Aspects of the Chiral and Partonic Structure of the Nucleon

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SOLI DEO GLORIA

Outline

- Generalized Polarizabilities of the Nucleon
  AM, Drechsel, ZPA356 (1996) 351; ZPA 359 (1997) 165
  Drechsel, Knöchlein, Korchin, AM, Scherer, PRC55 (1997) 424; PRC57 (1998) 941; PRC58 (1998) 1751

- Universality of TMD Fragmentation Functions
  AM, PLB549 (2002) 139 / Collins, AM, PRL93 (2004) 252001

- Transverse Single-Spin Asymmetries
  AM, Pitonyak, PLB723 (2013) 365 / Kanazawa, Koike, AM, Pitonyak, PRD89 (2014) 111501(R)

- Generalized TMDs and Wigner Functions
  Meißner, AM, Schlegel, Goeke, JHEP 0808 (2008) 038 / Meißner, AM, Schlegel, JHEP 0908 (2009) 056

- Decomposition of the Proton Mass
  Rodini, AM, Pasquini, JHEP 09 (2020) 067 / AM, Pasquini, Rodini, PRD102 (2020) 114042
  Lorcé, AM, Pasquini, Rodini, JHEP 11 (2021) 121

- Transversity Distributions and Tensor Charges of the Proton ($\rightarrow$ talks by Pitonyak, Cocuzza)
  Pitonyak, Cocuzza, AM, Prokudin, Sato, PRL132 (2024) 011902; arXiv:2502.15817
  Cocuzza, AM, Pitonyak, Prokudin, Sato, Seidl, PRL132 (2024) 091901 ; PRD109 (2024) 034024

Generalized Polarizabilities of the Nucleon

1. Introduction

- Electric ($\bar{\alpha}$) and magnetic ($\bar{\beta}$) polarizabilities in classical EM

$$\vec{D}_{\rm ind} = \bar{\alpha}\, \vec{E}_{\rm ext} \qquad\qquad \vec{M}_{\rm ind} = \bar{\beta}\, \vec{B}_{\rm ext}$$

- Calculation in quantum mechanics $\rightarrow$ excitation spectrum matters

$$\bar{\alpha} = 2 \sum_{n \neq 0} \frac{|\langle n | \hat{D}_z | 0 \rangle|^2}{E_n - E_0}$$

- Polarizabilities of the proton in real Compton scattering, $\gamma p \rightarrow \gamma p$

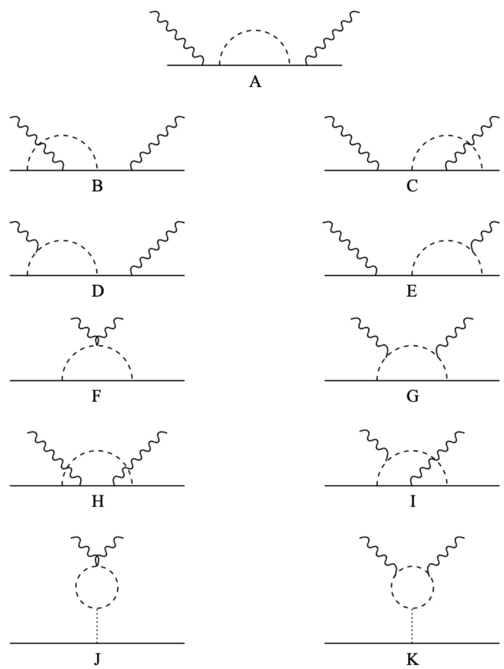
$$T^{\rm RCS} = T_B^{\rm RCS}(e, M, \kappa) + T_{NB}^{\rm RCS}$$

$$T_{NB}^{\rm RCS} = \alpha\, \omega\omega' \vec{\varepsilon} \cdot \vec{\varepsilon}\,'^{*} + \beta\, (\vec{\varepsilon} \times \vec{q}\,) \cdot (\vec{\varepsilon}\,'^{*} \times \vec{q}\,') + \mathcal{O}(\omega^3)$$

- PDG values $\rightarrow$ proton is stiff

$$\alpha_p = (11.2 \pm 0.4) \times 10^{-4}\, {\rm fm}^3 \qquad\qquad \beta_p = (2.5 \pm 0.4) \times 10^{-4}\, {\rm fm}^3$$

2. Polarizabilities in the Linear Sigma Model (LSM)



- non-resonant πN excited states important (described by chiral dynamics)
- LSM incorporates chiral symmetry (like ChPT)
- consider LSM for $m_\sigma \to \infty$
- results depend on $\mu = m_\pi / M$

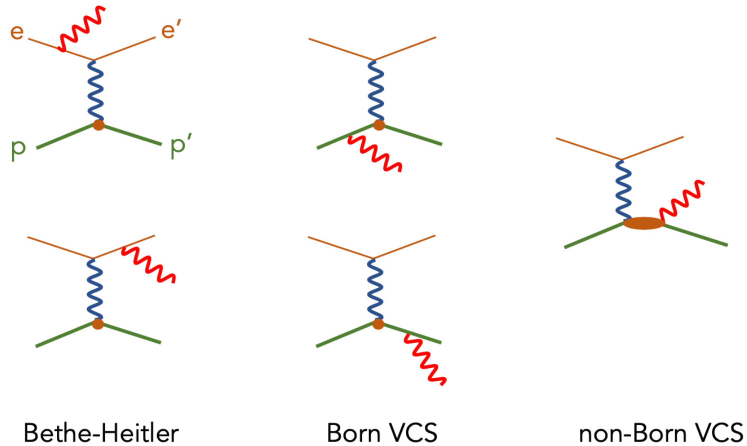
- Results

$$\alpha_p^{\mathrm{LSM}} = \frac{e^2 g_{\pi N}^2}{192\pi^3 M^3}\left[\frac{5\pi}{2\mu} + 18\ln\mu + \frac{33}{2} + \mathcal{O}(\mu)\right] = 12 \times 10^{-4}\,\mathrm{fm}^3 + \ldots$$

$$\beta_p^{\mathrm{LSM}} = \frac{e^2 g_{\pi N}^2}{192\pi^3 M^3}\left[\frac{\pi}{4\mu} + 18\ln\mu + \frac{63}{2} + \mathcal{O}(\mu)\right] = 1.2 \times 10^{-4}\,\mathrm{fm}^3 + \ldots$$

  – result first obtained in "relativistic ChPT" (Bernard, Kaiser, Meißner, 1991)
  – leading term matches result of HBChPT at $\mathcal{O}(p^3)$

3. Generalized Polarizabilities (GPs) in Virtual Compton Scattering (VCS)



- separation on T^{VCS} into T_B^{VCS} and T_{NB}^{VCS}
- parametrization of T_{NB}^{VCS} in terms of 10 GPs
  (Guichon, Liu, Thomas, 1995)
- generalizing α and β

$$\alpha(\bar{q}) = -\frac{e^2}{4\pi}\sqrt{\frac{3}{2}}\,P^{(01,01)0}(\bar{q})$$

$$\beta(\bar{q}) = -\frac{e^2}{4\pi}\sqrt{\frac{3}{8}}\,P^{(11,11)0}(\bar{q})$$

- 4 relations between 10 GPs $\longrightarrow$ only 6 independent GPs
  - examples

