Soft function study in $pp \to V + j + X$ processes

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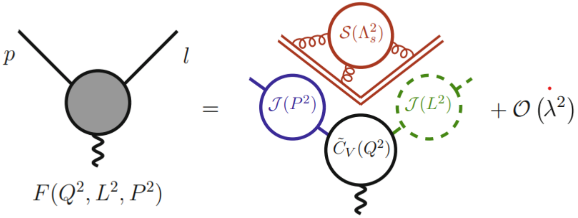
Why SCET?



p
l
$F(Q^2, L^2, P^2)$
$=$
$\mathcal{S}(\Lambda_s^2)$
$\mathcal{J}(P^2)$
$\mathcal{J}(L^2)$
$\tilde{C}_V(Q^2)$
$+\mathcal{O}(\lambda^2)$

Contents

Contents

NOTATION

Light-like vectors:

$$\left.\begin{array}{l} n^\mu = \frac{1}{\sqrt{2}}(1,0,0,1) \\ \bar{n}^\mu = \frac{1}{\sqrt{2}}(1,0,0,-1) \end{array}\right\} n^2 = \bar{n}^2 = 0, \;\; \bar{n}\cdot n = 1.$$

p^μ can be decomposed in terms of its light-cone components:

$$p^\mu = p_+ \bar{n}^\mu + p_- n^\mu + {p_\perp}^\mu = (p_+, p_-, p_\perp)_n;$$

$$\text{with} \;\; \begin{cases} p_+ = n\cdot p, \;\; p_- = \bar{n}\cdot p, \\ p^2 = 2p_+p_- + {p_\perp}^2 = 2p_+p_- - \mathbf{p}^2. \end{cases}$$

NOTATION

Expansion parameter λ:

$$p_1^2 \sim p_2^2 \sim \lambda^2 Q^2$$

Regions with a non-null contribution (see *Introduction to Soft-Collinear Effective Theory*, T. Becher *et al.* [1]):

- *Hard Region*, $h \to k^\mu \sim (1, 1, 1)Q$.
- *Collinear Region*, $c \to k^\mu \sim (\lambda^2, 1, \lambda)Q$.
- *Anti-Collinear Region*, $\bar{c} \to k^\mu \sim (1, \lambda^2, \lambda)Q$.
- *Soft Region*, $s \to k^\mu \sim (\lambda, \lambda, \lambda)Q$.
- *Ultra-Soft Region*, $us \to k^\mu \sim (\lambda^2, \lambda^2, \lambda^2)Q$.

Contents

FROM QCD TO SCET I

Main problems:

- $\psi(x)$ (quark field) and $A_\mu(x)$ (gluon field) scale with **different powers** of λ.
- We should check that the theory does **not modify** the fields **dependence** on λ.
- The non-local operator require **Wilson lines** to preserve the **gauge invariance**.

FROM QCD TO SCET I

- Gluon fields:

$$A^\mu(x) \to A_c^\mu(x) + A_{us}^\mu(x)$$

- Quarks fields:

$$\psi(x) \to \psi_c(x) + \psi_{us}(x)$$

$$\text{where } \psi_c(x) \equiv \xi(x) + \eta(x) \begin{cases} \xi = P_+\psi_c \equiv \frac{\not{n}\not{\bar{n}}}{2}\psi_c \\ \eta = P_-\psi_c \equiv \frac{\not{\bar{n}}\not{n}}{2}\psi_c \end{cases}$$

Quark fields

- Good collinear fields $\longrightarrow$ $\boxed{\xi(x) \sim \lambda}$
- Bad collinear fields $\longrightarrow$ $\boxed{\eta(x) \sim \lambda^2}$
- Ultra-soft fields $\longrightarrow$ $\boxed{\psi_{us} \sim \lambda^3}$

Gluon fields

$$\langle 0|T\{A^\mu(x)A^\mu(0)\}|0\rangle = \int \frac{d^4p}{(2\pi)^4}\frac{i}{p^2+i0}e^{-ip\cdot x}\left[-g^{\mu\nu}+\xi\frac{p^\mu p^\nu}{p^2}\right]$$

