Nucleon spin sum rules and spin polarizabilities at low Q^2

A. Deur

Thomas Jefferson National Accelerator Facility

- Why is the spin of the nucleon interesting?
- QCD at small and large distances: emerging properties and effective theories.
- Importance of effective descriptions.
- Nucleon spin polarizabilities: how much does the nucleon spin jiggle?
- Experimental results.
- Comparison with leading effective theory of QCD at large distance.
- Conclusion.

Context

Specific topic

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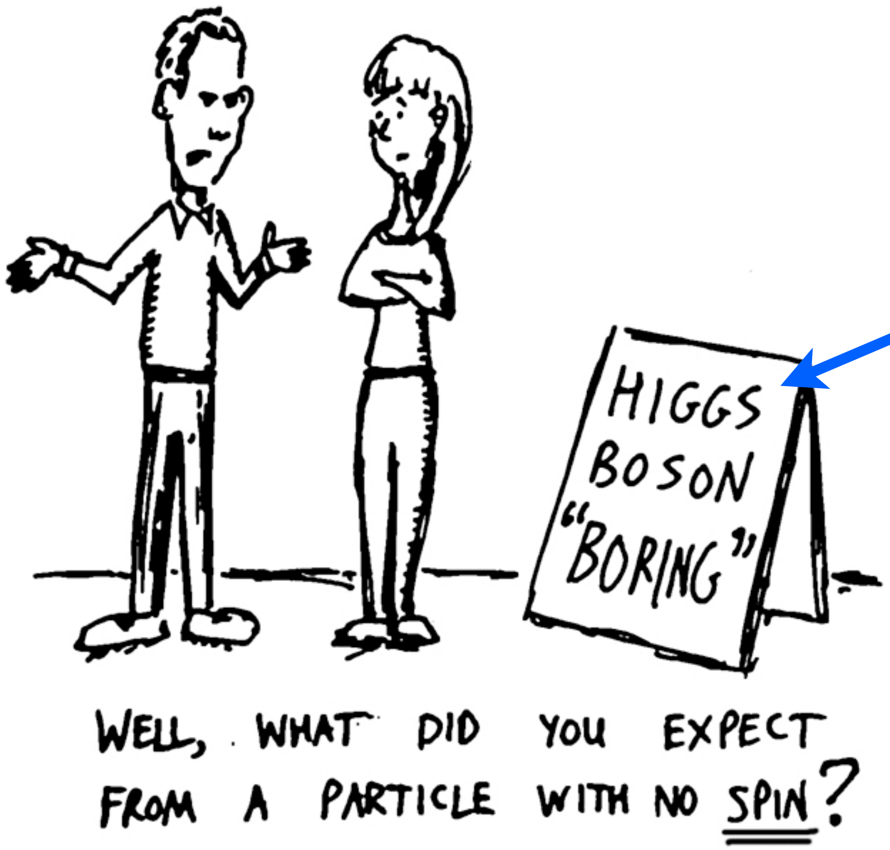

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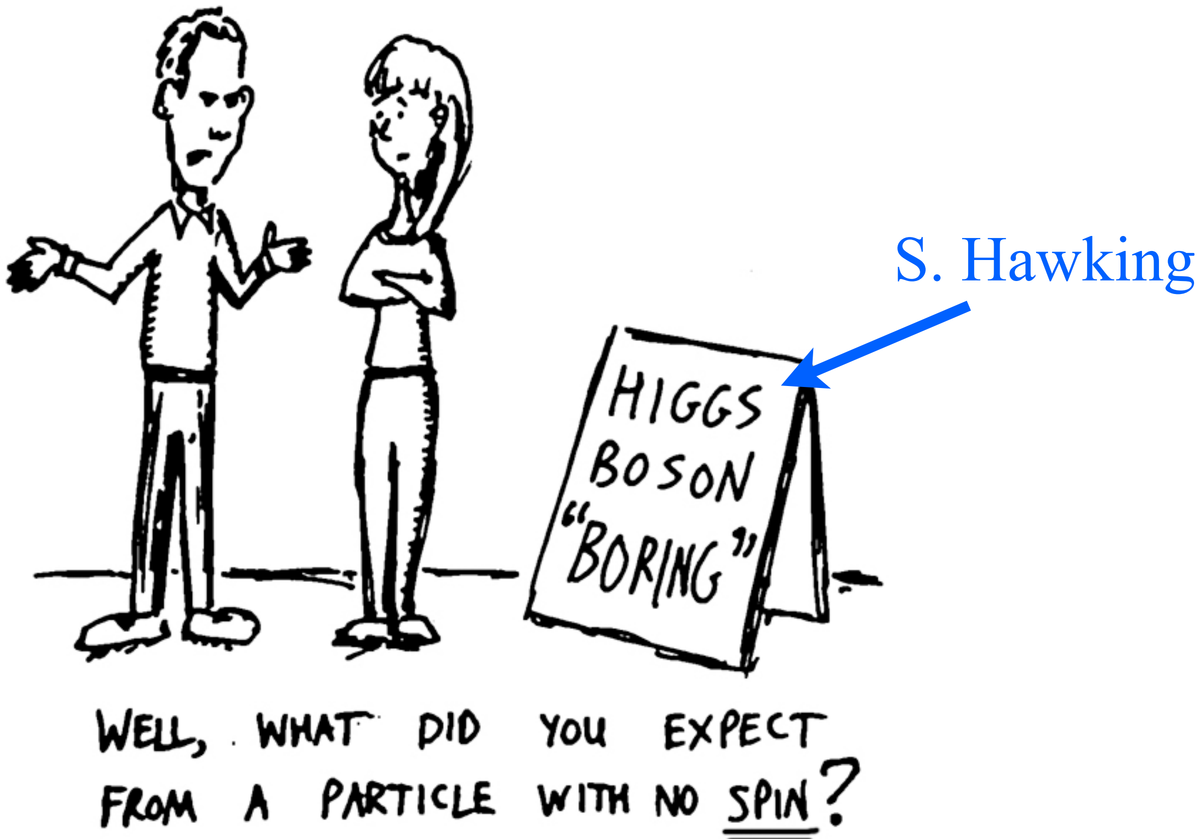

Spin is responsible for shaping world:

- fundamental components: spin ½
  ⇒ matter doesn't collapse.
- spin even bosons: attractive forces. e.g. nuclear force (pion), gravitation.
  ⇒stable nuclei, burning stars, structured universe...
- spin odd bosons: repulsive between like charges, attractive between opposite charges.
  ⇒ neutral atoms.

⇒Spin is key to the marvelous diversity of the universe

Why do we study the nucleon spin structure?

- Human curiosity (i.e. it's interesting): $S_N = \frac{1}{2} = \underbrace{\frac{1}{2}\Delta\Sigma}_{\text{quark spin contribution} \sim 0.15} + \underbrace{\Delta G + L_G}_{\text{gluon contribution} \sim 0.15?} + L_q$.

  quark spin contribution ~0.15
  gluon contribution ~0.15?
  quark orbital angular mom. ~0.2?

- Nucleon: most of mass of known matter in the universe. Spin: Fundamental observable.

  Fundamental understanding of matter.

  ⇒ understand its elementary bricks

- Spin degrees of freedom: additional handles to test theories.
  - Constituent quark model, Parity symmetry of physical laws, Ellis-Jaffe sum rule, …
  - Spin permits more complete study of QCD;
    - mechanism of confinement;
    - how effective degrees of freedom (hadrons) emerge from fundamental ones (quark and gluons);
    - Test effectives theories or models for nucleon/nuclear structures.

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1970s-1980s: success of constituent quark model. Suggests $S_N = \frac{1}{2}\Delta\Sigma$

CERN's EMC experiment (1987): $\Delta\Sigma \sim 0$

⇒Nucleon spin composition is not trivial. Thus it reveals interesting information on the nucleon structure and the mechanisms of the strong force

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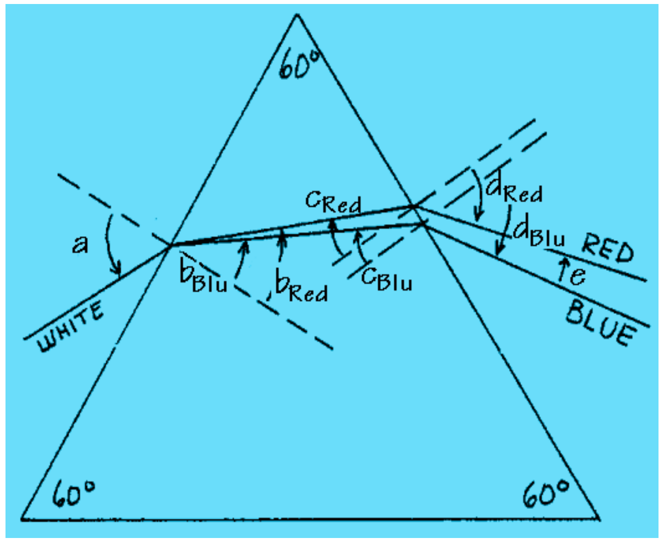
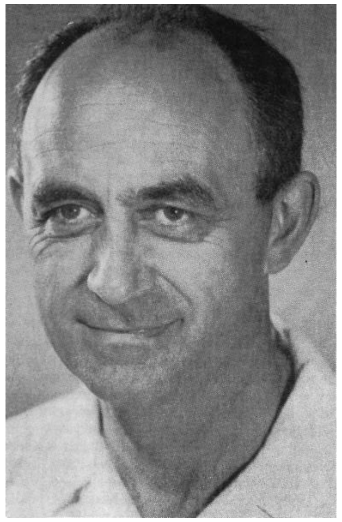
Effective descriptions of Nature

Fundamental forces: electromagnetic, weak, strong, gravitation
Fundamental particles: quarks, electrons, neutrinos…

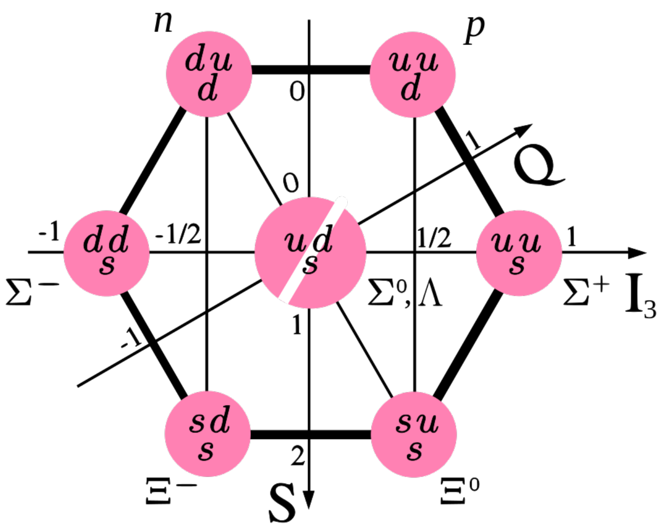
Effective descriptions of Nature



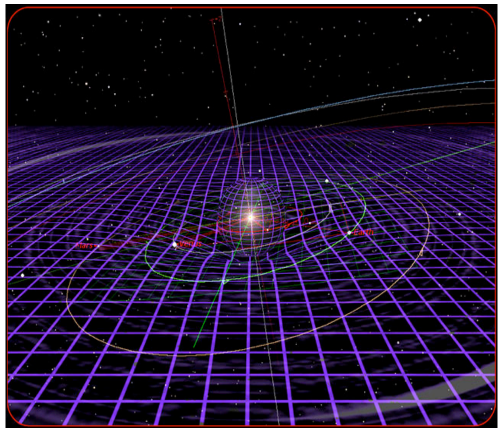
Geometric optics
d.o.f: rays



Fermi theory
d.o.f: hadrons, leptons



hadronic physics
d.o.f: hadrons



General Relativity(?)

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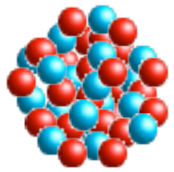
d.o.f: hadrons



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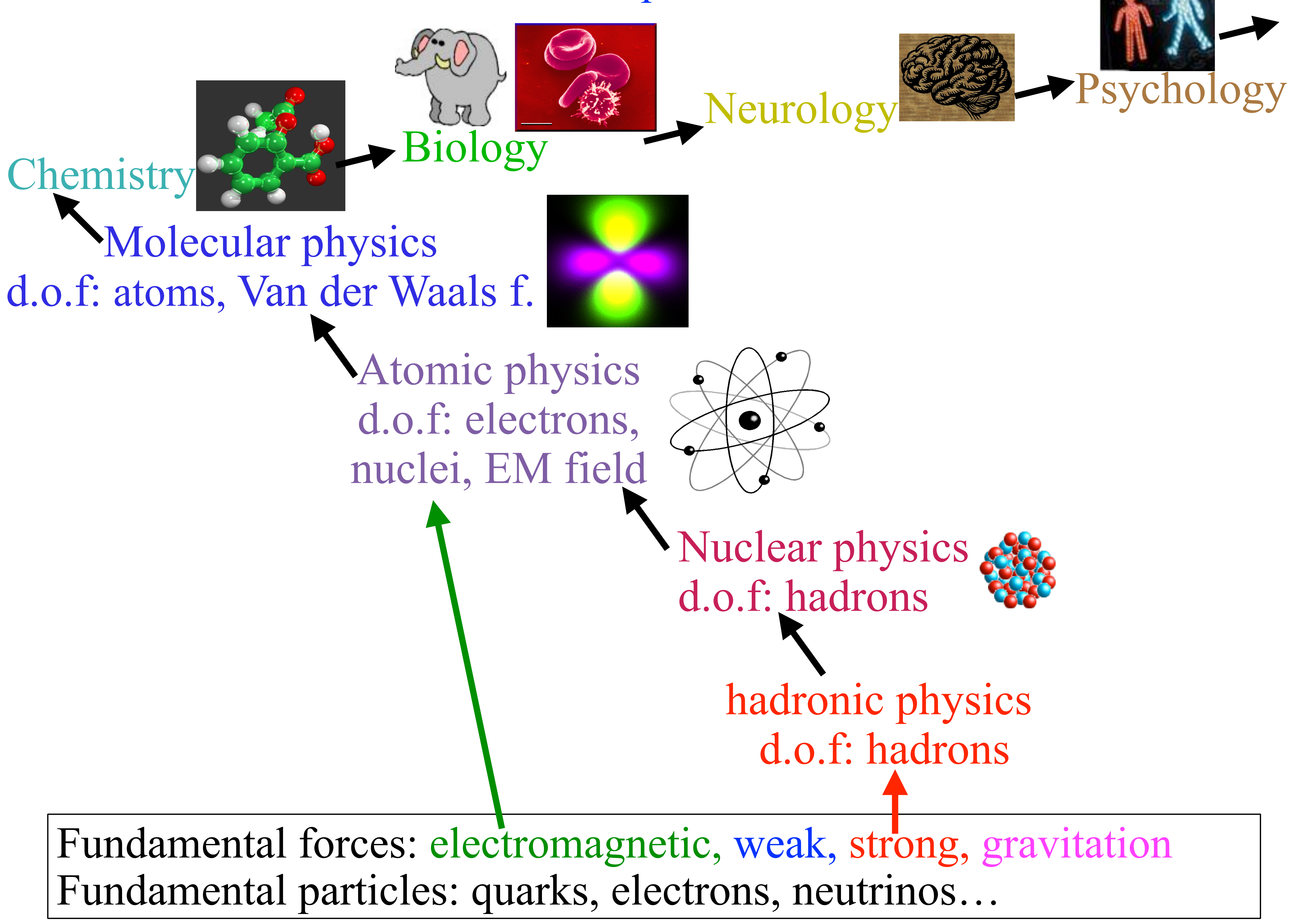

Effective descriptions of Nature

Nuclear physics
d.o.f: hadrons

hadronic physics
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← low Q^2

Fundamental forces: electromagnetic, weak, strong, gravitation
Fundamental particles: quarks, electrons, neutrinos…

Effective descriptions of Nature

Leading effective theory: Chiral Effective Field Theory (**χEFT**). Obtained using a Lagrangian consistent with QCD's chiral symmetry (neglecting quark masses). Typically valid for $Q^2 \ll m_\pi^2$.

⇒ Crucial piece of a complete understanding of Nature.

Nuclear physics
d.o.f: hadrons

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hadronic physics
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Nuclear physics
d.o.f: hadrons

χEFT has been very successful in describing many hadronic and nuclear phenomena. However, the late 1990s JLab experiments suggested that it did not describe well nucleon spin observables.

hadronic physics
d.o.f: hadrons

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Nuclear physics
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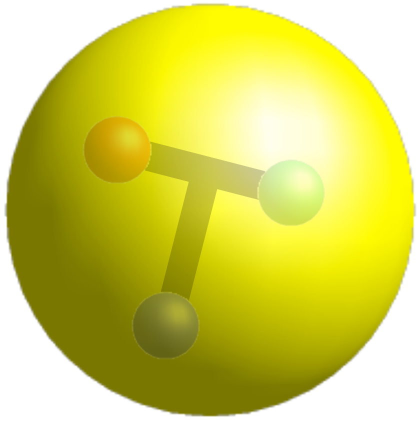

Emerging quantities that characterize the nucleon: charge, mass, anomalous magnetic moment, generalized spin polarizabilities…

Some hadronic quantities can be measured directly: charge, mass, (anomalous) magnetic moment…

Others cannot: e.g. generalized spin polarizabilities. To access them, we used **sum rules**.

Sum rule: relation (rule) between a static property of the target and an integral (sum) over a dynamical quantity

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Ex:

Bjorken sum rule (most famous QCD spin sum rule). Derived for infinite Q^2:

$$\int_0^1 g_1^p - g_1^n dx = \frac{1}{6} g_a$$

Axial charge (g_a)

Bjorken-x (x)

Neutron spin structure function (g_1^n)

Proton spin structure function (g_1^p)

(The axial charge is best measured directly, in β-decay, so this sum rule* is used to test QCD)

*or rather its generalization to finite Q^2

Spin polarizabilities sum rules:

Generalized forward spin polarizability:

$$\gamma_0 = \frac{4e^2M^2}{\pi Q^6}\int_0^1 x^2\left(g_1 - \frac{4M^2}{Q^2}\ x^2 g_2\right)dx$$

Longitudinal-Transverse polarizability:

$$\delta_{LT} = \frac{4e^2M^2}{\pi Q^6}\int_0^1 x^2(g_1+g_2)dx$$

We do not know how to experimentally access γ_0 and δ_{LT} directly, so sum rules are used to measure them.

What are spin polarizabilities ?

Polarizabilities encode the 2nd order reaction of a body subjected to a (bona-fide, i.e. $Q^2 = 0$) electromagnetic field.

The full reaction is described by two Compton scattering amplitudes, f_1 (spin-independent) and f_2 (spin-dependent).

At low photon energy ν, one can expand them in powers of ν:

Spin-independent ⟶ $$f_1(\nu) = -\frac{\alpha}{M} + \cdots$$

Spin-dependent ⟶ $$f_2(\nu) = -\frac{\alpha\kappa^2}{2M^2}\nu + \cdots$$

Purely elastic reaction (rigid object)

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Electric polarizability

Magnetic polarizability

Spin-independent ⟶ $$f_1(\nu) = -\frac{\alpha}{M} + (\alpha_E + \beta_M)\nu^2 + \mathcal{O}(\nu^4)$$ ⟵ Polarizabilities

Spin-dependent ⟶ $$f_2(\nu) = -\frac{\alpha\kappa^2}{2M^2}\nu + \gamma_0\nu^3 + \mathcal{O}(\nu^5)$$ ⟵ Spin-polarizabilities

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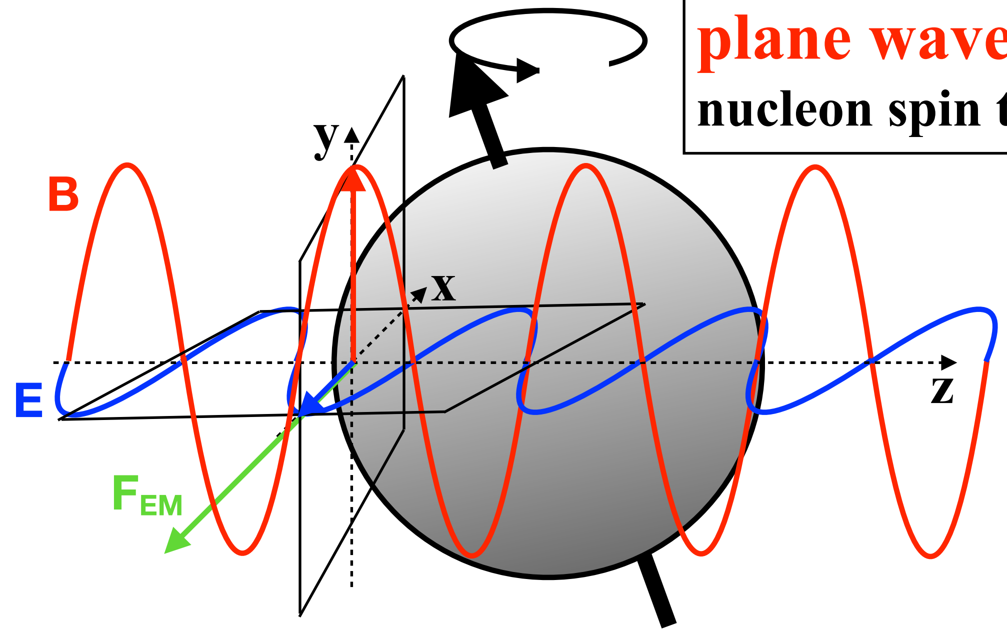
Reaction with deformation (internal rearrangement)

If $Q^2 \neq 0$, the virtual photon has a longitudinal spin component, and δ_{LT} appears (LT stands for Longitudinal-Transverse interference term).

Spin polarizabilities : classical picture

Real photons ($Q^2=0$)
wavelength≳nucleon size.
Hadronic d.o.f.



Standard electromagnetic plane waves make the nucleon spin to precesses: γ_0

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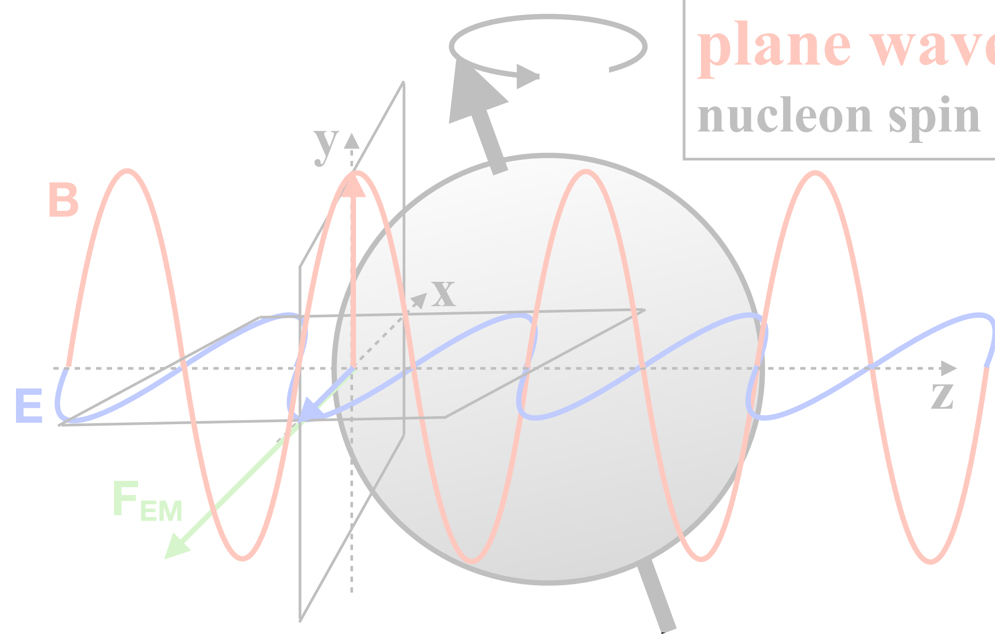
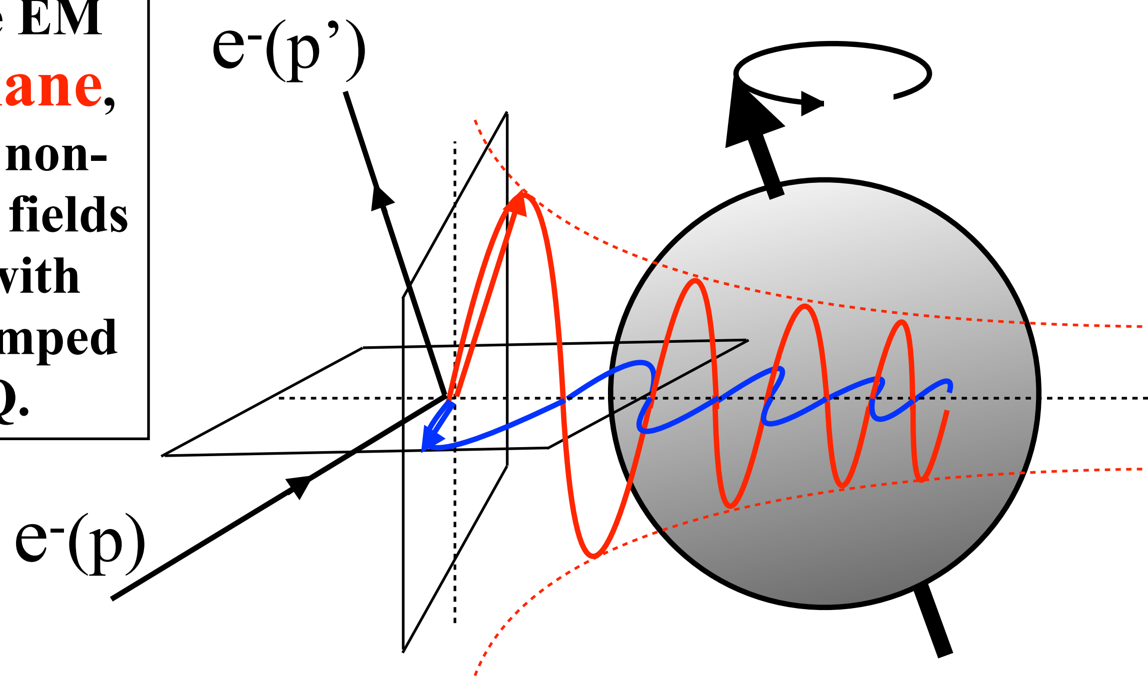


For $Q^2 \neq 0$, the EM wave is not plane, as pictured by non-planar E and B fields components with magnitudes damped as $1/m_\gamma \sim i/Q$.



