\gamma^+ q(y)$



From dPDFs to Density Matrices

dPDFs encode correlations between partons in the proton

$$f_{ij}(x_1, x_2, Q^2) = f_i(x_1, Q^2) f_j(x_2, Q^2) \cdot C_{ij}(x_1, x_2, Q^2)$$

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we always trace over flavor

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this factorized dPDF

- remains factorized using DGLAP to evolve to higher Q^2
- does not accurately model the correlations inside the proton

The Proton Wavefunction

Consider $x_i \gtrsim 0.1$ and $k_i^2 \lesssim \Lambda^2_{QCD}$,
$\Longrightarrow$ approximate the light cone state of the proton in terms of its leading Fock state

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$$|P\rangle = \int_{[0,1]^3} \prod_{i=1\cdots 3} \frac{dx_i}{2x_i}\, \delta\left(1 - \sum_i x_i\right) \int \prod_{i=1\cdots 3} \frac{d^2k_i}{(2\pi)^3}\; (2\pi)^3\, \delta^2\left(\sum_i \overrightarrow{k_i}\right)\; \Psi_{qqq}\left(k_1^\mu, k_2^\mu, k_3^\mu\right) |k_1^\mu; k_2^\mu; k_3^\mu\rangle$$

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We use the Brodsky and Schlumpf wavefunction

$$\Psi_{qqq}\left(x_i, \vec{k_i}\right) = N\sqrt{x_1 x_2 x_3}\, e^{-\mathcal{M}^2/2\beta^2},\ \mathcal{M}^2 = \sum \frac{k_i^2 + m_q^2}{x_i}$$

with $m_q = 0.26\,\text{GeV}$, $\beta = 0.55\,\text{GeV}$

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Density matrix: $\rho_{\alpha,\alpha'} = \langle\alpha'|P'\rangle\langle P|\alpha\rangle = \Psi^*_{qqq}\left(x_i', \overrightarrow{k_i}'\right)\Psi_{qqq}\left(x_i, \overrightarrow{k_i}\right)$

Center of Mass Variables

We are only interested in x d.o.f. so trace over k_i: $\rho_{x_1 x_2, x_1' x_2'} = \mathrm{tr}_{\,k_1, k_2, k_3}\ \rho_{\alpha, \alpha'}$

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$$\begin{matrix} x_1 + x_2 \le 1 \\ x_1' + x_2' \le 1 \end{matrix} \quad \not\Longrightarrow \quad \begin{matrix} x_1 + x_2' \le 1 \\ x_1' + x_2 \le 1 \end{matrix}$$

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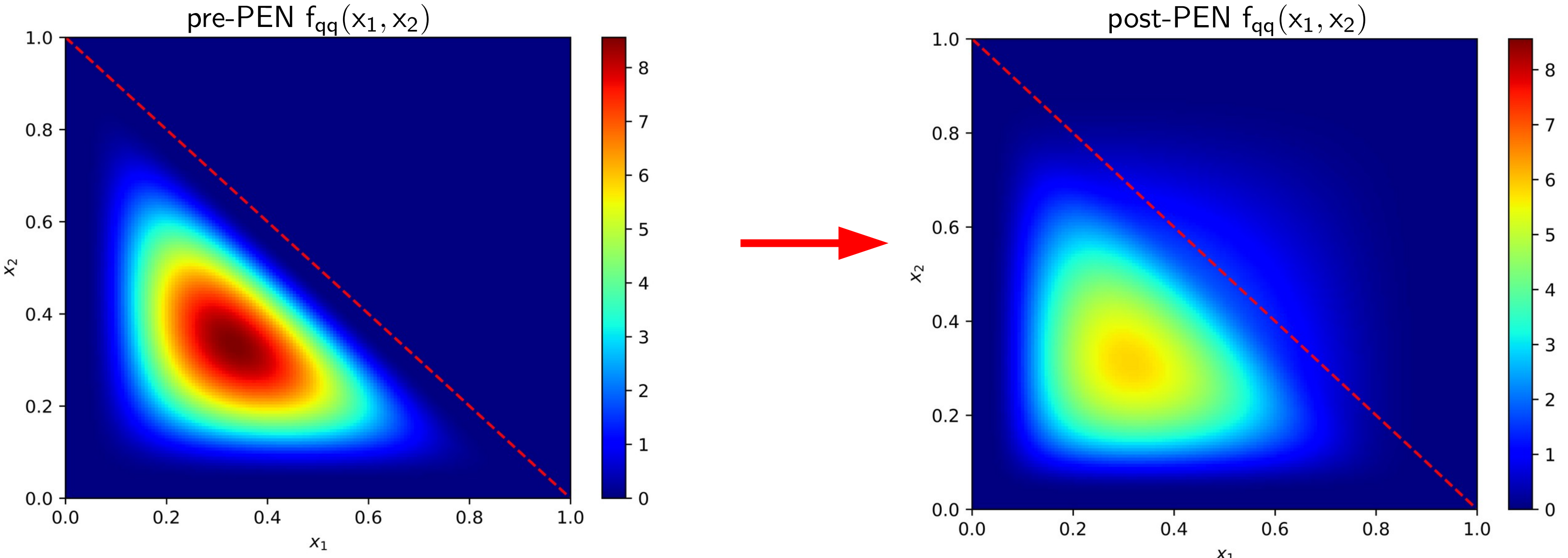

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Solution: variables that describe internal dynamics with $\delta(1 - x_1 - x_2 - x_3)$ constraint implicit

$$\xi = \frac{x_1}{x_1 + x_2}, \quad \eta = x_1 + x_2$$

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The support of x_i means $0 \leq \xi, \eta \leq 1$ with no other constraints, so we can partial transpose

$$\rho_{x_1 x_2, x_1' x_2'} \to \rho_{\xi\eta,\xi'\eta'} \xrightarrow{\text{PEN}} \rho'_{\xi\eta,\xi'\eta'} \to \rho'_{x_1 x_2, x_1' x_2'}$$