- Collinear fields $\begin{cases}(n\cdot A_c)\sim\lambda^2\\(\bar{n}\cdot A_c)\sim\lambda^0\\(A_c)_\perp\sim\lambda\end{cases}$
- Ultra-soft fields $\longrightarrow A^\mu_{us}\sim\lambda^2$

SCET effective Lagrangian

Gluon field: $A^\mu(x) \to (n \cdot A_c(x) + n \cdot A_{us}(x))\bar{n}^\mu + (\bar{n} \cdot A_c(x))n^\mu + (A_c^\mu)_\perp(x)$

SCET I Lagrangian

$$\mathcal{L}_{\text{SCET I}} = \bar{\psi}_{us} i \not{D}_{us} \psi_{us} - \frac{1}{4}(F_{\mu\nu}^{us,a})^2 - \frac{1}{4}(F_{\mu\nu}^{c,a})^2 + \\ + \bar{\xi} \not{\bar{n}} \left[i(n \cdot D) + i(\not{D}_c)_\perp \frac{1}{i2(\bar{n} \cdot D_c)} i(\not{D}_c)_\perp \right] \xi$$

- $iD_\mu \equiv (i\partial_\mu + gA_\mu)$
- $iD^\mu = i(n \cdot D)\bar{n}^\mu + i(\bar{n} \cdot D_c)n^\mu + i(D_c^\mu)_\perp$
- $i(n \cdot D) = i(n \cdot \partial) + g(n \cdot A_{us}^a(x_-)) + g(n \cdot A_c^a(x))$
- $igF_{\mu\nu}^{us} = \left[iD_\mu^{us}, iD_\nu^{us}\right]$; $igF_{\mu\nu}^{c} = [iD_\mu, iD_\nu]$

Wilson lines

We define an **infinite** Wilson line for each of both fields which form SCET:

- $W_c(x) \equiv [x, -\infty\bar{n}] = \mathbf{P}\exp\left[ig\int_{-\infty}^{0} ds\ \bar{n}\cdot A_c(x+s\bar{n})\right]$ (collinear)
- $S_n(x) = \mathbf{P}\exp\left[ig\int_{-\infty}^{0} ds\ n\cdot A_{us}(x+sn)\right]$ (ultra-soft)

$$W(x) \to V(x)W(x)V^\dagger(-\infty\bar{n}) \Longrightarrow \underbrace{\chi(x) \equiv W^\dagger(x)\psi(x),\ \bar{\chi}(x) \equiv \bar{\psi}(x)W(x)}_{\text{gauge invariant}}$$

Fields decoupling in SCET

Lagrangian with collinear and ultra-soft fields interaction:

$$\boxed{\mathcal{L}_{c+us} = \bar{\xi} i (n \cdot D) \xi}$$

We will consider the following transformations:

- $\xi(x) \to S_n(x_-)\xi^{(0)}(x)$,
- $A_c^{\mu}(x) \to S_n(x_-)A_c^{(0)\mu}(x)S_n^{\dagger}(x_-)$.

Thus,

$$i(n \cdot D)\xi(x) \to \; i(n \cdot D')S_n(x_-)\xi^{(0)}(x) =$$

$$= \left(in \cdot \partial + gn \cdot S_n(x_-)A_c^{(0)}(x)S_n^{\dagger}(x_-) + gn \cdot A_{us}(x_-)\right)S_n(x_-)\xi^{(0)}(x) =$$

$$= S_n(x_-)i\left(n \cdot D_c^{(0)}(x)\right)\xi^{(0)}(x).$$

Fields decoupling in SCET

We will work with **SCET II**, which implies: SCET I + soft fields.

$$\boxed{p_s + p_{SCETI} = (\lambda, \lambda, \lambda) + (\lambda, 1, \lambda^2) = (\lambda, 1, \lambda)}$$

This result does **not** match any of the variables which defines our cross section (**real modes**) and, when it comes to the Lagrangian, we do not consider the virtual modes, but the real ones $\Longrightarrow$ The collinear and soft components are independent from each other.