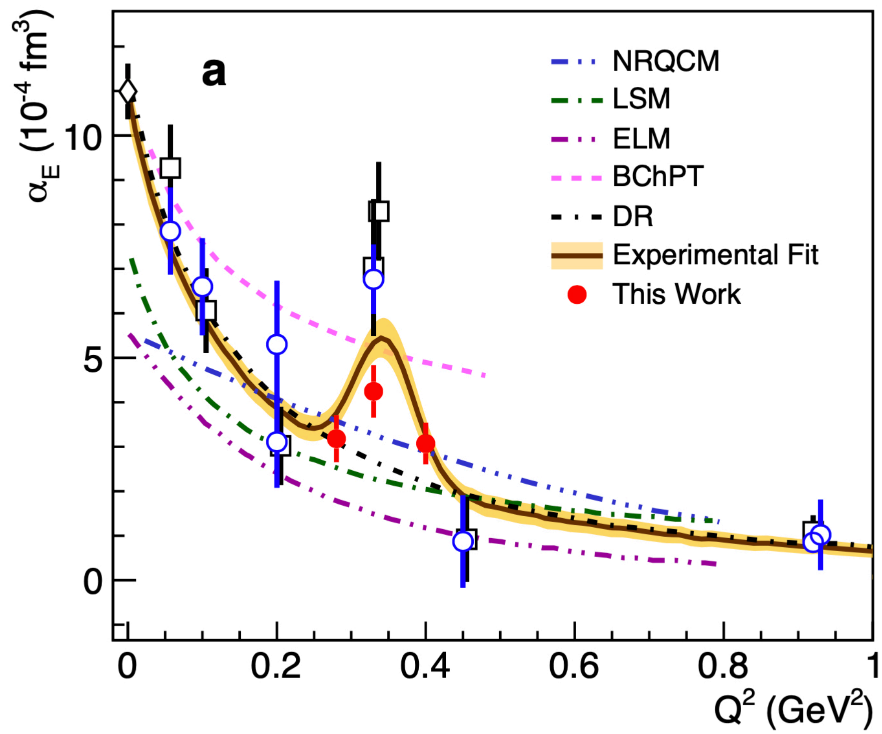
$$0 = \sqrt{\frac{3}{2}}\,P^{(01,01)0}(\bar{q}) + \sqrt{\frac{3}{8}}\,P^{(11,11)0}(\bar{q}) + \frac{3\bar{q}^2}{2\omega_0}\,\hat{P}^{(01,1)0}(\bar{q})$$

$$0 = P^{(11,11)1}(\bar{q}) + \sqrt{\frac{3}{2}}\,\omega_0\,P^{(11,02)1}(\bar{q}) + \frac{5}{2}\,\bar{q}^2\,\hat{P}^{(11,2)1}(\bar{q})$$

  - relations obtained in LSM and model-independent way

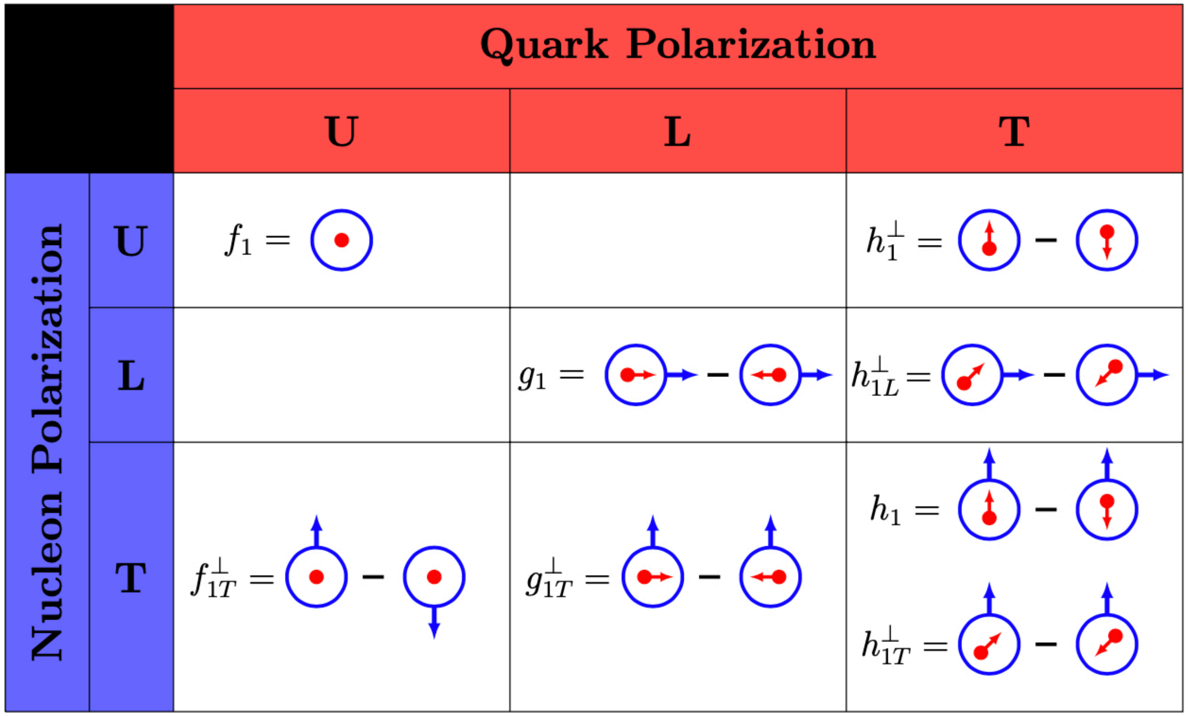
4. (Generalized) Polarizabilities is Active Field of Research

- Example: unexpected behavior of $\alpha(\bar{q})$ (Li et al, 2022)



Universality of TMD Fragmentation Functions

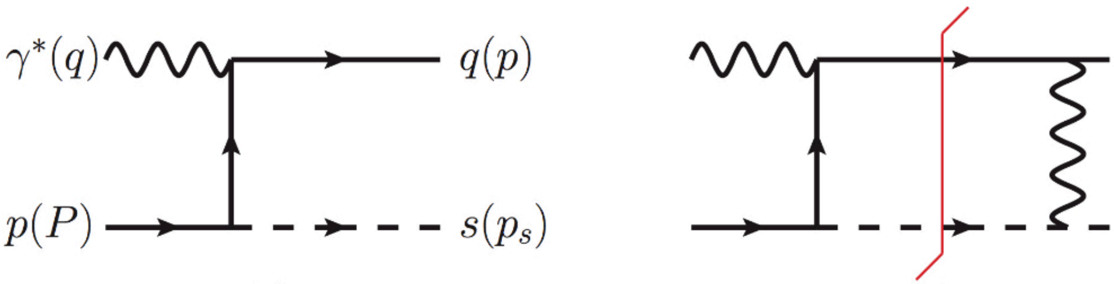
1. Overview of TMD Quark Distributions $F(x, k_\perp)$



- Sivers function $f_{1T}^\perp$ and Boer-Mulders function $h_1^\perp$ can gives rise to transverse single-spin asymmetries (Sivers, 1989, 1990 / Boer, Mulders, 1997)
- $f_{1T}^\perp$ and $h_1^\perp$ were believed to vanish (applying parity and time-reversal) (Collins, 1992)

2. The BHS Transverse Single-Spin Asymmetry (SSA)
(Brodsky, Hwang, Schmidt, 2002)

- Consider semi-inclusive DIS (SIDIS) in diquark spectator model



- Focus on transverse SSA

$$A_T^y = \frac{e_1^2}{8\pi} \frac{2(Mx + m_q)p_\perp^x}{(Mx + m_q)^2 + p_\perp^2} \frac{p_\perp^2 + \widetilde{M}^2}{p_\perp^2} \ln \frac{p_\perp^2 + \widetilde{M}^2}{\widetilde{M}^2} \neq 0$$

$$\widetilde{M}^2 = x(1-x)\left(-M^2 + \frac{m_q^2}{x} + \frac{m_s^2}{1-x} \right)$$

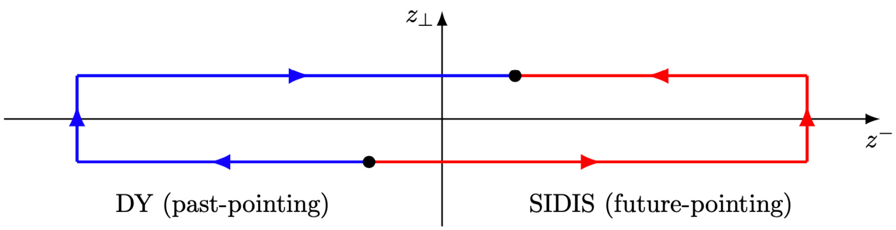
  – nonzero result, not suppressed in Bjorken limit
  – SSA arises from interference of tree-level diagram and imaginary part of one-loop diagram
  – does this effect break factorization ?

