

ACCESSING THE **STRANGE** MECHANICAL STRUCTURE OF THE PROTON AT THE EIC

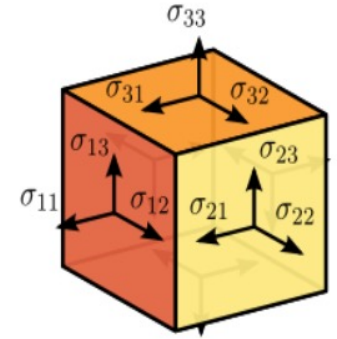
HENRY KLEST

EICUG EC Workshop
July 2024

ENERGY MOMENTUM TENSOR

$$T^{\mu\nu} = \begin{bmatrix} \text{Energy density} & \text{Momentum density} & & \\ T^{00} & T^{01} & T^{02} & T^{03} \\ T^{10} & T^{11} & T^{12} & T^{13} \\ T^{20} & T^{21} & T^{22} & T^{23} \\ T^{30} & T^{31} & T^{32} & T^{33} \\ \text{Energy flux} & \text{Momentum flux} & & \end{bmatrix}$$

Shear stress
Normal stress (pressure)



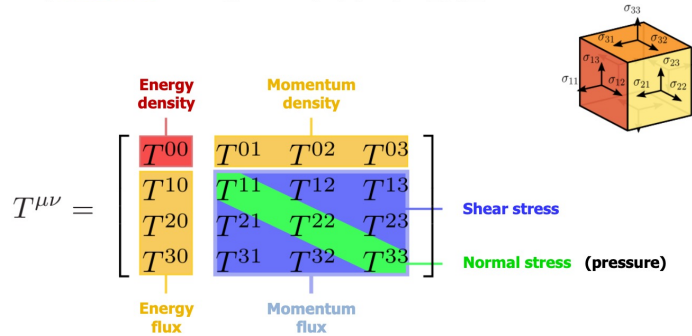
MOTIVATION

- Proton gravitational form factors (GFFs) encode information about the matrix elements of the QCD energy-momentum tensor

$$\langle p', \vec{s}' | T_a^{\mu\nu} | p, \vec{s} \rangle = \bar{u}(p', \vec{s}') \left[A_a(t) \frac{P^\mu P^\nu}{m_N} + D_a(t) \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2}{4m_N} + \bar{C}_a(t) m_N g^{\mu\nu} + J_a(t) \frac{P^{\{\mu} i \sigma^{\nu\} \lambda} \Delta_\lambda}{m_N} - S_a(t) \frac{P^{[\mu} i \sigma^{\nu] \lambda} \Delta_\lambda}{m_N} \right] u(p, \vec{s}),$$

This messy equation is just an informed parameterization of the terms in the QCD EMT

Spiritually similar to e.g. the Wolfenstein parameterization of the CKM matrix



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Form factors

Fourier transforms of t distributions

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Form factors

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“Gravitational”

Describing the energy-momentum tensor
I.e. what would be seen from proton-graviton scattering

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D-term at zero momentum transfer ($t = 0$) represents a fundamental property of the proton, on par with charge, spin, and mass!

em: $\partial_\mu J_{\text{em}}^\mu = 0$ $\langle N' | J_{\text{em}}^\mu | N \rangle \rightarrow G_E(t), G_M(t) \rightarrow Q, \mu, \dots$

weak: PCAC $\langle N' | J_{\text{weak}}^\mu | N \rangle \rightarrow G_A(t), G_P(t) \rightarrow g_A, g_p, \dots$

gravity: $\partial_\mu T_{\text{grav}}^{\mu\nu} = 0$ $\langle N' | T_{\text{grav}}^{\mu\nu} | N \rangle \rightarrow A(t), B(t), D(t) \rightarrow M, J, D, \dots$

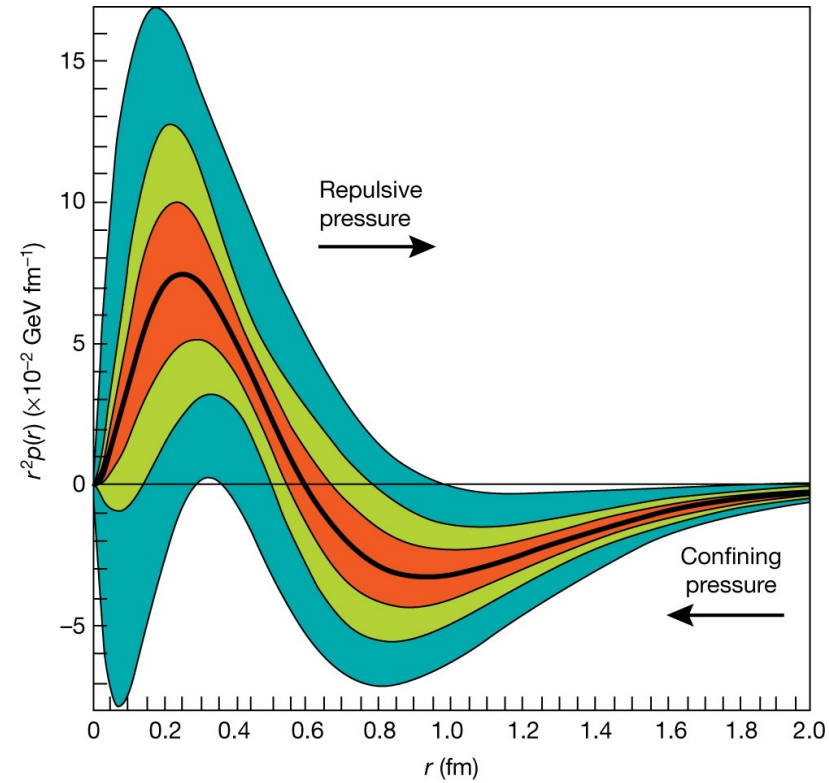
global properties:

Q_p	=	$1.602176487(40) \times 10^{-19} \text{C}$
μ_p	=	$2.792847356(23) \mu_N$
g_A	=	$1.2694(28)$
g_P	=	$8.06(0.55)$
M_p	=	$938.272013(23) \text{ MeV}$
J_p	=	$\frac{1}{2}$
D	=	$?$

MOTIVATION

- The D-term provides a gateway for extraction of various **mechanical** properties of the proton, including:
 - **Pressure distribution***
 - Mass & Mechanical radii*
 - Normal & shear force distributions

*only defined for the total D-term, not individual partonic components

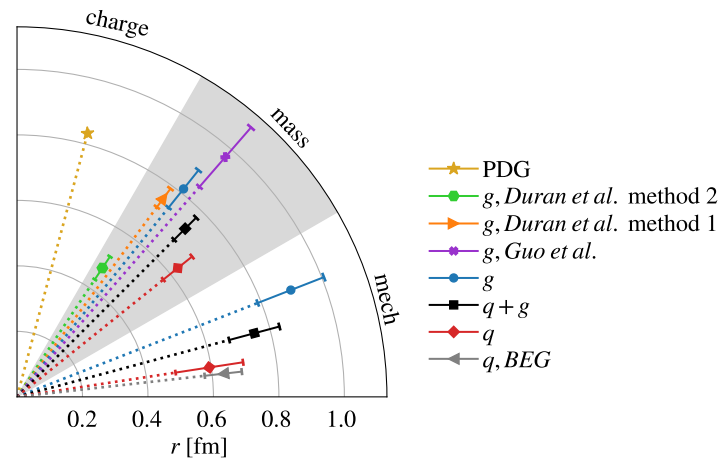
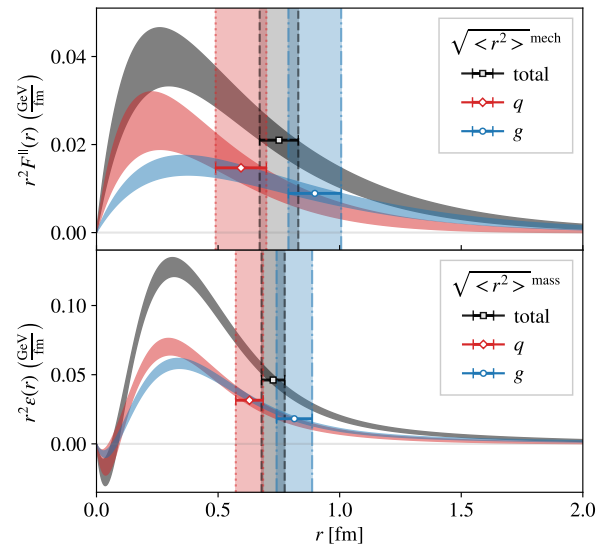


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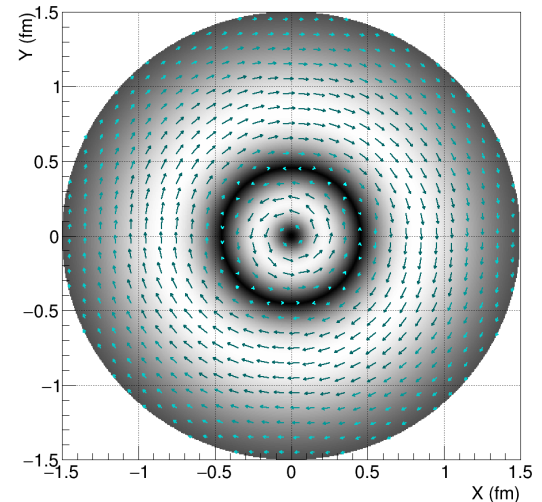
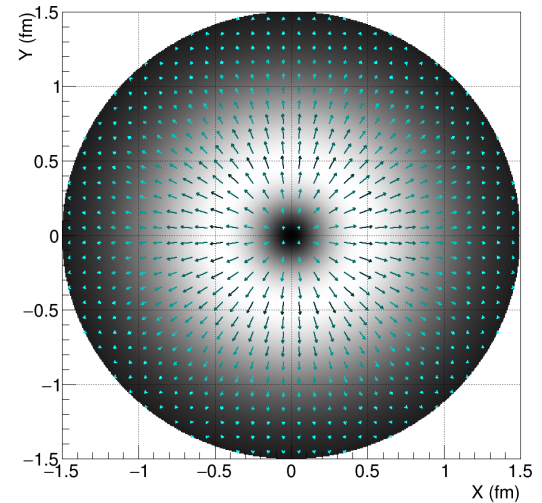
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*only defined for the total D-term, not individual partonic components

$$s(r) = -\frac{1}{4m} r \frac{d}{dr} \frac{1}{r} \frac{d}{dr} \tilde{D}(r)$$
$$p(r) = \frac{1}{6m} \frac{1}{r^2} \frac{d}{dr} r^2 \frac{d}{dr} \tilde{D}(r)$$
$$\tilde{D}(r) = \int \frac{d^3 \Delta}{(2\pi)^3} e^{-i\Delta r} D(-\Delta^2).$$



HOW DO WE MEASURE IT?