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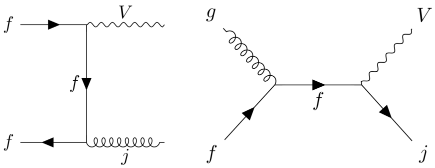
Our processes of interest:



(a) (b)

Figure 1

Aplications:

- Obtain some fundamental parameters.
- Physics beyond the SM.
- Improve our hadronic radiation knowledge.
- Study of the enviroment where the event happens: quarks and gluons plasma (QGP).

Other processes of interest

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TMD factorization for dijet and heavy-meson pair in DIS

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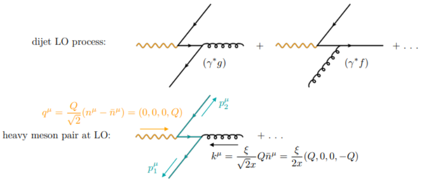

Factorization Theorem

We rewrite the cross section as

$$\int d\sigma = \int \frac{d^3\mathbf{p}_V}{2p_V^0} \int \frac{d^3\mathbf{p}_j}{2p_j^0} \int \frac{d^3\mathbf{p}_X}{2p_X^0} \tilde{\sigma} \longrightarrow d\hat{\sigma} = \frac{d\sigma}{d\eta_V d\eta_j dx_j dx_1 dx_2 d\mathbf{r}_T d\mathbf{p}_T}$$

with

$$\left.\begin{array}{l} \mathbf{r}_T = \mathbf{p}_{T,V} + \mathbf{p}_{T,j} \\ \mathbf{p}_T = \frac{\mathbf{p}_{T,V} - \mathbf{p}_{T,j}}{2} \end{array}\right\} |\mathbf{r}_T| \ll |\mathbf{p}_T| \text{, where we define } \lambda = \frac{|\mathbf{r}_T|}{|\mathbf{p}_T|}$$

Factorization Theorem

Factorized cross section:

$$d\hat{\sigma} = d\hat{\sigma}^a + d\hat{\sigma}^b + d\hat{\sigma}^c$$

- $$d\hat{\sigma}^a = \sum_f H_{ff\to\gamma j}(\hat{s},\hat{t},\hat{u},\mu)\int \frac{d^2\mathbf{b}}{(2\pi)^2} e^{i\mathbf{b}r_T} F_f(x_1,\mathbf{b},\mu,\zeta_1)\times \times F_f(x_2,\mathbf{b},\mu,\zeta_2) S_{ff}(\mathbf{b},\mu,\zeta_1,\zeta_2)\Big(\mathcal{C}_g(\mathbf{b},R,\mu) J_g(x_j,p_T,R,\mu)\Big)$$

- $$d\hat{\sigma}^b = \sum_f H^{\mu\nu}_{fg\to\gamma j}(\hat{s},\hat{t},\hat{u},\mu)\int \frac{d^2\mathbf{b}}{(2\pi)^2} e^{i\mathbf{b}r_T} F_f(x_1,\mathbf{b},\mu,\zeta_1)\times \times F_{g,\mu\nu}(x_2,\mathbf{b},\mu,\zeta_2) S_{fg}(\mathbf{b},\mu,\zeta_1,\zeta_2)\Big(\mathcal{C}_f(\mathbf{b},R,\mu) J_f(x_j,p_T,R,\mu)\Big).$$