Deformed electromagnetic waves select specific space-time windows for the measurement.
Additional out-of-plane degrees of freedom becomes available.

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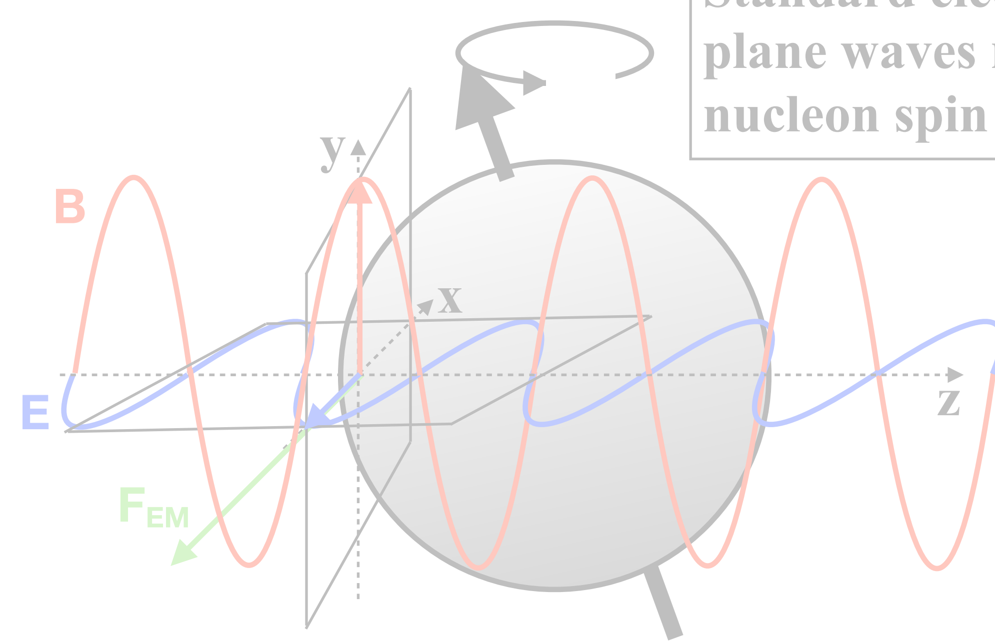

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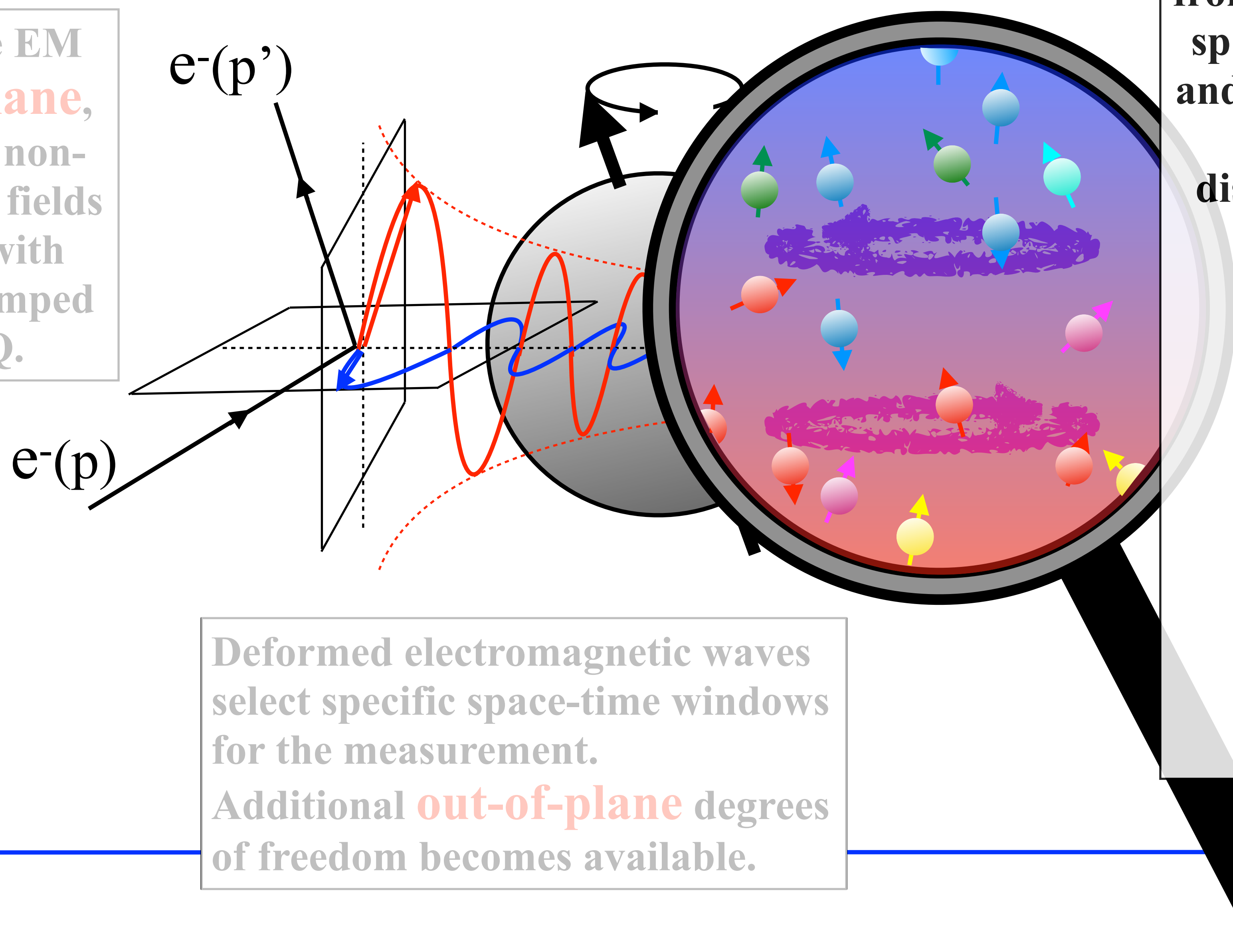

The nucleon spin ultimately arises from the quarks and gluons. At large spacetime scale, their arrangement and motions are revealed by nucleon charges and current density distributions.

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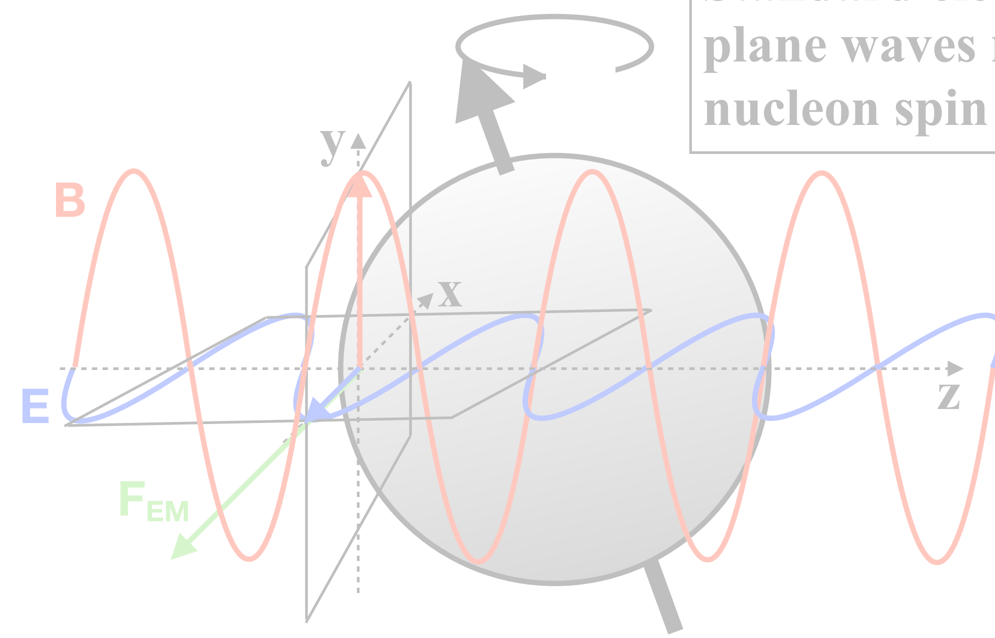

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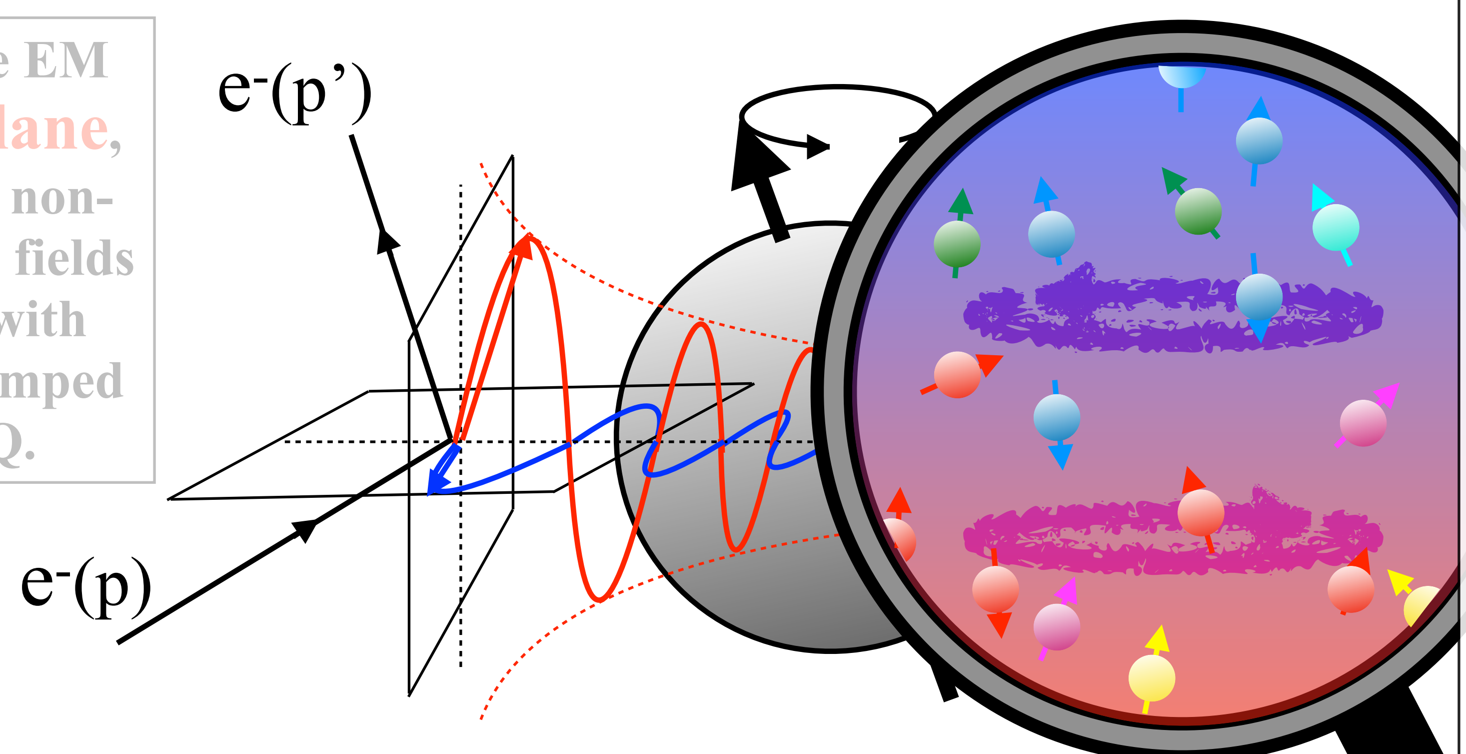

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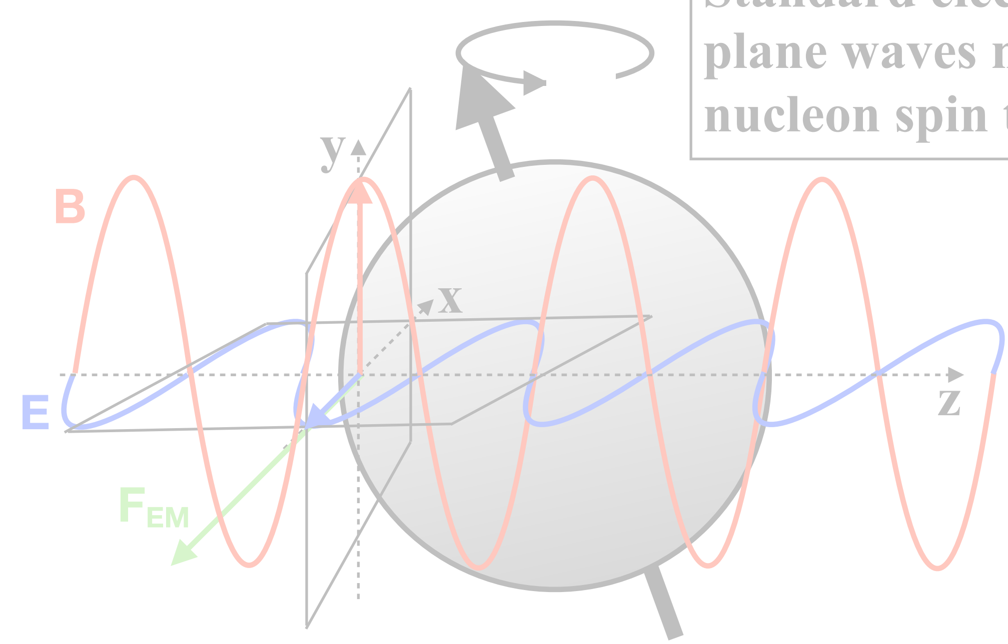

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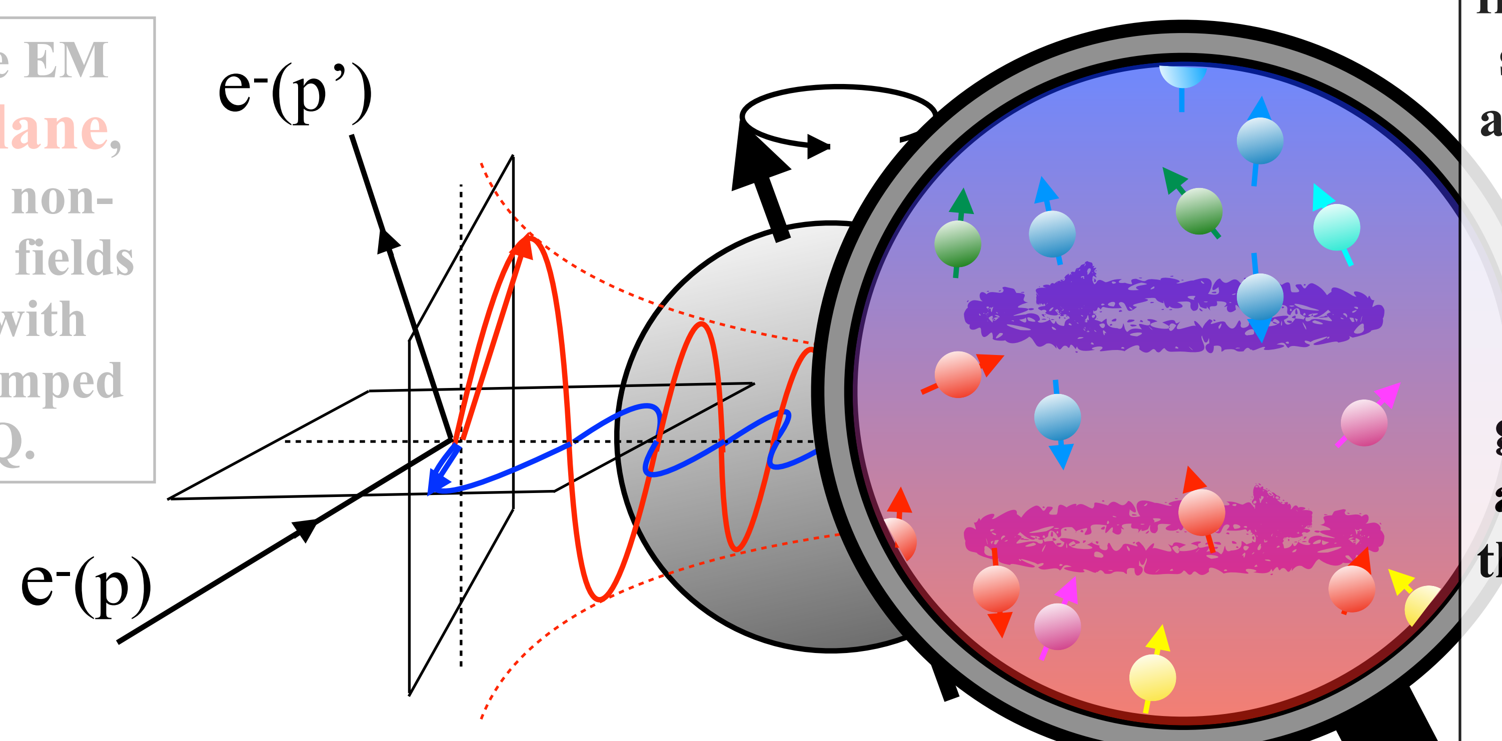

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Deformed electromagnetic waves select specific space-time windows for the measurement.
Additional **out-of-plane** degrees of freedom becomes available.

Results on measurements of spin polarizabilities

Study of the spin structure of the neutron and proton at low Q^2

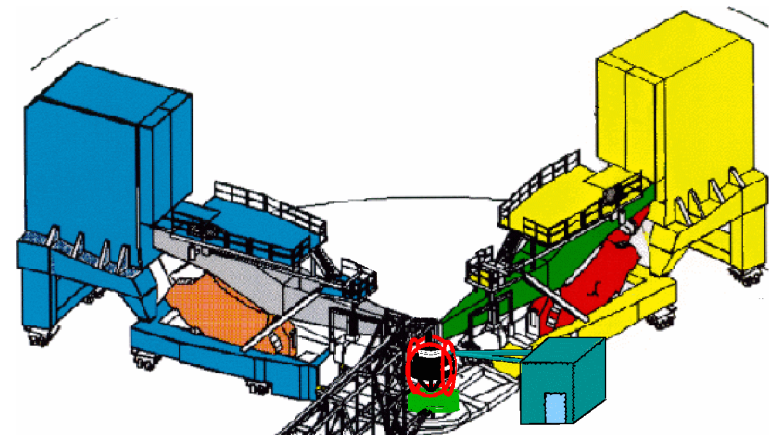
E97-110 (neutron, using longitudinally and transversally polarized ^{3}He):

Spokespeople: **J.P. Chen,** A.D., F. Garibaldi

E08-027 (NH_3, longitudinally and transversally polarized):

Spokespeople: A. Camsonne, J.P. Chen, D. Crabb, **K. Slifer**

JLab Hall A:



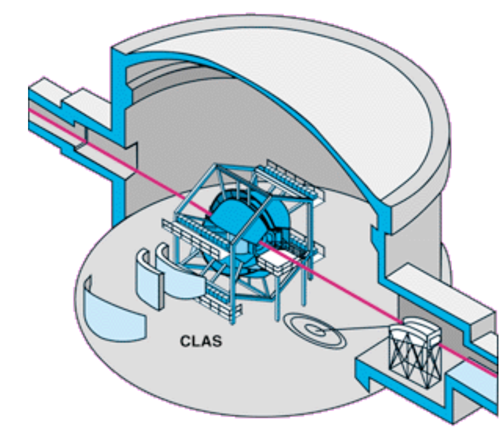
E03-006 (NH_3, longitudinally polarized):

Spokespeople: **M. Ripani**, M. Battaglieri, A.D., R. de Vita

E06-017 (ND_3, longitudinally polarized):

Spokespeople: **A.D.**, G. Dodge, M. Ripani, K. Slifer

JLab Hall B:



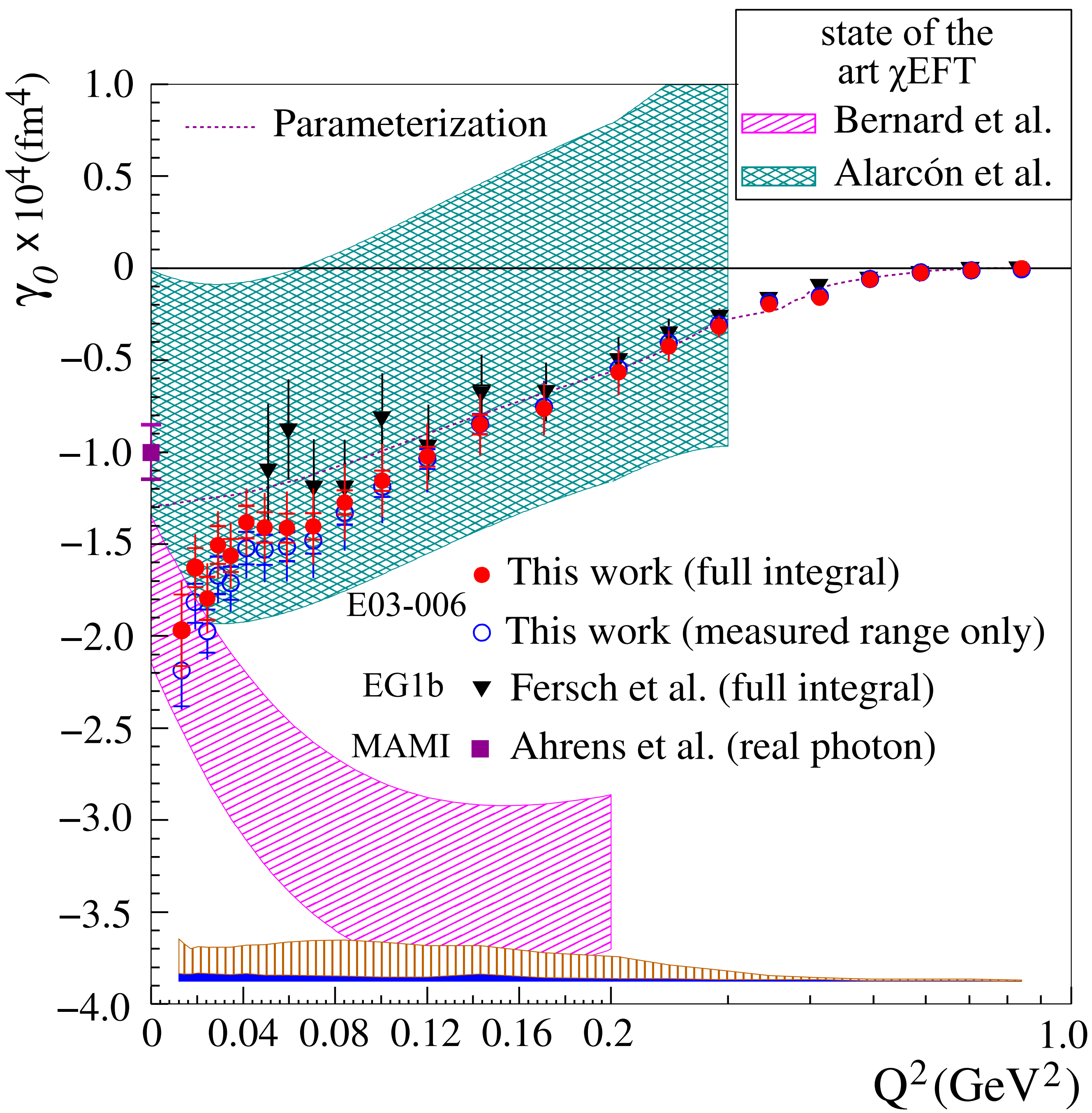
Low Q^2 + covering large ν range so that sum rule's integrals can be formed⇒forward angles

First nucleon spin structure JLab data reaching well into the χEFT applicability domain.

