DOE contracts:
DE-SC0007981
DE-AC05-06OR23177

Studies of Hadron Structure by the CJ Collaboration

Matteo Cerutti

CTEQ-JLab collaboration

Main focus: Investigate the internal structure of nucleons in their valence region

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collinear factorization

$$d\sigma_{\text{hadron}} = \sum_{f_1, f_2, i, j} \phi_{f_1} \otimes \hat{\sigma}^{f_1 f_2 \to ij}_{\text{parton}} \otimes \phi_{f_2}$$

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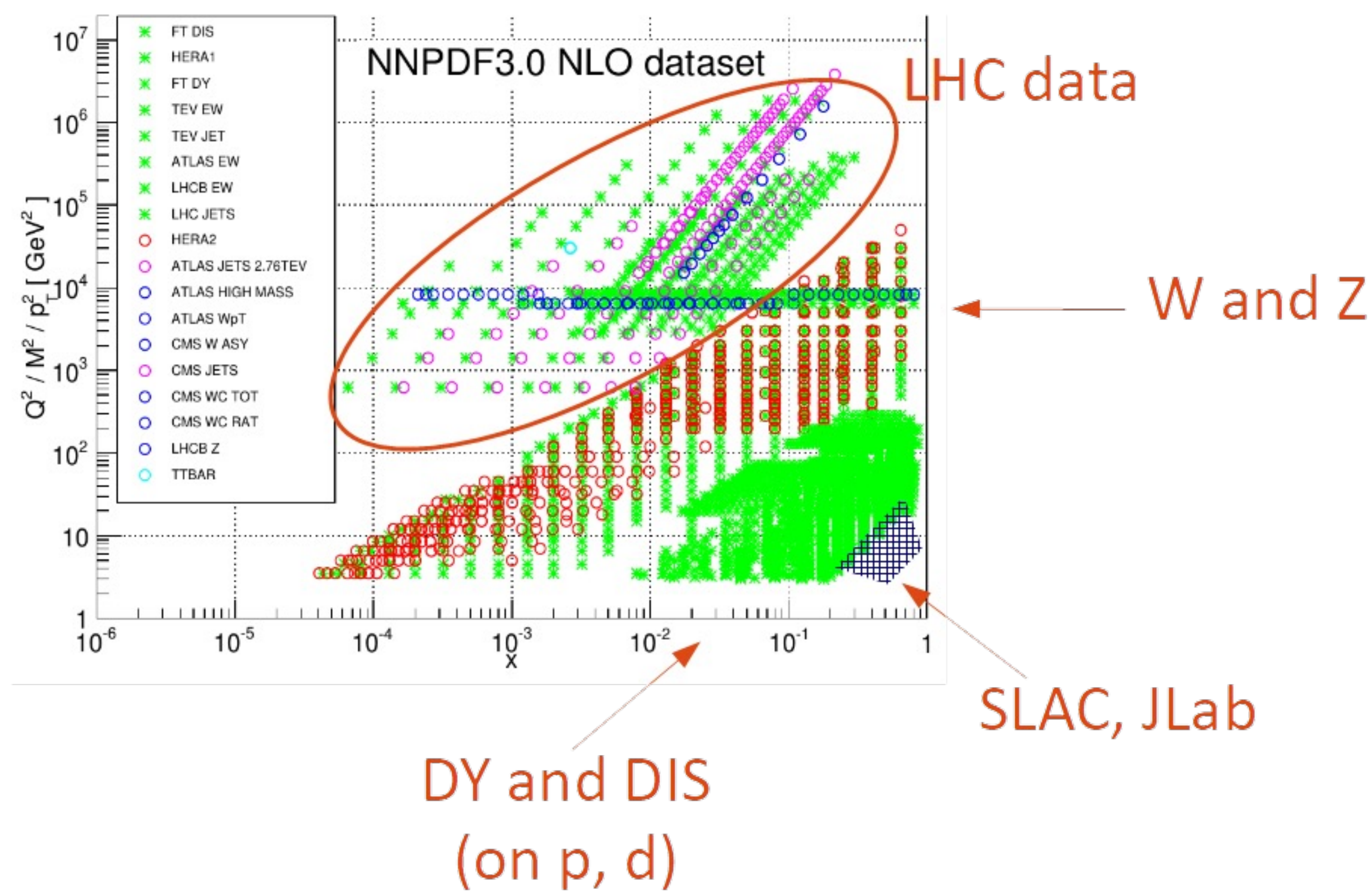

universality

- DIS — p, d targets
- pp collisions — Drell-Yan
  W/Z boson production
  Jets

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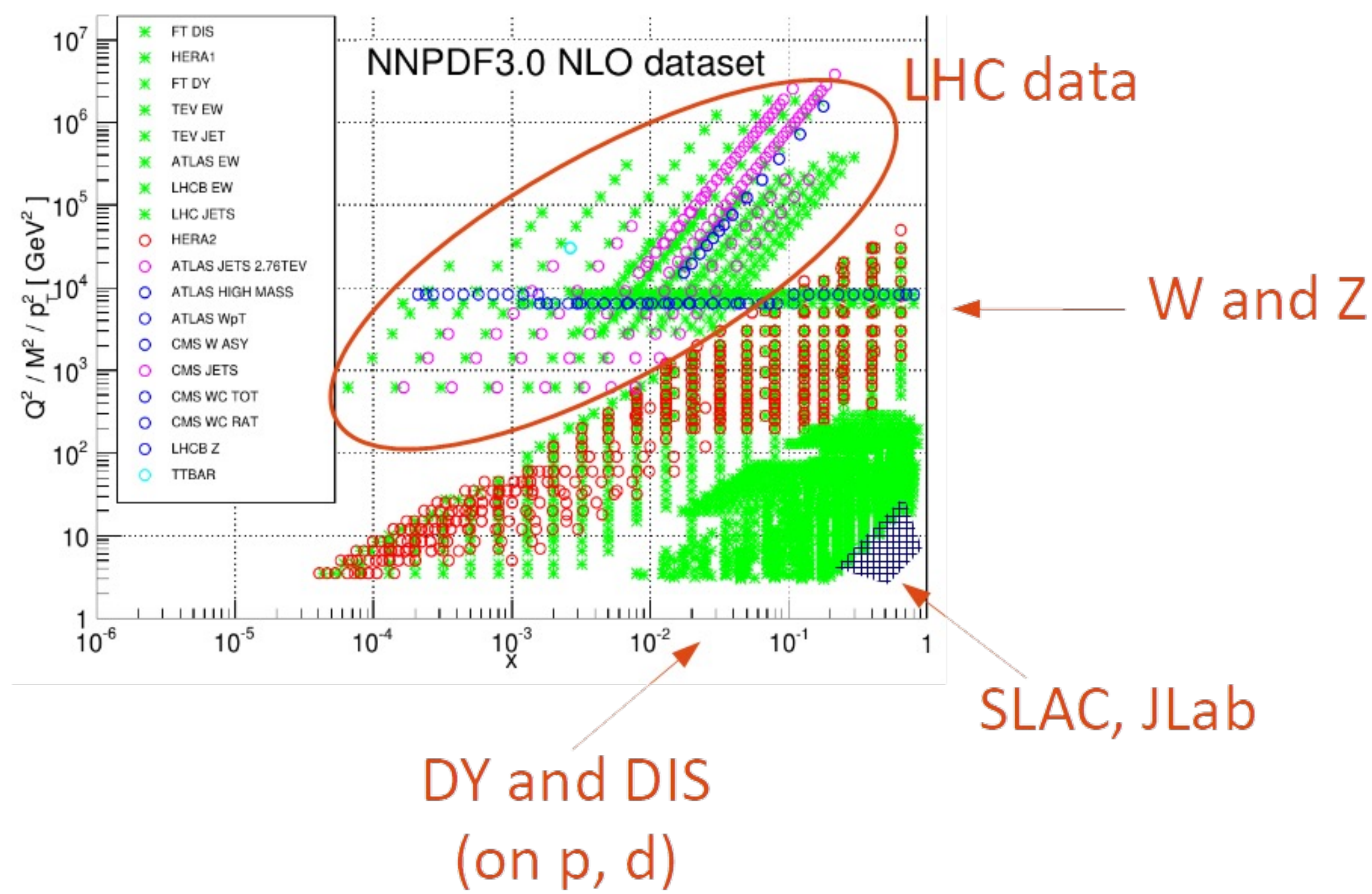

40+ years of experience

CTEQ-JLab collaboration

Coordinate theory+experiment effort within Jefferson Lab

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Recent works:

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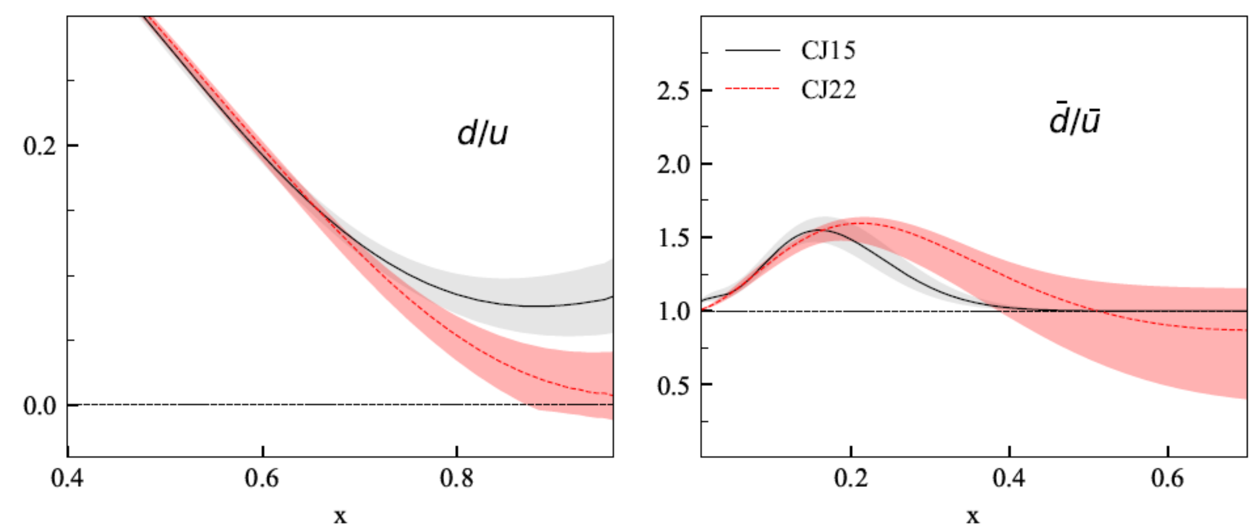

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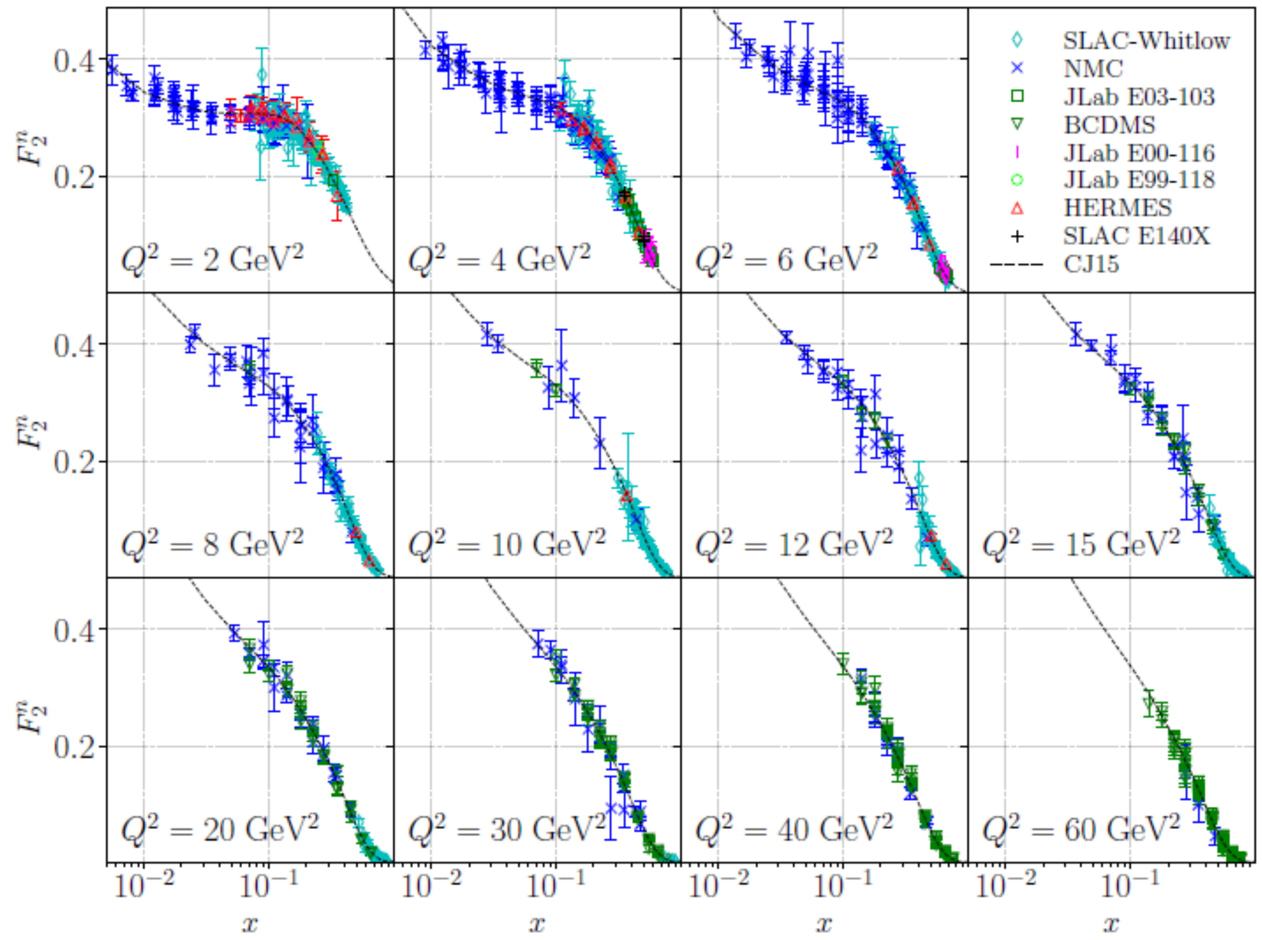

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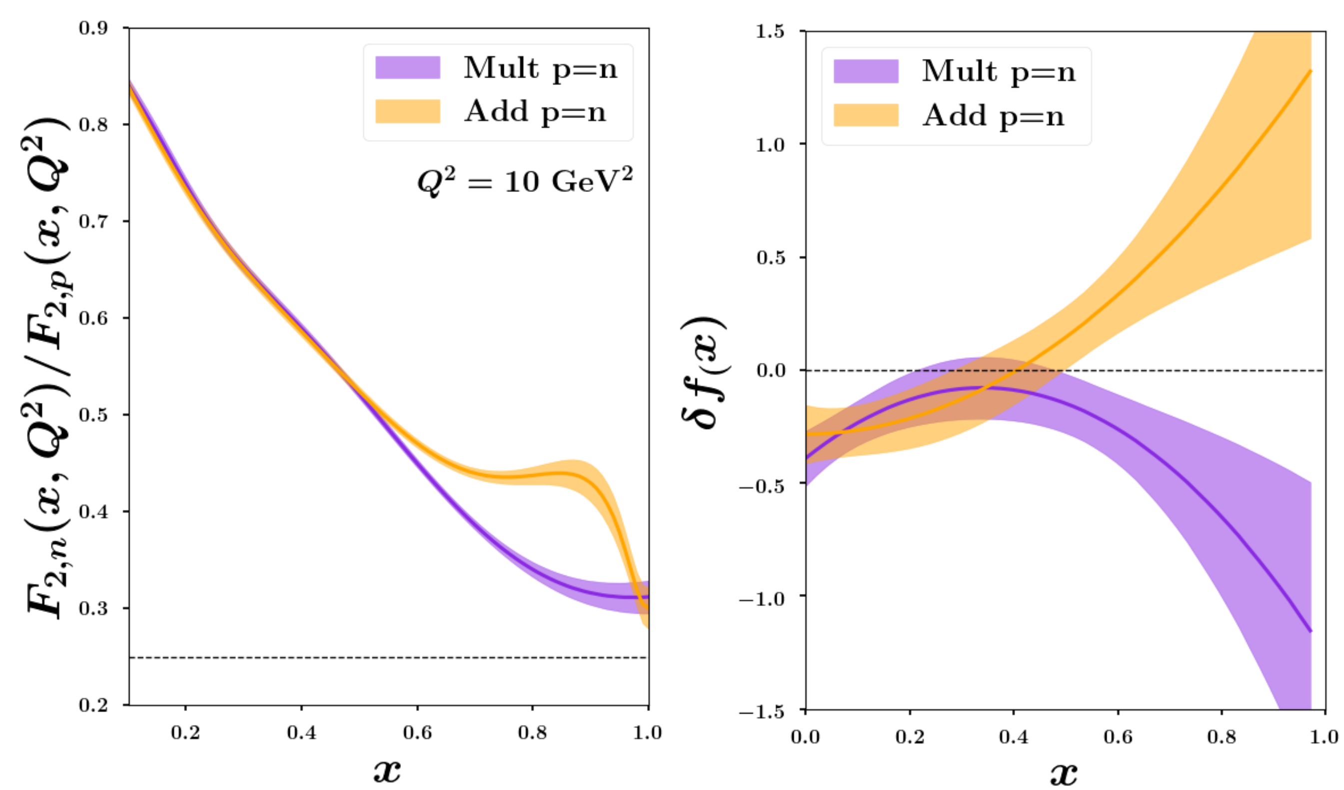

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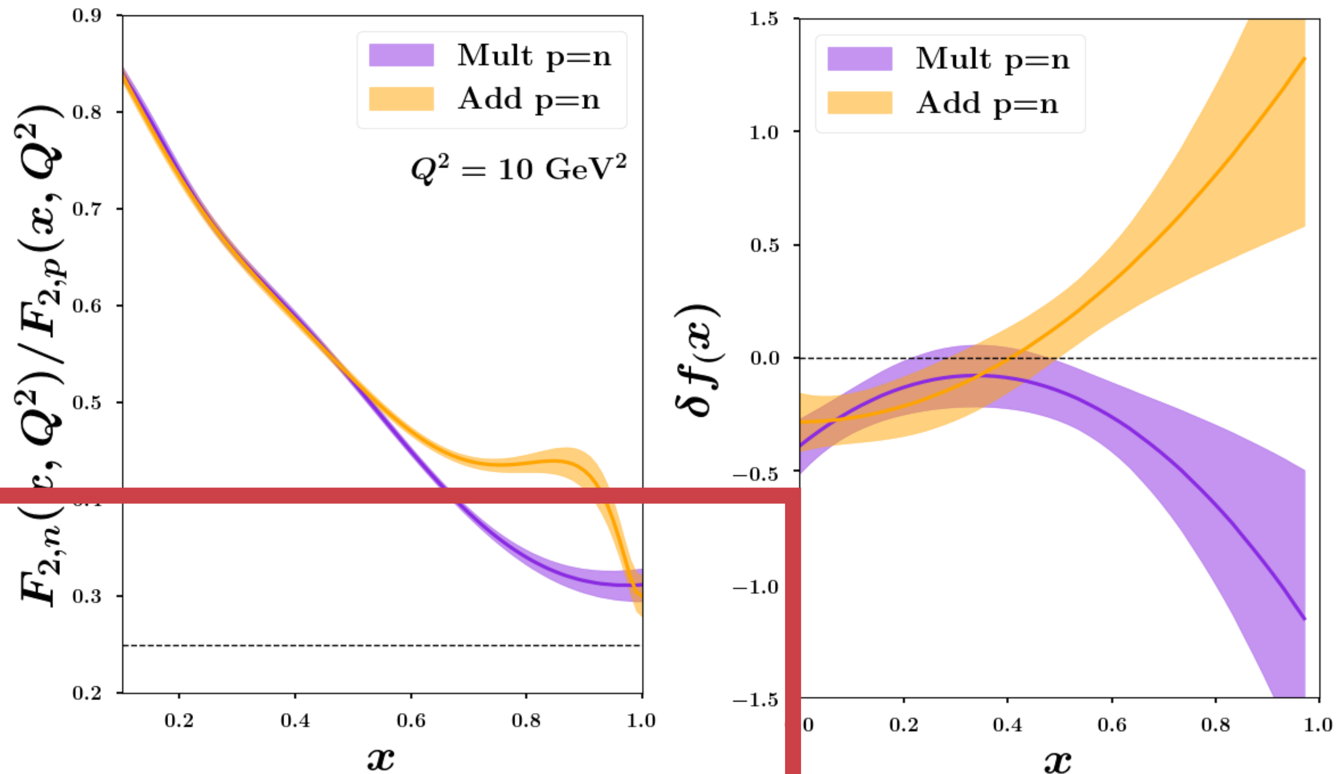


+ ...

