Hadron spectroscopy within HadSpec

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hadspec

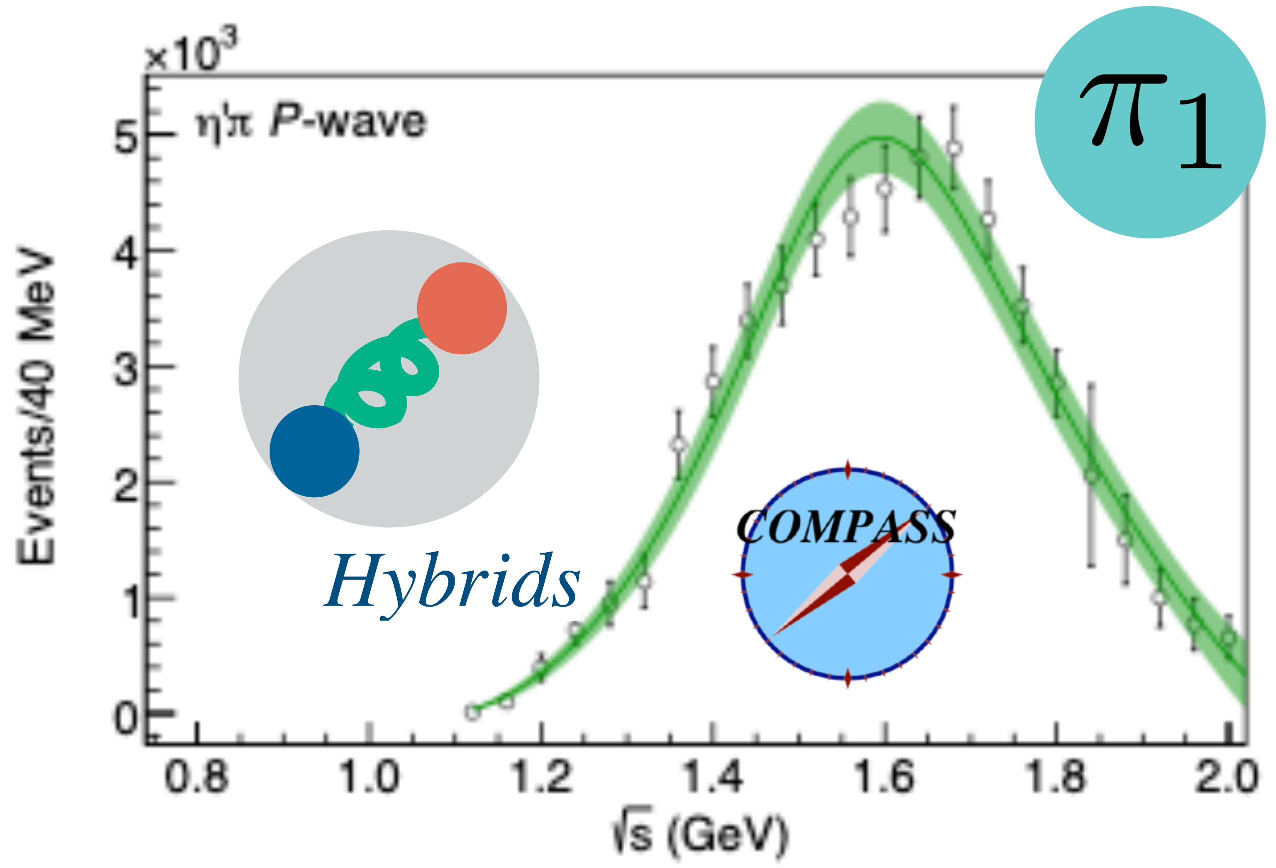
Spectroscopy in lattice QCD

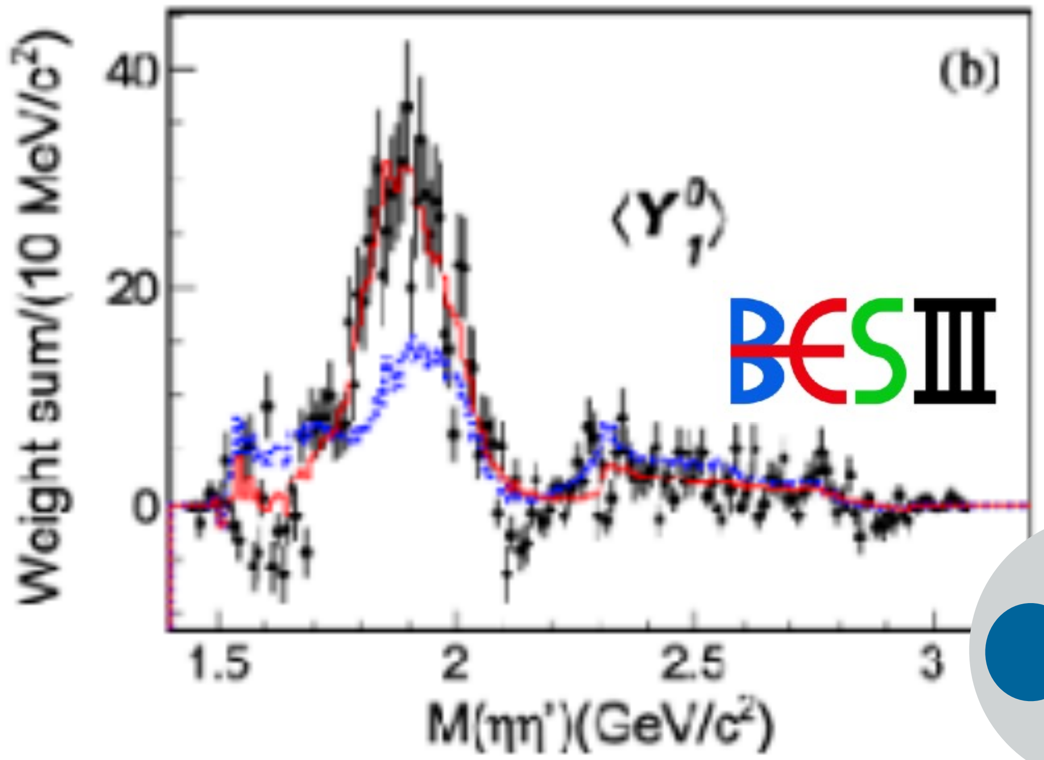
How do quark and gluons combine inside unstable hadrons?

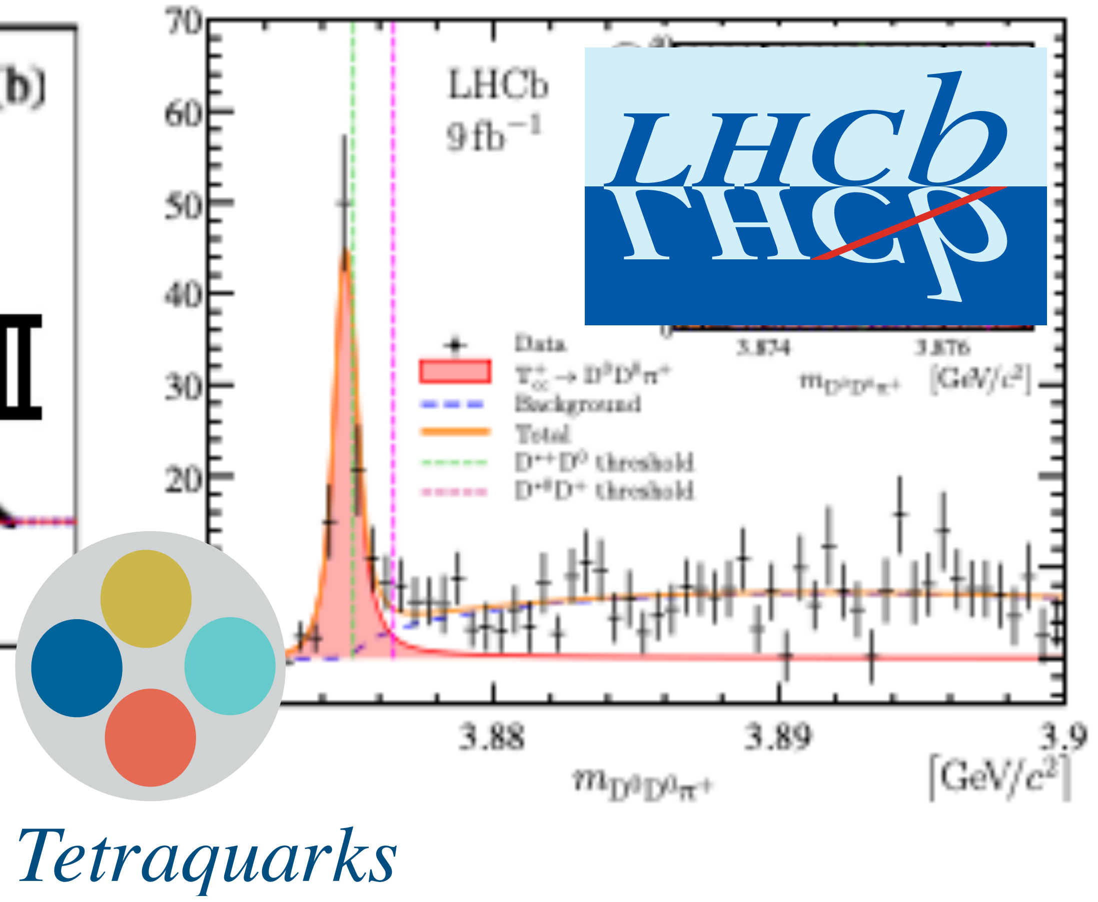
We need a combination of lattice QCD and experiment to answer that question

Guide experimental searches (π_1,η_1)

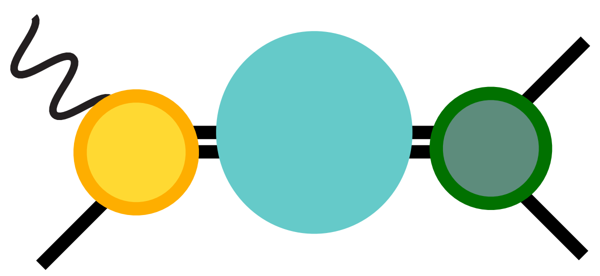
Confirm existence (tetraquarks, pentaquarks, glueballs)

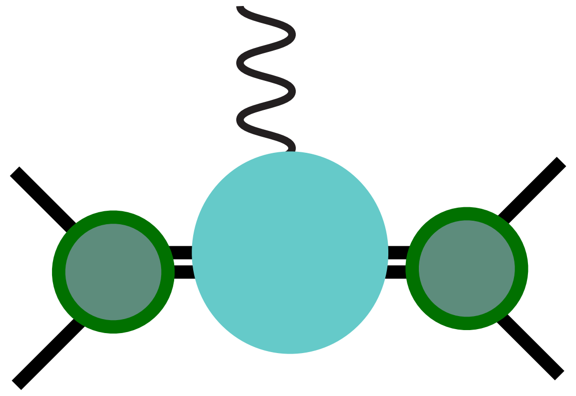




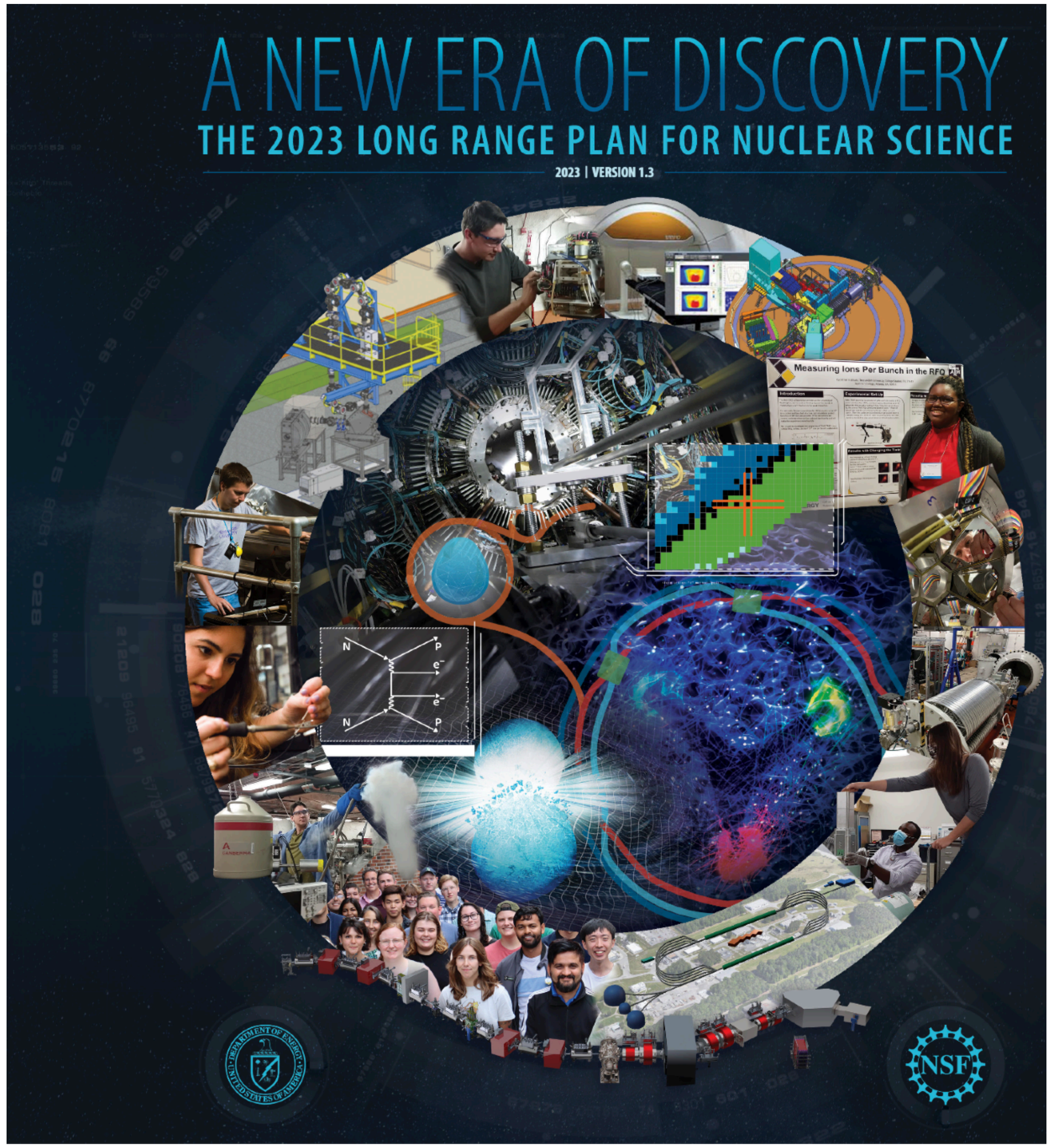


Understand their nature (*observations are not enough*!)





"hadron spectroscopy explores the possible bound combinations of quarks and gluons allowed by the interactions of QCD"



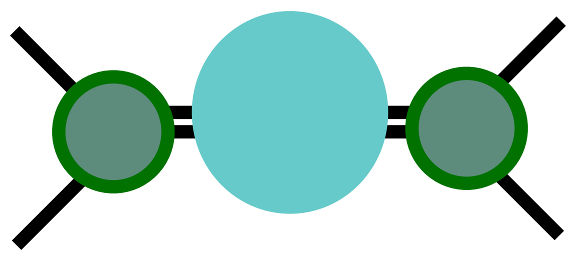
Spectroscopy in lattice QCD

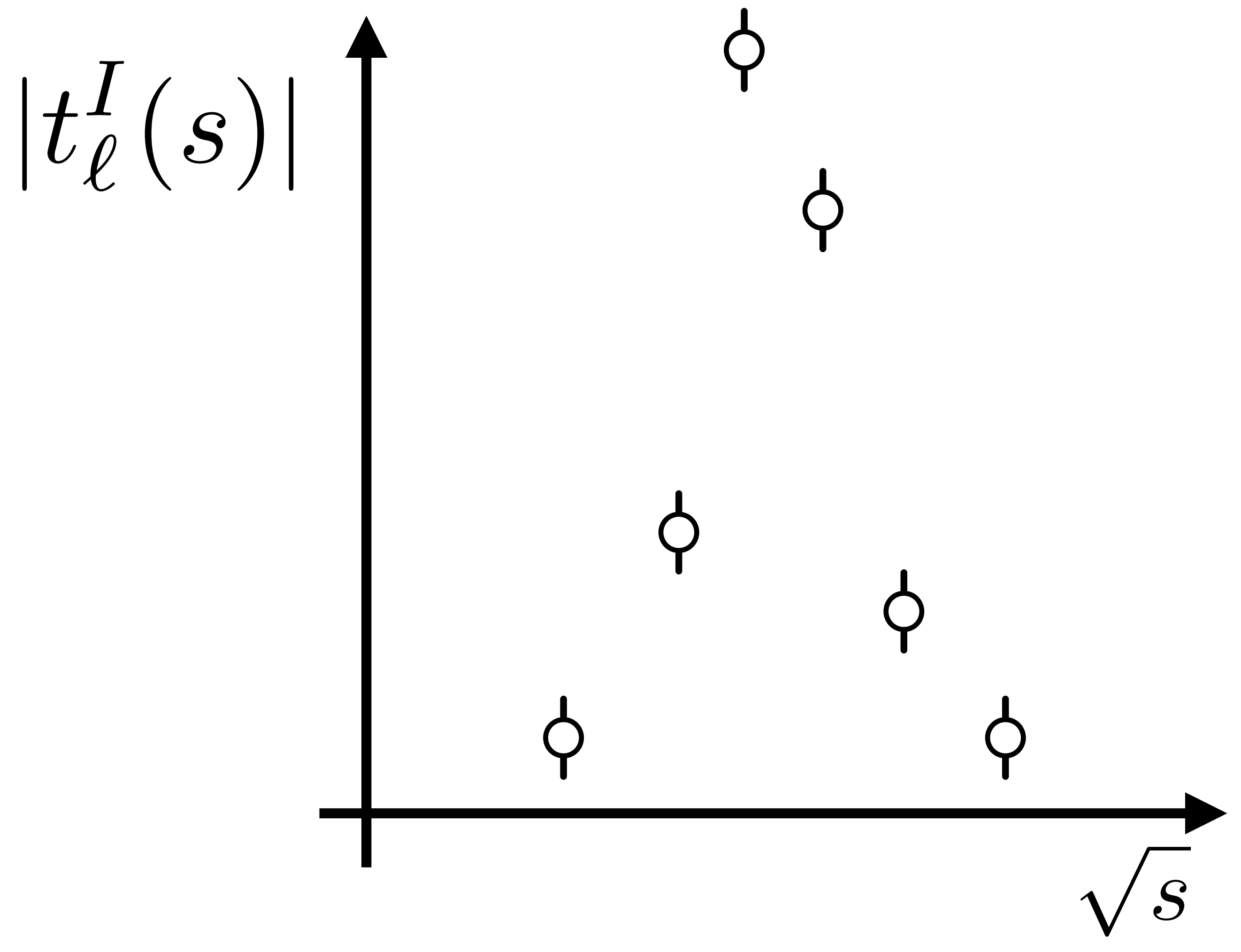
Extracting resonances from 2-body data 101

Assume we have scattering data for well-defined angular momentum

Assume the resonance is narrow and isolated

$$t_\ell^I(s) = \frac{1}{\rho(s)} \frac{\sqrt{s}\Gamma}{m_{\rm BW}^2 - s - i\sqrt{s}\Gamma}$$





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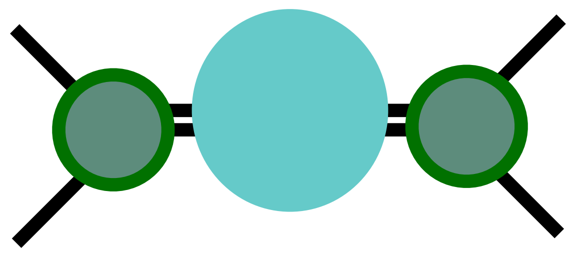

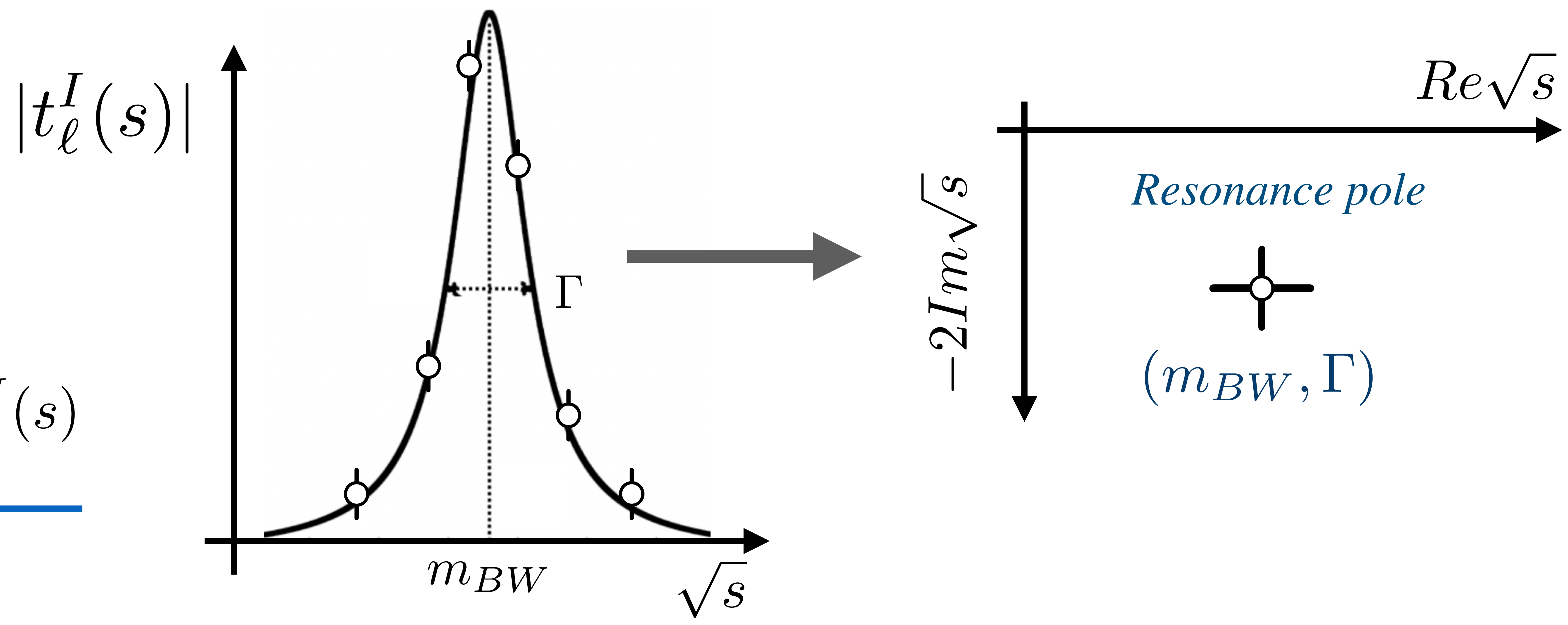

More general form for the amplitude

$$t_\ell^I(s) = \frac{1}{\rho(s)} \frac{K(s)}{1 - i\rho(s)K(s)} = \frac{1}{\rho(s)} e^{i\delta_\ell^I(s)} \sin \delta_\ell^I(s)$$

Elastic case

Spectroscopy in lattice QCD

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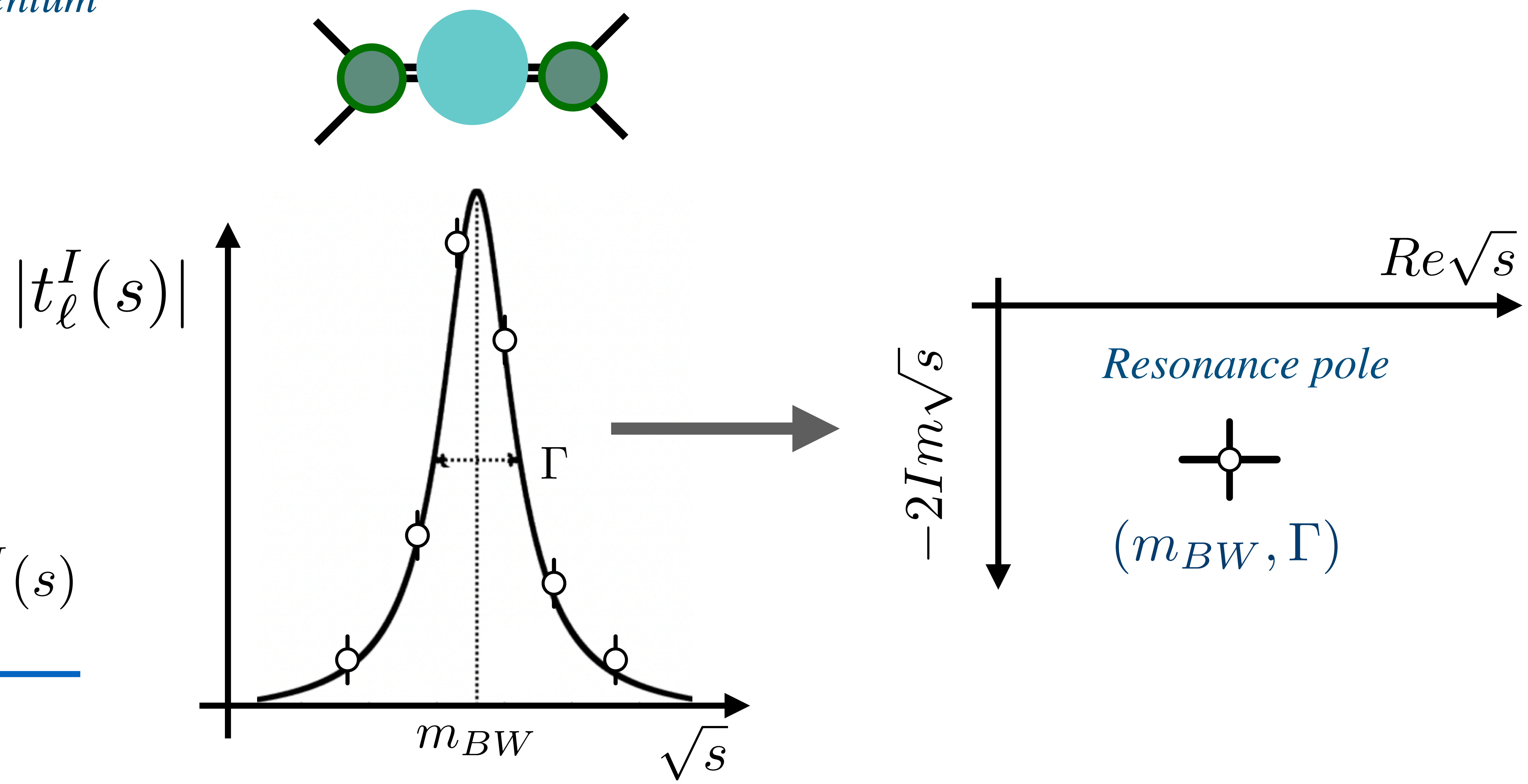

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Elastic case

In lattice QCD, our basic equation is the Lagrangian

$$\mathcal{L}_{\rm QCD} = \sum_f \bar{\psi}_f \left(i \not{D} - m_f\right) \psi_f - \frac{1}{2g_s^2} {\rm Tr}\, G_{\mu\nu}G^{\mu\nu}$$

Quark masses are a parameter for us → m_π is a "choice"

Our basic observables are correlation functions

$$\langle O_f(t) O_i^\dagger(0)\rangle = \frac{1}{Z_T} \int \mathcal{D}[\Phi] {\rm e}^{-S_E[\Phi]} O_f[\Phi] O_i^\dagger[\Phi]$$

Spectroscopy in lattice QCD

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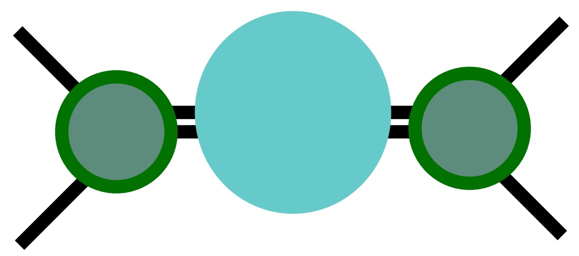

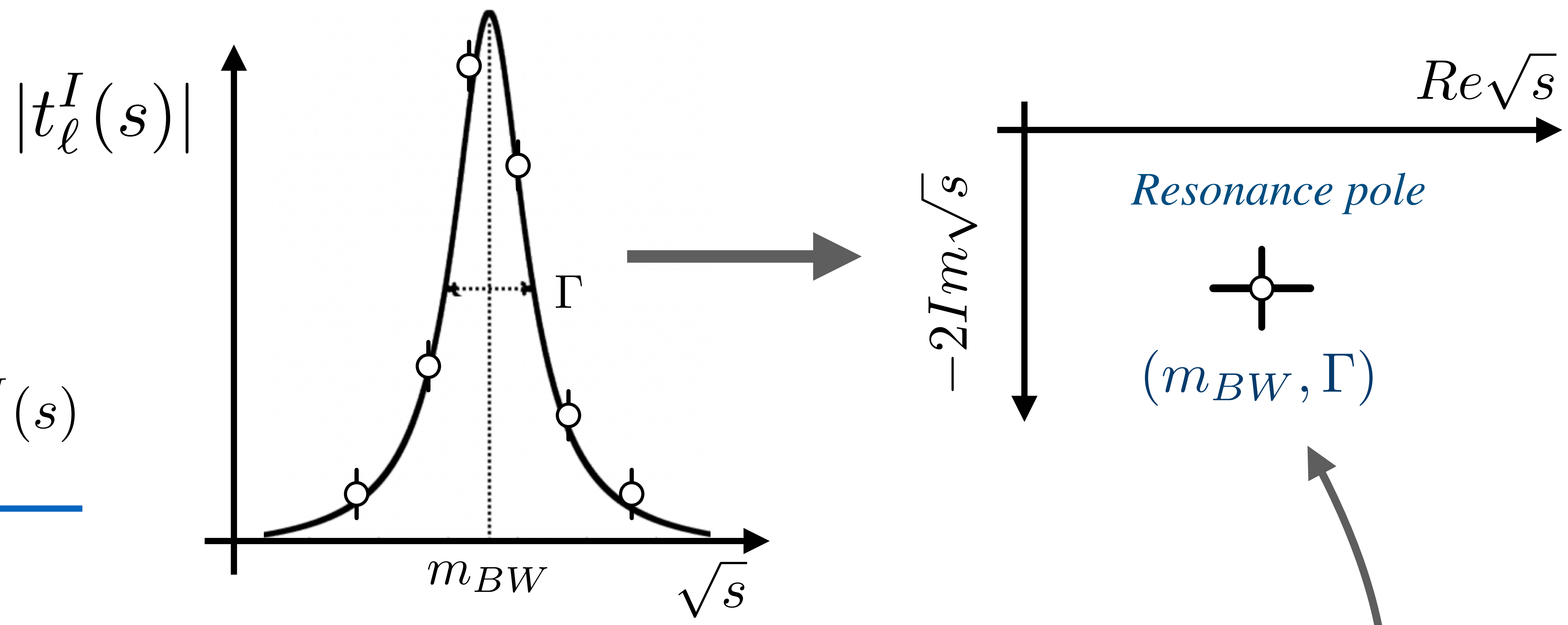

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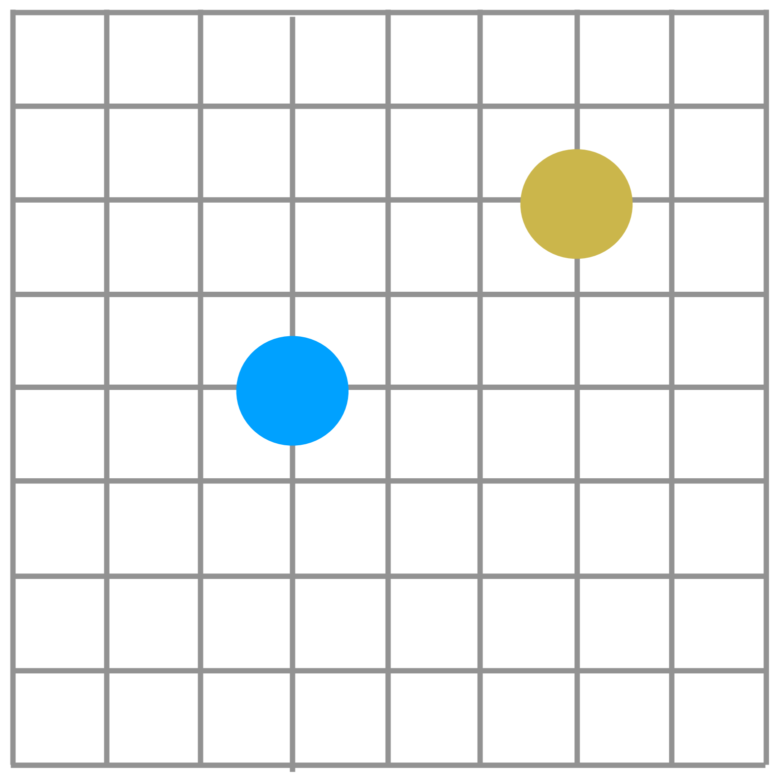
How do we go from here to there??