- The total D-term is related to the partonic D-terms by a simple sum rule:

$$D(0) = D_g(0) + D_u(0) + D_d(0) + D_s(0) + \dots$$


- Different physics processes provide insights into the various partonic D-terms
- Only know total D-term once all the partonic components are known!

$$D(0) = D_g(0) + \underbrace{D_u(0) + D_d(0)} + D_s(0) + \dots$$

Up & down quarks: Accessible via DVCS
cross section & beam-spin asymmetries

The pressure distribution inside the proton

[V. D. Burkert](#) , [L. Elouadrhiri](#) & [F. X. Girod](#)

$$D(0) = D_g(0) + D_u(0) + D_d(0) + D_s(0) + \dots$$


Glueons: Accessible via near-threshold
production of J/ψ and Υ

Determining the Proton's Gluonic Gravitational Form Factors

B. Duran^{3,1}, Z.-E. Meziani^{1,3**}, S. Joosten¹, M. K. Jones², S. Prasad¹, C. Peng¹,
W. Armstrong¹, H. Atac³, E. Chudakov², H. Bhatt⁵, D. Bhetuwal⁵, M. Boer¹¹,
A. Camsonne², J.-P. Chen², M. M. Dalton², N. Deokar³, M. Diefenthaler², J. Dunne⁵,
L. El Fassi⁵, E. Fuchey⁹, H. Gao², D. Gaskell², O. Hansen², F. Hauenstein⁶,
D. Higinbotham², S. Jia³, A. Karki⁵, C. Keppel², P. King⁷, H.S. Ko¹⁰, X. Li⁴, R. Li³,
D. Mack², S. Malace², M. McCaughan², R. E. McClellan⁸, R. Michaels², D. Meekins²,
M. Paolone³, L. Pentchev², E. Pooser², A. Puckett⁹, R. Radloff¹, M. Rehfuss³,
P. E. Reimer¹, S. Riordan¹, B. Sawatzky², A. Smith⁴, N. Sparveris³, H. Szumila-Vance²,
S. Wood², J. Xie¹, Z. Ye¹, C. Yero⁶, and Z. Zhao⁴

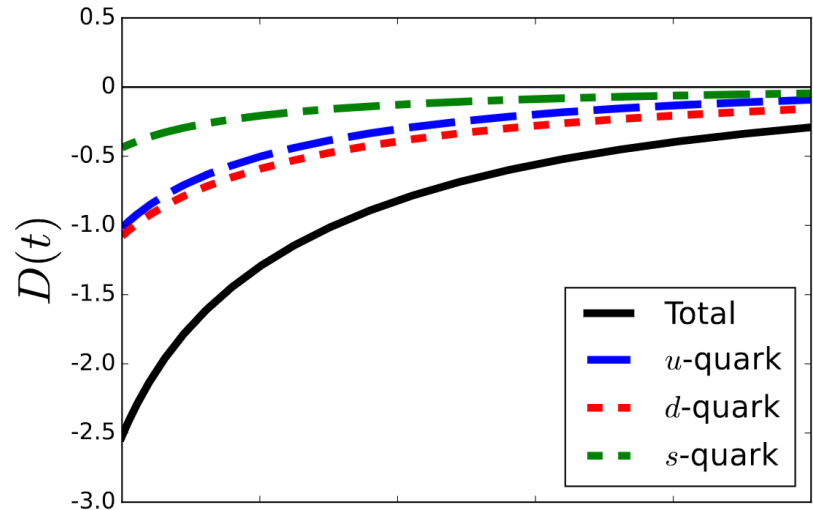
$$D(0) = D_g(0) + D_u(0) + D_d(0) + \underbrace{D_s(0)} + \dots$$

Strange quarks: Accessible via ?

WHO CARES ABOUT D_s ?

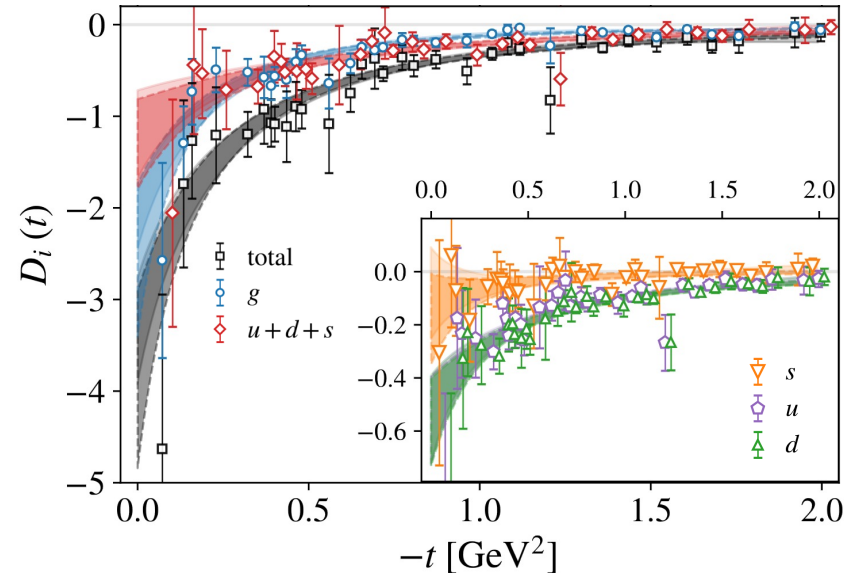
- Naively, D_s should be small
- However, large- N_c theory predicts that the D-term is "flavor-blind", i.e. $D_u \sim D_d$ despite their different number densities
 - $D_u \sim D_d$ is supported by lattice results
- Extending this argument, could $D_u \sim D_d \sim D_s$?
- Calculation by Won et al. in the χQSM suggests that $D_u \sim D_d \sim 2D_s$

This would make D_s a non-negligible contributor to the total D-term, and thus necessary for a full extraction of many of the mechanical properties of the proton!



WHO CARES ABOUT D_s ?

- On the other hand, the lattice results by Hackett et al. show that D_s is consistent with zero
- Uncertainties are still large, but the results do not exclude *positive* values of D_s
- $D_s > 0$ suggests the intriguing possibility that strange quarks exert forces in the **opposite direction** as up & down quarks!

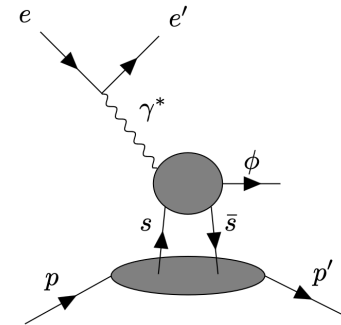


	Dipole	z -expansion
	D_i	D_i
u	-0.56(17)	-0.56(17)
d	-0.57(17)	-0.56(17)
s	-0.18(17)	-0.08(17)
$u + d + s$	-1.30(49)	-1.20(48)
g	-2.57(84)	-2.15(32)
Total	-3.87(97)	-3.35(58)

ACCESSING THE STRANGENESS D-TERM

- Information on strangeness in the valence region of the proton is limited in general
 - Disentangling it from up & down requires use of specialized processes, e.g. W/Z exchange or kaon SIDIS
- Recently, Hatta & Strikman proposed that *near-threshold electroproduction of ϕ mesons* could provide sensitivity to the strangeness D-term
 - Utilized a novel OPE framework that applies in the near-threshold region (unlike the collinear framework)

This is the only known process to access this potentially important piece of the sum rule!



ArXiv:2102.12631

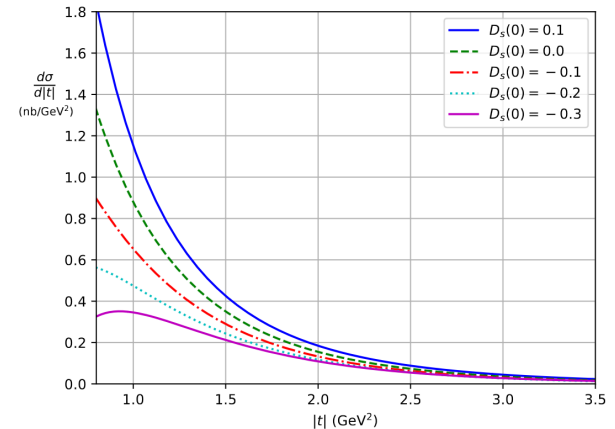


Figure 2: Theoretical predictions for $d\sigma/d|t|$ at $Q^2 = 3.4 \text{ GeV}^2$ and $W = 2.2 \text{ GeV}$ with different assumptions for $D_s(0)$. In this kinematic range $t_{\min} \approx 0.7 \text{ GeV}^2$. It can be seen that the introduction of a non-zero $D_s(0)$ has a large impact on the shape and size of the cross section.