$F_2(n)$

Li, Accardi, MC, Fernando et al., PRD 109 (2024)

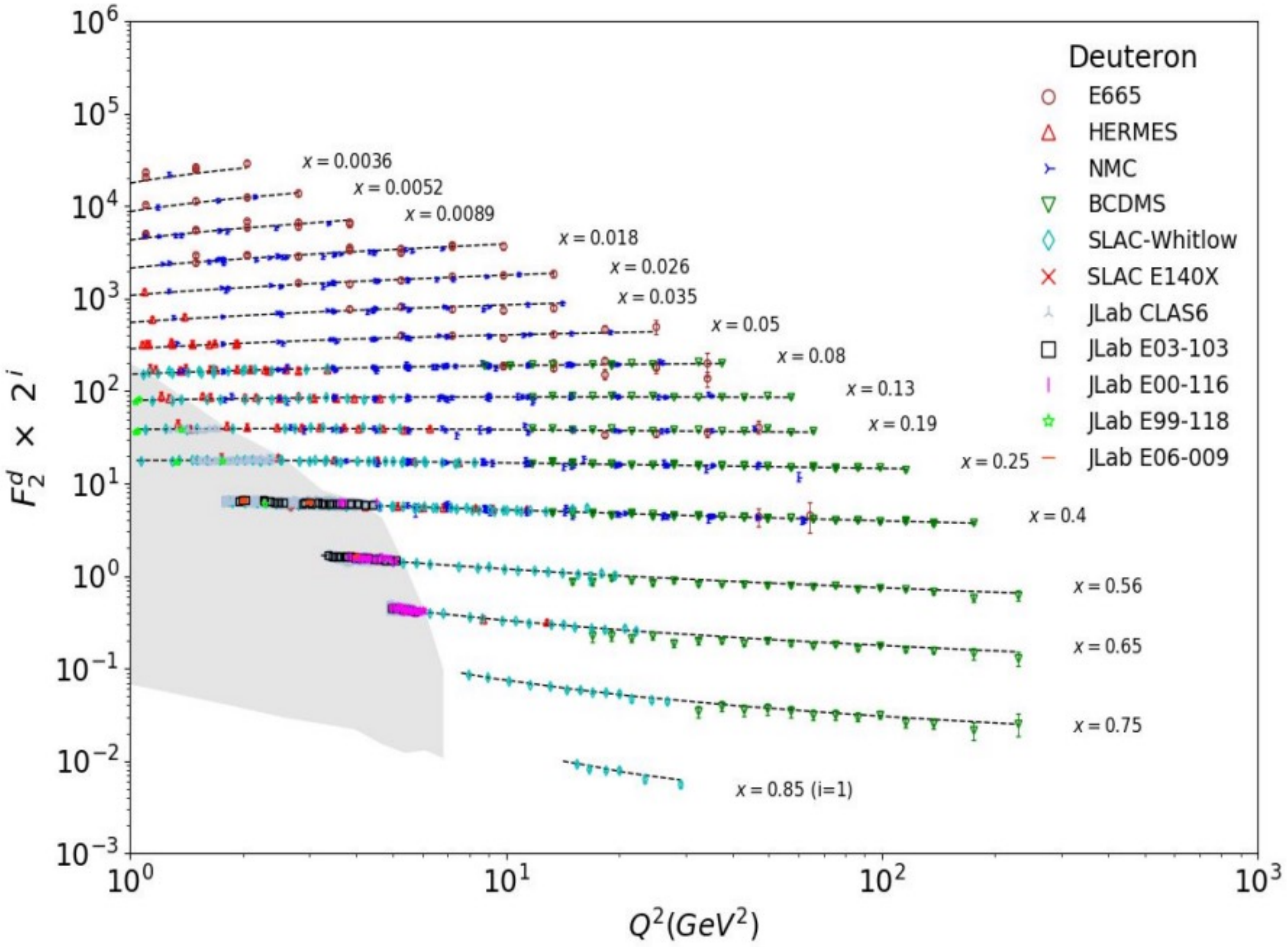
Data-driven generation of neutron data set

Extraction of neutron F_2 structure function

DIS on deuteron target

CJ global data set:

- 1000+ data points
- high-x and low-Q^2
- $W^2 > 3\,\mathrm{GeV}^2$, $Q^2 > 1.69\,\mathrm{GeV}^2$



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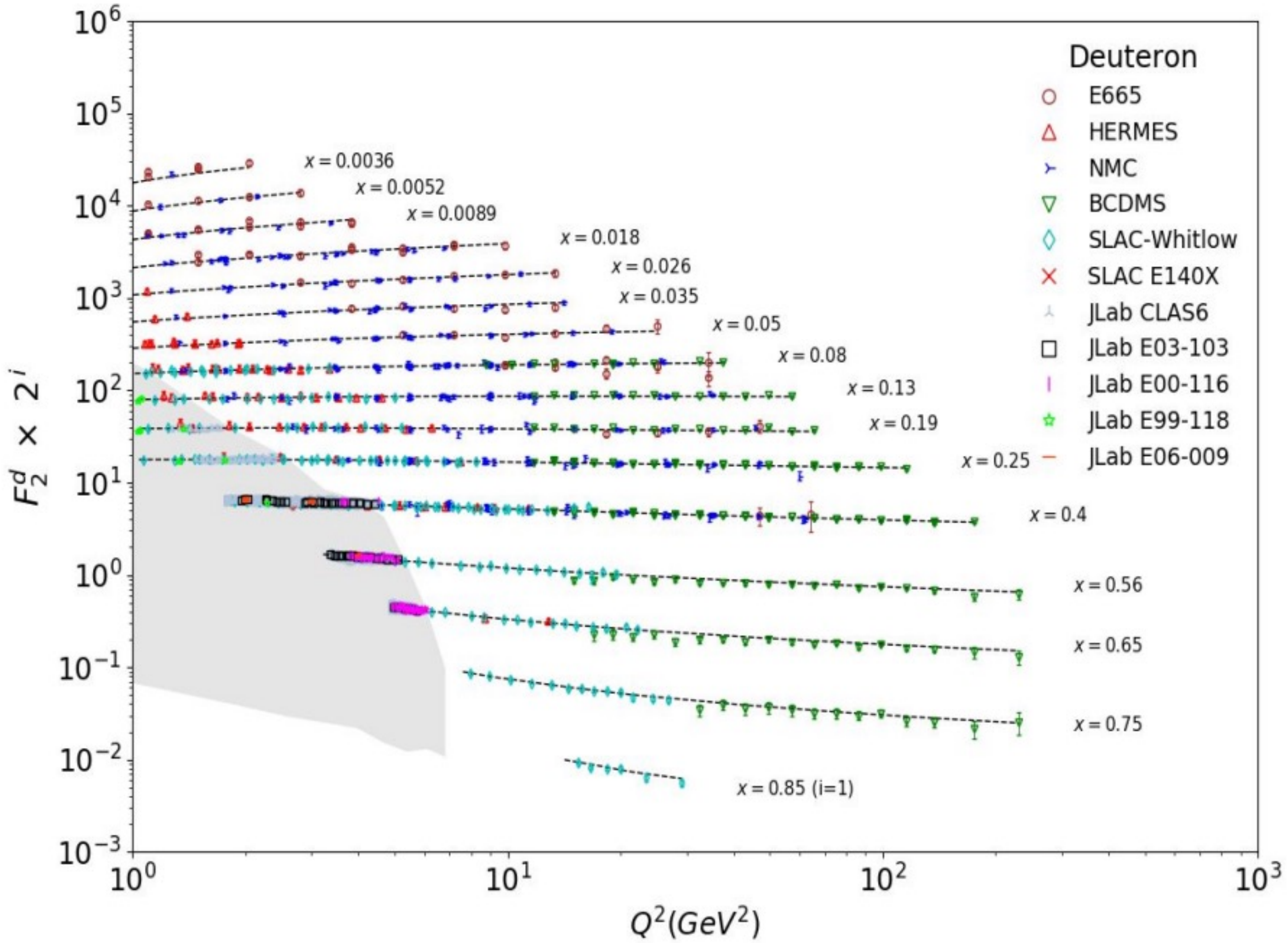

Full treatment of nuclear corrections

Binding effects, Fermi motion, off-shell corrections, Higher Twist (HT), Target Mass Corrections (TMC)

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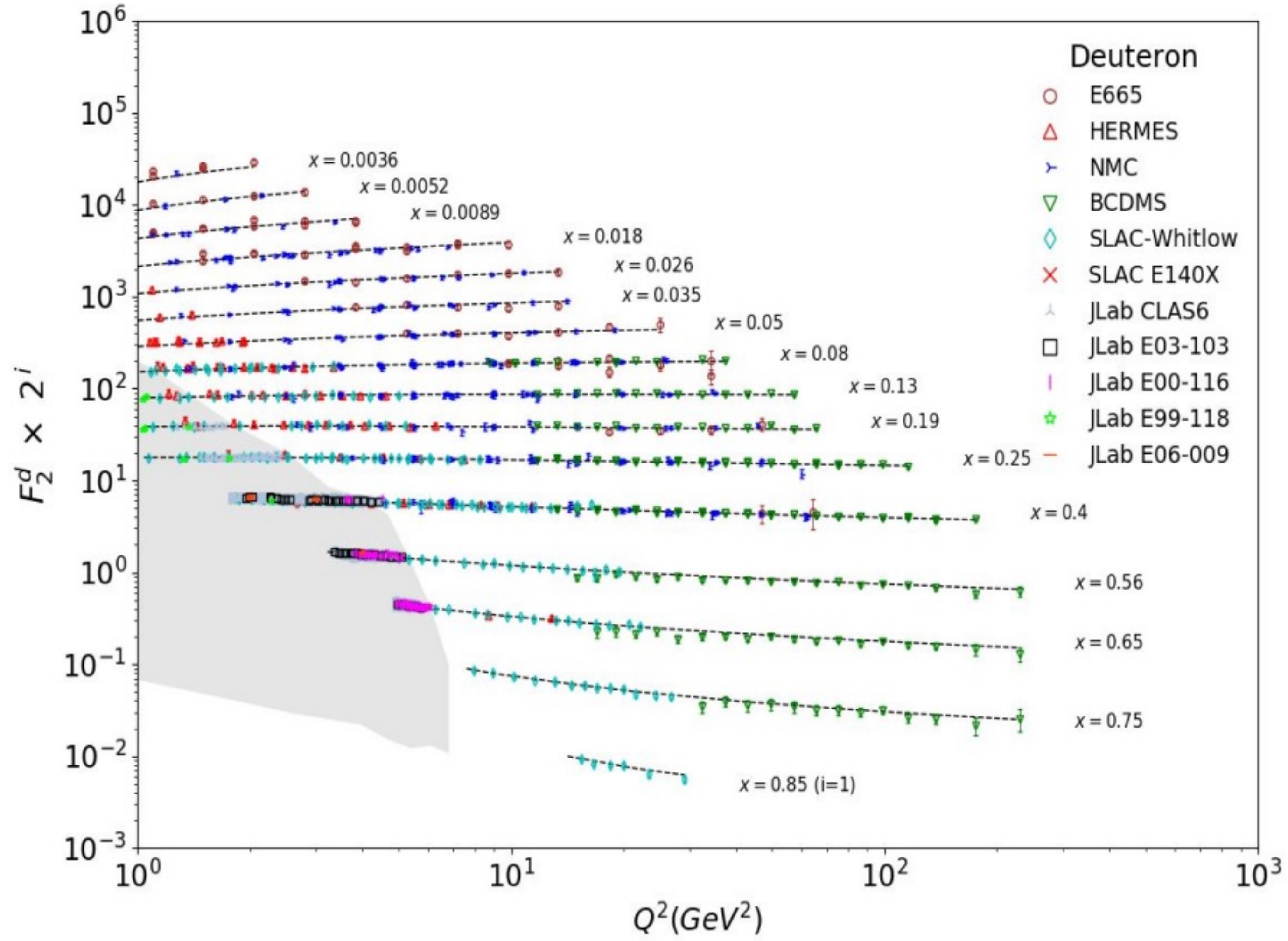

Full treatment of nuclear corrections

Binding effects, Fermi motion, off-shell corrections, Higher Twist (HT), Target Mass Corrections (TMC)

$$F_{2,D}(x_D,Q^2) = \int_{y_{Dmin}}^{y_{Dmax}} dy_D dp_T^2 f_{N/D}(y_D,p_T^2;\gamma) F_{2,N}\left(\frac{x_D}{y_D},Q^2,p^2\right)$$

Smearing function

Structure function of a bound, off-shell nucleon

Extraction of neutron F_2 structure function

Basic idea

$$\widehat{F}_2^{n(0)}(x,Q^2) = \frac{2\,\widehat{F}_2^{d(0)}(x,Q^2)_{\rm exp}}{R_{d/N}^{CJ}(x,Q^2)} - \widehat{F}_2^{p(0)}(x,Q^2)_{\rm exp}$$

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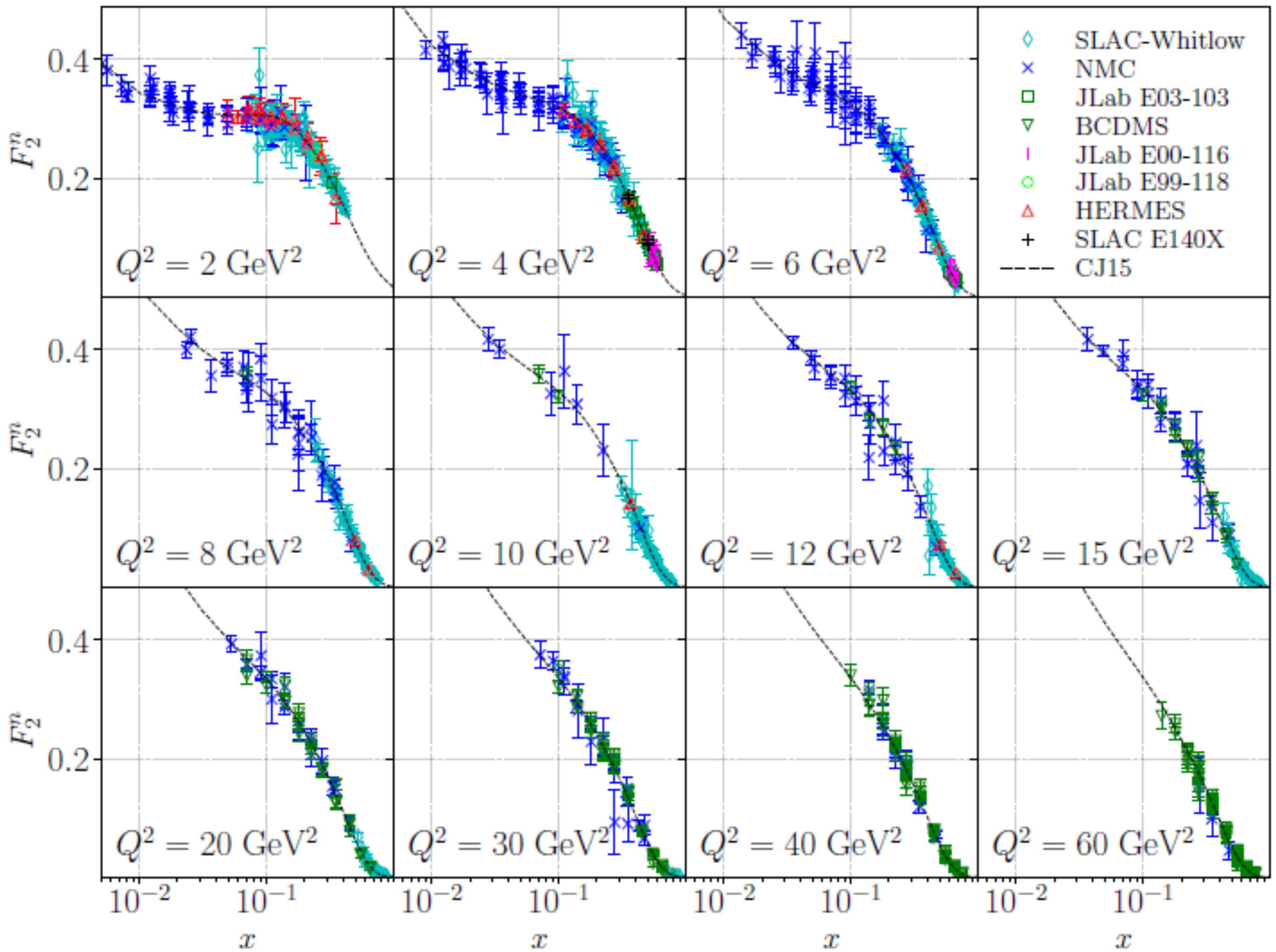
p, d data matching

data cross normalization

results based on CJ15 analysis

extracted experimental bins
centered for applications



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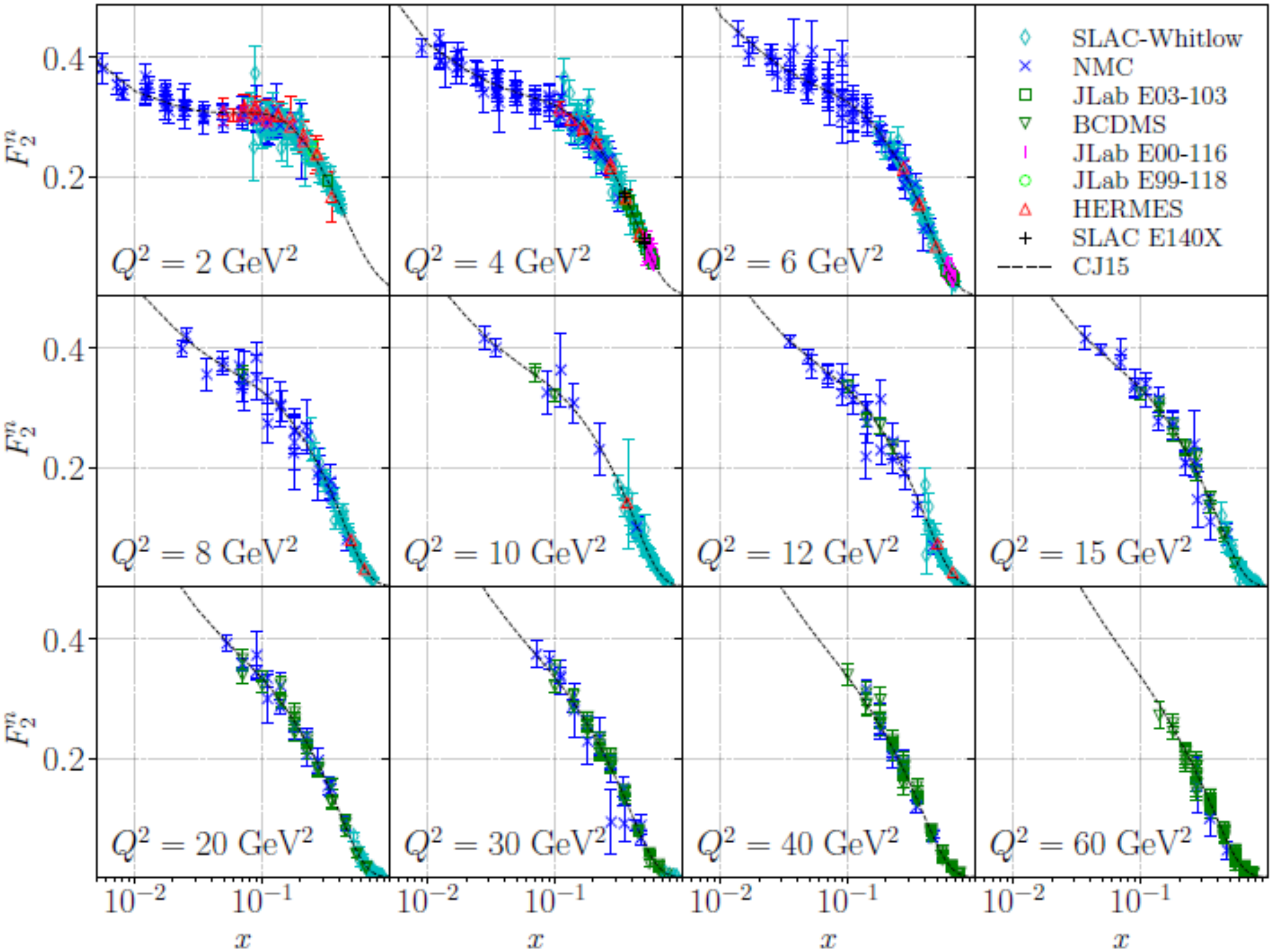
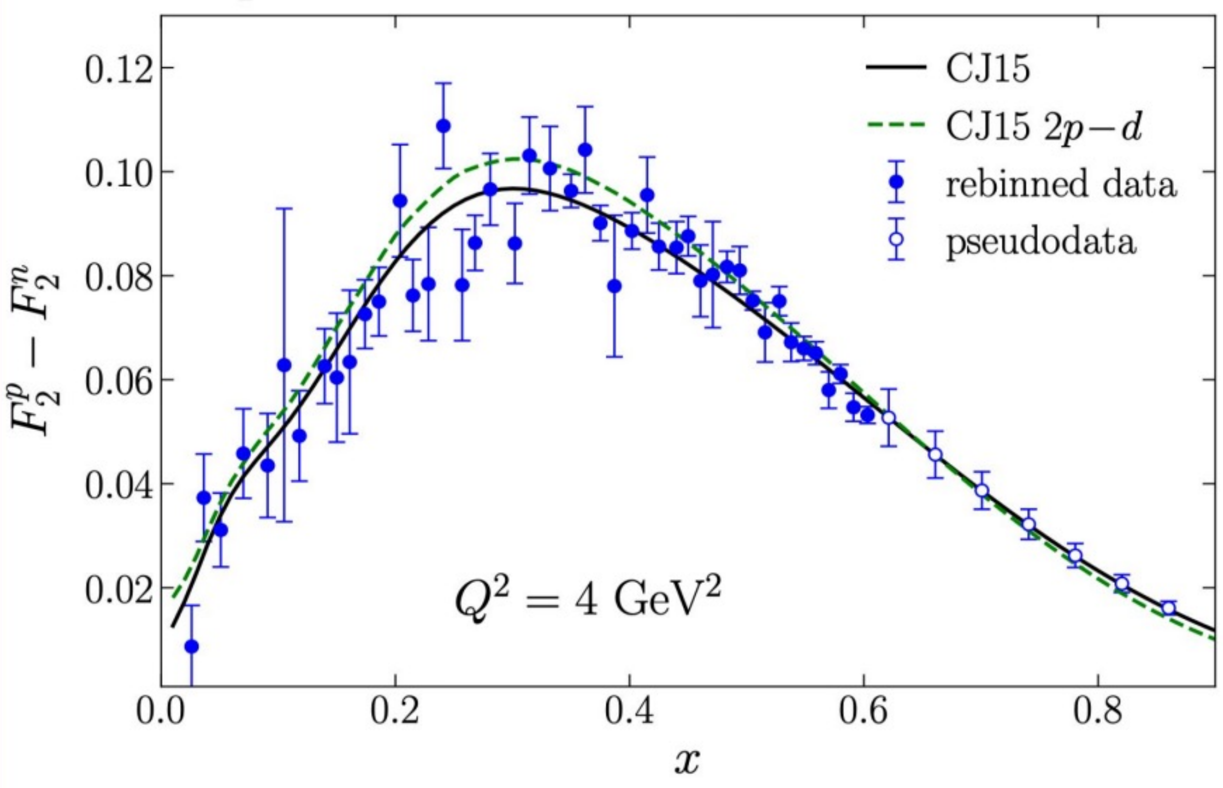


Neutron F2 data sets and grids available!!!

https://github.com/JeffersonLab/CJ-database/

Application: non-singlet moments

$$M_2^{p-n}(Q^2) = \int_0^1 dx \frac{\xi^3}{x^3}\left[\frac{3+9r+8r^2}{20}\right] F_2^{p-n}(x,Q^2)$$



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- **0.01<x<0.6**: Exp. data
- **x>0.6**: CJ15 model

