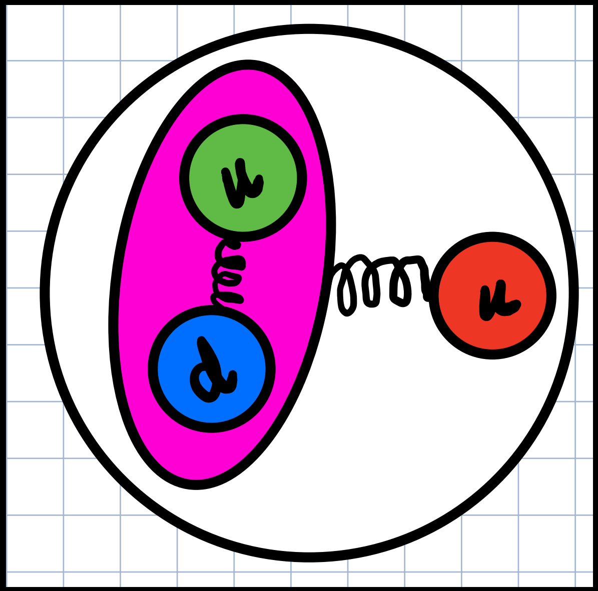
Quantum Chromodynamics Predictions:
Structure Formation in the Proton Sea

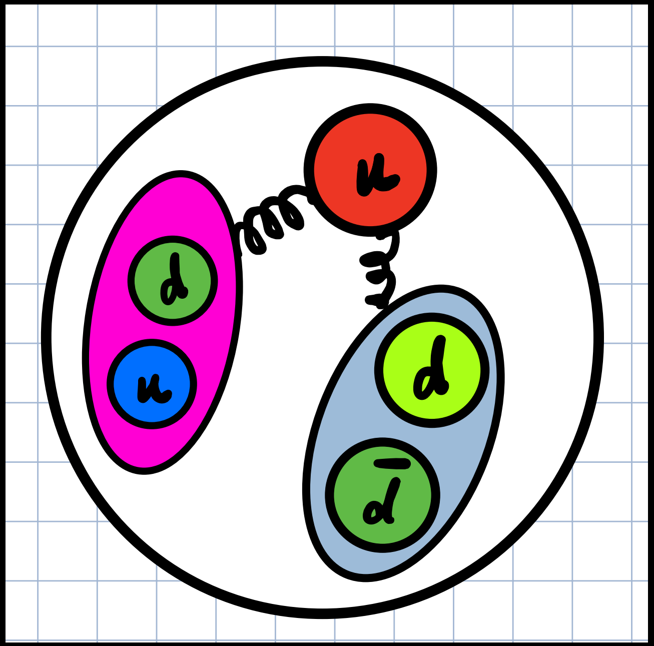
Jennifer Rittenhouse West
University of California, Berkeley
Electron-Ion Collider Early Career Workshop Talk
22 July 2024

Outline for talk on Fundamental QCD predictions for the internal structures of nucleons (& all baryons)

- Finding structure via quark behavior (Deep Inelastic Scattering overview)
- Quantum Chromodynamics definitions and predictions, including short-range QCD potentials
- Fock states in the proton: 3-quark, 5-quark, 7-quark, …
- “Diquark capture” model on higher Fock states in the nucleon - work in progress with Stan Brodsky
- Diquark capture model vs. data



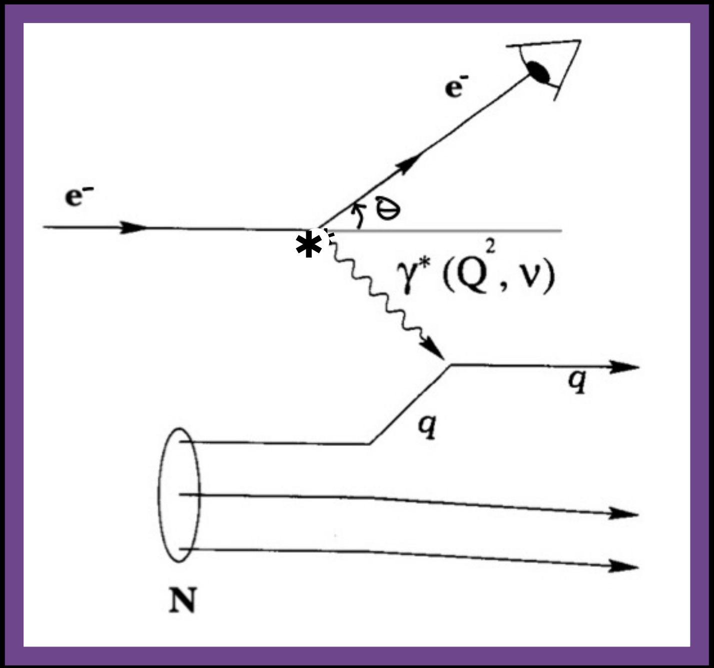
3-quark Fock state of proton



5-quark Fock state of proton

Finding Structure via Quark Behavior: Deep Inelastic Scattering variables

- Deep inelastic scattering (DIS) experiments
- Lepton scatters from target, exchanging virtual photon with 4-momentum q^2 given by: $Q^2 \equiv -q^2 = 2EE'(1-\cos\theta)$
- Fraction of nucleon momentum carried by struck quark is found via **Bjorken scaling variable** $x_B = \dfrac{Q^2}{2M_p\nu}$, where $\nu = E - E'$, M_p=mass of nucleon (lepton mass neglected)