DEEP NEAR-THRESHOLD ϕ KINEMATICS

- Deep = high momentum transfer = **high Q^2**
- Near-threshold = invariant mass of final-state hadrons **$W \sim M_\phi + M_p \sim 1.96$ GeV**
- Small momentum transfer to proton = **Low- $|t|$**
 - Most sensitive to strangeness D-term

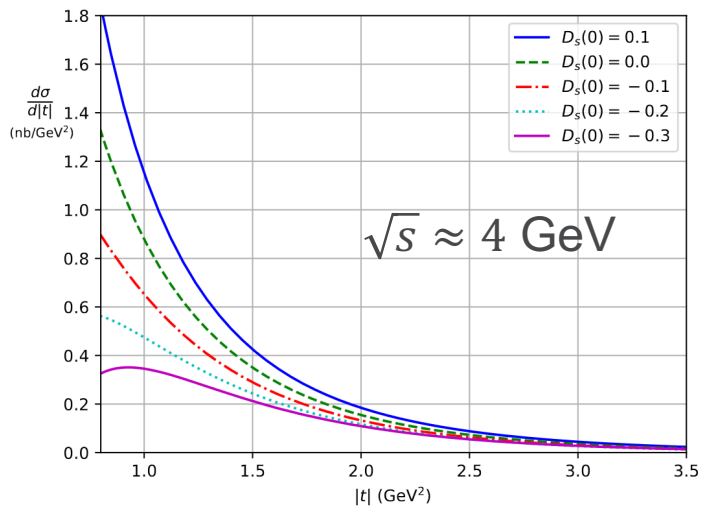
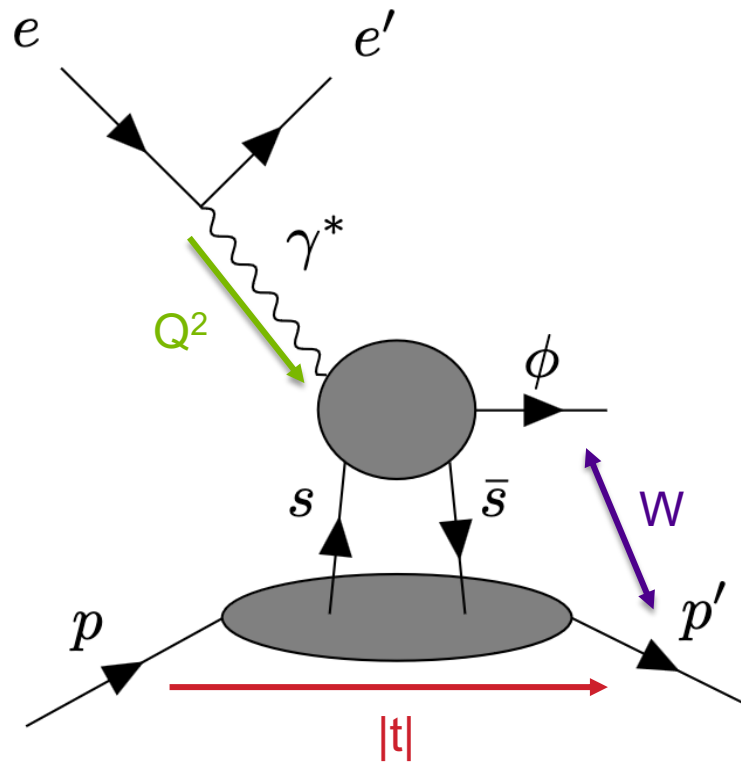
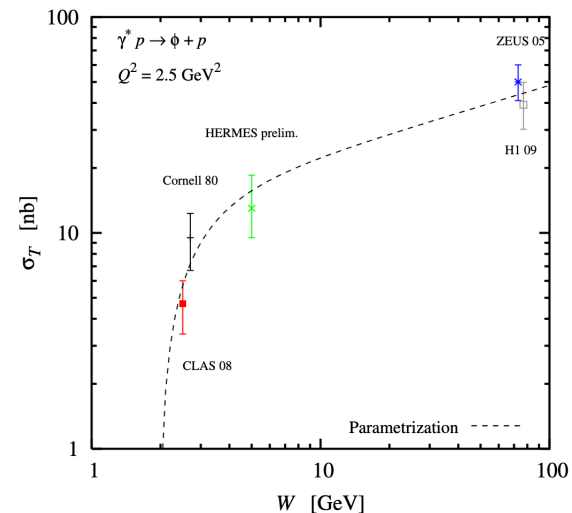
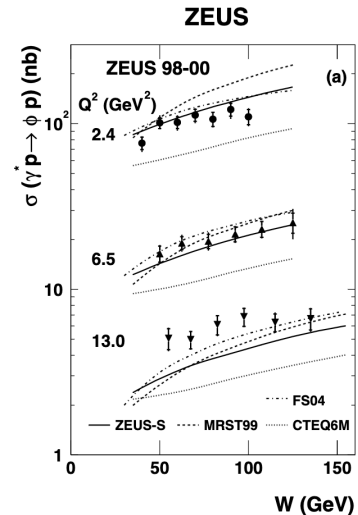
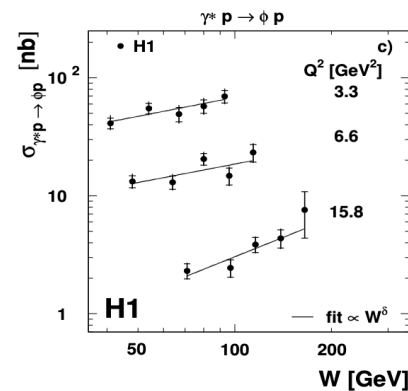
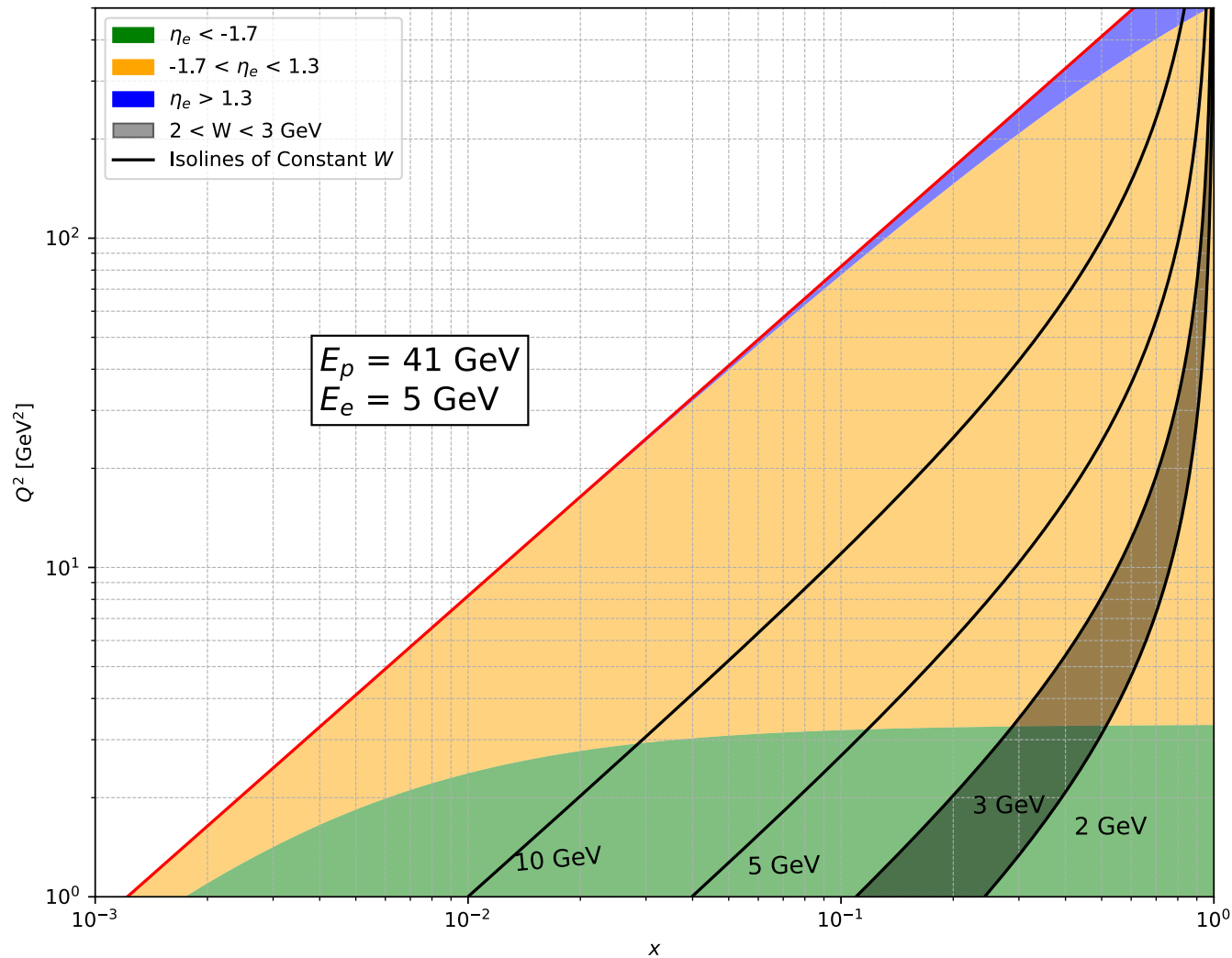


Figure 2: Theoretical predictions for $d\sigma/d|t|$ at $Q^2 = 3.4$ GeV² and $W = 2.2$ GeV with different assumptions for $D_s(0)$. In this kinematic range $t_{\min} \approx 0.7$ GeV². It can be seen that the introduction of a non-zero $D_s(0)$ has a large impact on the shape and size of the cross section.

NEAR-THRESHOLD ϕ AT EIC?