Spectroscopy in lattice QCD



We start by formulating our theory in a discretized box

$$\int \mathcal{D}[\phi] = \prod_x \int d\phi_x$$

Imagine our quark living on the sites

We perform a time rotation $\quad it \to t \qquad iS \to S_E$

$$\int \mathcal{D}[\phi] e^{-iS[\phi]} = \prod_x \int d\phi_x e^{-S_E[\phi_x]}$$

$$0 < e^{-S_E[\phi_x]} < 1$$

Probability-like function

Numerical, Montecarlo sampling of our gluon fields

$$\langle O \rangle = \lim_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} O\left[U_n\right]$$

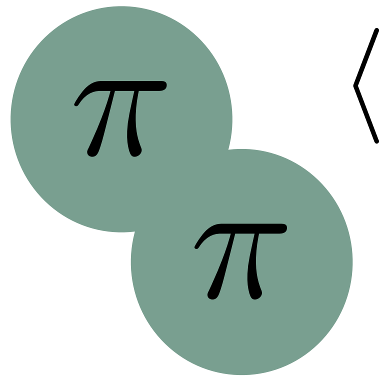
N is the number of samples

$$\langle O_f(t) O_i^\dagger(0) \rangle$$

Our observables come with a central value and error associated to the number of samples ("measurements")

Spectroscopy in lattice QCD

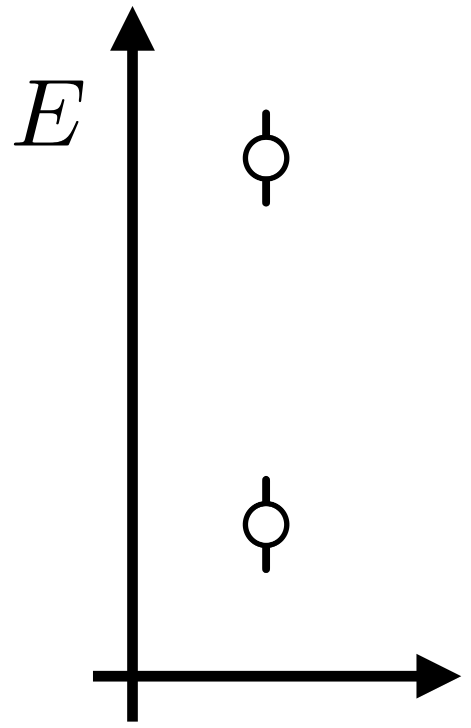
Quantum mechanical time evolution



$$\langle O_f(t) O_i^\dagger(0)\rangle \sim \sum_{\mathrm{n}} e^{-E_{\mathrm{n}} t} \langle 0 \,| O_{\mathrm{f}}(0) |\, \mathrm{n}\rangle \langle \mathrm{n} | O_{\mathrm{i}}^\dagger(0) | 0\rangle$$

Time is imaginary

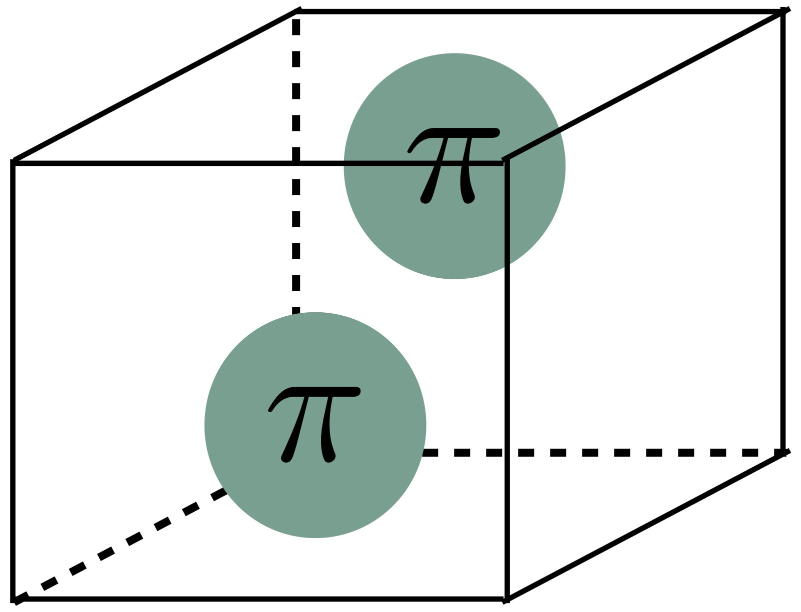
Extract E_n



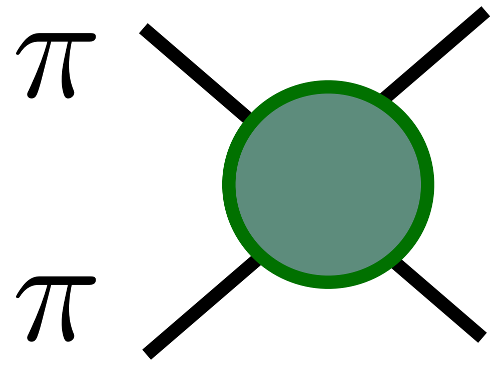
We determine the strength of the reaction from the difference between non-interacting and interacting energies

Attraction reduces energies, repulsion increases it

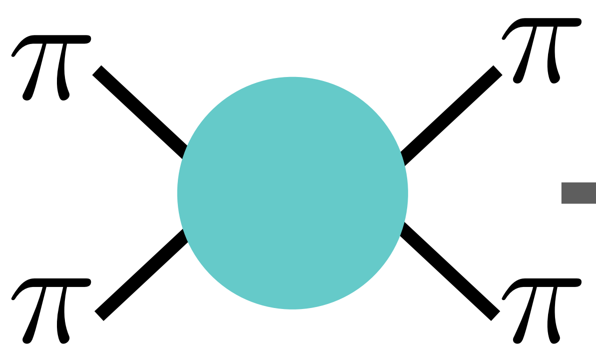
Lüscher, Nucl. Phys. B 354 (1991)

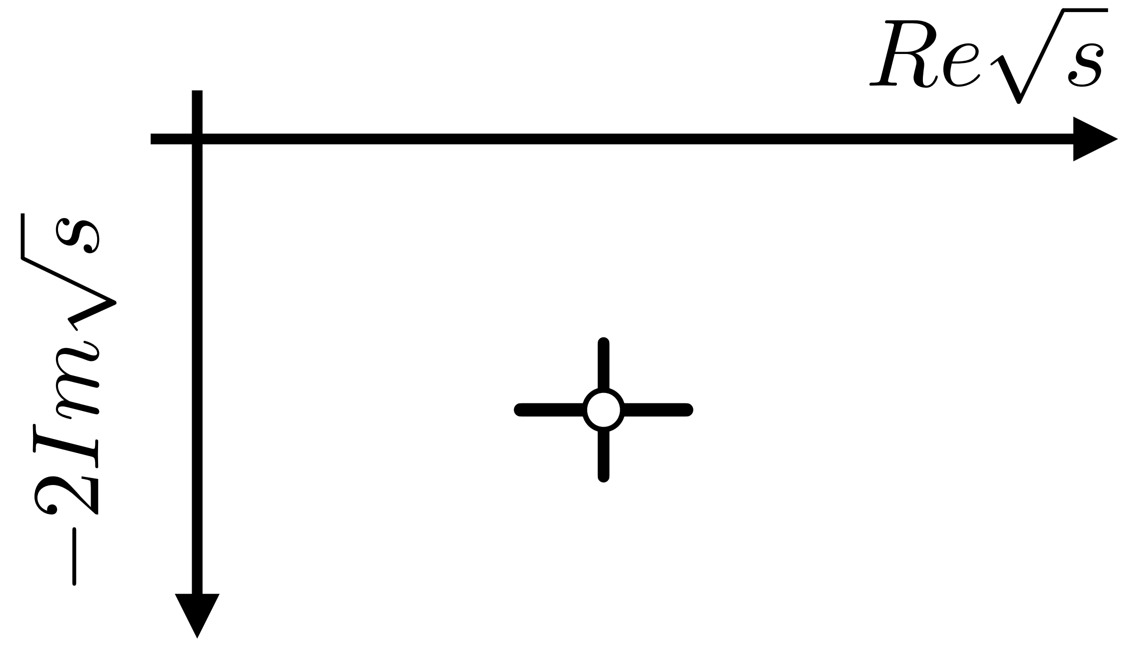


Short-distance dynamics



Full scattering amplitude





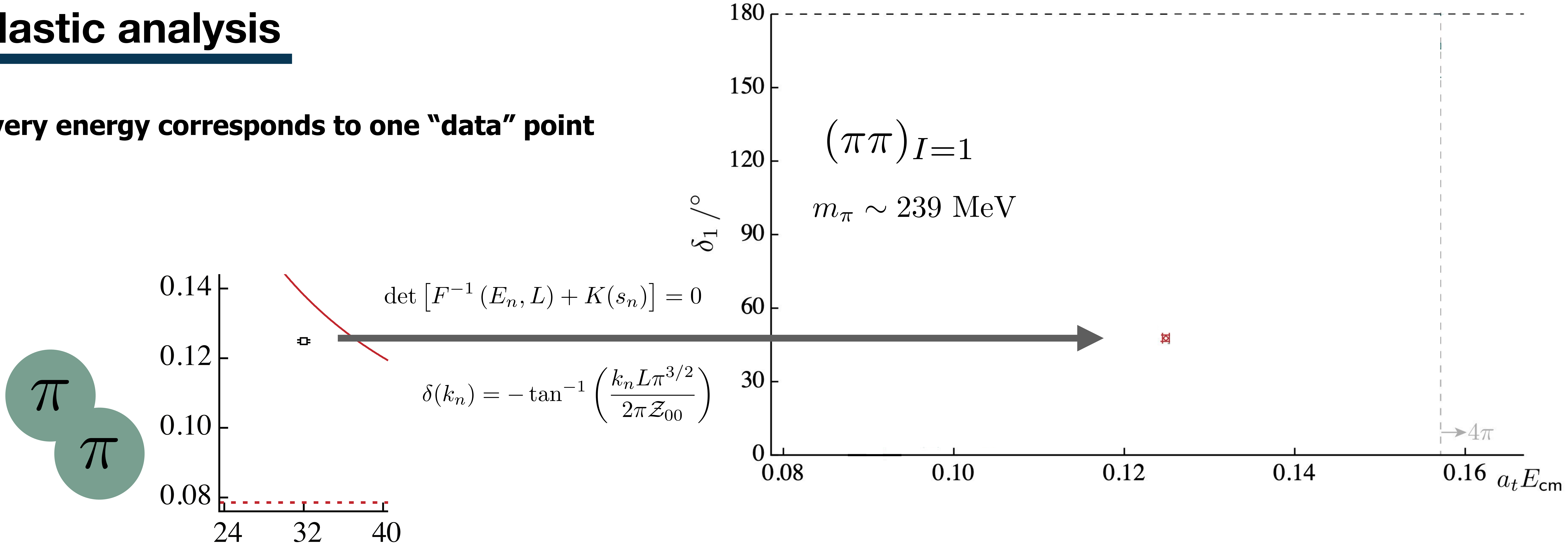
General $\det\left[F^{-1}(E_n, L) + K(s_n)\right] = 0$ $\qquad t_\ell^I(s) = \frac{1}{\rho(s)} \frac{K(s)}{1 - i\rho(s)K(s)}$

Elastic $\delta(k_n) = -\tan^{-1}\left(\frac{k_n L \pi^{3/2}}{2\pi \mathcal{Z}_{00}}\right)$

Known kinematic functions

Elastic analysis

Every energy corresponds to one "data" point



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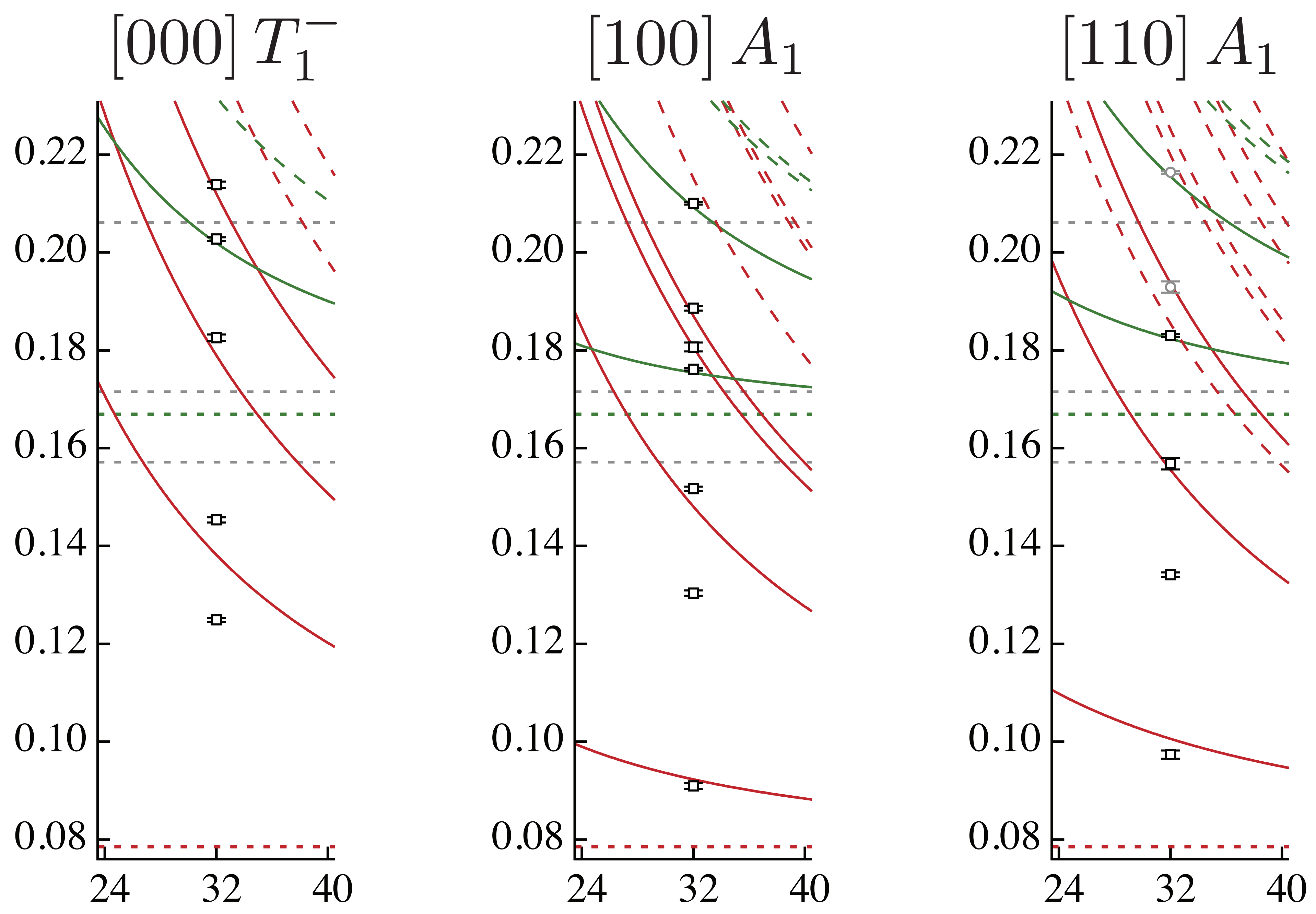
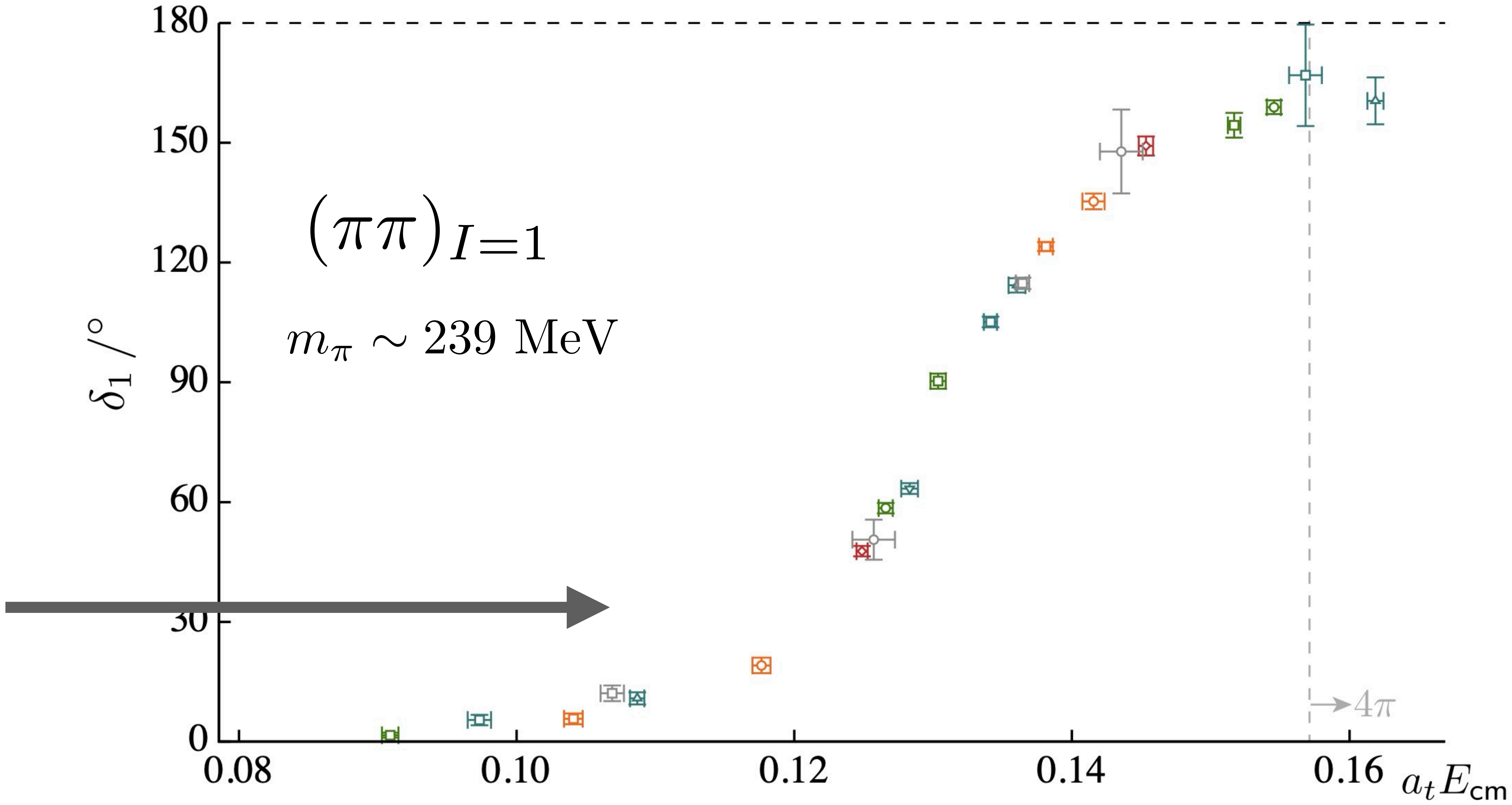




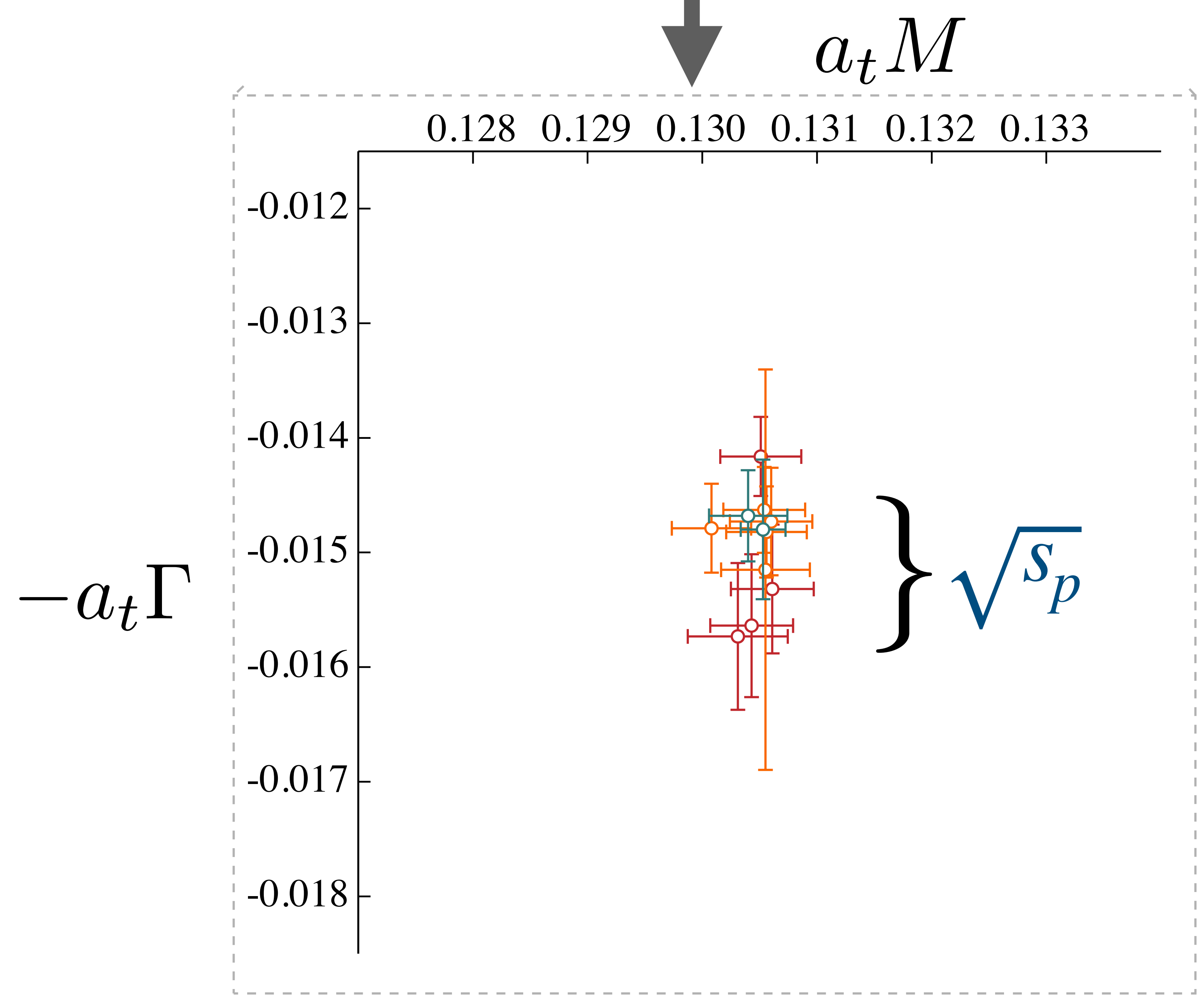
Elastic analysis

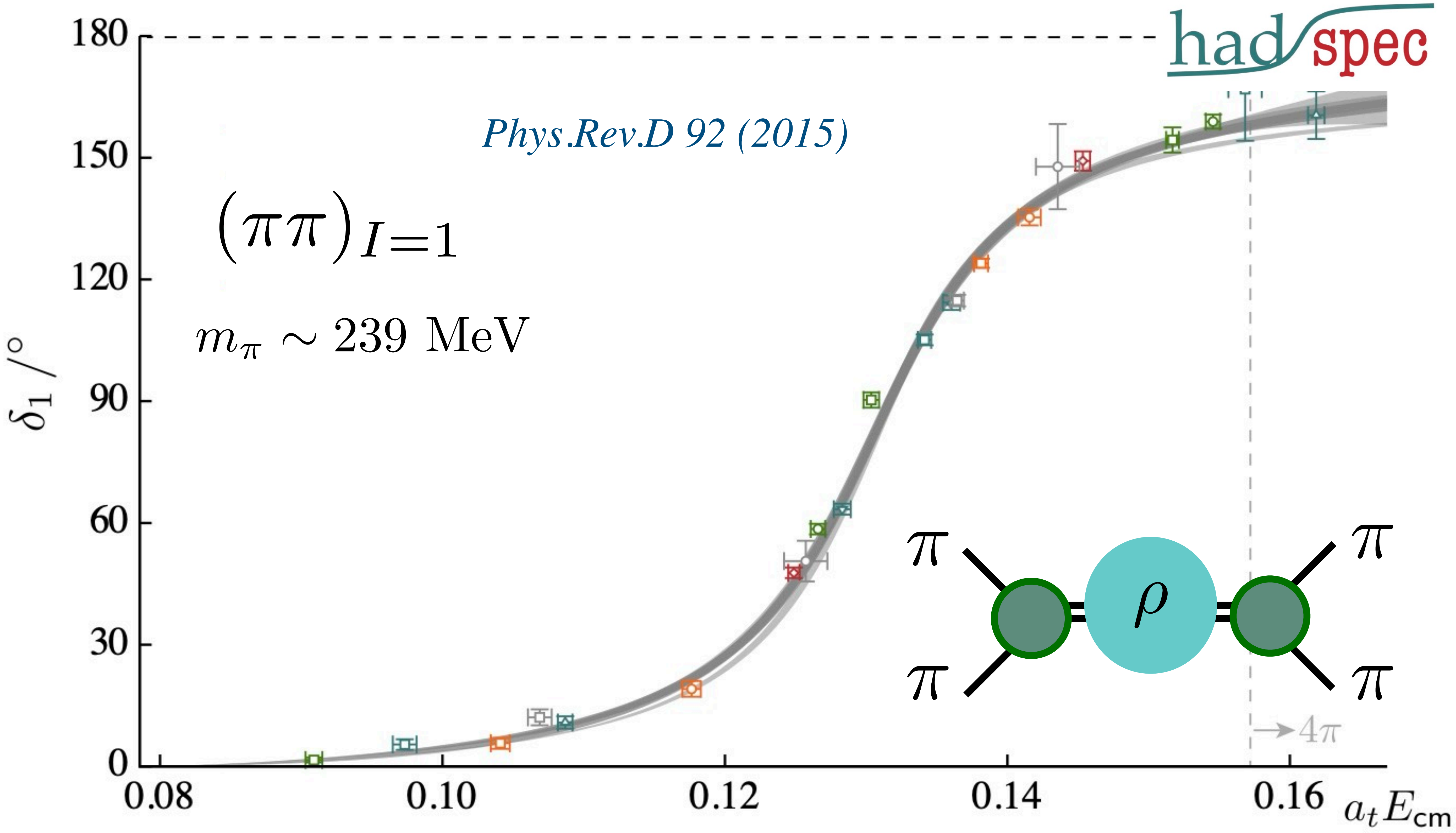
This amplitude can be easily fitted

$$t_\ell^I(s) = \frac{1}{\rho(s)} \frac{\sqrt{s}\Gamma}{m_{\rm BW}^2 - s - i\sqrt{s}\Gamma}$$