JLab small angle experimental results $\gamma_0(Q^2)$ and $\delta_{LT}(Q^2)$

E03-006 proton,

$\gamma_0^p(Q^2)$: X. Zheng et al, Nature Phys.**17** 736 (2021)



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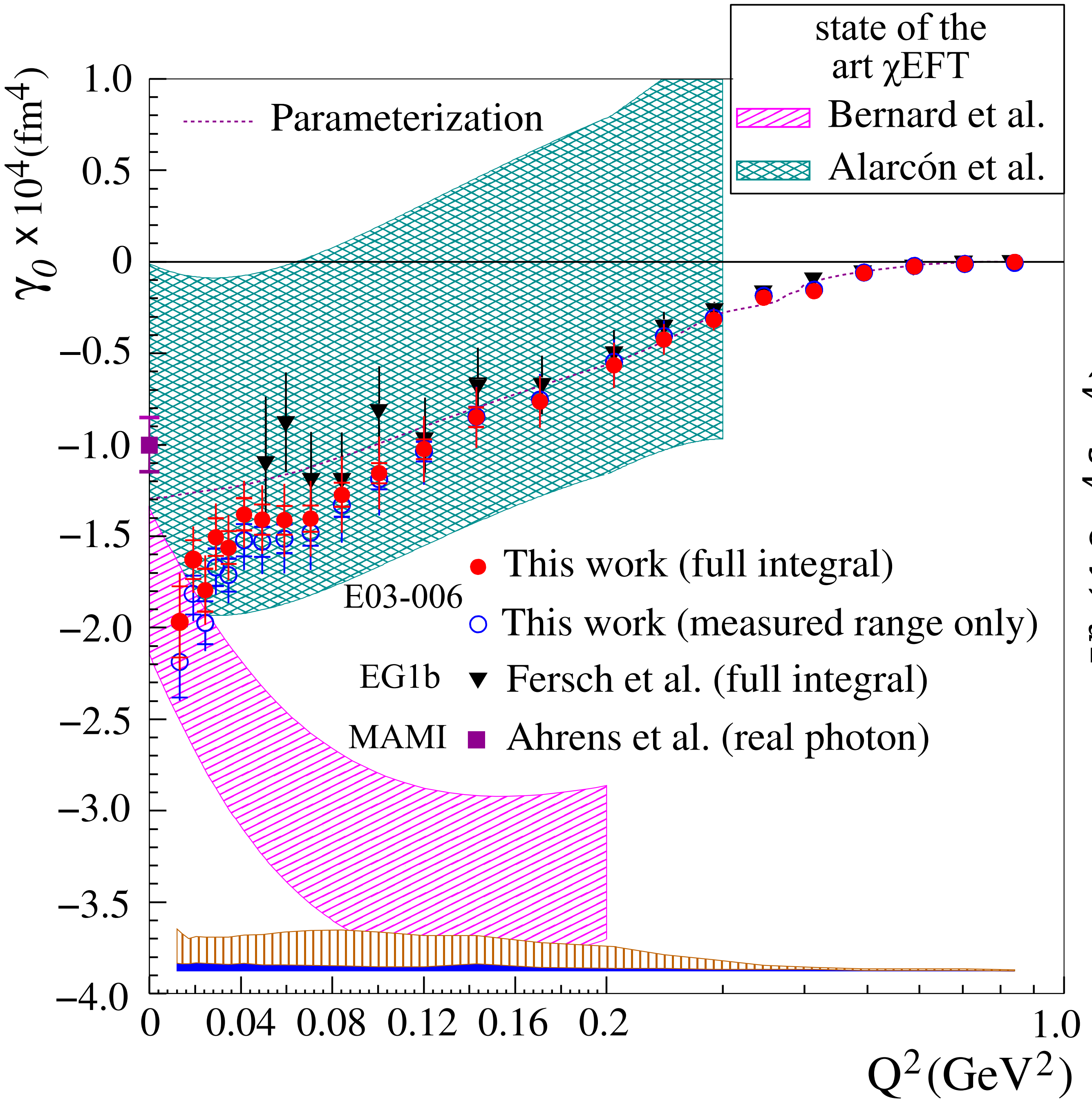
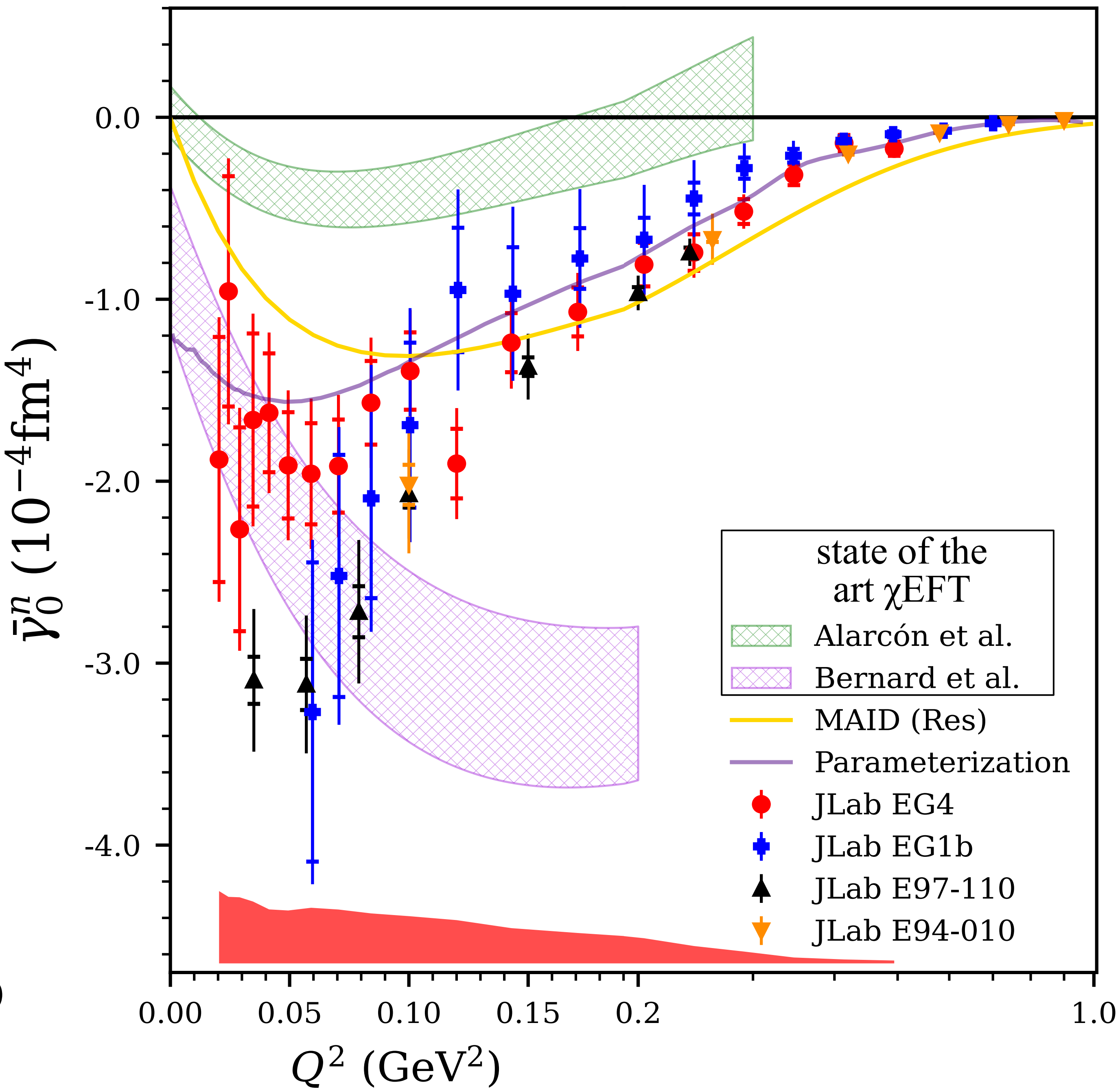


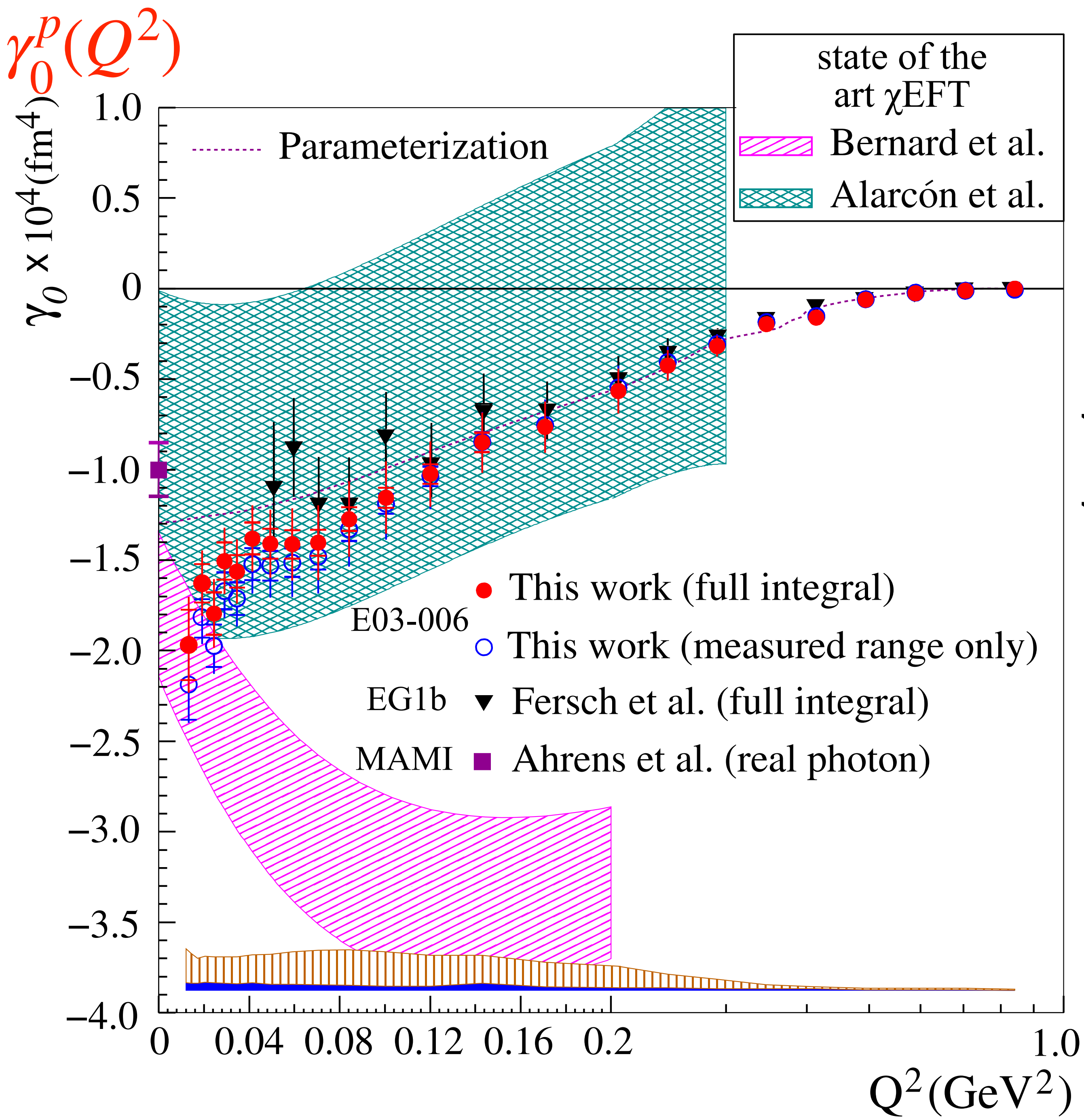
E97-110 neutron,

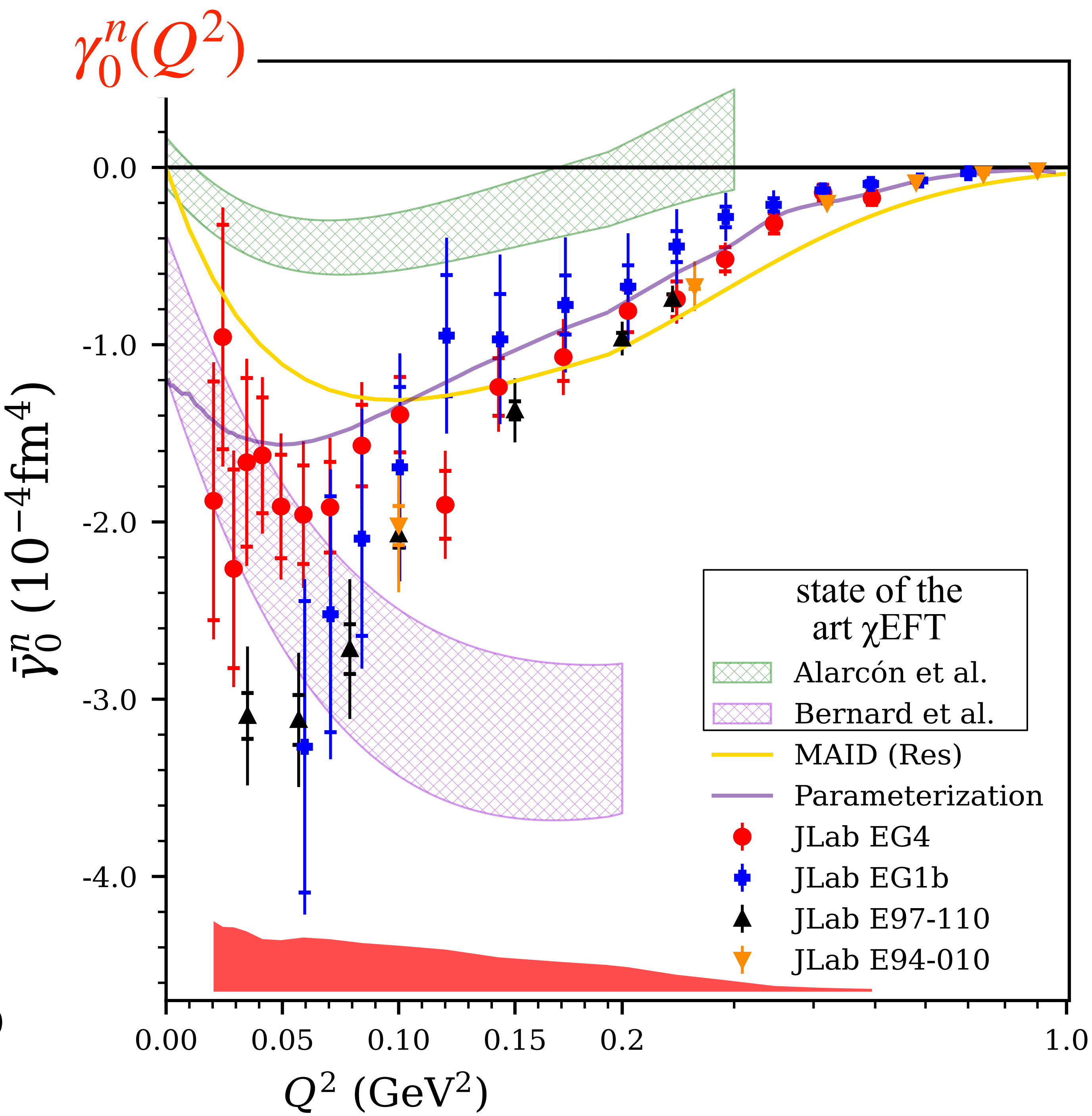
$\gamma_0^n(Q^2)$: V. Sulkosky et al. Nature Physics, **17** 687 (2021)
A. D. et al, (2024)
Article under review



JLab small angle experimental results $\gamma_0(Q^2)$ and $\delta_{LT}(Q^2)$

- New data agree with previous data at larger Q^2 (E94-010/EG1b).
- χEFT calculation of Alarcón et al agrees with data for proton, but large uncertainty, and disagree for neutron.
- Bernard et al. χEFT result agrees for lowest Q^2 points. Proton: large slope at low Q^2 supported by the MAMI+EG4 data

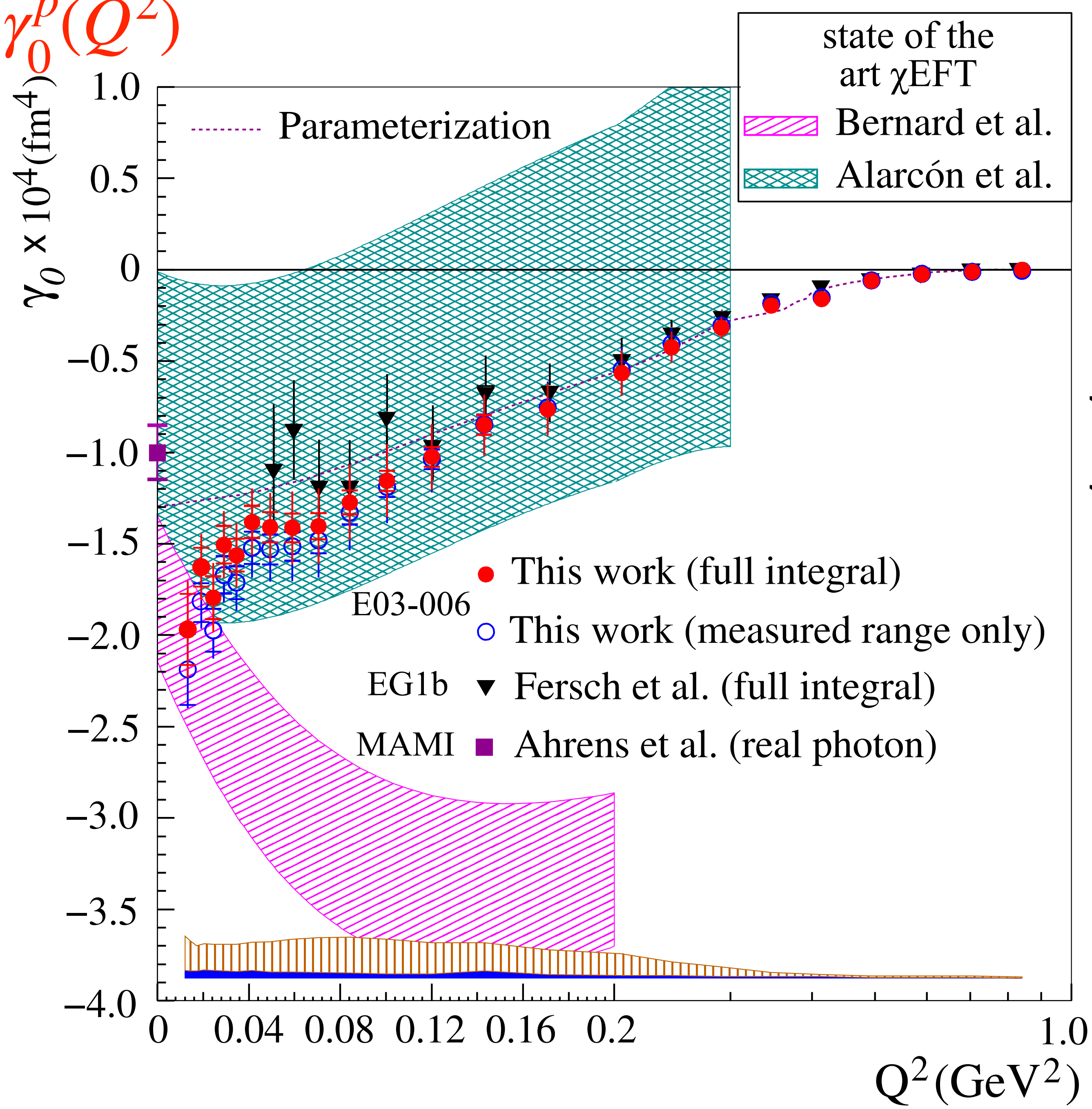




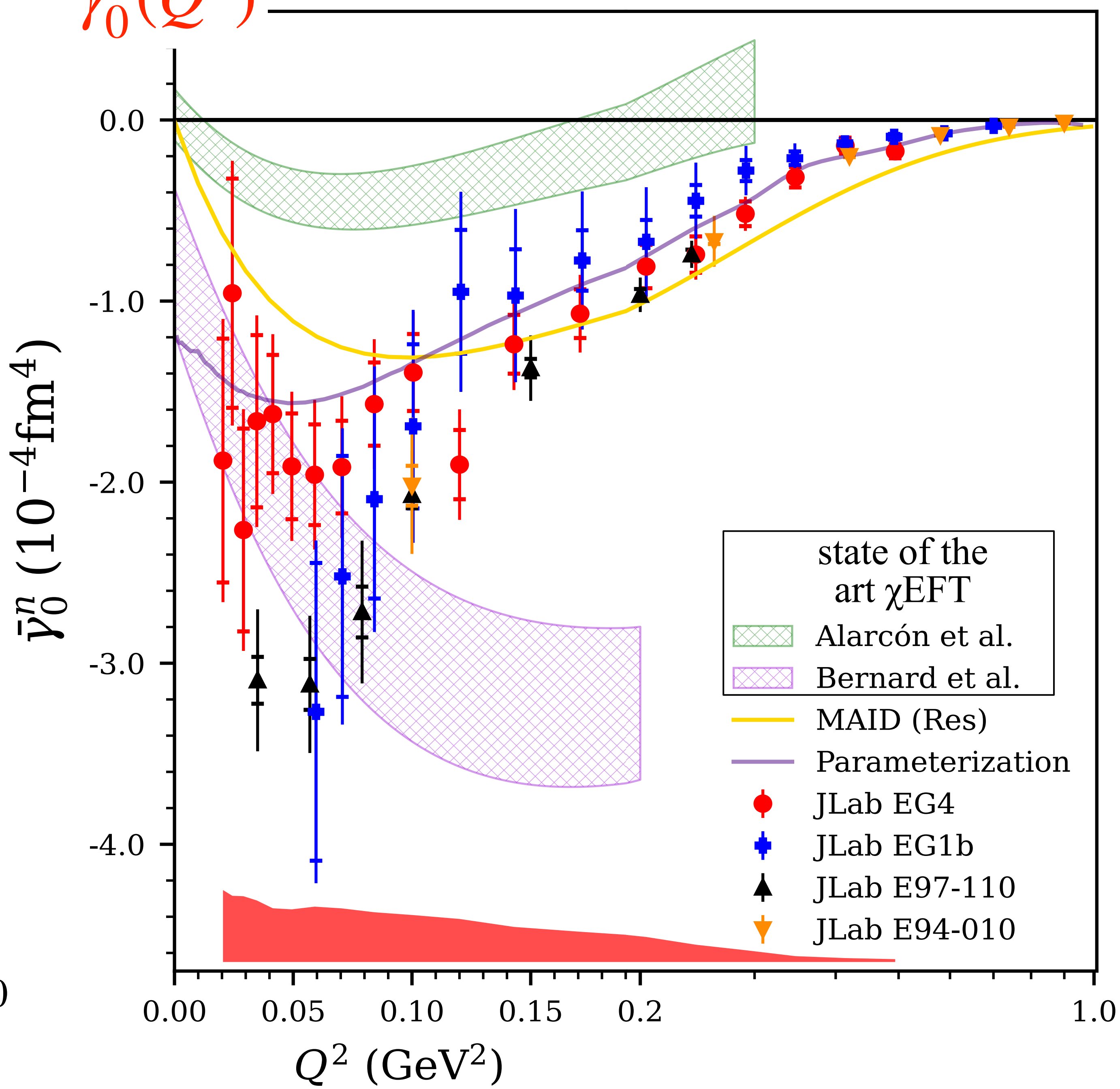
JLab small angle experimental results $\gamma_0(Q^2)$ and $\delta_{LT}(Q^2)$

Interpretation from effective theory (hadronic d.o.f freedom): γ_0 is mostly the difference between contributions from the Δ resonance (negative) and from the nucleon's pion cloud (positive). At $Q^2=0$, the Δ dominates. As the spacetime resolution becomes finer (larger Q^2) the contribution from to the (extended) pion cloud becomes even smaller. At larger Q^2, γ_0 vanishes since it is a global property of the nucleon.

$\gamma_0^p(Q^2)$



$\gamma_0^n(Q^2)$



JLab small angle experimental results $\gamma_0(Q^2)$ and $\delta_{LT}(Q^2)$

$\delta_{LT}(Q^2)$:

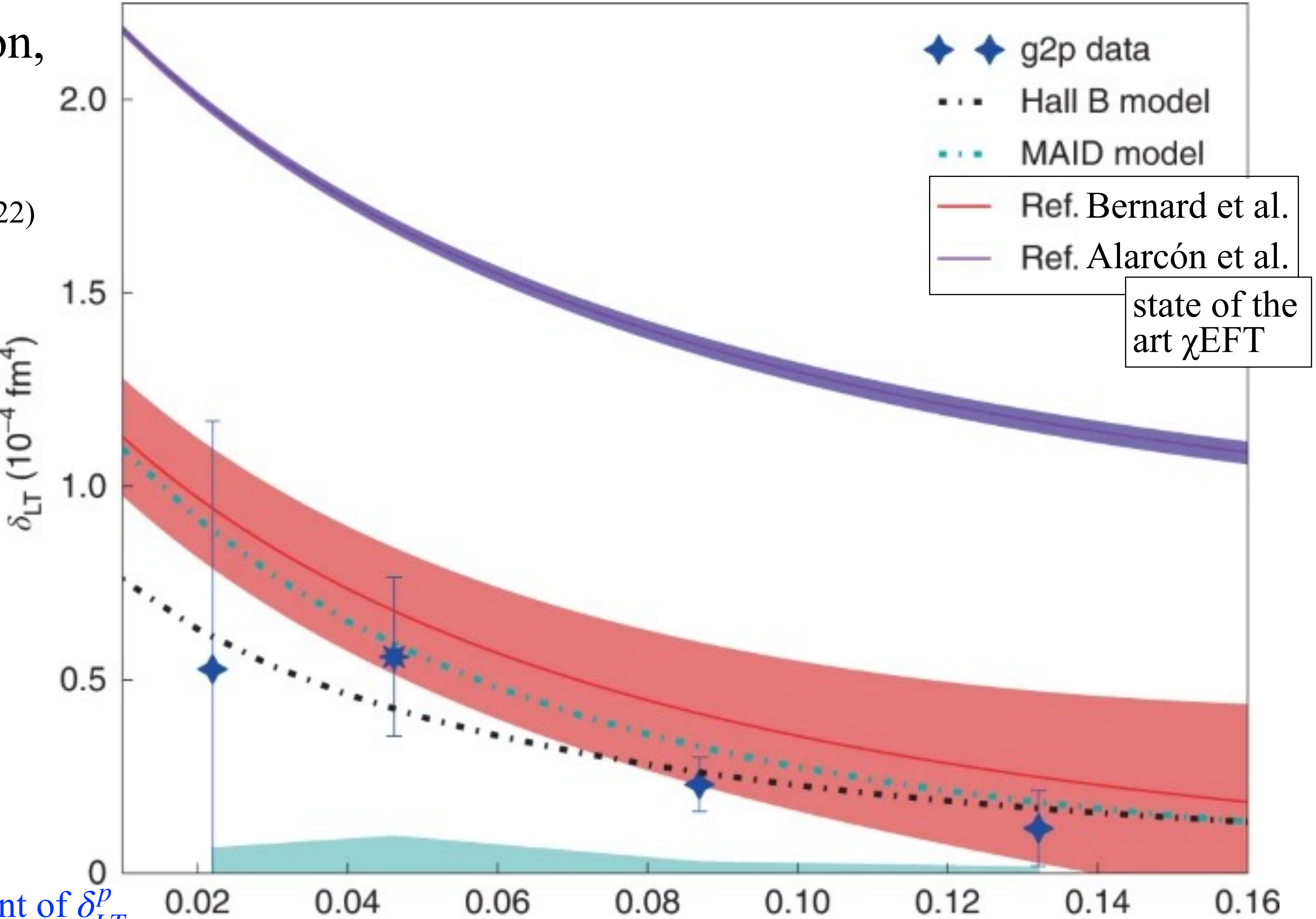
- *Δ* resonance contribution suppressed: Expect to be a robust χEFT prediction (*Δ* d.o.f difficult to include in χEFT calculations);
- Higher moment: Expect to be a robust moment measurement (essentially no unmeasured low-*x* issue).