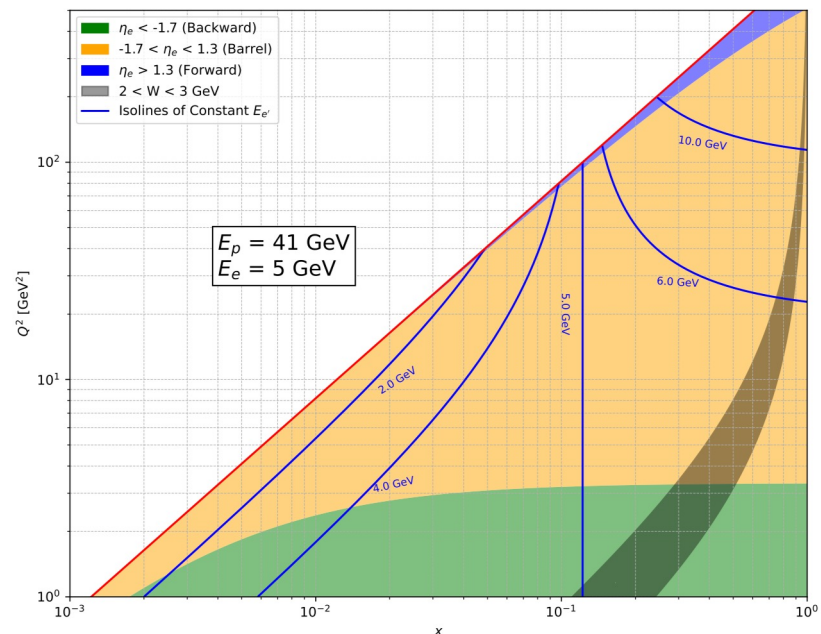
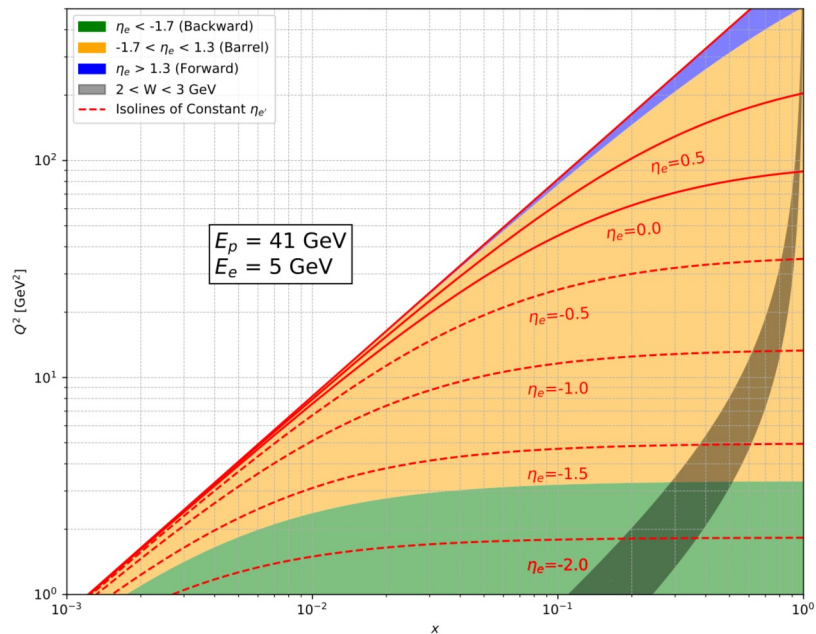
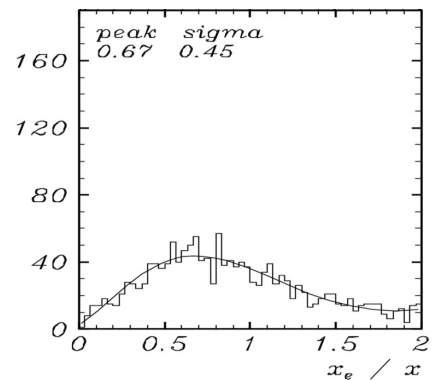
- Explore EIC's capabilities to measure exclusive ϕ near-threshold
- Near-threshold reactions are hard at colliders
 - This reaction wasn't measurable for H1 or ZEUS
 - Near-threshold implies that the invariant mass of the proton + produced particles is small
 - Requires acceptance for particles near the beam
- Several aspects make the EIC superior to HERA for these measurements
 - Ability to go to lower proton beam energies, enabling measurement of ϕ decay in central region
 - High luminosity to measure small cross sections
 - Better far forward acceptance





ASIDE: KINEMATICS

- Near-threshold kinematics for ϕ production sits at high x , low y
- Scattered electron is well contained in central detector
 - **However, this is the dreaded low y kinematic region!**
 - Near the kinematic peak of $E_{e'} = E_{e,\text{beam}}$, tiny mis-measurements of $E_{e'}$ have a huge impact on where the kinematics are reconstructed (x in particular)
 - The only way out is to use information from the hadronic final state to reconstruct the kinematics, e.g. double-angle method, Σ method



EIC ANALYSIS STRATEGY

- How much information do we need to reconstruct these exclusive ϕ events?
- Option 1: Full missing mass
 - Only reconstruct $e + p$
 - Severe backgrounds
- Option 2: One missed particle
 - $e + p + K$ or $e + K^+ + K^-$
 - Use missing mass technique to reconstruct the remaining particle
 - Resolution on M_x is not great, expect non-trivial background
- Option 3: Fully exclusive reconstruction
 - Require $e + p + K^+ + K^-$

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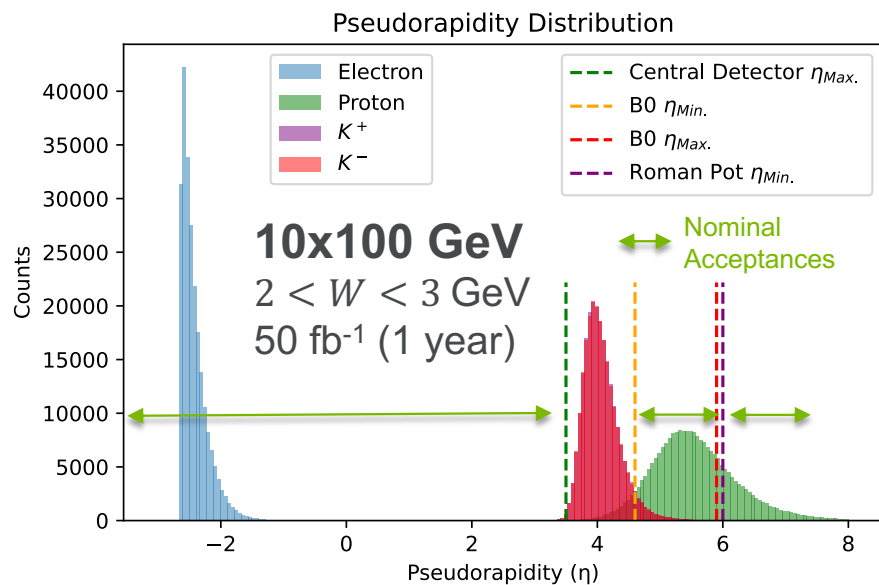
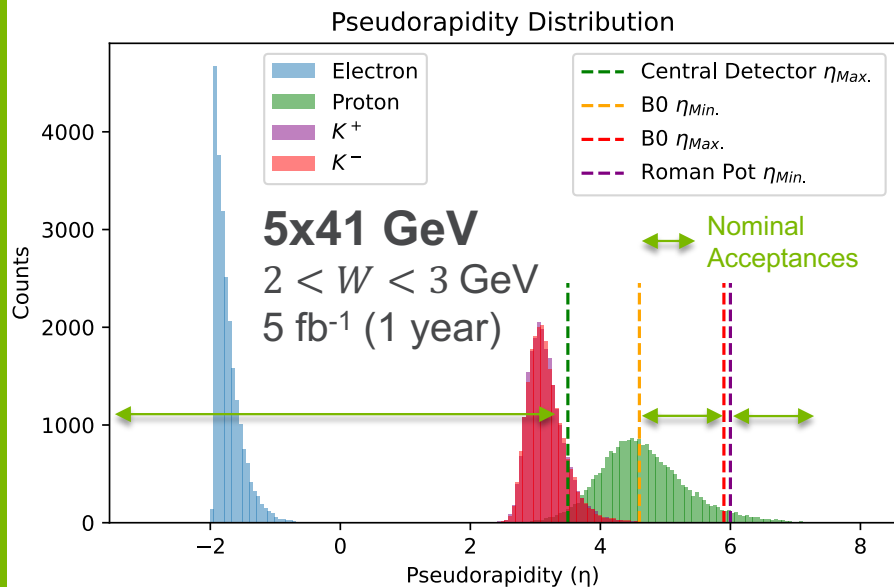
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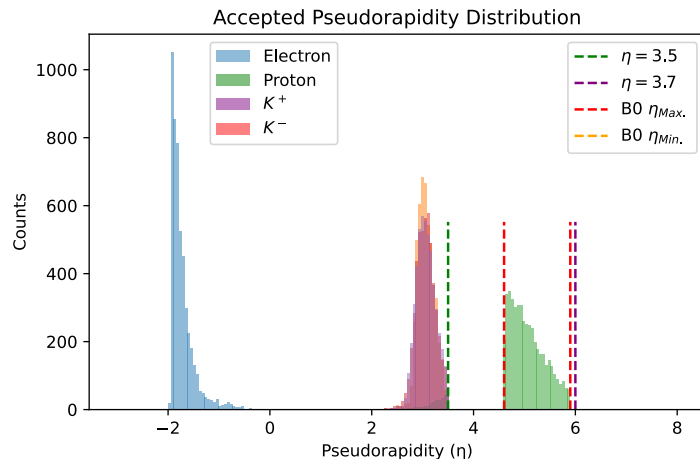
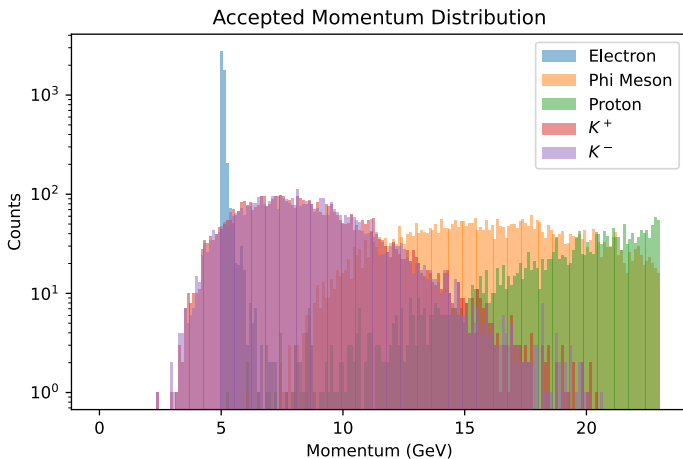