- $$d\hat{\sigma}^c = d\hat{\sigma}^b(f\leftrightarrow g)$$

Soft function calculation

$$1a \longrightarrow \hat{S}_{ff}(\mathbf{b}) = \frac{1}{C_A C_F} \langle 0 | \mathcal{S}_v^{\dagger}(\mathbf{b}, +\infty)_{ca'} \, Tr\{ S_n(\mathbf{b}, -\infty) T^{a'} \times$$

$$\times \, S_{\bar{n}}^{\dagger}(\mathbf{b}, -\infty) S_{\bar{n}}(0, -\infty) T^a S_n^{\dagger}(0, -\infty) \} \mathcal{S}_v(0, +\infty)_{ac} | 0 \rangle$$

$$1b \longrightarrow \hat{S}_{fg}(\mathbf{b}) = \frac{1}{C_A C_F} \langle 0 | \mathcal{S}_n(\mathbf{b}, -\infty)_{ca'} \, Tr\{ S_v^{\dagger}(\mathbf{b}, +\infty) T^{a'} \times$$

$$\times \, S_{\bar{n}}^{\dagger}(\mathbf{b}, -\infty) S_{\bar{n}}(0, -\infty) T^a S_v(0, +\infty) \} \mathcal{S}_n^{\dagger}(0, -\infty)_{ac} | 0 \rangle$$

Wilson lines

$$S_v(+\infty, \xi) = P \ exp\Big[- ig \int_0^{+\infty} d\lambda \ v \cdot A(\lambda v + \xi) \Big]$$

$$S_{n(\bar{n})}(-\infty, \xi) = P \ exp\Big[ig \int_{-\infty}^{0} d\lambda \ n(\bar{n}) \cdot A(\lambda n(\bar{n}) + \xi) \ e^{\delta^+ (\delta^-) \lambda}) \Big]$$

Soft function calculation

Soft function **perturbative** expansion[1]:

$$\hat{S} = \sum_{m=0}^{\infty} a_s^m \hat{S}^{[m]} \longrightarrow \hat{S}^{[1]} = \frac{1}{2} \sum_{i \neq j} C^{ij} \hat{S}_{ij}^{[1]} \tag{3.1}$$

Coefficients for each interaction[2]:

$$C_{ff}^{n\bar{n}} = 2C_F - C_A, \qquad C_{fg}^{n\bar{n}} = C_A,$$
$$C_{ff}^{vn} = C_A, \qquad C_{fg}^{vn} = C_A,$$
$$C_{ff}^{v\bar{n}} = C_A, \qquad C_{fg}^{v\bar{n}} = 2C_F - C_A.$$

[1] $a_s = \frac{\alpha_s}{4\pi} = \frac{g^2}{(4\pi)^2}$.

[2] $C_F \equiv \frac{(N_c^2-1)}{2N_c}$ y $C_A \equiv N_c$, for $SU(N_c)$ algebras with (N_c^2-1) generators associated to $N_c \times N_c$ matrices.

Soft function calculation

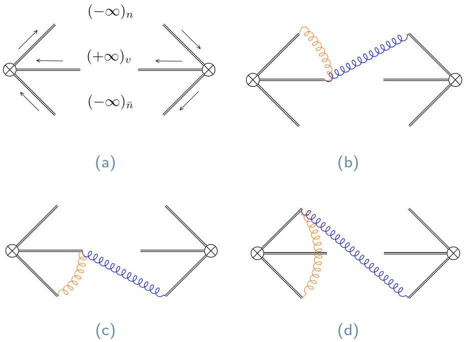

Figure 2

$$(2b) \longrightarrow \hat{S}^{[1]}_{vn} = 2(4\pi)^2(n\cdot v)(\pi I^{R}_{vn} + iI^{V}_{vn}) + h.c.$$

$$(2c) \longrightarrow \hat{S}^{[1]}_{v\bar{n}} = 2(4\pi)^2(\bar{n}\cdot v)(\pi I^{R}_{v\bar{n}} + iI^{V}_{v\bar{n}}) + h.c.$$

$$(2d) \longrightarrow \hat{S}^{[1]}_{n\bar{n}} = 2(4\pi)^2(n\cdot \bar{n})(\pi I^{R}_{n\bar{n}} + iI^{V}_{n\bar{n}}) + h.c.$$