Adapted from Kerry Whisnant, 2002 book chapter

Differential cross section for DIS:

$$\frac{d\sigma}{dxdy}\left(e^-p \rightarrow e^-X\right) = \sum_f x\, e_f^2 \left[\, q_f(x) + \bar{q}_{\bar{f}}(x)\right] \cdot \frac{2\pi\alpha^2 s}{Q^4}\left(1 + (1-y)^2\right)$$

where $y = \dfrac{\nu}{E}$ is the fraction of lepton energy transferred to target. $F_2(x)$ is the **nucleon structure function**, defined as:

$$F_2(x_B) \equiv \sum_f x_B\, e_f^2 \left(\, q_f(x_B) + \, \bar{q}_{\bar{f}}(x_B)\right)$$

in terms of quark distribution functions $q_f(x)$: probability to find a quark with momentum $x_i \in [x, x+dx]$.

Standard Model of Particle Physics is Group theory!

- Mathematical definition of SM is group theory - these are symmetries of the Lagrangian & path integral:

$$SU(3)_{Color} \otimes SU(2)_{Weak} \otimes U(1)_{Y}$$

- But ↑ is in the early Universe, prior to electroweak symmetry breaking by the Higgs boson

- Today, the Standard Model of the cold Universe in a blackbody soup of photons peaked at 2.7 K with the following symmetry structure:

$$SU(3)_{color} \otimes U(1)_{EM}$$

$$\mathscr{L}_{\text{QCD}} = \sum_q \left(\bar{\psi}_{qi} \gamma^\mu \left[\delta_{ij} \partial_\mu + ig \left(t_a G^a_\mu \right)_{ij} \right] \psi_{qj} - m_q \bar{\psi}_{qi} \psi_{qi} \right) - \frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu}$$

QCD confines color charged objects to distance scales of ~ 1 femtometer. We never detect color charge.

Only red+green+blue = colorless, the 3 quark **baryons**

and, e.g., red+antired=colorless, the quark-antiquark **mesons** are observed (*plus tetraquarks and pentaquarks now!*)

Diquarks & Hidden-Color: Background QCD

Building blocks: Quantum chromodynamics, Spin-statistics constrained

- Begin with group theory mathematics of the strong interaction: $\mathrm{SU}(3)_\mathrm{C}$

- Next, degrees of freedom (particles carrying strong force charge) - indices run over the 3 colors:

$$q_a \text{ (triplet, } 3_\mathrm{C}), \quad q^a \text{ (antitriplet, } \overline{3}_\mathrm{C}), \quad g^b_c \text{ (octet, } 8_\mathrm{C})$$

- All combinations of D.o.F. predicted, color charged & color singlet. Combine via $\delta^a_b,\, d_{abc},\, \epsilon_{abc}$:

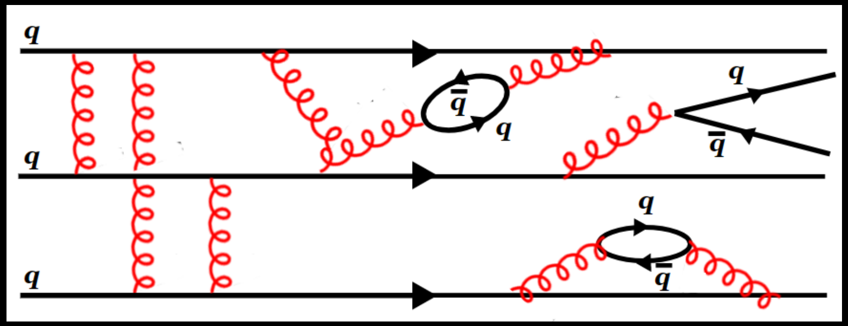
$$(\bar{q}^a q_a)_{1_\mathrm{C}} \quad (\epsilon^{abc}\, q_a q_b q_c)_{1_\mathrm{C}} \quad (\epsilon_{abc}\, \bar{q}^a \bar{q}^b \bar{q}^c)_{1_\mathrm{C}}, \quad (q_a q_b \epsilon^{abc})_{\bar{3}_C} \quad \leftarrow (qq)^c$$