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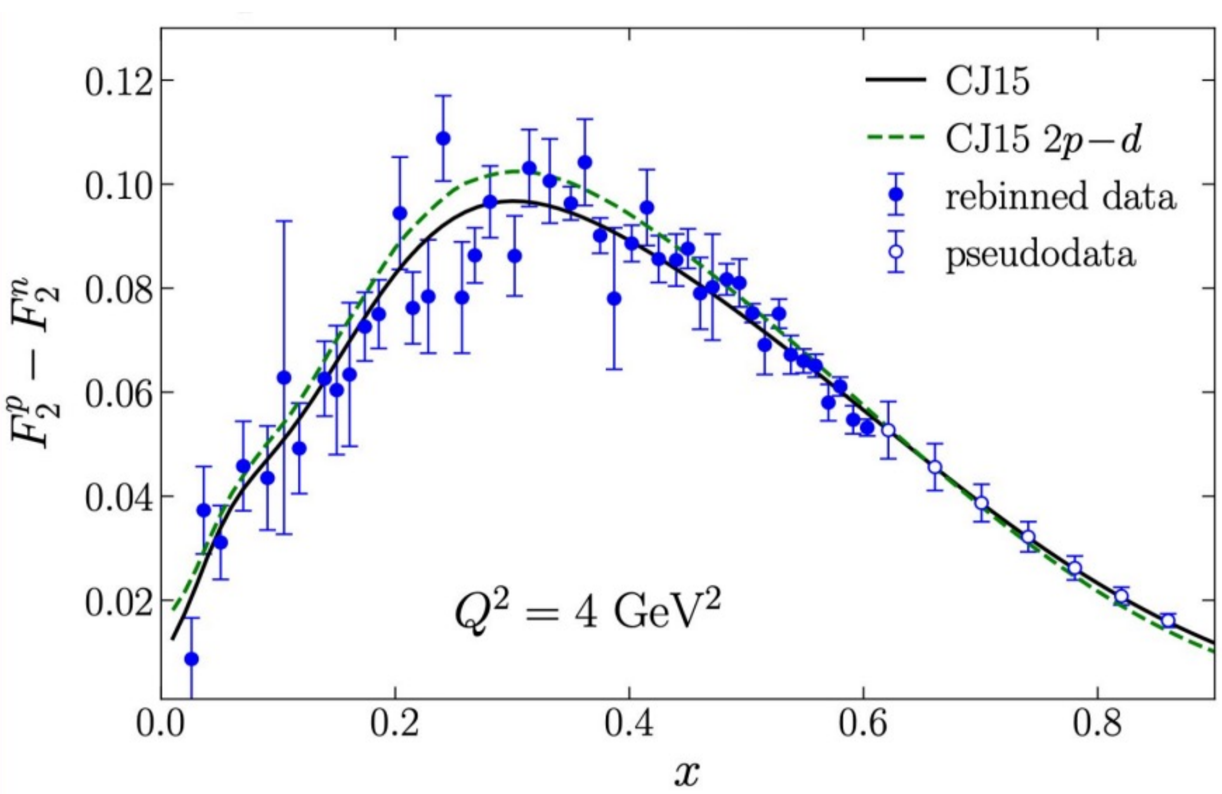
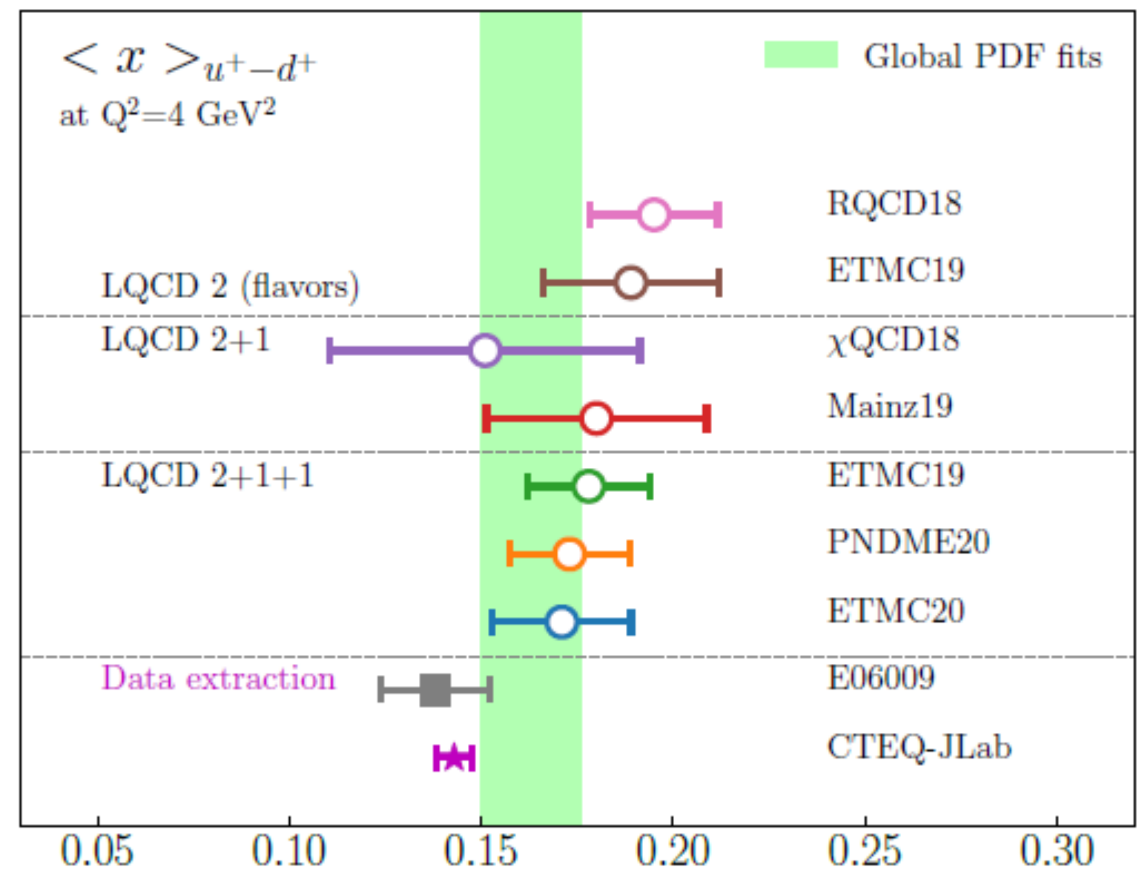
$$\frac{3}{C_2} M_2^{p-n} = \langle x \rangle_{u^+-d^+} + \text{ HT}$$





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$$\langle x \rangle_{u^+-d^+} = \int_0^1 dx x[u(x) + \bar{u}(x) - d(x) - \bar{d}(x)]$$

Application: isoscalar corrections

$$f_A^{iso}(x,Q^2) \simeq \left(\frac{A}{2}\right)\frac{1+F_2^n/F_2^p}{Z+NF_2^n/F_2^p}$$

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EMC effect

Neutrino scattering

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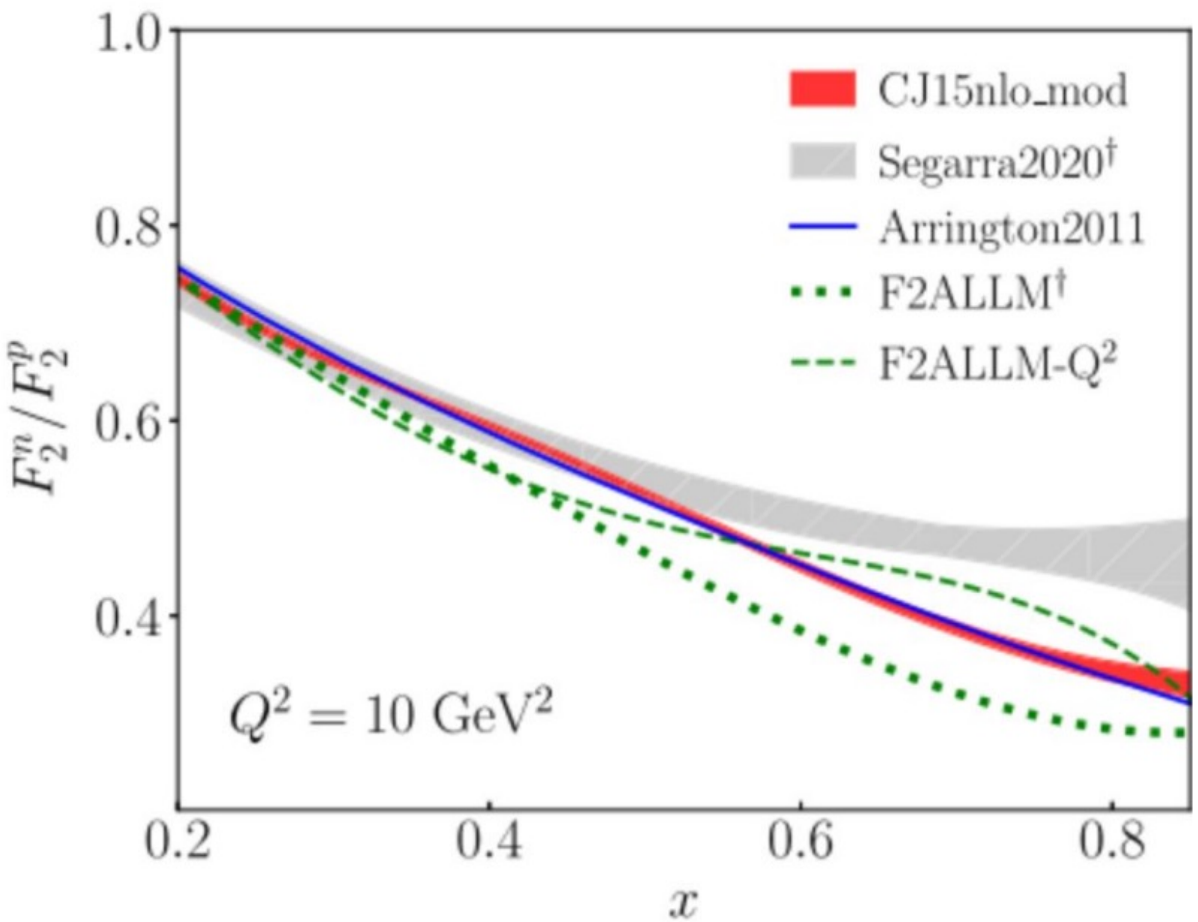

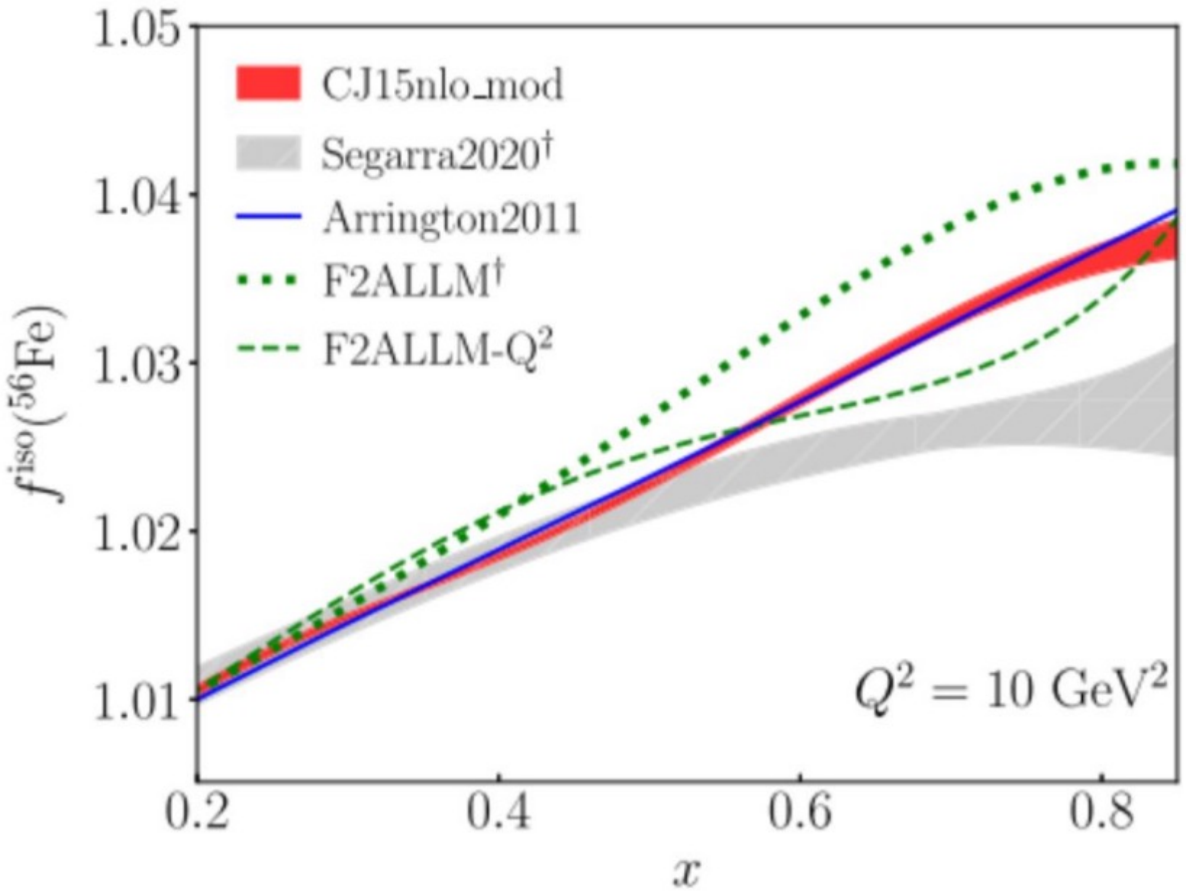

Open database on GitHub

https://github.com/JeffersonLab/CJ-database/

CJ Unpolarized DIS Database Homepage

Reference: arXiv:2309.16851.

See also

- CTEQ-JLab collaboration website.
- note for reduced cross section and F2 calculation.

World DIS data tables

World **proton** and **deuteron** data of unploarized DIS cross sections, F2 structure functions, and the longitudinal to transverse cross section ratio R are collected or extracted from various experiments. Data were collected for the CJ global fit and related analysis. Now open for general use. See details under the data directory.

Neutron F2 extraction

Based on the collected F2 data, we performed a data-driven extraction of **neutron F2** and **neutron-to-proton F2n/F2p ratio** within the CJ15 framework (see eq. 7-9 in reference for details). Data from all experiemnts are cross-normalized and combined into a single Excel file, both in the original kineamtics, as well as rebinned in Q^2. Check the f2n directory.

Structure function grids

Within CJ framework, we calculated various structure functions (F2, F3, FL, etc) at given x, Q^2 grids. Results are provided under folder SFN_grids in the LHAPDF format. An example plotting script is available at `src/plot_sfn.py`

Experiment	σ_r	F2	R
SLAC-Whitlow	p: 10014	p: 10010	p: 10064
	d: 10015	d: 10011	d: 10065
	d/p: 10034	d/p (*): 10034	
SLAC-Whitlow(rebinned)		rebinned p: 10012	
		rebinned d: 10013	
SLAC-E140			d: 10066
SLAC-E140x	p: 10037	p: 10035	p: 10067
	d: 10038	d: 10036	d: 10068
NMC	p: 10022	p: 10020	
	d: 10040	d: 10039	
	d/p:10021	d/p (*):10021	
BCDMS	p: 10018	p: 10016	p: 10069
	d: 10019	d: 10017	d: 10070
JLab E06-009	d: 10042	d: 10041	d: 10071
(includes E04-001, E02-109)			
JLab E94-110	p: 10044	p: 10043	p: 10074
JLab E03-103	p:10047	p:10045	
	d:10048	d:10046	
JLab E99-118	p:10052	p:10049	p: (A)
	d:10053	d:10050	p-d: (A)
	d/p:10054	d/p:10051	
JLab JLCEE96	p: 10055	p: 10072	
	d: 10056	d: 10073	
JLab E00-116	p: 10003	p: 10001	
	d: 10004	p: 10002	
CLAS6	p: 10059	p: 10057	
	d: 10060	d: 10058	
BONUS		n: 10061	
		n/d: 10033	
HERA I+II	p: 10026 - 10032		
HERMES	p: 10007	p: 10005	
	d: 10008	d: 10006	
	d/p: 10009		
E665		p: 10062	
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LHAPDF grids

F_2, F_L, F_3 $\quad$ $\gamma, \gamma Z, Z$

w/, w/o HT

HTvsOffshell

in preparation

Bias in the approach identified

…and solved!

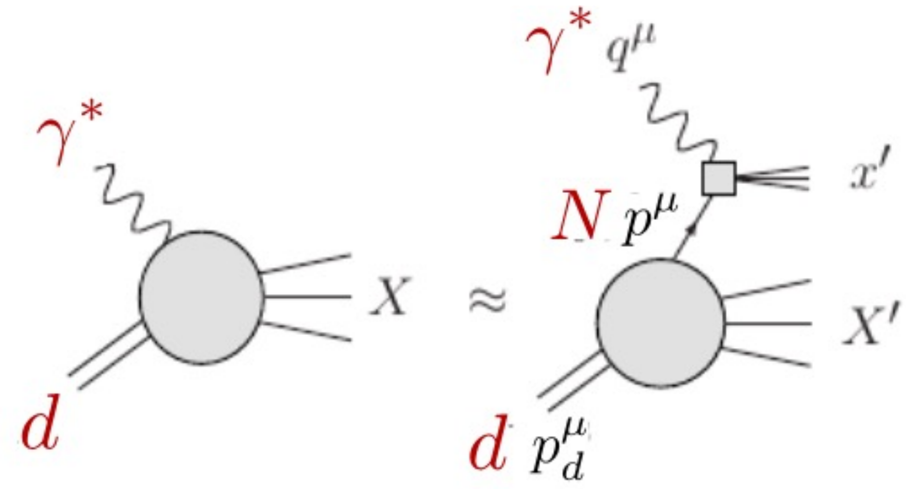
Deuterium: off-shell corrections

Deuterium: off-shell corrections

Bound, off-shell nucleon inside the deuteron

$$p^2 < m_N^2$$

Structure functions are deformed at large x



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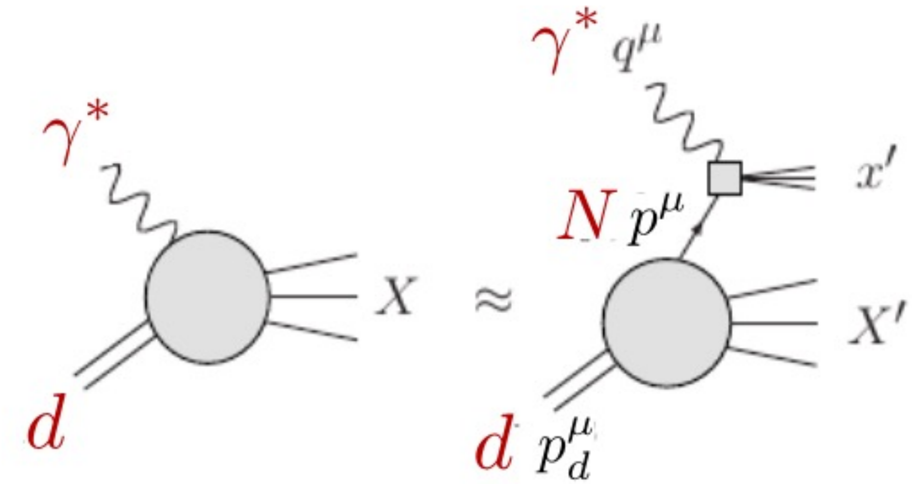

Off-shell expansion (in nucleon virtuality p^2)

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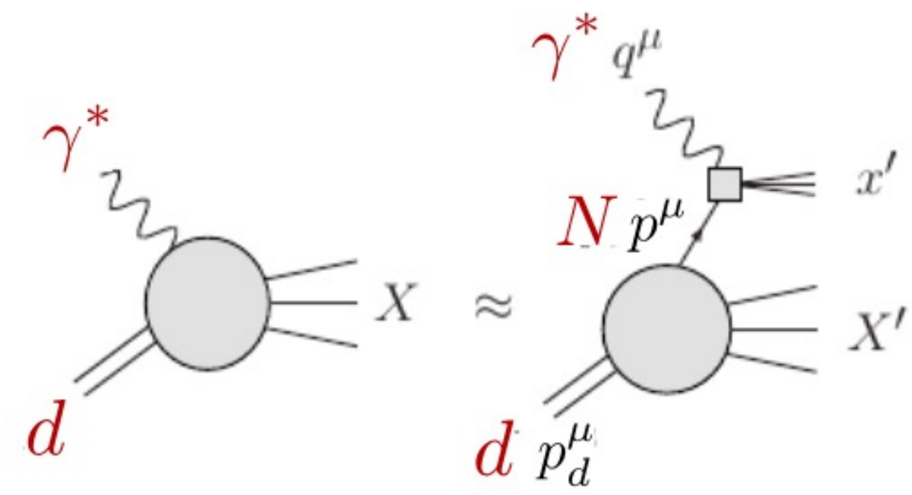

Off-shell expansion (in nucleon virtuality p^2)

parton level

$$q_N(x, Q^2, p^2) = q_N^{\text{free}}(x, Q^2)\left[1 + \frac{p^2 - M^2}{M^2}\delta f(x)\right]$$

Kulagin, Piller, Weise, PRC 50 (1994)
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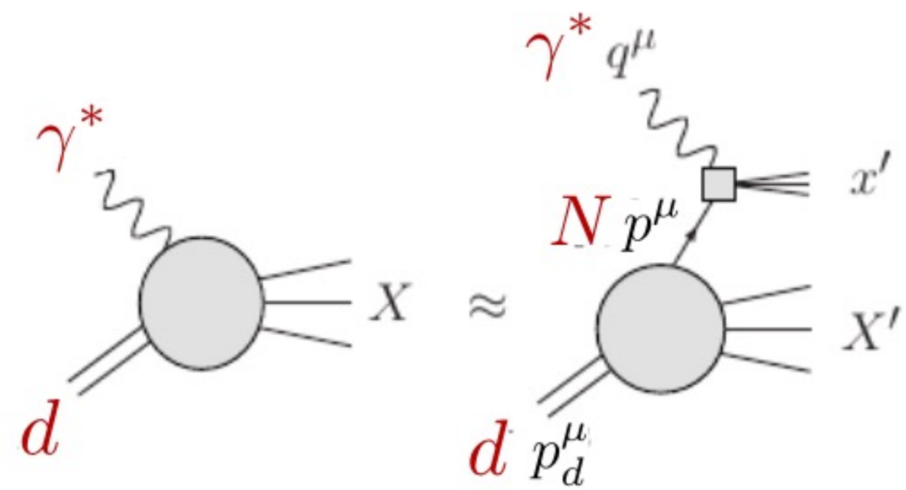

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struct. func level

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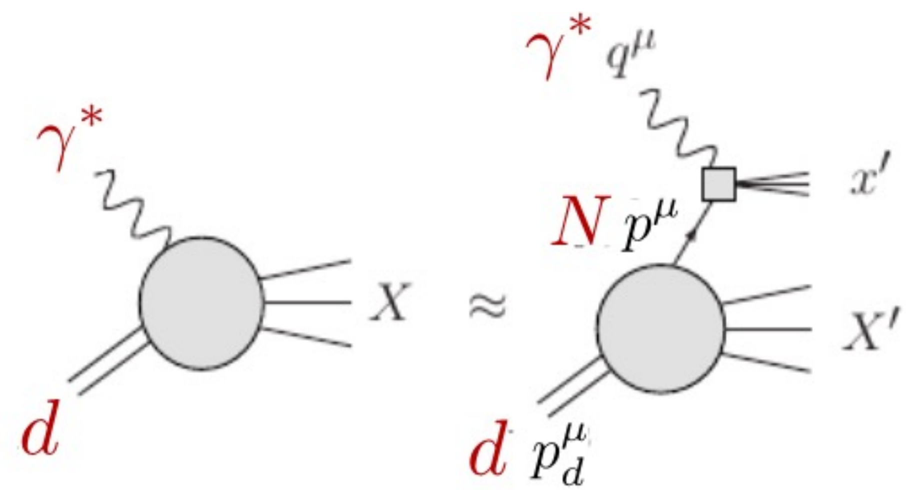

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Free nucleon pdfs/SFs

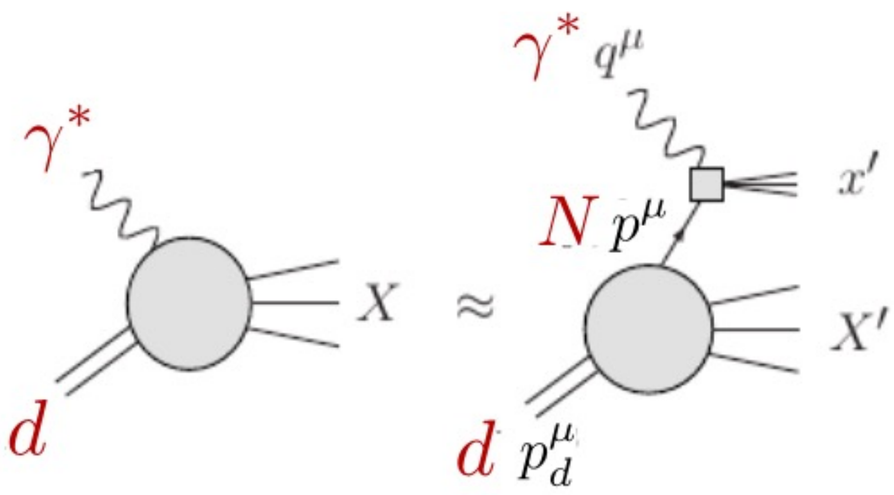
$$p^2 = m_N^2$$

Deuterium: off-shell corrections

Bound, off-shell nucleon inside the deuteron

$$p^2 < m_N^2$$

Structure functions are deformed at large x



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Free nucleon pdfs/SFs

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Off-shell function

(To be fitted)