Soft function calculation

Soft function for (1a):

$$\hat{S}_{ff}^{bare}(\mathbf{b}) = \hat{S}_{ff}^{finite}(\mathbf{b}) + 2a_s\Bigg\{ - C_A\left[\frac{1}{\epsilon^2} + \frac{1}{\epsilon}\left(\ln\left(B\mu^2 e^{\gamma_E}\right) + \ln\left(-\frac{(\mathbf{v}\cdot\mathbf{b})^2}{2v_+v_-B}\right)\right)\right] + $$
$$+ 2C_F\left[\frac{1}{\epsilon^2} + \frac{1}{\epsilon}\left(\ln(B\mu^2 e^{-\gamma_E}) - \ln(2\delta^+\delta^- B)\right)\right]\Bigg\} \text{ with } B = \left(\frac{b_\perp}{2}\right)^2$$

$$\hat{S}_{ff}^{finite}(\mathbf{b}) = 1 - 2a_s\Bigg\{ C_A\left[-\frac{\pi^2}{12} + \ln\left(B\mu^2 e^{\gamma_E}\right)\left(\ln\left(-\frac{(\mathbf{v}\cdot\mathbf{b})^2}{2v_+v_-B}\right) + \frac{1}{2}\ln\left(B\mu^2 e^{\gamma_E}\right)\right)\right] + $$
$$+ 2C_F\left[\frac{\pi^2}{12} + \ln(B\mu^2 e^{\gamma_E})\left(\ln(2\delta^+\delta^- B e^{\gamma_E}) - \frac{1}{2}\ln(B\mu^2 e^{-\gamma_E})\right)\right]\Bigg\}$$

Soft function calculation

Soft function for (1b):

$$\hat{S}^{bare}_{fg}(\mathbf{b}) = \hat{S}^{finite}_{fg}(\mathbf{b}) + 2a_s\Bigg\{C_A\left[\frac{1}{\epsilon^2} + \frac{1}{\epsilon}\left(\ln\left(B\mu^2 e^{-\gamma_E}\right) - \ln\left(\frac{v_+}{v_-}\right) - \ln(2(\delta^-)^2 B)\right)\right] - $$

$$- 2C_F\frac{1}{\epsilon}\left[\ln(\delta^+\sqrt{B}e^{\gamma_E}) + \ln\left(-i\frac{\mathbf{v}\cdot\mathbf{b}}{v_+\sqrt{B}}\right)\right]\Bigg\}$$

$$\hat{S}^{finite}_{fg}(\mathbf{b}) = 1 - 2a_s\Bigg\{2C_F\ln(B\mu^2 e^{\gamma_E})\left[\ln(\delta^+\sqrt{B}e^{\gamma_E}) + \ln\left(-i\frac{\mathbf{v}\cdot\mathbf{b}}{v_+\sqrt{B}}\right)\right] +$$

$$+ C_A\left[\frac{\pi^2}{12} + \ln(B\mu^2 e^{\gamma_E})\left(\ln(2(\delta^-)^2 B e^{\gamma_E}) + \ln\left(\frac{v_+}{v_-}\right) - \frac{1}{2}\ln(B\mu^2 e^{-\gamma_E})\right)\right]$$

TMDs definition

$$\hat{B}_f(x_1,\mathbf{b},\mu,\zeta_1)\hat{B}_i(x_2,\mathbf{b},\mu,\zeta_2)\hat{S}_{fi}(\mathbf{b},\mu,\delta^+,\delta^-) = F_f(x_1,\mathbf{b},\mu,\zeta_1)F_i(x_2,\mathbf{b},\mu,\zeta_2)S_{fi}(\mathbf{b},\mu,\zeta_1,\zeta_2)$$

- $F_f(x_1,\mathbf{b},\mu,\zeta_1) = \dfrac{B_i^{un.}(x_1,\mathbf{b},\mu,\delta^+)}{S_f^{\frac{1}{2}}(x_1,\mathbf{b},\mu,(\delta^1)^2)}$
- $F_i(x_2,\mathbf{b},\mu,\zeta_2) = \dfrac{B_i^{un.}(x_2,\mathbf{b},\mu,\delta^-)}{S_i^{\frac{1}{2}}(x_2,\mathbf{b},\mu,(\delta^2)^2)}$
- $S_{fi}(\mathbf{b},\mu,\zeta_1,\zeta_2) = \dfrac{\hat{S}_{fi}(\mathbf{b},\mu,\delta^+,\delta^-)}{S_f^{\frac{1}{2}}(\mathbf{b},\mu,(\delta^1)^2)S_i^{\frac{1}{2}}(\mathbf{b},\mu,(\delta^2)^2)}$