PEN on $\rho_{\xi\eta,\xi'\eta'}$ respects the momentum sum rule

Effects of PEN

- structure of distribution is preserved
- peak less pronounced with information spread out over the full distribution

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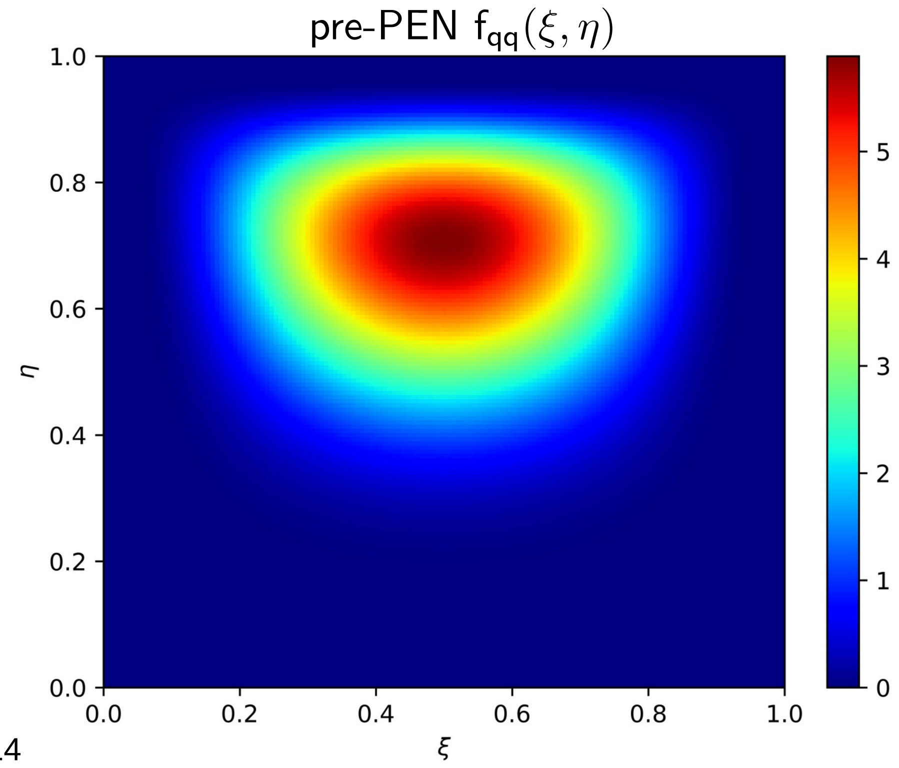

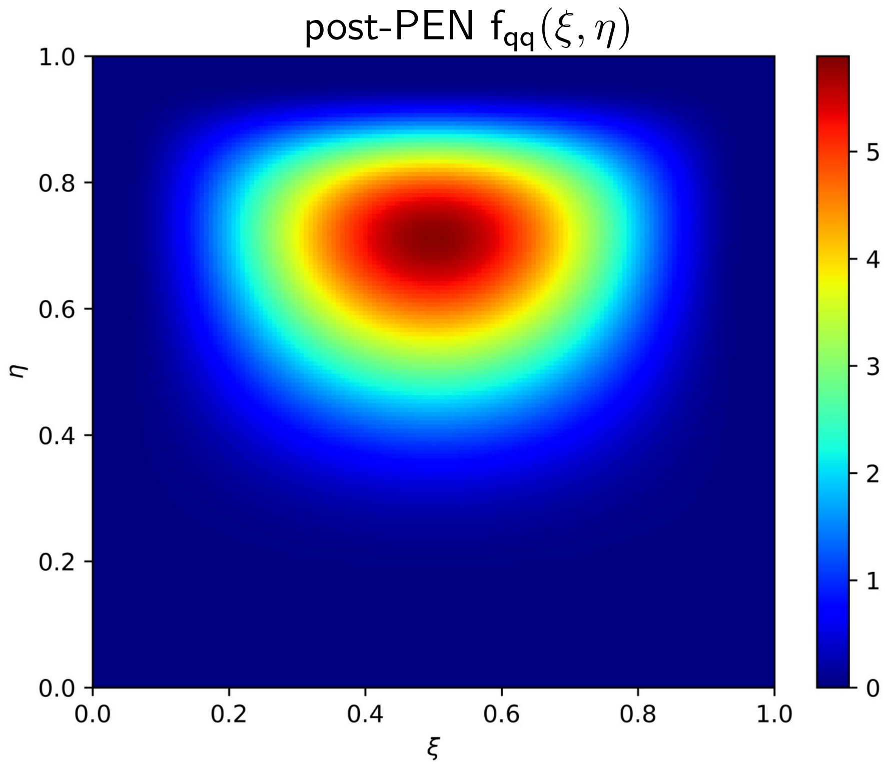

Effects of PEN

- structure of distribution is preserved
- peak less pronounced with information spread out over the full distribution
- largest effects are far from the peak! (asymmetric momenta with small $x_1 + x_2$)