Assume only this technique in this talk

EIC SIMULATION SETUP

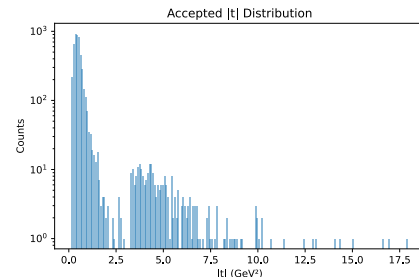
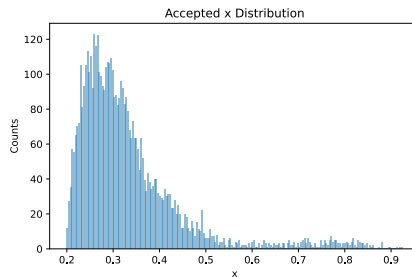
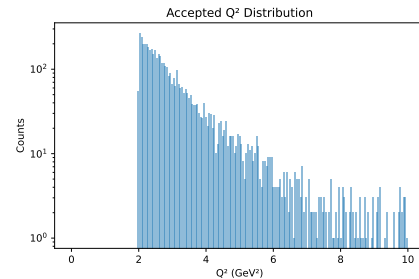
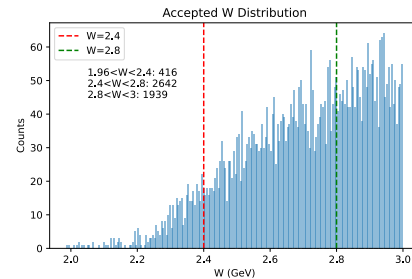
- 5 fb⁻¹ of 5 GeV e⁻ on 41 GeV p data
 - Q² > 2 GeV²
 - 1 year of data taking
 - Use I_Ager event generator
 - Cross section model based on a parameterization of all existing world data, developed for CLAS12
- Only the 5x41 energy setting allows the kaons from the ϕ decay to make it into the central detector acceptance
 - Boost from proton beam is too large even for 100 GeV proton beam energy





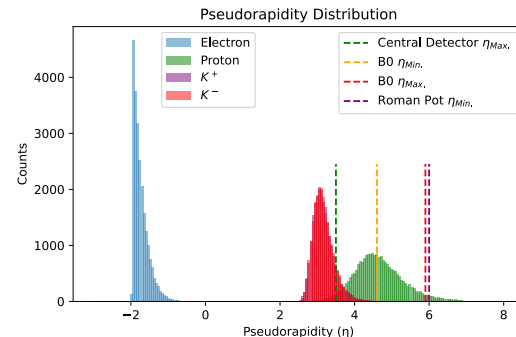
- Pessimistically assume 50% $B0$ acceptance & 0% RP & OMD acceptance
 - Fairly few detectable events send proton to RPs or OMDs for $Q^2 > 2 \text{ GeV}^2$
 - $|t|$ distribution is steep!

- Statistics seem reasonable in this case!

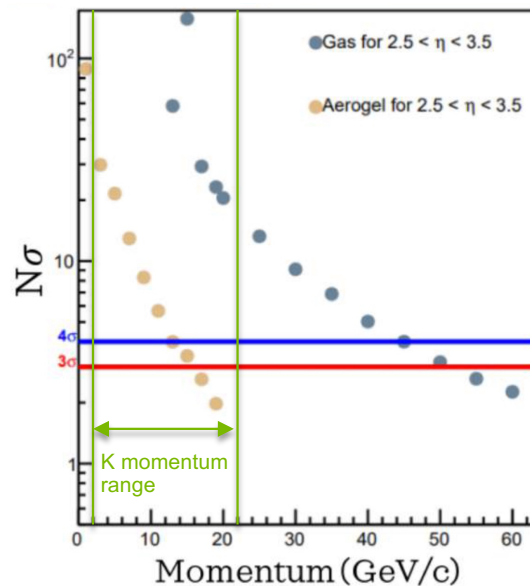
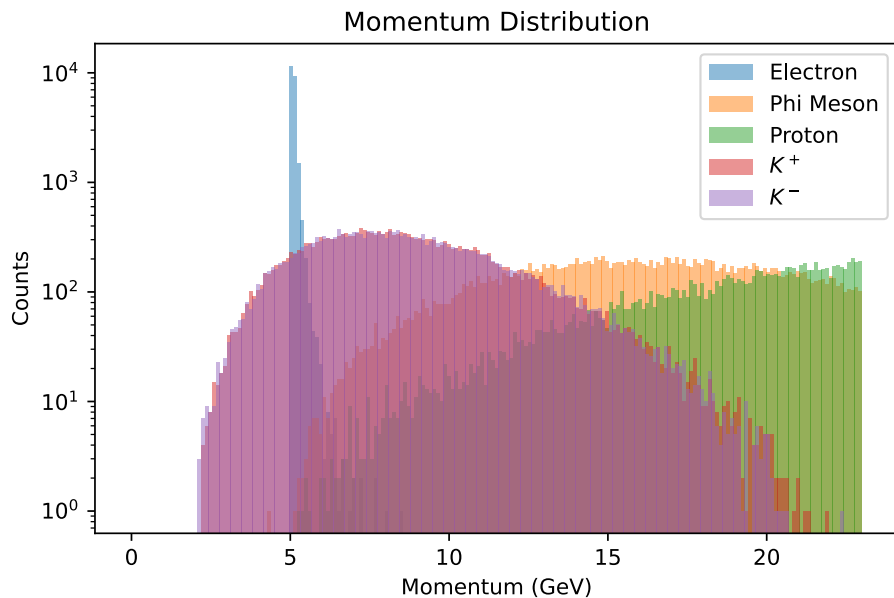


PID

- Kaon momenta range from a few GeV to ~20 GeV
- dRICH can separate pions from kaons consistently in this momentum & pseudorapidity range
 - Kaons begin to radiate in aerogel ($n = 1.026$) around 2.5 GeV
 - Kaons in C_2F_6 produce rings above 15 GeV



Expect greater than 4σ separation power for all kaons from this process!



- Multi-differential measurement possible!

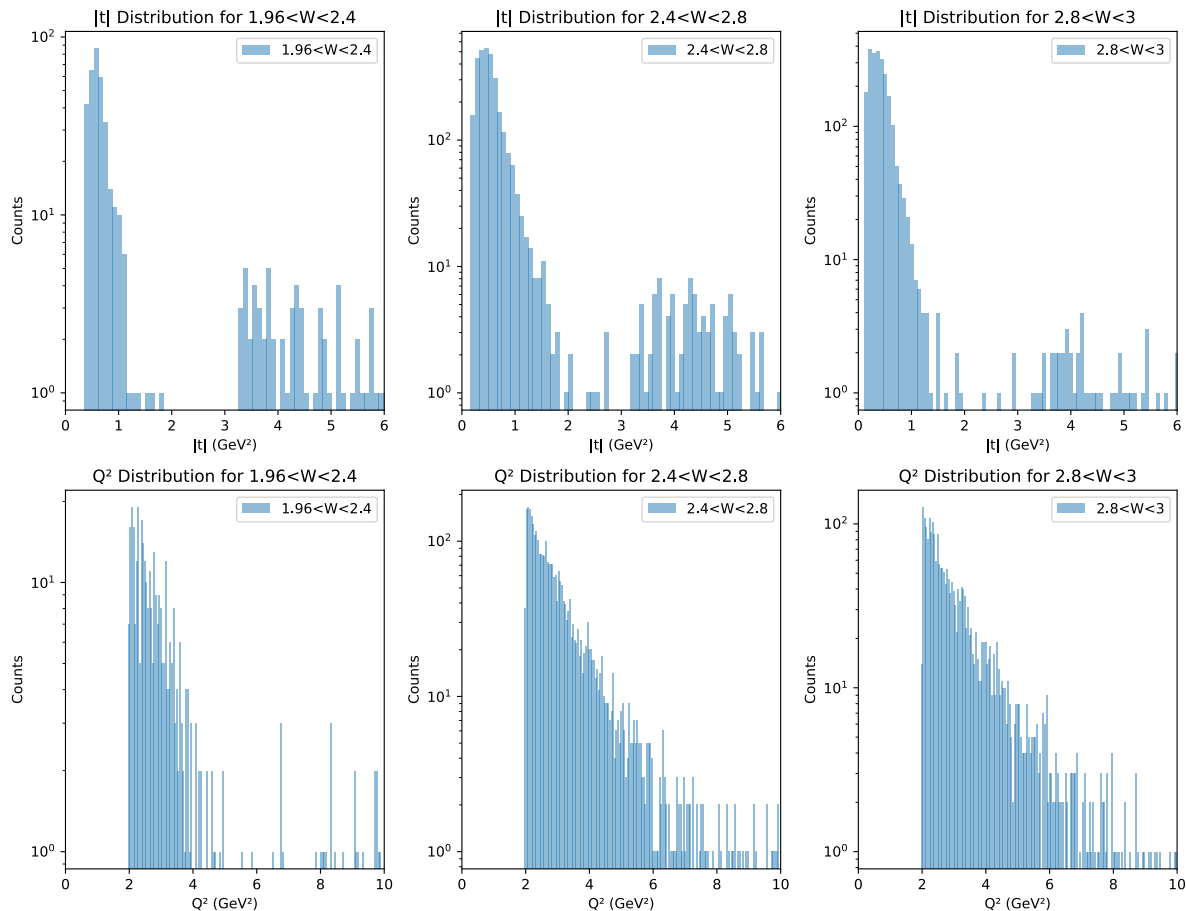
- E.g. 3 different bins in W

- Evolution of cross section with W is important to rule out contamination from baryonic resonances decaying to $\phi + p$