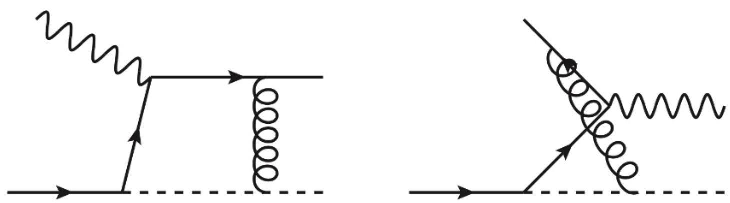
3. Analysis/Interpretation by Collins and some of the Aftermath (Collins, 2002)

- Factorization in terms of nonzero Sivers function $f_{1T}^{\perp}$
- Gauge link in TMD operator definition is crucial (generated by gluon exchange)

$$\int \frac{dz^- \, d^2\vec{z}_\perp}{2\,(2\pi)^3} \, e^{ik\cdot z} \, \langle P, S| \, \bar{\psi}^q(-\tfrac{z}{2}) \, \Gamma \, \mathcal{W}_{\mathrm{TMD}}[-\tfrac{z}{2}, \tfrac{z}{2}] \, \psi^q(\tfrac{z}{2}) \, |P, S\rangle \Big|_{z^+=0}$$

- Non-trivial universality: SIDIS vs Drell-Yan





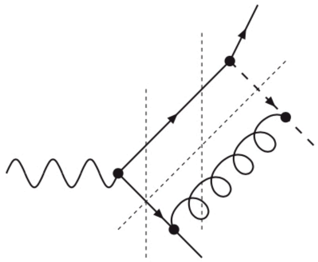
$$f_{1T}^{\perp}|_{\mathrm{DY}} = - \, f_{1T}^{\perp}|_{\mathrm{SIDIS}} \qquad h_1^{\perp}|_{\mathrm{DY}} = - \, h_1^{\perp}|_{\mathrm{SIDIS}}$$

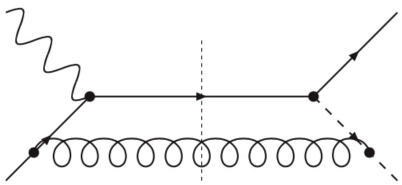
- Developments gave tremendous boost to studies of transverse SSAs and TMD field
- Sign change of $f_{1T}^{\perp}$ confirmed experimentally (STAR, 2015 / COMPASS, 2019 / ...)

4. Are TMD Fragmentation Functions (FFs) Universal ?

- Compare fragmentation in electron-positron annihilation and SIDIS
  - a priori, future-pointing Wilson line vs past-pointing Wilson line
  - unlike for TMD-PDFs, parity and time-reversal cannot be used to relate two cases

- Spectator model calculation (BHS for fragmentation)





  - important cancellation for electron-positron annihilation

$$A_T^{q\bar{q}}\big|_{e^+e^-} = -\, A_T^{\bar{q}s}\big|_{e^+e^-}$$

  - contribution from third cut agrees with SIDIS result $\rightarrow$ universality of TMD-FFs

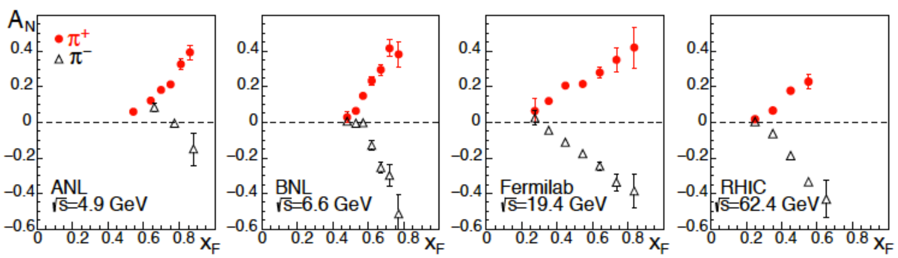
$$A_T^{qg}\big|_{\mathrm{SIDIS}} = +\, A_T^{qg}\big|_{e^+e^-} \quad \rightarrow \quad D_{1T}^{\perp}\big|_{\mathrm{SIDIS}} = D_{1T}^{\perp}\big|_{e^+e^-} \qquad H_1^{\perp}\big|_{\mathrm{SIDIS}} = H_1^{\perp}\big|_{e^+e^-}$$

- Universality of TMD-FFs (direction of Wilson line does not matter) (i) due to kinematics, (ii) holds beyond one-loop model calculations, (iii) heavily used

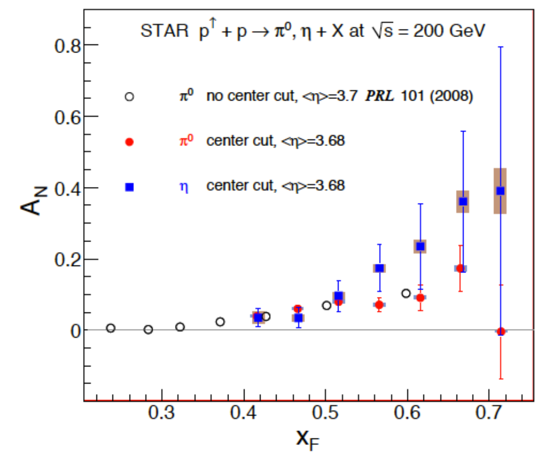
Transverse Single-Spin Asymmetries

1. Sample Data for Single-Spin Asymmetries (SSAs) in $p^{\uparrow}p \to \pi\, X$

$$A_N = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}}$$



(from Aidala, Bass, Hasch, Mallot, 2012)



STAR, 2012 $\sqrt{s} = 200\,\mathrm{GeV}$

- General features
  - very striking effects at large x_F
  - A_N survives at large $\sqrt{s}$
  - A_N is twist-3 observable, cannot be explained in collinear parton model (Kane, Pumplin, Repko, 1978 / Efremov, Teryaev, 1983)
  - data on transverse SSAs represented 40-year old puzzle

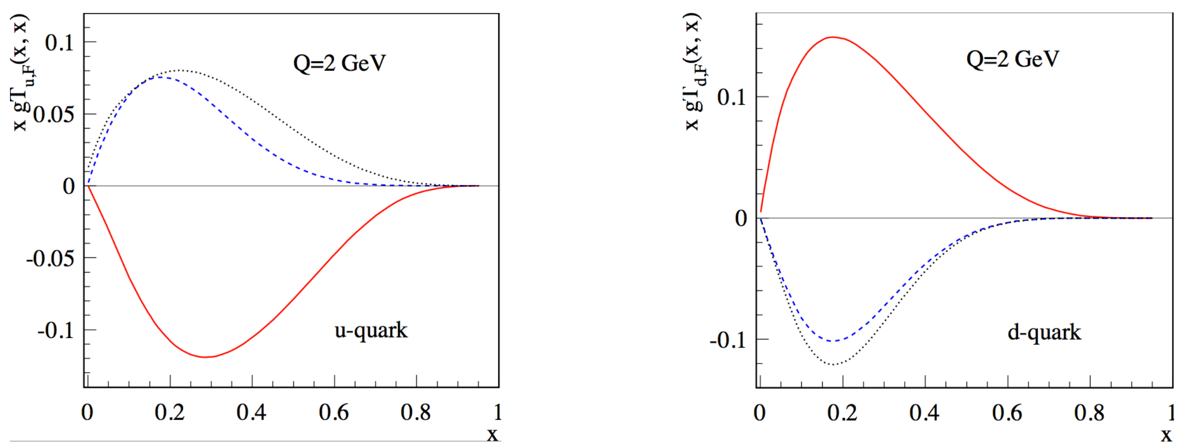
2. Transverse Spin Puzzle

- Collinear twist-3 factorization for $p^{\uparrow} p \to \pi\, X$ in full glory

(Efremov, Teryaev, 1983, 1984 / Qiu, Sterman, 1991, 1998 / Koike et al, 2000, ... / ...)

$$
\begin{aligned}
d\sigma^{\uparrow} &= H \otimes f_{a/A(3)} \otimes f_{b/B(2)} \otimes D_{C/c(2)} &\to&\quad \text{Sivers-type} \\
&+ H' \otimes f_{a/A(2)} \otimes f_{b/B(3)} \otimes D_{C/c(2)} &\to&\quad \text{Boer-Mulders-type} \\
&+ H'' \otimes f_{a/A(2)} \otimes f_{b/B(2)} \otimes D_{C/c(3)} &\to&\quad \text{“Collins-type”}
\end{aligned}
$$

- Focus on Sivers-type contribution (Kang, Qiu, Vogelsang, Yuan, 2011)
  - relation between pp and SIDIS due to $T_F \leftrightarrow f_{1T}^{\perp}$
  - sign mismatch



  * which of the signs for T_F is correct ?
  * Boer-Mulders type contribution small (Koike, Kanazawa, 2000)
  * large A_N in $p^{\uparrow} p \to H\, X$ caused by the “Collins-type” contribution ?