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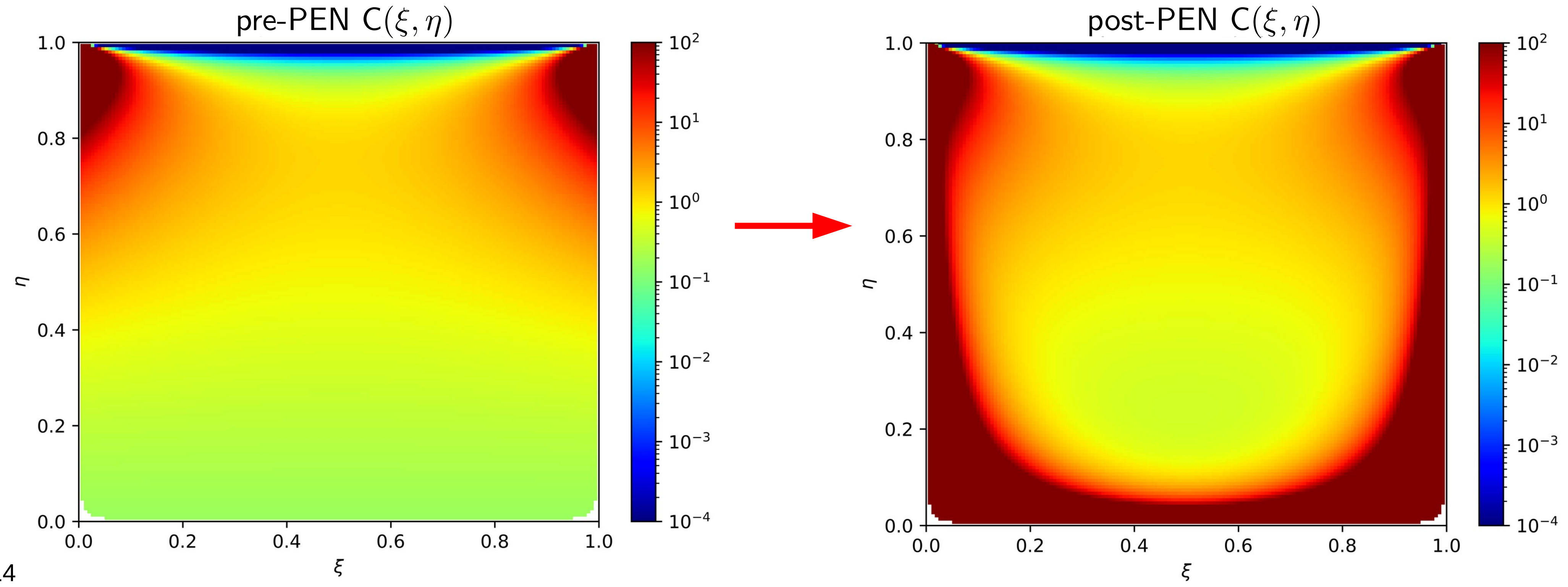

Entanglement Correlations

The differences in $C(x_1, x_2)$ are easiest to see by looking at slices of constant x_2

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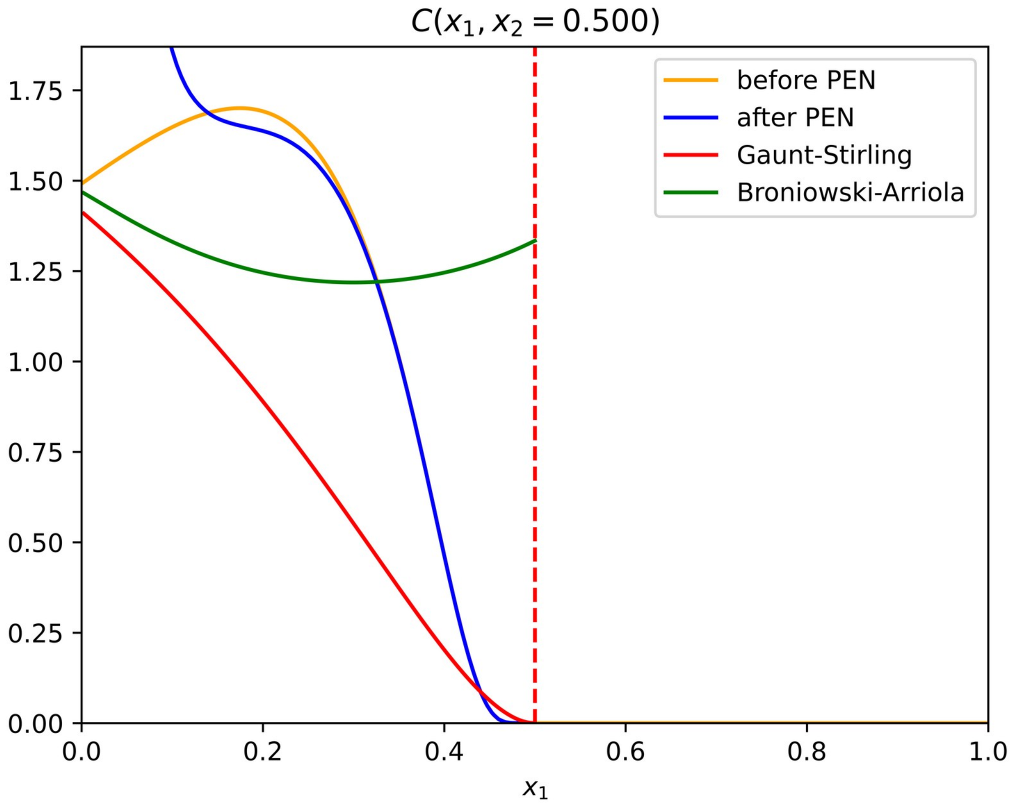
Broniowski-Arriola: (arXiv:1310.8419)
Gaunt-Stirling: (arXiv:0910.4347)

$x_2 = 0.5$:

- for $x_1 \ll x_2$:
  - pre-PEN BS increasing
  - post-PEN decreasing!
- maximum at $x \sim 0.2$ becomes saddle point