We fix: $\delta^1 = \dfrac{\zeta_1}{\left(p_1^+\right)^2}\delta^+,\ \delta^2 = \dfrac{\zeta_2}{\left(p_2^-\right)^2}\delta^-;\ \zeta_1\zeta_2 = \left(p_1^+\right)^2\left(p_2^-\right)^2.$

TMDs definition

Renormalized soft function for (1a):

- $$S_{ff}(\mathbf{b},\mu,\zeta_1,\zeta_2) = 1 - 2a_s\Bigg\{2C_F \ln(B\mu^2 e^{\gamma_E}) \ln\left(\frac{(p_1^+)^2(p_2^-)^2}{\zeta_1\zeta_2}\right) + \\ + C_A\left[-\frac{\pi^2}{12} + \ln\left(B\mu^2 e^{\gamma_E}\right)\left(\ln\left(-\frac{(\mathbf{v}\cdot\mathbf{b})^2}{2v_+v_-B}\right) + \frac{1}{2}\ln\left(B\mu^2 e^{\gamma_E}\right)\right)\right]\Bigg\} + \mathcal{O}(a_s^2)$$

Renormalization function for (1a):

- $$Z_{ff}^S(\mathbf{b},\mu,\zeta_1,\zeta_2) = 1 - 2a_s\Bigg\{C_A\left[\frac{1}{\epsilon^2} + \frac{1}{\epsilon}\left(\ln\left(B\mu^2 e^{\gamma_E}\right) + \ln\left(-\frac{(\mathbf{v}\cdot\mathbf{b})^2}{2v_+v_-B}\right)\right)\right] + \\ + 2C_F\frac{1}{\epsilon}\ln\left(\frac{(p_1^+)^2(p_2^-)^2}{\zeta_1\zeta_2}\right)\Bigg\} + \mathcal{O}(a_s^2)$$

TMDs definition

Renormalized soft function for (1b):

- $S_{fg}(\mathbf{b}, \mu, \zeta_1, \zeta_2) = 1 - 2a_s\Bigg\{ C_A \ln(B\mu^2 e^{\gamma_E})\left[\ln\left(\frac{v_+}{v_-}\right) + 2\ln\left(\frac{(p_2^-)^2}{\zeta_2}\right)\right] + C_F\Bigg[-\frac{\pi^2}{12} + \\ + \ln(B\mu^2 e^{\gamma_E})\left(2\ln\left(\frac{(p_1^+)^2}{\zeta_1}\right) + 2\ln\left(-i\frac{\mathbf{v}\cdot\mathbf{b}}{v_+\sqrt{2B}}\right) + \frac{1}{2}\ln(B\mu^2 e^{\gamma_E})\right)\Bigg]\Bigg\} + \mathcal{O}(a_s^2)$

Renormalization function for (1b):

- $Z^S_{fg}(\mathbf{b}, \mu, , \zeta_1, \zeta_2) = 1 - 2a_s\Bigg\{ C_A \frac{1}{\epsilon}\left[\ln\left(\frac{v_+}{v_-}\right) + 2\ln\left(\frac{(p_2^-)^2}{\zeta_2}\right)\right] + \\ + C_F\left[\frac{1}{\epsilon^2} + \frac{1}{\epsilon}\left(\ln(B\mu^2 e^{\gamma_E}) + 2\ln\left(\frac{(p_1^+)^2}{\zeta_1}\right) + 2\ln\left(-i\frac{\mathbf{v}\cdot\mathbf{b}}{v_+\sqrt{2B}}\right)\right)\right]\Bigg\} + \mathcal{O}(a_s^2)$

Soft function anomalous dimension

Soft function anomalous dimension [3] *soft*

$$\gamma_{S_{fi}} = (Z^S_{fi})^{-1} \frac{d}{d \ln \mu} Z^S_{fi}$$

We can write the **perturbative expansion** of the anomalous dimension as:

$$\boxed{\gamma = \sum_{n=1} a_s^n \gamma^{[n]}}$$

[3] Each element of the factorized cross section satisfies the RG equation: $\frac{d}{d \ln \mu} G(\mu) = \gamma_G(\mu) G(\mu)$, since they depend on the factorization scale μ.