3. Twist-3 Fragmentation Contribution: Analytical Result

- General structure of result

$$\frac{P_h^0 d\sigma(\vec{S}_\perp)}{d^3\vec{P}_h} = -\frac{2\alpha_s^2 M_h}{S}\epsilon_{\perp,\alpha\beta}\, S_\perp^\alpha P_{h\perp}^\beta$$

$$\times \sum_i \sum_{a,b,c} \int_{z_{min}}^1 \frac{dz}{z^3} \int_{x'_{min}}^1 \frac{dx'}{x'} \frac{1}{x} \frac{1}{x'S + T/z} \frac{1}{-x'\hat{t} - x\hat{u}} h_1^a(x)\, f_1^b(x')$$

$$\times \left\{ \left[\hat{H}^c(z) - z\frac{d\hat{H}^c(z)}{dz} \right] S^i_{\hat{H}} + \frac{1}{z} H^c(z)\, S^i_H \right.$$

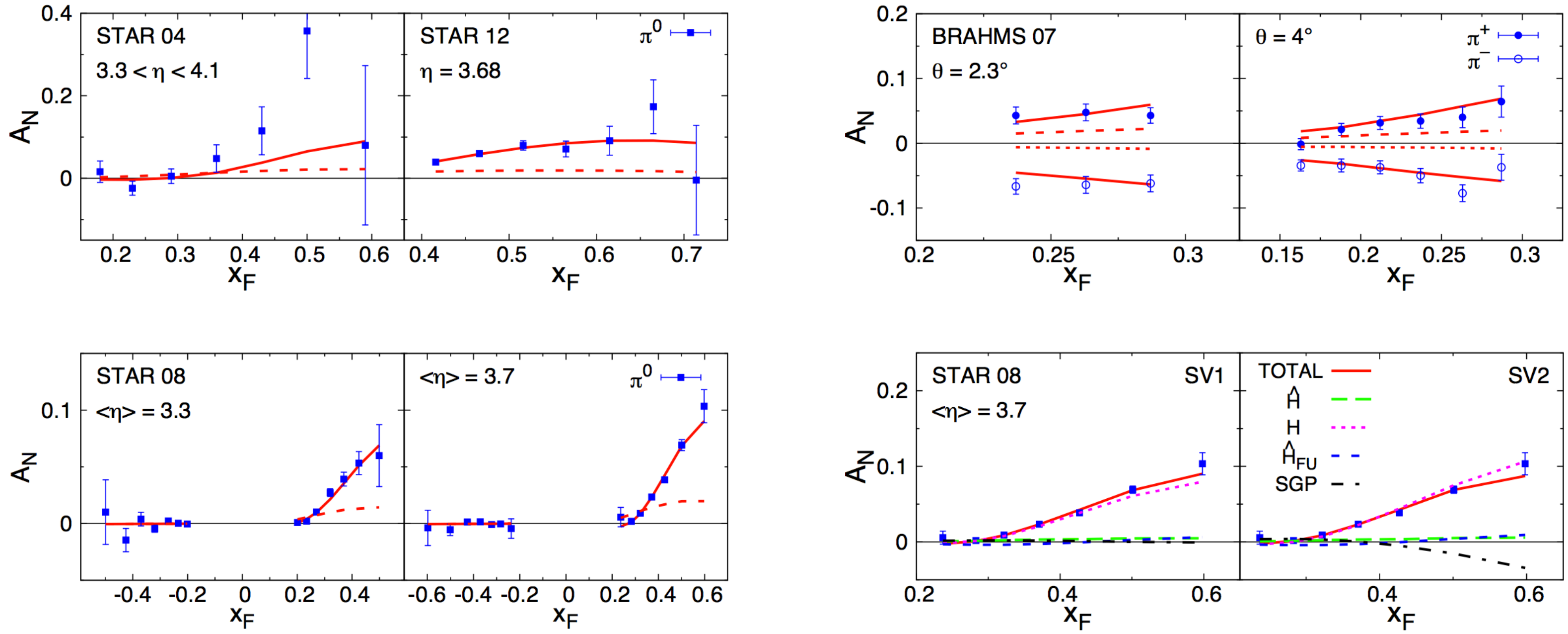
$$\left. + 2z^2 \int_z^\infty \frac{dz_1}{z_1^2} \frac{1}{\frac{1}{z} - \frac{1}{z_1}} \hat{H}_{FU}^{c,\Im}(z, z_1)\, \frac{1}{\xi}\, S^i_{\hat{H}_{FU}} \right\}$$

- $\hat{H}$ related to Collins function $H_1^\perp$
- $\hat{H},\ H,\ \hat{H}_{FU}^{\Im}$ related, but in twist-3 approach dynamics of fragmentation contribution to A_N goes beyond Collins effect
- many Feynman diagrams
- derivative term for $\hat{H}$ was computed first (Kang, Yuan, Zhou, 2010)
  $\rightarrow$ does not necessarily dominate

4. Numerical Description of Transverse SSAs

- Input for transversity h_1, Collins function $H_1^\perp$ $(\hat{H})$, and Sivers function $f_{1T}^\perp$ from $A_{\mathrm{SIDIS}}^{\mathrm{Siv}}$, $A_{\mathrm{SIDIS}}^{\mathrm{Col}}$, $A_{e^+e^-}^{\cos(2\phi)}$ (Anselmino et al, 2008, 2013)

- Sample results



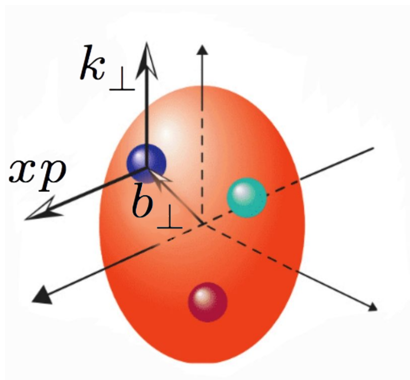
- Main outcome: simultaneous description of A_N, and $A_{\mathrm{SIDIS}}^{\mathrm{Siv}}$, $A_{\mathrm{SIDIS}}^{\mathrm{Col}}$, $A_{e^+e^-}^{\cos(2\phi)}$

- More recent (numerical) results available
  (Cammarota, Gamberg, Kang, Miller, Pitonyak, Prokudin, Rogers, Sato, 2020 / ...)