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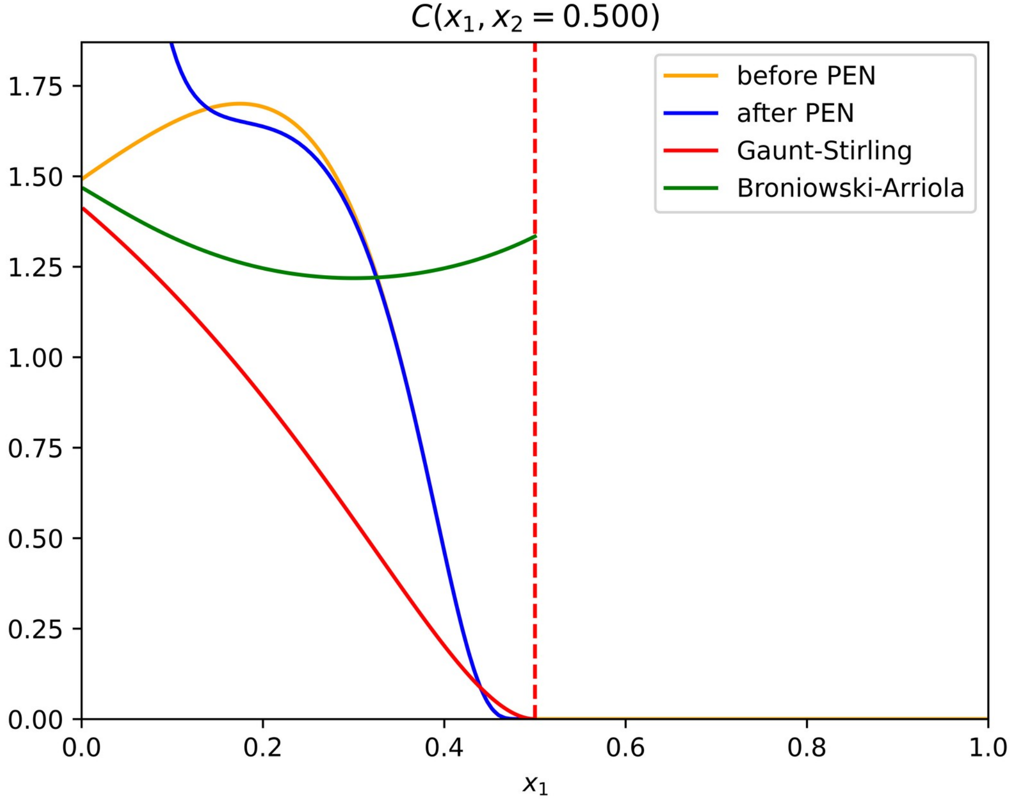
The only increasing plot for small x_1 is the lightcone w.f. with quantum correlations due to negativity



QCD Scale Evolution

So far everything is at an "initial condition" energy scale Q_0^2

How do classical vs quantum correlations evolve to higher scales?
We need to evolve the entire density matrix $\rho_{\xi\eta,\xi'\eta'}$ (not just the diagonal = dPDF)

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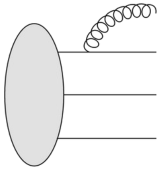

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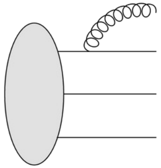
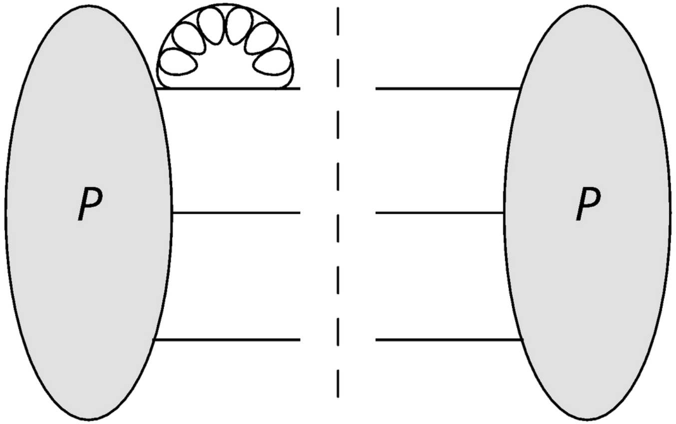


$\rho^{(1'1')}_{\xi\eta,\xi'\eta'} =$



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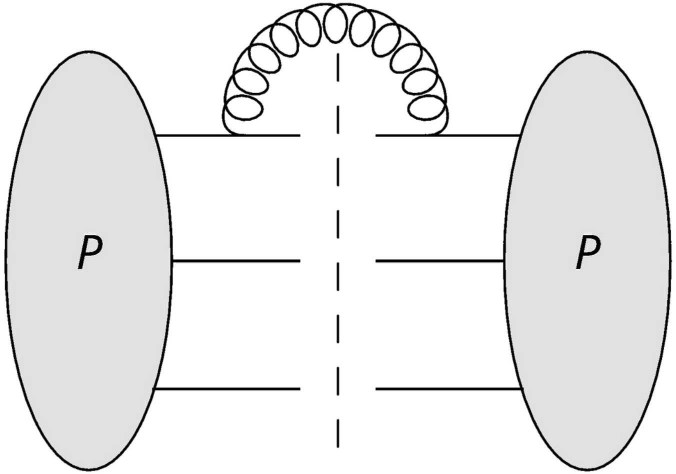

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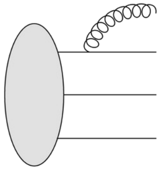
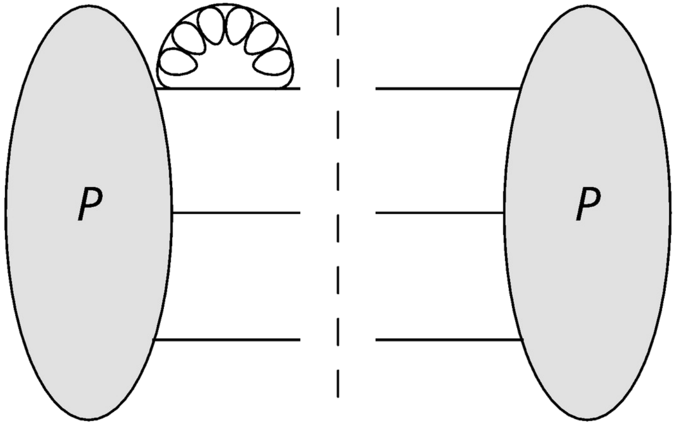