⇒ The disagreement between $\delta^n_{LT}(Q^2)$ data from earlier experiment (E94-010) and χEFT was particularly surprising: "δ_{LT} puzzle".

JLab small angle experimental results $\gamma_0(Q^2)$ and $\delta_{LT}(Q^2)$

E08-027 proton, $\delta^p_{LT}(Q^2)$:

D. Ruth, et al,
Nat. Phys.**18** 1441 (2022)



- First measurement of δ^p_{LT}
- Agree with χEFT (Bernard et al).
- Agree with other state-of the χEFT (Alarcón et al) for relative Q^2-behavior, but not absolute value.
- "$\delta_{LT}(Q^2)$ puzzle" solved?

JLab small angle experimental results $\gamma_0(Q^2)$ and $\delta_{LT}(Q^2)$

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V. Sulkosky et al.
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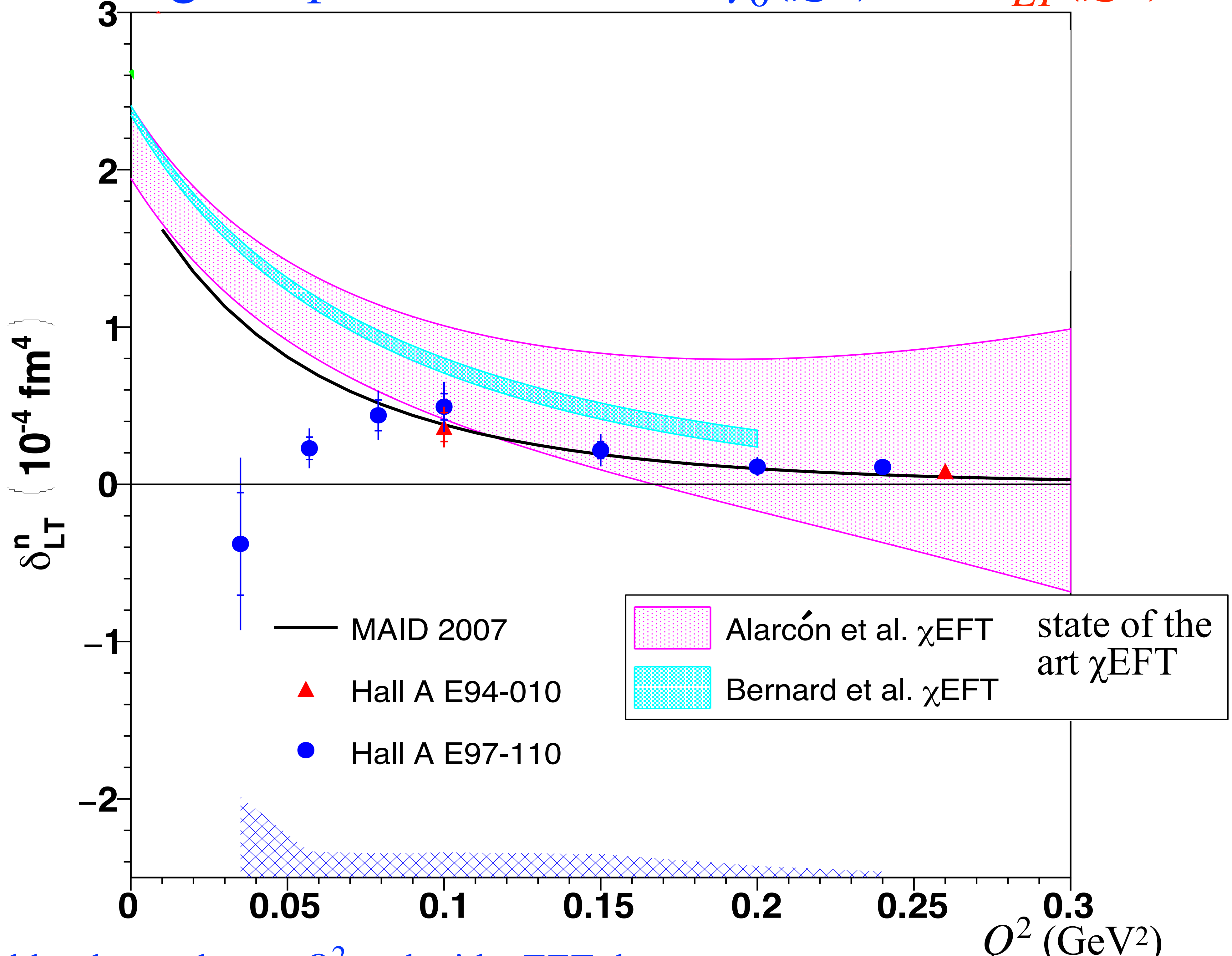
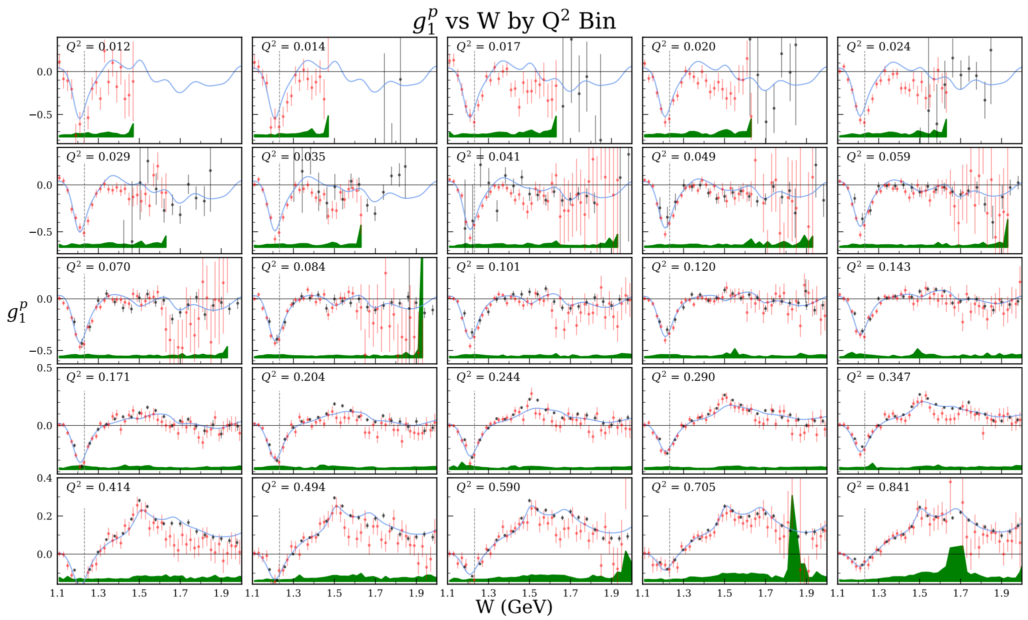
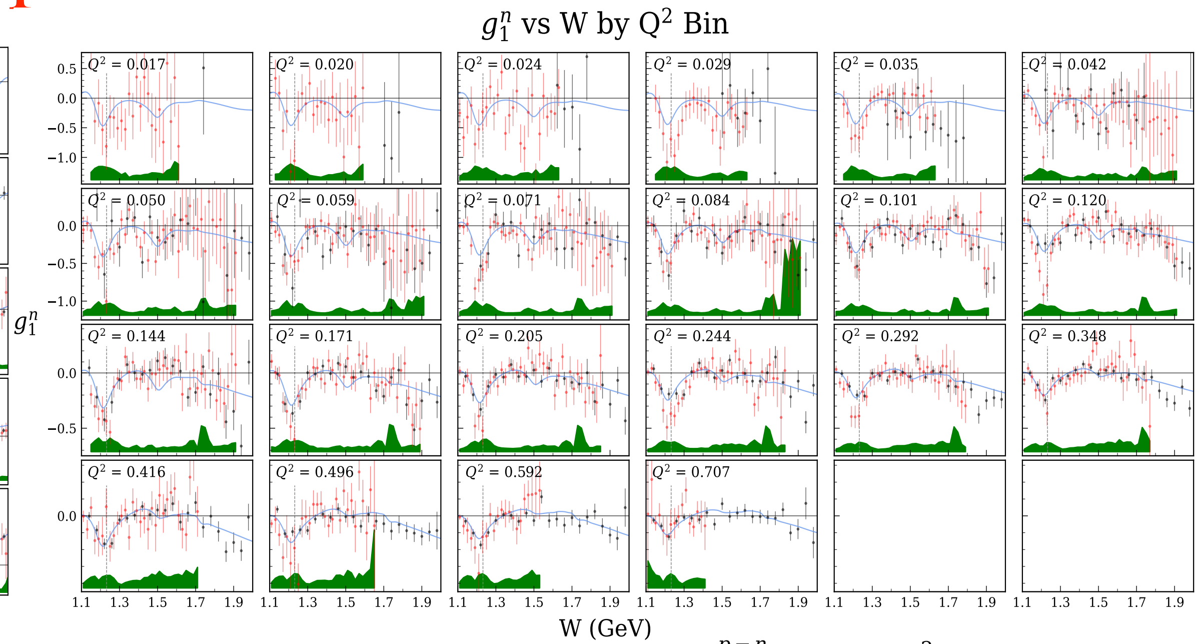
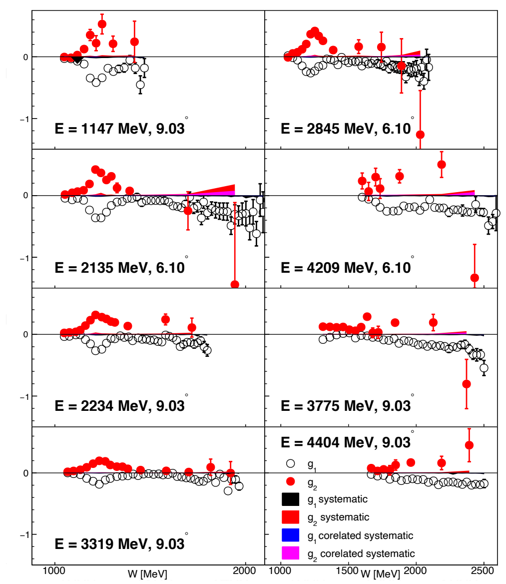


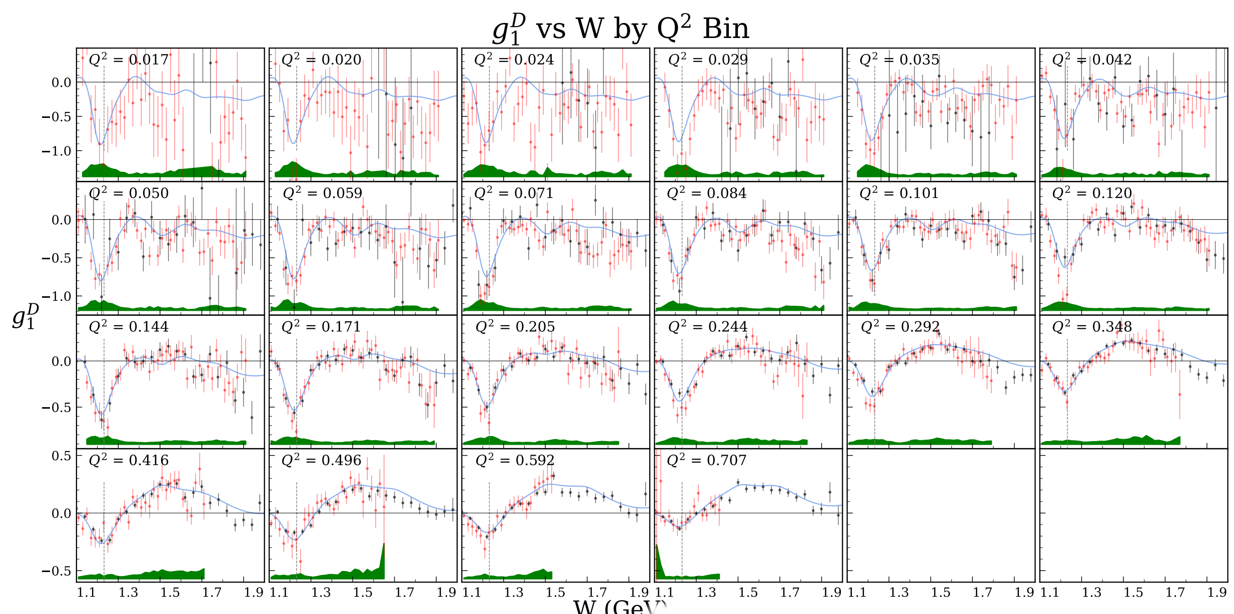
- Good agreement with older data at larger Q^2 and with χEFT there.
- Disagreement with χEFT at lower Q^2, although first moment $\int[g_1 + g_2]dx$ agrees with Schwinger sum rule, see back-up slides.
- ⇒"$\delta^n_{LT}(Q^2)$ puzzle" still remains.

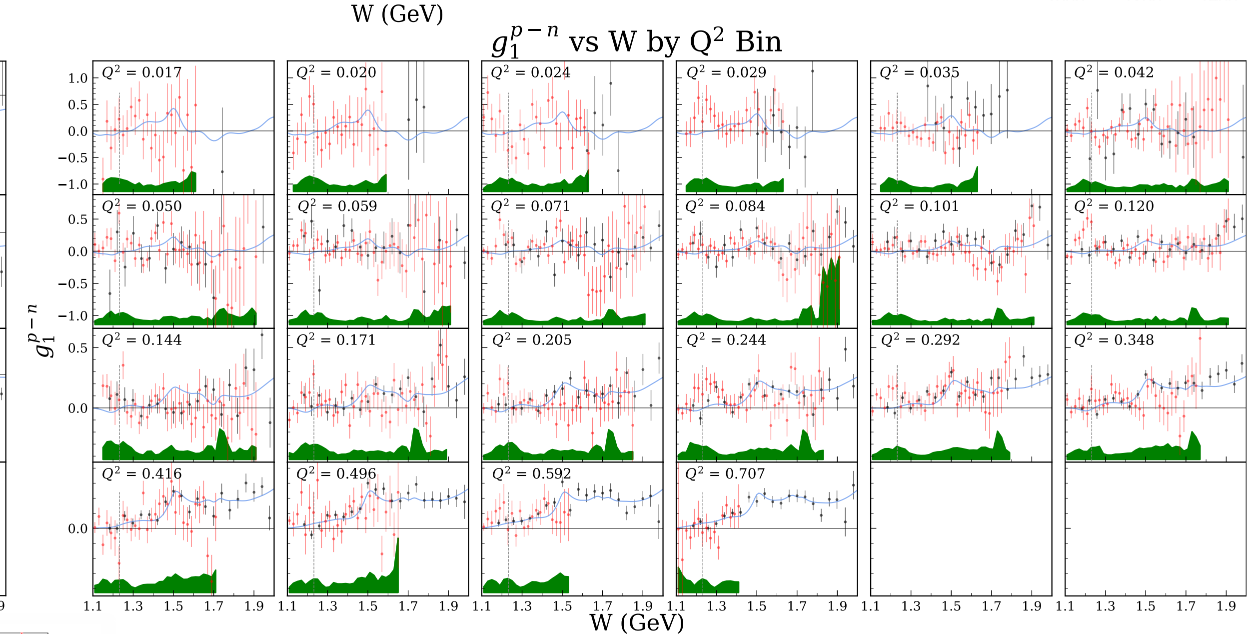
Lots more data on spin structure functions and their moments

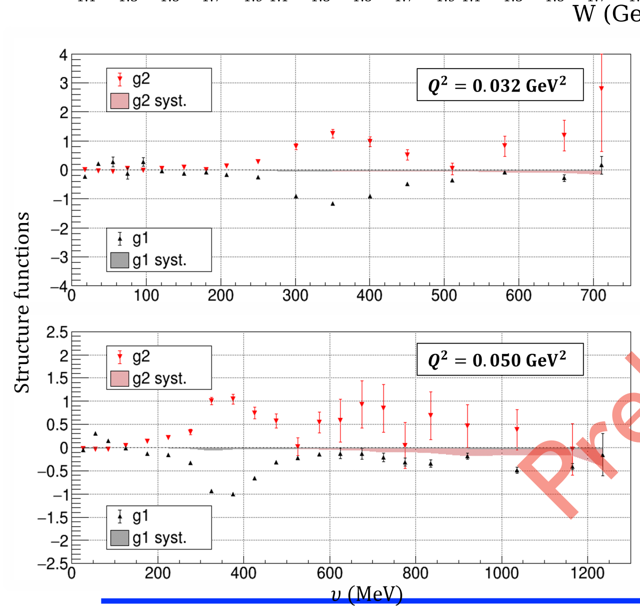


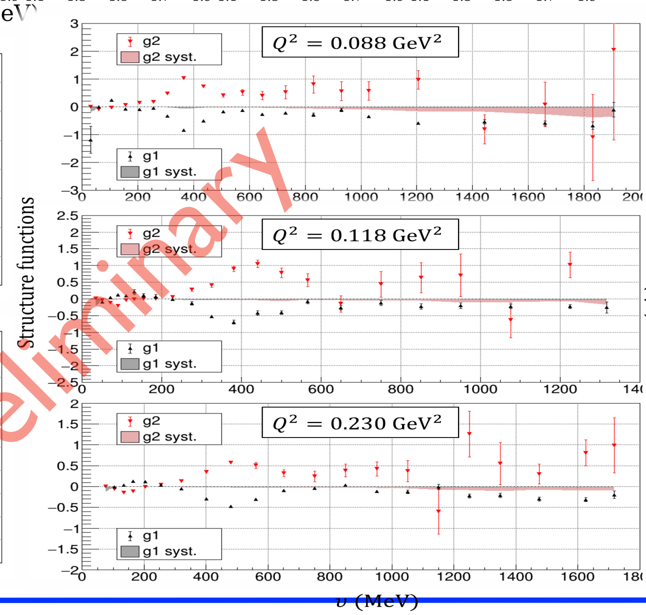


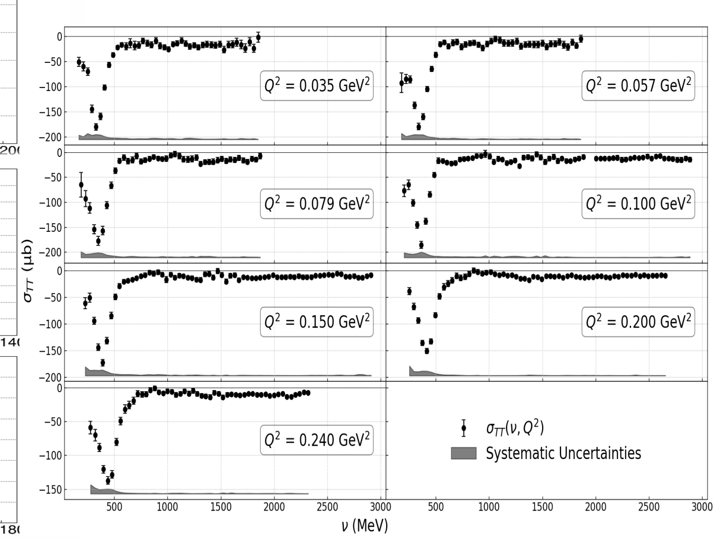


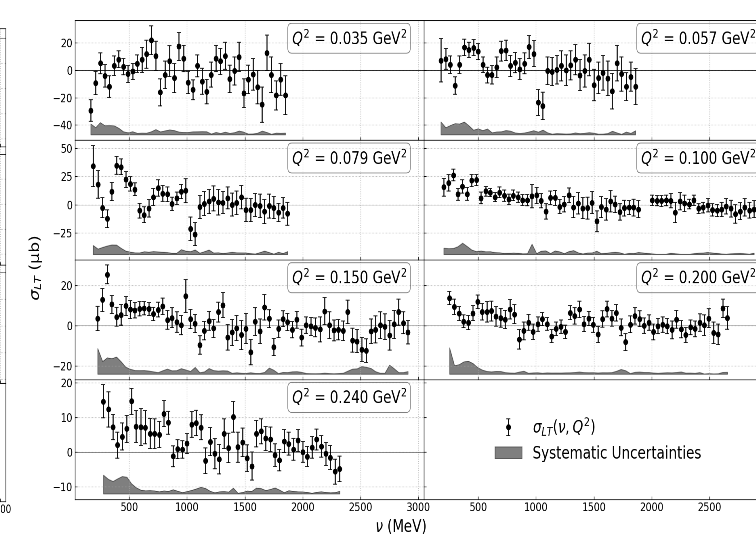




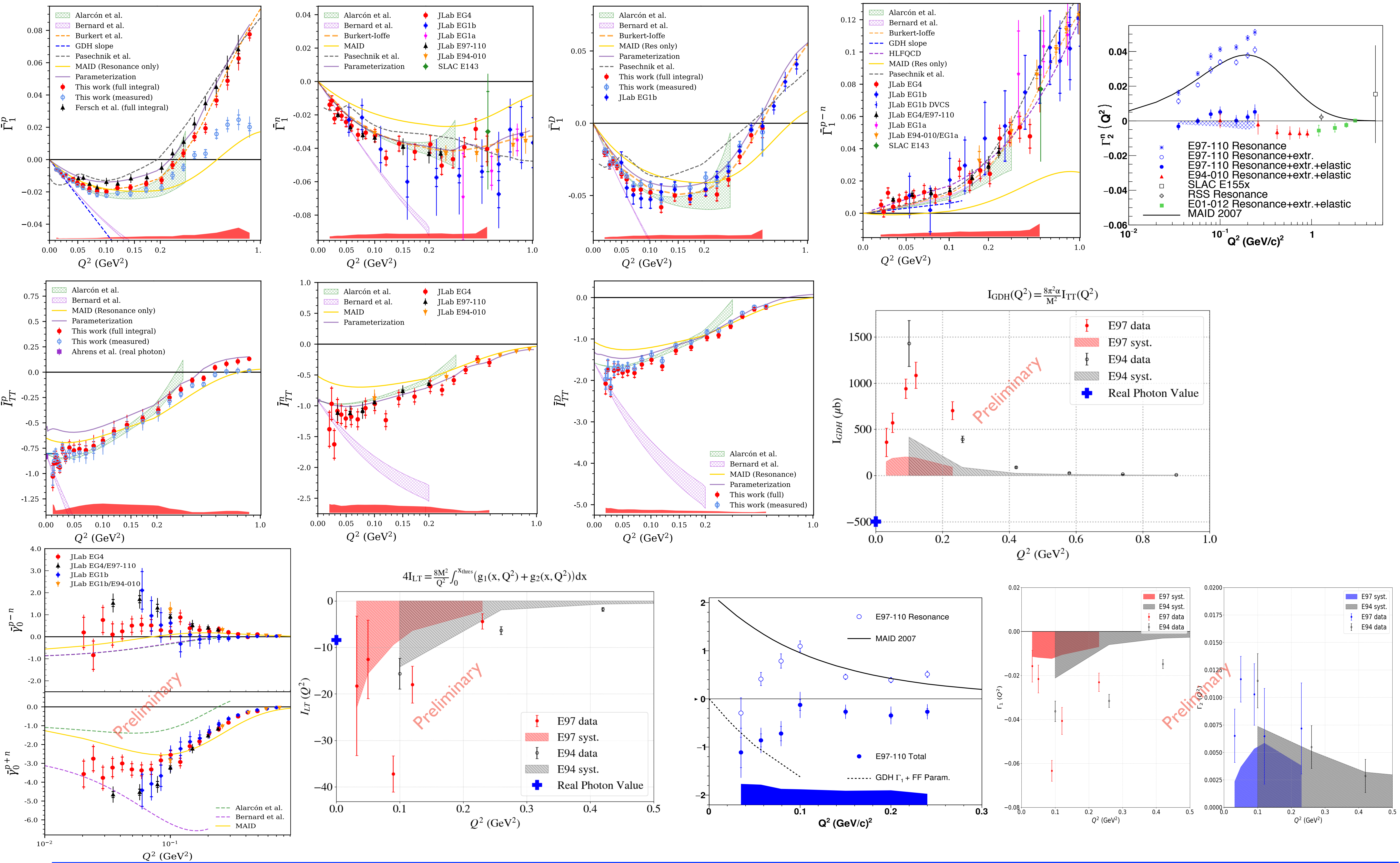








Lots more data on spin structure functions and their moments



Extensive test of χEFT with spin degrees of freedoms

A: agree over range $0<Q^2 \lesssim 0.1$ GeV^2
X: disagree over range $0<Q^2 \lesssim 0.$ GeV^2
- : No prediction available

No significant low-x contribution (More robust measurements)

Ref.	Γ_1^p	Γ_1^n	$\boldsymbol{\Gamma_1^{p-n}}$	Γ_1^{p+n}	γ_0^p	γ_0^n	$\boldsymbol{\gamma_0^{p-n}}$	γ_0^{p+n}	$\boldsymbol{\delta_{LT}^p}$	$\boldsymbol{\delta_{LT}^n}$
Ji 1999	X	X	A	X	-	-	-	-	-	-
Bernard 2002	X	X	A	X	X	A	X	X		X
Kao 2002	-	-	-	-	X	X	X	X		X
Bernard 2012	X	X	~A	X	X	A	X	X	X	X
Alarcon 2020	A	A	~A	A	~A	X	X	X	A	X

} 1^{st} generation experiments and χEFT predictions (Ji 1999, Bernard 2002, Kao 2002)

} 2^{nd} generation (this talk) (Bernard 2012, Alarcon 2020)

Nucleon resonance Δ_{1232} contribution is suppressed (more robust χpt calculations)

Extensive test of χEFT with spin degrees of freedoms

A: agree over range $0<Q^2 \lesssim 0.1$ GeV^2
X: disagree over range $0<Q^2 \lesssim 0.$
GeV^2
- : No prediction available

Ref.	Γ_1^p	Γ_1^n	$\boldsymbol{\Gamma_1^{p-n}}$	Γ_1^{p+n}	γ_0^p	γ_0^n	$\boldsymbol{\gamma_0^{p-n}}$	γ_0^{p+n}	$\boldsymbol{\delta_{LT}^p}$	$\boldsymbol{\delta_{LT}^n}$
Ji 1999	X	X	A	X	-	-	-	-	-	-
Bernard 2002	X	X	A	X	X	A	X	X		X
Kao 2002	-	-	-	-	X	X	X	X		X
Bernard 2012	X	X	~A	X	X	A	X	X	X	X
Alarcon 2020	A	A	~A	A	~A	X	X	X	A	X

Improvement compared to the state of affaires of early 2000s.