Polynomial off-shell function

Polynomial off-shell function

$$\delta f^N = C(x - x_0)(x - x_1)(1 + x_0 - x)$$

KP-like model

Kulagin and Petti, NPA 765 (2006)

+ valence sum rule

$$\int_0^1 dx\, \delta f^N(x)\ [q(x) - \bar{q}(x)] = 0$$

Polynomial off-shell function

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Release the assumption of the valence sum rule

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$$\int_0^1 dx\, \delta f^N(x)\, [q(x) - \bar{q}(x)] = 0$$

Release the assumption of the valence sum rule

C, x_0 and x_1 fitted $\Rightarrow$ $x_1 \simeq x_0$

Polynomial off-shell function

$$\delta f^N = C(x - x_0)(x - x_1)(1 + x_0 - x)$$ KP-like model Kulagin and Petti, NPA 765 (2006)

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Alekhin, Kulagin, Petti, PRD 96 (2017)

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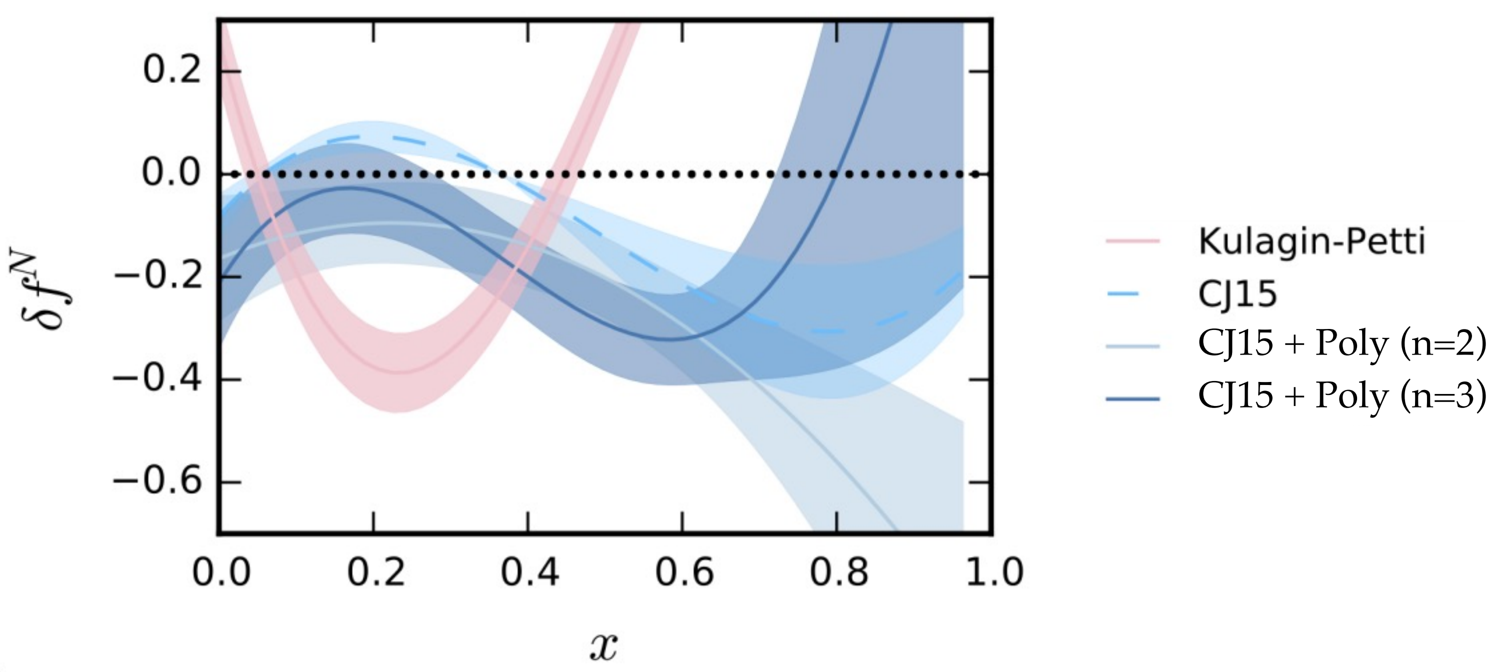

Better agreement with the data w/o imposing nodes a priori to the off-shell function

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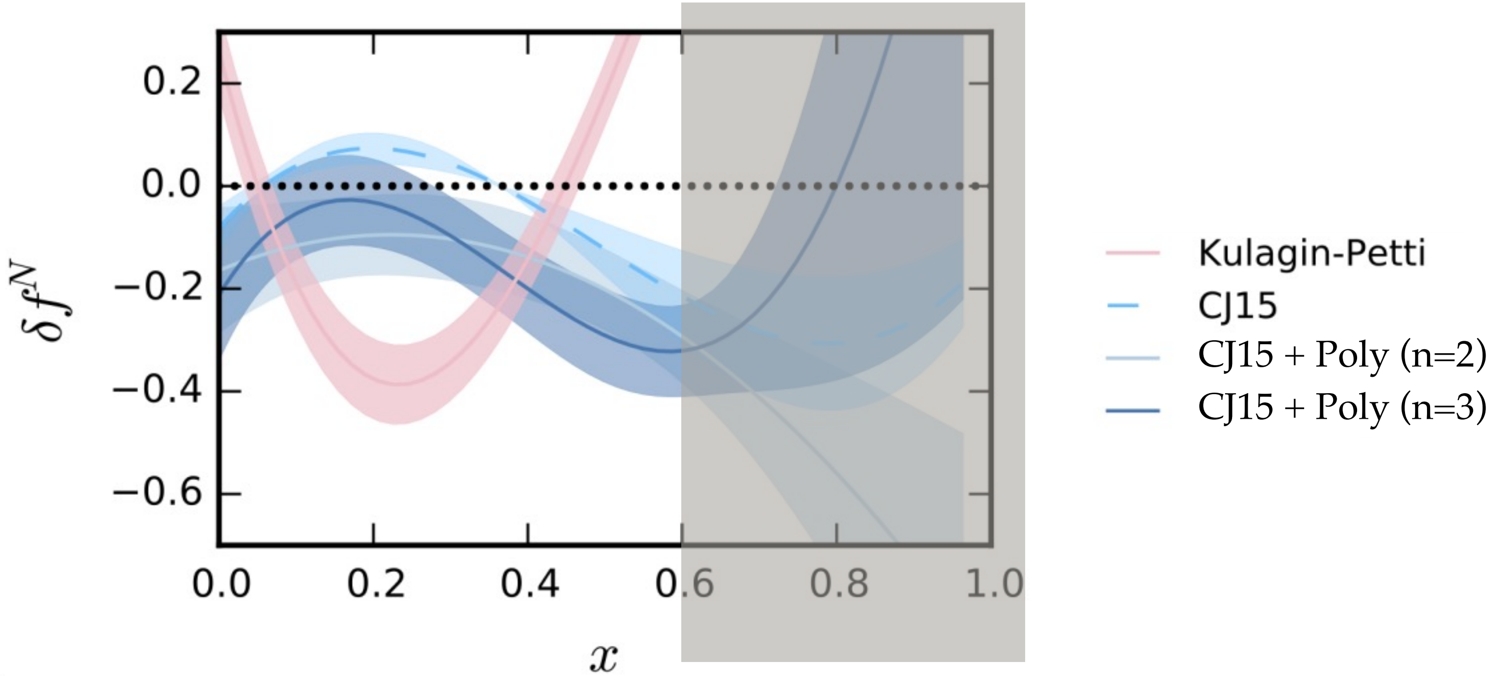

Better agreement with the data w/o imposing nodes a priori to the off-shell function

Constrain power of CJ15 dataset only up to x = 0.6

Higher-Twist function

Higher Twist correction

Higher-Twist function

Higher Twist correction

Multiplicative

$$F_2(x,Q^2) = F_2^{LT}(x,Q^2)\left(1+\frac{\boldsymbol{C}(x)}{Q^2}\right)$$

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Additive

$$F_2 = F_2^{LT}(x,Q^2) + \frac{\boldsymbol{H}(x)}{Q^2}$$

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Are experimental observables independent of the choice of the HT?

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Bias in n/p function

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structure function is smaller

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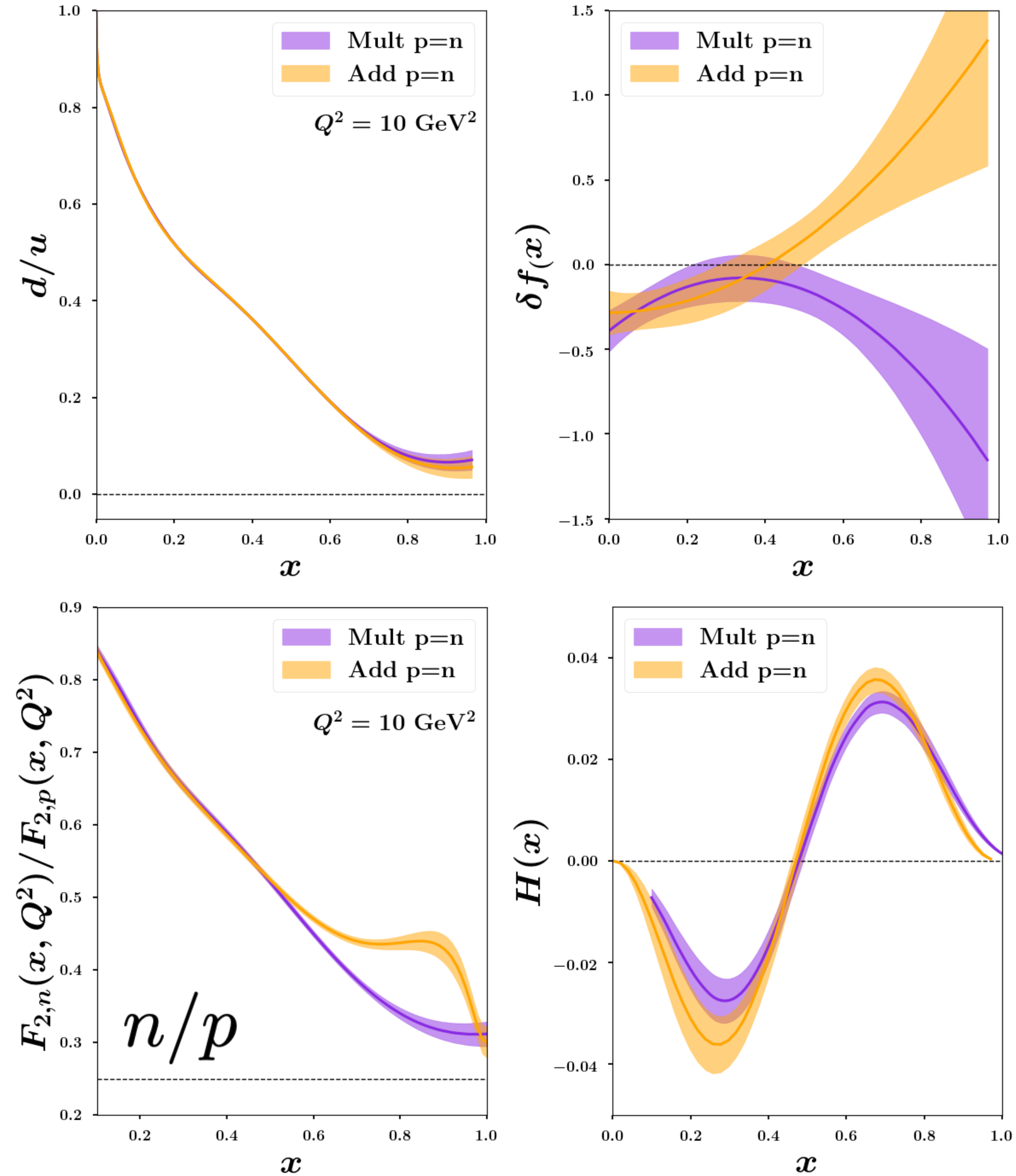
same as Add

Bias not present!

Results in the CJ fitting framework

Case 1: isospin symmetry



Results in the CJ fitting framework

Case 1: isospin symmetry

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Unnaturally large n/p

BUT smaller d/u than Mult

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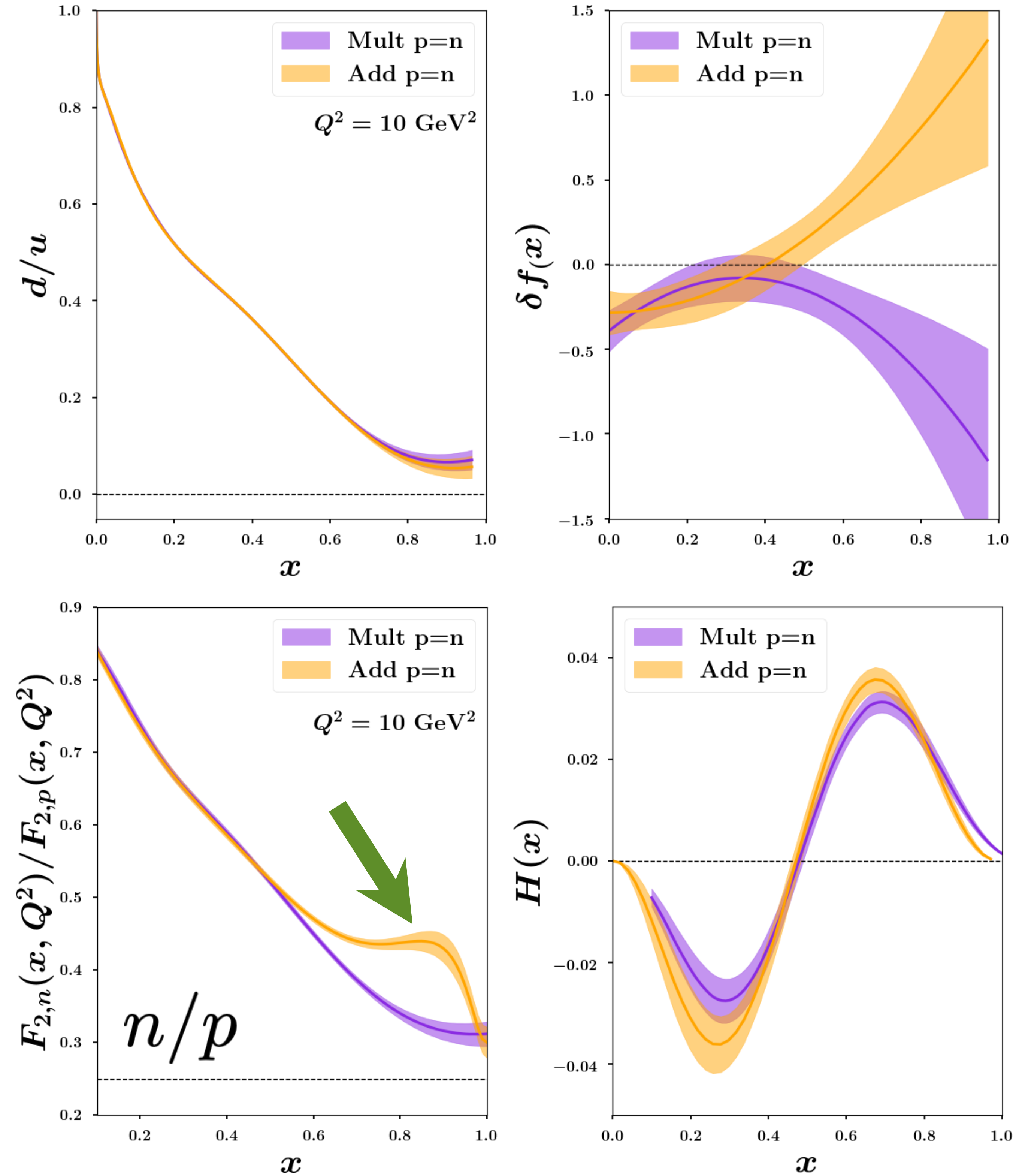

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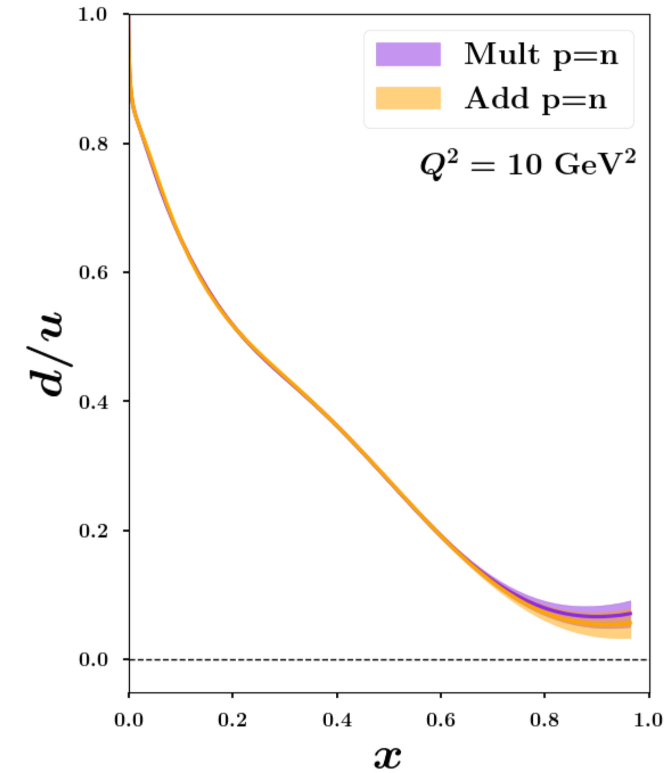
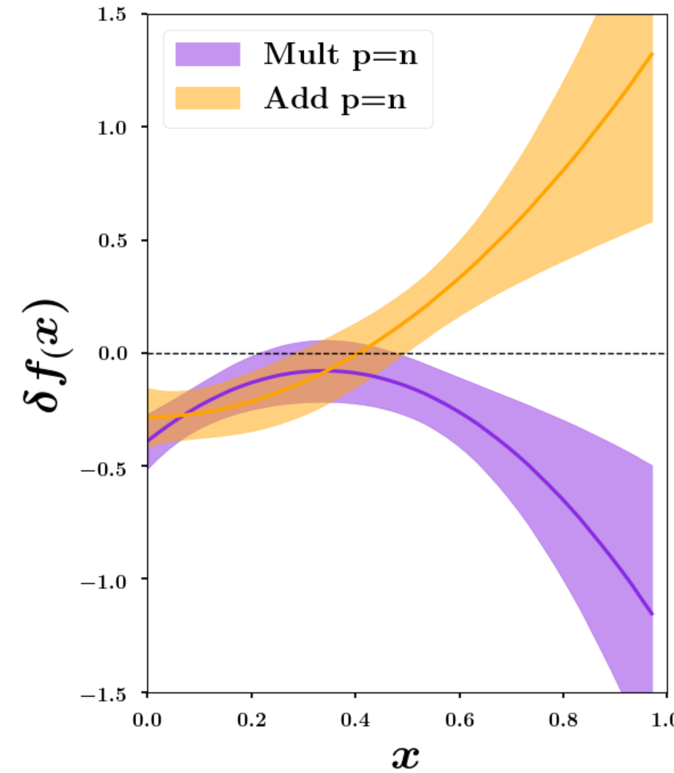
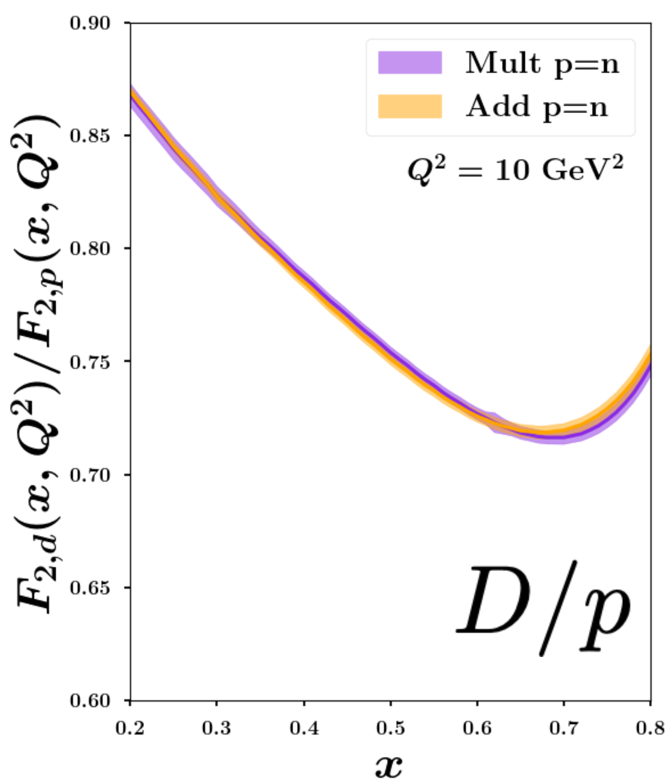
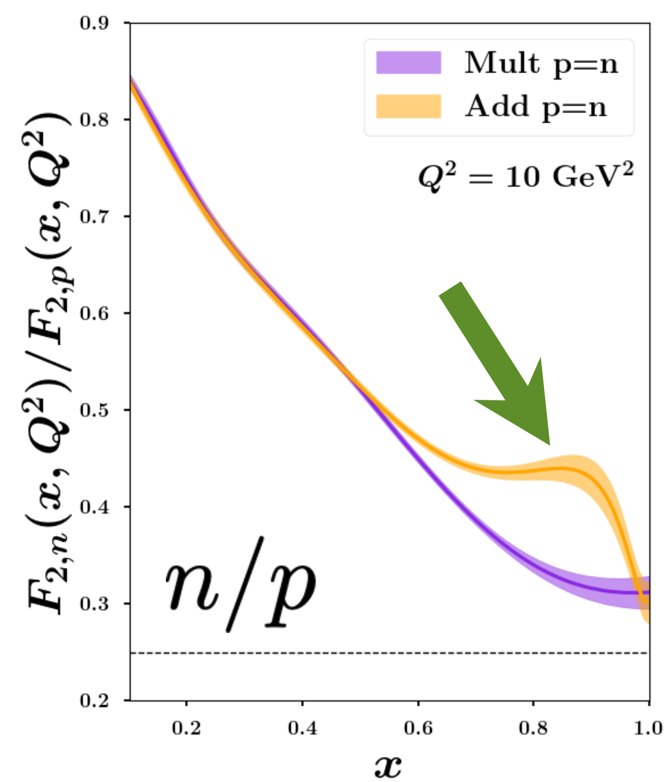
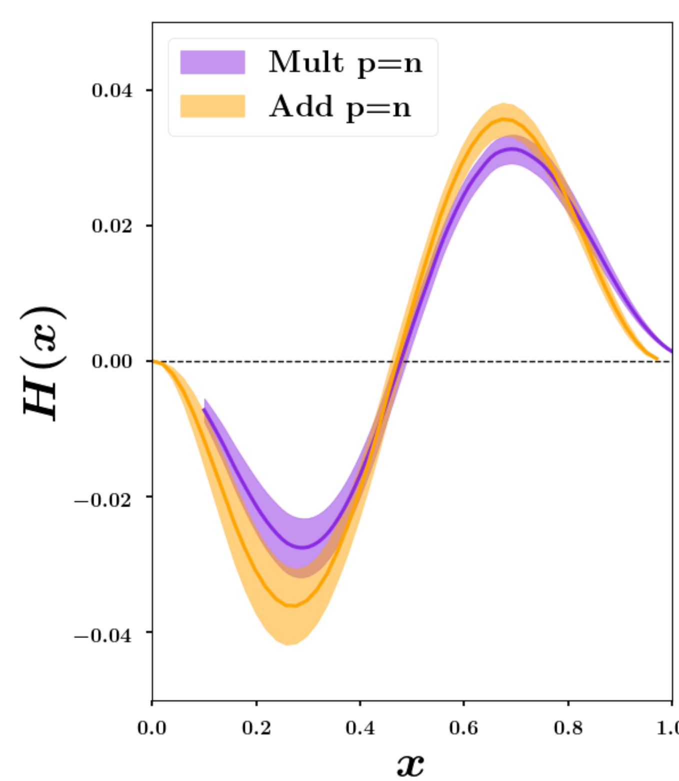
Off-shell compensates n/p bias