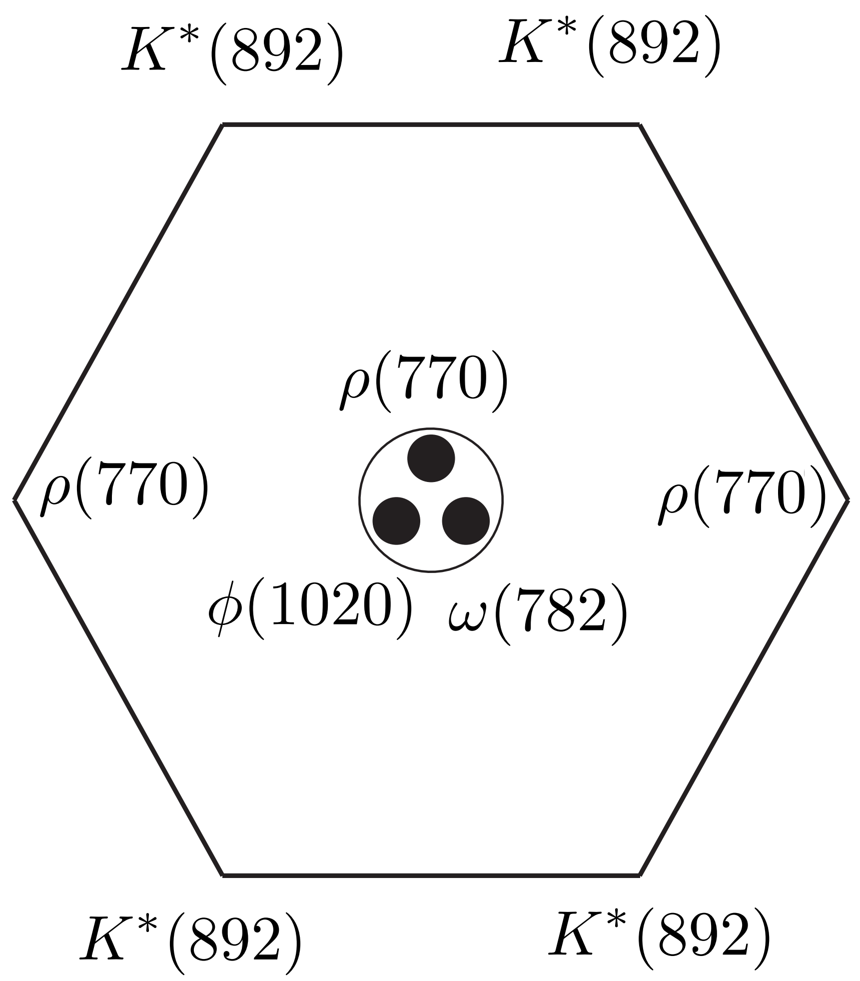
Pole at $\sqrt{s_p} \sim (m_{BW} - i\Gamma/2)$

We can fit other parameterizations



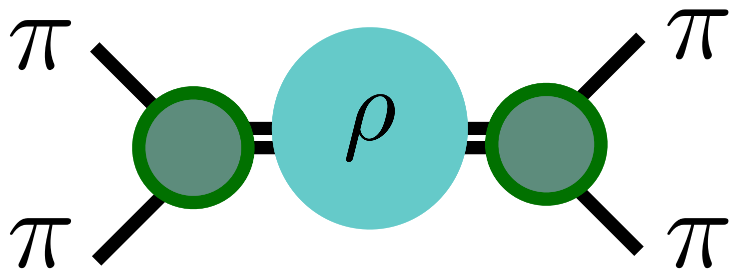


The ρ is an ordinary $q\bar{q}$, narrow, isolated resonance



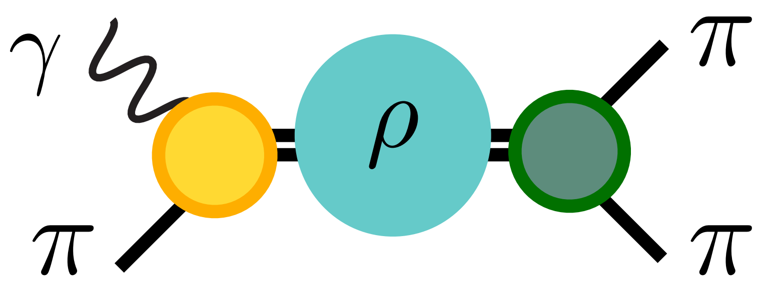
Form factors

Our partial-wave amplitude is defined by



$$\langle \pi\pi | T | \pi\pi \rangle_{I,\ell=1} \propto t_1^1(s)$$

Photoproduction process given by

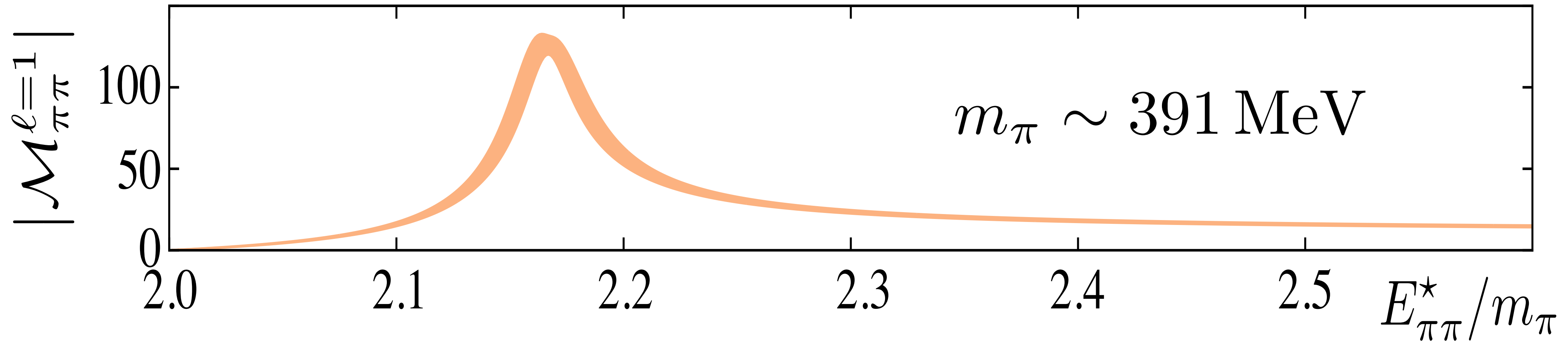


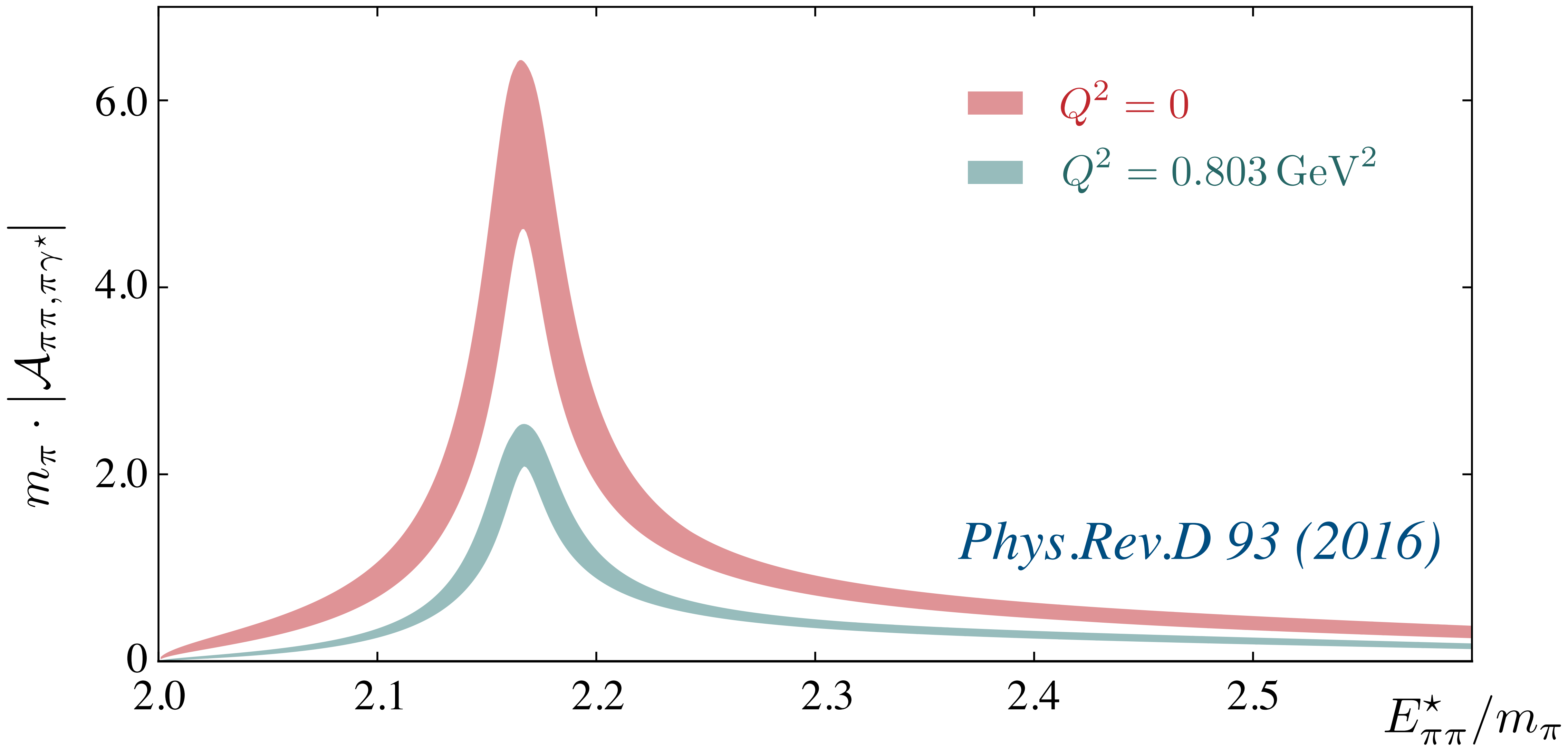
$$\langle \pi \,|J_\gamma|\, \pi\pi \rangle_{I,\ell=1} \propto f\left(Q^2, s\right) t_1^1(s)$$

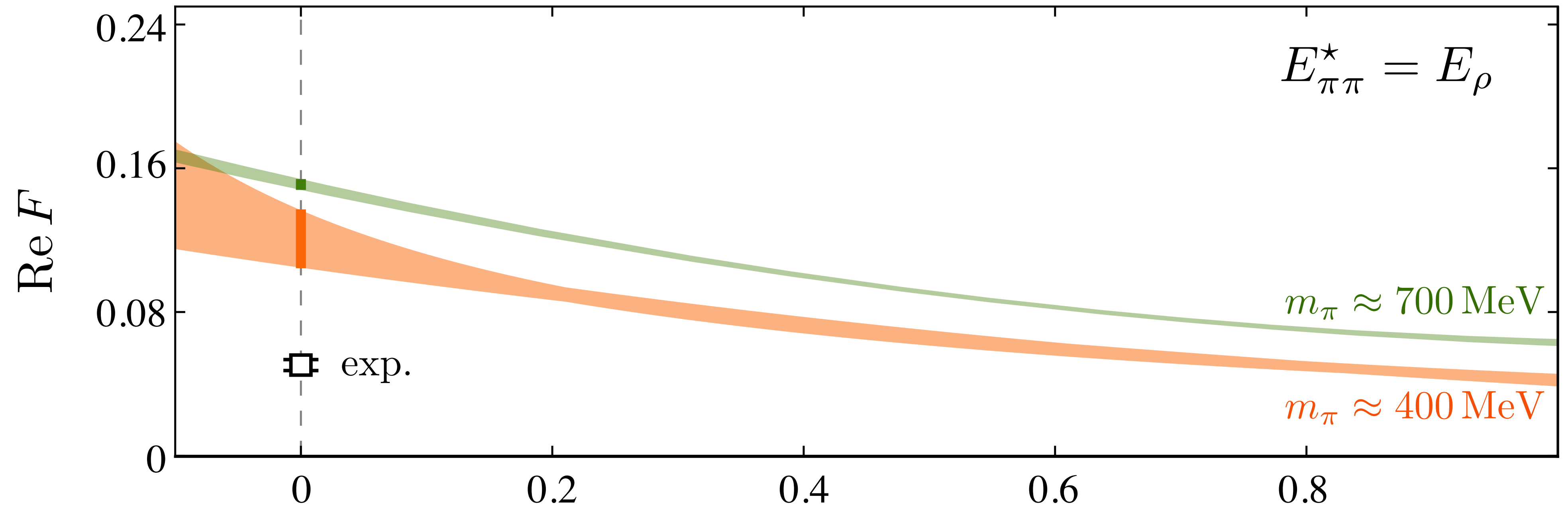
Smooth function

When continuing to the pole location, we recover the transition form factor of the resonance

$$f(Q^2, s_p) \propto f_R(Q^2)$$







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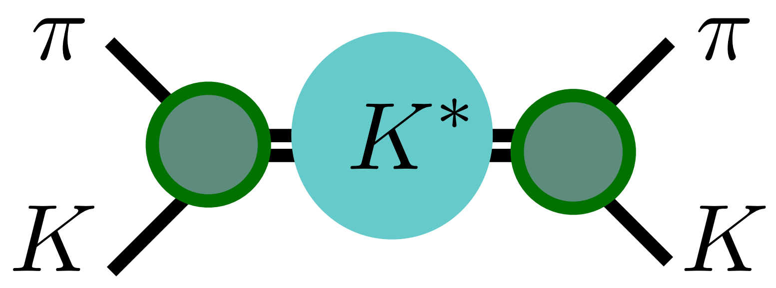
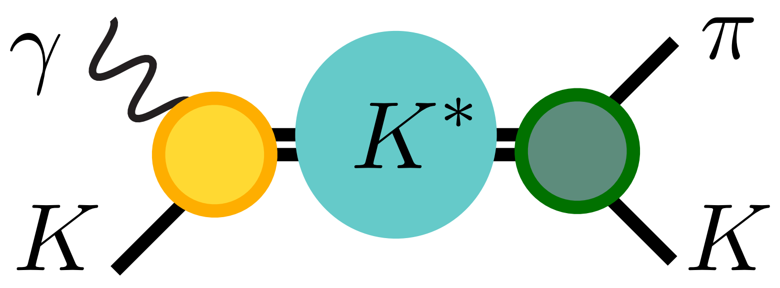


$$\langle \pi K | T | \pi K \rangle_{I=1/2,\ell=1} \propto t_1^{1/2}(s)$$

Photoproduction process given by



$$\langle K \, | J_\gamma | \, \pi K \rangle_{I=1/2,\ell=1} \propto f\left(Q^2, s\right) t_1^{1/2}(s)$$

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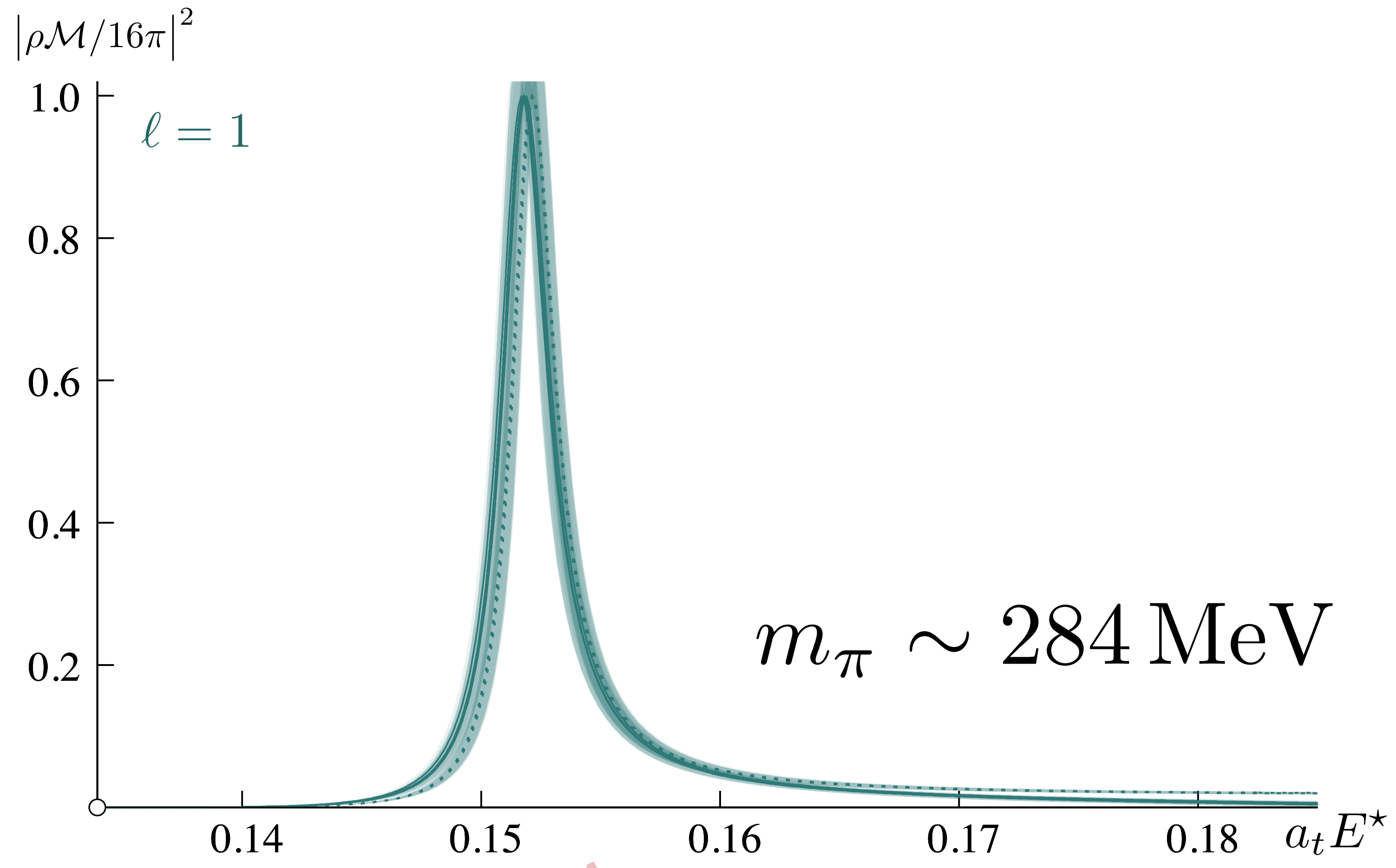
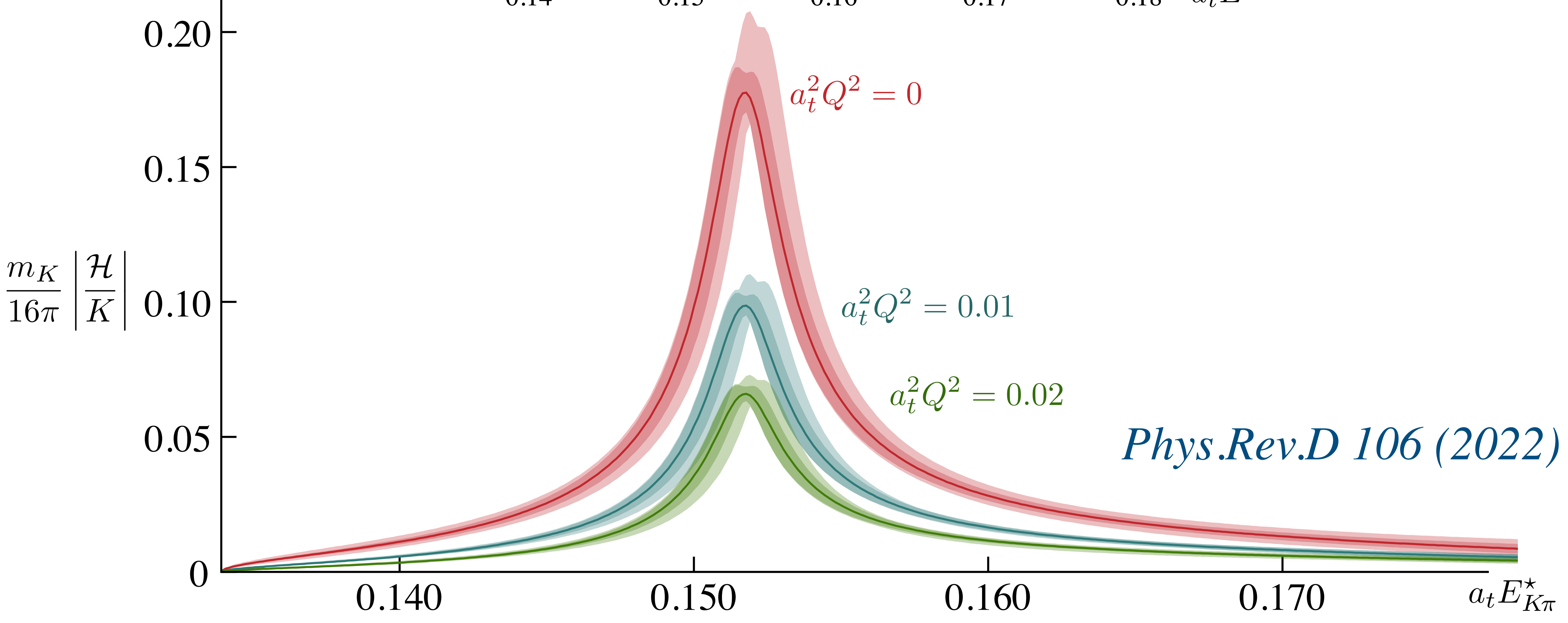
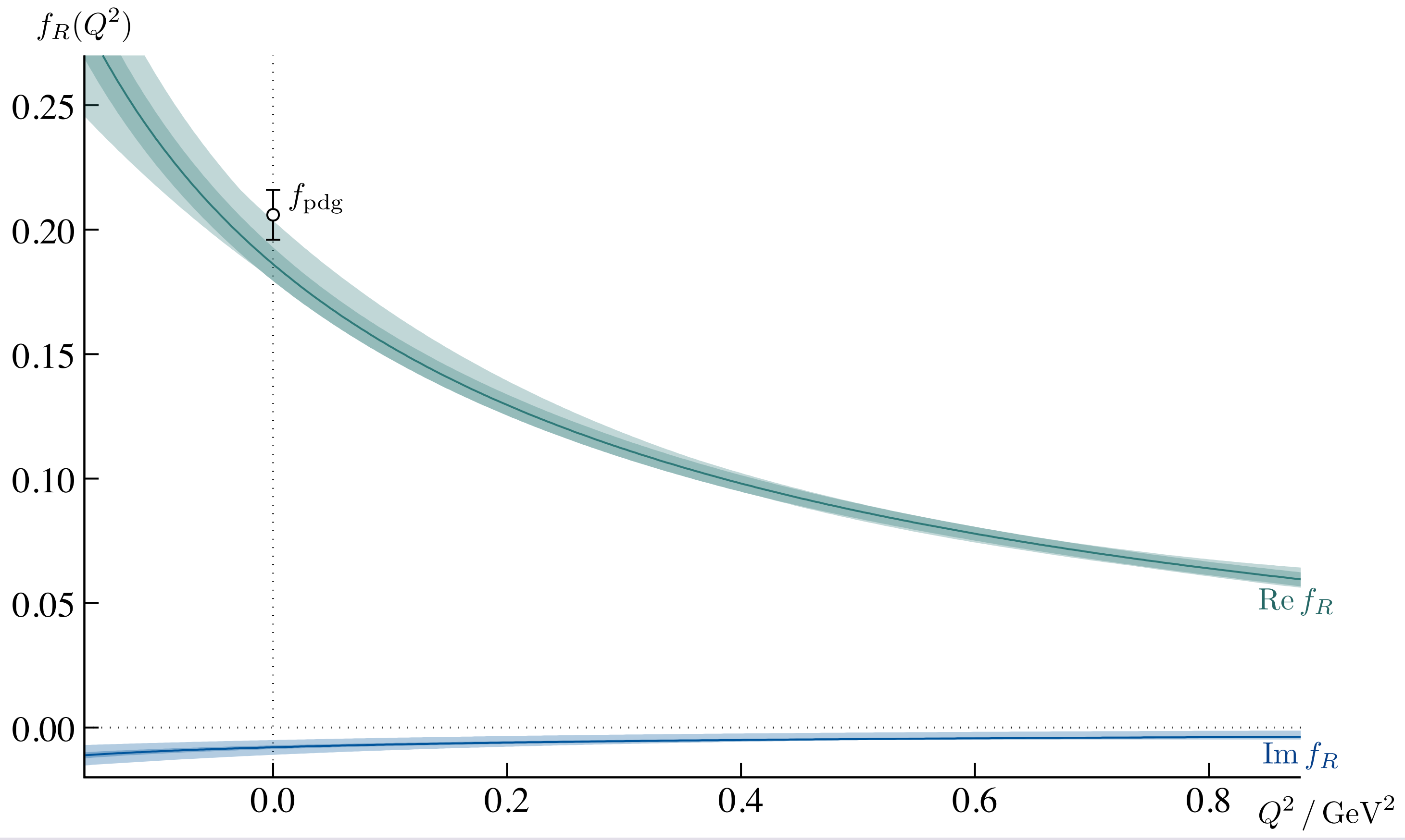
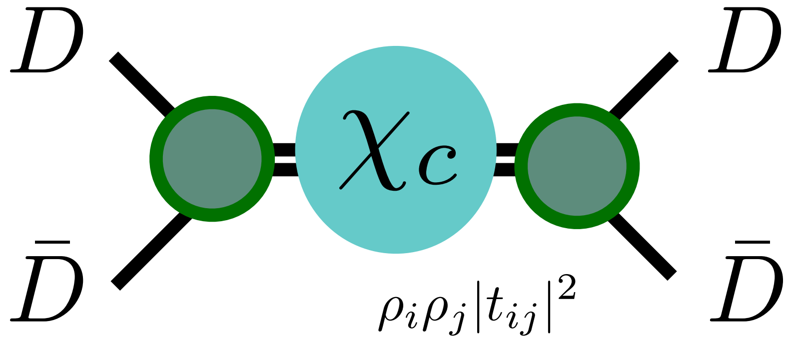






Coupled channels



$m_\pi \sim 391\,\text{MeV}$

For heavier m_π, the χ_{c0} and χ_{c2} can be studied as a 2-body coupled channel scattering process

$$\det\left[F^{-1}(E_n, L) + K(s_n)\right] = 0$$

$N \times N$ matrix (N=number of decay channels)

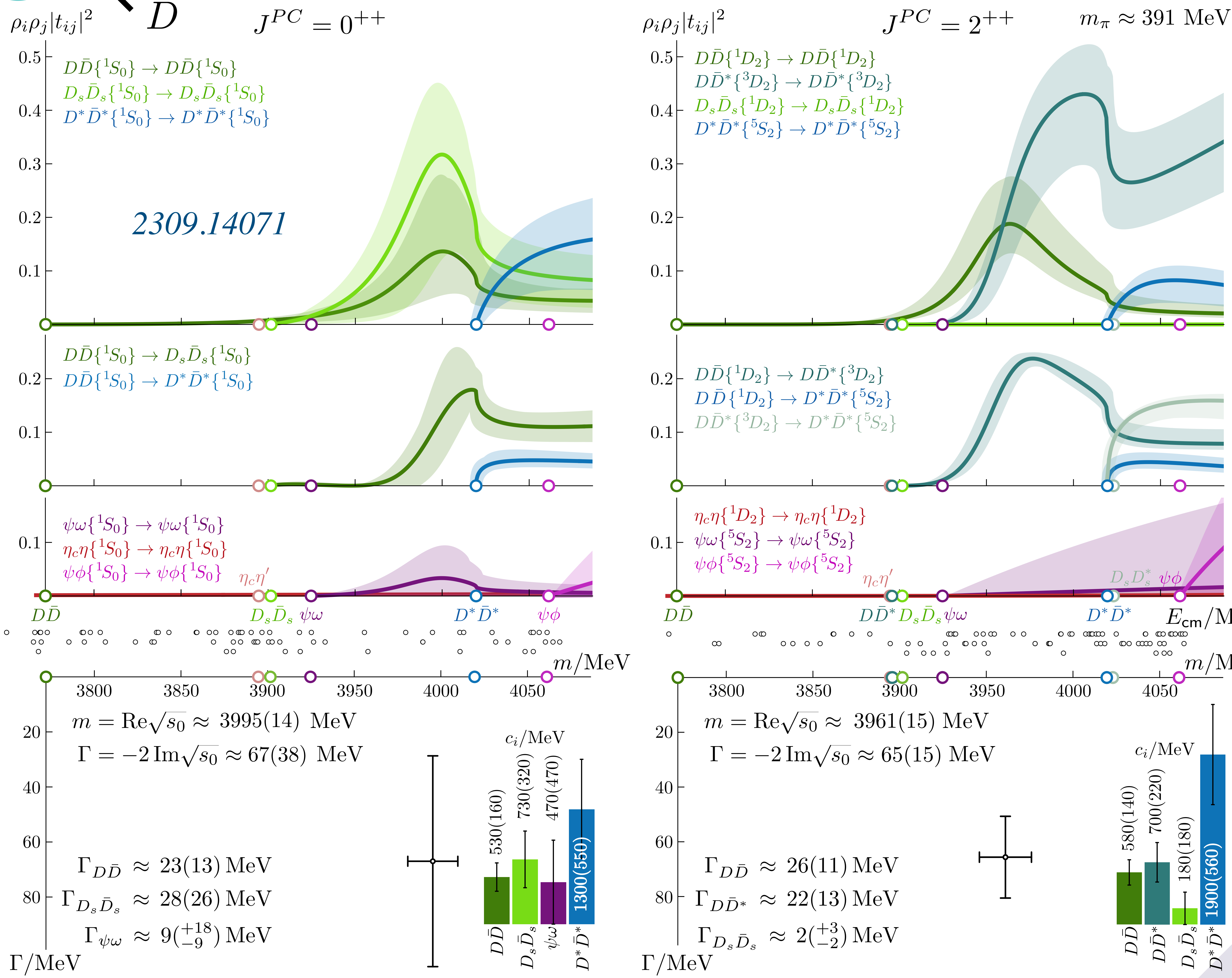
For given angular momenta

As usual with XYZ resonances, both theory and experimental results are not so clear