Yet, mixed agreement, depending on the observable, despite χEFT refinements (new expansion scheme, including the Δ_{1232} d.o.f,…) and despite data now being well into the expected validity domain of χEFT.

Well-controlled χEFT description of spin observables at large distance remains challenging.

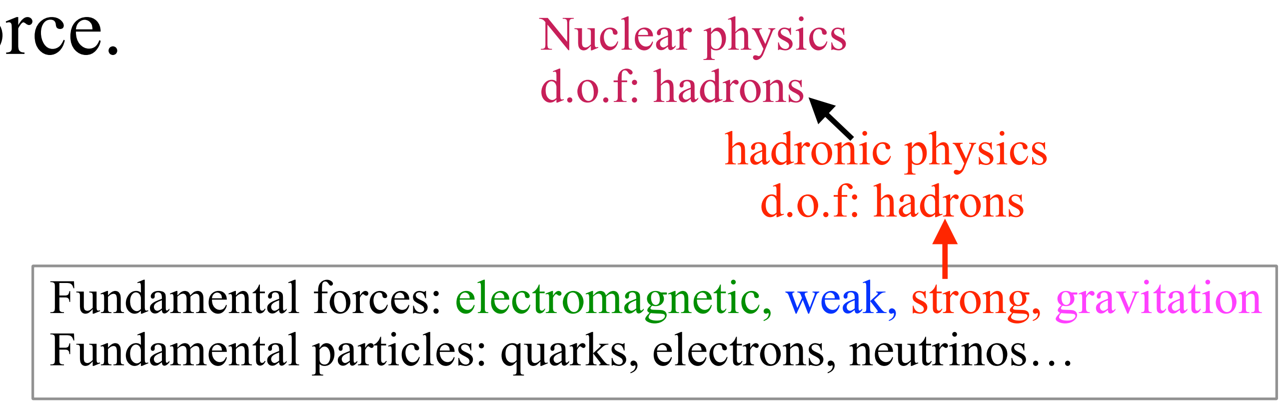
Conclusion

χEFT, although successful in many instances, is challenged by results from dedicated (low Q^2, χEFT domain) spin experiments.

To be sure, low Q^2 sum rule measurements are challenging (forward angles, low-x extrapolation, high-x contamination). But the experiments were run independently with very different detectors and methods. $\Rightarrow$ We seem to be verifying James Bjorken's statement:

> "Polarization data has often been the graveyard of fashionable theories. If theorists had their way they might well ban such measurements altogether out of self protection."

This is a problem: χEFT is the leading approach to manage the first level of complexity of the strong force.



Back-up slides

χEFT series

Domain of applicability: Q^2=0 to somewhere between $m_\pi^2 \approx 0.02$ GeV^2 and $\Lambda_\chi^2 \approx 1$ GeV^2 (the chiral symmetry breaking scale).
Depends on the order at which the series is expanded.

Main χPT expansion (π-N loops): small parameter m_π/Λ_χ.

Including Δ effects (Δ-N loops): additional expansion parameter(s). Two schemes:

- $\delta_{N\Delta} \equiv M_\Delta - M_N$ considered to be of same order as m_π (Bernard et al)
- $\delta_{N\Delta}$ considered as intermediate scale > m_π (Alarcon et al.)

⇒ various Δ contributions may arise at different order in the two schemes.

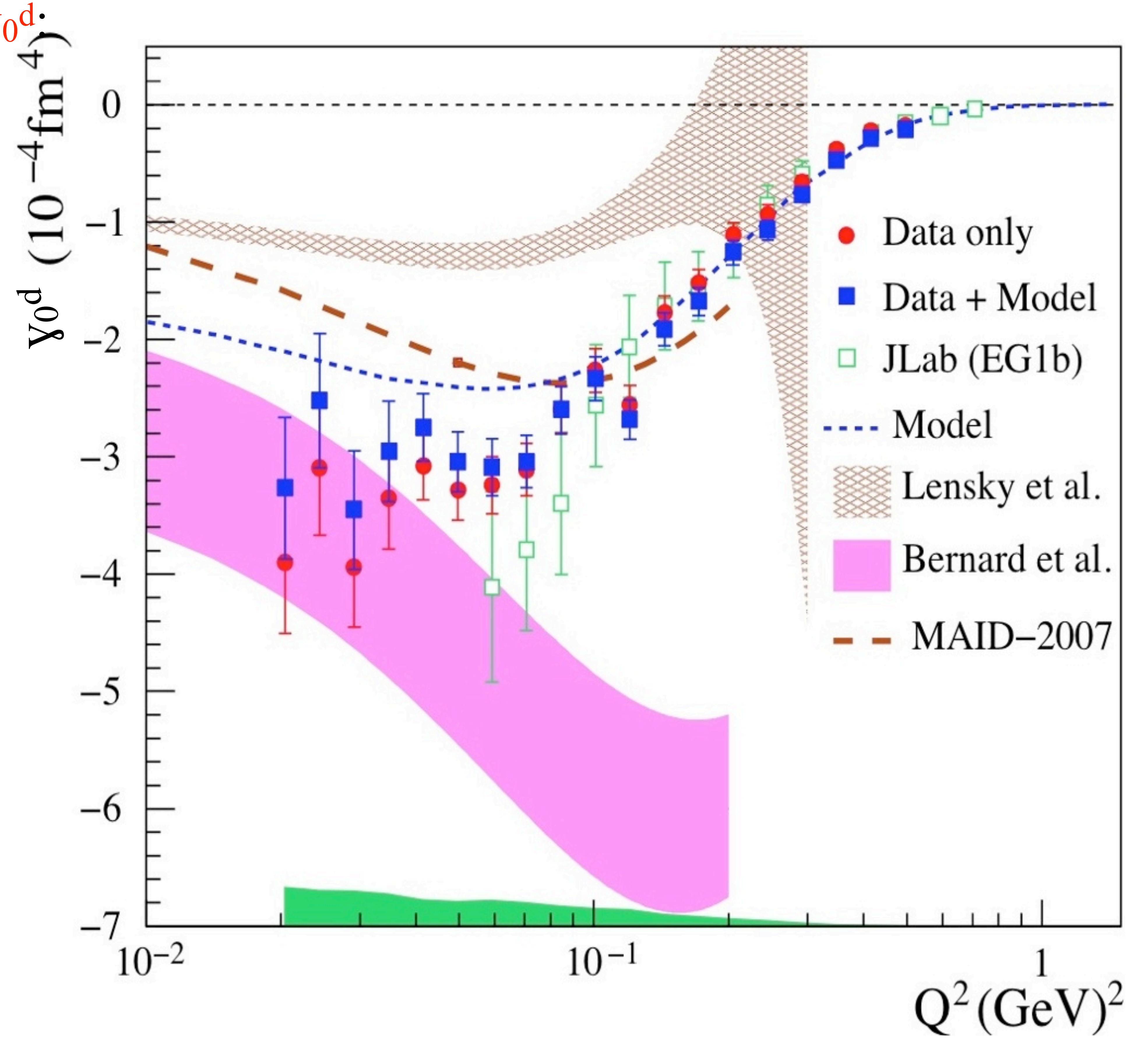
At high enough order, the scheme difference should be negligible.

Bigger difference between two state of the art calculations:
Alarcón et al. includes empirical form factors to the relevant couplings to approximate some of the high-order contributions. Accounts for the suppression of γ_0 and δ_{LT} at large Q^2.
Bernard et al. is a purer calculation, with no such empirical addition, but does not account well for large Q^2 suppression.

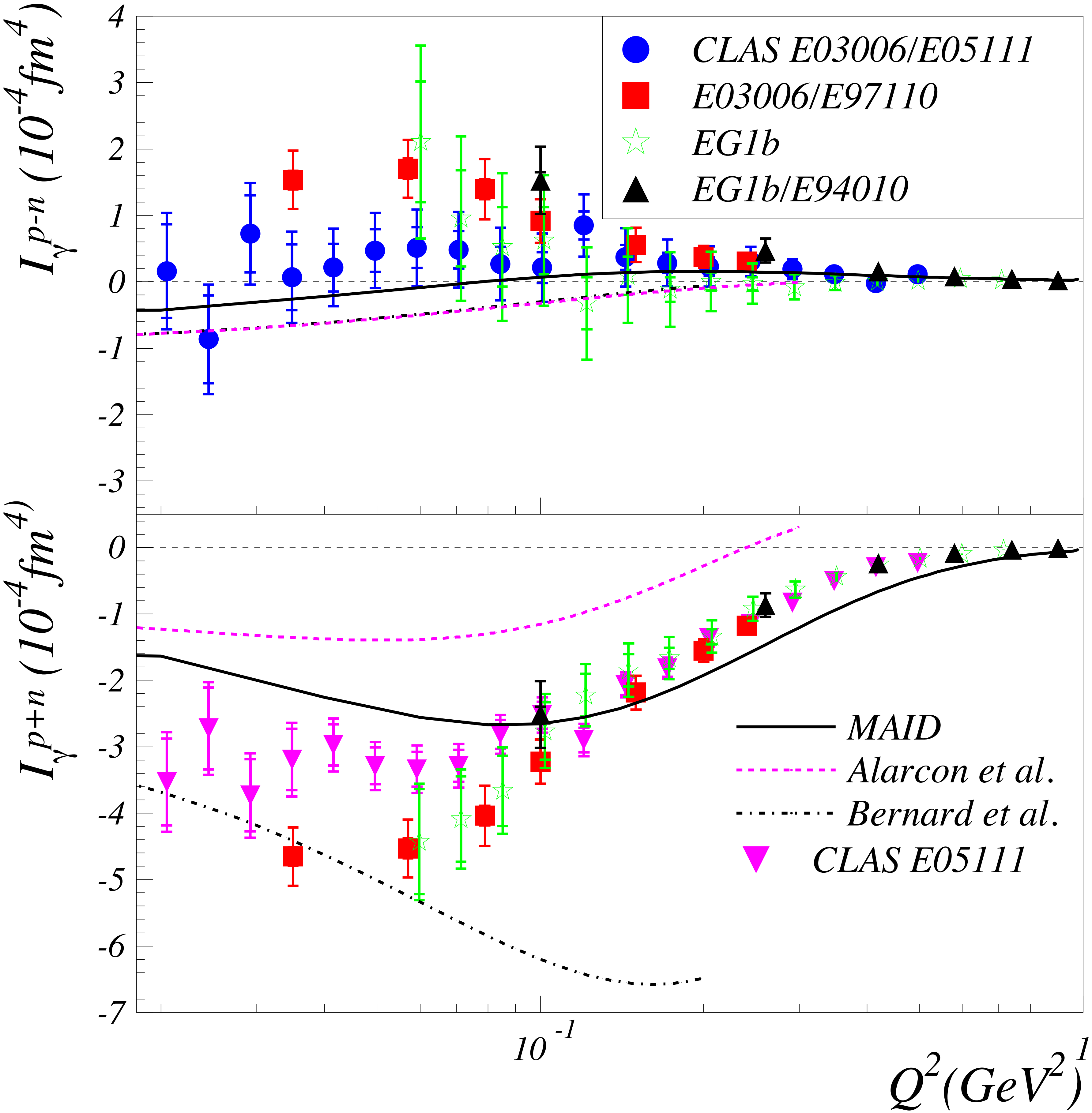
JLab small angle experimental results $\gamma_0(Q^2)$ and $\delta_{LT}(Q^2)$

E05-111 deuteron, γ_0^d:

K. Adhikari et al.
PRL **120**, 062501 (2018)



Isospin decomposition of γ_0



Γ_1 measurements from E97-110 and EG4

$\Gamma_1 \equiv \int g_1(x,Q^2)dx.$

Bjorken sum rule (most famous QCD spin sum rule). Derived for infinite Q^2:

$$\int g_1^p - g_1^n dx = \frac{1}{6} g_a$$

Axial charge (g_a)

(The axial charge is best measured directly, in β-decay, so this sum rule is used to test QCD)

The Gerasimov-Drell-Hearn sum rule:

$$\int_0^{1^-} g_1 \, dx \xrightarrow[Q^2 \to 0]{} \frac{-Q^2 \kappa^2}{8M^2}$$

κ: anomalous magnetic moment

M: Hadron mass

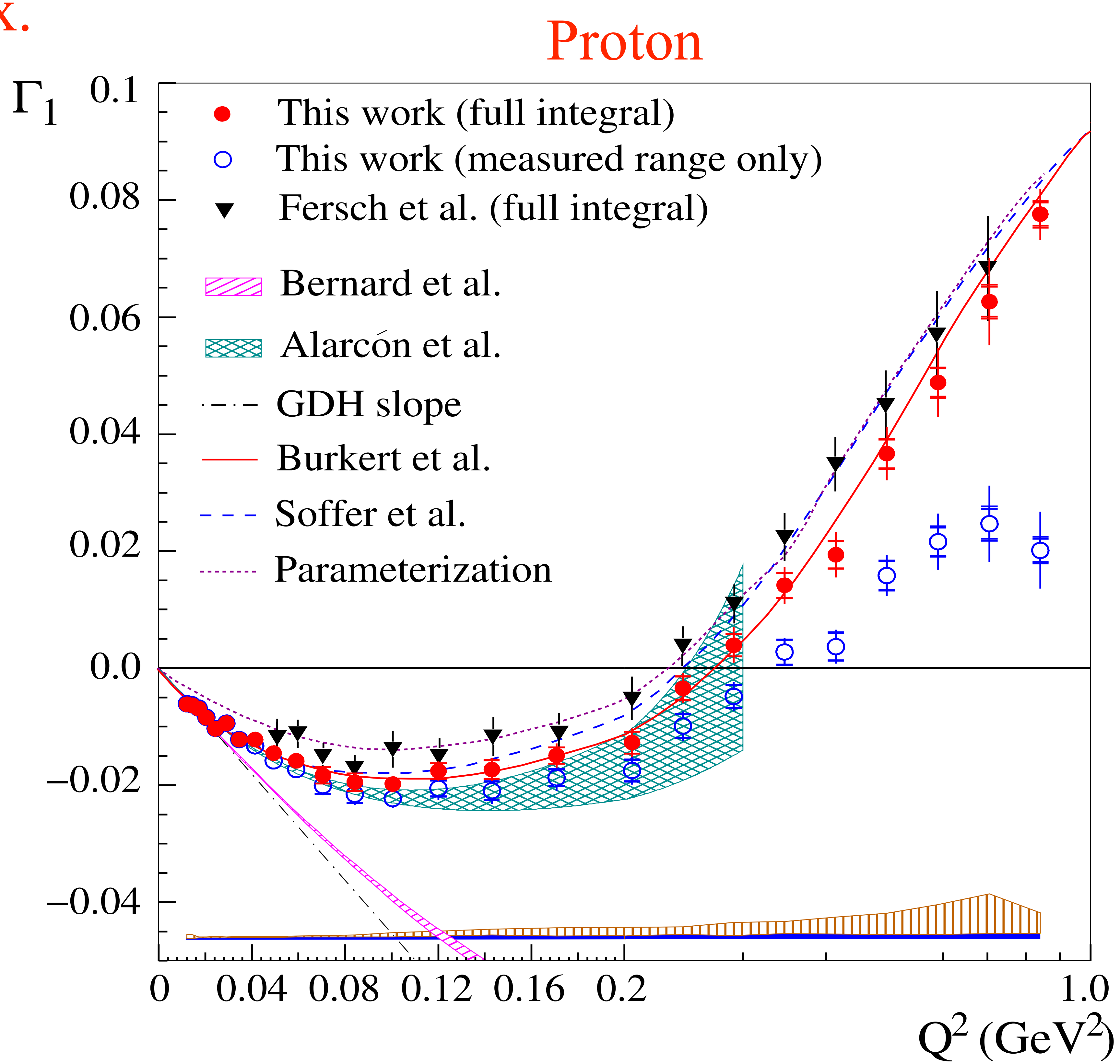
(The anomalous magnetic moment is best measured directly, so this sum rule is used to study the hadron structure)

Both sum rules can be generalized to finite and non-zero Q^2.

Γ_1 measurements from JLab

$\Gamma_1 \equiv \int g_1(x,Q^2)dx.$

X. Zheng et al,
Nature Phys.**17** 736 (2021)



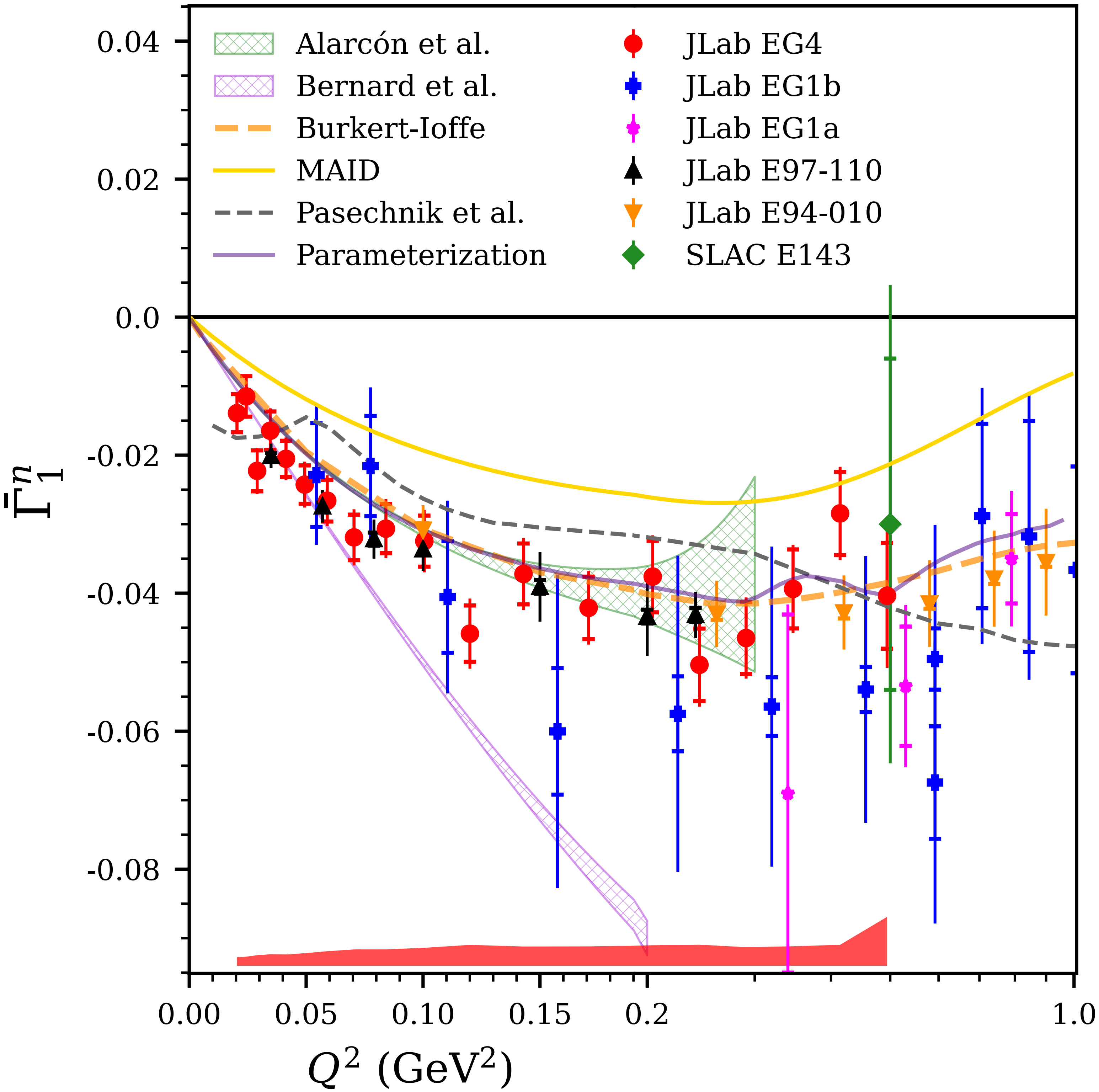
Γ_1 measurements from JLab

$\Gamma_1 \equiv \int g_1(x,Q^2)dx.$

V. Sulkosky et al.
PLB 805 135428 (2020)

A. D. et al, (2024)
Article under review

Neutron

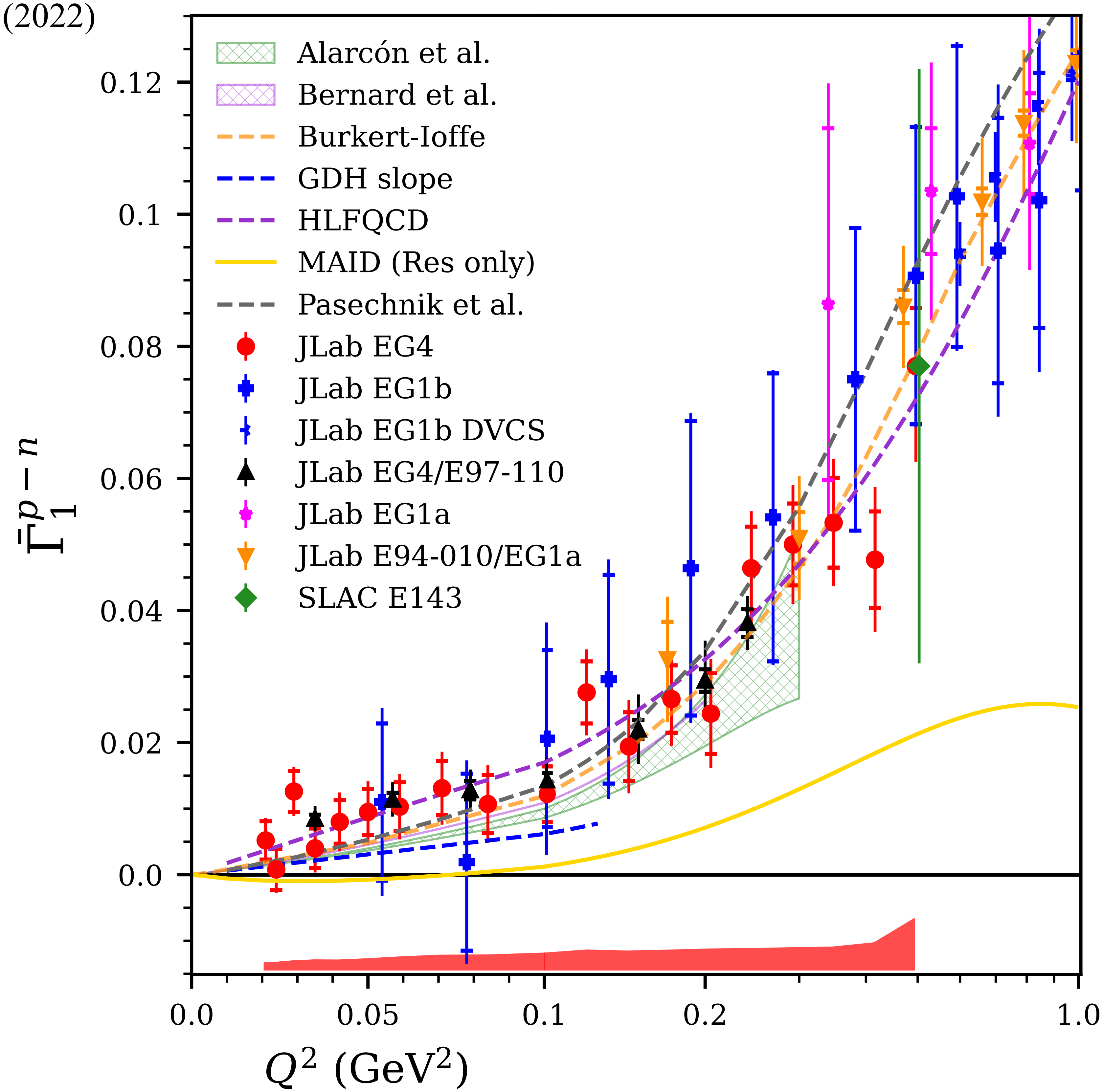


Γ_1 measurements from JLab

Proton-neutron = Bjorken sum

A.D. *et al.* PLB 825 136878 (2022)