goes into wavefunction renormalization

$$\rho^{(1'1')}_{\xi\eta,\xi'\eta'} =$$



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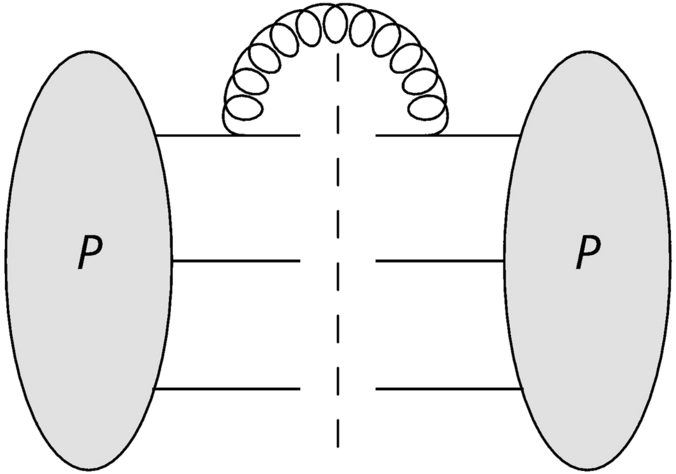

QCD Scale Evolution

So far everything is at an "initial condition" energy scale Q_0^2

How do classical vs quantum correlations evolve to higher scales?
We need to evolve the <u>entire</u> density matrix $\rho_{\xi\eta,\xi'\eta'}$ (not just the diagonal $=$ dPDF)

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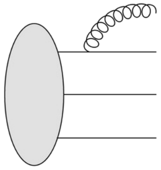
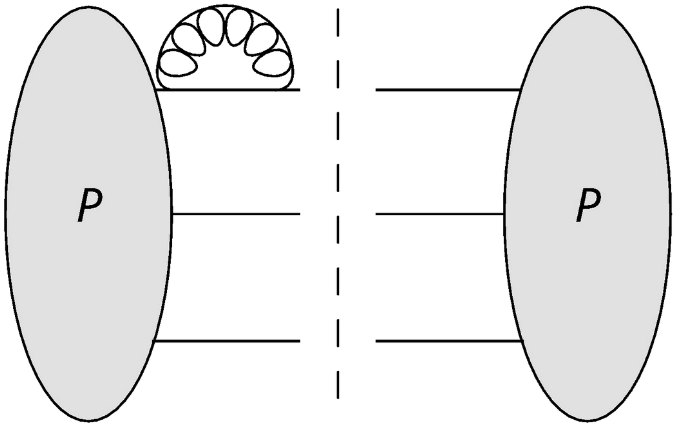


$x_g' = x_g$

$k_g' = k_g$

goes into wavefunction renormalization

$\rho^{(1'1')}_{\xi\eta,\xi'\eta'} =$



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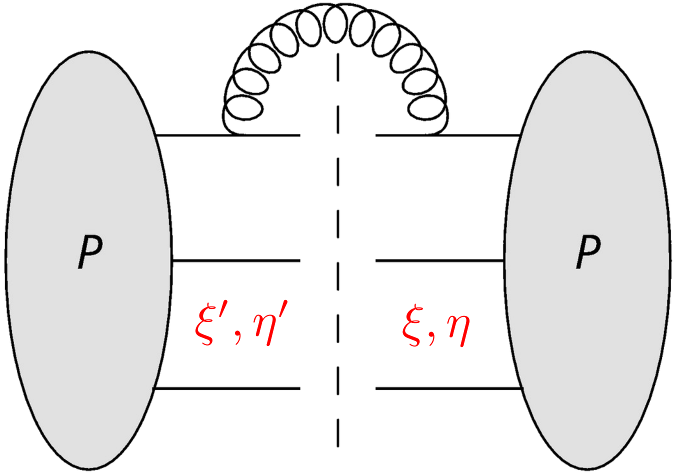


QCD Scale Evolution

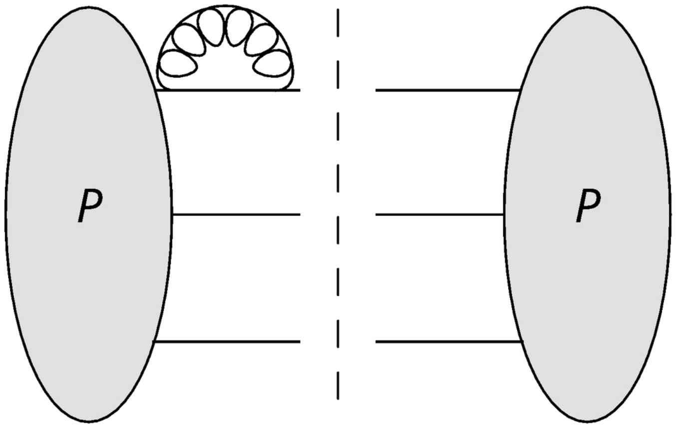
Six $\mathcal{O}(g^2)$ corrections to the three-quark density matrix, e.g.

$$\rho^{(1'1')}_{\xi\eta,\xi'\eta'} = -\frac{g^2 C_F N_C}{3} \frac{d\xi}{\sqrt{4\xi(1-\xi)\xi'(1-\xi')}} \frac{d\eta}{\sqrt{4\eta(1-\eta)\eta'(1-\eta')}} \frac{1}{4} \int \prod_{i=1\cdots 3} \frac{d^2 k_i}{(2\pi)^3} (2\pi)^3 \delta^2\left(\sum_i \vec{k_i}\right)$$

$$\times \int_x^1 \frac{dx_g}{x_g} \frac{d^2 k_g}{16\pi^3} \Theta(\eta'\xi' - x_g) \left[1 + (1-z')^2\right] \left[\frac{1}{k_g^2 + \Delta'^2} - \frac{1}{k_g^2 + \Lambda'^2}\right] \Psi^*(x_i'; \vec{k_i}) \Psi(x_i; \vec{k_i})$$