χ_{c0} not seen in some expected final states by LHCb

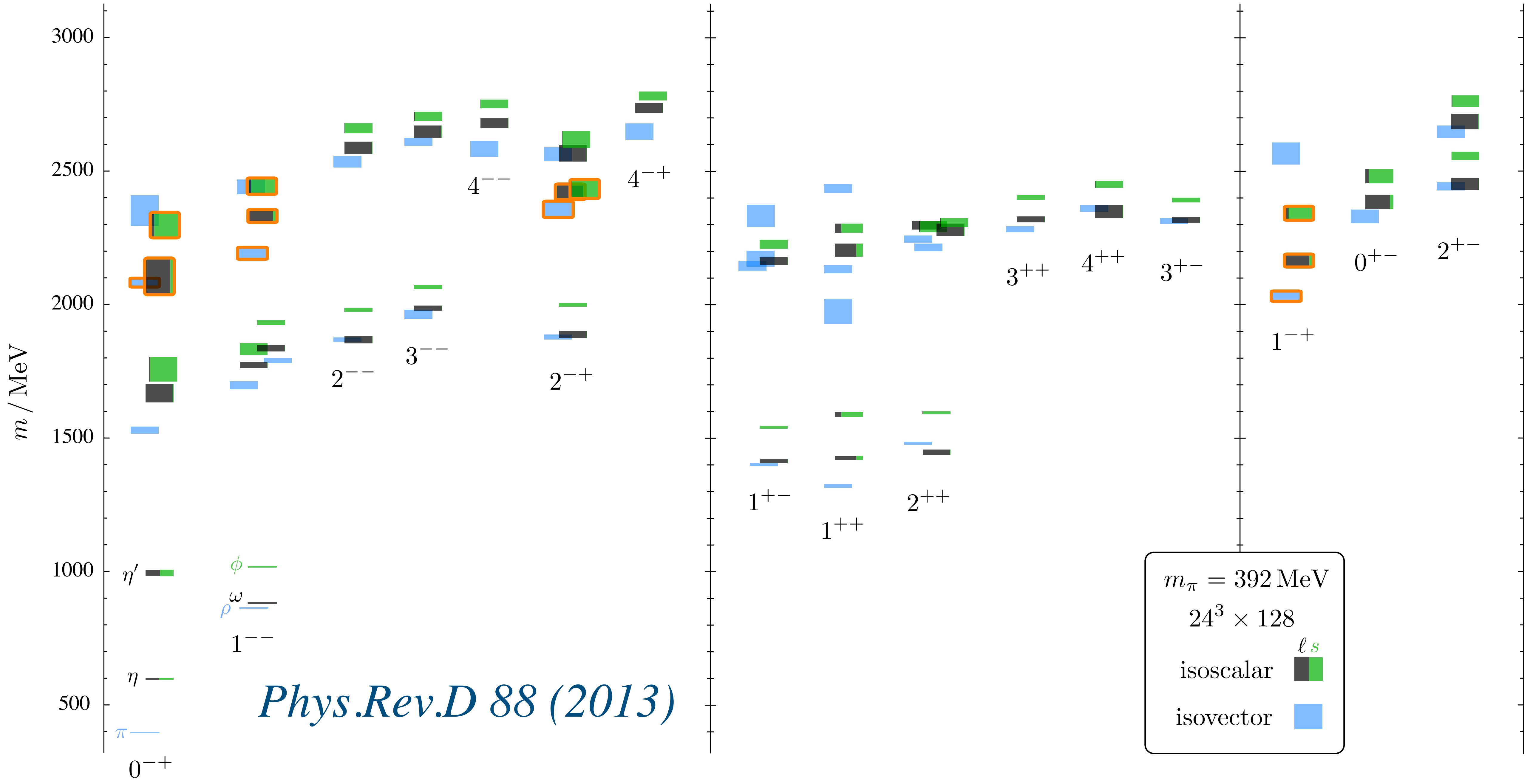
HadSpec finds both resonances coupling to open-charm channels

HadSpec does not find S-wave states bound or near the $D\bar{D}$ threshold



Exotic mesons: The hybrid

Lattice QCD (and models) predicts a lightest $J^{PC} = 1^{-+}$, isolated hybrid



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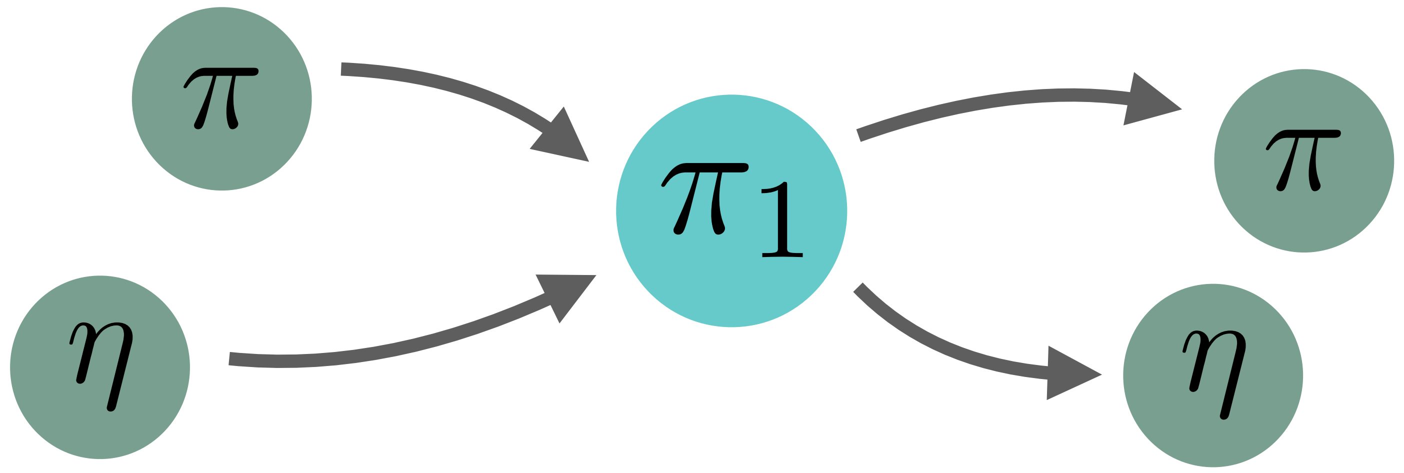
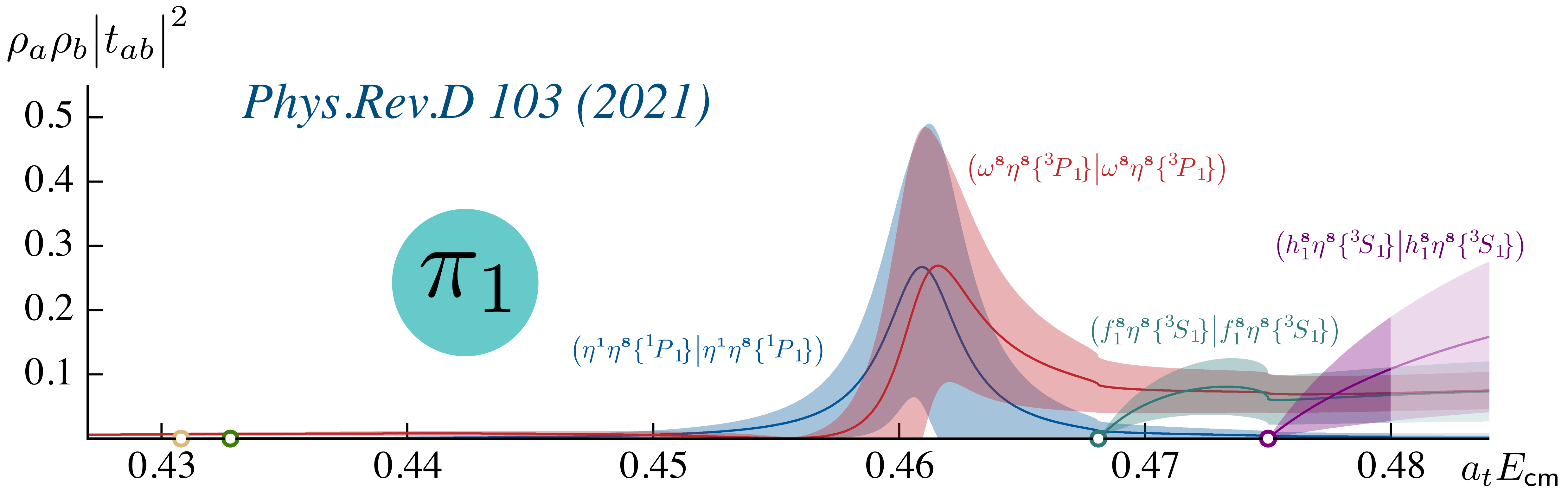
Not well known from experiment !!

A single pole was mistaken by 2 states

Coupled-channel analyses were required to resolve this issue

Most analyses are based on simple mesonic decays

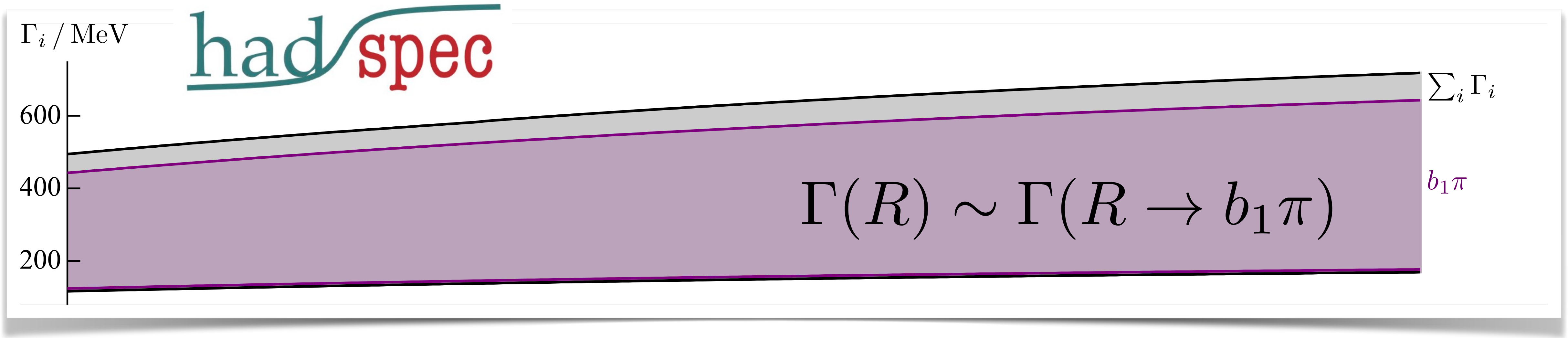




HadSpec extracted it using 8 different decay channels (at $m_\pi \sim 700$ MeV)

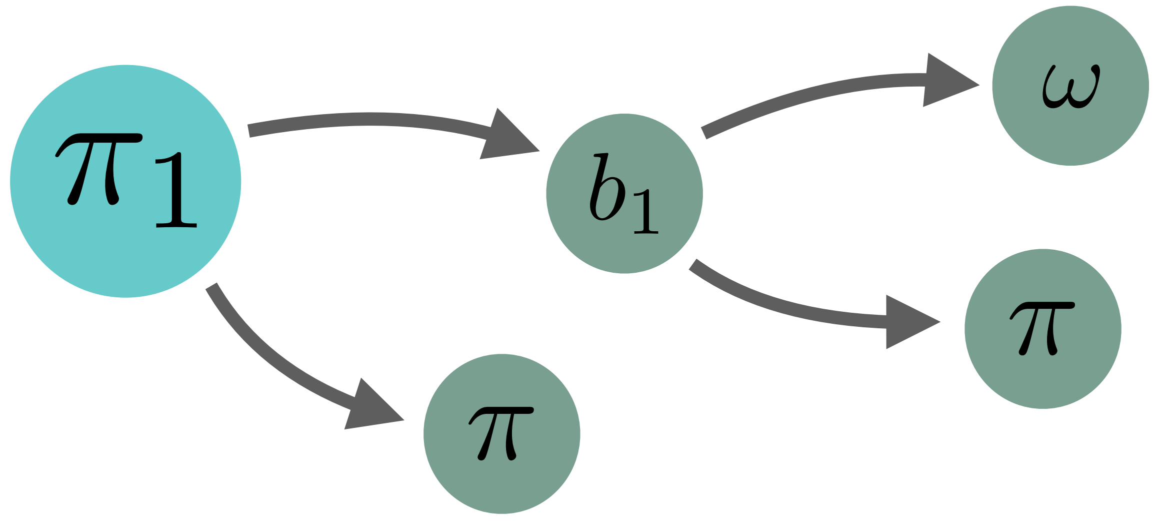
Exotic mesons: The hybrid

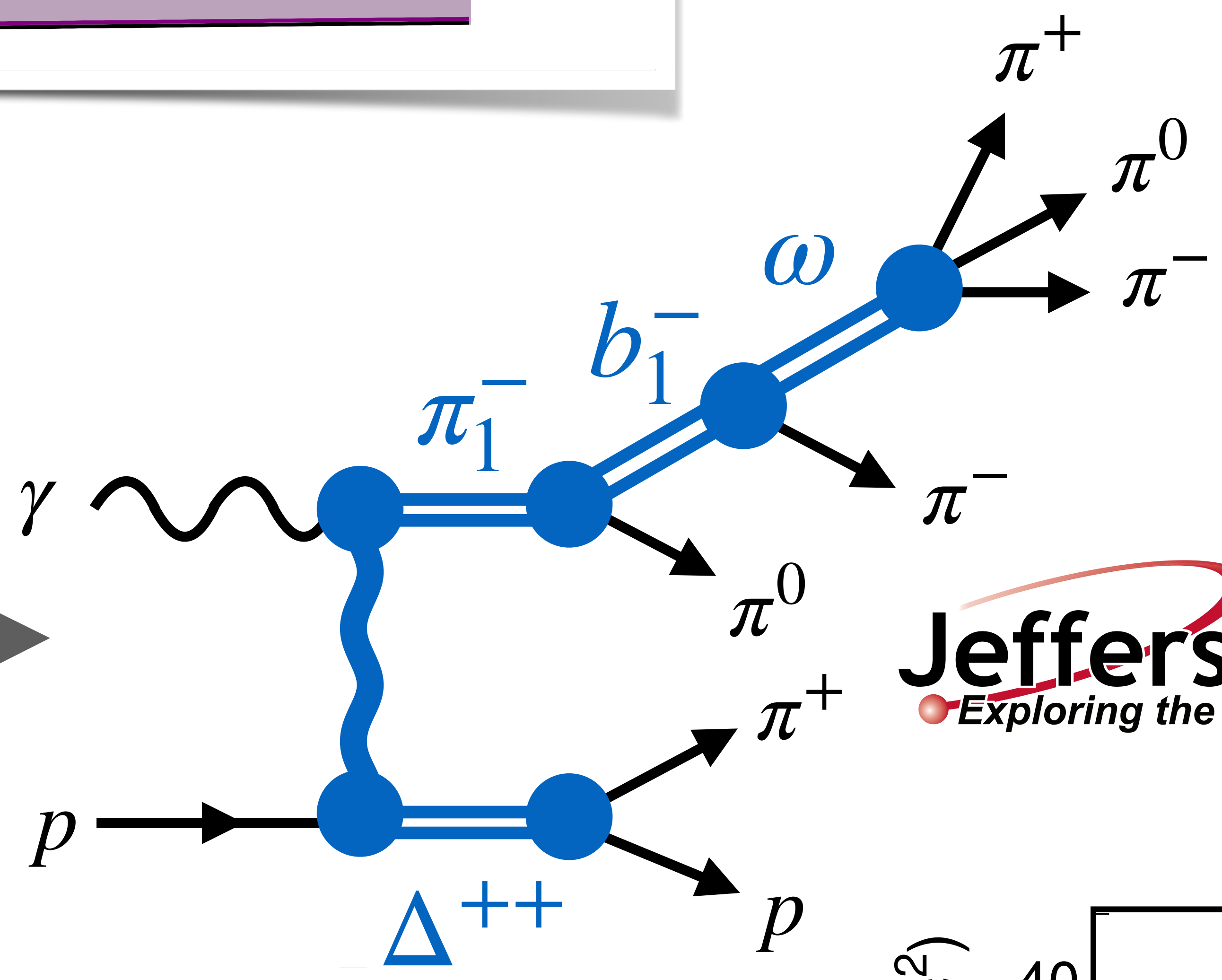
Out of 8 possible decay modes, Lattice QCD predicts a dominant one



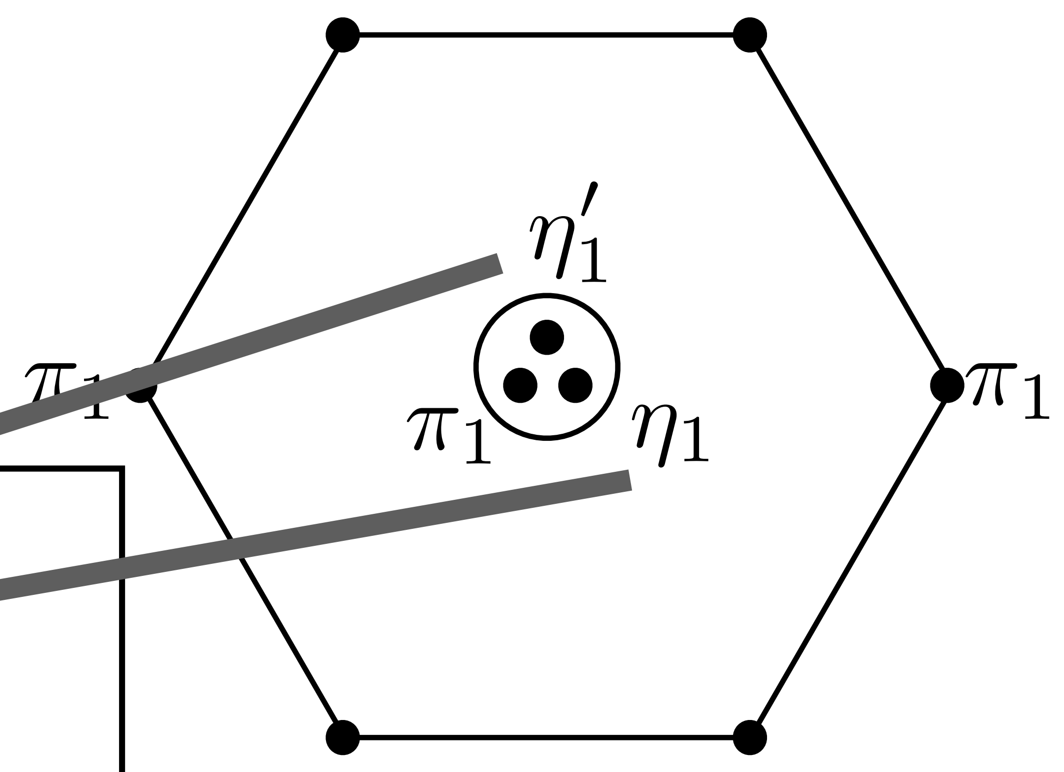
	thr./MeV	$\lvert c_i^{\text{phys}}\rvert$/MeV	Γ_i/MeV
$\eta\pi$	688	$0 \to 43$	$0 \to 1$
$\rho\pi$	910	$0 \to 203$	$0 \to 20$
$\eta'\pi$	1098	$0 \to 173$	$0 \to 12$
$b_1\pi$	1375	$799 \to 1559$	$139 \to 529$
K^*K	1386	$0 \to 87$	$0 \to 2$
$f_1(1285)\pi$	1425	$0 \to 363$	$0 \to 24$
$\rho\omega\{^1P_1\}$	1552	$\lesssim 19$	$\lesssim 0.03$
$\rho\omega\{^3P_1\}$	1552	$\lesssim 32$	$\lesssim 0.09$
$\rho\omega\{^5P_1\}$	1552	$\lesssim 19$	$\lesssim 0.03$
$f_1(1420)\pi$	1560	$0 \to 245$	$0 \to 2$
			$\Gamma = \sum_i \Gamma_i = 139 \to 590$

We know most productive decay chain

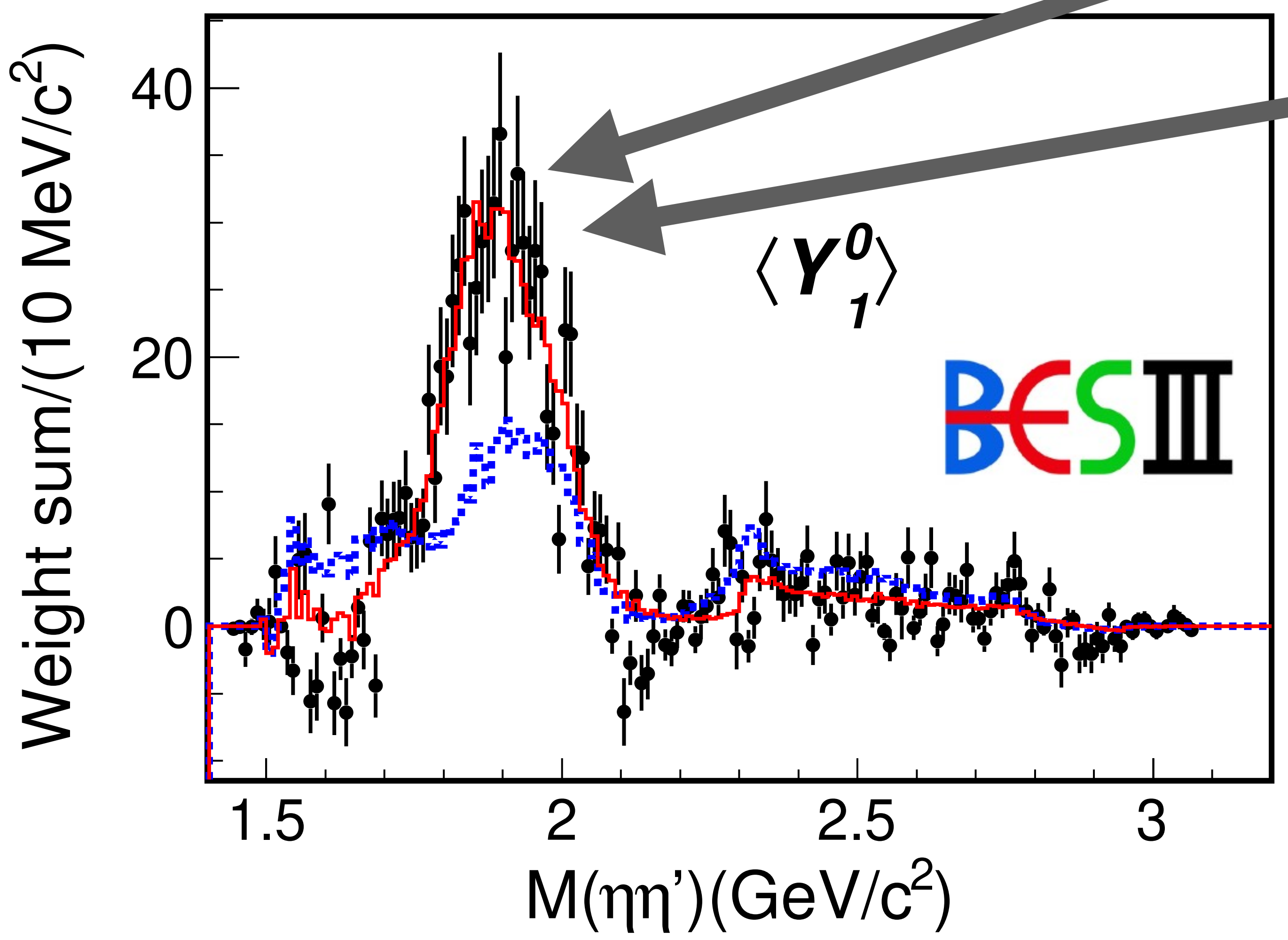




Jefferson Lab
Exploring the Nature of Matter



Octet partner found by BESIII !!



Not-so-expected discovery (there should be two!!!)

Opportunity to capitalize on previously used lattices for extraction!