- Higher Fock states (baryon with 3 valence quarks is lowest order Fock states), e.g., the 5-quark Fock state for baryons - gluon splitting to meson cloud:

$$|N\rangle \subset (\epsilon^{abc} q_a q_b q_c\, \bar{q}^e q_e)_{1_\mathrm{C}}$$

- 5-quark Fock using color octets, gluon splitting:

$$|N\rangle \subset \Big((\epsilon^{abf} q_a q_b q_c)_{8_\mathrm{C}}\, (q^c q_f)_{8_\mathrm{C}} \Big)_{1_\mathrm{C}}$$



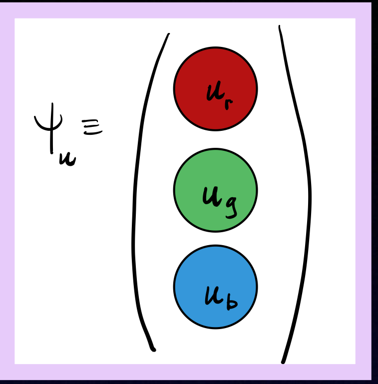
Quantum Chromodynamics prediction: Diquarks

- 3 color charges in $\mathrm{SU}(3)_C$, the local gauge symmetry ≡ QCD

- QCD ⟹ Diquark creation: Quark-quark bond with single gluon exchange **&** group theory transformation into a fundamentally different object:

$$3_C \otimes 3_C \rightarrow \bar{3}_C$$

Up quark in the fundamental rep of $\mathrm{SU}(3)_C$:



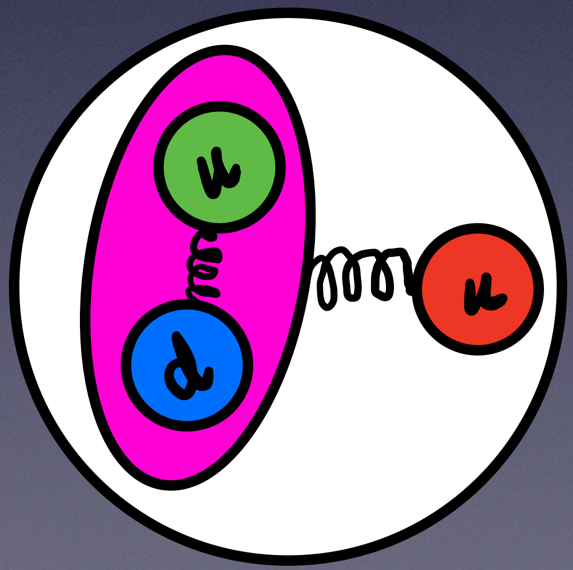
Diquark wavefunction :

$$\psi_a^{[ud]} = \frac{1}{\sqrt{2}} \epsilon_{abc} \left(u^b_\uparrow d^c_\downarrow - d^b_\uparrow u^c_\downarrow \right)$$

Like quarks and gluons, diquarks carry color charge. They cannot be seen directly due to color confinement. Only 1_C (red+green+blue or red-antired etc.) observed.

Therefore there is no direct evidence for diquarks. Work in progress for diquark detection experimental proposals (*e.g.*, diquark jets from DIS increase Λ production)

Strong indirect evidence exists (baryon mass splittings, Regge slopes).

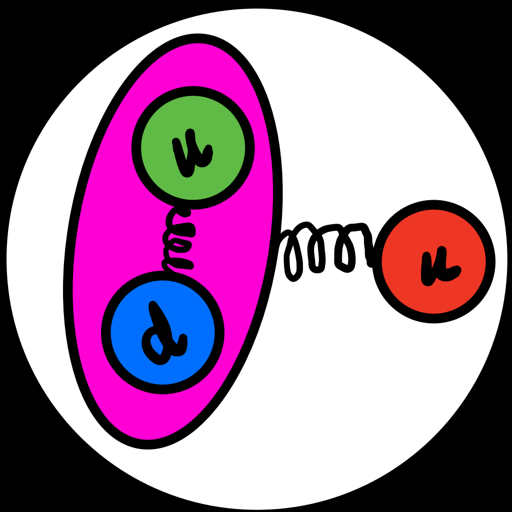


Diquark with anti-color charge:
green x blue -> anti-red

Quark-quark potential in QCD: $V(r)$ calculation

- $\mathrm{SU}(3)_C$ invariant QCD Lagrangian:

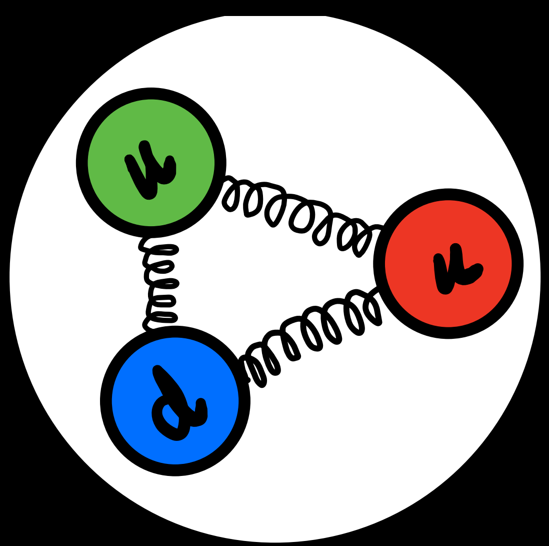
$$\mathscr{L}_{\mathrm{QCD}} = -\frac{1}{4} G_a^{\mu\nu} G_{\mu\nu}^a + \bar{\Psi}_f \left(i\gamma^\mu D_\mu - m \right) \Psi_f$$



where covariant derivative $D_\mu = \partial_\mu - i g_s A_\mu^a t^a$ acts on quark fields, t^a are the 3x3 traceless Hermitian matrices (e.g., the 8 Gell-Mann matrices), g_s the strong interaction coupling, $\alpha_s \equiv \frac{g_s^2}{4\pi}$

- QCD potential for states in representations R and R' is given by:

$$V(r) = \frac{g_s^2}{4\pi r} t_R^a \otimes t_{R'}^a$$



- To compute $V(r)$ for a $3_c \otimes 3_c \to \bar{3}_c$, we use the definition of the scalar $C_2(R)$, $t_R^a\, t_R^a \equiv C_2(R)\, \mathbf{1}$, the *quadratic Casimir operator* (NB: R_f is the final state representation):

$$V(r) = \frac{g_s^2}{4\pi r} \cdot \frac{1}{2} \cdot \left(C_2\left(R_f\right) - C_2(R) - C_2(R') \right)$$

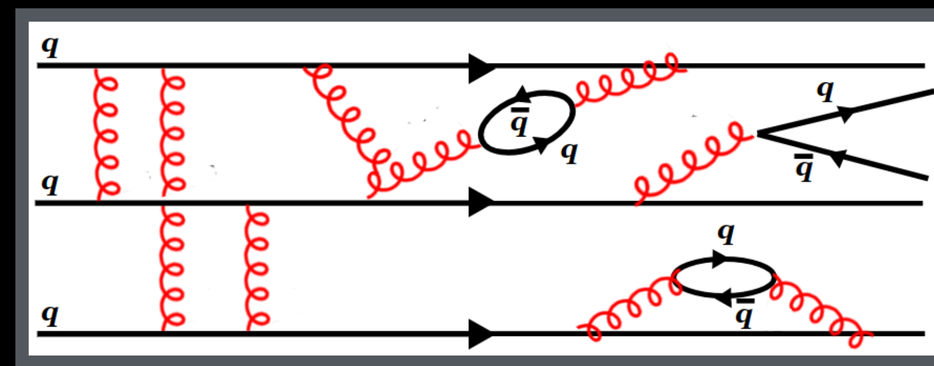
- Diquarks combine 2 fundamental representation quarks into an anti-fundamental, $3_C \otimes 3_c \to \bar{3}_C$:

$$V(r) = -\frac{2}{3} \frac{g_s^2}{4\pi r} \Longrightarrow \text{Diquark is bound!}$$

Compare to color singlet attractive potential:

$$q\bar{q}: \quad V(r) = -\frac{4}{3} \frac{g_s^2}{4\pi r}$$

Higher QCD Fock states in the Proton



- Sea quark distributions contribute most at low-x_B
  Sea quarks created by gluon splitting, creating higher Fock states

$$|p\rangle_{3\text{q}} = |uud\rangle \;\; = C_1|uud\rangle + C_2|u[ud]\rangle$$

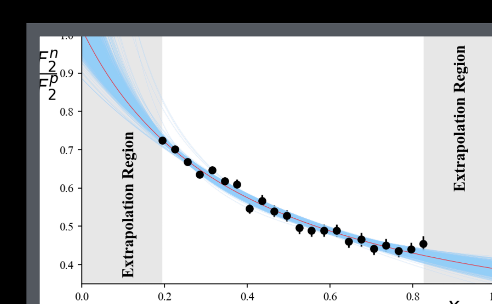
$$|p\rangle_{5\text{q}} = |uud\rangle + |uud\;\bar{d}d\rangle$$

$$|p\rangle_{7\text{q}} = |uud\rangle + |uud\;\bar{d}d\rangle + |uud\;\bar{d}d\;\bar{u}u\rangle$$

$$|p\rangle = |p\rangle_{3q} + |p\rangle_5 q + |p\rangle_{7q} + |p\rangle_{9q} + \ldots$$