- Measurement likely to be statistics limited

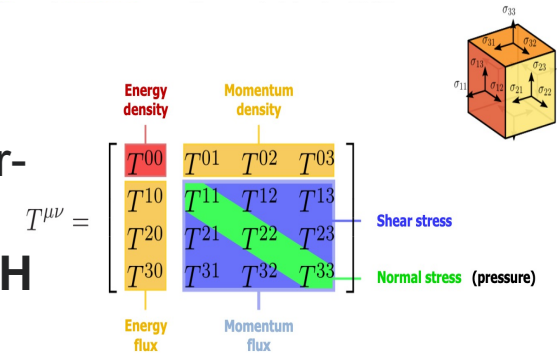
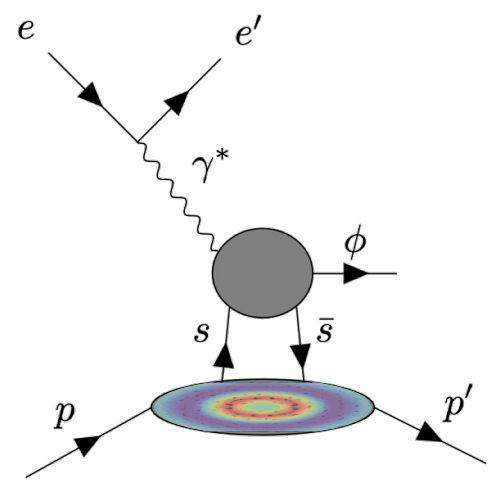
- Integrated luminosity greater than 5 fb^{-1} would improve the situation
 - Let's run more than one year of 5×41 😊

Accepted $|t|$ -distributions

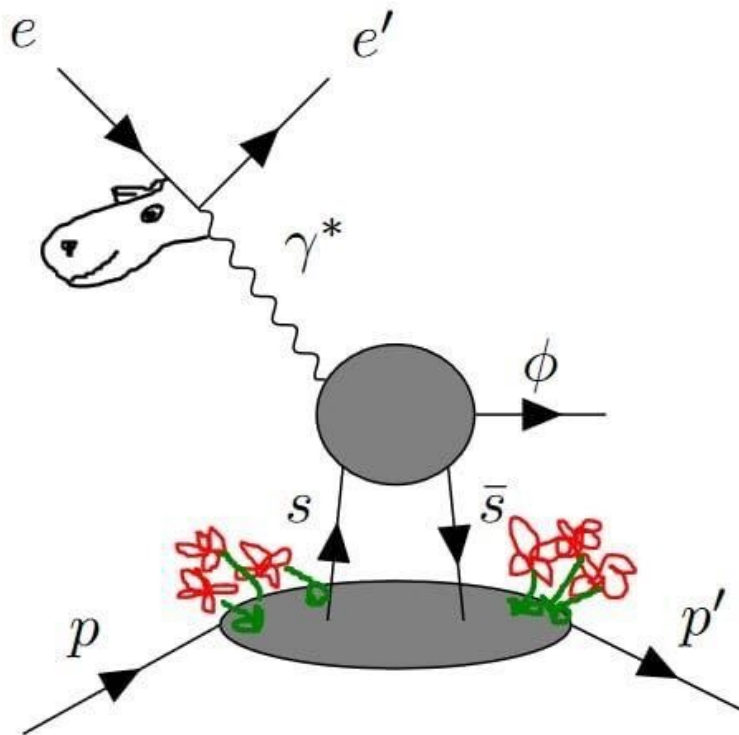


CONCLUSION

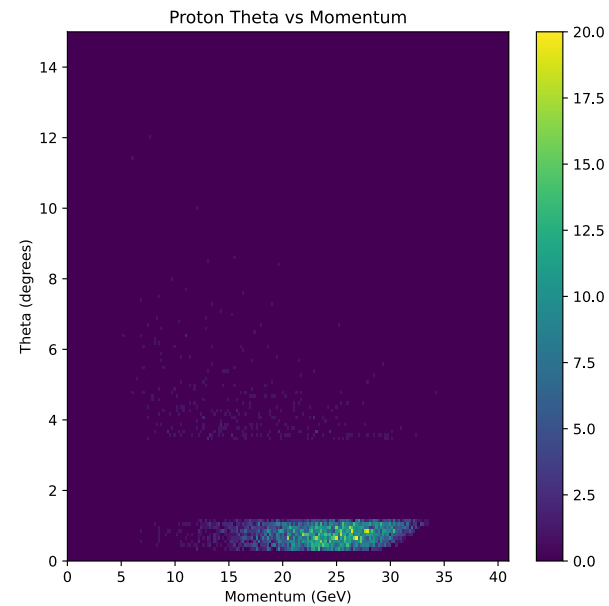
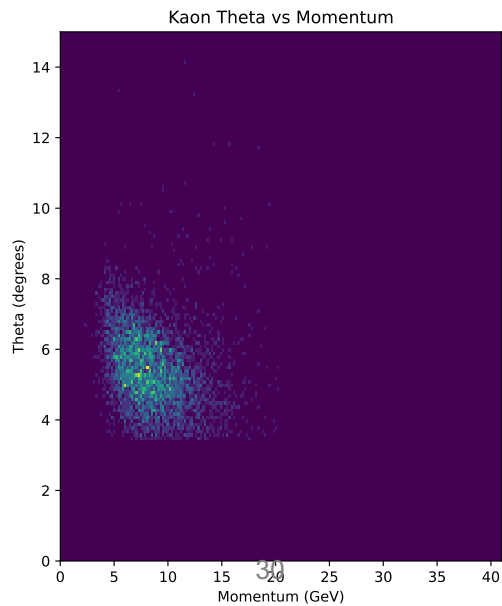
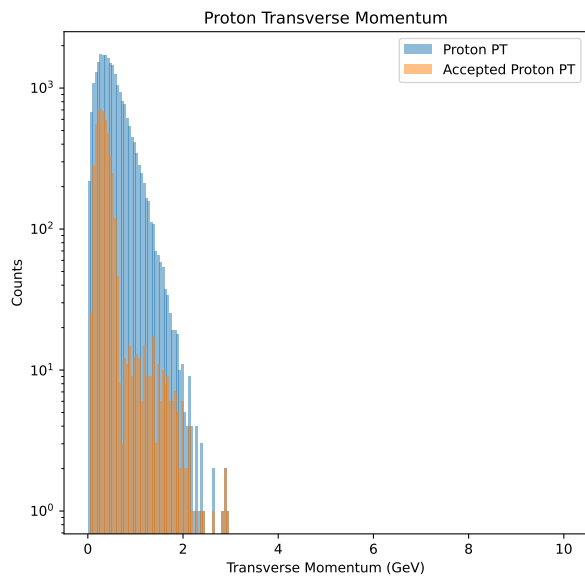
- If we ever want a complete experimental picture of the mechanical forces in the proton, will need to measure the strangeness D-term
 - More theoretical & phenomenological input is needed!
- Deep exclusive ϕ production offers the opportunity to learn about the forces exerted by strange quarks inside the proton!
- EIC 5x41 energy configuration can provide access to near-threshold ϕ electroproduction!
 - **Sensitive to high η acceptance for tracking & dRICH**