$m_\pi \sim 700\,\text{MeV}$

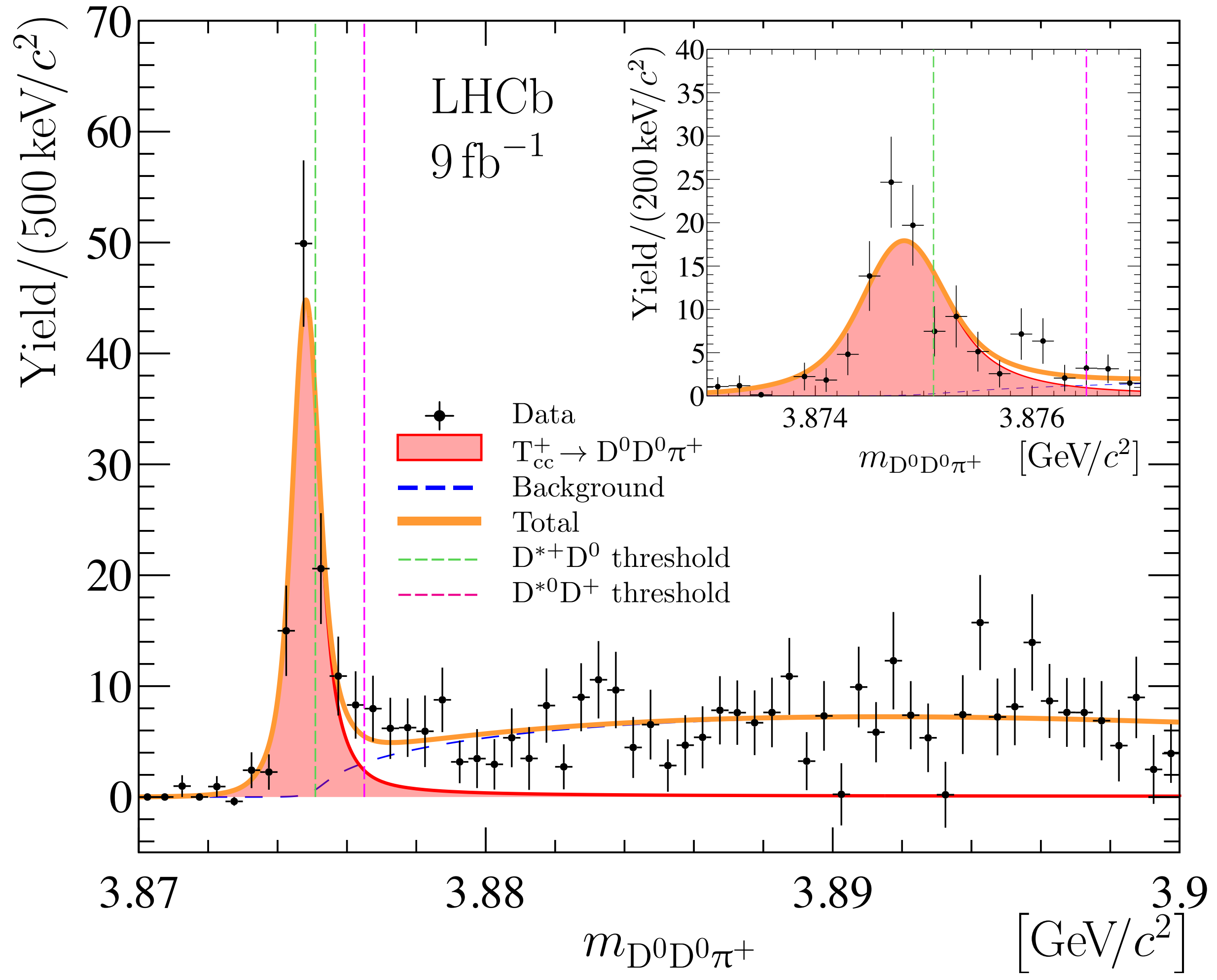
Exotic mesons: T_{cc}

Exotic resonance, first observed at LHCb

Considered as a doubly-charmed tetraquark

Narrow resonance in DD^*

D^* decays, so it is actually a 3-body process

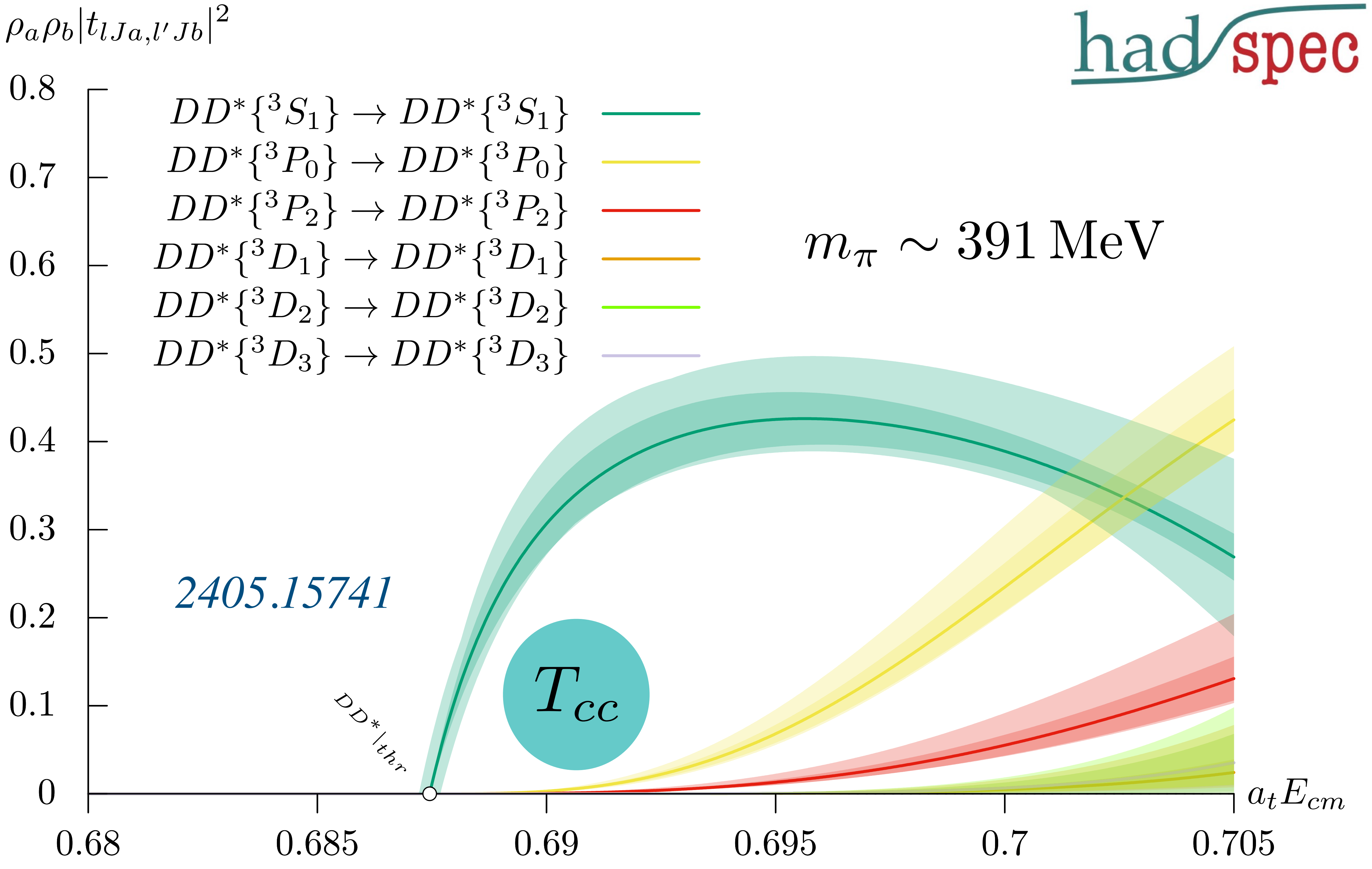


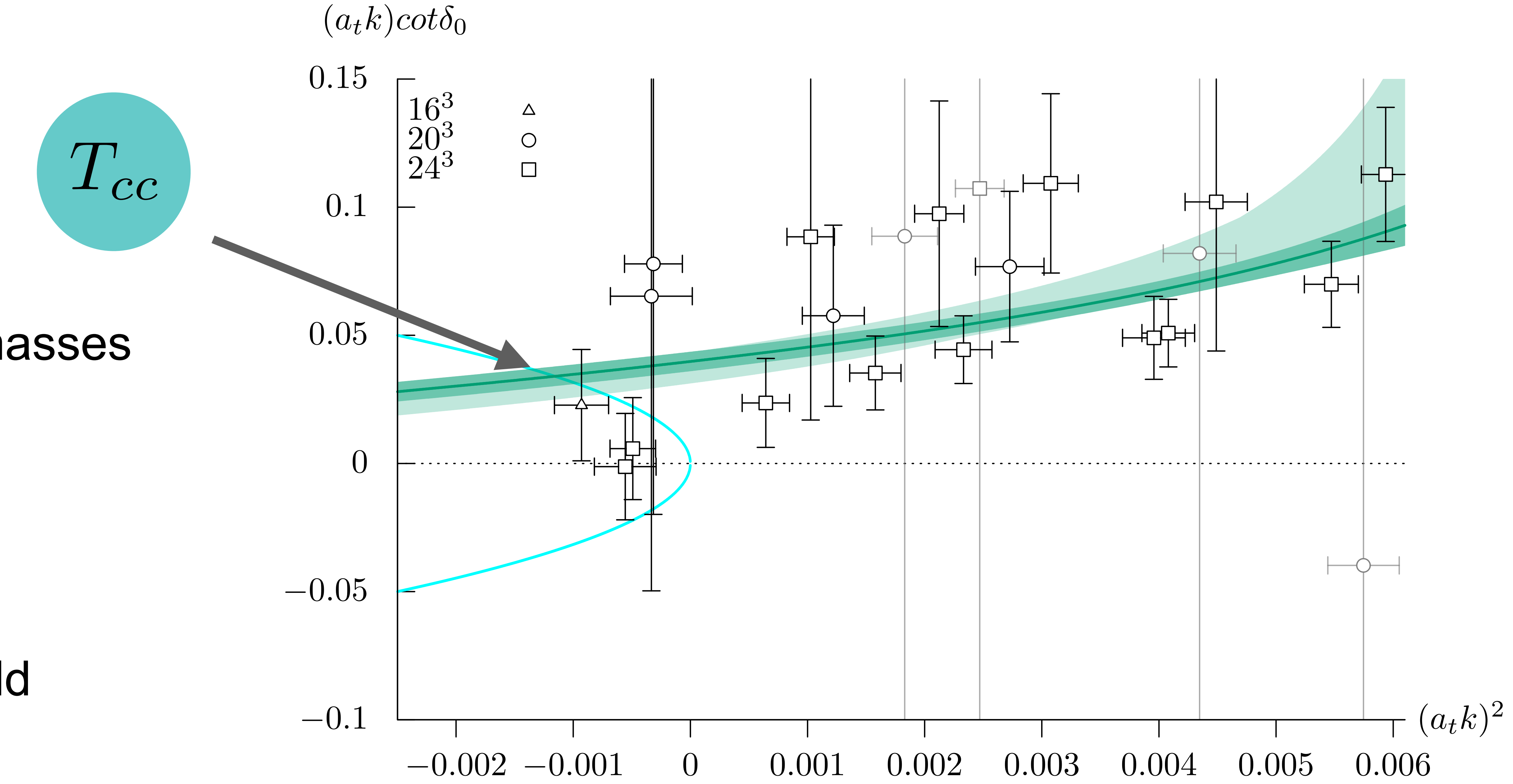
We can study it as a true 2-body process for heavier pion masses

D^* stable for $m_\pi \sim 391$ MeV

All amplitudes produce a virtual bound state pole

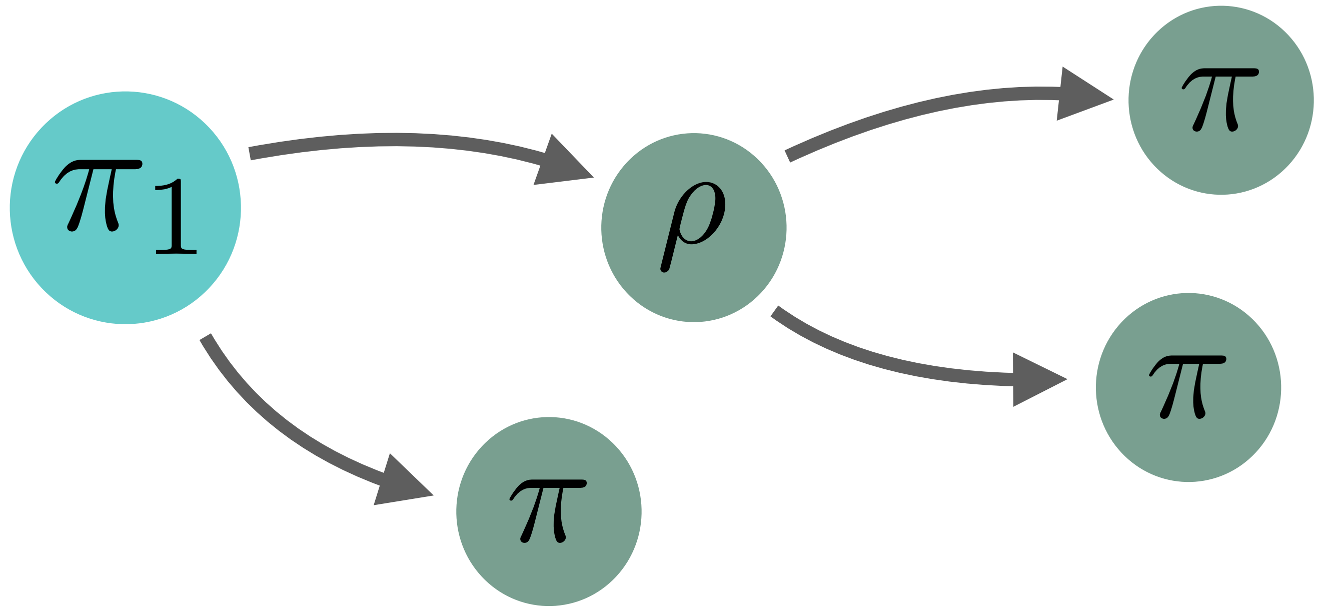
A second partner, T'_{cc} is also found near the D^*D^* threshold





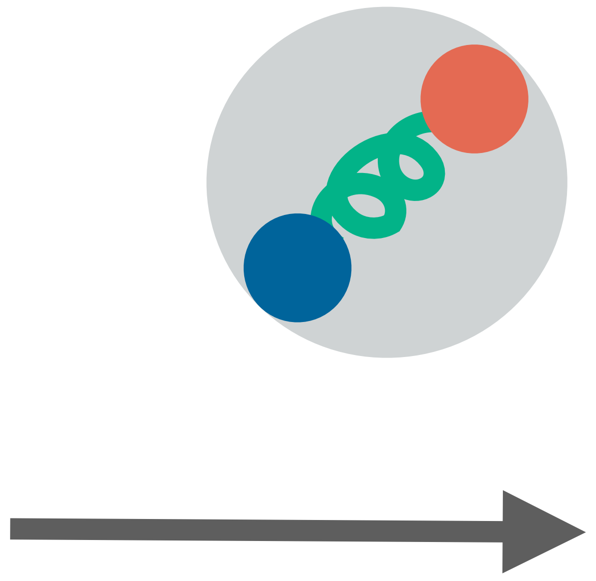
Multi-body decays

When lowering m_π → more phase space→ massive particles start decaying to lighter ones



Going beyond two-body effects is crucial to understanding exotic candidates of matter

$\pi_1(1600)$

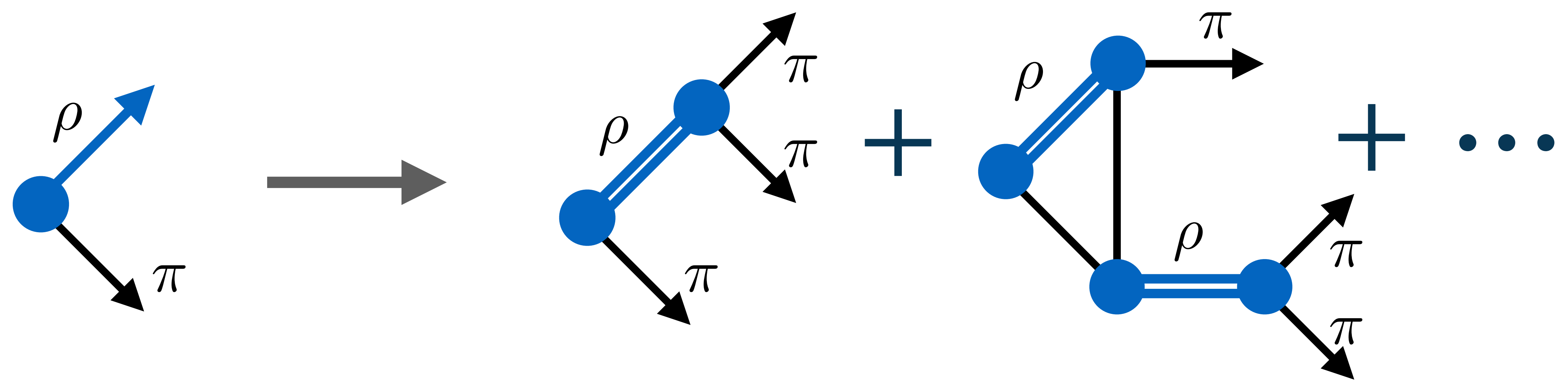


	thr./MeV	$\lvert c_i^{\text{phys}}\rvert$/MeV	Γ_i/MeV
$\eta\pi$	688	$0 \to 43$	$0 \to 1$
$\rho\pi$	910	$0 \to 203$	$0 \to 20$
$\eta'\pi$	1098	$0 \to 173$	$0 \to 12$
$b_1\pi$	1375	$799 \to 1559$	$139 \to 529$
$K^*\overline{K}$	1386	$0 \to 87$	$0 \to 2$
$f_1(1285)\pi$	1425	$0 \to 363$	$0 \to 24$
$\rho\omega\{^1P_1\}$	1552	$\lesssim 19$	$\lesssim 0.03$
$\rho\omega\{^3P_1\}$	1552	$\lesssim 32$	$\lesssim 0.09$
$\rho\omega\{^5P_1\}$	1552	$\lesssim 19$	$\lesssim 0.03$
$f_1(1420)\pi$	1560	$0 \to 245$	$0 \to 2$
		$\Gamma = \sum_i \Gamma_i = 139 \to 590$	

Three body decays

When decreasing m_π → multi-body thresholds open

Stable for $m_\pi \sim 700$ MeV *Decays for $m_\pi \lesssim 500$ MeV*

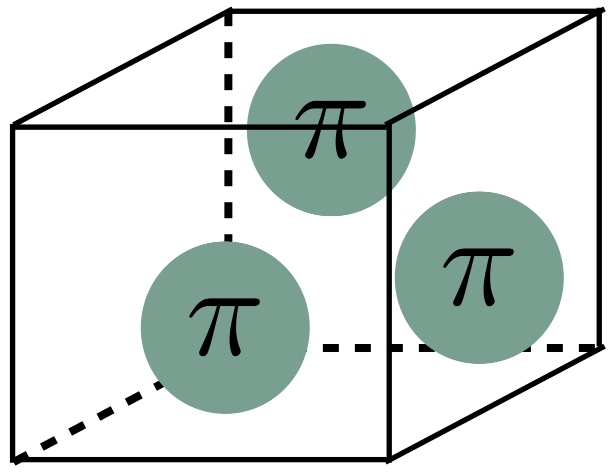


$$\rho(s) \to \int d\sigma_1 \int d\sigma_3 \, F(s, \sigma_1, \sigma_3)$$

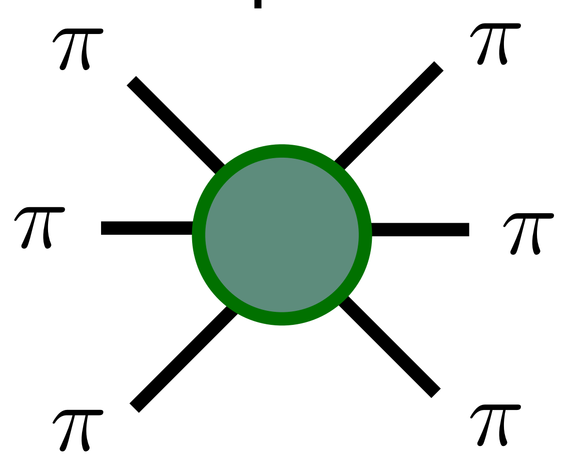
$2 \to 2$ amplitudes inside

Much more cumbersome

Plethora of formalism works on how to extract three-body amplitudes from the spectrum (not discussed here)

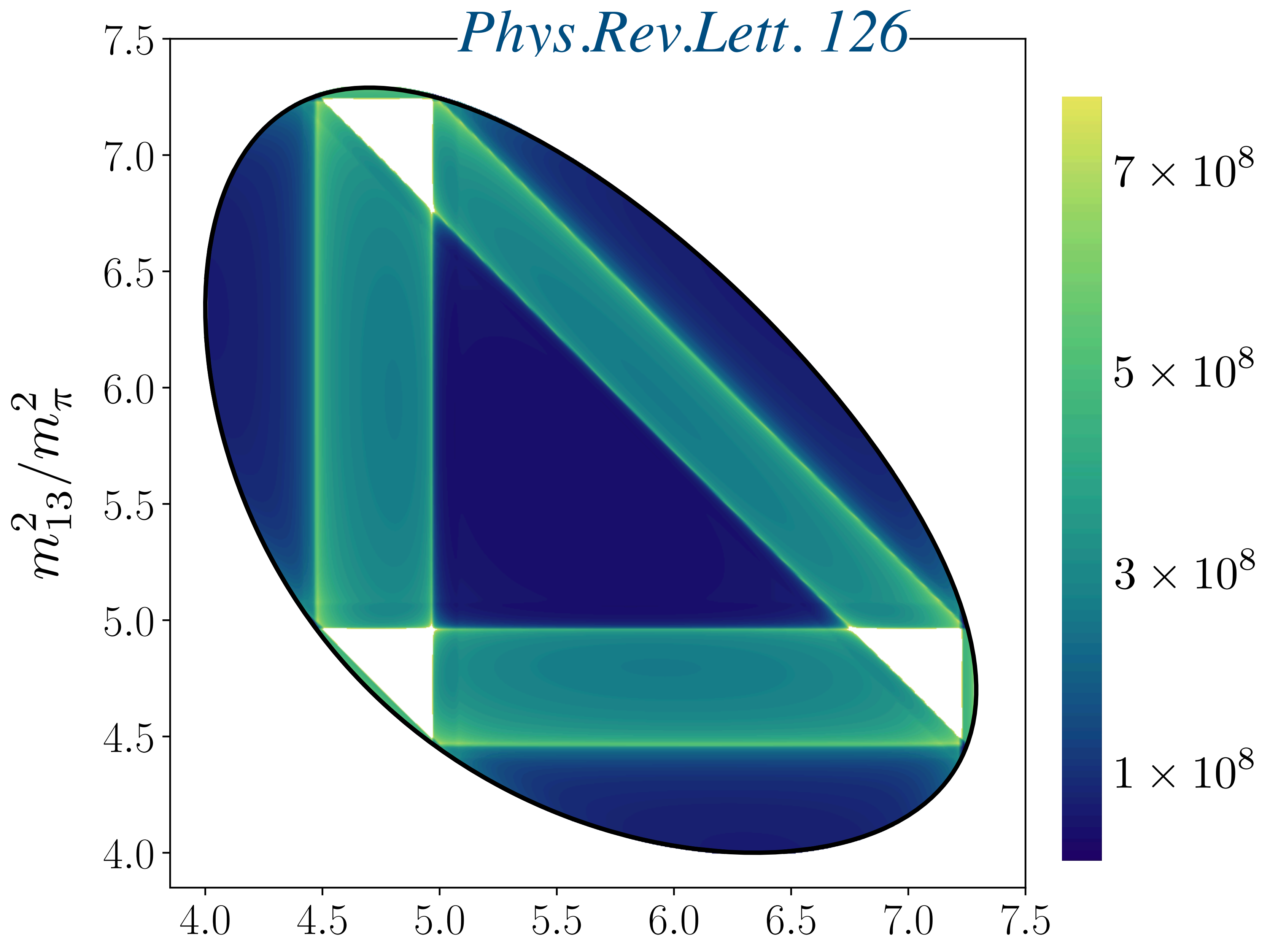


$$\det\left[F^{-1}(E_n, L) + K_3\right] = 0$$

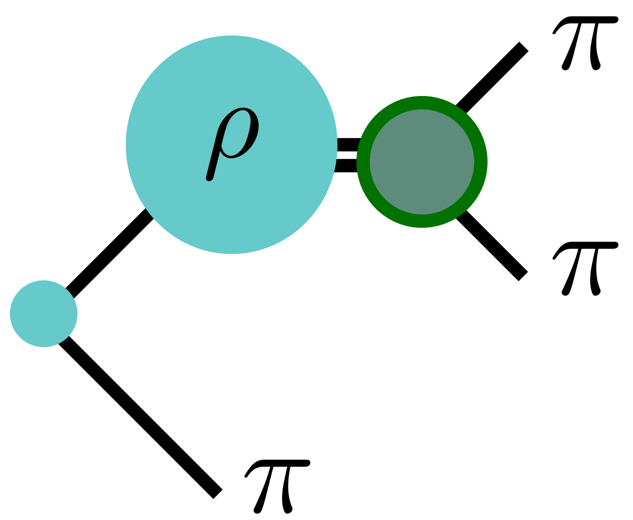


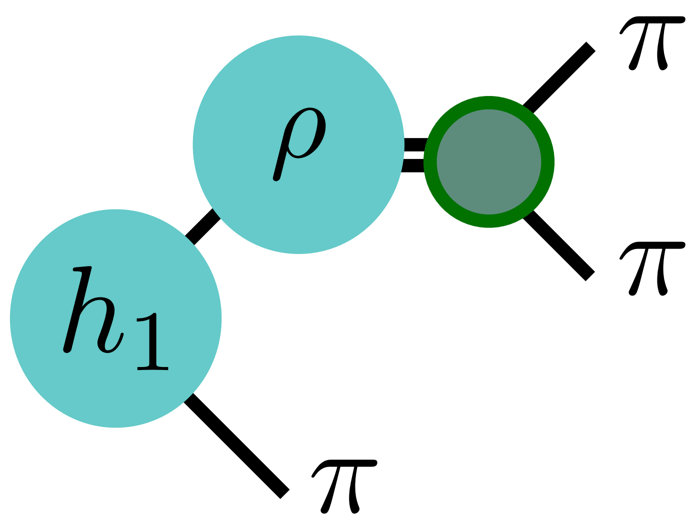
HadSpec has also been leading numerical calculations

Full 3-body amplitude for non-resonant 3 pions system



Future projects include three-body states including intermediate resonances





Lowering m_π: Pushing amplitude analyses

When lowering m_π → more phase space→ decay widths become larger

Resonance extraction becomes more challenging

We need a better infinite volume formalism than “simple” amplitude fitting

$$t^I_\ell(s) = \frac{e^{i\delta^I_\ell(s)}\sin\delta^I_\ell(s)}{\rho(s)}$$

Unitarity

Implement a full dispersive approach

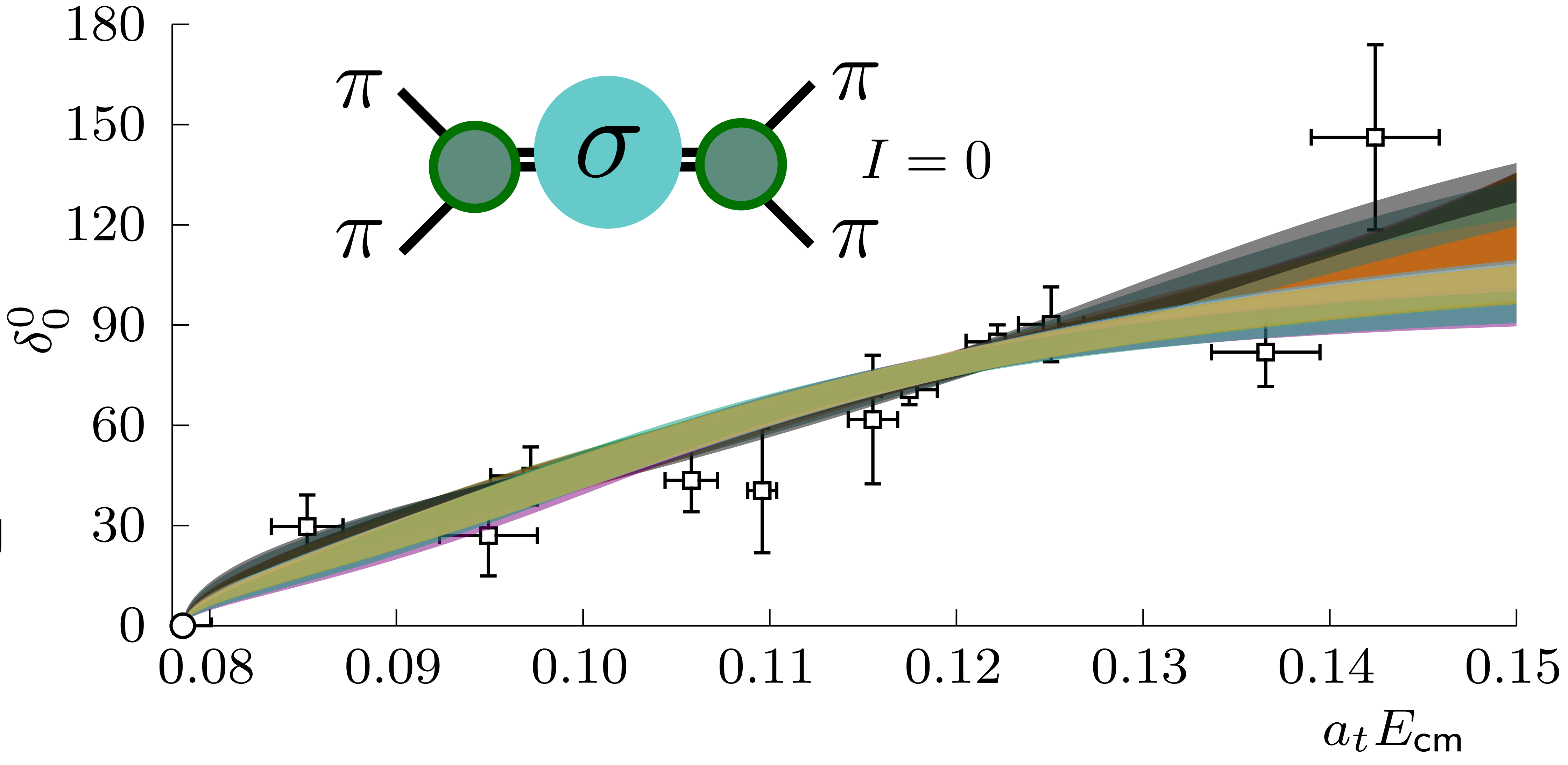
$$\tilde{t}^I_\ell(s) = \tau^I_\ell(s) + \sum_{I',\ell'}\int_{4m_\pi^2}^{\infty} ds' K^{II'}_{\ell\ell'}(s',s)\,\mathrm{Im}\,t^{I'}_{\ell'}(s')$$

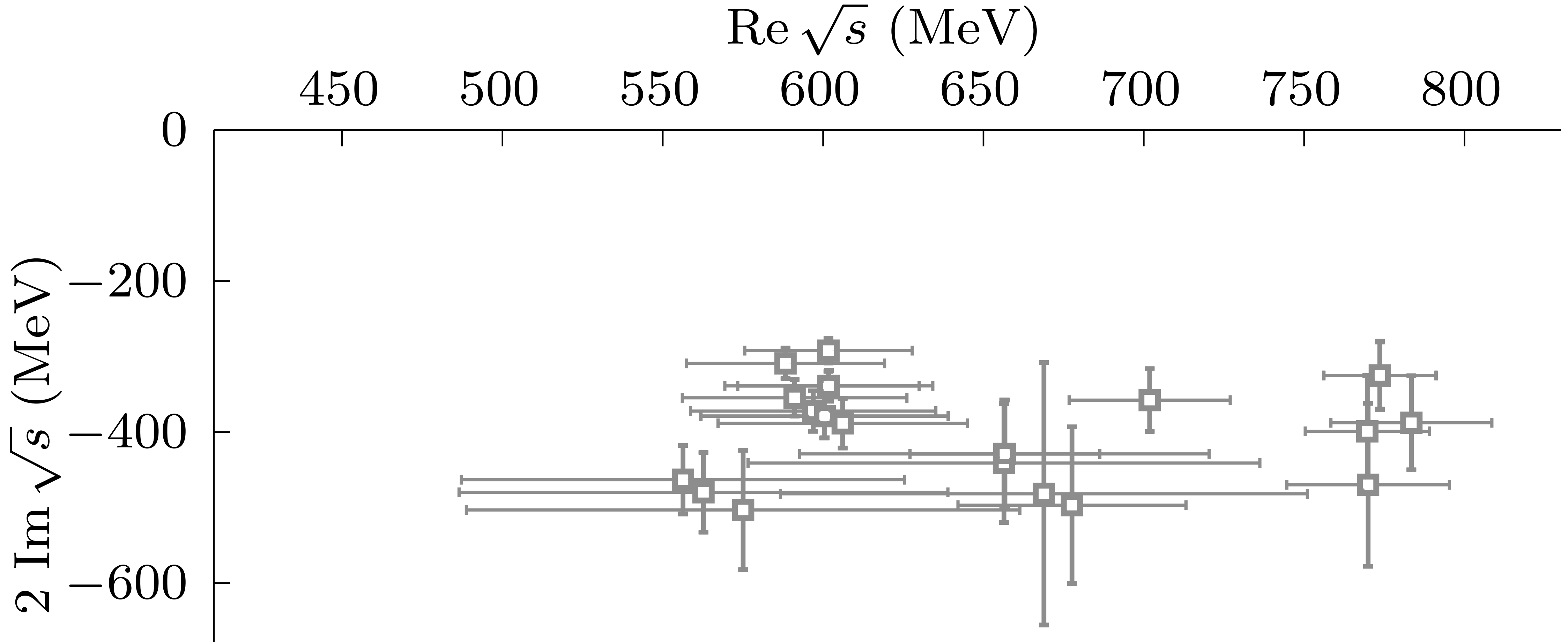
Sum over waves and isospins

Analyticity *Crossing symmetry*

Once these dispersive constrains are imposed, the systematic error is drastically reduced