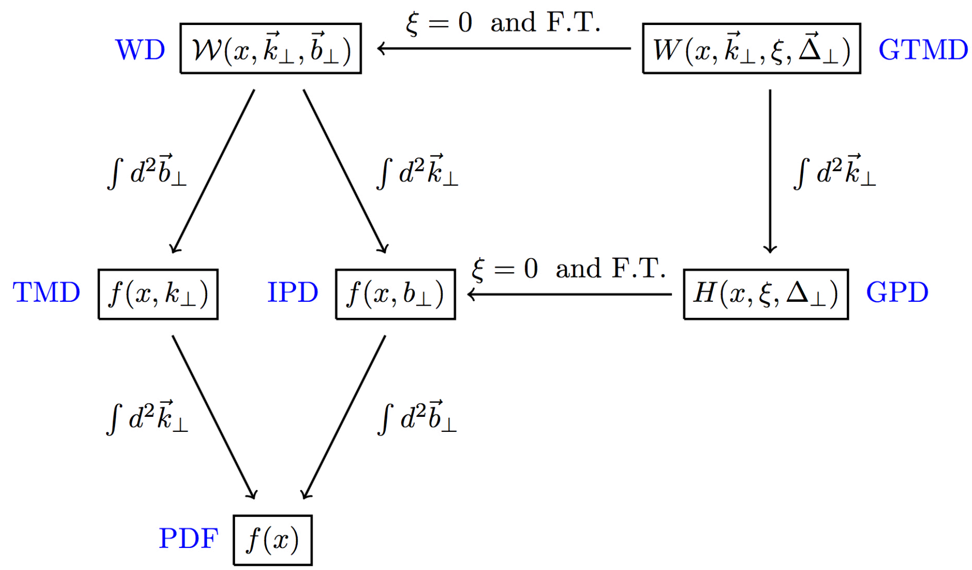
Generalized TMDs and Wigner Functions

1. Big Picture

   Information about x, $\vec{k}_\perp$, $\vec{b}_\perp$



Overview of partonic functions



- Some features of generalized TMDs (GTMDs)
  - like TMDs, they depend on $\vec{k}_\perp$
  - like GPDs, they depend on ξ, $\vec{\Delta}_\perp$
  - Fourier transform gives Wigner functions (5D quasi-probability distributions)
  - GTMDs/Wigner functions describe most general one-parton structure of hadrons

2. Wigner functions in non-relativistic QM (Wigner, 1932)

- calculable from wave function (1D)

$$\mathcal{W}(x,k) = \int \frac{dx'}{2\pi}\, e^{-\,i\,k\,x'} \psi^*\left(x - \frac{x'}{2}\right) \psi\left(x + \frac{x'}{2}\right)$$

$$= \int \frac{dk'}{2\pi}\, e^{+\,i\,k' x}\, \tilde{\psi}^*\left(k - \frac{k'}{2}\right) \tilde{\psi}\left(k + \frac{k'}{2}\right)$$

- relation to densities and observables

$$|\psi(x)|^2 = \int dk\, \mathcal{W}(x,k) \qquad |\tilde{\psi}(k)|^2 = \int dx\, \mathcal{W}(x,k)$$

$$\langle O(x,k)\rangle = \int dx\, dk\, O(x,k)\, \mathcal{W}(x,k)$$

3. Partonic Wigner functions $\mathcal{W}(x, \vec{k}_\perp, \vec{b}_\perp)$

- first highlighted about two decades ago (Belitsky, Ji, Yuan, 2003)
- significant progress in the field since
- partonic Wigner functions open up unique opportunities (e.g., spin-orbit correlations)

4. Orbital Angular Momentum of Partons

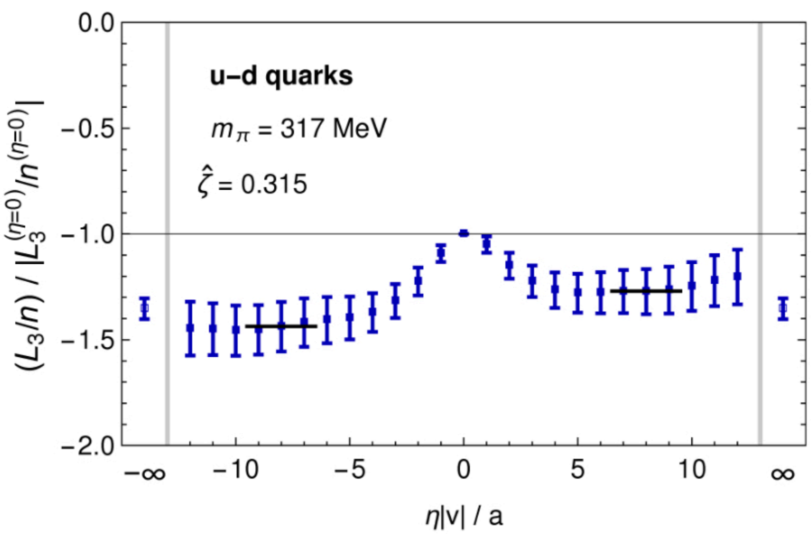
- Jaffe-Manohar spin sum rule (Jaffe, Manohar, 1989)

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L^q_{\rm JM} + L^g_{\rm JM}$$

- parton OAM in longitudinally polarized nucleon (Lorcé, Pasquini, 2011 / Hatta, 2011)

$$L_z^{q/g} = \int dx\, d^2\vec{k}_\perp\, d^2\vec{b}_\perp\, (\vec{b}_\perp \times \vec{k}_\perp)_z\, \mathcal{W}_L^{q/g\,[\rm unp]}(x, \vec{k}_\perp, \vec{b}_\perp)$$

- exploratory calculation of $L^{u-d}_{\rm JM}$ in lattice QCD (Engelhardt et al, 2017, 2019)



- figure shows $L^{u-d}_{\rm JM}/|L^{u-d}_{\rm Ji}|$ (for large $\eta|v|/a$)
- considerable numerical difference btw $L^{u-d}_{\rm JM}$ and $L^{u-d}_{\rm Ji}$

- developments in this area are milestone in spin physics

5. Parameterization of GTMD Correlator

- Correlator definition

$$W^{q\,[\Gamma]} = \int \frac{dz^- \, d^2\vec{z}_\perp}{2\,(2\pi)^3} \, e^{ik\cdot z} \, \langle p' \,|\, \bar{\psi}^q(-\tfrac{z}{2}) \, \Gamma \, \mathcal{W}_{\rm TMD}[-\tfrac{z}{2}, \tfrac{z}{2}] \, \psi^q(\tfrac{z}{2}) \,|\, p\rangle \Big|_{z^+=0}$$

- Examples

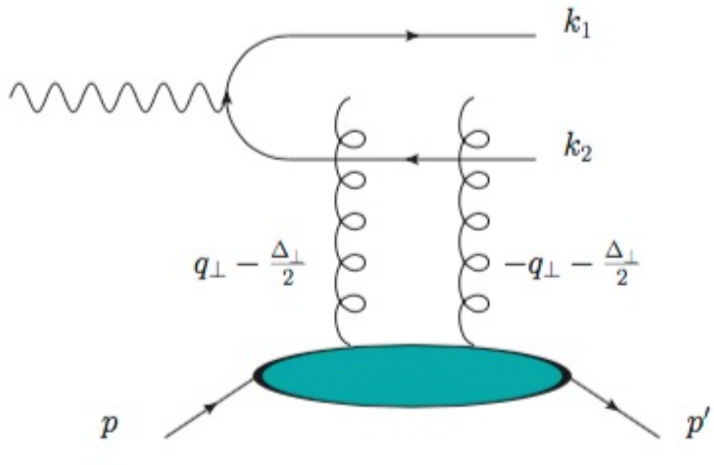
$$W^{[\gamma^+]} = \frac{1}{2M} \, \bar{u}(p') \left[F_{1,1} + \frac{i\sigma^{i+} \, k_\perp^i}{P^+} \, F_{1,2} + \frac{i\sigma^{i+} \, \Delta_\perp^i}{P^+} \, F_{1,3} + \frac{i\sigma^{ij} \, k_\perp^i \, \Delta_\perp^j}{M^2} \, F_{1,4} \right] u(p)$$

$$W^{[\gamma^+\gamma_5]} = \frac{1}{2M} \, \bar{u}(p') \left[- \frac{i\varepsilon_\perp^{ij} \, k_\perp^i \, \Delta_\perp^j}{M^2} \, G_{1,1} + \frac{i\sigma^{i+} \, \gamma_5 \, k_\perp^i}{P^+} \, G_{1,2} + \frac{i\sigma^{i+} \, \gamma_5 \, \Delta_\perp^i}{P^+} \, G_{1,3} + i\sigma^{+-} \, \gamma_5 \, G_{1,4} \right] u(p)$$