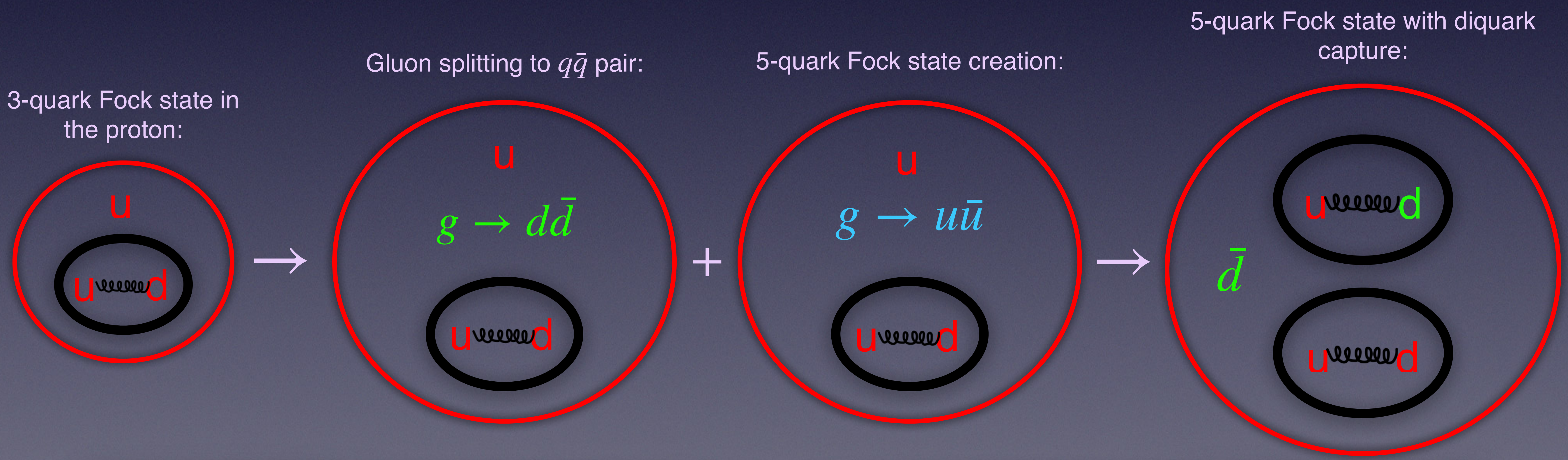
"Physical implications of the extrapolation and statistical bootstrap of nucleon structure function ratios $\frac{F_2^n}{F_2^p}$ *for mirror nuclei* ^{3}He *and* ^{3}H*"*

H.Valenty, JRW, F.Benmokhtar, D.Higinbotham, A.Parker, E.Seroka, Phys.Rev.C 2023



Work in progress: Small-x_B physics in proton via Diquark Capture $\bar{d}/\bar{u}$ effects

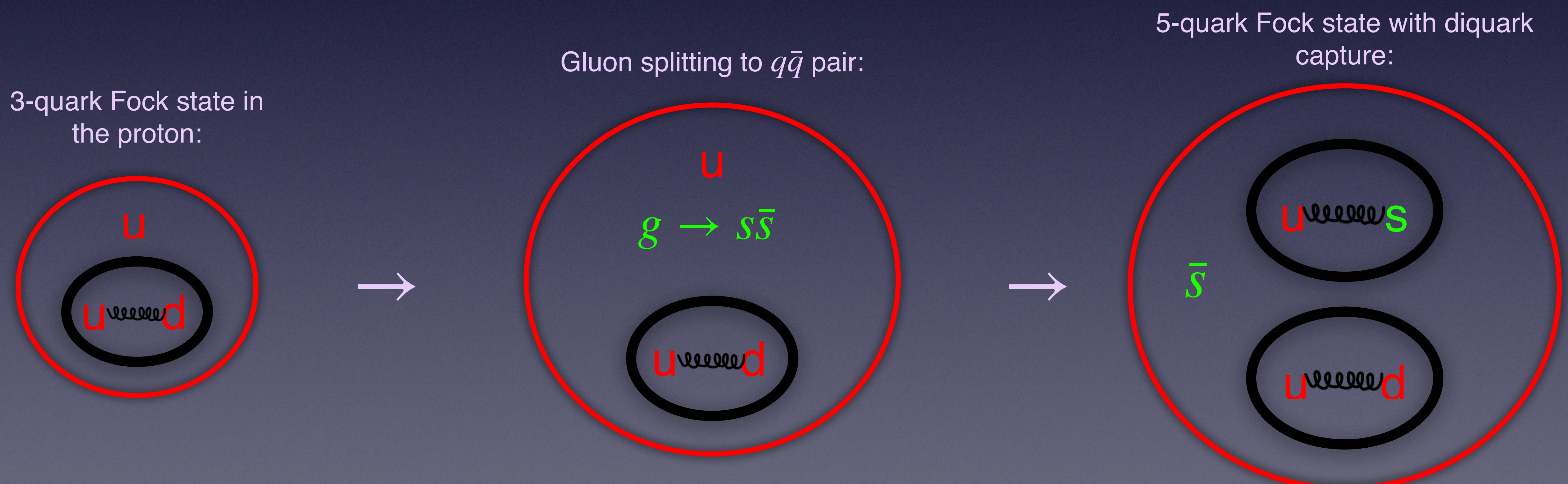
- Gluon in nucleon splits into $q\bar{q}$, adding to the 3-valence quarks
- Creates 5-quark Fock state in the nucleon wavefunction
- $g \to u\bar{u}$, $g \to d\bar{d}$ produced equally
- Diquark formation $[ud]$ has ~150 MeV binding energy - valence u may capture the d quark from $d\bar{d}$ pair - affects $\bar{d}/\bar{u}$ ratio
- Creates residual $\bar{d}$ in the quark sea - in the quark-diquark configuration of the proton, $\bar{d}$ must carry more of the proton's spin than $\bar{u}$ - STAR experiment seems to contradict this