$m_\pi \sim 239\,\mathrm{MeV}$





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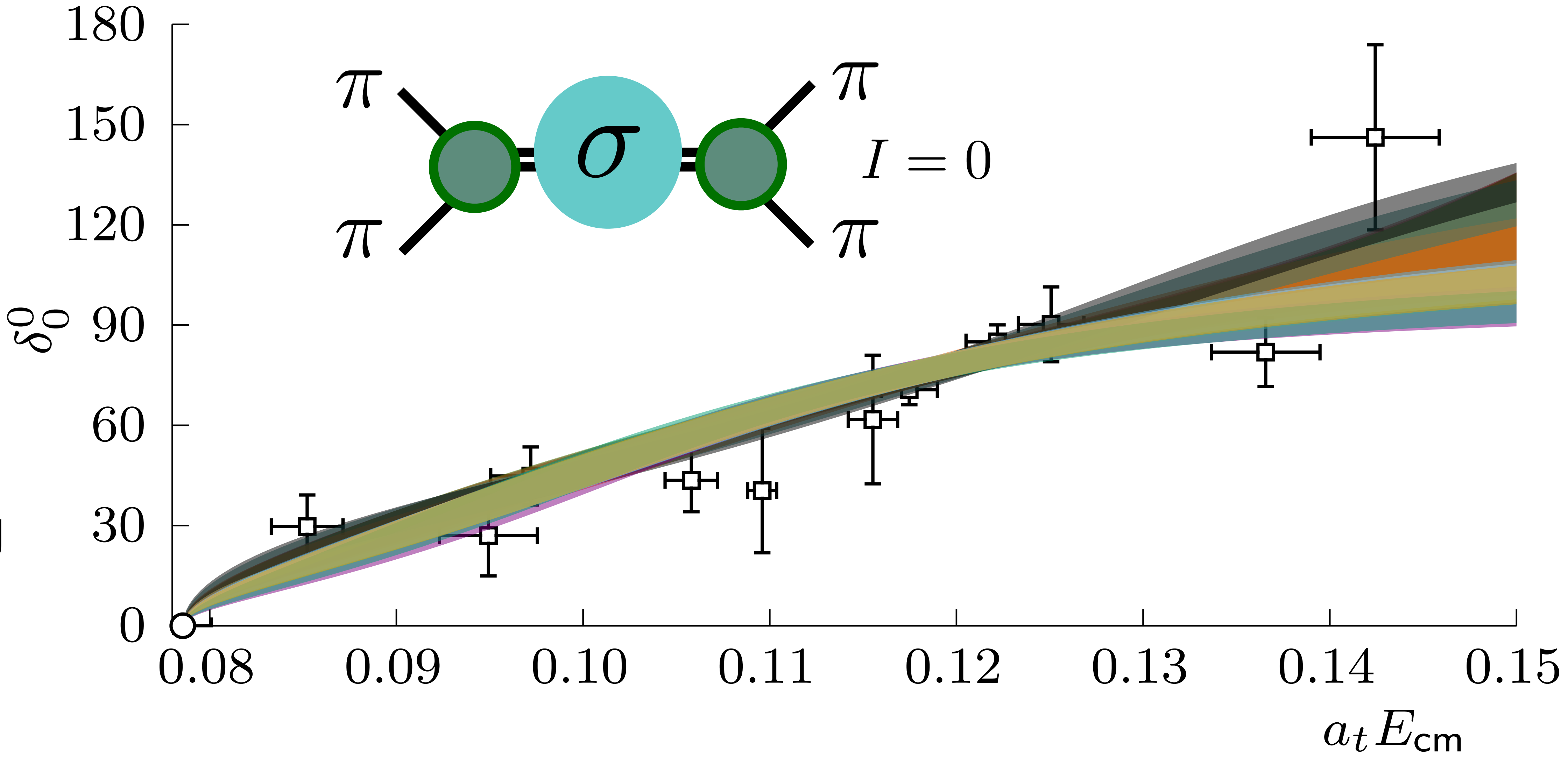
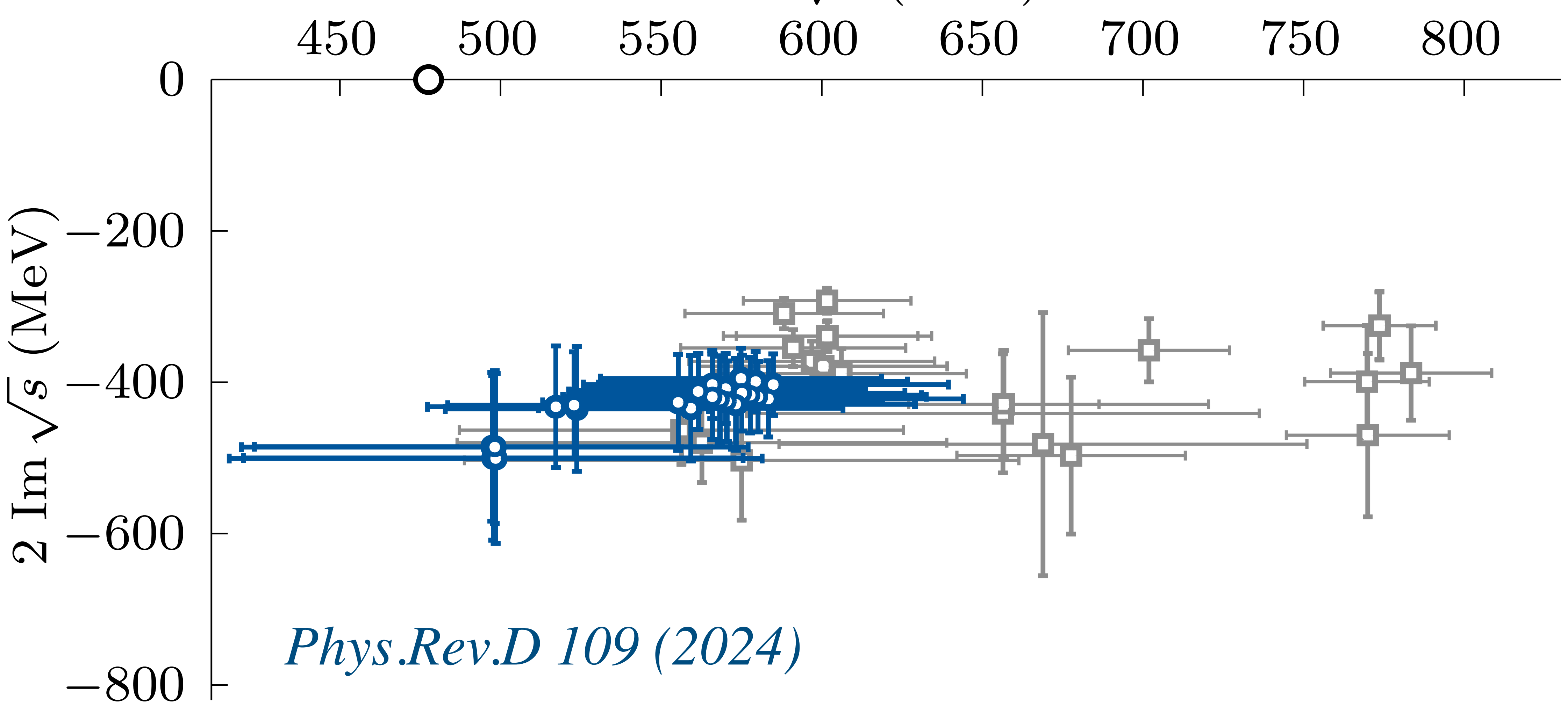




Phys.Rev.D 109 (2024)

Summary

hadspec

- Studying reactions beyond scattering processes → resonance form factors

  First photo production, then current insertions on resonances

- Significant progress made for different exotic searches

  Both in the light and charm sector

  Capitalizing on heavier m_π lattices

- Working on 3-body decays from theoretical and phenomenological side to perform first 3-body resonance extraction

  Crucial to identify exotic states for lower m_π

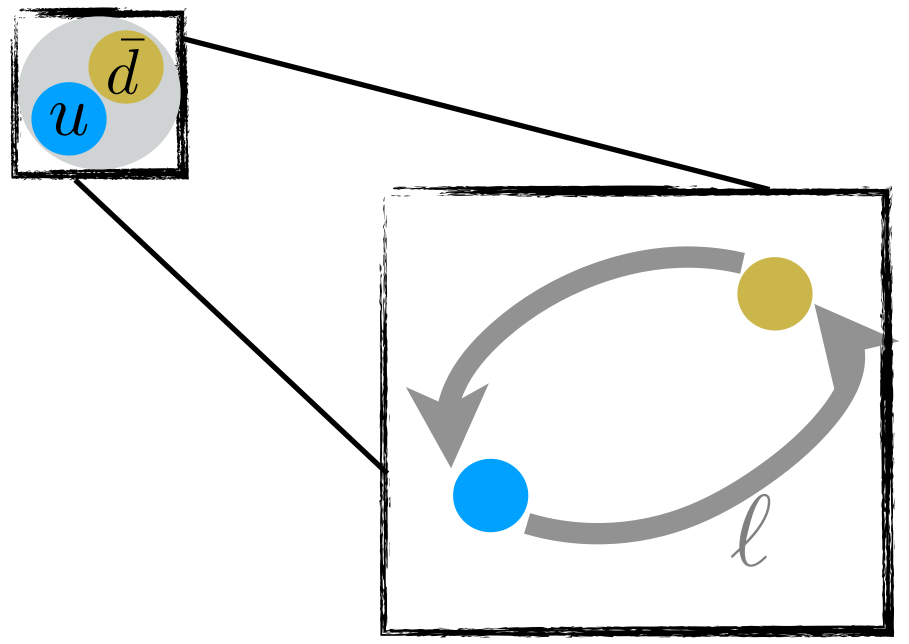
- Pushing the amplitude analysis boundaries when lowering m_π

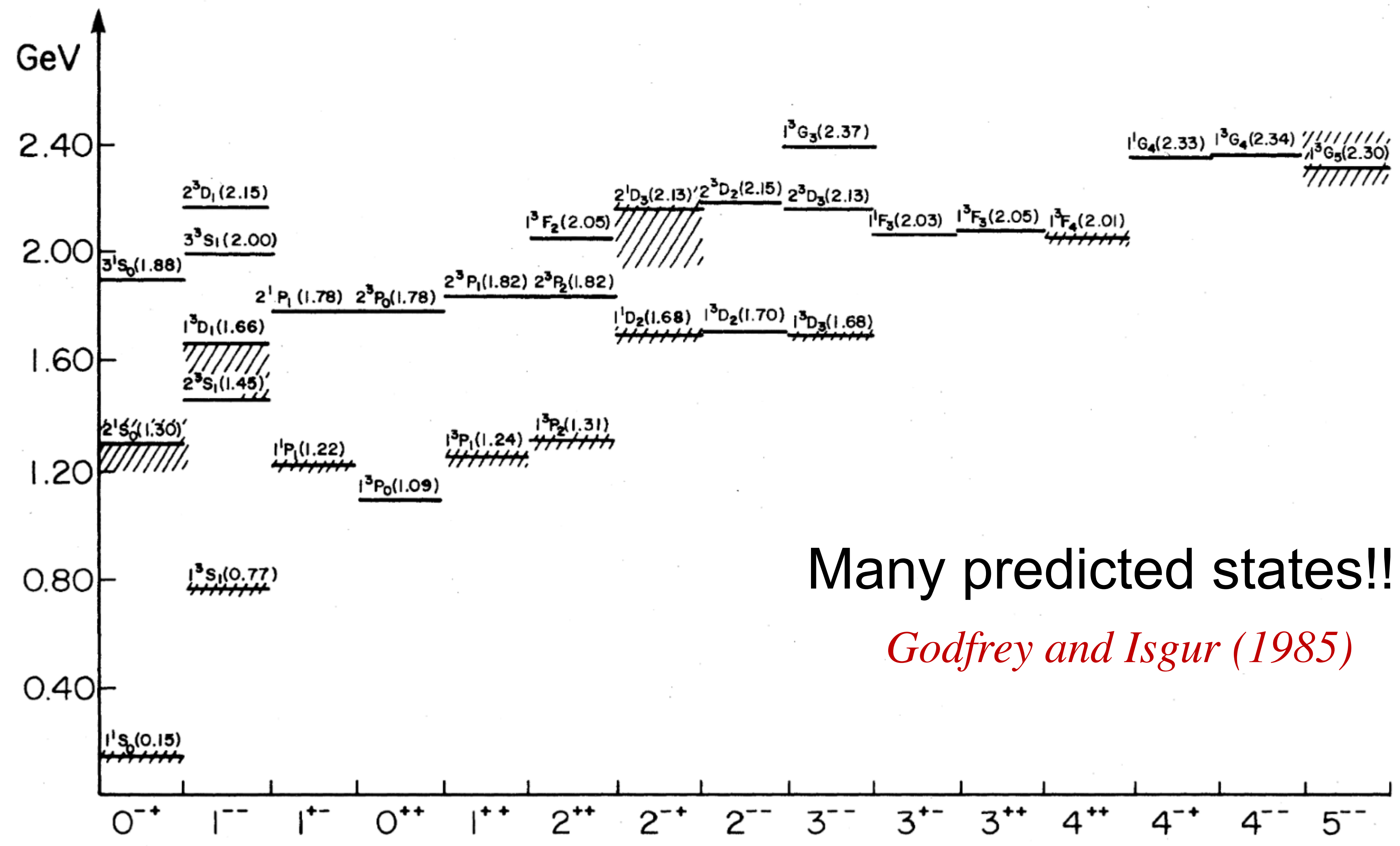
  Dispersive approaches required for light hadron spectroscopy

Spare slides

Simple quark model interpretation

Assume they are $q\bar{q}$ (meson) bound states

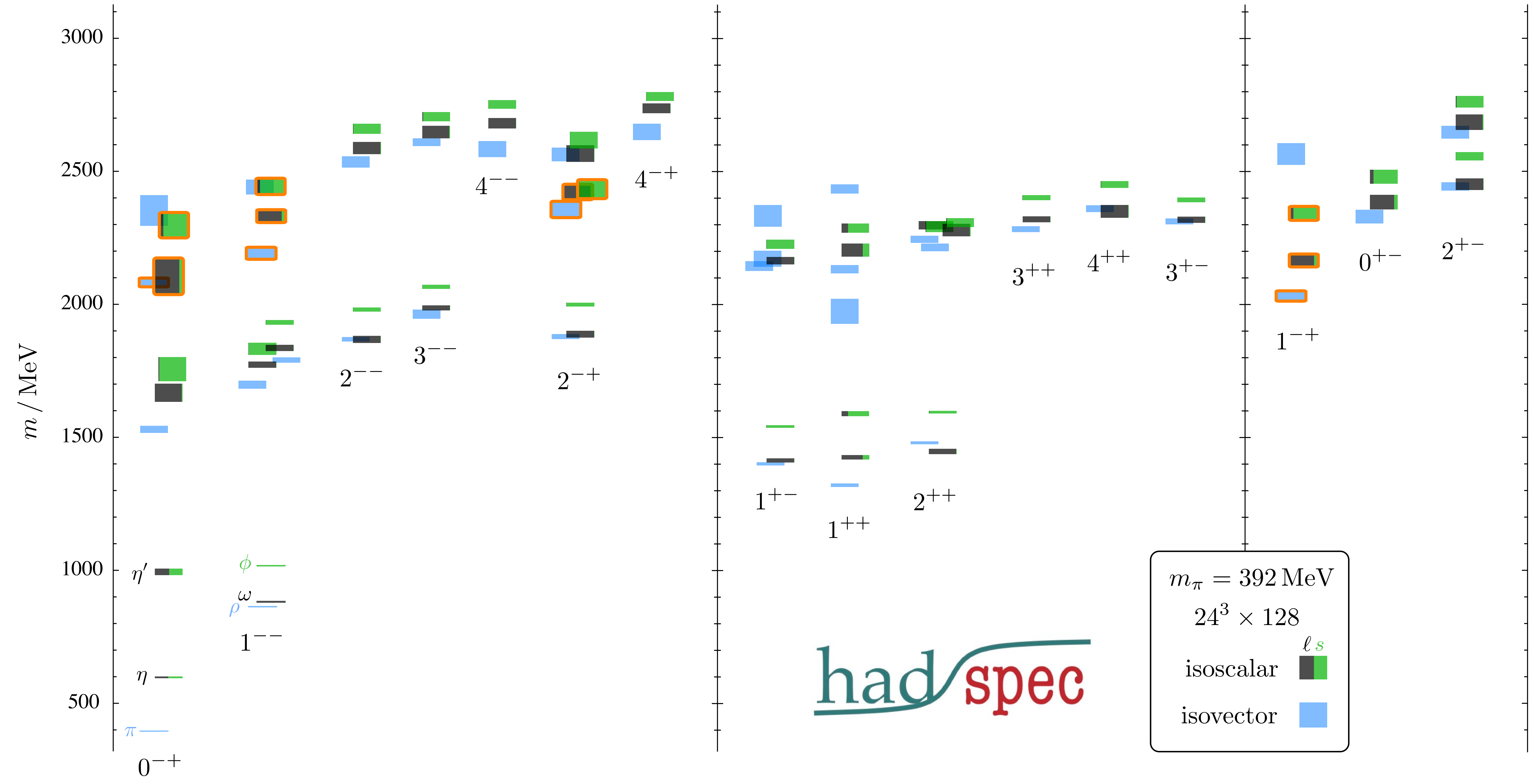




Many predicted states!!

Godfrey and Isgur (1985)

Many more states exist!!



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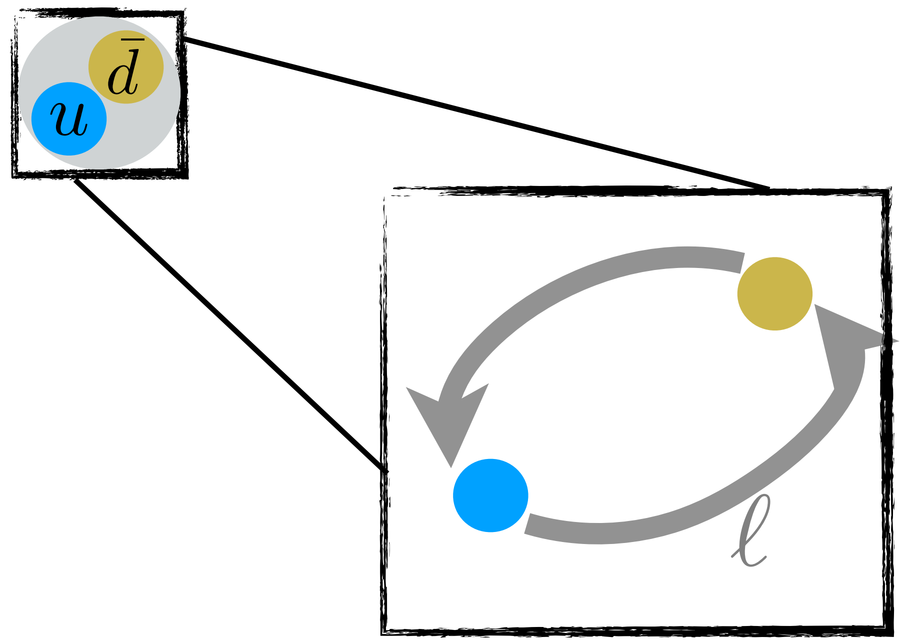

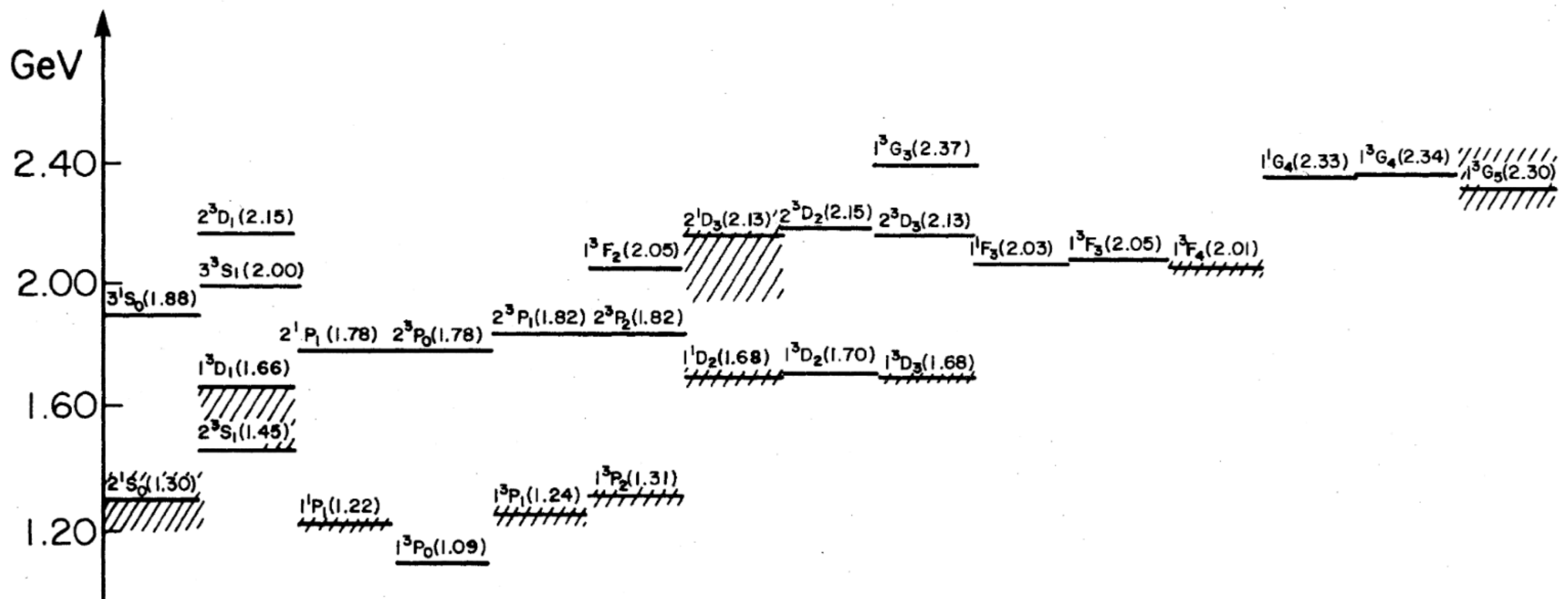

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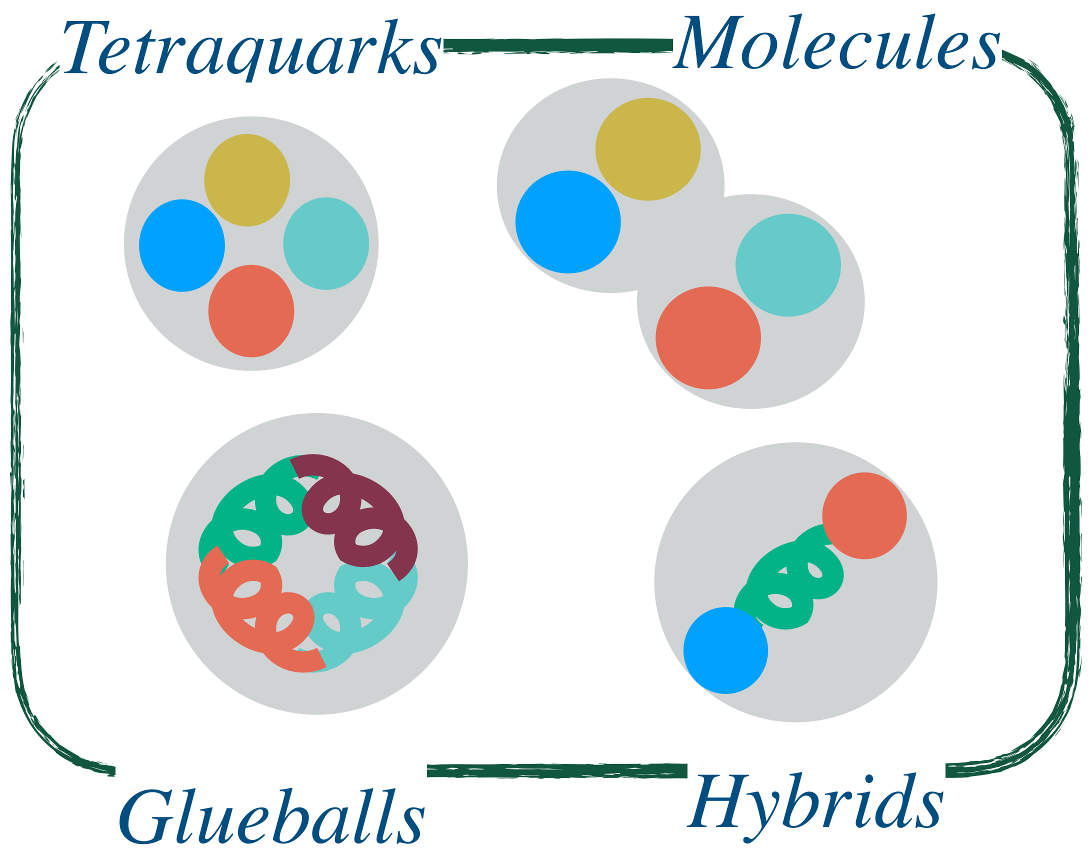

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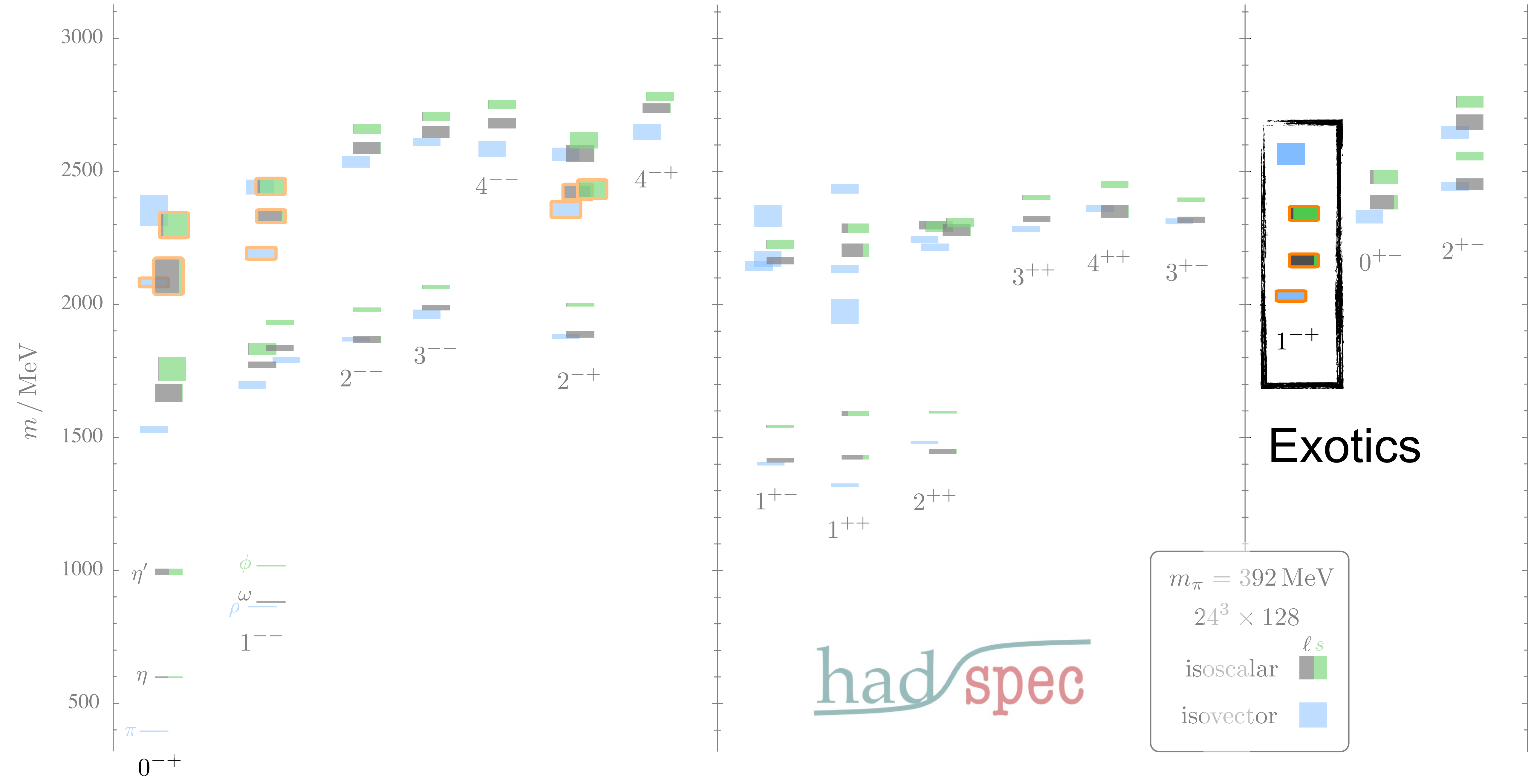
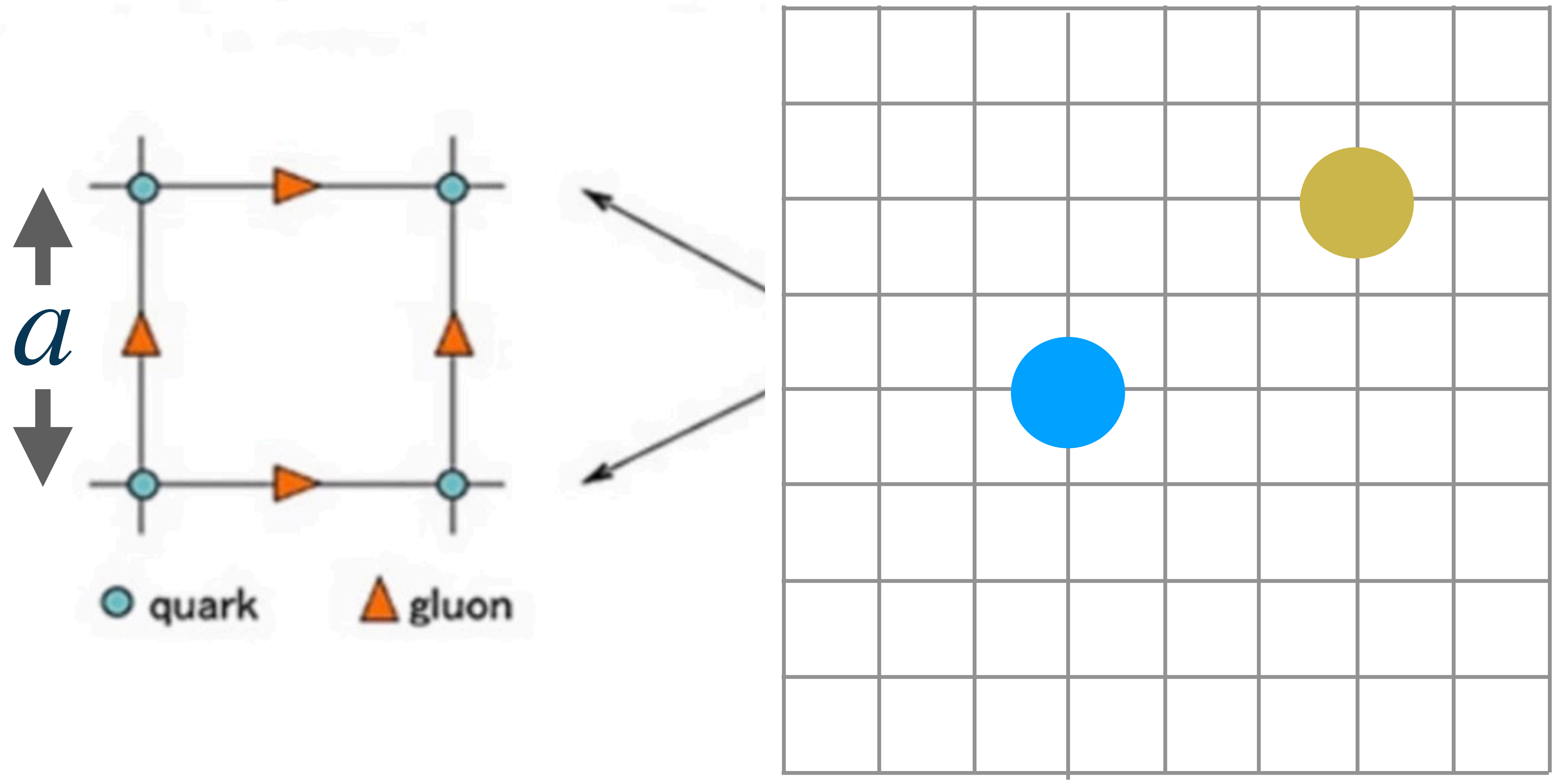