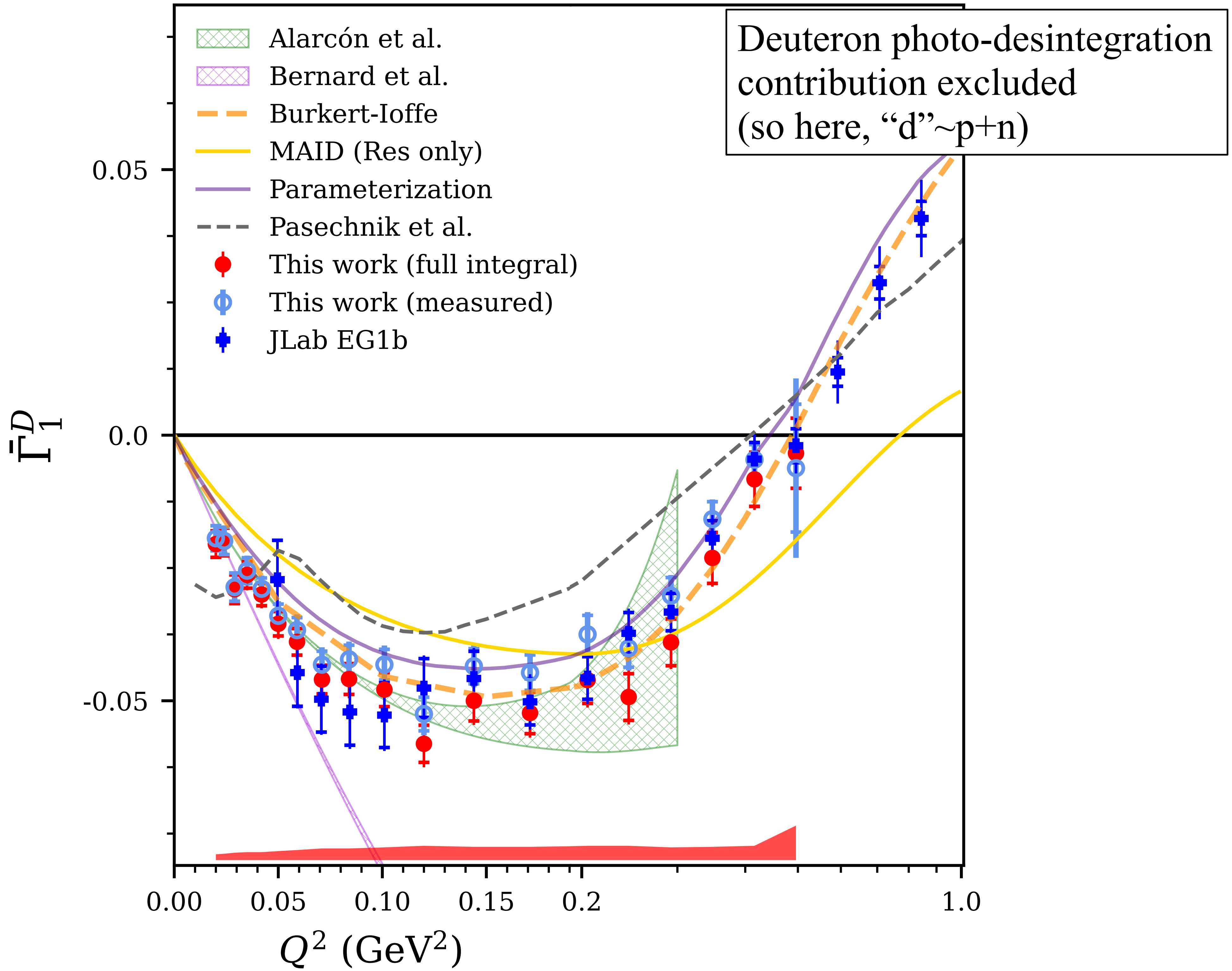
Δ-resonance contribution suppressed for the Bjorken sum



First moments: generalized GDH sum $\Gamma_1^d(Q^2)$ measurement from EG4

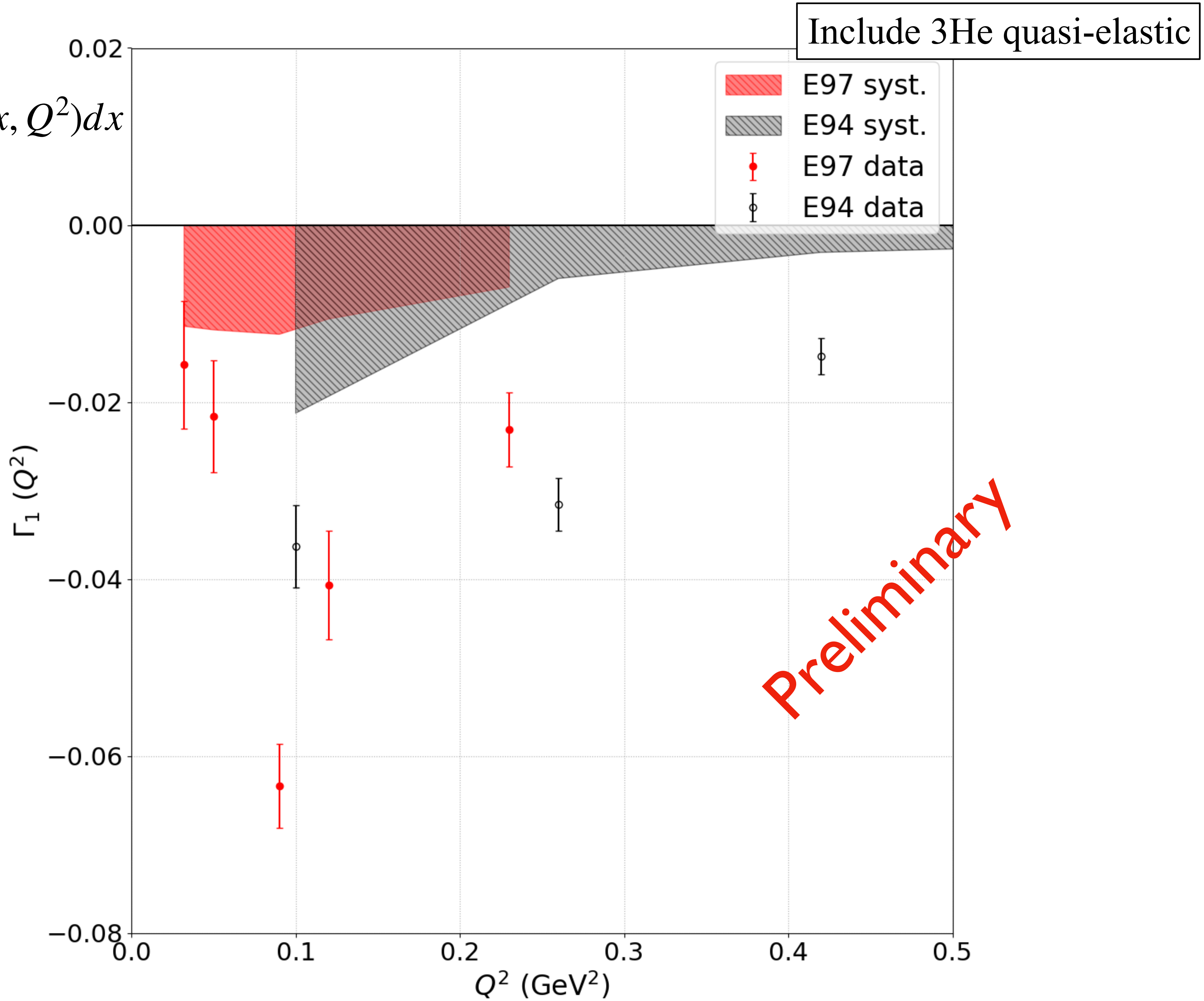
$$\Gamma_1^d = \int_0^{1^-} g_1^d(x, Q^2)dx$$

K. Adhikari et al.
PRL **120**, 062501 (2018)



First moments: generalized GDH sum $\Gamma_1^{3He}(Q^2)$ from E97-110

$$\Gamma_1^{3He} = \int_0^{1^-} g_1^{3He}(x, Q^2)dx$$

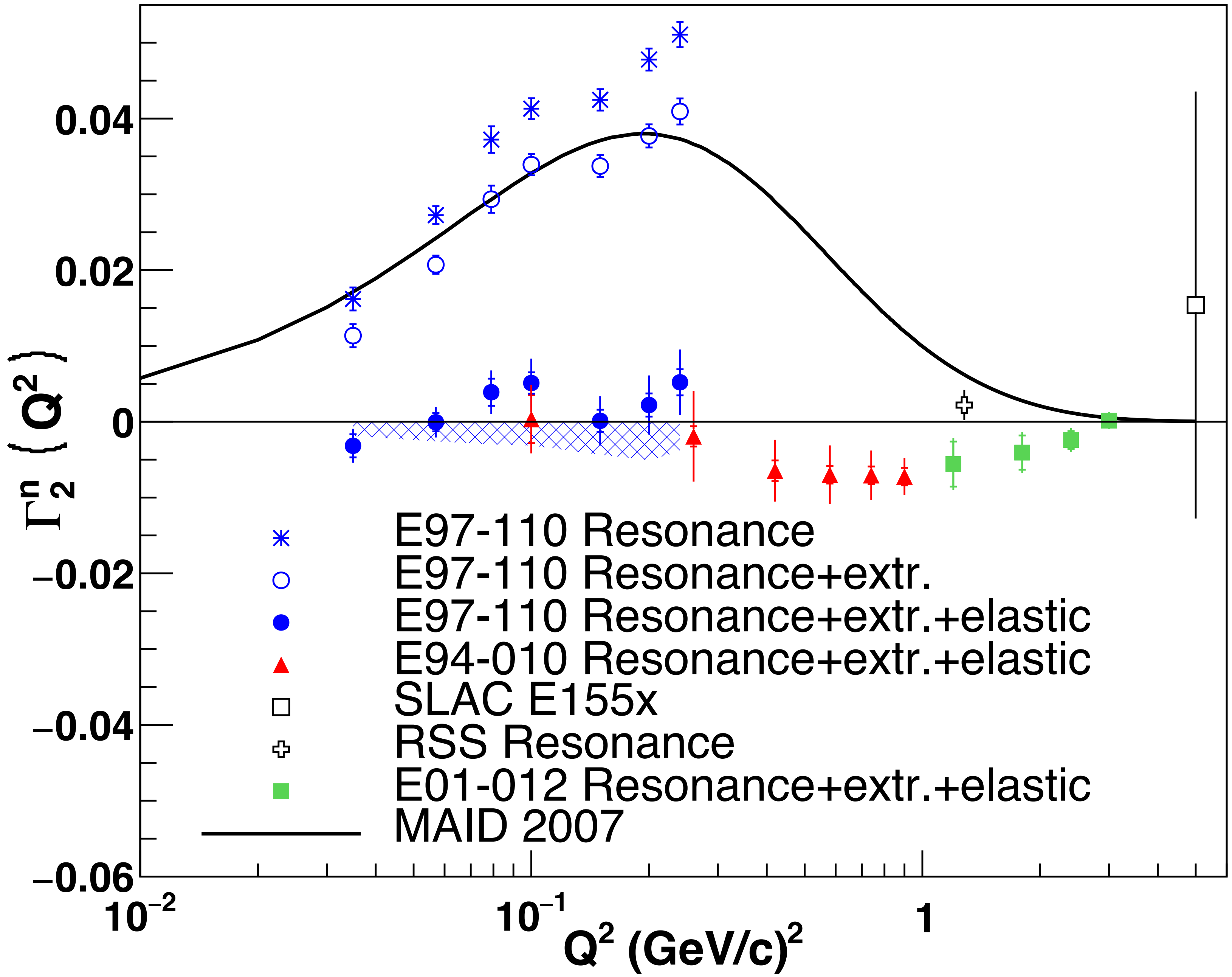


First moments: Burkhardt–Cottingham sum rule on neutron from E97-110

$$\Gamma_2(Q^2) \equiv \int_0^1 g_2\, dx = 0$$

B-C sum rule

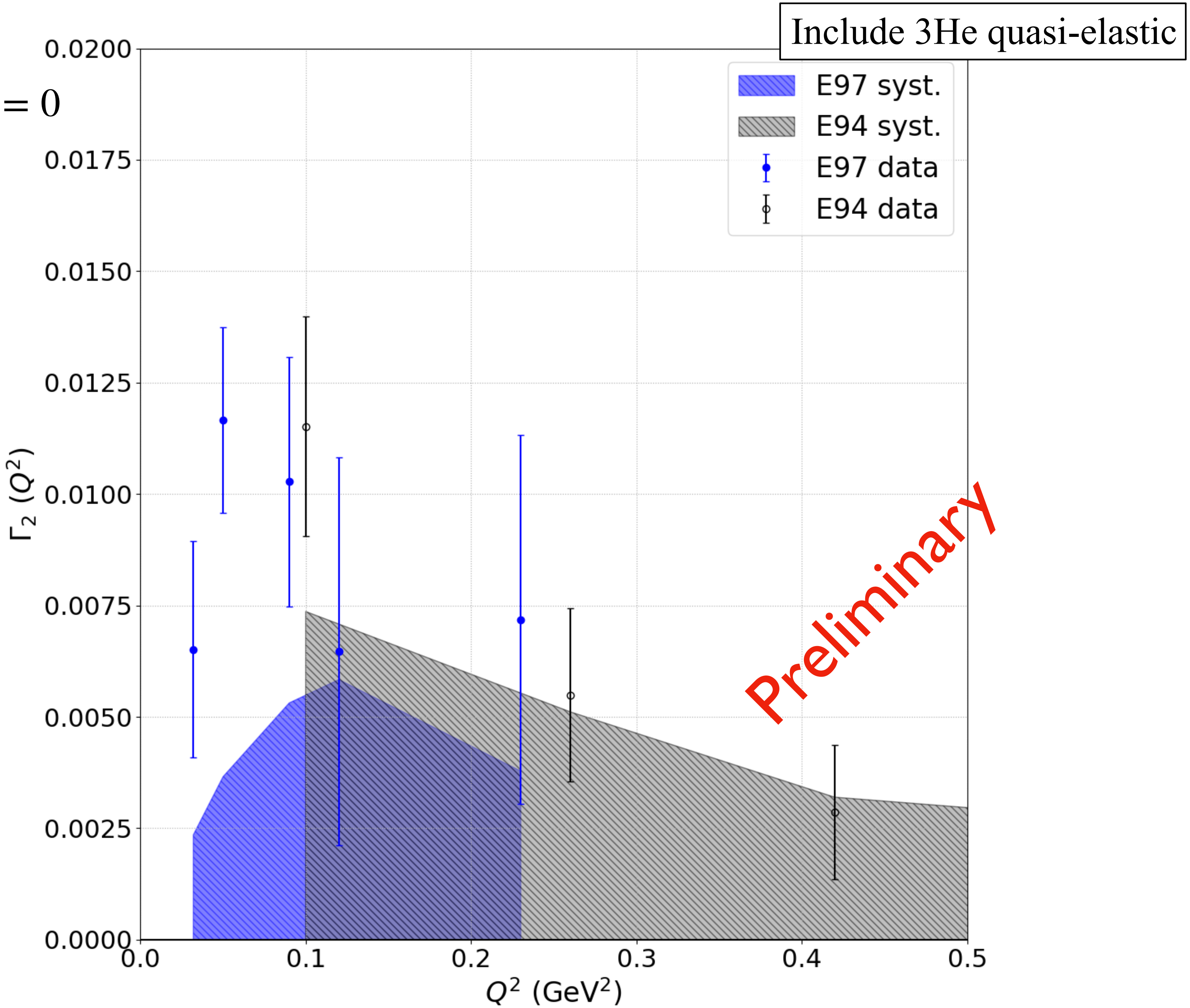
V. Sulkosky et al.
PLB 805 135428 (2020)



E97-110 verifies the B-C sum rule at low Q^2. Older experiments at higher Q^2 also verify it.

First moments: Burkhardt–Cottingham sum rule on ^{3}He from E97-110

$$\Gamma_2(Q^2) \equiv \int_0^1 g_2\, dx = 0$$

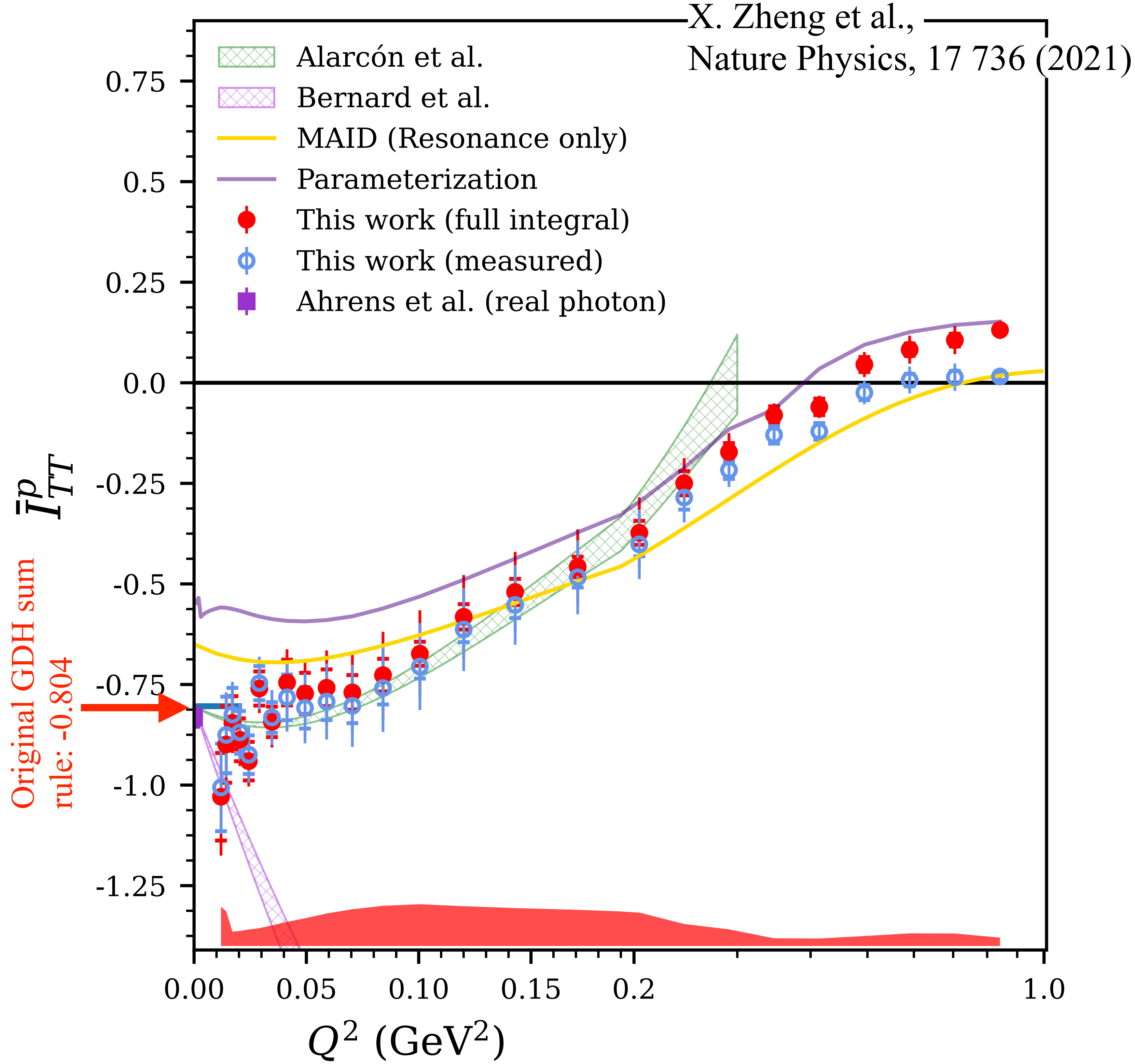


Another generalization of GDH sum: $I_{TT}^p(Q^2)$. EG4 Data

$$I_{TT}(Q^2) \equiv \frac{M^2}{8\pi^2\alpha}\int_{\nu_{thr}}^{\infty}\frac{K}{\nu}\frac{\sigma_A-\sigma_P}{\nu}d\nu$$

K: virtual photon flux

No suppressing Q^2 factor.
Contains g_2 (not measured by EG4)



Extrapolating the (very low Q^2) data to Q^2=0 provides an independent check of the GDH SR validity, with a different method (inclusive data) than photoproduction experiments (exclusive data).

$$I_{TT}^{p\ \mathrm{EG4}}(0) = -0.798 \pm 0.042$$

Agrees with the GDH SR, with precision similar to photoproduction method: $I_{TT}^{p\ \mathrm{MAMI}}(0) = -0.832 \pm 0.023(stat)$\$ $\pm 0.063(syst)$

Another generalization of GDH sum: $I_{TT}^{n}(Q^2)$. E97-110 & EG4 Data

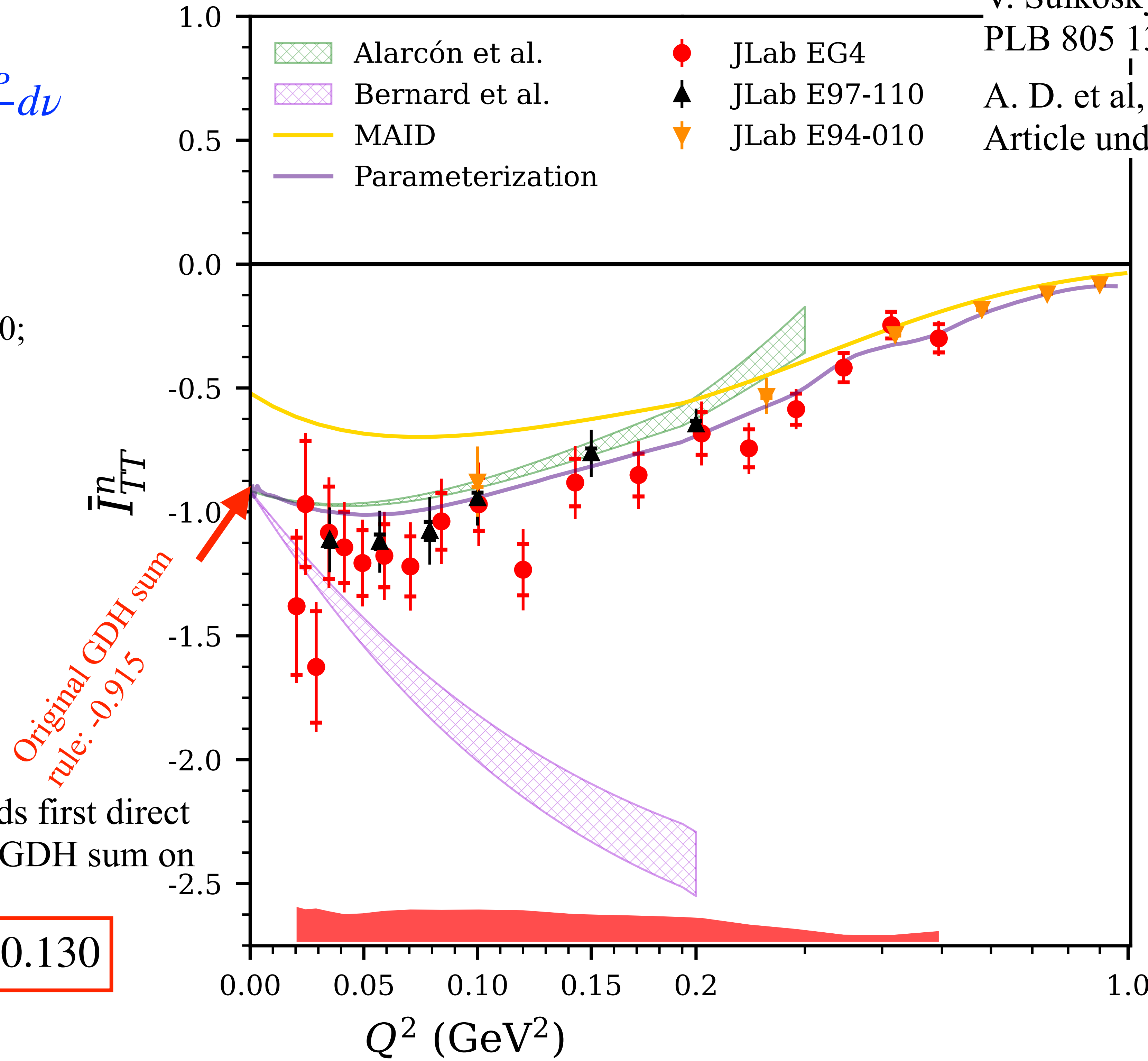
$$I_{TT}(Q^2) \equiv \frac{M^2}{8\pi^2\alpha}\int_{\nu_{thr}}^{\infty}\frac{K}{\nu}\frac{\sigma_A-\sigma_P}{\nu}d\nu$$

K: virtual photon flux

No suppressing Q^2 factor.
Contains g_2 (measured by E97-110; not measured by EG4)

V. Sulkosky et al. PLB 805 135428 (2020)

A. D. et al, (2024) Article under review



Extrapolation (EG4 data only) yields first direct experimental check of the original GDH sum on the neutron.

$$I_{TT}^{n\ \mathrm{EG4}}(0) = -1.084 \pm 0.130$$

- E97-110 and EG4 agree with each other and with older data at larger Q^2.
- E97-110, EG4 and χEFT:
  - agree for lowest data point ($Q^2 \sim 0.04$ GeV^2) for Bernard *et al.*
  - disagree with Alarcón *et al*. except at the higher Q^2.
- Maid disagrees with the data.