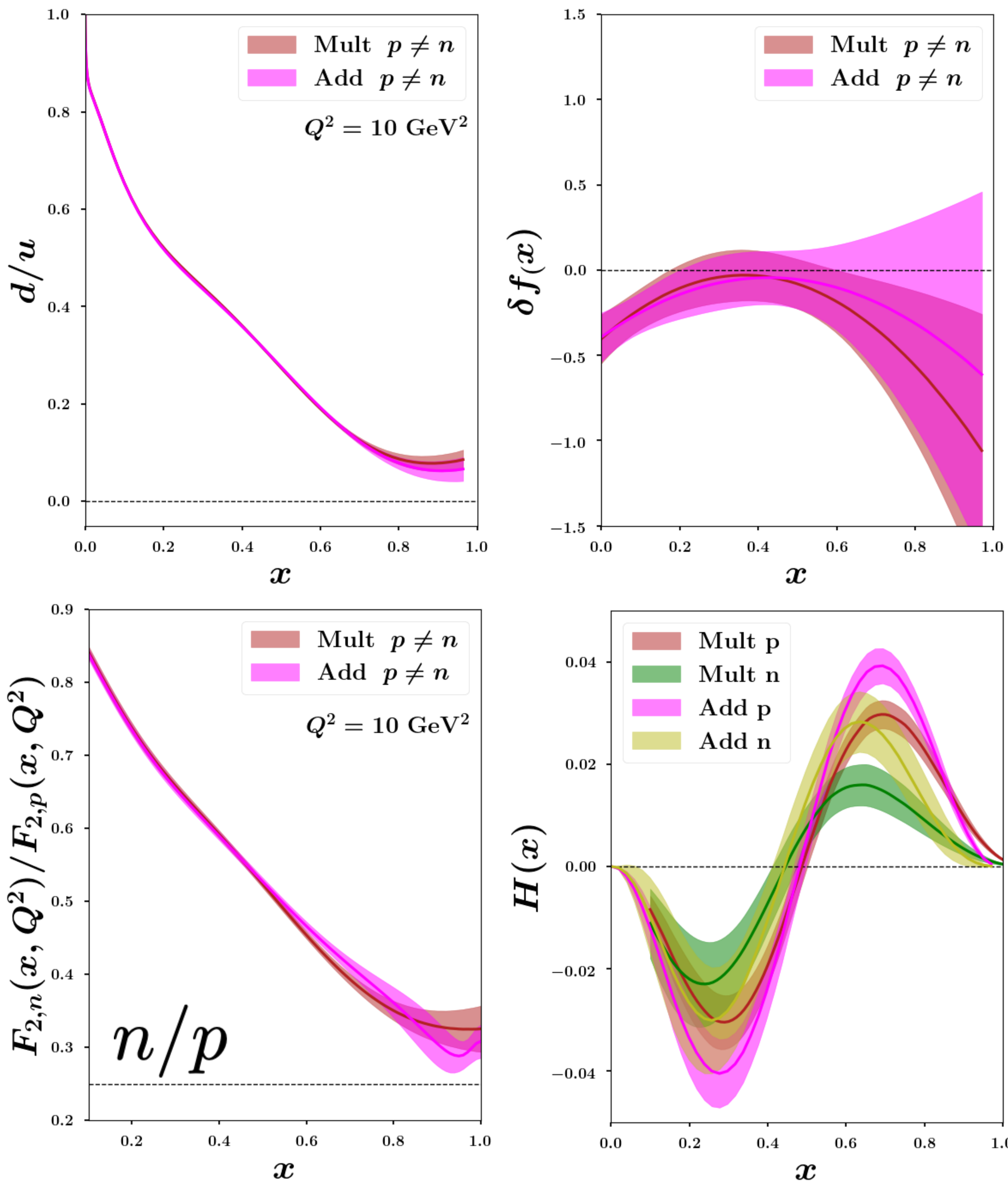






Results in the CJ fitting framework

Case 2: isospin breaking

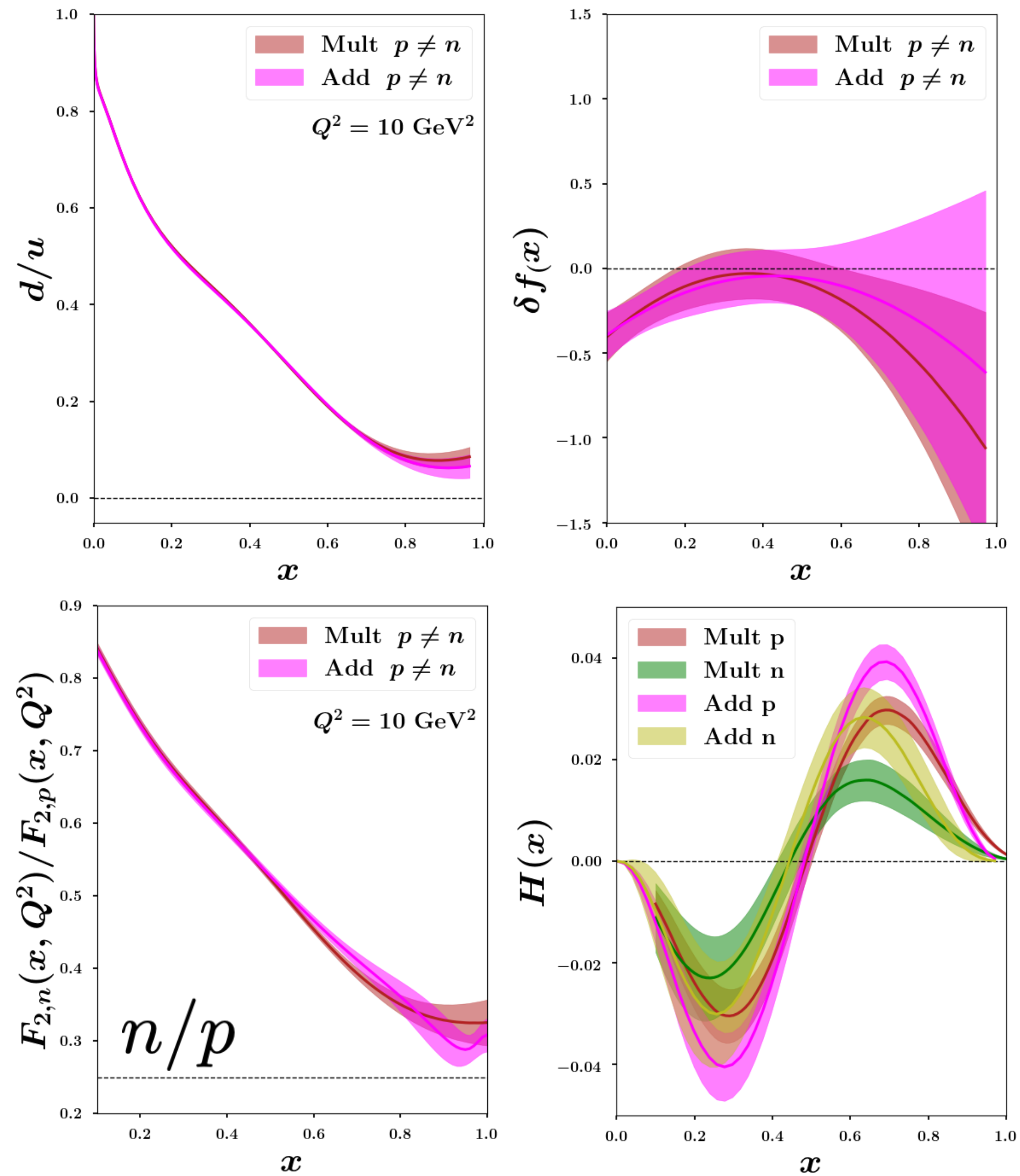


Results in the CJ fitting framework

Case 2: isospin breaking

Compatible n/p

$$H_n(x) \simeq \frac{1}{2} H_p(x)$$



Results in the CJ fitting framework

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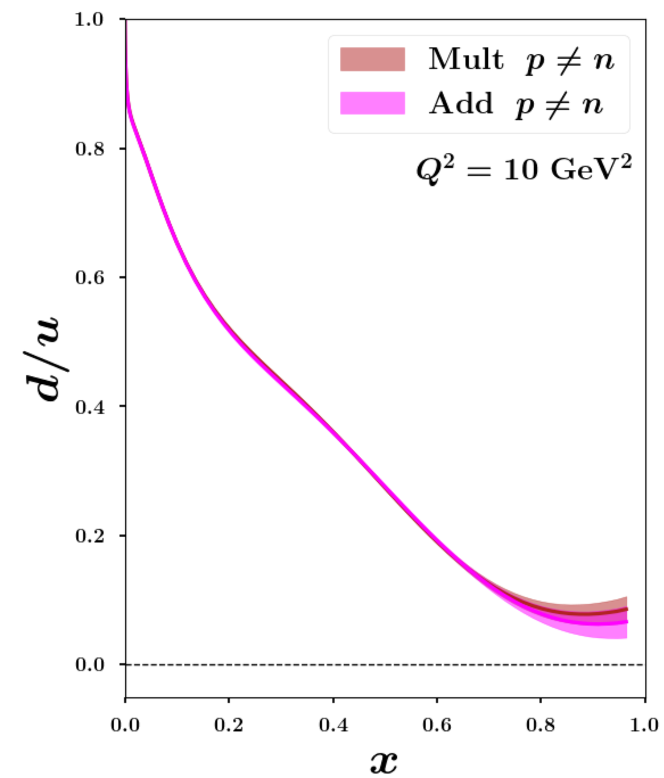
Compatible n/p

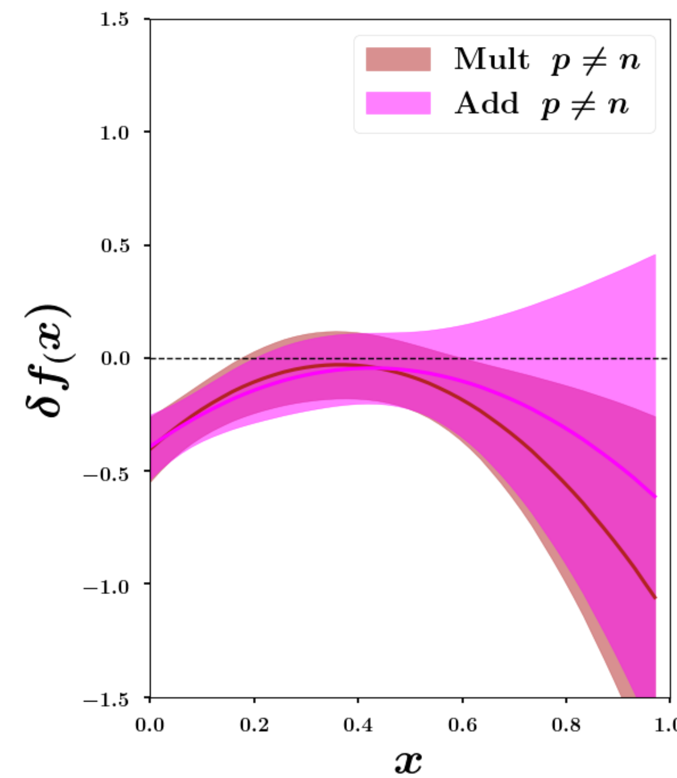
$$H_n(x) \simeq \frac{1}{2} H_p(x)$$

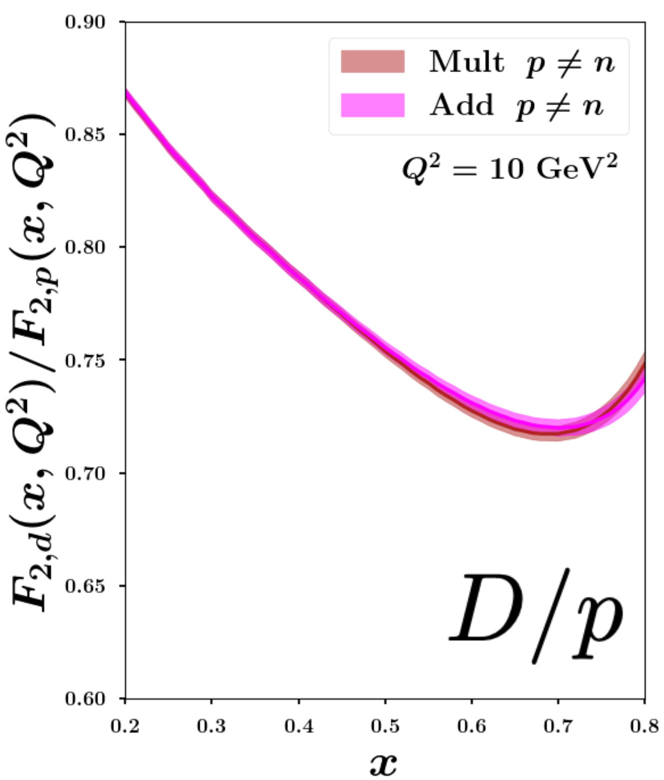
Bias removed

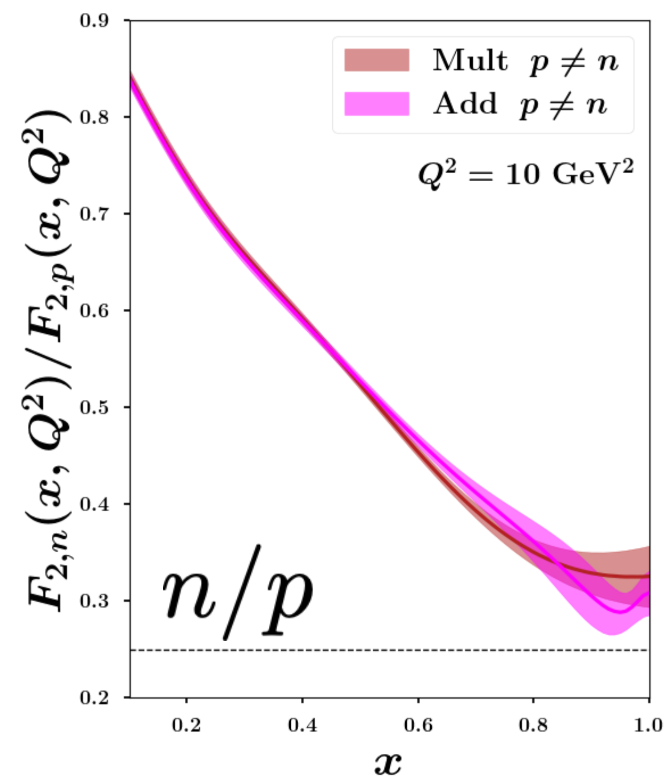
No need of compensation by off-shell

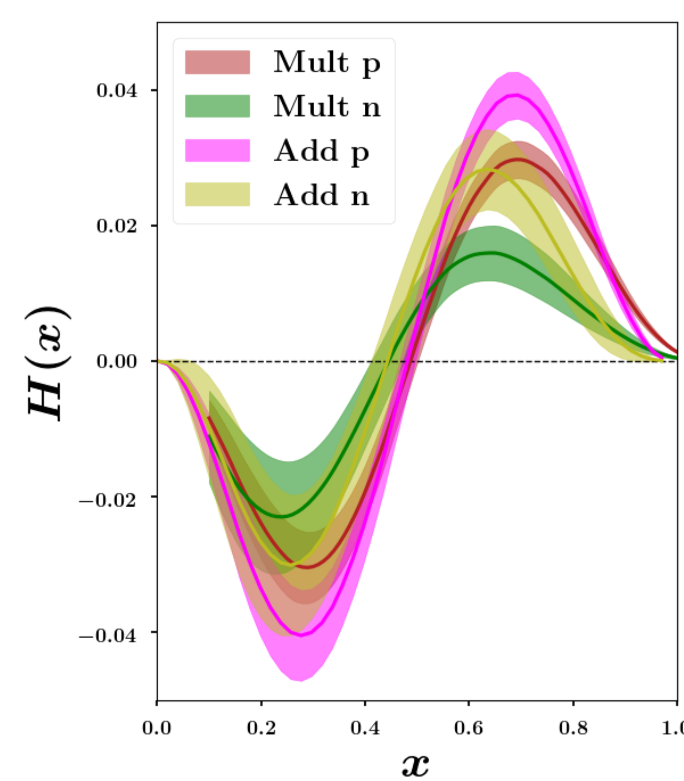
Theory calculation confirmed!





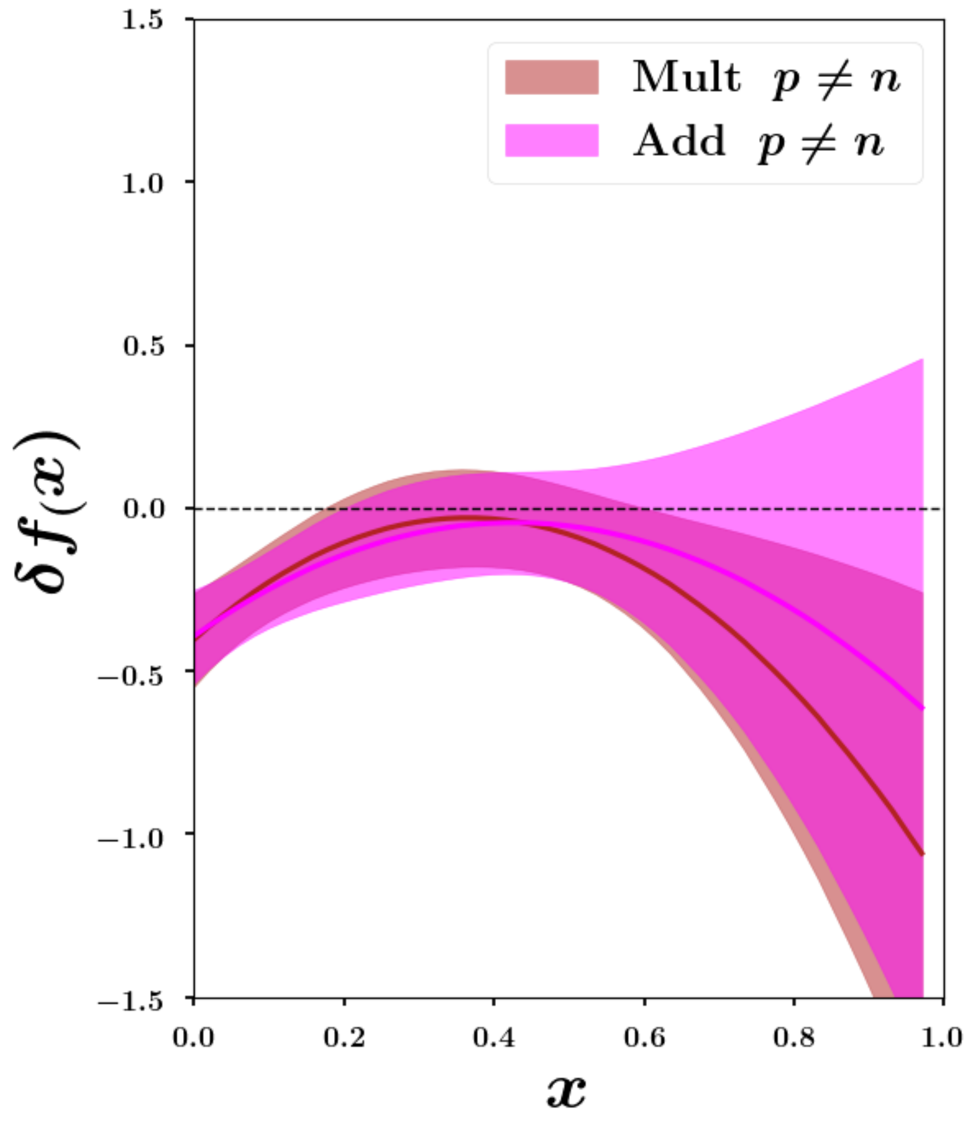






Results in the CJ fitting framework

After removing the bias $\delta f(x) \simeq 0$



Is the nucleon inside the deuterium almost on-shell?