BACKUP

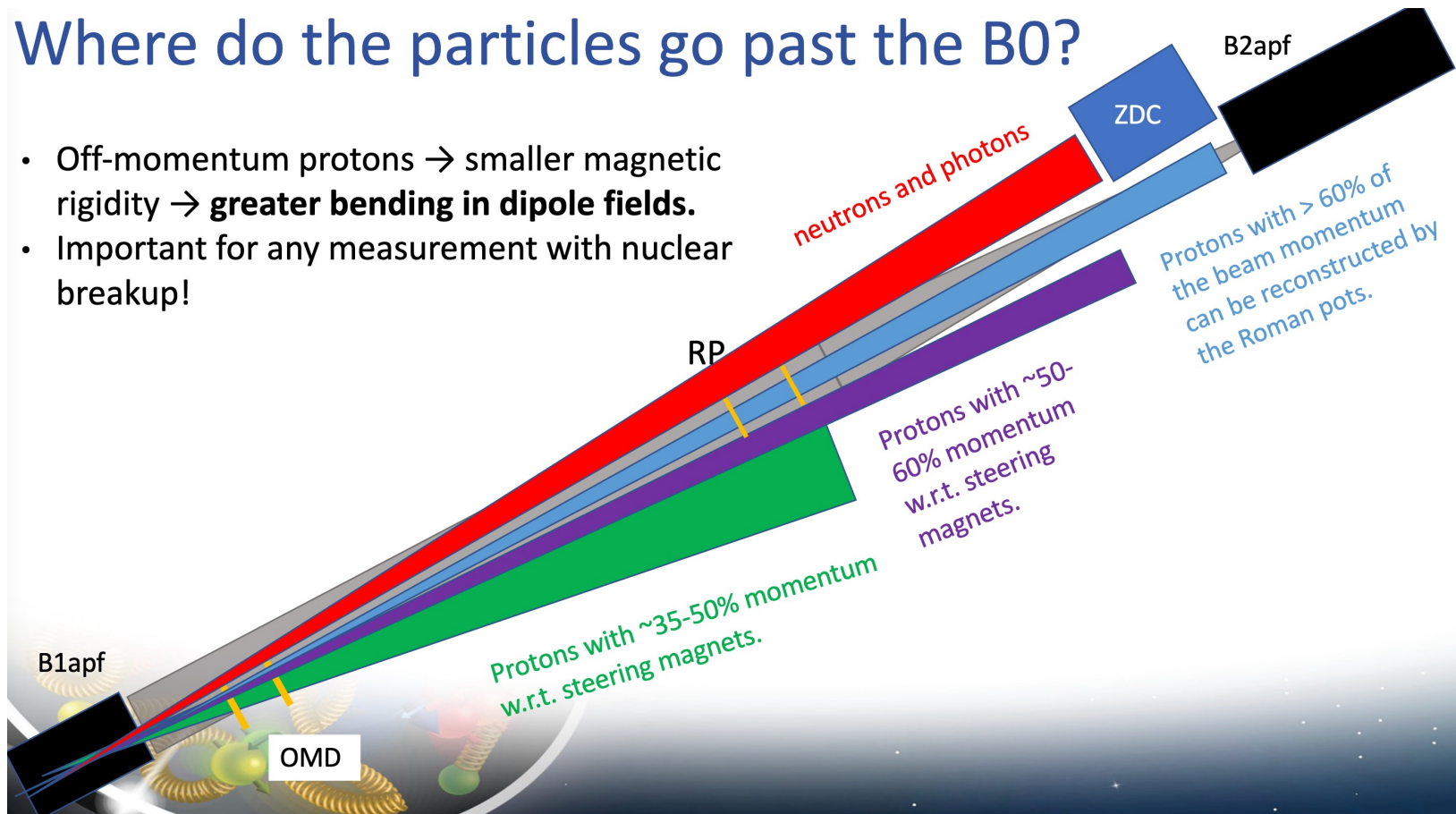


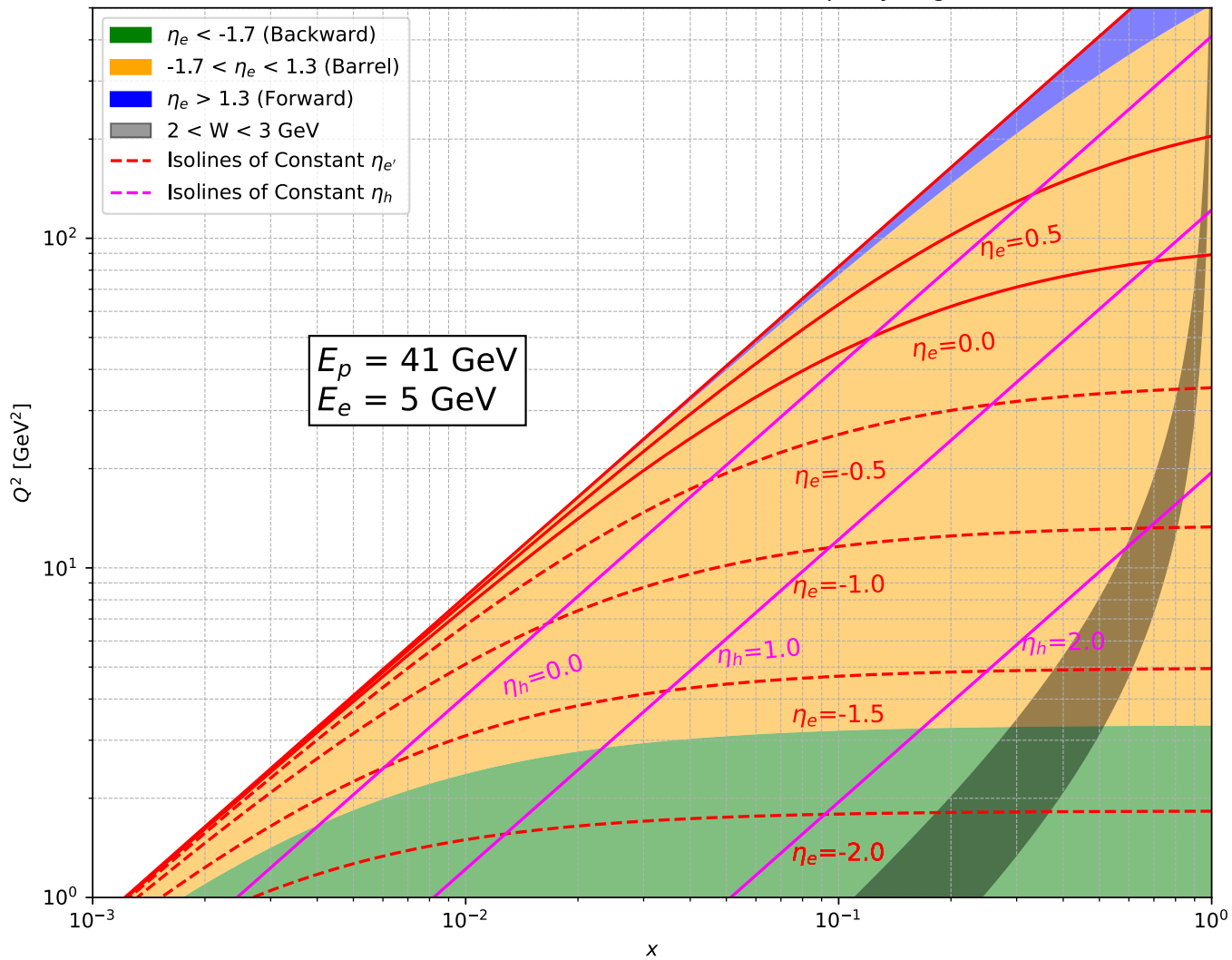
Accepted particle distributions



Where do the particles go past the B0?

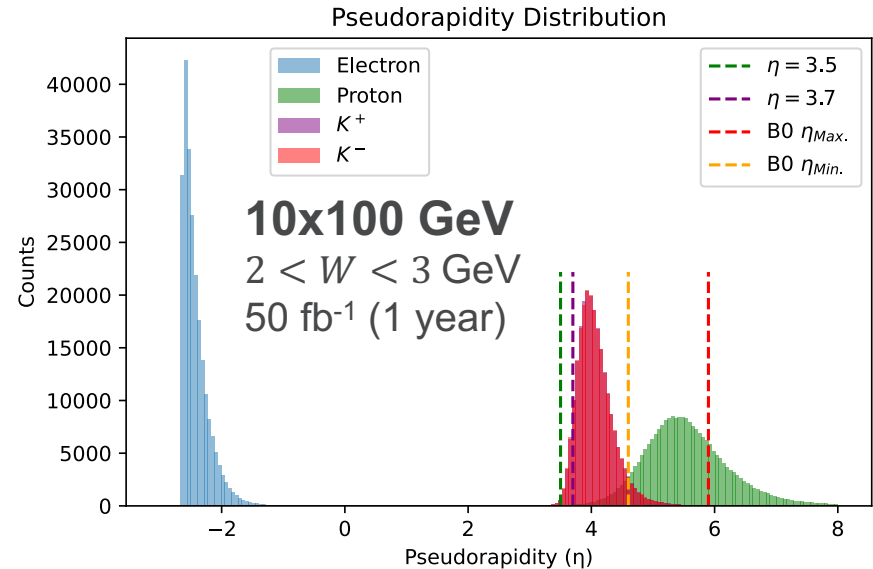
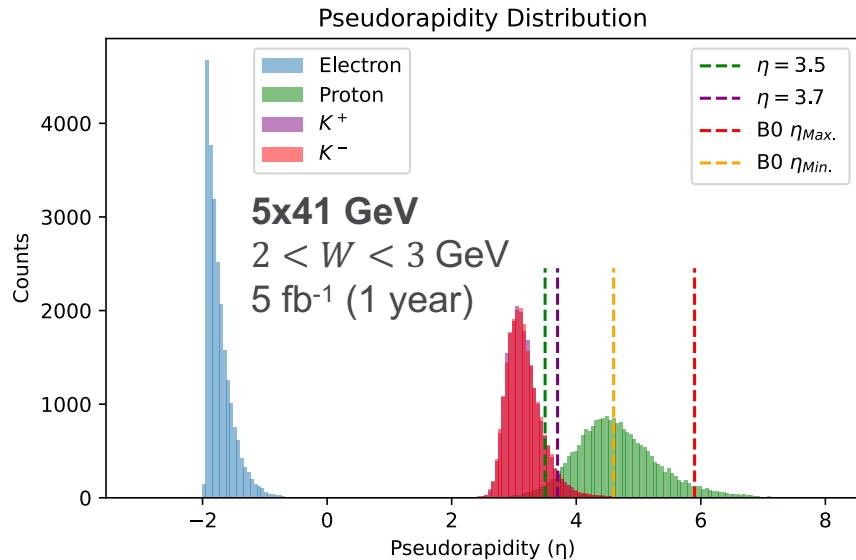
- Off-momentum protons \rightarrow smaller magnetic rigidity \rightarrow **greater bending in dipole fields.**
- Important for any measurement with nuclear breakup!

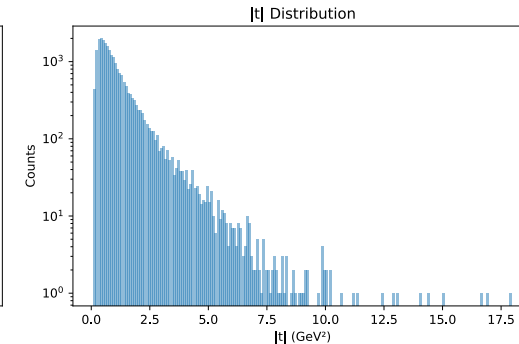
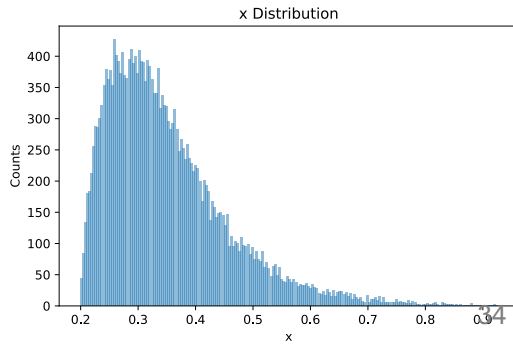
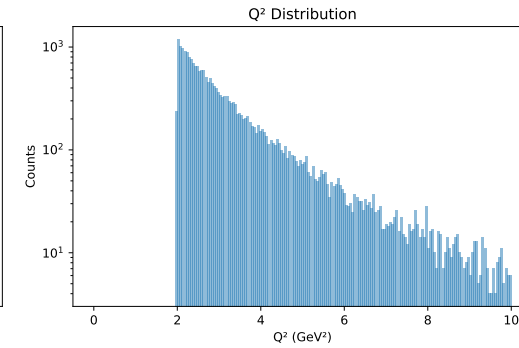
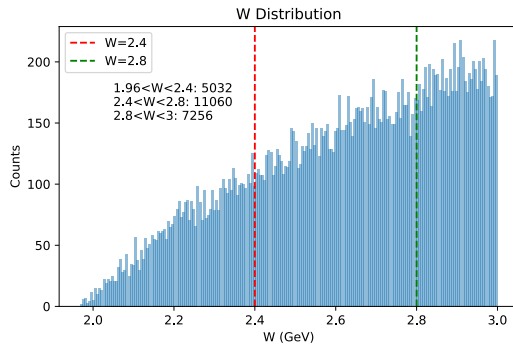
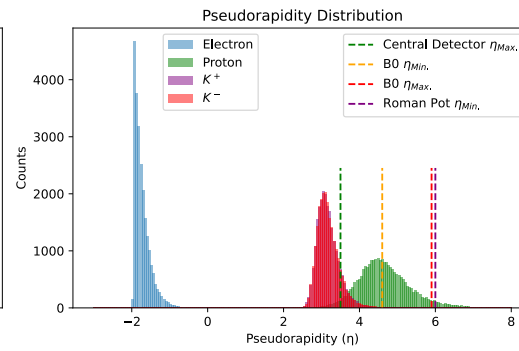
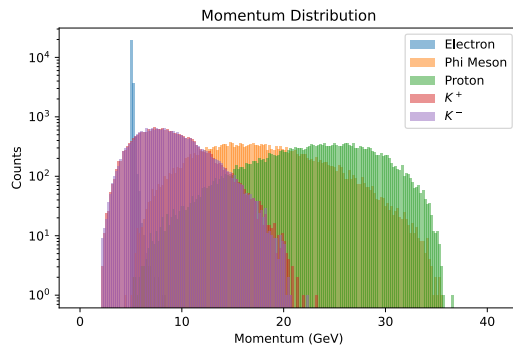