Soft function anomalous dimension

$$(1a) \longrightarrow \bullet\ \gamma^{[1]}_{S_{ff}} = -4a_s\left\{2C_F \ln\left(\frac{(p_1^+)^2(p_2^-)^2}{\zeta_1\zeta_2}\right) + \right.$$
$$\left. + C_A\left[\ln\left(B\mu^2 e^{\gamma_E}\right) + \ln\left(-\frac{(\mathbf{v}\cdot\mathbf{b})^2}{2v_+v_-B}\right)\right]\right\}$$

$$(1b) \longrightarrow \bullet\ \gamma^{[1]}_{S_{fg}} = -4a_s\left\{C_F\left[\ln(B\mu^2 e^{\gamma_E}) + 2\ln\left(-i\frac{\mathbf{v}\cdot\mathbf{b}}{v_+\sqrt{2B}}\right) + \right.\right.$$
$$\left.\left. + 2\ln\left(\frac{(p_1^+)^2}{\zeta_1}\right)\right] + C_A\left[\ln\left(\frac{v_+}{v_-}\right) + 2\ln\left(\frac{(p_2^-)^2}{\zeta_2}\right)\right]\right\}$$

Contents

CONCLUSIONS

- For the full calculation, we should also consider:
  - All the TMDs involved in the process $\longrightarrow$ **gluon** and **quark** TMDs (polarized/non-polarized up to NNLO and N^3LO).
  - Jet structure $\longrightarrow$ *Winner-Take-All* (WTA) scheme: 4-momentum null mass limit and it points towards the harder particle direction of the pair.
- Our initial solution for the soft function is compatible with Ec. (3.29) obtained in *JOURNAL OF HIGH ENERGY PHYSICS*, Y. CHIEN *ET AL.* [2].
- As in *JOURNAL OF HIGH ENERGY PHYSICS*, Y. CHIEN *ET AL.* [3], the final results have a dependence on the angle $\widehat{\mathbf{v} \cdot \mathbf{b}}$.
- Aim: To estimate the **non-perturbative** effects of QCD in the TMDs factorization formalism.

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THANK YOU!

SCET effective Lagrangian

$$\not{n}\xi = \bar{\xi}\not{n} = 0,\ \not{\bar{n}}\eta = \bar{\eta}\not{\bar{n}} = 0,\ \bar{\xi}\not{D}_\perp\xi = 0 \text{ y } \bar{\eta}\not{D}_\perp\eta = 0\ (\{\not{n}, \not{D}_\perp\} = \{\not{\bar{n}}, \not{D}_\perp\} = 0)$$

Collinear Lagrangian

$$\mathcal{L}_c = \bar{\psi}_c i\not{D}\psi_c = (\bar{\xi} + \bar{\eta})i\left[(\bar{n}\cdot D)\not{n} + (n\cdot D)\not{\bar{n}} + \not{D}_\perp\right](\xi + \eta)$$

Through Euler-Lagrange equations:

$$\eta = -\frac{\not{\bar{n}}}{(\bar{n}\cdot D)}\not{D}_\perp\xi\ ; \qquad \bar{\eta} = -\bar{\xi}\not{D}_\perp\frac{\not{\bar{n}}}{(\bar{n}\cdot D)}.$$

$$\mathcal{L}_c = \bar{\xi}i(n\cdot D)\not{\bar{n}}\xi + \bar{\xi}i\not{D}_\perp\frac{1}{i2(\bar{n}\cdot D)}i\not{D}_\perp\not{\bar{n}}\xi$$