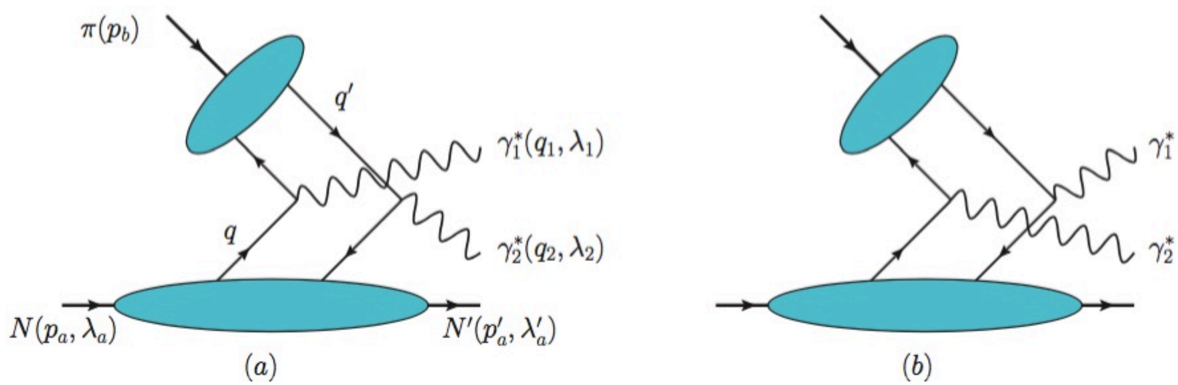
  - Main challenge: include all possible structures and avoid redundant structures
  - 16 leading-twist GTMDs for quarks
  - 16 leading-twist GTMDs for gluons (Lorcé, Pasquini, 2013)
  - GTMDs have real and imaginary part

6. Observables for GTMDs

- Gluon GTMDs: Diffractive exclusive dijet production in $\ell N / \ell A$
  (Hatta, Xiao, Yuan, 2016 / Altinoluk, Armesto, Beuf, Rezaeian, 2015 / ...)



- Quark GTMDs: Exclusive double Drell-Yan process: $\pi\, N \to (\ell_1^- \ell_1^+)\, (\ell_2^- \ell_2^+)\, N'$

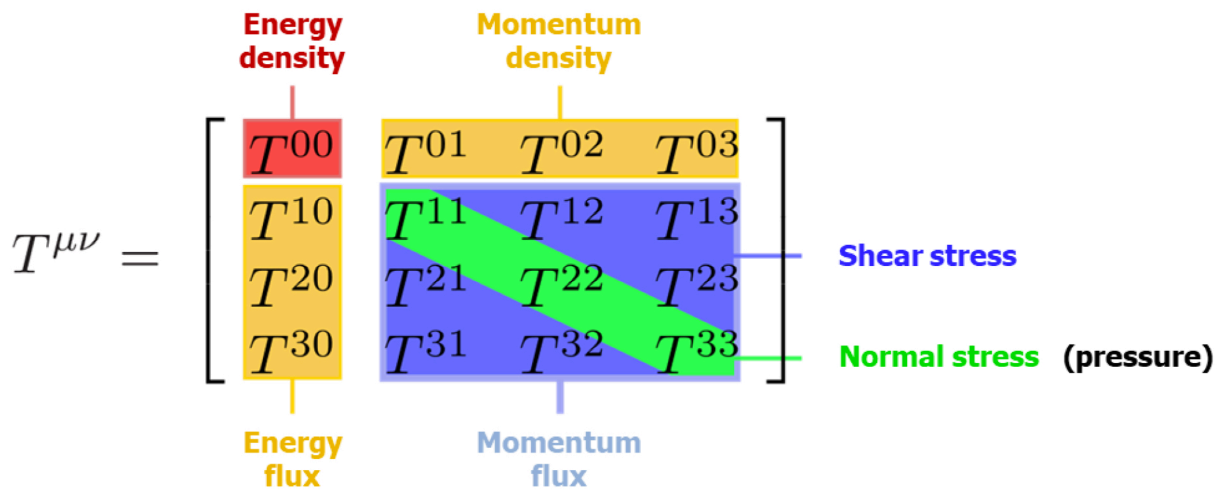


- Various more recent works on observables for GTMDs

Decomposition of the Proton Mass

1. Energy Momentum Tensor and Hadron Mass

- Interpretation of EMT



(courtesy, C. Lorcé)

- Symmetric (gauge invariant) EMT in QCD

$$T^{\mu\nu} = T_q^{\mu\nu} + T_g^{\mu\nu}$$

$$T_q^{\mu\nu} = \frac{i}{4}\,\bar{\psi}\,\gamma^{\{\mu}\overleftrightarrow{D}^{\nu\}}\,\psi \qquad \left(\gamma^{\{\mu}\overleftrightarrow{D}^{\nu\}} = \gamma^{\mu}\overleftrightarrow{D}^{\nu} + \gamma^{\nu}\overleftrightarrow{D}^{\mu}\right)$$

$$T_g^{\mu\nu} = -F^{\mu\alpha}F^{\nu}{}_{\alpha} + \frac{g^{\mu\nu}}{4}\,F^2$$

  – total EMT not renormalized, but $T_i^{\mu\nu}$ require renormalization

- Trace (anomaly) of EMT in QCD
  (Collins, Duncan, Joglekar, 1977 / Nielsen, 1977 / ...)

$$T^{\mu}{}_{\mu} = \underbrace{(m\bar{\psi}\psi)_R}_{\text{classical trace}} + \underbrace{\gamma_m\,(m\bar{\psi}\psi)_R + \frac{\beta}{2g}(F^2)_R}_{\text{trace anomaly}}$$

  – $T^{\mu}{}_{\mu}$, classical trace (quark mass term), and trace anomaly are UV-finite

- Quark and gluon contribution to EMT trace (Hatta, Rajan, Tanaka, 2018 / Tanaka 2018)

$$T^{\mu}{}_{\mu} = (T_{q,R})^{\mu}{}_{\mu} + (T_{g,R})^{\mu}{}_{\mu}$$

$$(T_{q,R})^{\mu}{}_{\mu} = (1+y)(m\bar{\psi}\psi)_R + x\,(F^2)_R$$

$$(T_{g,R})^{\mu}{}_{\mu} = (\gamma_m - y)(m\bar{\psi}\psi)_R + \Big(\frac{\beta}{2g} - x\Big)(F^2)_R$$

  x and y related to finite parts of renormalization constants $\rightarrow$ scheme dependence

- Different scheme choices
  – MS scheme / $\overline{\text{MS}}$ scheme (Hatta, Rajan, Tanaka, 2018 / Tanaka 2018)
  – D1 scheme: $x = 0$, $y = \gamma_m$
  – D2 scheme: $x = y = 0$

  D-type schemes look natural

- Forward matrix element of total EMT (for spin-0 and spin-$\frac{1}{2}$)

$$\langle\, T^{\mu\nu} \,\rangle \equiv \langle P |\, T^{\mu\nu} \,| P \rangle = 2 P^\mu P^\nu$$

- Relation to proton mass ($n = \frac{1}{2M}$, depends on normalization of state)

$$M = n \,\langle\, T^{\mu}{}_{\mu} \rangle = n \,\langle\, T^{00} \,\rangle \Big|_{\mathbf{P}=0} = \frac{\langle\, H_{\rm QCD} \,\rangle}{\langle P | P \rangle} \Big|_{\mathbf{P}=0} \qquad \left(\int d^3\vec{x}\, T^{00} = H_{\rm QCD} \right)$$