Lattice QCD

$$\left\langle \varphi_{\mathrm{f}} \left| e^{-i\hat{H}(t_{\mathrm{f}}-t_{\mathrm{i}})} \right| \varphi_{\mathrm{i}} \right\rangle = \int \mathcal{D}\varphi(x) e^{-iS[\varphi(x)]}$$

Regulator



Discretization

Sum over all paths

$$\int \mathcal{D}\varphi(x) = \prod_x \int d\varphi_x$$

$$\mathcal{L}_{\mathrm{E}} = \bar{\psi}\left(\gamma_\mu D_\mu + m\right)\psi + \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu}$$

Euclidean action $t \to -it$

$$-iS = -i\int d^3x dt \mathcal{L} \to -\int d^3x dt \mathcal{L}_{\mathrm{E}} = -S_{\mathrm{E}}$$

$0 < \qquad < 1$

$$\left\langle \varphi_{\mathrm{f}} \left| e^{-i\hat{H}(t_{\mathrm{f}}-t_{\mathrm{i}})} \right| \varphi_{\mathrm{i}} \right\rangle = \int \mathcal{D}\varphi(x) e^{-iS[\varphi(x)]} = \int \mathcal{D}\varphi(x) e^{-S_{\mathrm{E}}[\varphi(x)]}$$

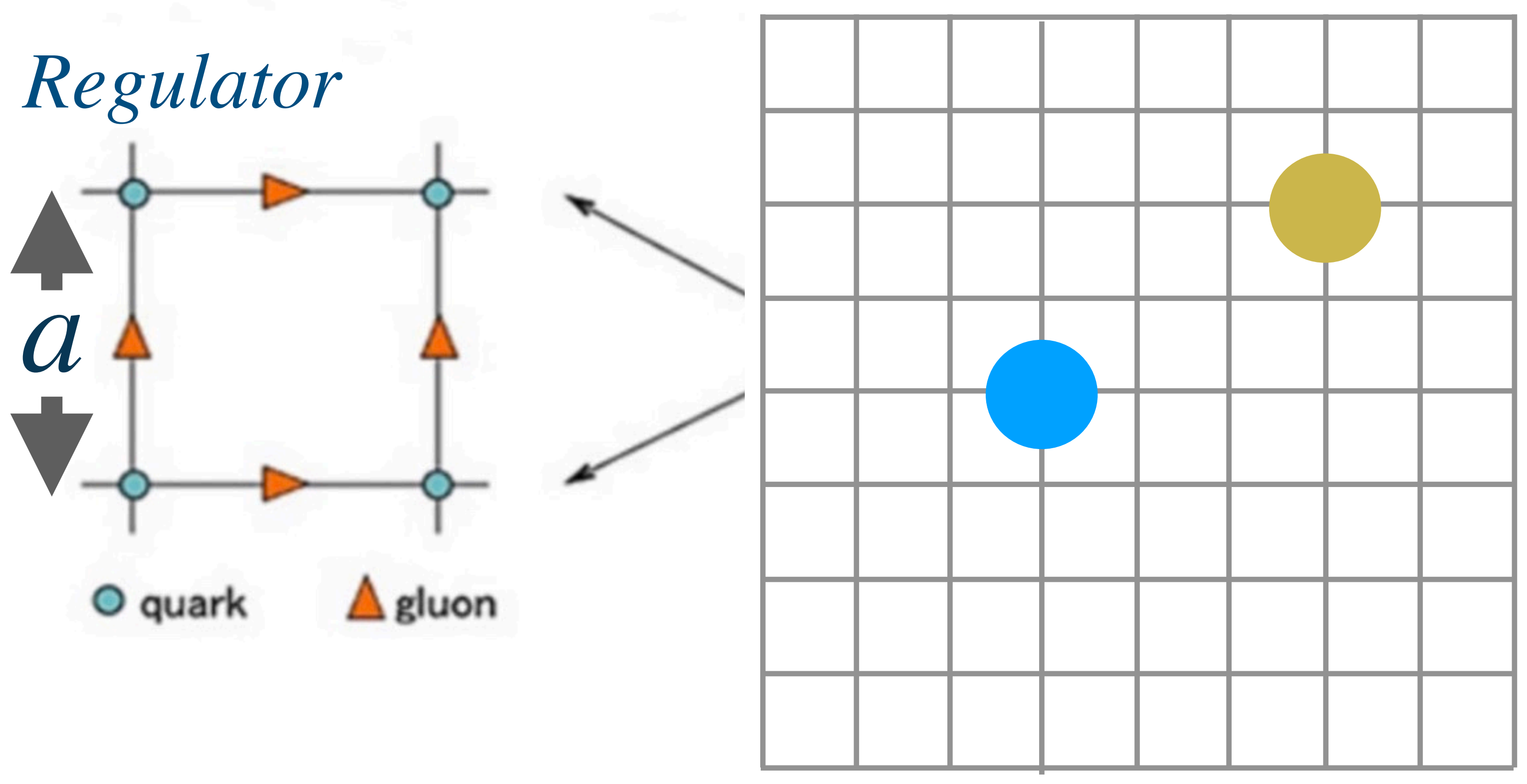
Probability like

Lattice QCD

Discretized, euclidean spacetime

$$\int \mathcal{D}\varphi(x) = \prod_x \int d\varphi_x$$

$$-iS = -i\int d^3x dt \mathcal{L} \to -\int d^3x dt \mathcal{L}_E = -S_E$$

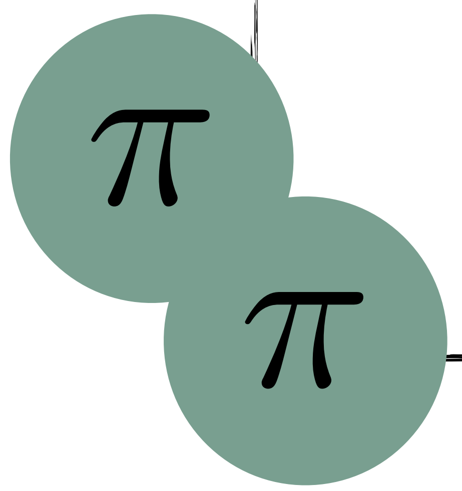


Numerical, Montecarlo sampling of our gluon fields

$$\langle O\rangle = \lim_{N\to\infty}\frac{1}{N}\sum_{n=1}^{N} O\left[U_n\right]$$

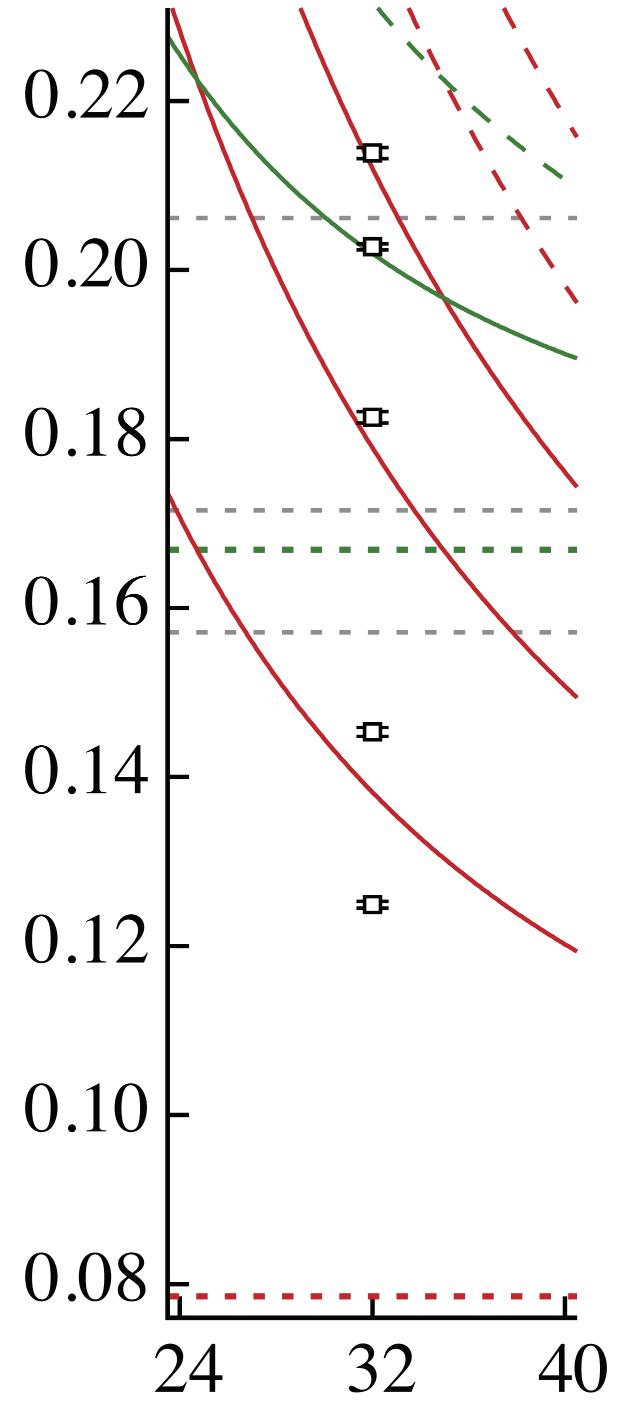
States Time evolution $|\psi(t)\rangle = e^{-Ht}|\psi(0)\rangle$

$$\langle O_f(t) O_i^\dagger(0)\rangle \sim \sum_{\mathrm{n}} e^{-E_{\mathrm{n}}t}\langle 0\,|O_{\mathrm{f}}(0)|\,\mathrm{n}\rangle\langle \mathrm{n}|O_{\mathrm{i}}^\dagger(0)|0\rangle$$



Desired energies

E_n



Hadspec: The basics

Optimal lattice setup for spectroscopy

Improved, stout-smeared lattices

High anisotropy $\xi \sim 3.5$

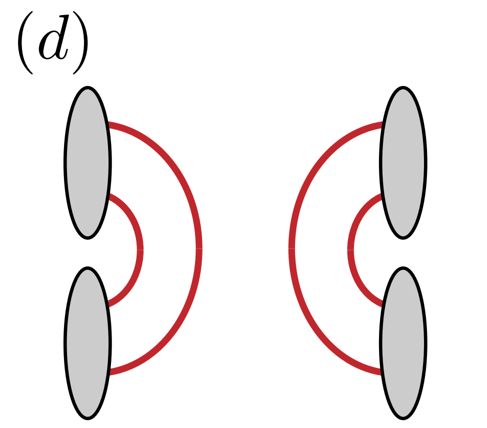
Time evolution

$$\sum_n e^{-E_n t} \langle 0 | O_f(0) | n \rangle \langle n | O_i^\dagger(0) | 0 \rangle$$

Efficient energy level extraction

Full distillation (very large N_v *available)*

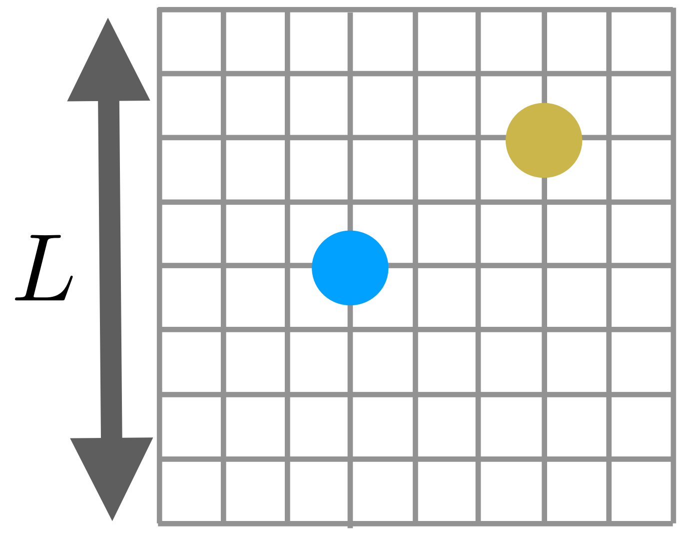
Annihilation lines averaged over many time slices (full time extent)



Large number of interpolators $\mathcal{O}_b \sim \bar{q}\,\Gamma_b\, q, \pi\pi, KK, \ldots, 4\pi, \ldots$

Full GEVP solution (many excited states)

Finite volume formalism



K-matrix

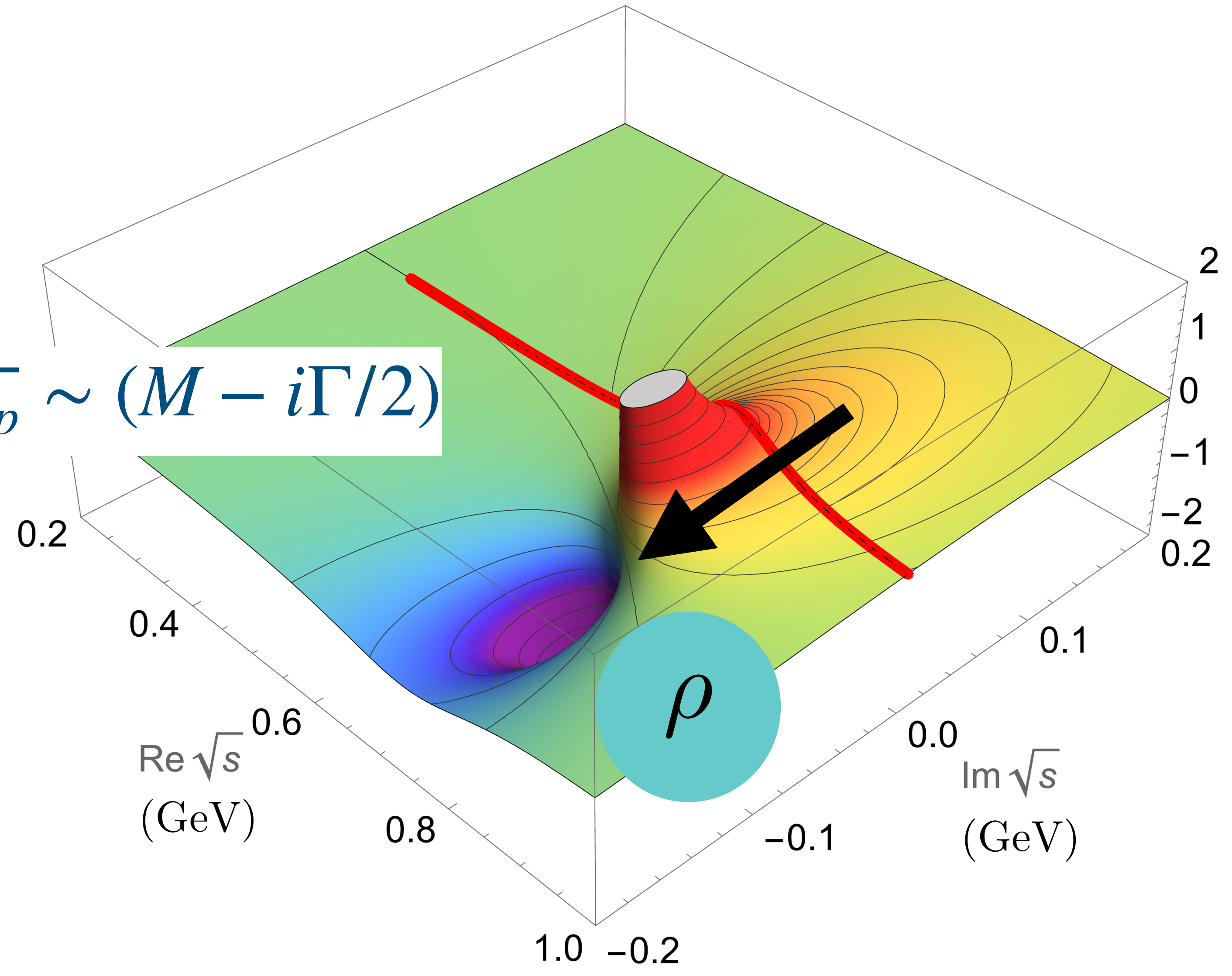
$$\det\left[F^{-1}(P,L) + K\right] = 0$$

Fit-parameters

Infinite volume formalism

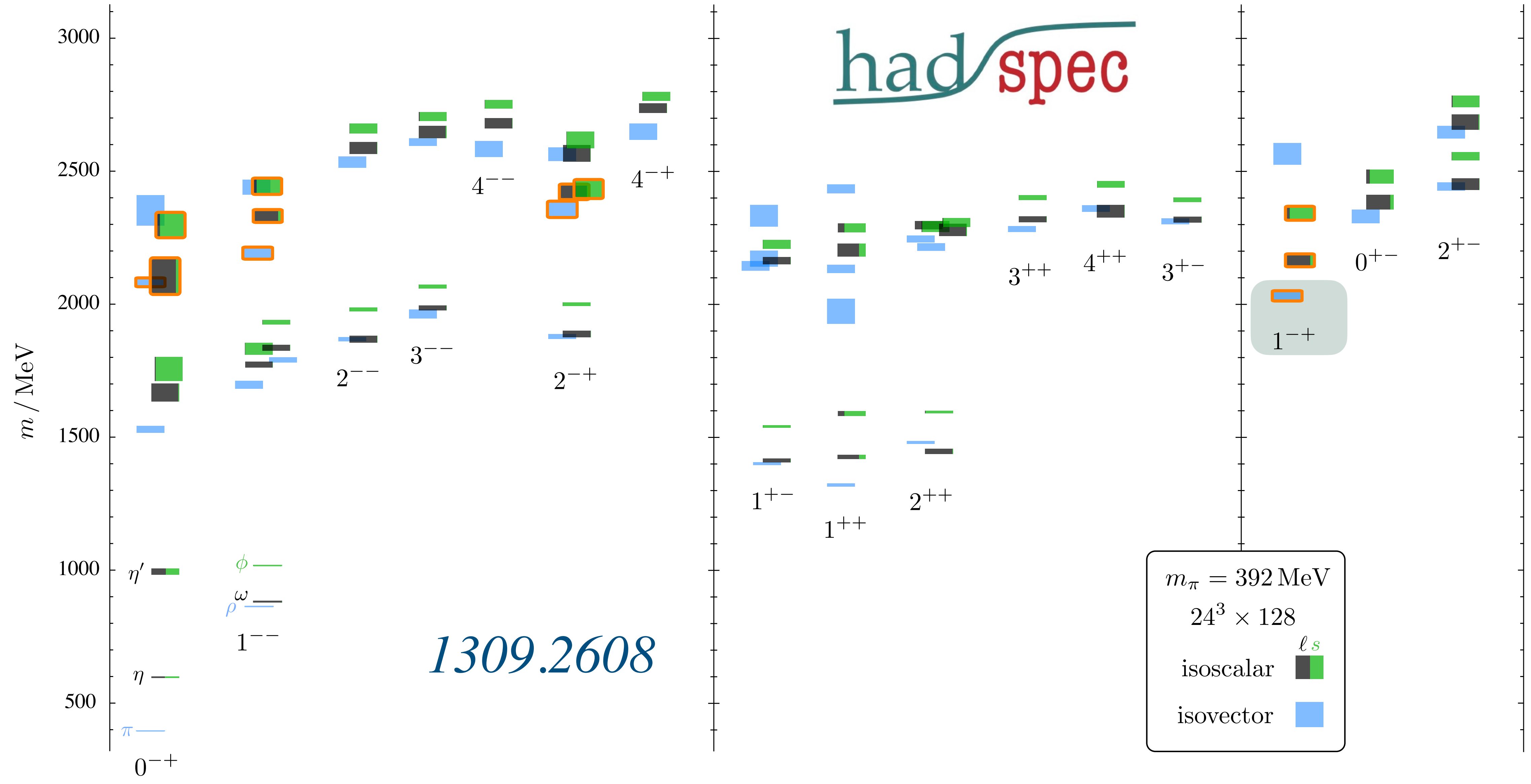
$$t(s) = \frac{K}{1 - i\rho(s)K}$$

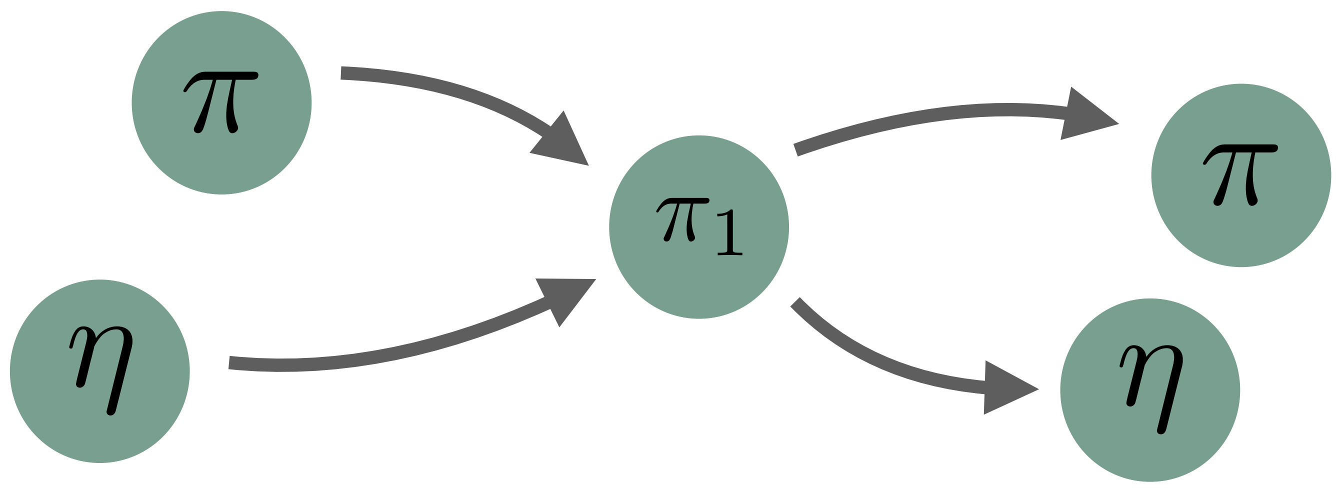
Pole at $\sqrt{s_p} \sim (M - i\Gamma/2)$



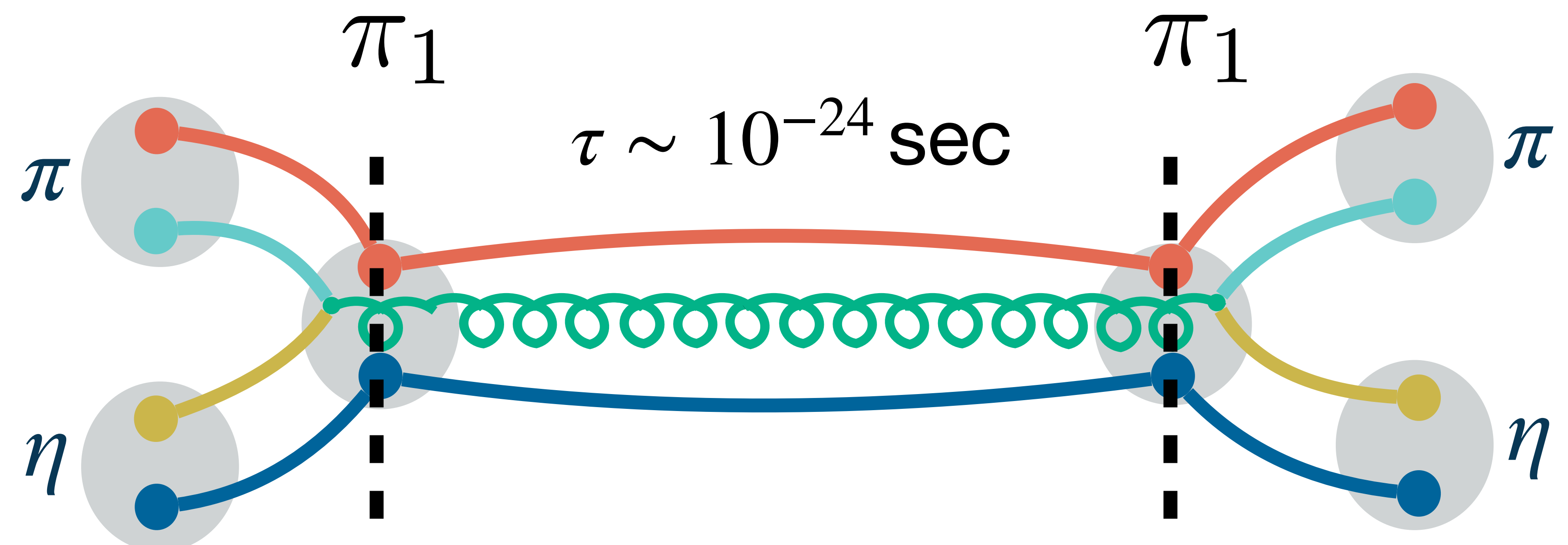
Hybrid exotic candidates

Lattice QCD (and models) predicts a lightest $J^{PC} = 1^{-+}$, isolated hybrid

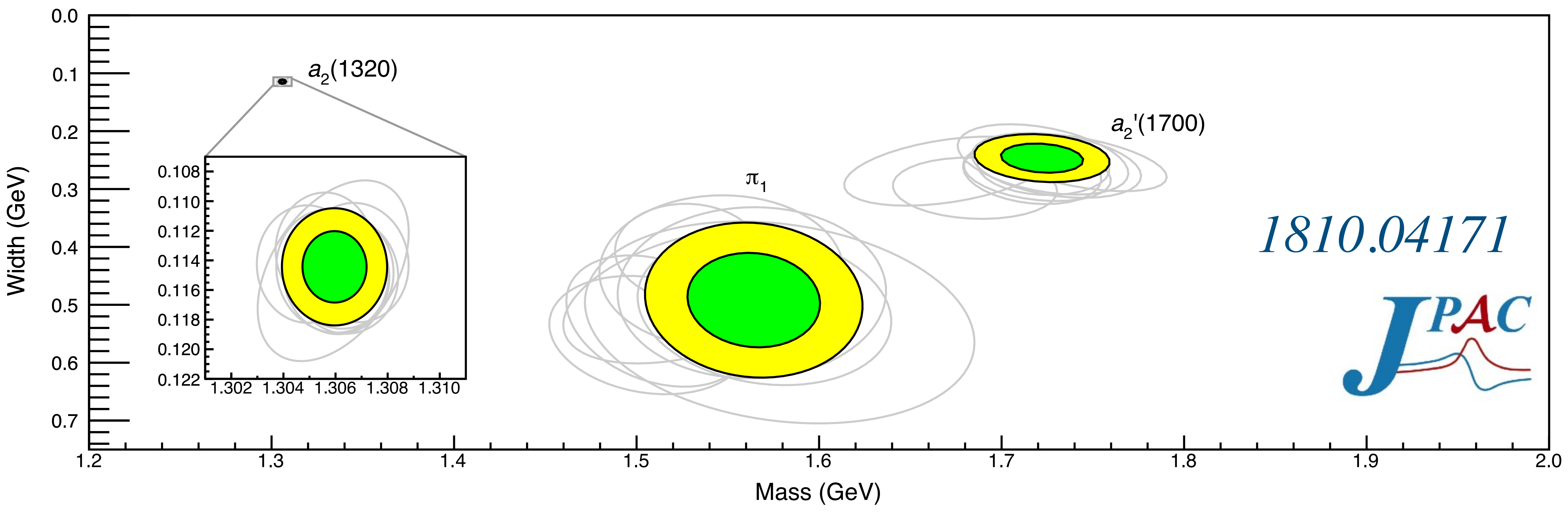


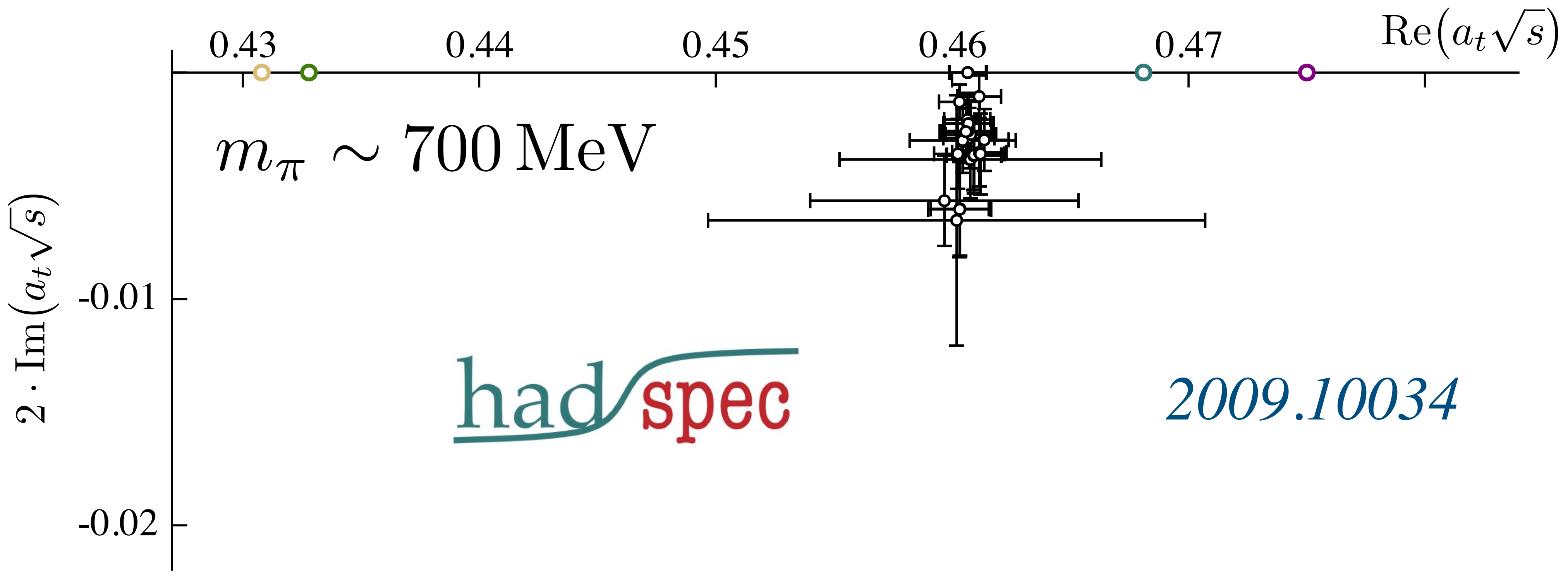


It decays to two pseudo-scalar mesons



Extracted, recently, both from experiment (JPAC/COMPASS) and Lattice QCD (HadSpec)