L=1 duo-diquark, S=1/2 $\bar{d}$, L=1 between them $\Longrightarrow$ S=1/2 proton (high virtuality)

Strange quarks: Small-x_B physics in proton via Diquark Capture of s

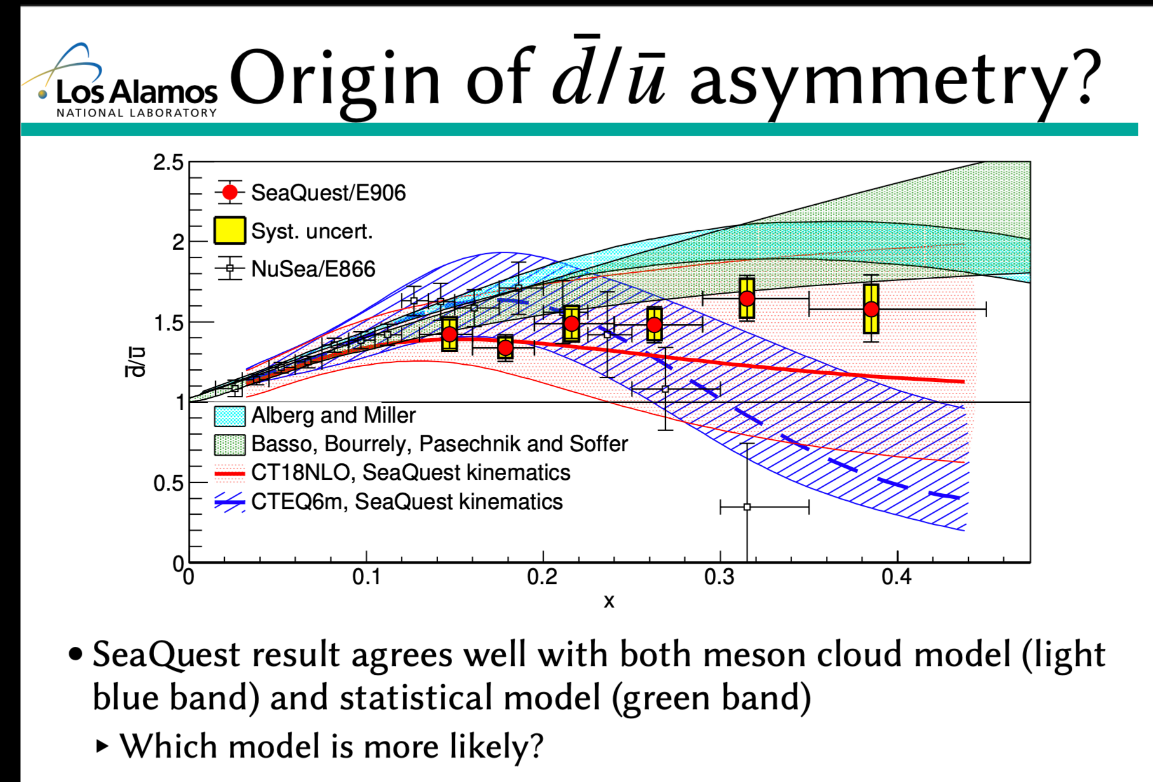
- Gluon in nucleon splits into $q\bar{q}$, adding to the 3-valence quarks
- Creates 5-quark Fock state in nucleon wavefunction
- $g \rightarrow s\bar{s}$
- Diquark capture of s from $s\bar{s}$ creates $[us]$ - work in progress to check low mass, high binding energy
- Predicts strange seaquark contributions to mass and spin of proton - but not to EM properties!