with the same c.o.m. variables

$$\begin{aligned} \xi &= \tfrac{x_1}{x_1+x_2} \\ \eta &= x_1 + x_2 \end{aligned} \quad \Longrightarrow \quad \begin{aligned} x_1 &= \eta\xi \\ x_2 &= \eta(1-\xi) \\ x_3 &= 1-\eta \end{aligned}$$



in addition to

$$z' = \frac{x_g}{x_1'}$$

$$\Delta'^2 = z'^2 m_{col}^2$$

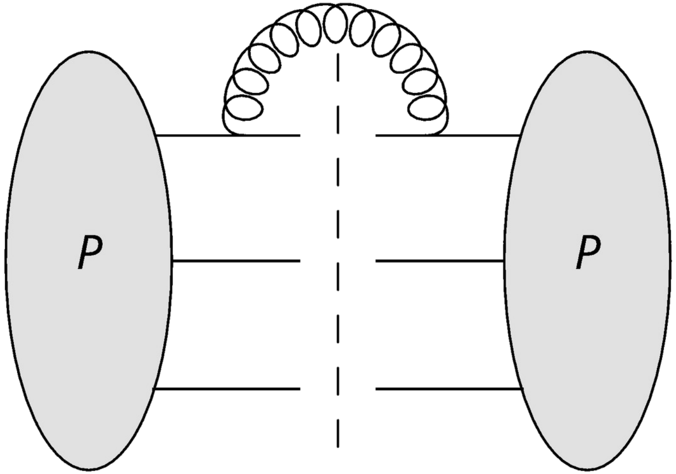
$$\Lambda'^2 = z'^2 M_{UV}^2$$

QCD Scale Evolution

For the other diagrams we no longer have $\sum x_i = 1$ in the daughter quarks

Shift momenta, keep ξ and η

$$\begin{array}{l} \xi = \frac{x_1}{x_1+x_2} \\ \eta = x_1 + x_2 \end{array} \quad \Longrightarrow \quad \begin{array}{ll} x_1 & = \eta\xi \\ x_2 & = \eta(1-\xi) \\ x_3 & = 1 - \eta - x_g \end{array}$$



so now e.g. $z' = \dfrac{x_g}{x_1' + x_g}$, also $\vec{n} = \vec{k_g} - z(\vec{k_1} + \vec{k_g})$

$$\rho^{(11')}_{\xi\eta,\xi'\eta'} = \frac{2g^2 C_F N_c}{3} \frac{d\xi}{\sqrt{4\xi(1-\xi)\xi'(1-\xi')}} \frac{d\eta}{\sqrt{4\eta(1-\eta)\eta'(1-\eta')}} \frac{1}{4} \int \prod_{i=1\cdots 3} \frac{d^2 k_i}{(2\pi)^3} (2\pi)^3 \delta^2\left(\sum_i \vec{k_i}\right)$$

$$\times \int_x^1 \frac{dx_g}{x_g} \frac{d^2 k_g}{16\pi^3} \frac{\Theta(1-\eta-x_g)}{\sqrt{1-\frac{x_g}{1-\eta}}} \frac{\Theta(1-\eta'-x_g)}{\sqrt{1-\frac{x_g}{1-\eta'}}} \frac{1}{\sqrt{1+\frac{x_g}{\xi\eta}}} \frac{1}{\sqrt{1+\frac{x_g}{\xi'\eta'}}} (2 - z - z' + zz')$$

$$\times \left[\frac{\vec{n} \cdot \vec{n}'}{(n^2+\Delta^2)(n'^2+\Delta'^2)} - \frac{1}{2}\frac{1}{k_g^2+\Lambda^2} - \frac{1}{2}\frac{1}{k_g^2+\Lambda'^2} \right] \Psi^*(x_1'+x_g, x_2', x_3'; \vec{k_i}) \Psi(x_1+x_g, x_2, x_3; \vec{k_i})$$

DGLAP Evolution

The diagonal (dPDF) evolves according to the dPDF DGLAP equations (convolution of dPDF with splitting functions)

virtual corrections:

$$Q^2 \frac{\partial}{\partial Q^2} \rho^{(11)}_{x_1 x_2, x_1 x_2} = Q^2 \frac{\partial}{\partial Q^2} \rho^{(1'1')}_{x_1 x_2, x_1 x_2} = -\frac{\alpha_s}{4\pi} \int_{x/x_1}^{1} dz \, P_{g \leftarrow q}(z) \, \rho^{qqq}_{x_1 x_2, x_1 x_2}$$

$$Q^2 \frac{\partial}{\partial Q^2} \rho^{(22)}_{x_1 x_2, x_1 x_2} = Q^2 \frac{\partial}{\partial Q^2} \rho^{(2'2')}_{x_1 x_2, x_1 x_2} = -\frac{\alpha_s}{4\pi} \int_{x/x_2}^{1} dz \, P_{g \leftarrow q}(z) \, \rho^{qqq}_{x_1 x_2, x_1 x_2}$$

$$Q^2 \frac{\partial}{\partial Q^2} \rho^{(33)}_{x_1 x_2, x_1 x_2} = Q^2 \frac{\partial}{\partial Q^2} \rho^{(3'3')}_{x_1 x_2, x_1 x_2} = -\frac{\alpha_s}{4\pi} \int_{\frac{x}{1-x_1-x_2}}^{1} dz \, P_{g \leftarrow q}(z) \, \rho^{qqq}_{x_1 x_2, x_1 x_2}$$

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real emission corrections:

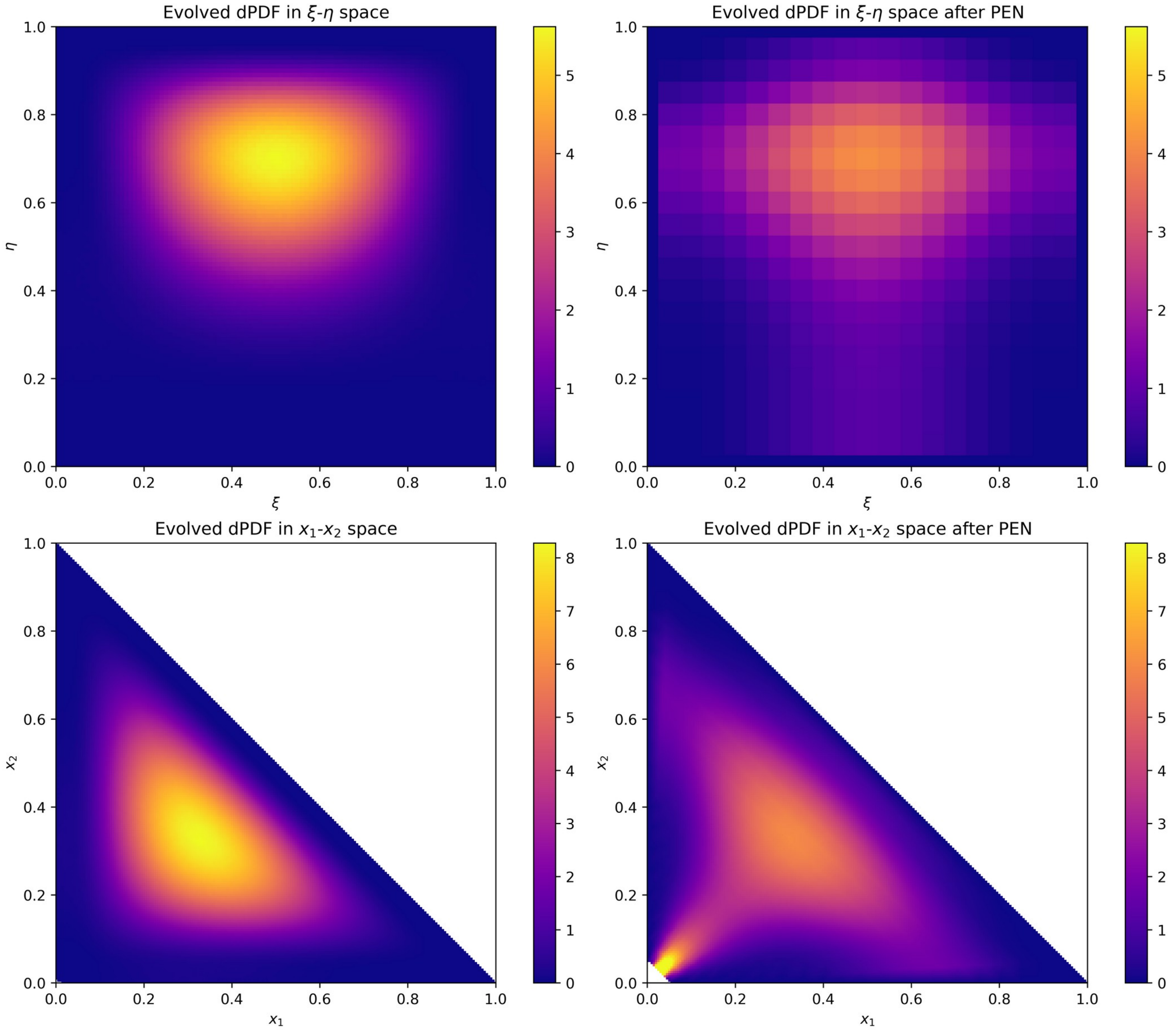
$$Q^2 \frac{\partial}{\partial Q^2} \rho^{(11')}_{x_1 x_2, x_1 x_2} = \frac{2\alpha_s}{4\pi} \int\limits_{x_1/(1-x_2)}^{x_1/(x_1+x)} \frac{\mathrm{d}z}{z} P_{g\leftarrow q}(1-z)\, \rho^{qqq}_{\frac{x_1}{z} x_2, \frac{x_1}{z} x_2}$$

$$Q^2 \frac{\partial}{\partial Q^2} \rho^{(22')}_{x_1 x_2, x_1 x_2} = \frac{2\alpha_s}{4\pi} \int\limits_{x_2/(1-x_1)}^{x_2/(x_2+x)} \frac{\mathrm{d}z}{z} P_{g\leftarrow q}(1-z)\, \rho^{qqq}_{x_1 \frac{x_2}{z}, x_1 \frac{x_2}{z}}$$

$$Q^2 \frac{\partial}{\partial Q^2} \rho^{(33')}_{x_1 x_2, x_1 x_2} = \frac{2\alpha_s}{4\pi} \int\limits_{\frac{x}{1-x_1-x_2}}^{1} \mathrm{d}z\, P_{g\leftarrow q}(z)\, \rho^{qqq}_{x_1 x_2, x_1 x_2}$$

dPDF after one collinear gluon emission

The main effect of removing entanglement correlations is now at small $x_1 \sim x_2$



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- Brodsky and Schlumpf wavefunction has strong quantum correlations for asymmetric and small momenta
- Single step of scale evolution (collinear gluon emission) for the entire density matrix now has entanglement negativity correlations primarily for small and similar x_1, x_2

dPDF Initial Conditions

Some models from the literature:

1. Gaunt-Stirling: (arXiv:0910.4347)

$$C(x_1, x_2) = \frac{(1 - x_1 - x_2)^2}{(1 - x_1)^{2+n}(1 - x_2)^{2+n}}$$

   - $n = 0.5$ for valence quarks, 0 for sea quarks
   - violates the quark number sum rule

2. Broniowski-Arriola: (arXiv:1310.8419)

$$f_q(x) = \frac{168}{145}(1 - x)^3(1 + 6x + 16x^2 + 6x^3 + x^4)$$
$$f_{qq}(x_1, x_2) = \frac{1008}{29}(1 - x_1)^2(1 - x_2)^2(x_1 + x_2)^2$$