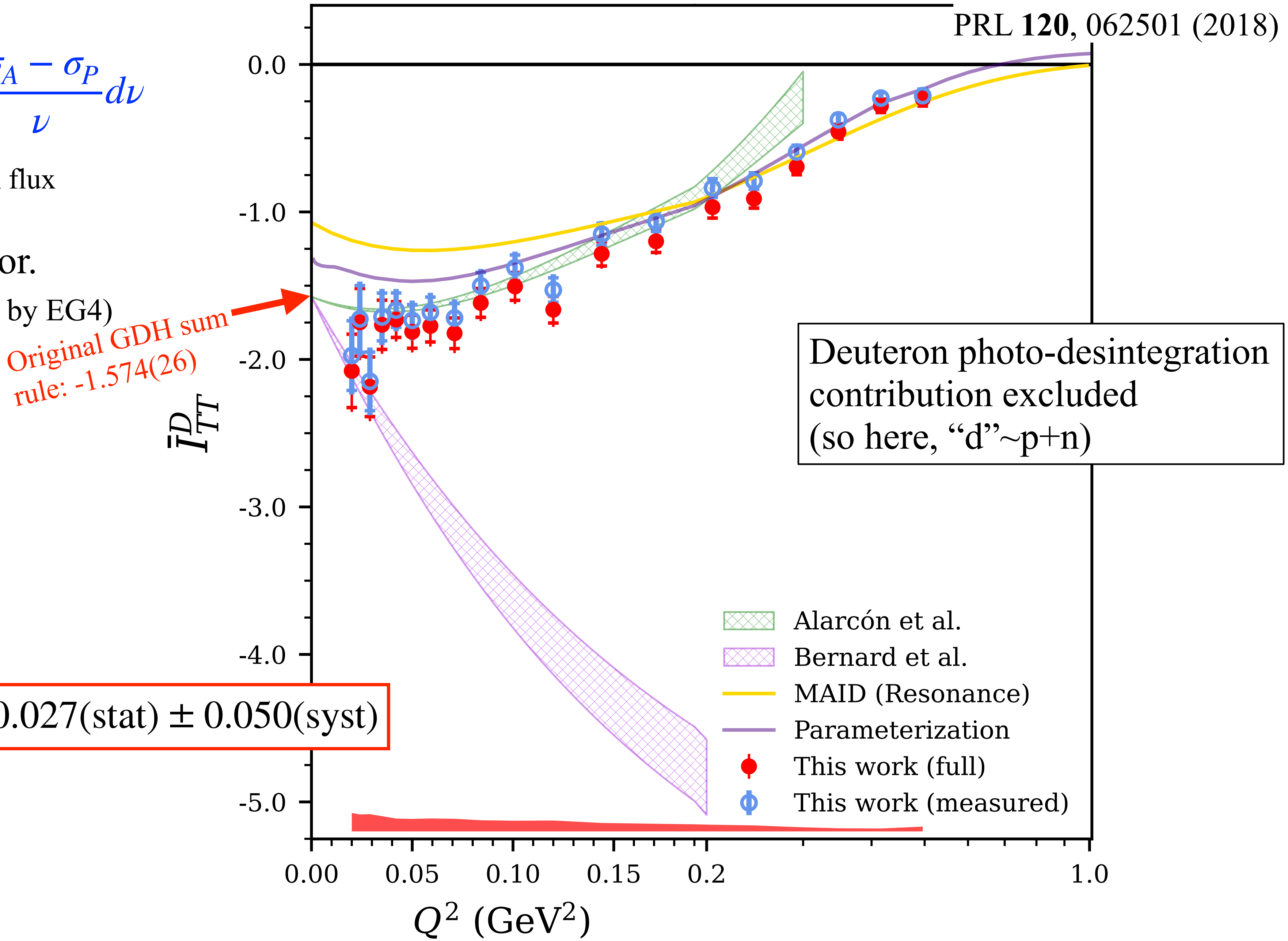
Another generalization of GDH sum: $\bar{I}_{TT}^{d}(Q^2)$. EG4 Data

K. Adhikari et al.
PRL **120**, 062501 (2018)

$$I_{TT}(Q^2) \equiv \frac{M^2}{8\pi^2\alpha}\int_{\nu_{thr}}^{\infty}\frac{K}{\nu}\frac{\sigma_A-\sigma_P}{\nu}d\nu$$

K: virtual photon flux

No suppressing Q^2 factor.
Contains g_2 (not measured by EG4)



Extrapolating EG4 yields:

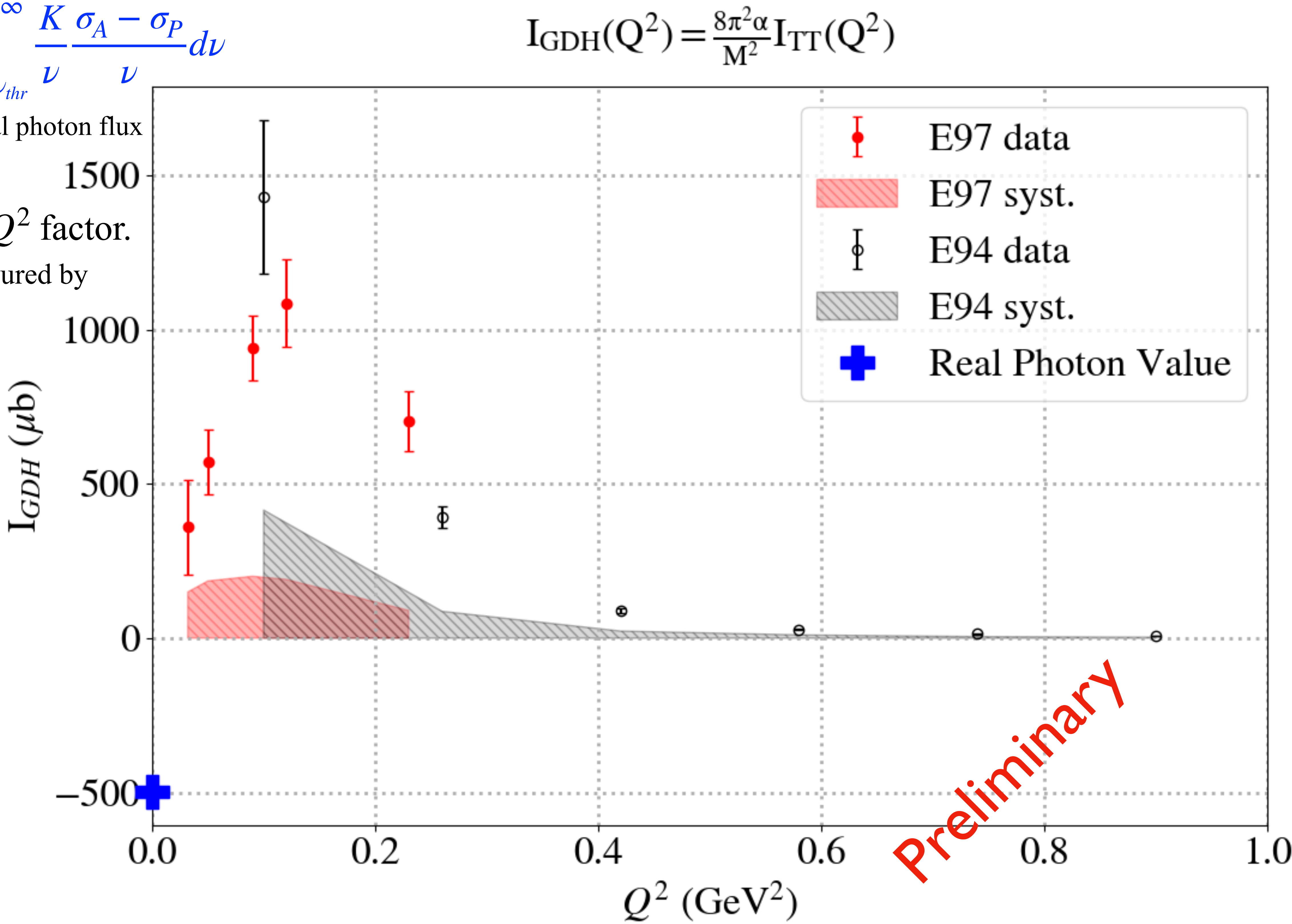
$$\bar{I}_{TT}^{d\,\text{EG4}}(0) = -1.724 \pm 0.027(\text{stat}) \pm 0.050(\text{syst})$$

Another generalization of GDH sum: $\bar{I}_{TT}^{3He}(Q^2)$, E97-110 Data

$$I_{TT}(Q^2) \equiv \frac{M^2}{8\pi^2\alpha}\int_{\nu_{thr}}^{\infty}\frac{K}{\nu}\frac{\sigma_A - \sigma_P}{\nu}d\nu$$

K: virtual photon flux

No suppressing Q^2 factor.
Contains g_2 (measured by E97-110)



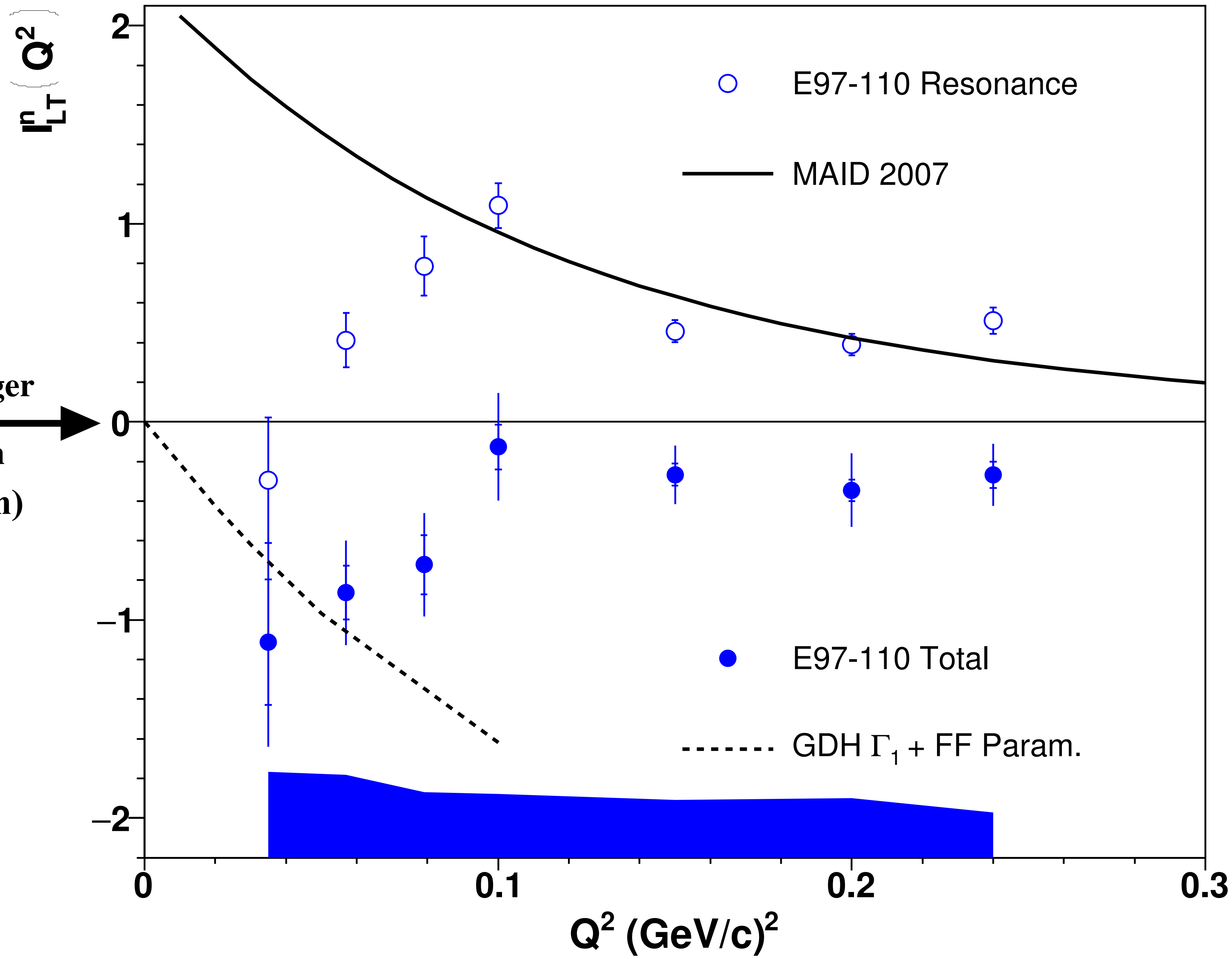
First moments: Schwinger sum rule on neutron from E97-110

$$I_{LT}(Q^2) = \frac{8M^2}{Q^2}\int_0^{1^-}(g_1+g_2)dx \xrightarrow[Q^2\to 0]{} \kappa e_t$$

anomalous magnetic moment×charge

V. Sulkosky et al.
Nature Physics, **17** 687 (2021)

Schwinger Relation (neutron)

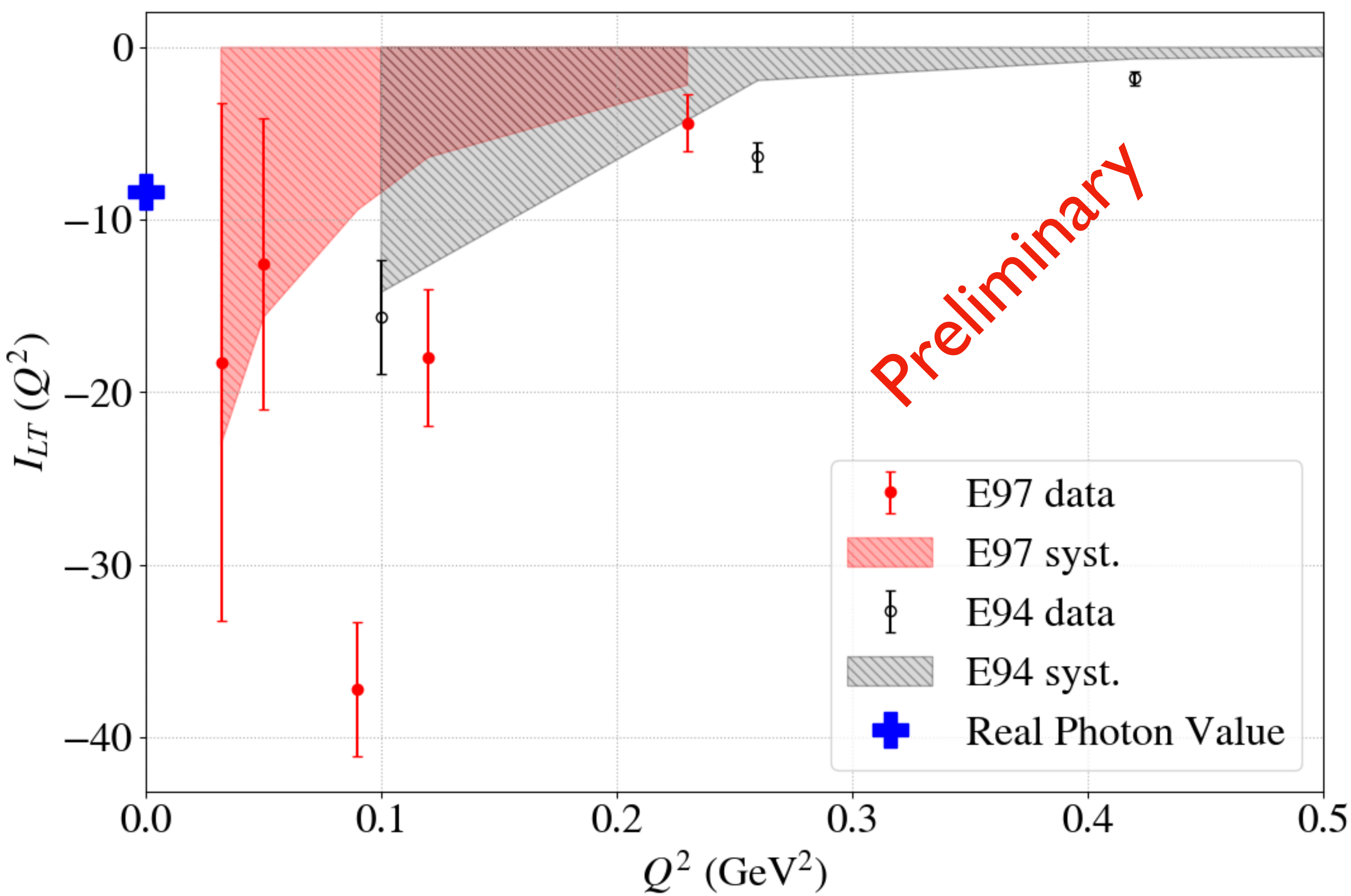


E97-110 (+GDH+BC sum rule+known neutron elastic form-factor) agrees with Schwinger sum rule.

First moments: Schwinger sum rule on ^{3}He from E97-110

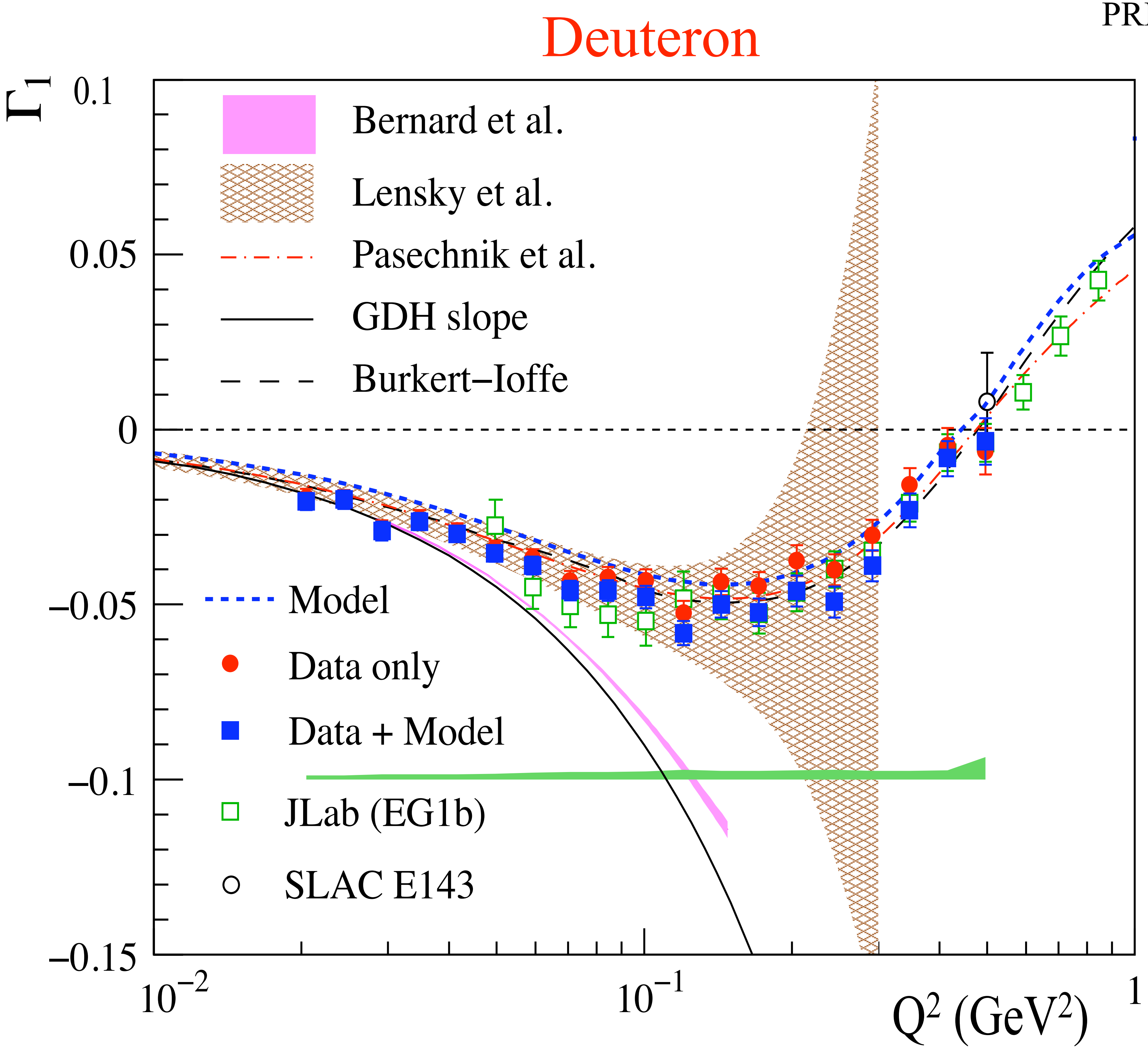
$$I_{LT}(Q^2) = \frac{8M^2}{Q^2}\int_0^{1^-}(g_1+g_2)dx \xrightarrow[Q^2\to 0]{} \kappa e_t$$