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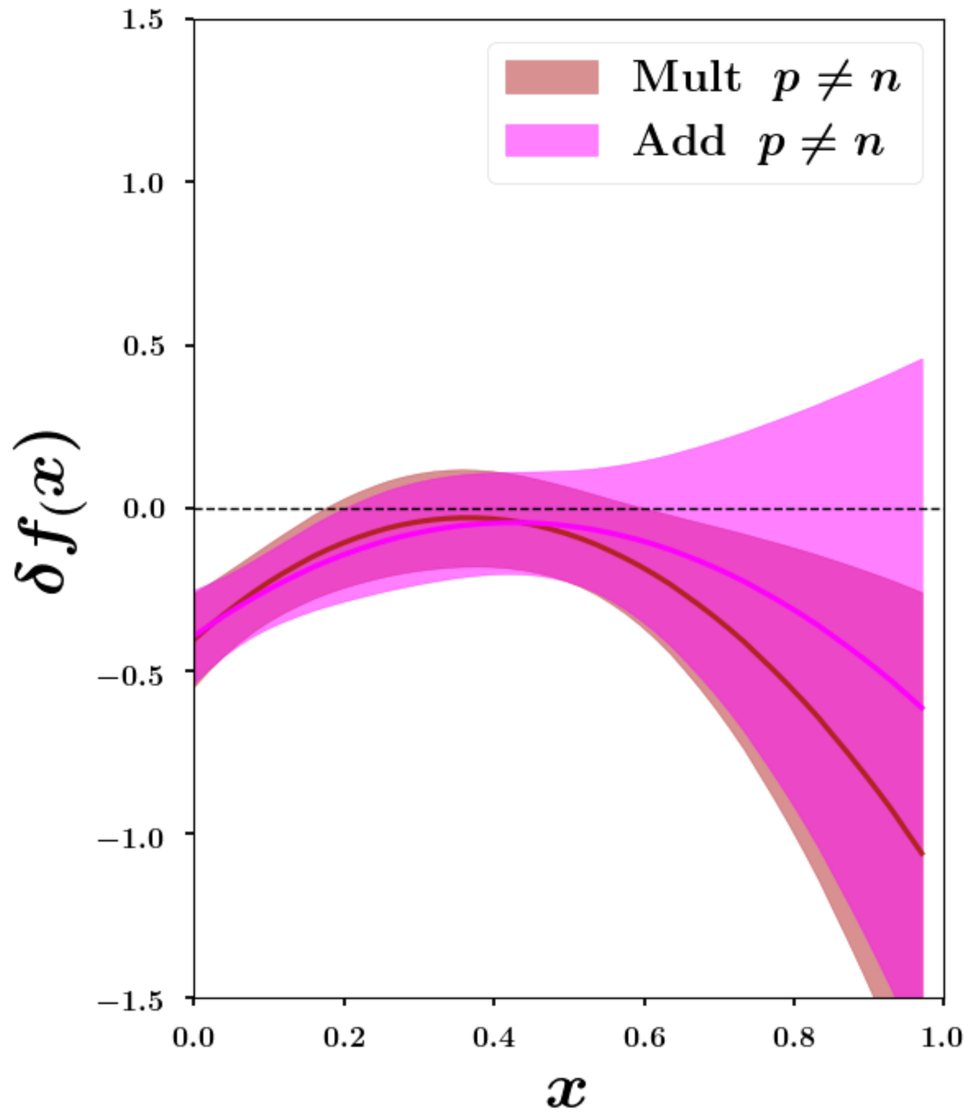

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Need A=3 data to assess flavour dependence of off-shell function

MARATHON data
Adams, et al., PRL 128 (2022)

Other extractions of the off-shell correction

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AKP Alekhin, Kulagin, Petti, PRD 107 (2023)

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JAM JAM Collaboration, PRL 127 (2021)

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Fit to A=3 data: $\delta f_u(x) \neq \delta f_d(x)$

Need more information

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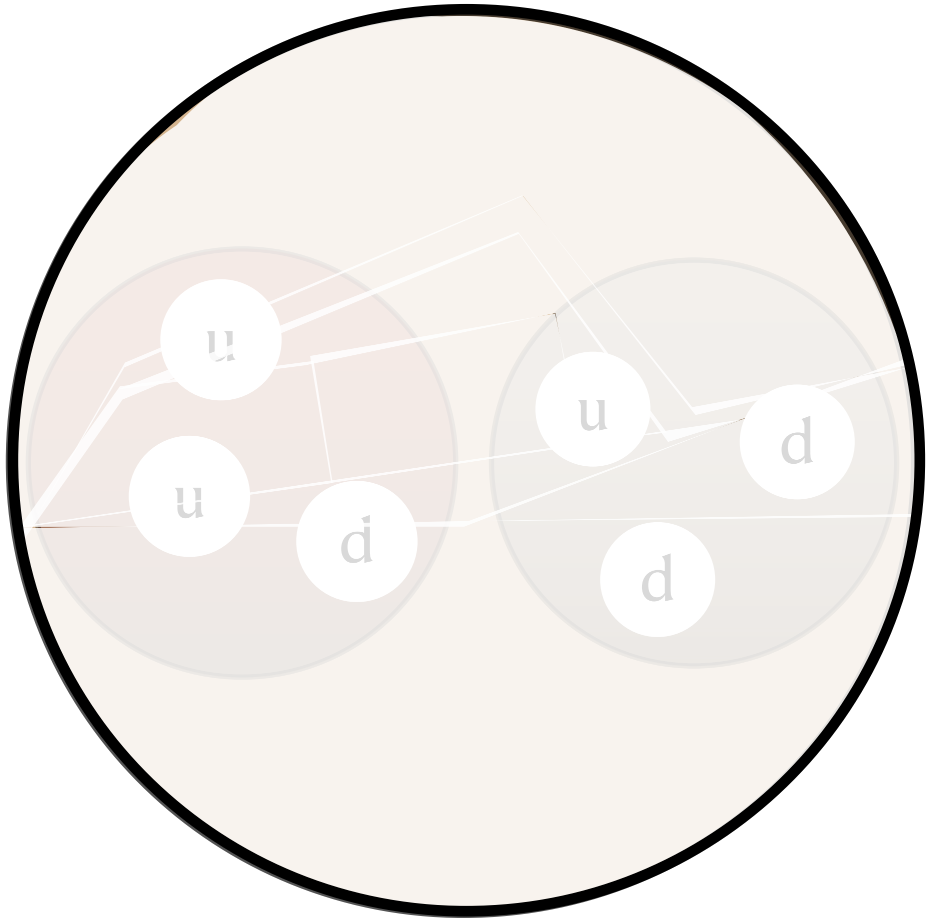
Experimental data differential on the off-shell proton virtuality p^2 would allow us to pin down the off-shell correction in a more clean way

Outlook - the CJ studies



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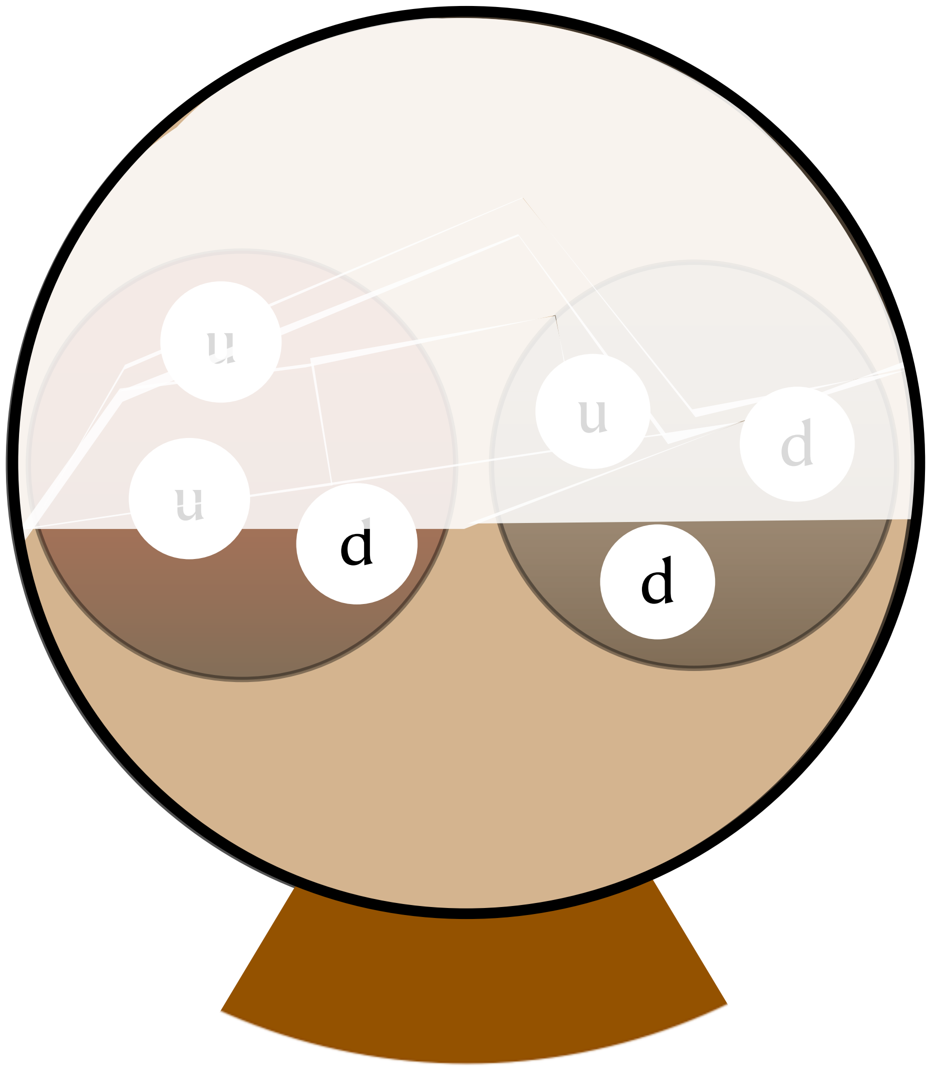
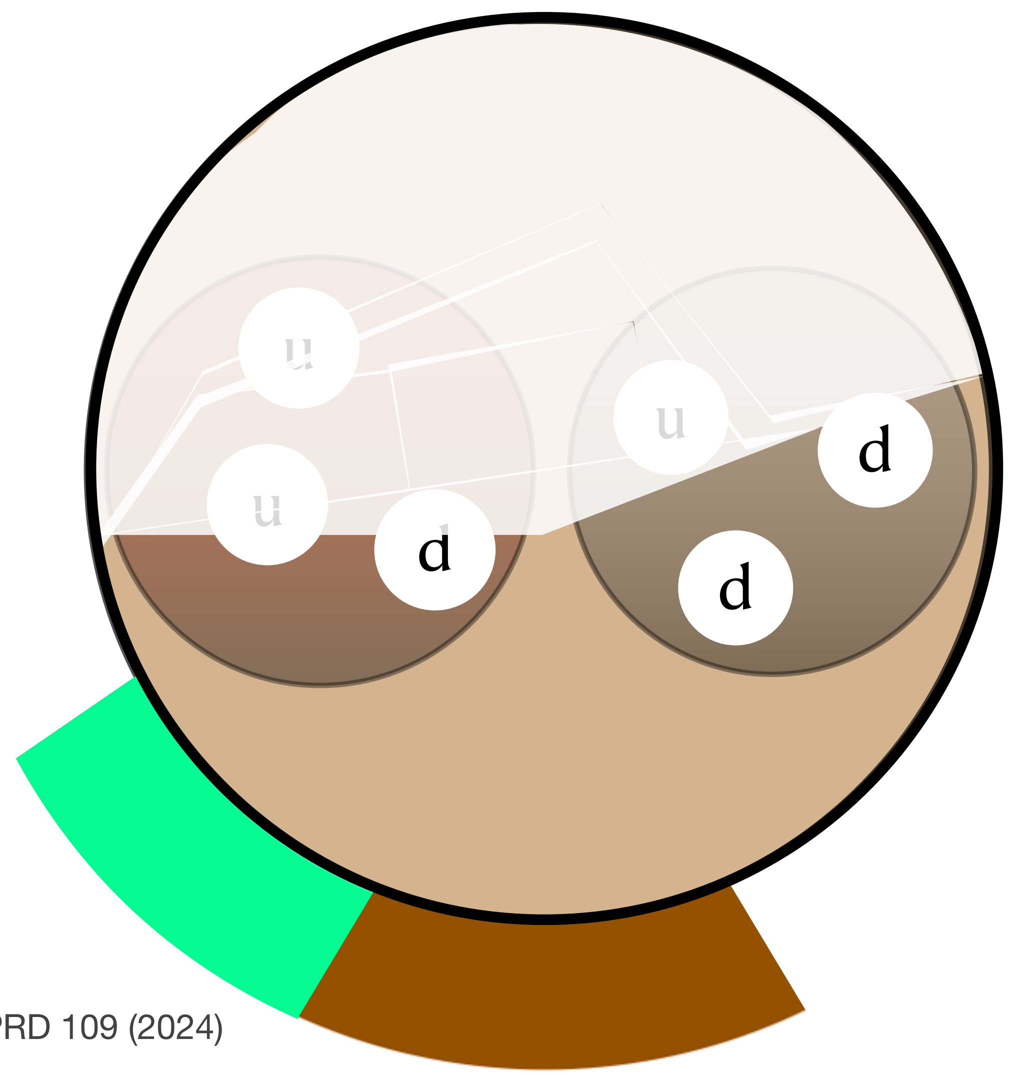


CJ22

Accardi, Jing, Owens et al., PRD 107 (2023)

Outlook - the CJ studies



F2(n)

Li, Accardi, MC, Fernando et al., PRD 109 (2024)

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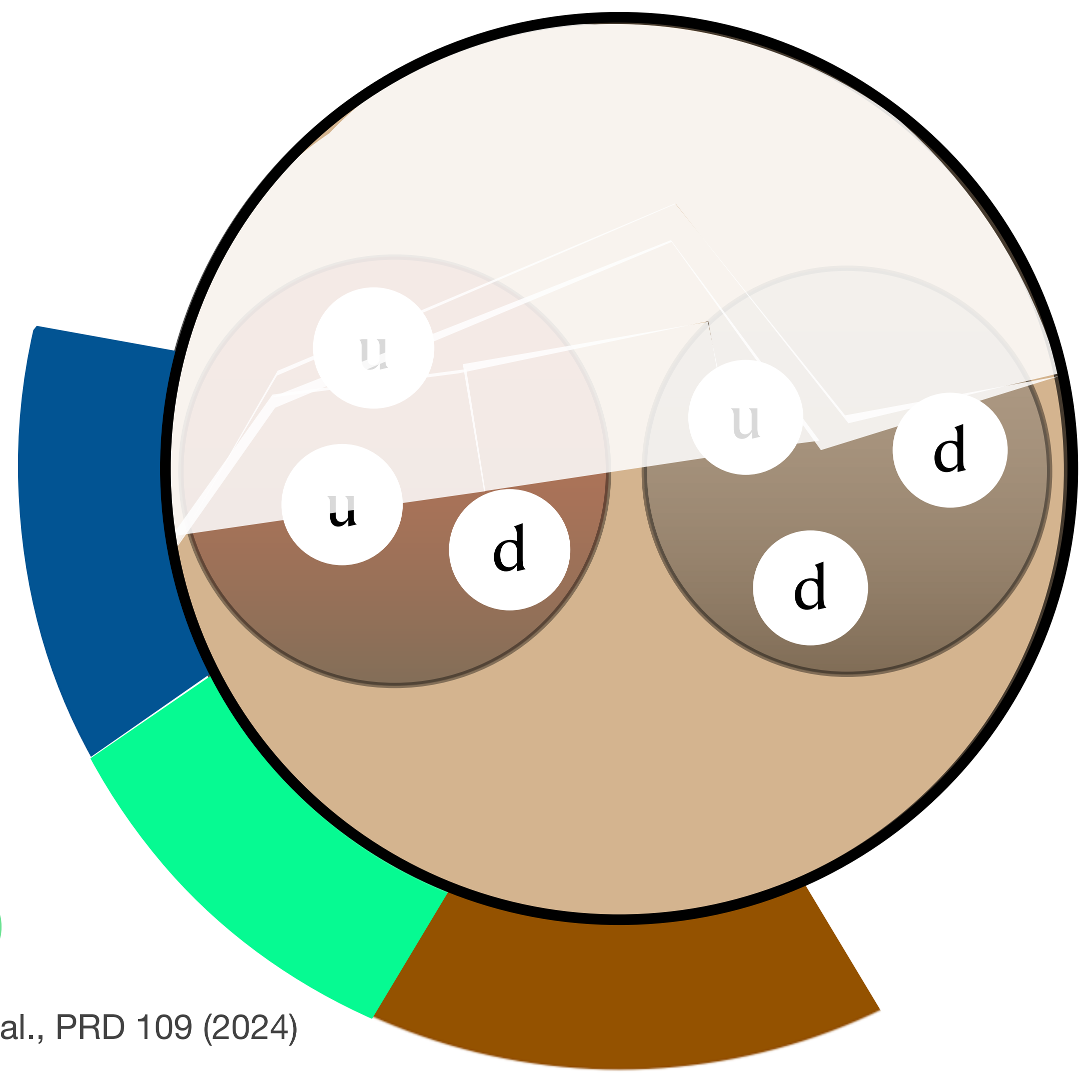


HTvsOS

In preparation (see DIS2024 talk)

F2(n)

Li, Accardi, MC, Fernando et al., PRD 109 (2024)

CJ22

Accardi, Jing, Owens et al., PRD 107 (2023)

Outlook - the CJ studies

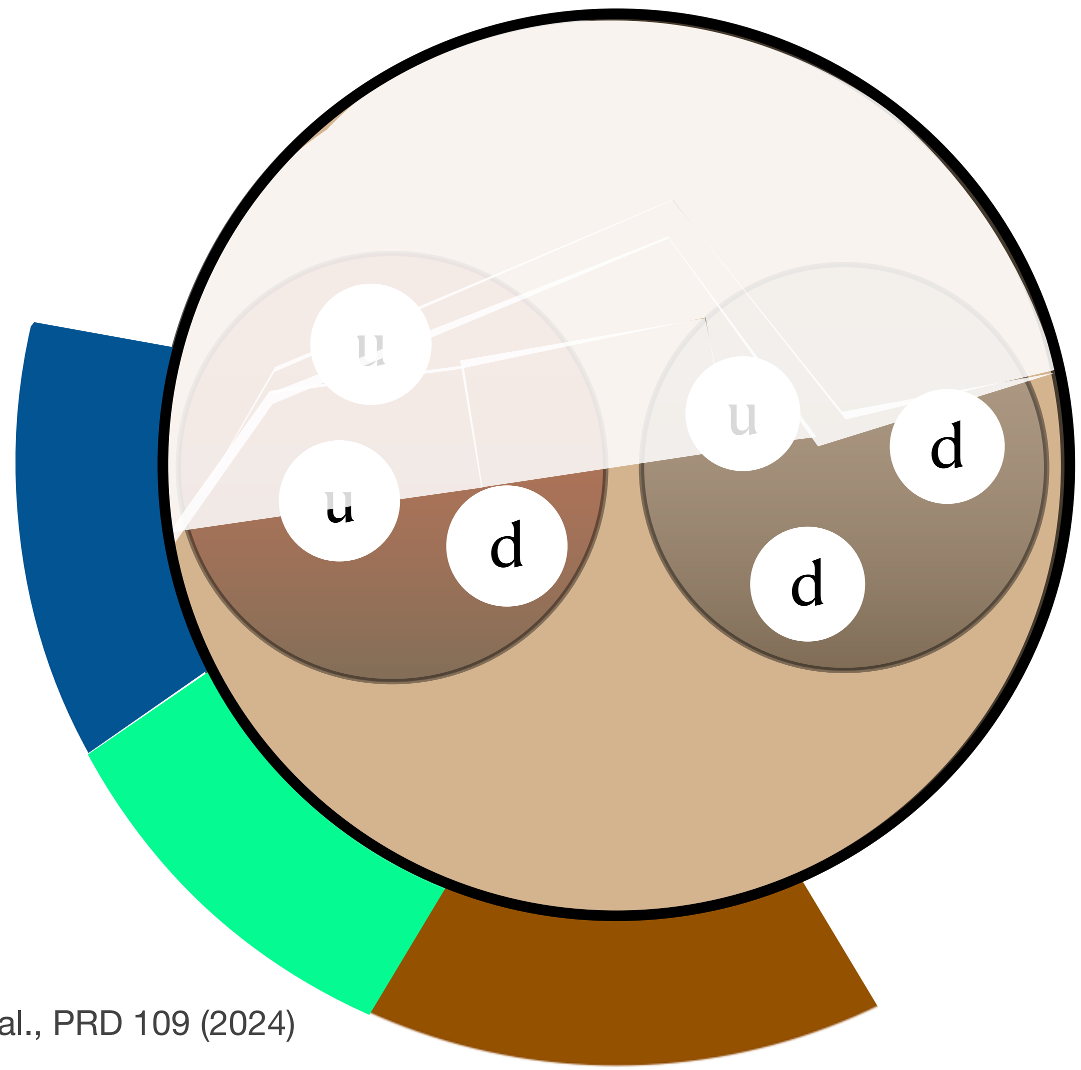
P
R
E
S
E
N
T

HTvsOS

In preparation (see DIS2024 talk)

F2(n)

Li, Accardi, MC, Fernando et al., PRD 109 (2024)



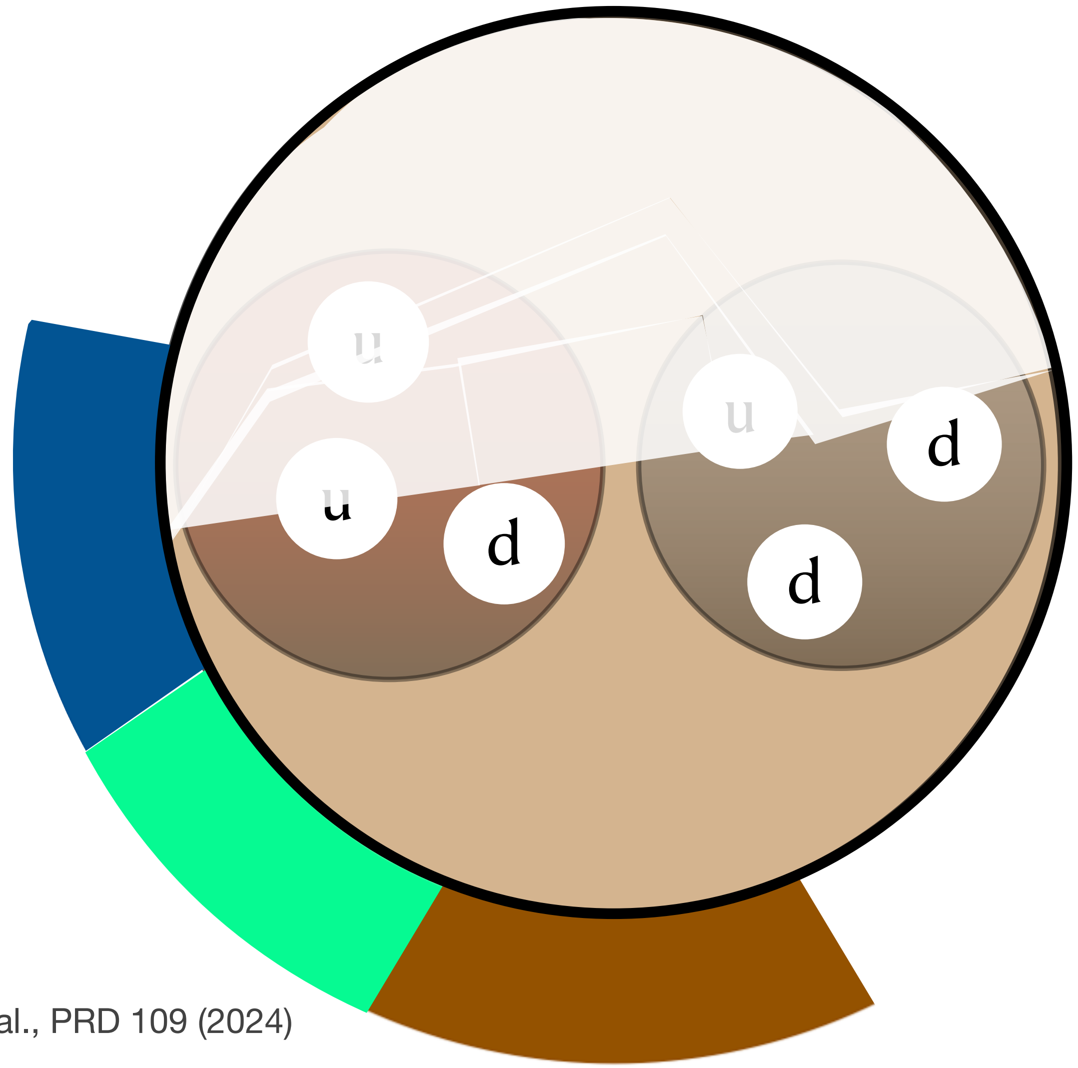
CJ22

Accardi, Jing, Owens et al., PRD 107 (2023)

Outlook - the CJ studies

F
U
T
U
R
E

P
R
E
S
E
N
T



HTvsOS

In preparation (see DIS2024 talk)

F2(n)

Li, Accardi, MC, Fernando et al., PRD 109 (2024)

CJ22

Accardi, Jing, Owens et al., PRD 107 (2023)