Exotic mesons

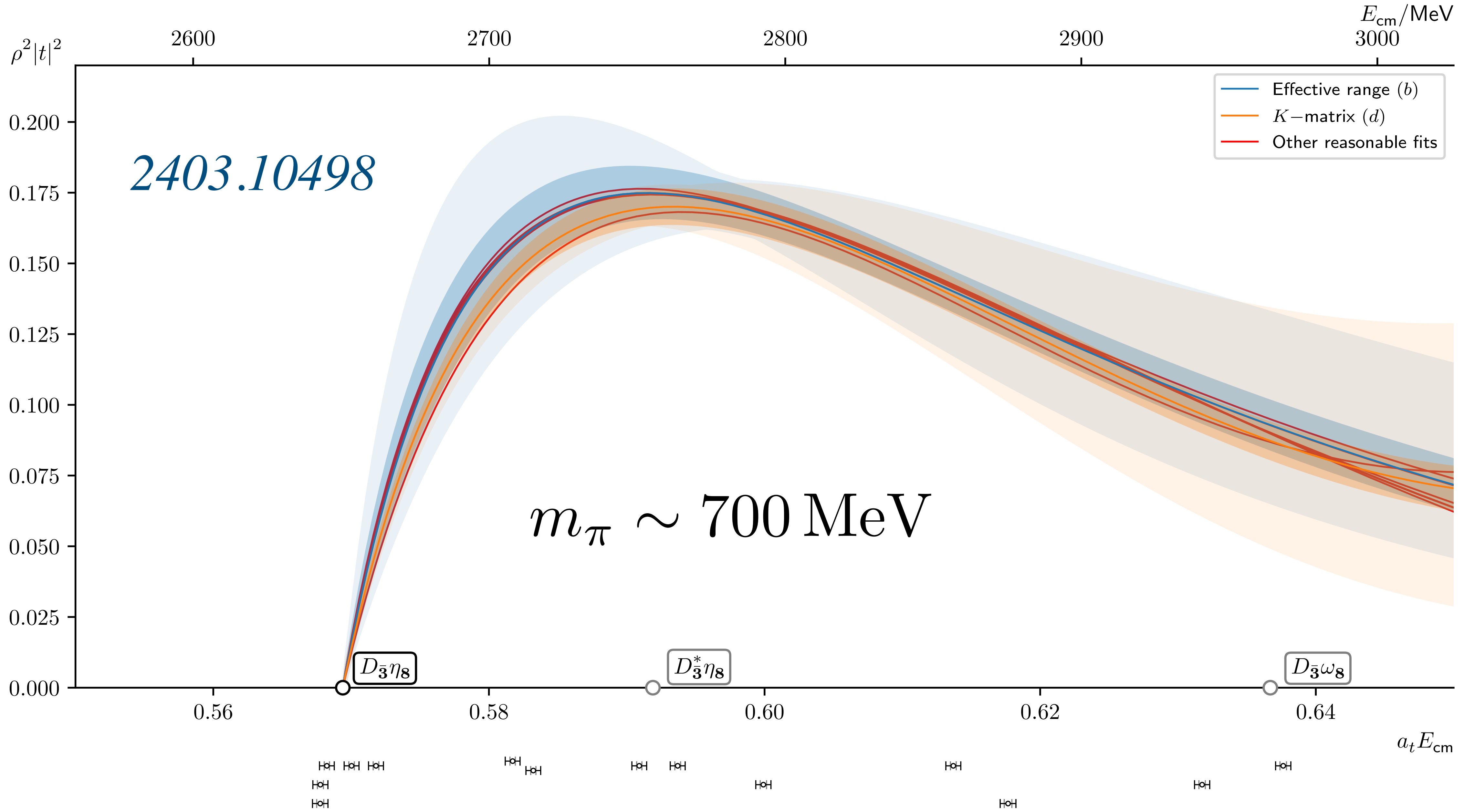
Exotic ($J^P = 0^+$) found at $m_\pi \sim 700$ MeV in $D\pi/DK$ scattering

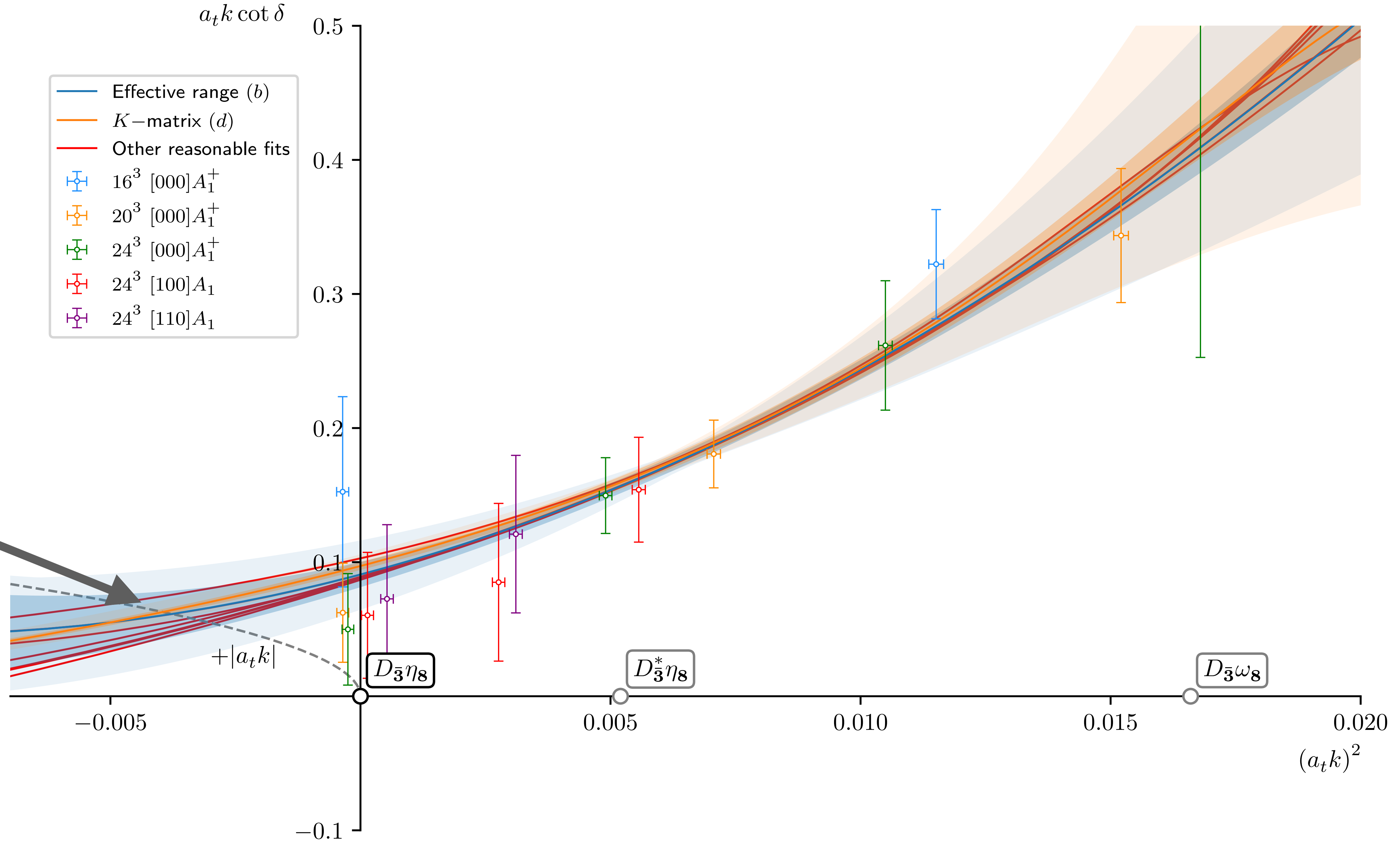
Once again, it appears as a virtual-bound state

Fir this pion mass, we set $m_u = m_d = m_s$

Many channels coalesce to a smaller subset

This state seems virtual bound also at lower pion masses

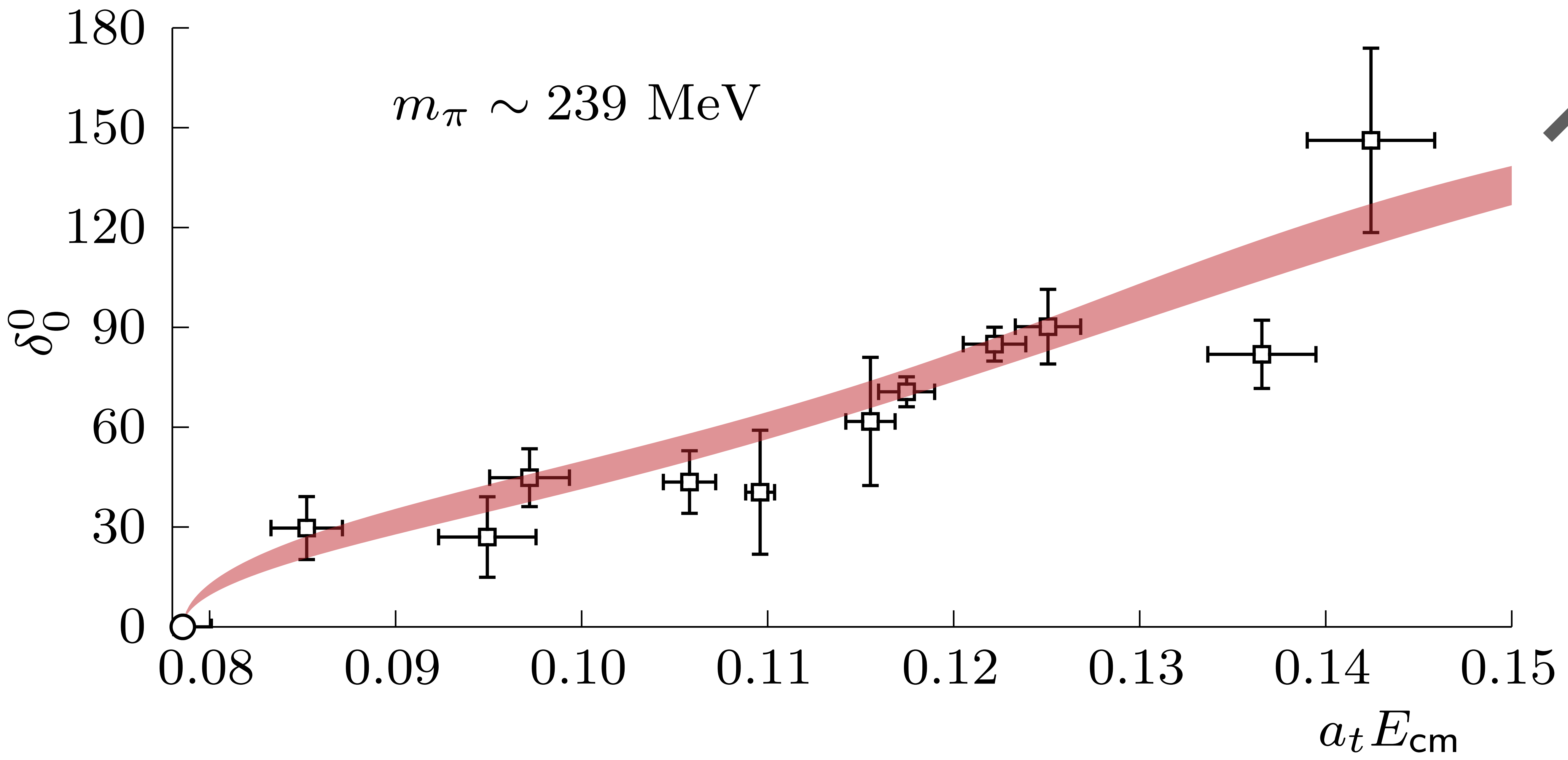


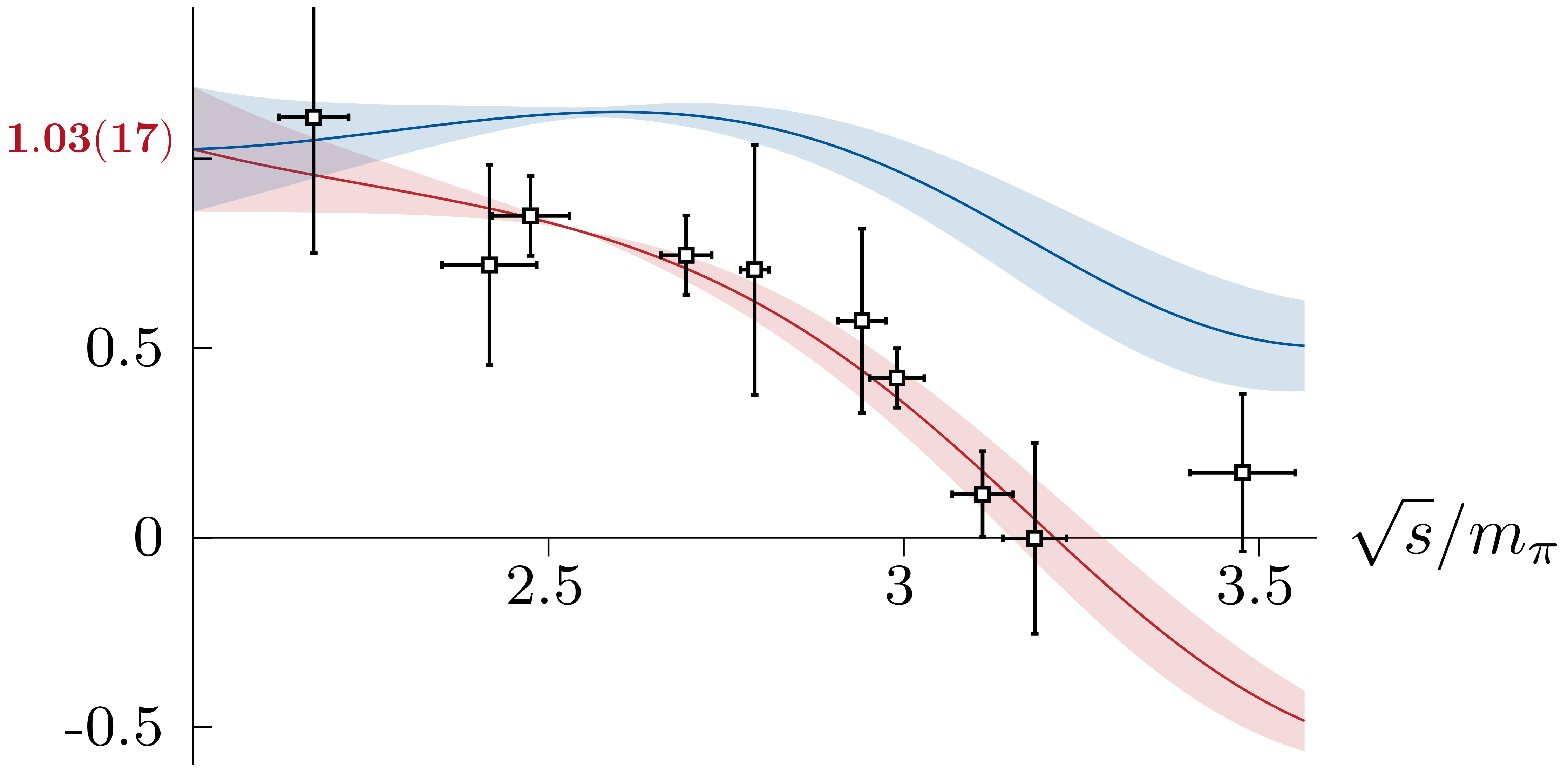


Select best combinations

Model 1

$$a_0^0 + \frac{1}{12m_\pi^2}\left(2a_0^0 - 5a_0^2\right)\left(s-4m_\pi^2\right) + \sum_{I',\ell'}\int_{4m_\pi^2}^{\infty} ds'\, K_{0\ell'}^{0I'}(s',s)\,\mathrm{Im}\, t_{\ell'}^{I'}(s') = \tilde{t}_0^0(s)$$

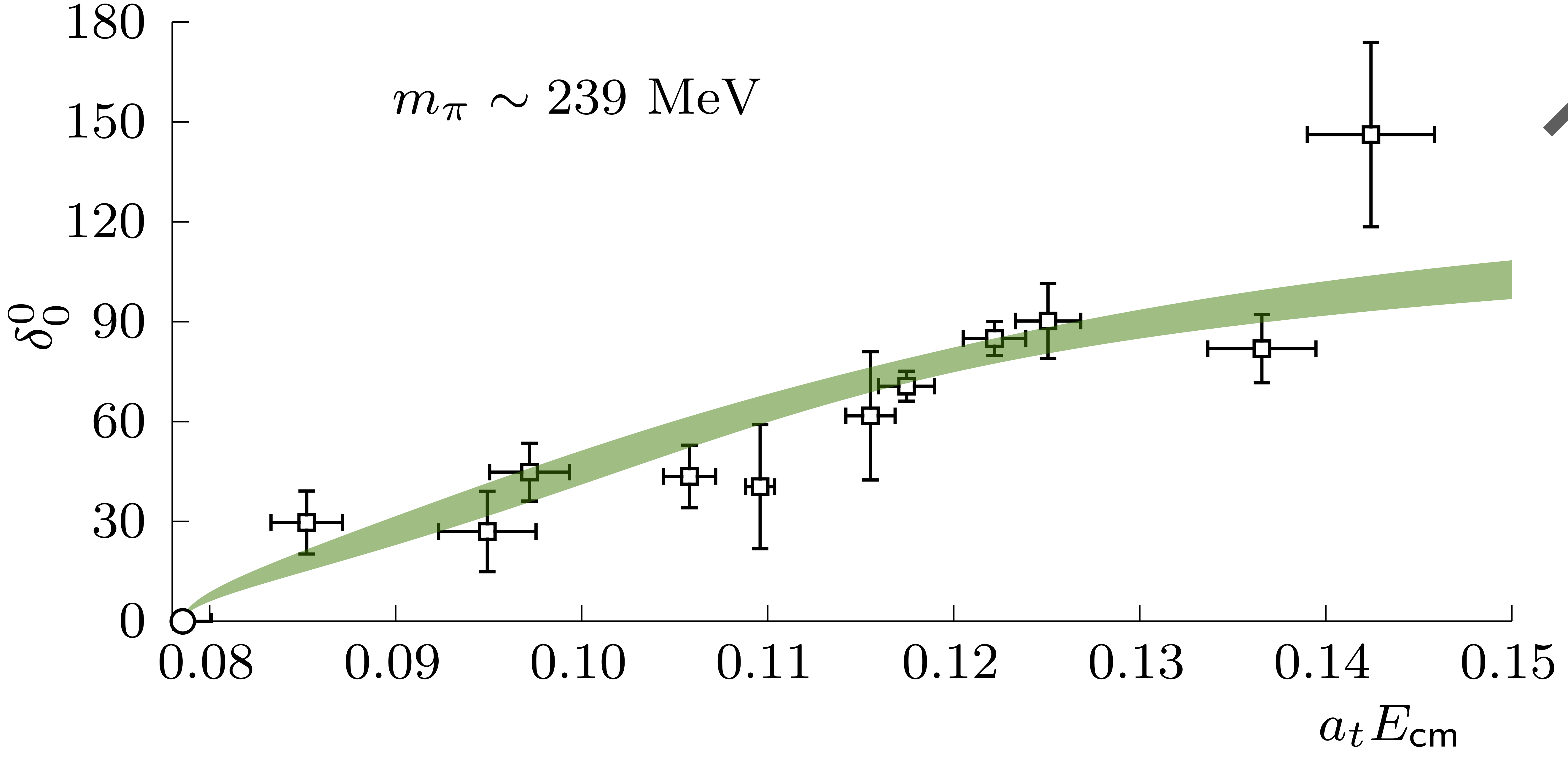


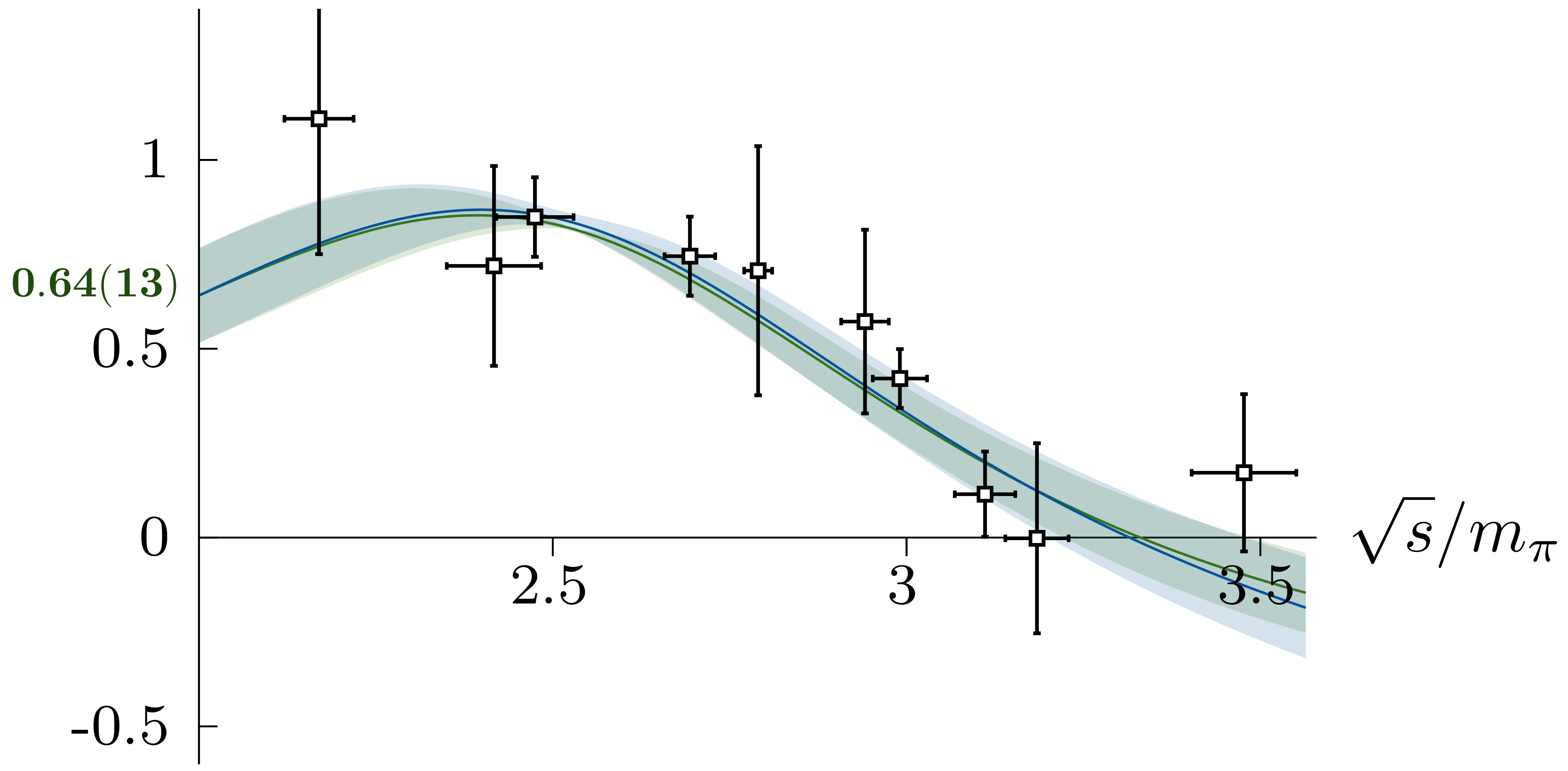


Select best combinations

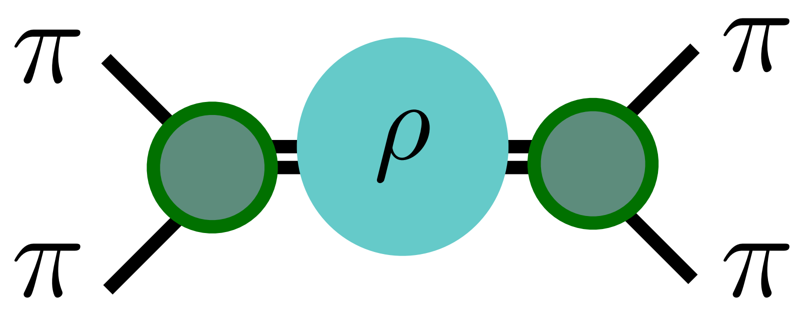
Model 2

$$a_0^0 + \frac{1}{12m_\pi^2}\left(2a_0^0 - 5a_0^2\right)\left(s - 4m_\pi^2\right) + \sum_{I',\ell'} \int_{4m_\pi^2}^{\infty} ds' K_{0\ell'}^{0I'}\left(s',s\right) \operatorname{Im} t_{\ell'}^{I'}\left(s'\right) = \tilde{t}_0^0(s)$$





Future projects

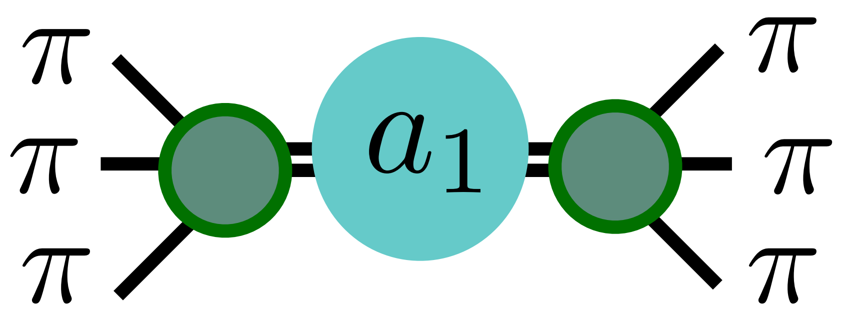


1 Determine the spectrum of ordinary and non-ordinary hadrons

Ready to compute exotic reactions at higher m_π
Pushing lower m_π calculations for meson-meson scattering processes

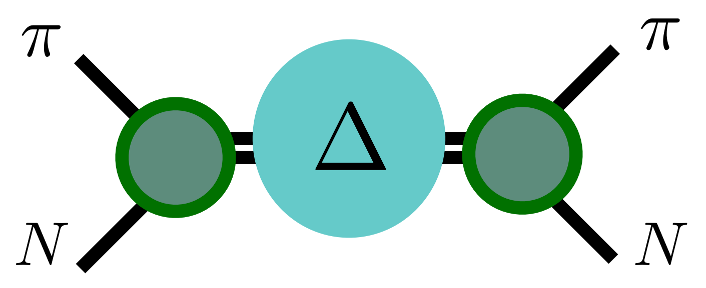
2 Develop and implement multi-body decay formalisms to the extraction of resonances

Technology ready for 3π systems with intermediate resonances
Getting closer to 3b baryonic systems!



3 Kick start our meson-baryon program

First explorations for Δ resonances are underway
The Roper resonance will also be extracted, in the longer future



4 Continue our EM analyses

Working on photo production processes for coupled-channels
Working on elastic form factors of scattering processes