- Forward matrix element of $T^{\mu\nu}_{i,R}$ (Ji, 1996)

$$\langle\, T^{\mu\nu}_{i,R} \,\rangle = 2 P^\mu P^\nu A_i(0) + 2 M^2 g^{\mu\nu} \overline{C}_i(0)$$

  - $A_i(0),\ \overline{C}_i(0)$ are gravitational FFs at $t = 0$
  - conservation of (full) EMT implies

$$A_q(0) + A_g(0) = 1 \qquad \overline{C}_q(0) + \overline{C}_g(0) = 0$$

  - in forward limit, matrix elements of EMT fully determined by two numbers only

2. Different Decompositions of the Proton Mass

- 2-term decomposition of $T^{\mu}{}_{\mu}$ into quark and gluon contributions
  (Hatta, Rajan, Tanaka, JHEP 12 (2018) 008 / Tanaka, JHEP 01 (2019) 120)

$$M = n\Big(\langle\,(T_{q,R})^{\mu}{}_{\mu}\,\rangle + \langle\,(T_{g,R})^{\mu}{}_{\mu}\,\rangle\Big)$$

recall operators

$$(T_{q,R})^{\mu}{}_{\mu} = (1+y)(m\bar{\psi}\psi)_R + x\,(F^2)_R$$

$$(T_{g,R})^{\mu}{}_{\mu} = (\gamma_m - y)(m\bar{\psi}\psi)_R + \Big(\frac{\beta}{2g} - x\Big)(F^2)_R$$

- 2-term decomposition of T^{00} into total quark and gluon contributions
  (Lorcé, EPJC 78, 120 (2018))

$$M = n\Big(\langle\,T^{00}_{q,R}\,\rangle + \langle\,T^{00}_{g,R}\,\rangle\Big)$$

- 3-term decomposition of T^{00}

$$M = n\Big(\langle\mathcal{H}_q\rangle + \langle\mathcal{H}_m\rangle + \langle\mathcal{H}_g\rangle\Big)$$

$\mathcal{H}_q = (\psi^\dagger\, i\boldsymbol{D}\cdot\boldsymbol{\alpha}\,\psi)_R$ quark kinetic plus potential energy

$\mathcal{H}_m = (m\bar{\psi}\psi)_R$ quark mass term

$\mathcal{H}_g = \frac{1}{2}(E^2 + B^2)_R$ gluon energy

- 4-term decomposition of T^{00} (Ji, PRL 74, 1071 (1995) and PRD 52, 271 (1995))

$$M = n\Big(\langle\mathcal{H}_{q[\mathrm{Ji}]}\rangle + \langle\mathcal{H}_m\rangle + \langle\mathcal{H}_{g[\mathrm{Ji}]}\rangle + \langle\mathcal{H}_a\rangle\Big)$$

$\mathcal{H}_{q[\mathrm{Ji}]} = (\psi^\dagger\, i\boldsymbol{D}\cdot\boldsymbol{\alpha}\,\psi)_{R[\mathrm{Ji}]}$ (quark kinetic plus potential energy)$_{[\mathrm{Ji}]}$

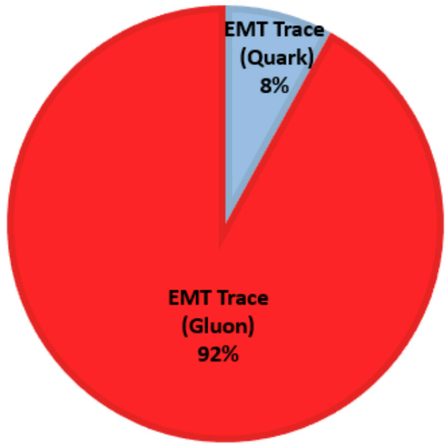
$\mathcal{H}_m = (m\bar{\psi}\psi)_R$ quark mass term

$\mathcal{H}_{g[\mathrm{Ji}]} = \frac{1}{2}(E^2 + B^2)_{R[\mathrm{Ji}]}$ (gluon energy)$_{[\mathrm{Ji}]}$

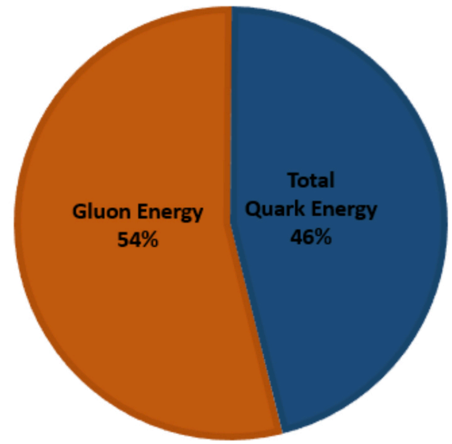
$\mathcal{H}_a = \frac{1}{4}\Big(\gamma_m\,(m\bar{\psi}\psi)_R + \frac{\beta}{2g}(F^2)_R\Big)$ anomaly contribution

3. Numerical Results (for D2 Scheme only)

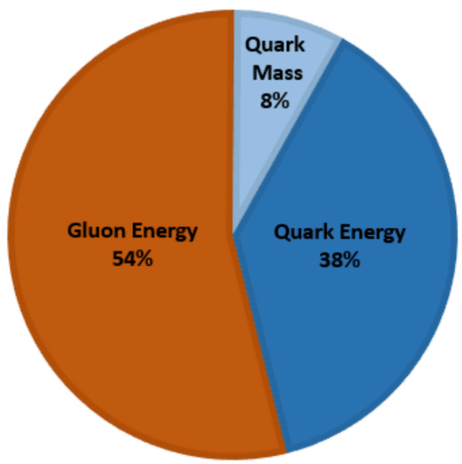
2 terms $T^{\mu}{}_{\mu}$



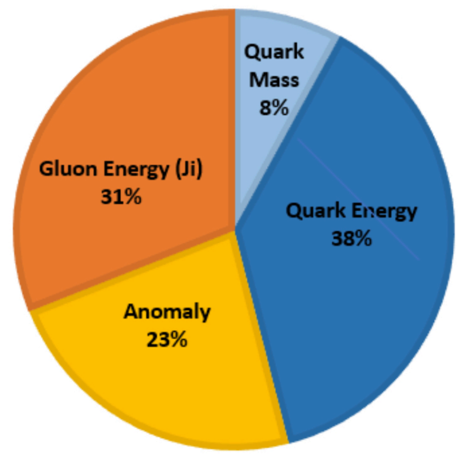
2 terms T^{00}



3 terms T^{00}



4 terms T^{00}



- Quark mass contribution in QCD (much) larger than frequently quoted 1-2%
- We already know numerics of the mass decompositions relatively well

Transversity Distributions and Tensor Charges of the Proton

1. Definition of Dihadron Fragmentation Functions (DiFFs)
   - Single-Hadron FFs as number densities

$$D_1^{h/q}(z, \vec{P}_{h\perp}) = \frac{1}{4z} \int \frac{d\xi^+ d^2\vec{\xi}_T}{(2\pi)^3} e^{ik\cdot\xi} \, \mathrm{Tr}\Big[\langle 0|\psi_q(\xi)|h, X\rangle\langle h, X|\bar{\psi}_q(0)|0\rangle \, \gamma^-\Big]_{\xi^-=0}$$

     – leading-order cross section for $e^-e^+ \to hX$

$$\frac{d\sigma}{dz} = \sum_{q,\bar{q}} \hat{\sigma}^q \, D_1^{h/q}(z) \quad \text{with} \;\; \hat{\sigma}^q = \hat{\sigma}(e^-e^+ \to \gamma^{(*)} \to q\bar{q}) = \frac{4\pi e_q^2 \alpha_{\mathrm{em}}^2 N_c}{3Q^2}$$