PSEUDORAPIDITY ACCEPTANCE

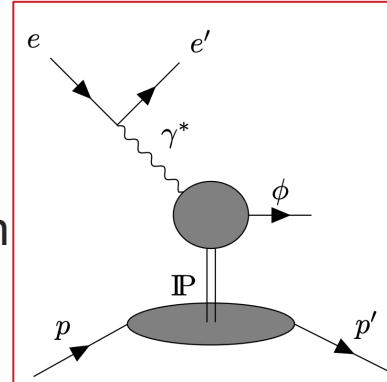
- If the η range of the dRICH PID extends even to $\eta = 3.7$, could potentially reconstruct this final state also in the 10x100 energy configuration
 - **Note y-axes!** 10x more integrated luminosity available in 10x100
 - Possible that the tail of the 10x100 statistics could provide a better measurement than the whole 5x41 dataset



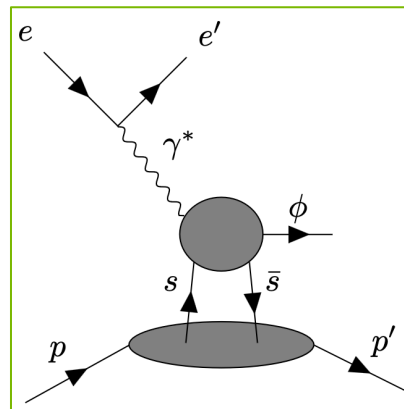


REALITY CHECK

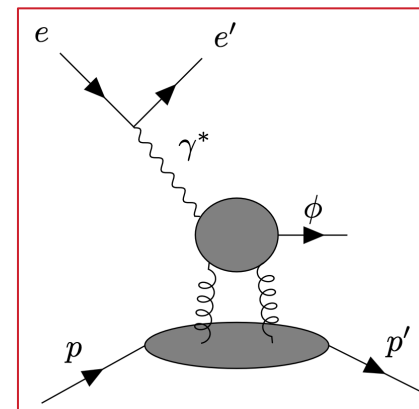
- The reality is (as always) that it's not so simple!
- Other physics processes can contribute to ϕ electroproduction
 - This will dilute the sensitivity to the D-term
- **Needs more phenomenological input before we can really claim an extraction of D_s**
 - E.g. calculation of gluon exchange contribution within the same framework
- Additional caveats:
 - Calculation wants $Q^2 \gg |t|$
 - For $|t| \sim 1 \text{ GeV}^2$ is $Q^2 \sim 3.5 \text{ GeV}^2$ high enough?
 - Non-linear behavior observed in the photoproduction cross section for $W < 2.4 \text{ GeV}$, resonances?



Pomeron exchange
(insensitive to D_s)



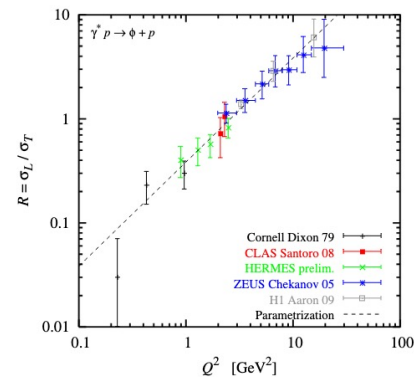
Strange exchange
(sensitive to D_s)



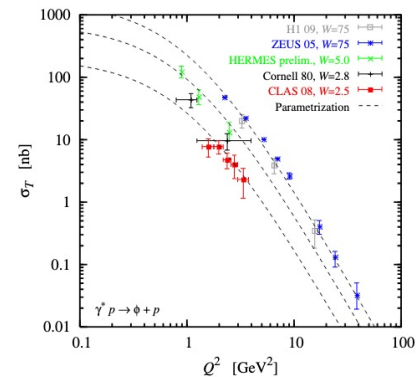
Two gluon exchange
(insensitive to D_s)

CROSS SECTION MODEL

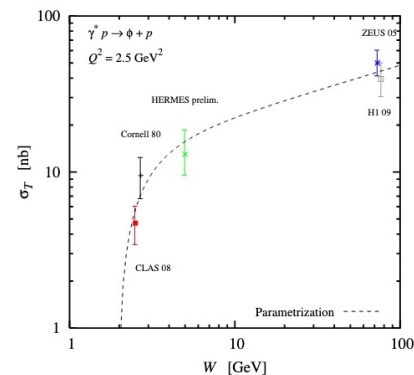
- Utilize the parameterization of world data developed for CLAS12 exclusive ϕ proposal to PAC39
- Events generated according to these parameterizations using the IAgger event generator
- t -dependence parameterized as a dipole



(a)



(b)



(c)

FIG. 12: (a) Ratio $R = \sigma_L/\sigma_T$ as function of Q^2 , combining measurements at different W . (b) Transverse cross section σ_T as function of Q^2 , for measurements at different W . (c) Transverse cross section σ_T as function of W , at $Q^2 = 2.5 \text{ GeV}^2$. The curves represents the parameterization of Eqs. (41)–(50).

CROSS SECTION MODEL

The differential cross section is given by the general expression

$$\frac{d\sigma_{L,T}}{dt} = \frac{\sigma_{L,T} F(t)}{F_{\text{int}}} \quad (51)$$

$$F(0) = 1, \quad (52)$$

$$F_{\text{int}} \equiv \int_{t_{\text{max}}}^{t_{\text{min}}} dt F(t), \quad (53)$$

where different physical models are considered for the function $F(t)$ implementing the t -dependence.

1. Exponential t -dependence

$$F(t) = e^{Bt} \quad (54)$$

$$F_{\text{int}} = e^{Bt_{\text{min}}}/B \quad (55)$$

The exponential slope B is parametrized as a function of W :

$$B(W) = B_0 + 4\alpha' \ln \frac{W}{\text{GeV}} \quad (56)$$

$$B_0 = 2.2 \text{ GeV}^{-2}, \quad (57)$$

$$\alpha' = 0.24. \quad (58)$$

2. Power-like t -dependence (dipole at amplitude level):

$$F(t) = \frac{m_g^8}{(m_g^2 - t)^4} \quad (59)$$

$$F_{\text{int}} = \frac{m_g^8}{3(m_g^2 - t_{\text{min}})^3} \quad (60)$$

The mass parameter at $W \sim \text{few GeV}$ is chosen as

$$m_g^2 = 1.0 \text{ GeV}^2. \quad (61)$$

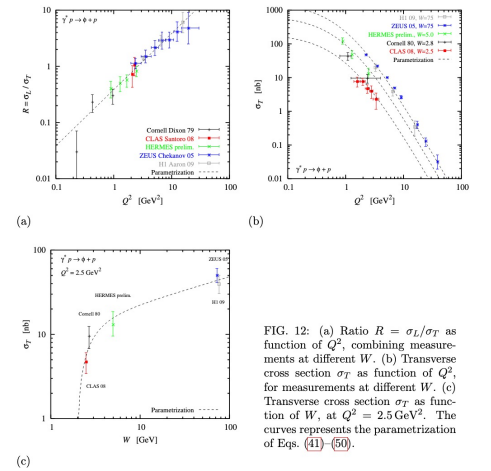


FIG. 12: (a) Ratio $R = \sigma_L/\sigma_T$ as function of Q^2 , combining measurements at different W . (b) Transverse cross section σ_T as function of Q^2 , for measurements at different W . (c) Transverse cross section σ_T as function of W , at $Q^2 = 2.5 \text{ GeV}^2$. The curves represent the parametrization of Eqs. (41)–(50).

The parametrization was constricted by fitting data on the transverse cross section $\sigma_T(W, Q^2)$ and the ratio $R = \sigma_L(W, Q^2)/\sigma_T(W, Q^2)$; the differential cross sections and their t -dependence were then parametrized according to different physical models for the t -dependence (exponential, dipole) [50]. The transverse cross section is parametrized as

$$\sigma_T(W, Q^2) = \frac{c_T(W)}{(1 + Q^2/m_\phi^2)^{\nu_T}}, \quad (41)$$

$$\nu_T = 3.0 \quad (\text{independent of } W) \quad (42)$$

$$c_T(W) = \alpha_1 \left(1 - \frac{W^2}{W^2_{\text{th}}}\right)^{\alpha_2} \left(\frac{W}{\text{GeV}}\right)^{\alpha_3} \text{ nb} \quad (43)$$

$$W_{\text{th}} = m_N + m_\phi = 1.96 \text{ GeV} \quad (44)$$

$$\alpha_1 = 400, \quad (45)$$

$$\alpha_2 = 1.0, \quad (46)$$

$$\alpha_3 = 0.32. \quad (47)$$

The longitudinal cross section is parametrized as

$$\sigma_L(W, Q^2) = R(W, Q^2) \sigma_T(W, Q^2) \quad (48)$$

$$R(W, Q^2) = \frac{c_R Q^2}{m_\phi^2}, \quad (49)$$

$$c_R = 0.4 \quad (\text{independent of } W) \quad (50)$$