Wilson lines

Finite Wilson line

$$[z, y]_A \equiv \mathbf{P}\exp\left[ig\int_{s_y}^{s_z} ds \frac{dx^\mu}{ds} A_\mu\left(x(s)\right)\right] \text{ with } y \equiv x(s_y),\ z \equiv x(s_z).$$

Gauge transformation: $V(x) = e^{i\alpha(x)} \Longrightarrow A_\mu \to A'_\mu(x) = A_\mu(x) + \frac{1}{g}\partial_\mu \alpha(x)$

Finite Wilson lines

$$[z, y]_A \to [z, y]_{A'}$$
$$= exp\left[ig\int_{s_y}^{s_z} ds\frac{dx^\mu}{ds}A_\mu\left(x(s)\right) + i\int_{s_y}^{s_z} ds\frac{dx^\mu}{ds}\partial_\mu\alpha\left(x(s)\right)\right]$$
$$= exp\left[ig\int_{s_y}^{s_z} ds\frac{dx^\mu}{ds}A_\mu\left(x(s)\right) + i\alpha(z) - i\alpha(y)\right]$$
$$= V(z)\,[z, y]_A\,V^\dagger(y)$$

Wilson lines

Finite Wilson lines

$$[x + s\bar{n}, x] \equiv \mathbf{P}\exp\left[ig \int_0^s ds' \; \bar{n} \cdot A(x + s'\bar{n})\right]$$

$$[x + s\bar{n}, x] \to V(x + s\bar{n})\,[x + s\bar{n}, x]\,V^\dagger(x) \Longrightarrow \underbrace{\bar{\psi}(x + s\bar{n})\,[x + s\bar{n}, x]\,\psi(x)}_{\text{gauge inveriant}}$$

Factorization toy model

$$\langle 0 \,| J^\mu(x) J^\nu(0) |\, p_1 p_2 \rangle \longrightarrow \langle 0 \left| \overline{\psi}_{\bar{n}} \gamma^\mu \psi_n(x) \overline{\psi}_n \gamma^\nu \psi_{\bar{n}}(0) \right| p_1 p_2 \rangle$$

FIERZING: $\langle 0 | \overline{\chi}^a \gamma^\mu_{ab} \eta^b(x) \overline{\eta}^c \gamma^\nu_{cd} \chi^d(0) | p_1 p_2 \rangle$

If we consider: $\boxed{\gamma^\mu_{ab} \gamma^\nu_{cd} = \sum_\Gamma c_\Gamma \gamma_{ad} \gamma_{cb}}$

$$\Longrightarrow \langle 0 | \textstyle\sum_\Gamma c_\Gamma \left(\overline{\chi}^a(x) \gamma^\mu_{ad} \chi^d(0) \right) \left(\overline{\eta}^c(0) \gamma^\nu_{cb} \eta^b(x) \right) | p_1 p_2 \rangle$$

$$\Longrightarrow \langle 0 | \textstyle\sum_\Gamma c_\Gamma \left(\overline{\psi}_{\bar{n}}(x) \gamma^\mu \psi_{\bar{n}}(0) \right) \left(\overline{\psi}_n(0) \gamma^\nu \psi_n(x) \right) | p_1 p_2 \rangle$$

Factorization toy model

Gauge transformation:

- $\psi_{\bar{n}} \to S_{\bar{n}}\xi_{\bar{n}}$
- $\overline{\psi}_{\bar{n}} \to \overline{\xi}_{\bar{n}}S^{\dagger}_{\bar{n}}$

$$\sum_{\Gamma} c_{\Gamma} \langle 0|S^{\dagger}_{\bar{n}}S_{n}(x)S^{\dagger}_{n}S_{\bar{n}}(0)|0\rangle \times$$

$$\times \langle 0|\overline{\xi}_{n}(0)\gamma^{\nu}\xi_{n}(x)|p_1\rangle \langle 0|\overline{\xi}_{\bar{n}}(x)\gamma^{\mu}\xi_{\bar{n}}(0)|p_2\rangle$$