   - Dihadron FFs as number densities

$$\frac{1}{64\pi^3 z_1 z_2} \int \frac{d\xi^+ d^2\vec{\xi}_T}{(2\pi)^3} e^{ik\cdot\xi} \, \mathrm{Tr}\Big[\langle 0|\psi_q(\xi)|h_1, h_2, X\rangle\langle h_1, h_2, X|\bar{\psi}_q(0)|0\rangle \, \gamma^-\Big]_{\xi^-=0}$$

$$= D_1^{h_1 h_2/q}(z_1, z_2, \vec{P}_{1\perp}, \vec{P}_{2\perp})$$

     – leading-order cross section for $e^-e^+ \to (h_1h_2)X$ (example)

$$\frac{d\sigma}{dz\, dM_h} = \sum_{q,\bar{q}} \hat{\sigma}^q \, D_1^{h_1 h_2/q}(z, M_h) \quad \text{with} \;\; \hat{\sigma}^q = \frac{4\pi e_q^2 \alpha_{\mathrm{em}}^2 N_c}{3Q^2}$$

2. Simultaneous Extraction of DiFFs and Transversity PDFs

- Main observables/input for DiFFs and transversity PDFs
  - unpolarized cross section in $e^-e^+ \to (h_1 h_2) X$ (data from Belle)

$$\frac{d\sigma}{dz\, dM_h} = \frac{4\pi\alpha_{\rm em}^2 N_c}{3Q^2} \sum_{q,\bar{q}} e_q^2\, D_1^q(z, M_h)$$

  - Artru-Collins asymmetry in $e^-e^+ \to (h_1 h_2)(\bar{h}_1 \bar{h}_2) X$ (data from Belle)

$$A^{e^-e^+} = \frac{\sin^2\theta \sum_{q,\bar{q}} e_q^2\, H_1^{\sphericalangle,q}(z, M_h) H_1^{\sphericalangle,\bar{q}}(\bar{z}, \overline{M}_h)}{(1+\cos^2\theta) \sum_{q,\bar{q}} e_q^2\, D_1^q(z, M_h) D_1^{\bar{q}}(\bar{z}, \overline{M}_h)}$$

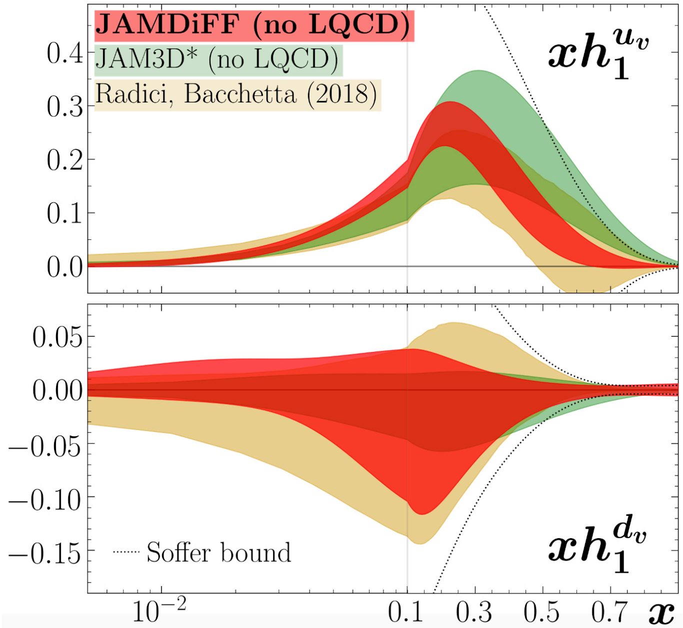
  - transverse SSA in SIDIS (data from HERMES and COMPASS)

$$A_{UT}^{\rm SIDIS} = c(y) \frac{\sum_{q,\bar{q}} e_q^2\, h_1^q(x)\, H_1^{\sphericalangle,q}(z, M_h)}{\sum_{q,\bar{q}} e_q^2\, f_1^q(x)\, D_1^q(z, M_h)}$$

  - transverse SSA in pp collisions (data from STAR)

$$A_{UT}^{pp} = \frac{\mathcal{H}(M_h, P_{hT}, \eta)}{\mathcal{D}(M_h, P_{hT}, \eta)}$$

- Extracted transversity PDFs



– fit of $h_1^{u_v}, \ h_1^{d_v}, \ h_1^{\bar{u}} = -h_1^{\bar{d}}$
large-N_c constraint for antiquarks (Pobylitsa, 2003)

– Soffer bound (Soffer, 1995)

$$h_1^q(x) \leq \tfrac{1}{2}\left|f_1^q(x) + g_1^q(x)\right|$$

– small-x constraint (Kovchegov, Sievert, 2019)

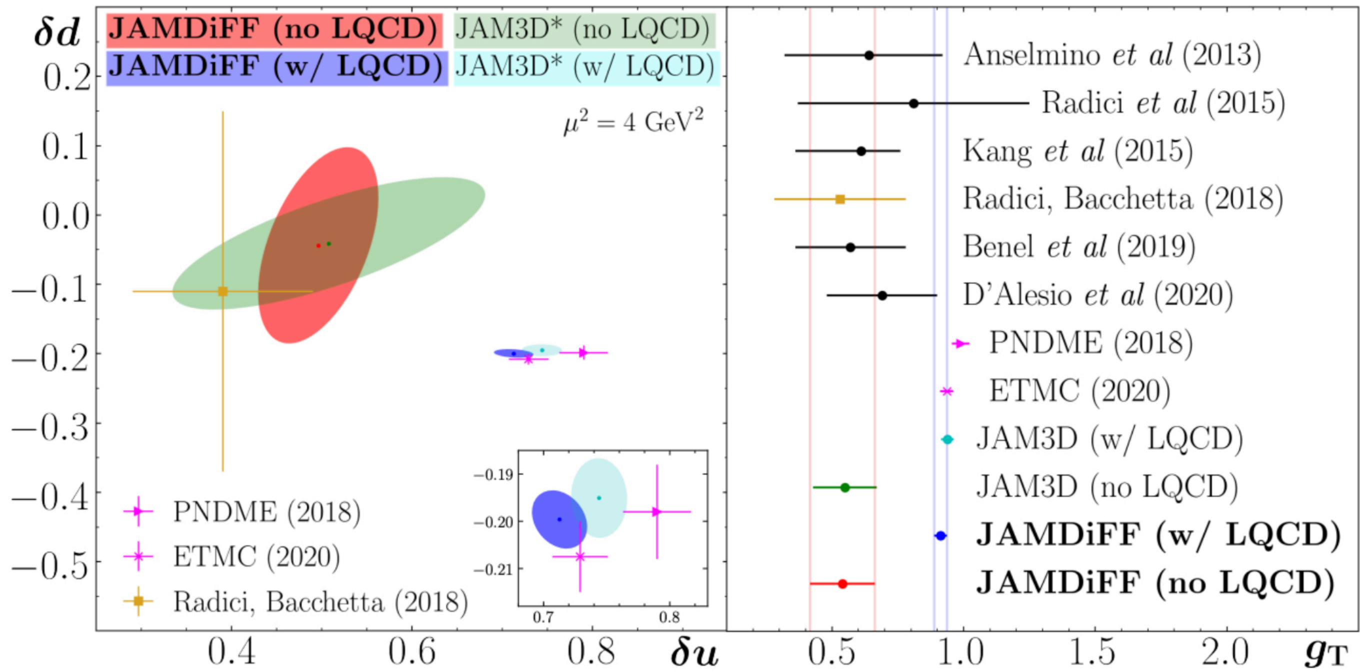
$$h_1^q \xrightarrow{x \to 0} x^{\alpha_q} \qquad \alpha_q \approx 0.17 \pm 0.085$$

– JAM3D* = JAM3D-22 (no LQCD)
+ antiquarks with $h_1^{\bar{u}} = -h_1^{\bar{d}}$
+ small-x constraint

– agreement between all three analyses within errors

- Tensor charges (no LQCD vs w/ LQCD)

$$\delta q(\mu) = \int_0^1 dx \left(h_1^q(x,\mu) - h_1^{\bar{q}}(x,\mu)\right) \qquad g_T = \delta u - \delta d$$



Overall finding: universal nature of all available information on h_1^q —
(1) data for di-hadron production, (2) data for single-hadron production,
(3) LQCD results for tensor charge, (4) Soffer bound, (5) small-x constraint