   - nonzero on the boundaries of phase space (fixed with DGLAP)

Discretization

The QIT discussion, PEN, etc. is for discrete systems!
$\implies$ need to make sure continuum limit is well defined for $\rho_{\xi\eta,\xi'\eta'}$

Discretize $[0,1]$ into bins of size $\Delta\xi$ and $\Delta\eta$ and include the Jacobian so $\mathrm{tr}\,\tilde{\rho} = \sum \lambda_i$

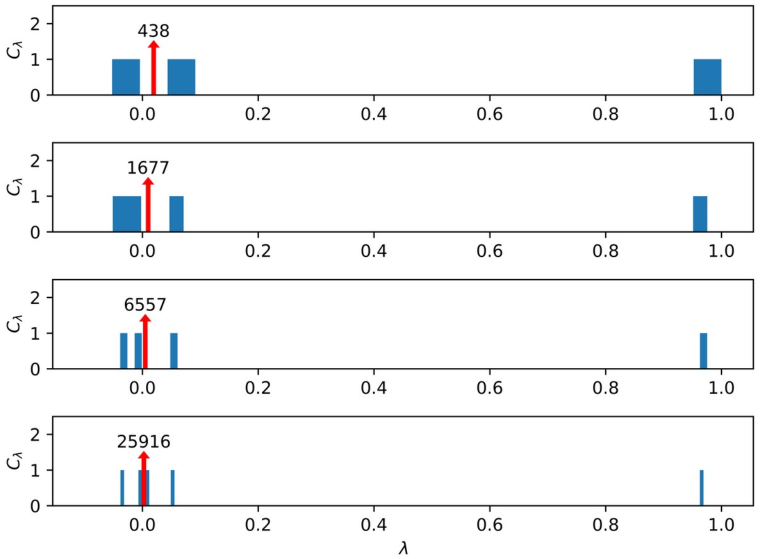
$$\tilde{\rho}_{\xi\eta,\xi'\eta'} = \frac{\Delta\xi\,\Delta\eta}{\sqrt{4\eta(1-\eta)\xi(1-\xi)\;4\eta'(1-\eta')\xi'(1-\xi')}}\,\rho_{\xi\eta,\xi'\eta'}$$

For

- $\Delta\xi = \Delta\eta = N^{-1}$
- $N = 20, 40, 80, 160$

we find the eigenvalue distr. of $\tilde{\rho}^{T_B}$ approaches

$$\frac{dN_\lambda}{d\lambda} = \left((N+1)^2 - \sum_{i=1}^{n} C_i\right)\delta(\lambda) + \sum_{i=1}^{n} C_i\,\delta(\lambda - \lambda_i)$$

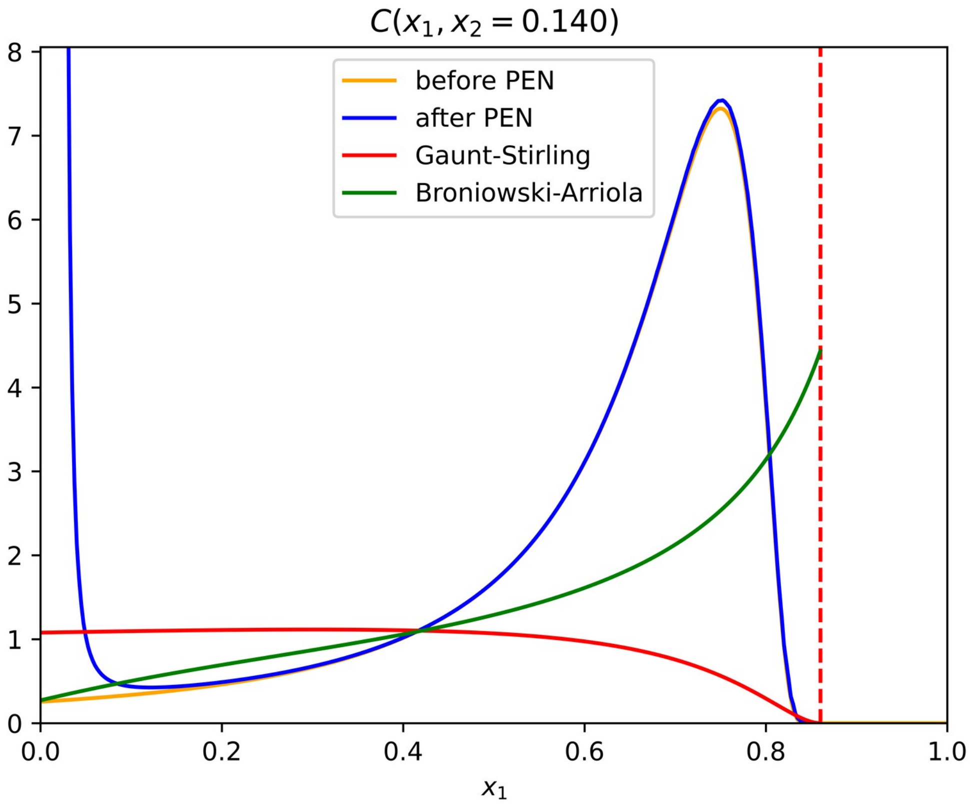


Comparison to Models

The differences in $C(x_1, x_2)$ are easiest to see by looking at slices of constant x_2

$x_2 = 0.14$:

- for $x_1 \ll x_2$, PEN effects are large
- at moderate $x_1 \sim 0.1$–0.5
  - PEN effects are much smaller
  - BA model similar to BS, increasing
  - GS model always decreasing at fixed x_2
- for $x_1 \gg x_2$, small effects (η large)



Comparison to Models

The differences in $C(x_1, x_2)$ are easiest to see by looking at slices of constant x_2

$x_2 = 0.31$:

- again PEN effects are large for $x_1 \ll x_2$
- smaller for moderate, large x_1
- still good agreement of BA and BS