L=0 duo-diquark, S=1/2 $\bar{s}$ $\Longrightarrow$ S=1/2 proton, low virtuality

Matching diquark capture in the proton sea to data:

- Quark spectator counting rules do **not** give a flat slope from $0.15 \lesssim x_B \lesssim 0.4$
- Diquark capture (at this point) predicts a downturn in the slope towards $\dfrac{\bar{d}}{\bar{u}} = 1$
- Disagrees with SeaQuest and agrees with earlier experiments…



- Spin asymmetries match for negative $\bar{d}$ spin contribution, as mandated by spin-statistics constraints on 5-quark Fock state in proton wavefunction - work in progress

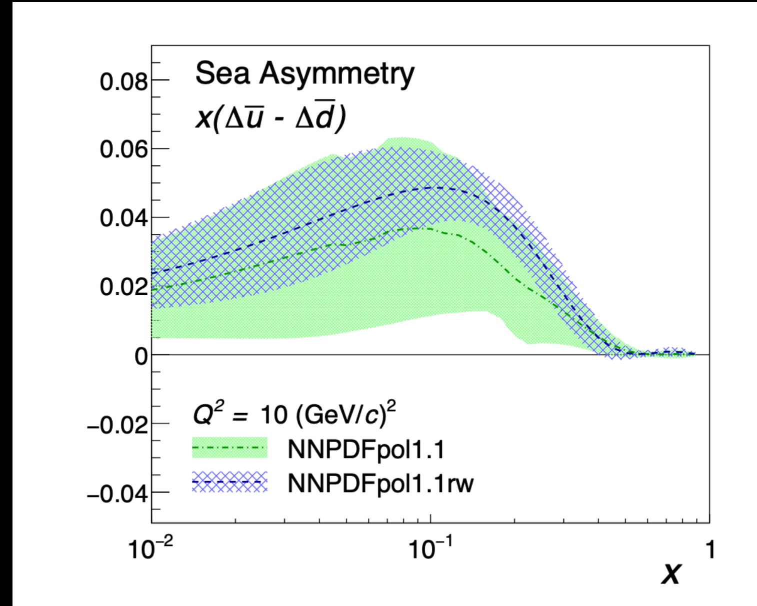


FIG. 6. The difference of the light sea-quark polarizations as a function of x at a scale of $Q^2 = 10\,(\mathrm{GeV}/c)^2$. The green band shows the NNPDFpol1.1 results [1] and the blue hatched band shows the corresponding distribution after the STAR 2013 $W^{\pm}$ data are included by reweighting.

Fin

Jennifer Rittenhouse West

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EIC Center at Jefferson Lab

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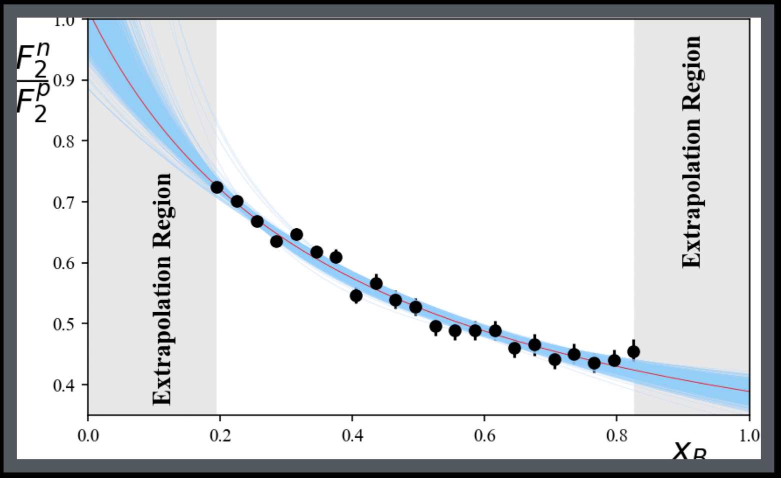
Structures in the proton at both high and low x_B

$$\text{Nucleon structure function}: \; F_2(x_B) = \sum_i Q_i^2 x(q_i(x_B) + \bar{q}_i(x_B))$$

- Structure functions in ^{3}He & ^{3}H from JLab's MARATHON experiment
  Ratio of deep inelastic (DIS) structure functions provides fundamental information about the quark distributions of nucleons - including in our extrapolated regions



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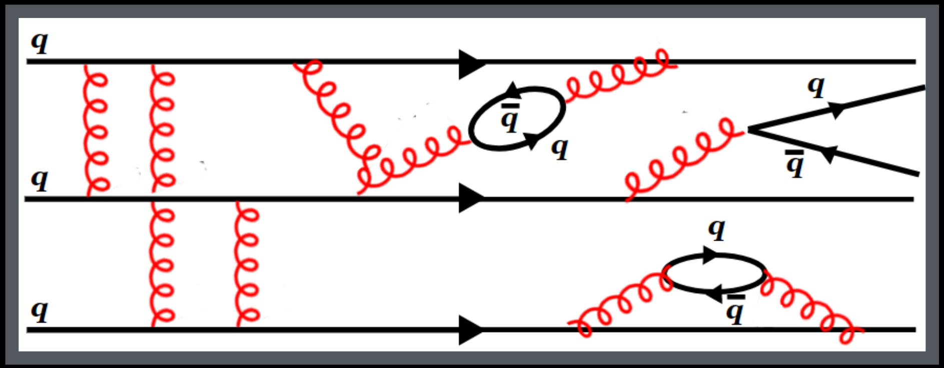

$$|p\rangle_{5q} = |uud\rangle + |uud\,\bar{d}d\rangle$$

$$|p\rangle_{7q} = |uud\rangle + |uud\,\bar{d}d\rangle + |uud\,\bar{d}d\,\bar{u}u\rangle$$