Outlook - the CJ studies

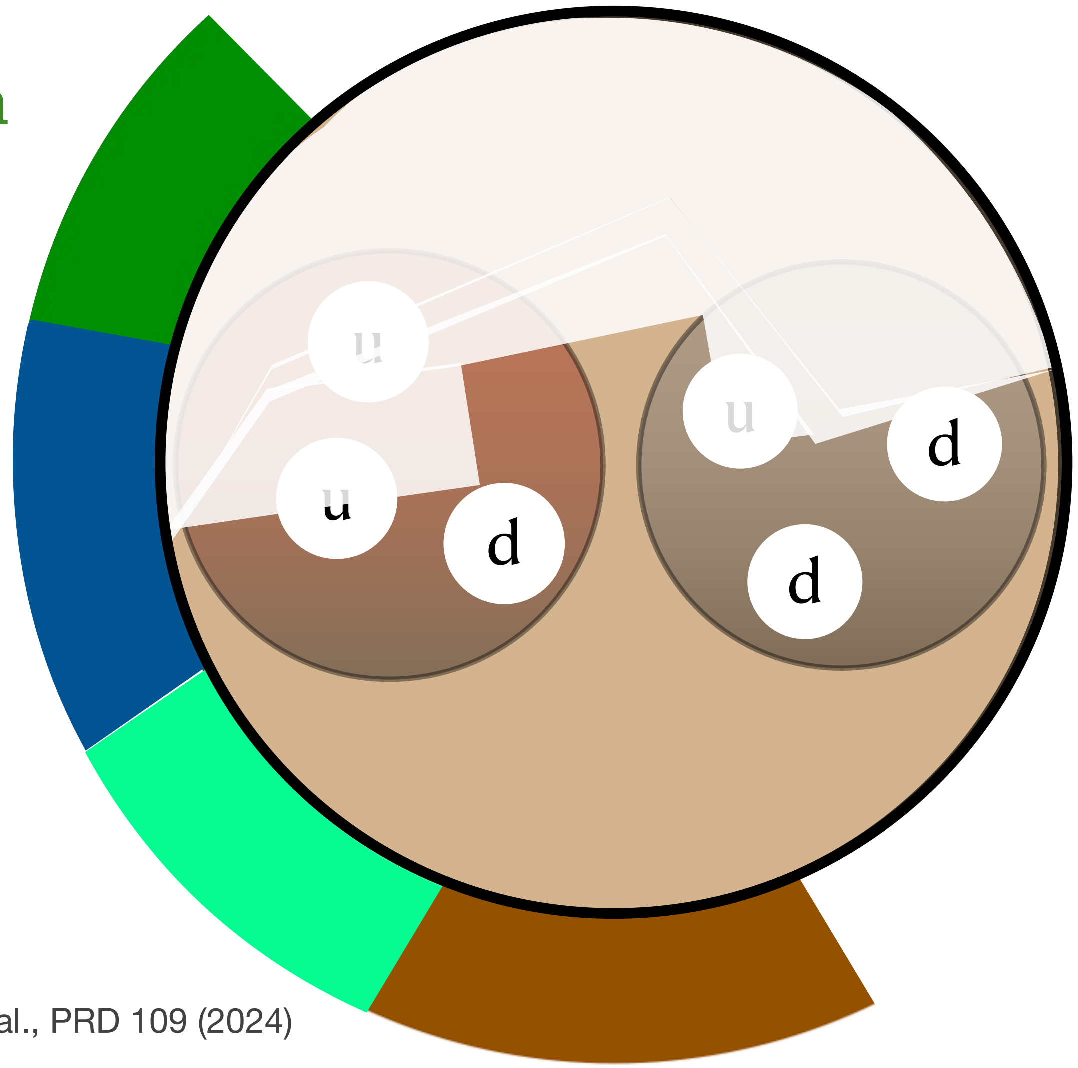
F U T U R E

MARATHON data

HTvsOS

In preparation (see DIS2024 talk)

P R E S E N T



F2(n)

Li, Accardi, MC, Fernando et al., PRD 109 (2024)

CJ22

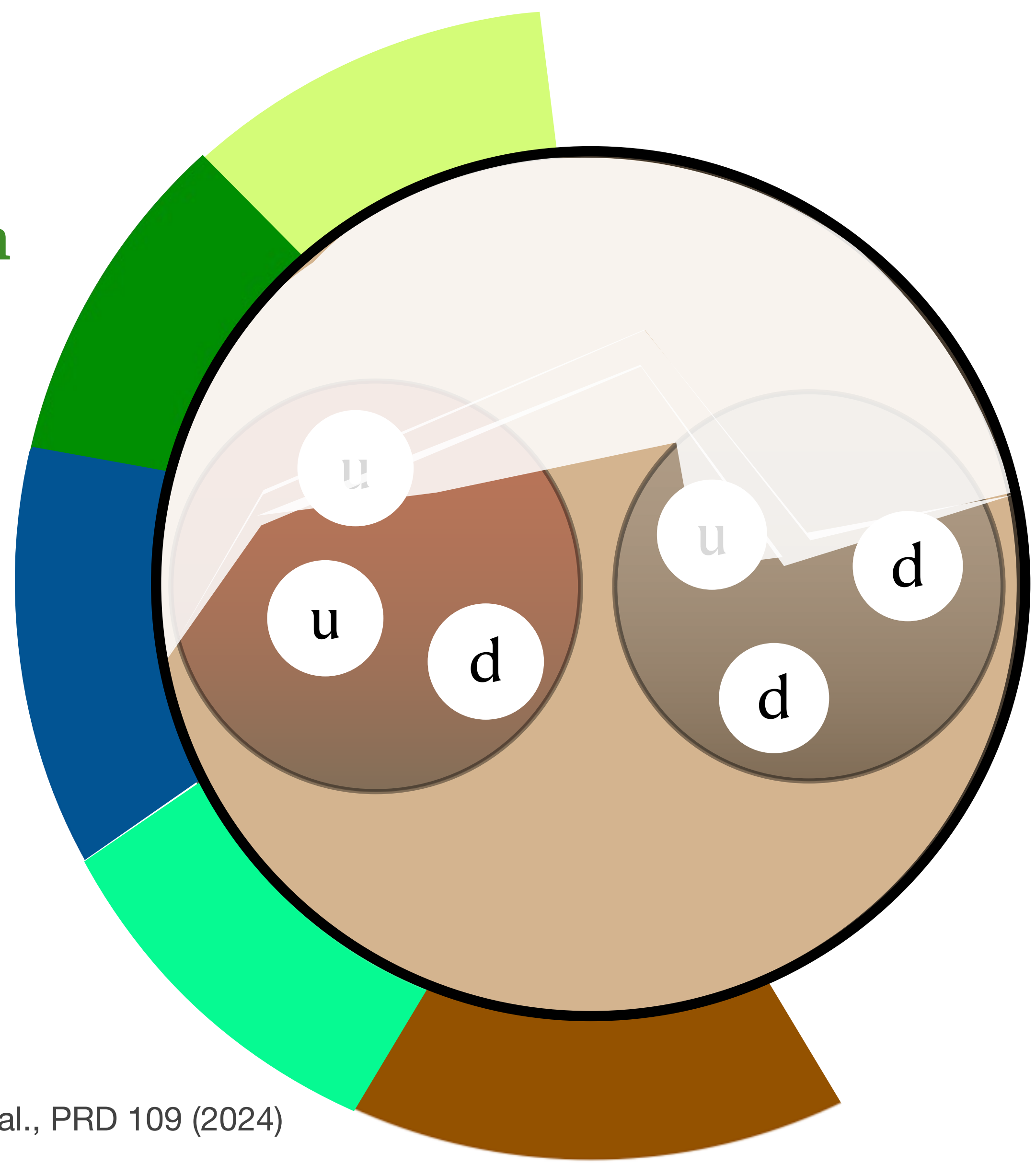
Accardi, Jing, Owens et al., PRD 107 (2023)

Outlook - the CJ studies

F U T U R E

LHC data

MARATHON data



HTvsOS

In preparation (see DIS2024 talk)

P R E S E N T

F2(n)

Li, Accardi, MC, Fernando et al., PRD 109 (2024)

CJ22

Accardi, Jing, Owens et al., PRD 107 (2023)

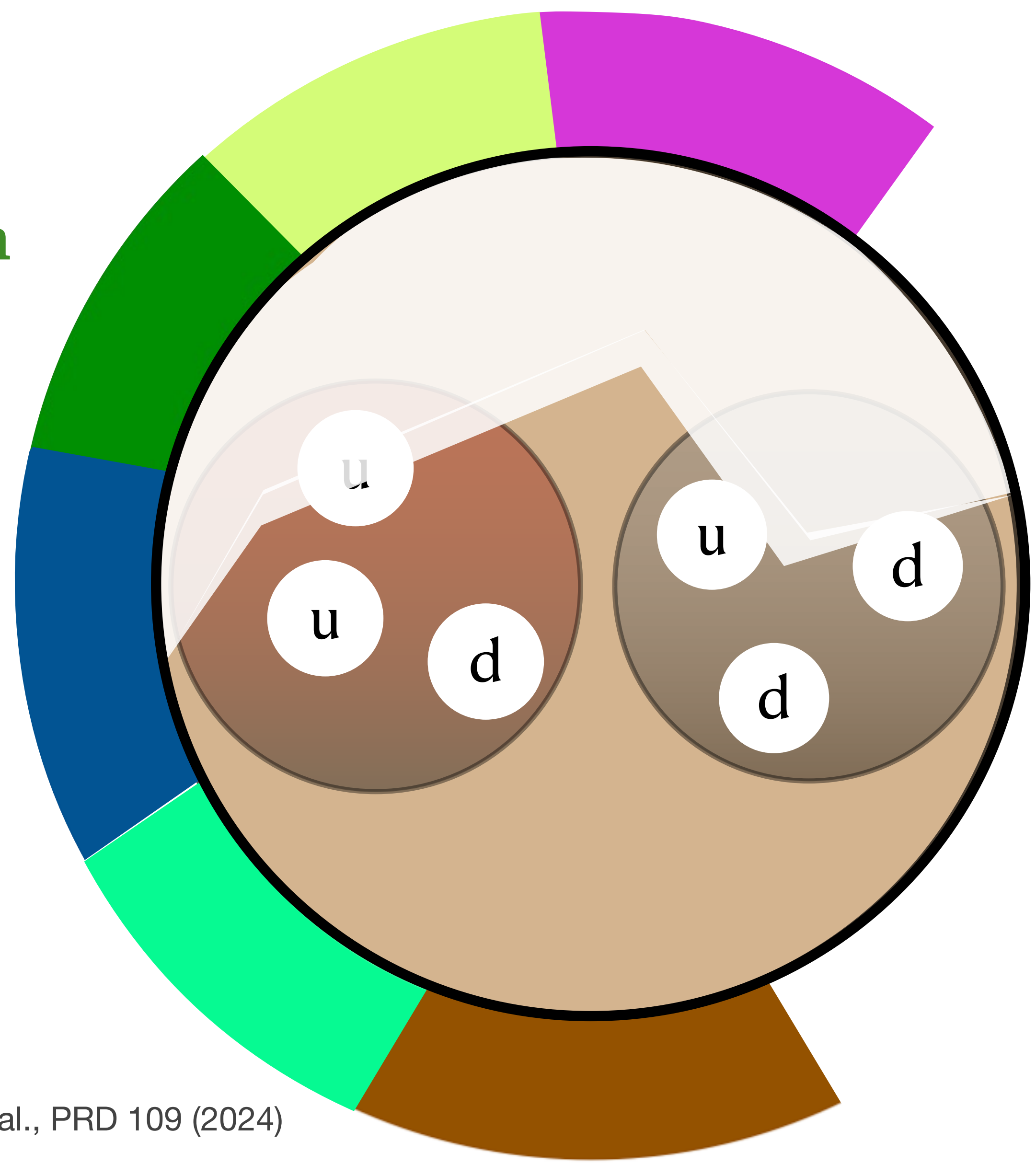
Outlook - the CJ studies

FUTURE

LHC data

PVDIS

MARATHON data



HTvsOS

In preparation (see DIS2024 talk)

PRESENT

F2(n)

Li, Accardi, MC, Fernando et al., PRD 109 (2024)

CJ22

Accardi, Jing, Owens et al., PRD 107 (2023)

Outlook - the CJ studies

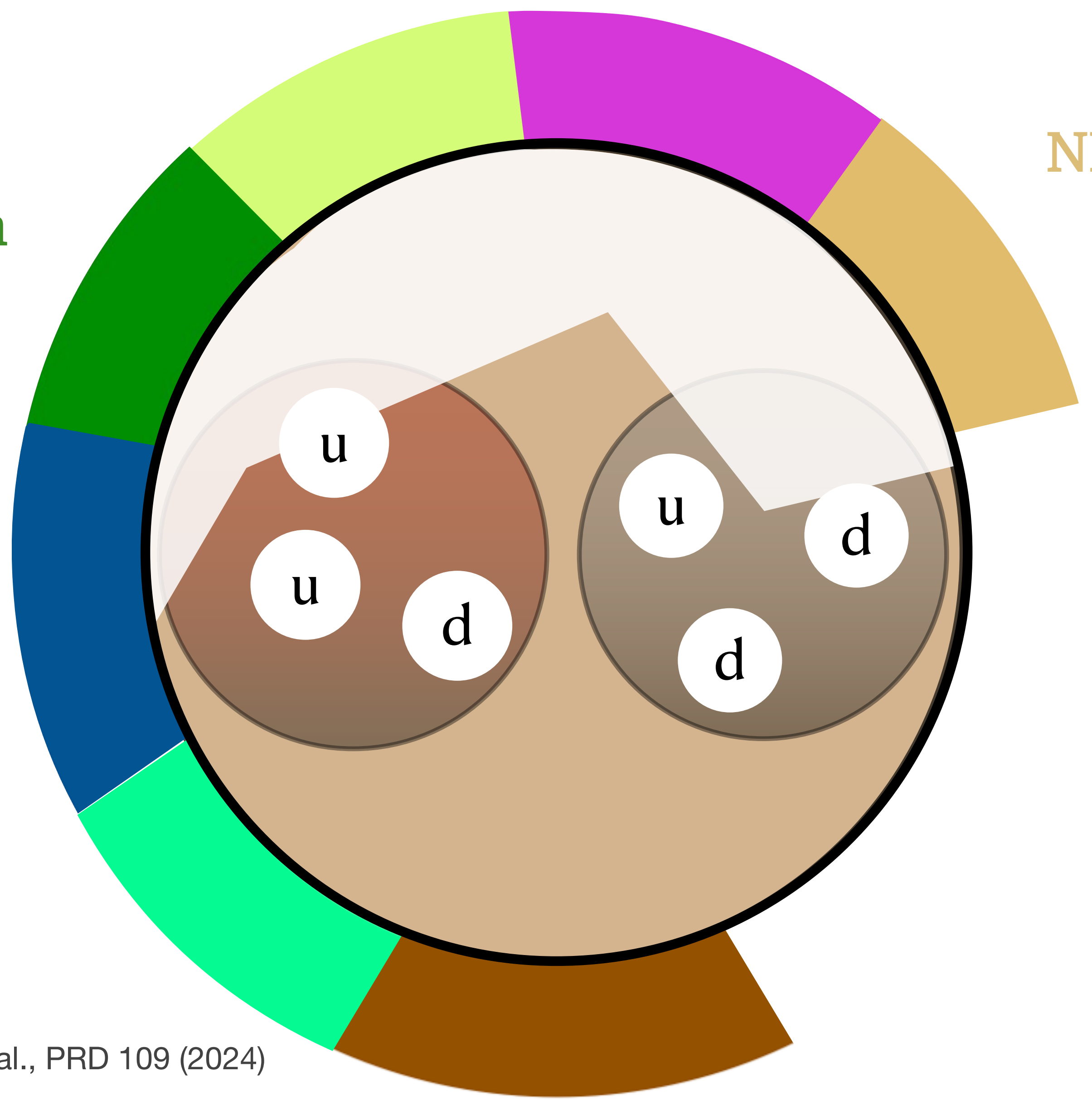
F
U
T
U
R
E

LHC data

PVDIS

NNLO

MARATHON data



HTvsOS

In preparation (see DIS2024 talk)

P
R
E
S
E
N
T

F2(n)

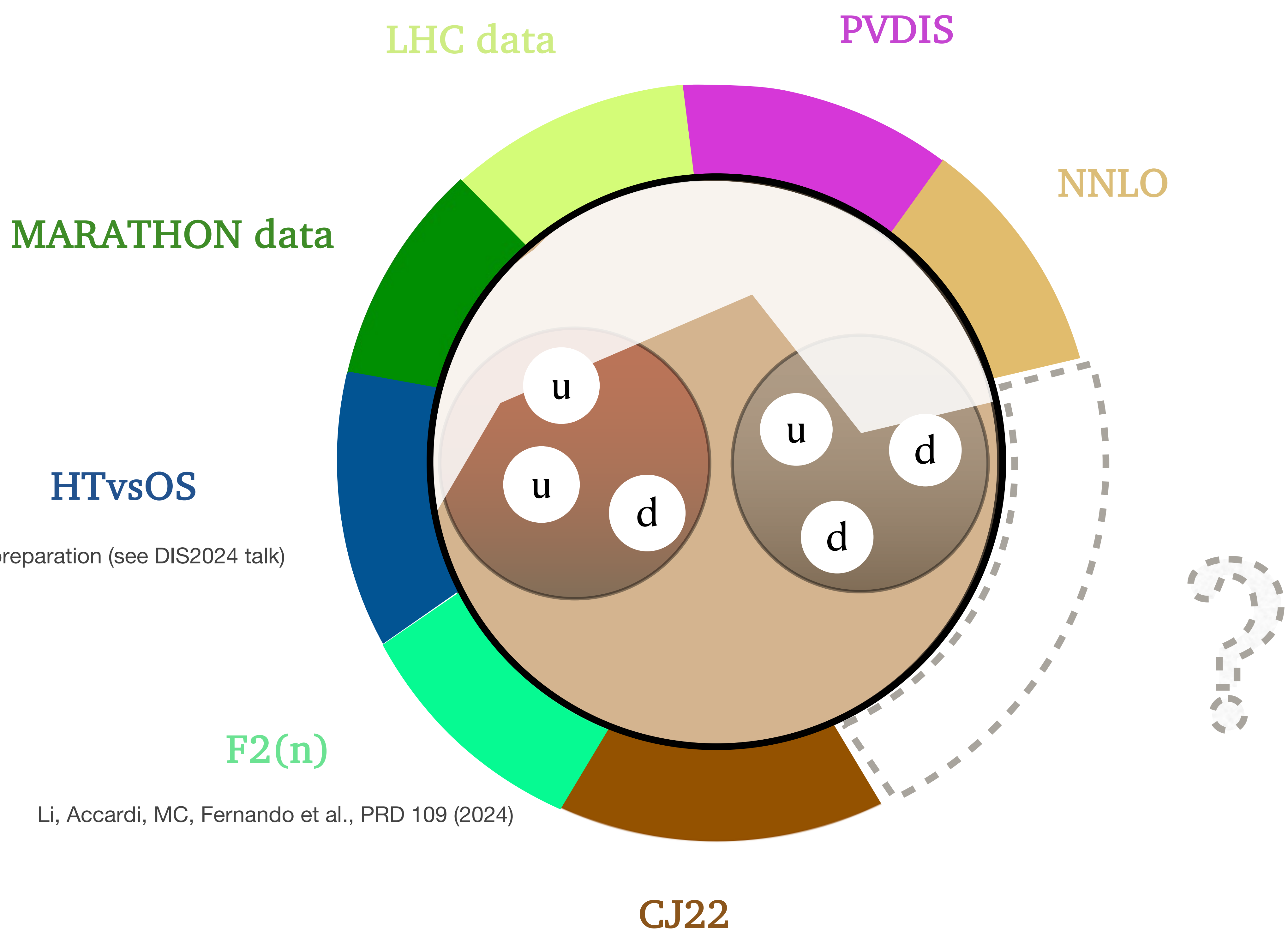
Li, Accardi, MC, Fernando et al., PRD 109 (2024)

CJ22

Accardi, Jing, Owens et al., PRD 107 (2023)

Outlook - the CJ studies

FUTURE



In preparation (see DIS2024 talk)

Li, Accardi, MC, Fernando et al., PRD 109 (2024)

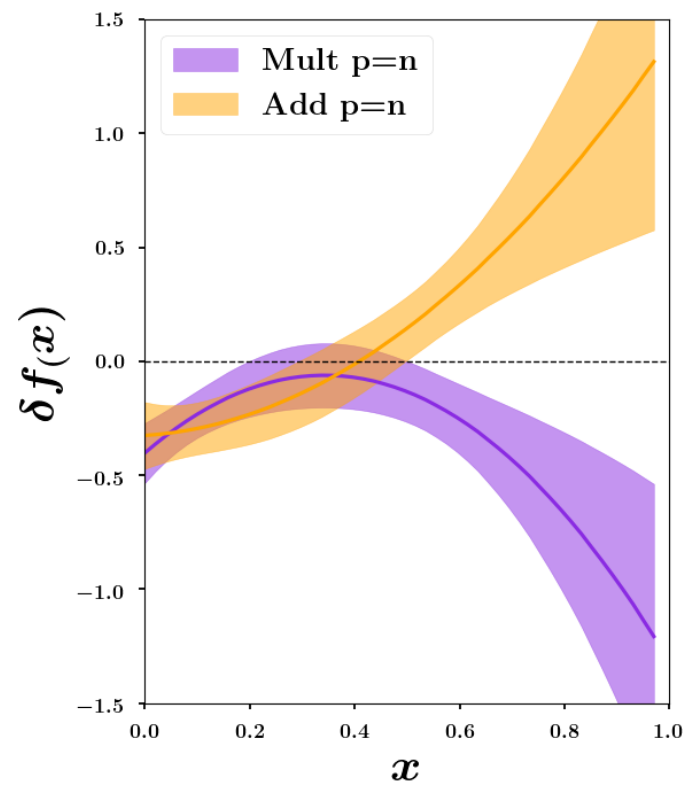
Accardi, Jing, Owens et al., PRD 107 (2023)