anomalous magnetic moment×charge

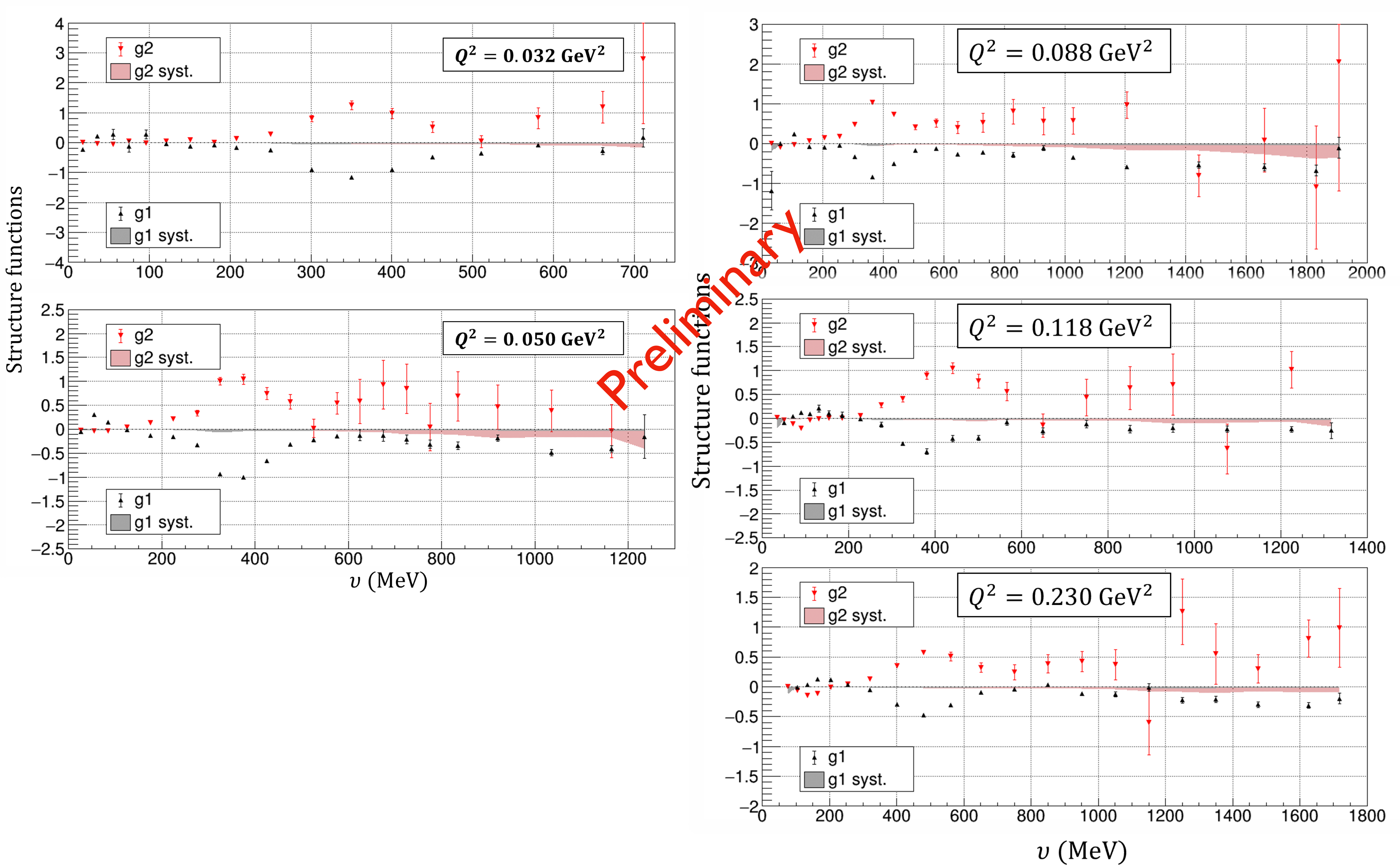


Γ_1 measurements from JLab

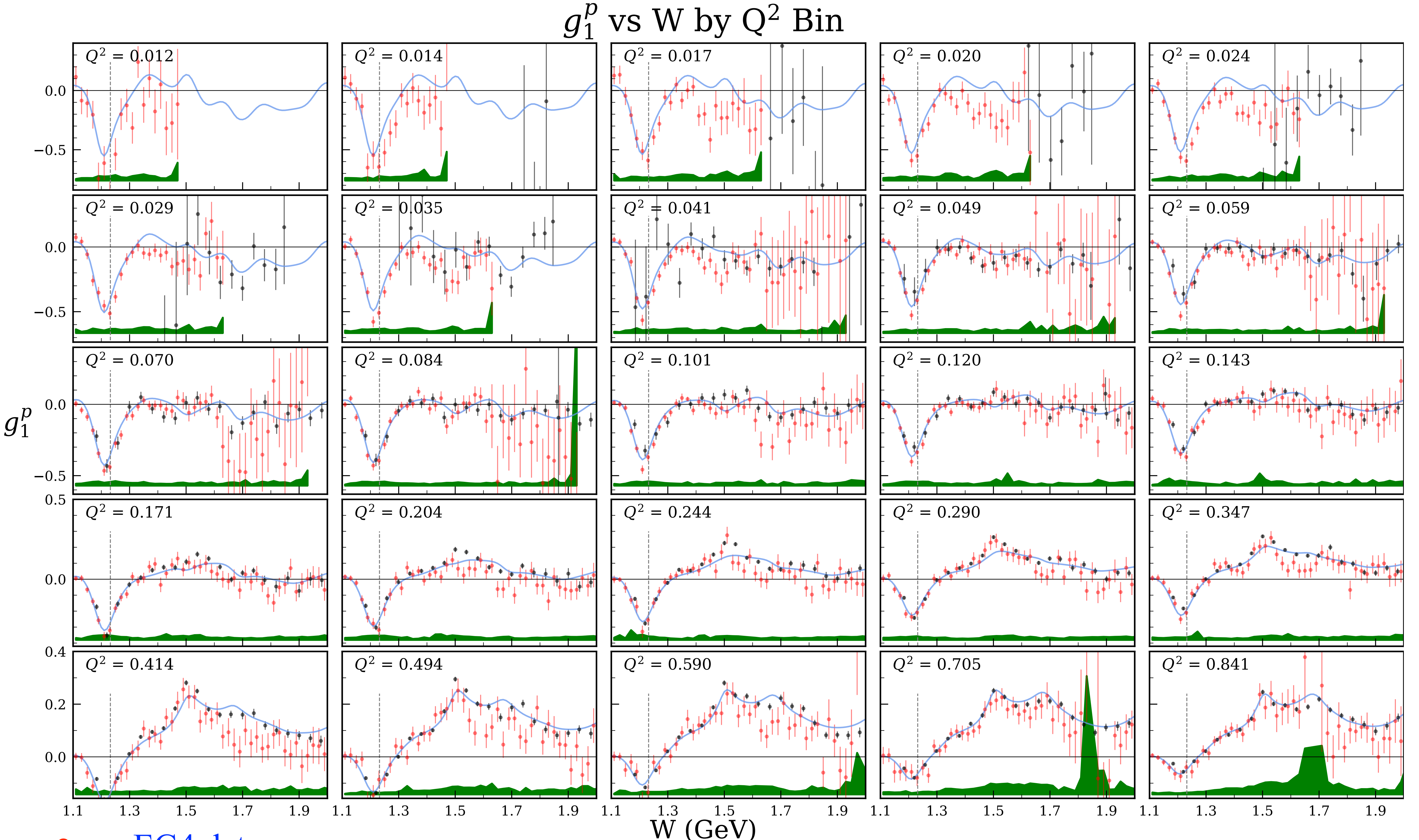
K. Adhikari et al.
PRL **120**, 062501 (2018)



$g_1^{^3He}(\nu, Q^2)$ and $g_2^{^3He}(\nu, Q^2)$ with quasi-elastic, from E97-110



Spin structure function $g_1^p(W, Q^2)$ data from EG4

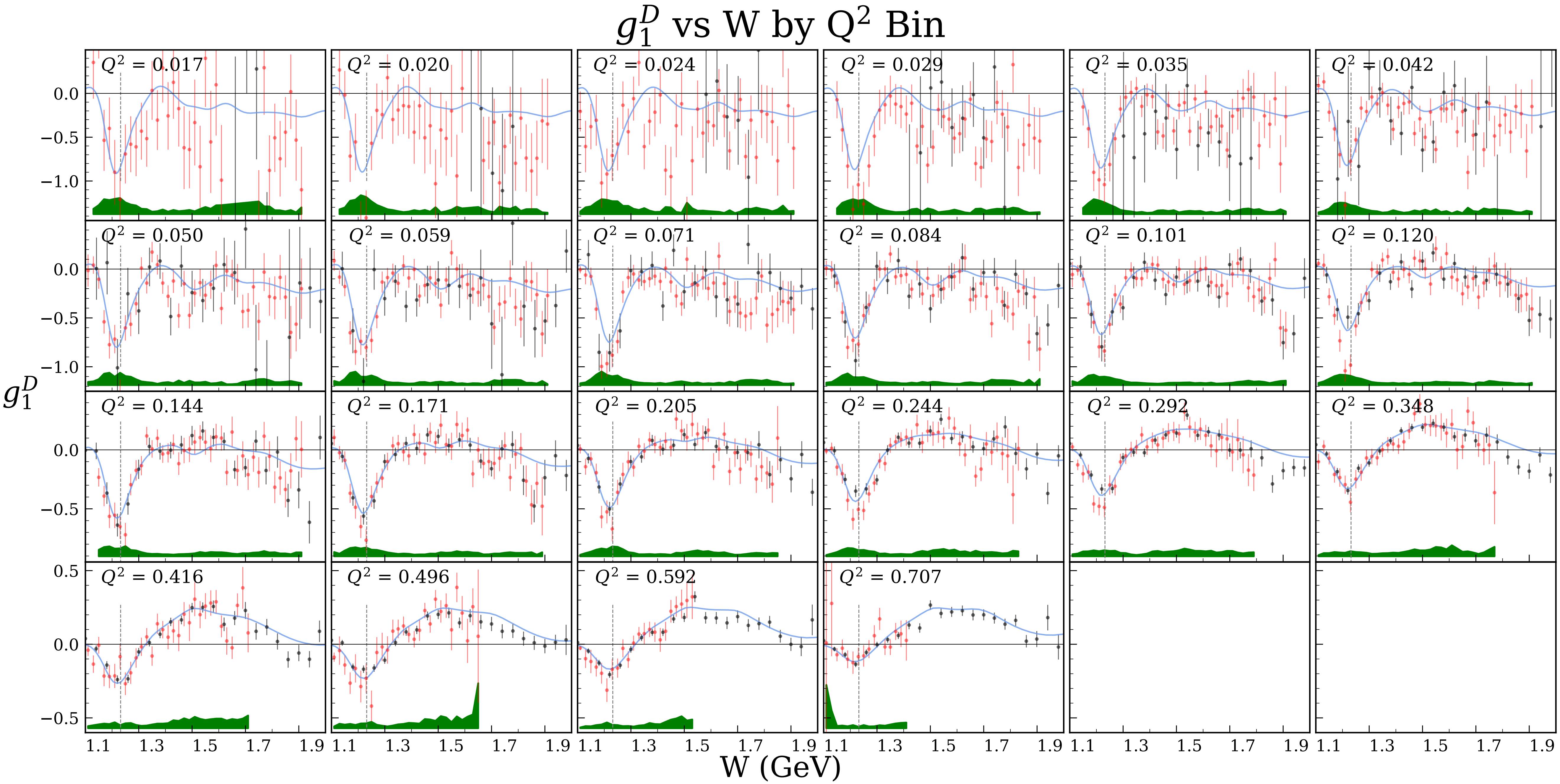


- EG4 data
- EG1b data

"Model" (Fit to EG1b + other published data).

X. Zheng et al.,
Nature Physics, 17 736 (2021)

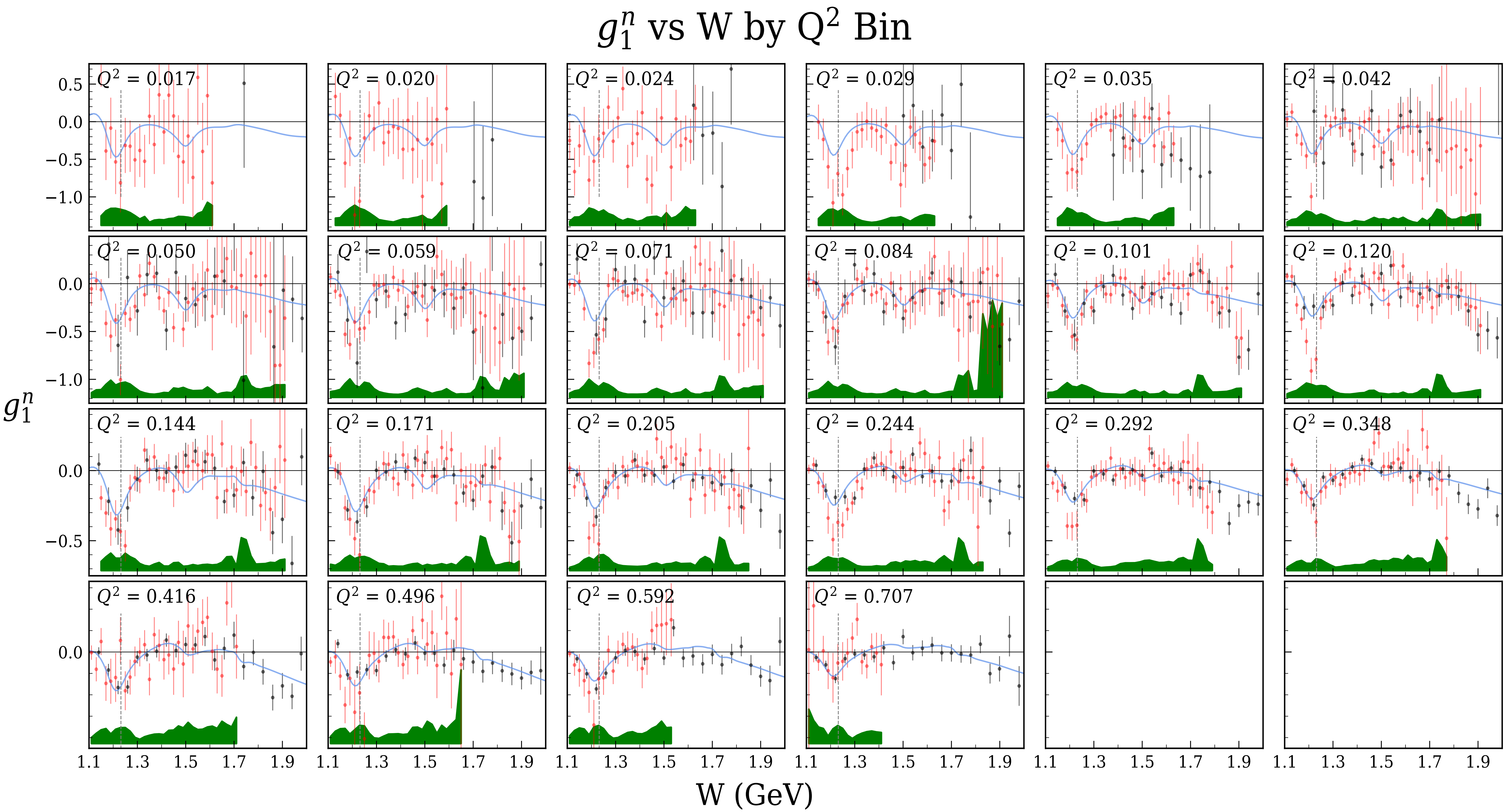
Spin structure function $g_1^d(W, Q^2)$ data from EG4



K. Adhikari et al.
PRL **120**, 062501 (2018)

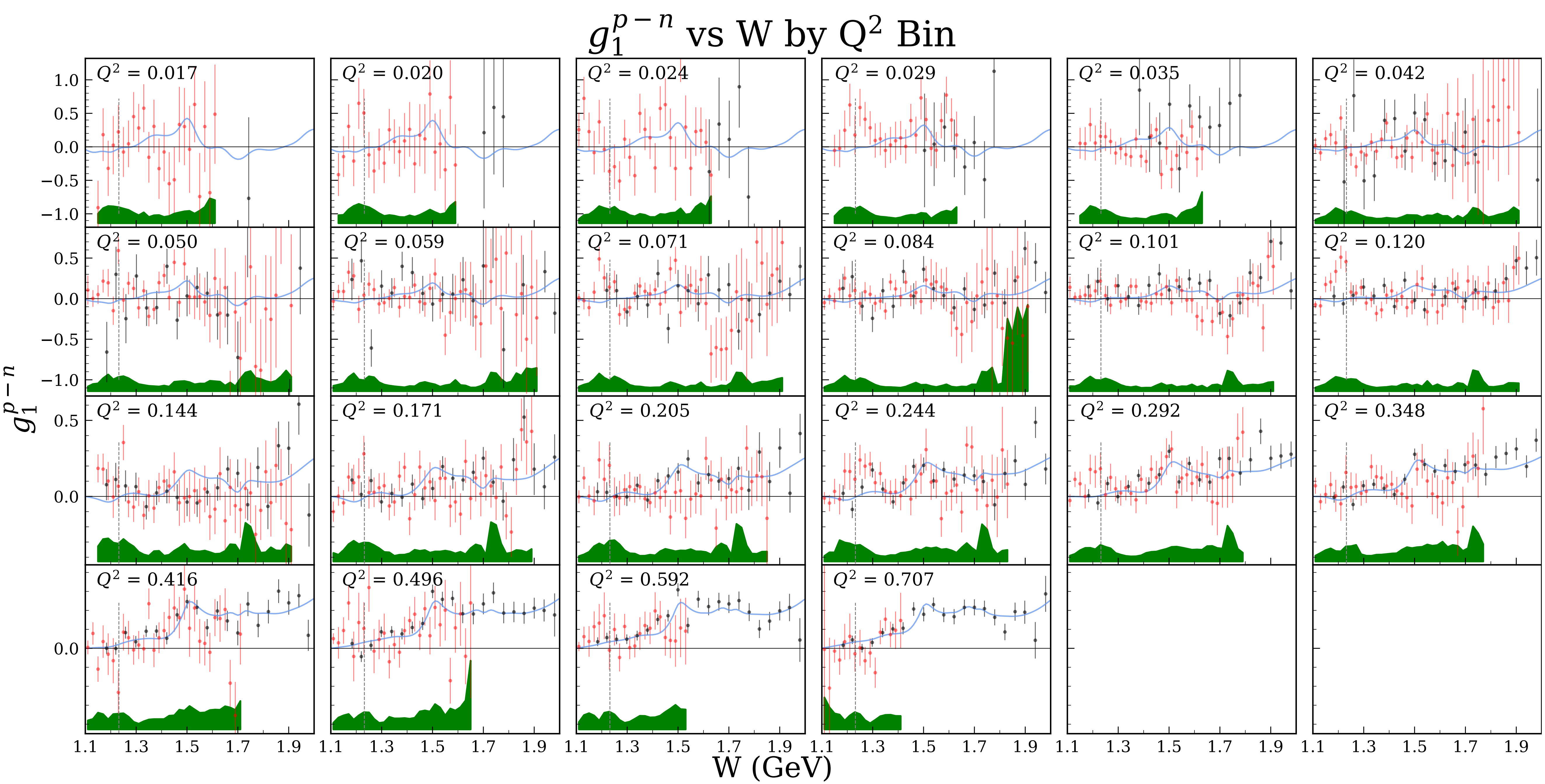
- EG4 data
- EG1b data
- "Model" (Fit to EG1b + other published data).

Spin structure function $g_1^n(W, Q^2)$ data from EG4



- EG4 data
- EG1b data
- "Model" (Fit to EG1b + other published data).

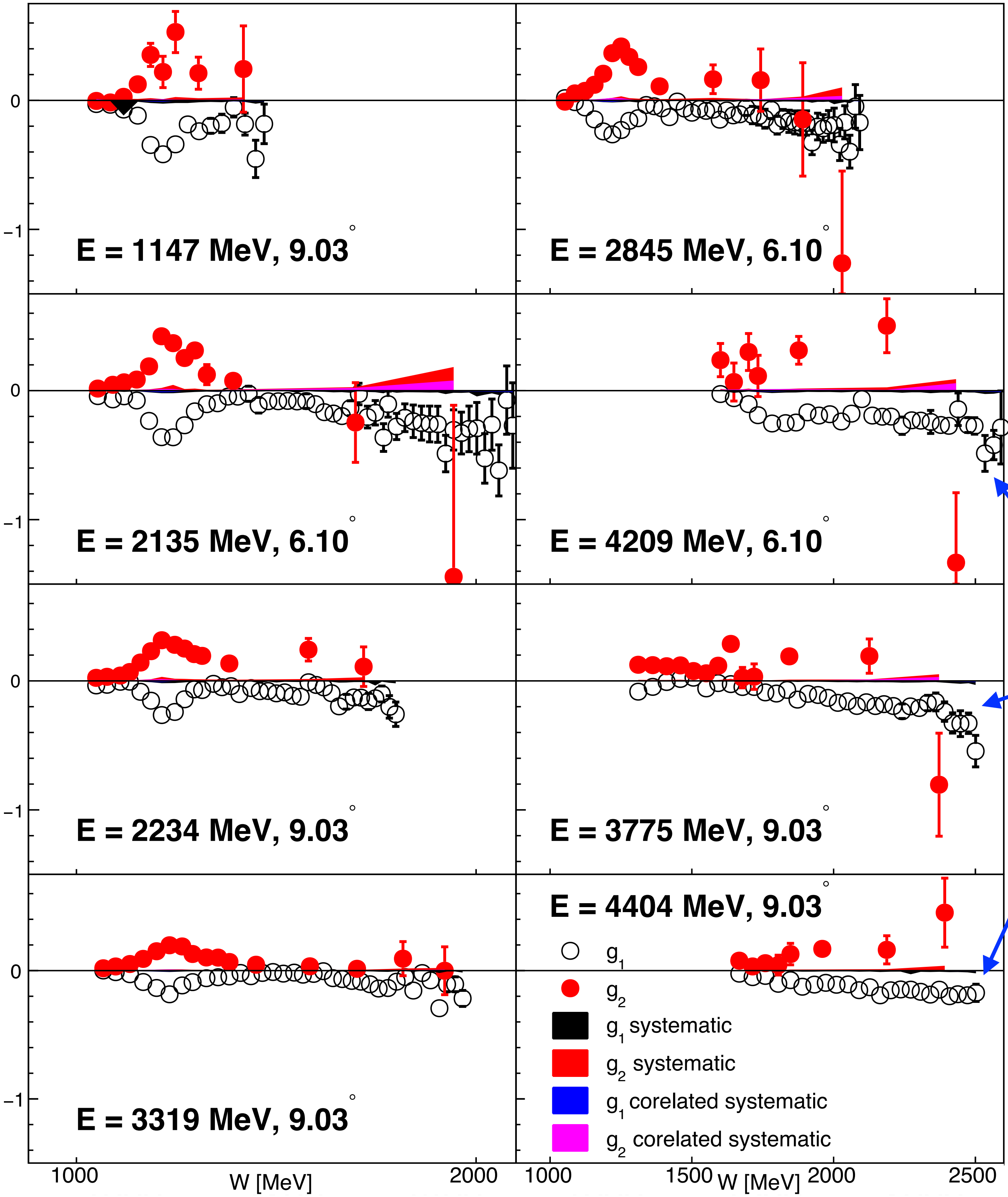
Spin structure function $g_1^{p-n}(W, Q^2)$ data from EG4



- EG4 data
- EG1b data
- "Model" (Fit to EG1b + other published data).

Spin structure function $g_1^{^3He}(W, Q^2)$ and $g_2^{^3He}(W, Q^2)$ data from E97-110

We do not know how to reliably extract neutron information from ³He for non-integrated quantities (e.g., spin structure functions, polarized cross-section difference…)



V. Sulkosky et al.
PLB 805 135428 (2020)

We observe the expected $g_1 \simeq -g_2$ symmetry near the Δ_{1232}.
Δ: ~M₁ transition ⇒ $\sigma_{LT} \propto g_1 + g_2 \simeq 0$

Large W coverage to test sum rule convergency

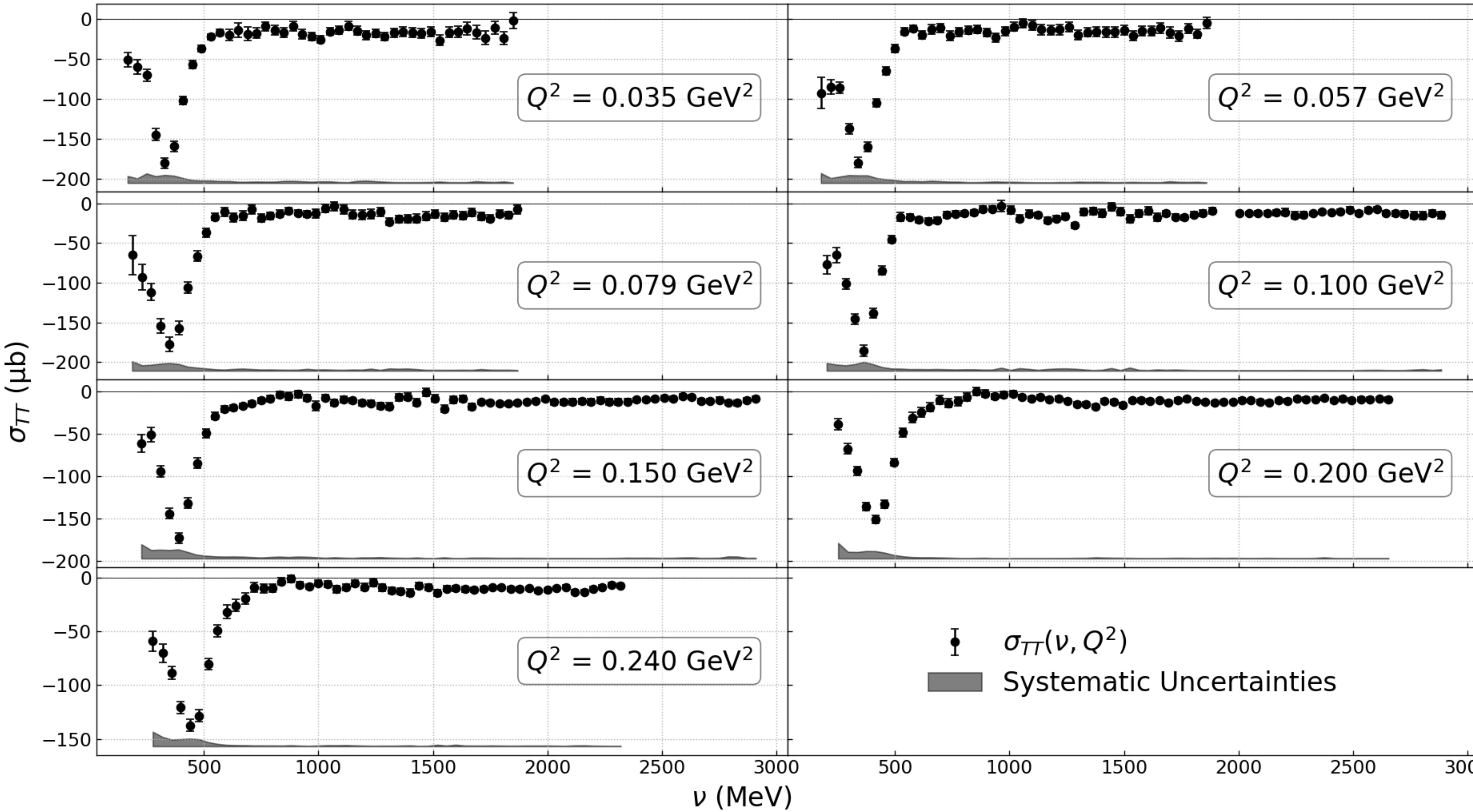
Polarized cross-section $\sigma_{TT}^{^3\text{He}}(\nu, Q^2)$ data from E97-110

We do not know how to reliably extract neutron information from ³He for non-integrated quantities (e.g., spin structure functions, polarized cross-section difference…)

$$\sigma_{TT} = \frac{\sigma_A - \sigma_p}{2} = \frac{4\pi^2\alpha}{MK}(g_1 - \gamma^2 g_2)$$

K: virtual photon flux

V. Sulkosky et al.
PLB 805 135428 (2020)



Polarized cross-section $\sigma_{LT}^{^3He}(\nu, Q^2)$ data from E97-110

We do not know how to reliably extract neutron information from ³He for non-integrated quantities (e.g., spin structure functions, polarized cross-section difference…)

$$\sigma_{LT} = \frac{4\pi^2\alpha}{MK}\gamma(g_1 + g_2)$$

V. Sulkosky et al.
Nature Physics, **17** 687 (2021)