"Physical implications of the extrapolation and statistical bootstrap of nucleon structure function ratios $\frac{F_2^n}{F_2^p}$ for mirror nuclei ^{3}He and ^{3}H"

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Diquark binding energy from Color hyperfine structure

Use Λ^0 baryon to find binding energy of $[ud]$:

$$\text{B.E.}_{[ud]} = m_u^b + m_d^b + m_s^b - M_{\Lambda^0}$$

Spin-spin interaction contributes to hadron mass;
QCD hyperfine interactions:

1. $M_{(\text{baryon})} = \sum_{i=1}^{3} m_i + a' \sum_{i<j} \left(\sigma_i \cdot \sigma_j\right)/m_i m_j$

2. $M_{(\text{meson})} = m_1 + m_2 + a\left(\sigma_1 \cdot \sigma_2\right)/m_1 m_2$

(de Rujula, Georgi & Glashow 1975, Gasiorowicz & Rosner 1981, Karliner & Rosner 2014)

Effective masses of light quarks are found using Eq.1 and fitting to measured baryon masses:

$$m_u^b = m_d^b \equiv m_q^b = 363 \text{ MeV}, \quad \mathrm{m_s^b} = 538 \text{ MeV}$$

$$\text{B.E.}_{[ud]} = m_u^b + m_d^b + m_s^b - M_\Lambda = 148 \ \pm \ 9 \text{ MeV}$$

[*NB: Diquark-carrying baryons* $\Lambda_c, \Sigma_c^+, \Sigma_c^0, \Sigma_c^- \Longrightarrow \sim 159 \ \pm 10$ MeV]

Relevant diquark-carrying baryons: $\Lambda, \Sigma^+, \Sigma^0, \Sigma^-$

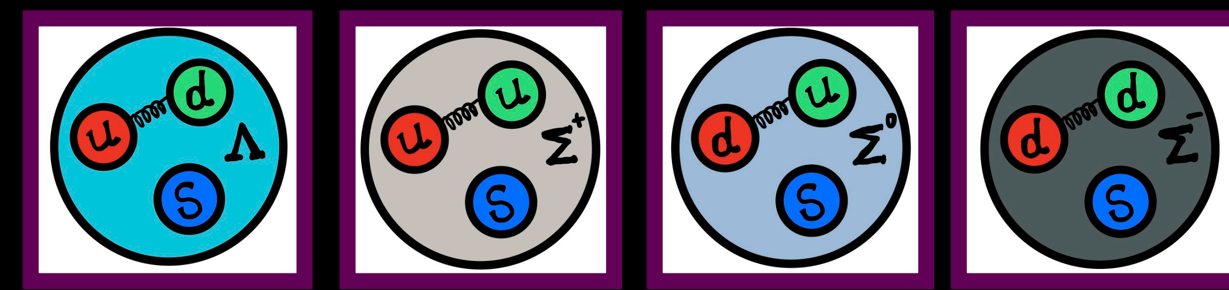

TABLE I: Diquark properties

Diquark	Binding Energy (MeV)	Mass (MeV)	Isospin I	Spin S
$[ud]$	148 ± 9	578 ± 11	0	0
(ud)	0	776 ± 11	1	1
(uu)	0	776 ± 11	1	1
(dd)	0	776 ± 11	1	1

Uncertainties calculated using average light quark mass errors $\Delta m_q = 5\ MeV$ [37]

TABLE II: Relevant $\mathrm{SU}(3)_\mathrm{C}$ hyperfine structure baryons [28]

Baryon	Diquark-Quark content	Mass (MeV)	$I\ (J^P)$
Λ	$[ud]s$	1115.683 ± 0.006	$0\ \left(\frac{1}{2}^+\right)$
Σ^+	$(uu)s$	1189.37 ± 0.07	$1\ \left(\frac{1}{2}^+\right)$
Σ^0	$(ud)s$	1192.642 ± 0.024	$1\ \left(\frac{1}{2}^+\right)$
Σ^-	$(dd)s$	1197.449 ± 0.030	$1\ \left(\frac{1}{2}^+\right)$

$I\ (J^P)$ denotes the usual isospin I, total spin J and parity P quantum numbers, all have $L=0$ therefore $J = S$

"Diquark Induced Short-Range Correlations & the EMC Effect," JRW, Nucl.Phys.A 2023