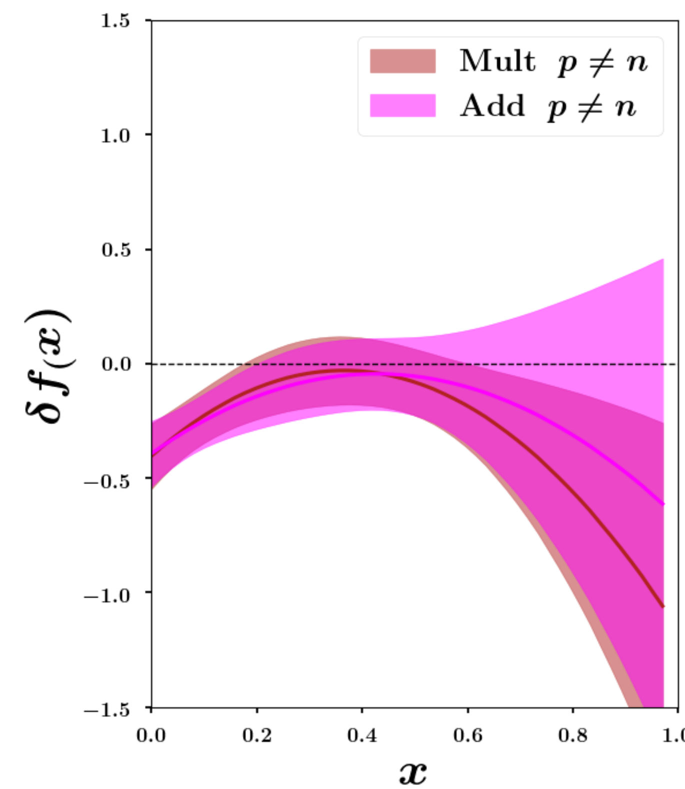
PRESENT

Backup

Off-shell table

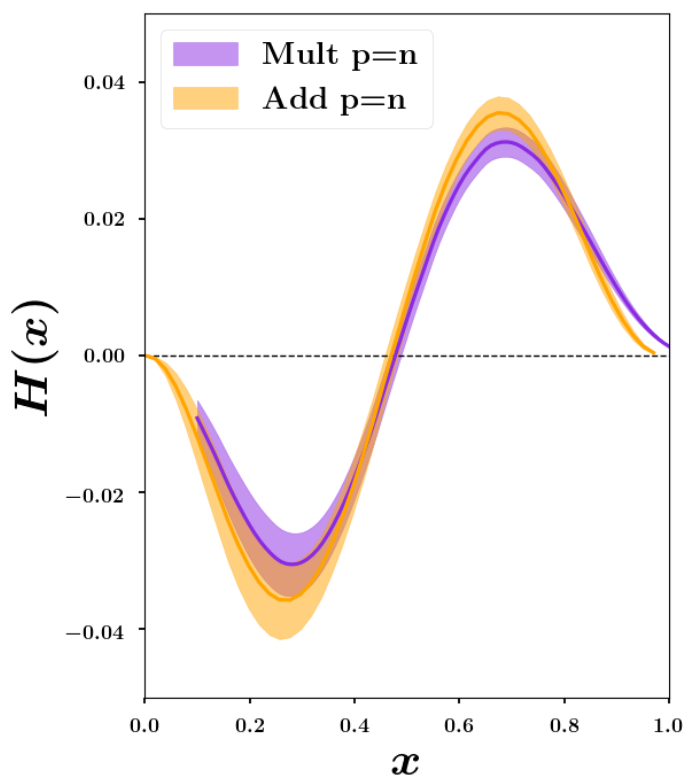
higher-twist \ isospin	Symmetric	Non symmetric
Multiplicative		
Additive		

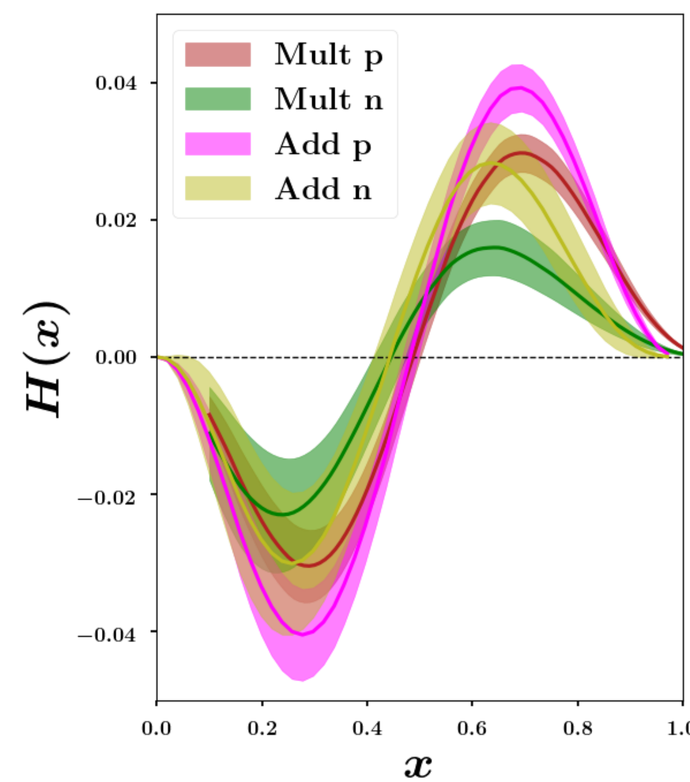




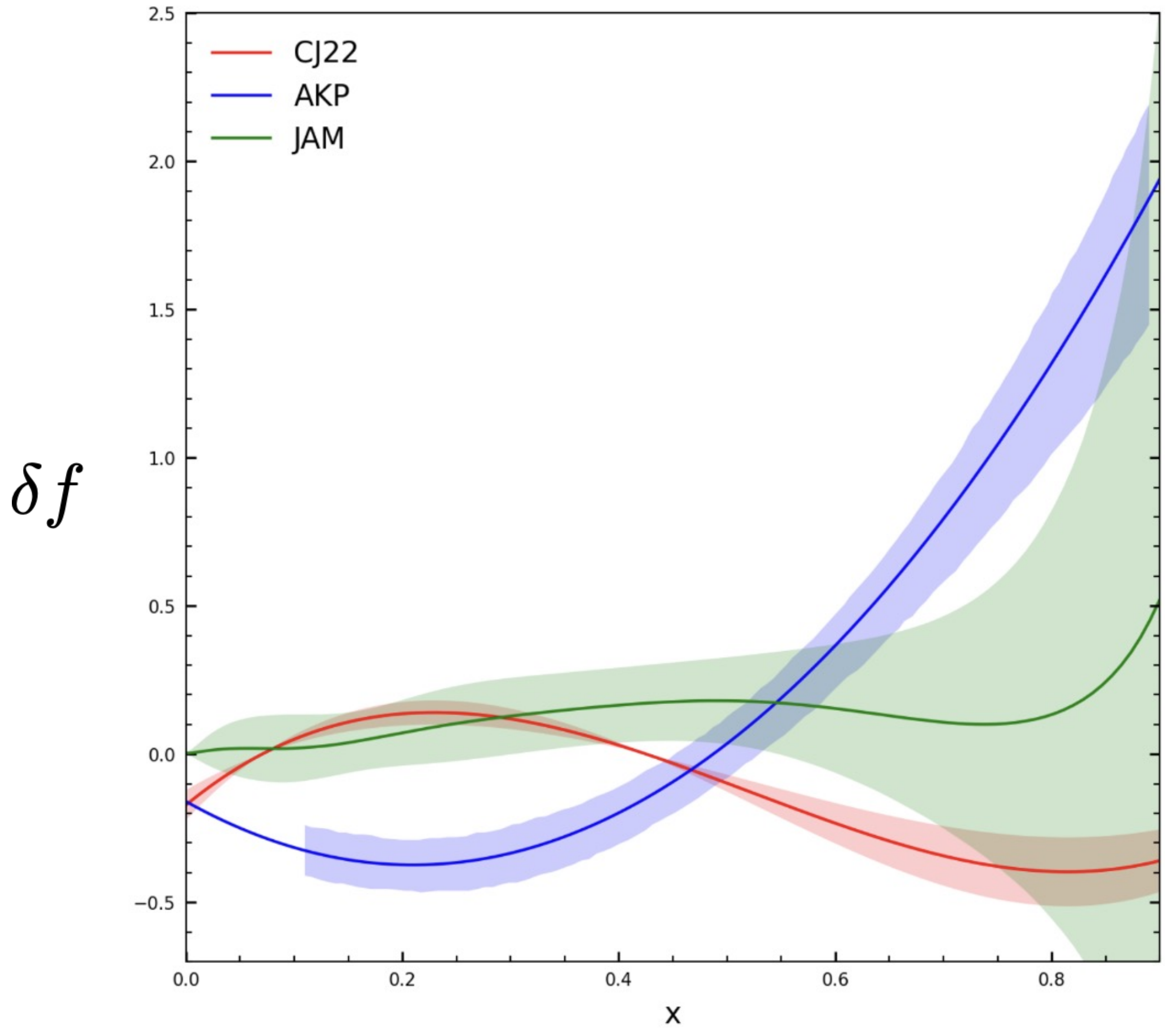
Higher-Twist table

isospin / higher-twist	Symmetric	Non symmetric
Multiplicative	$\tilde{H} = F_{2,N}(x,Q^2)H_{\rm Mult}(x)$	$\delta\tilde{H} = F_{2,N}(x,Q^2)\delta H_{\rm Mult}(x)$
Additive		





AKP vs CJ



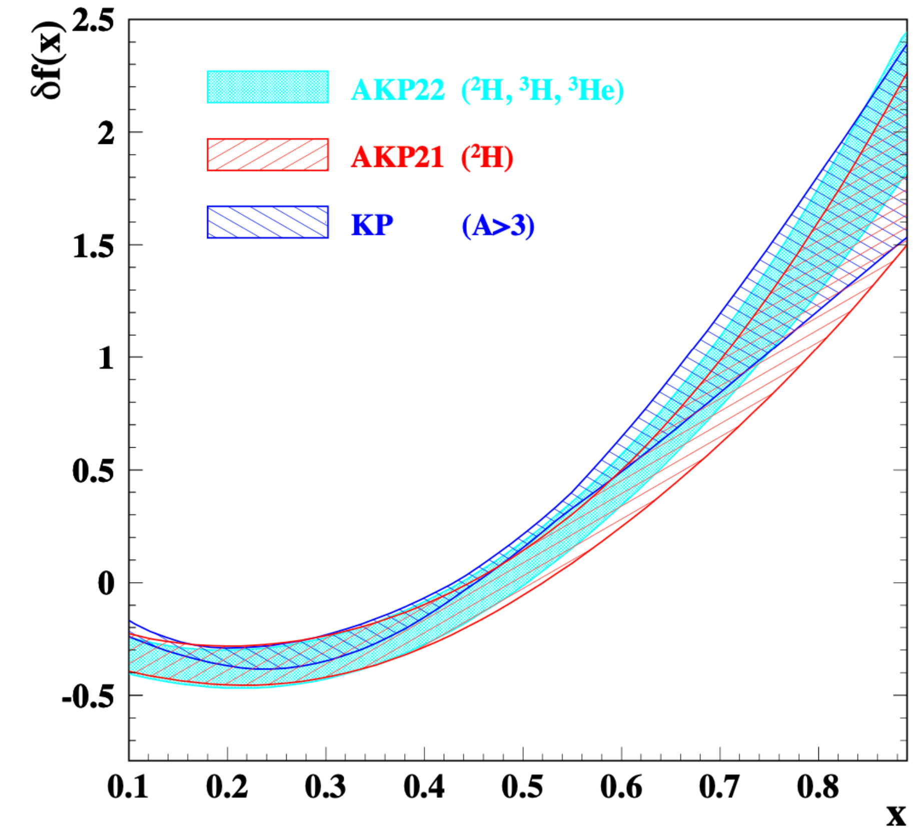
AKP results

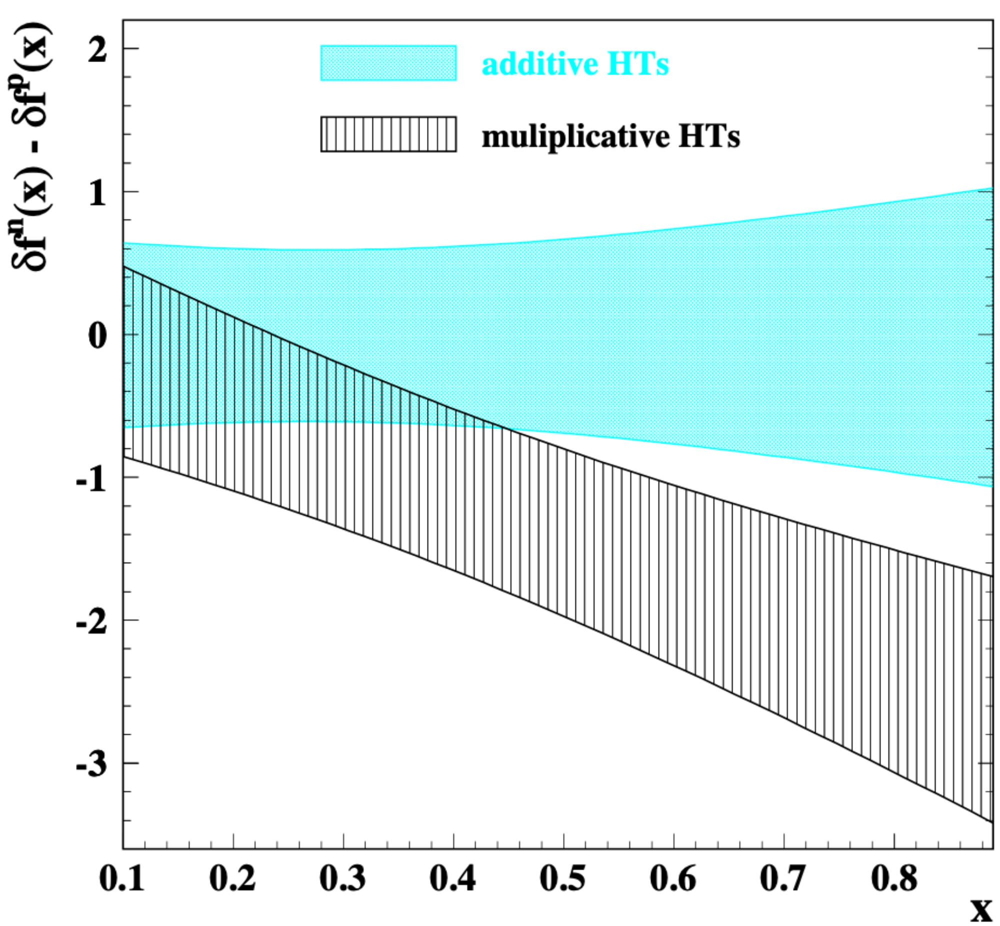
AKP Alekhin, Kulagin, Petti, PRD 107 (2023)

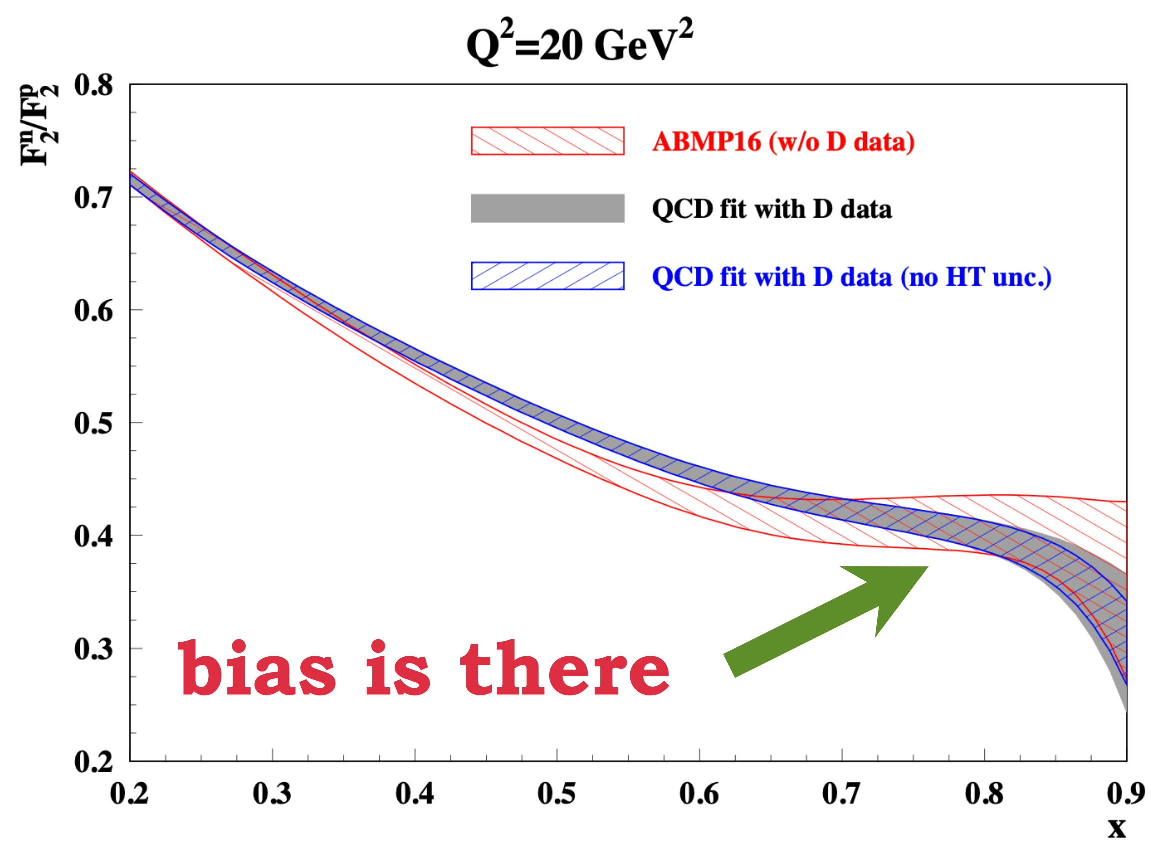
Add HT (p=n) as baseline choice

H_2, H_T parametrized

Fit to A=3 data $\delta F_p \delta F_n$





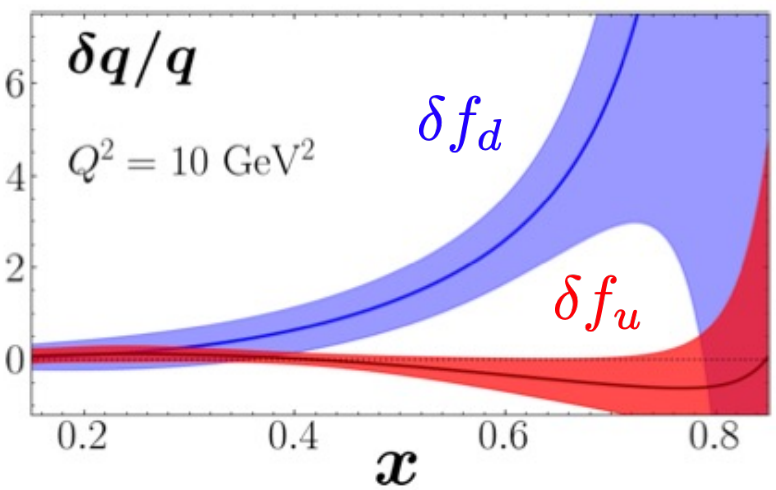


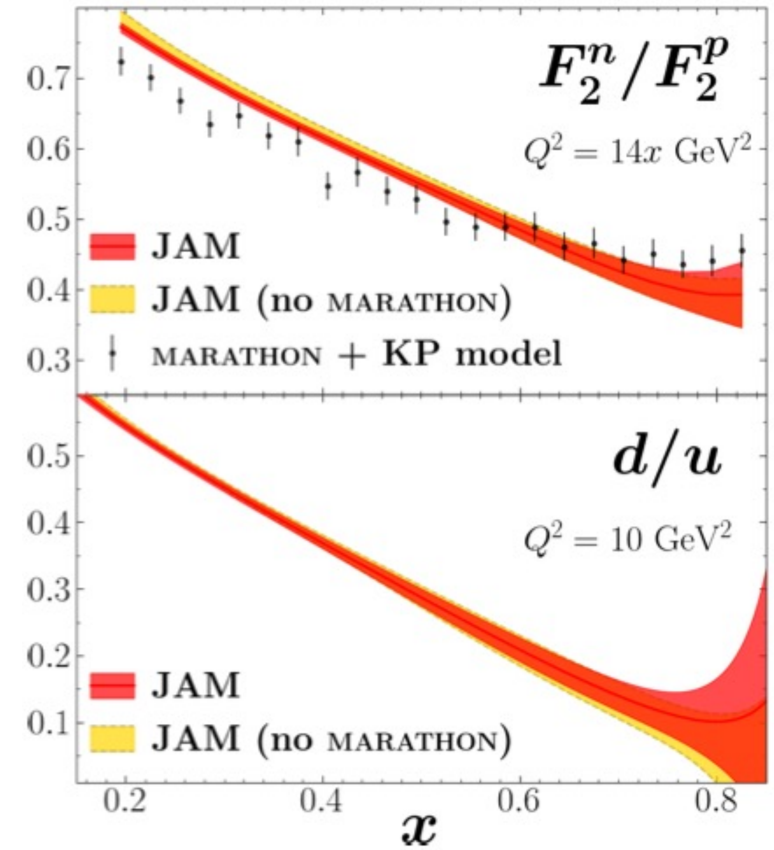
JAM results

JAM *Fit including A=3 data* $\delta f_u \delta f_d$

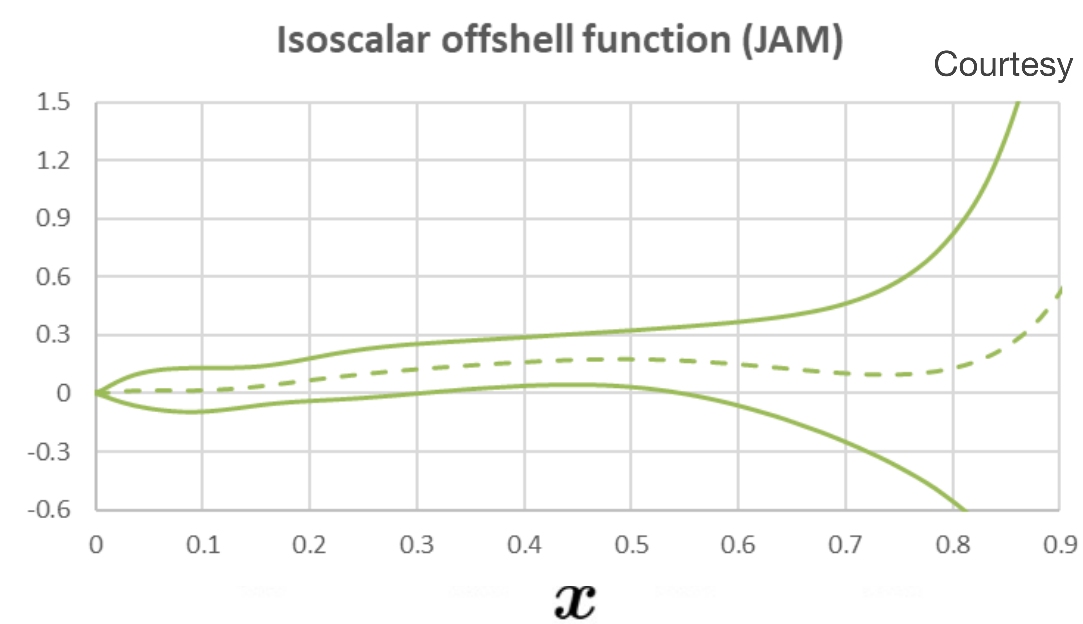
JAM Collaboration, PRL 127 (2021)

Mult HT (p=n) as default choice





$$\delta f(x)|_{\text{CJ-like}} = \frac{u\delta f_u + d\delta f_d}{u+d}$$



Courtesy of C. Cocuzza

Some implementation differences

Theoretical choices →

Corrections (increasing-x) ↓

	KP	AKP	CJ15	AKP-like
shadowing	yes	yes (which one?)	MST x<0.1	(same)
smearing	Paris	AV18	AV18 x>0.1	(same)
pi-cloud	yes	yes	----	----
TMC	GP O(Q4)?	GP O(Q4)??	GP approx.	(same)
HT	H (p=n ??)	H (p=n)	C (p=n)	H & C, p=n & p!=n
HT(x)	??	5 pt. spline	parametrized	parametrized
off-shell	O(p2-M2)	O(p2-M2)	O(p2-M2)	(same)
df(x)	factorized	polyn. 2nd/3rd	factorized + sum rule	polyn. 2nd/3rd
pi thresh.	yes	yes	----	----