Higher Twist Proton Distributions in the Light-Front Quark-Diquark Model

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in collaboration with

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Light-Front Quark-Diquark Model

TMD Correlator and Parameterization

Result Discussion

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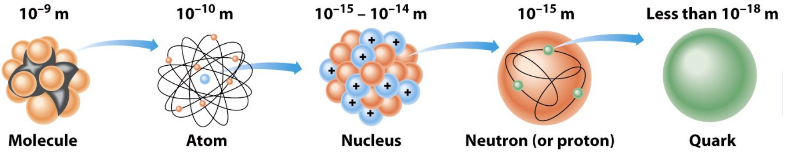
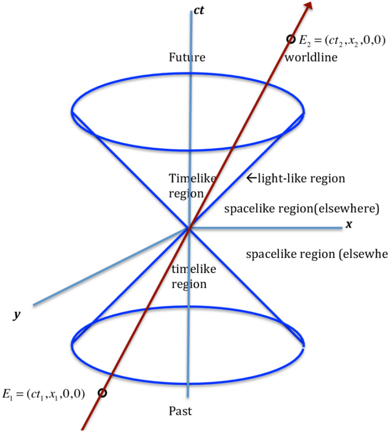
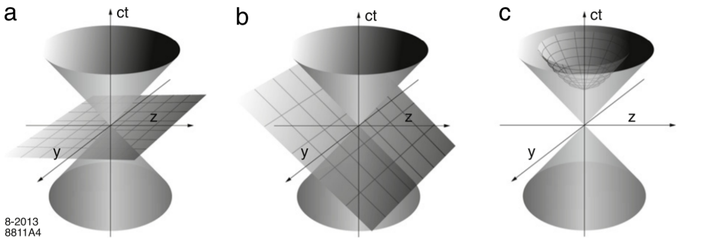
Introduction



- The theory of the strong interaction which provides the fundamental description of hadronic structure and dynamics in terms of their elementary quarks and gluons degrees of freedom is Quantum Chromodynamics (QCD).
- The foremost problem of hadron physics is to unravel the internal structure of hadron.

- From Special Theory of Relativity:
  - Space and time independently are not invariant quantities.
  - Rather space-time is an invariant object.





(a) the instant form, (b) the front form, (c) the point form.

Their initial surfaces are

a) $x^0 = 0$
b) $x^0 + x^3 = 0$
c) $x^2 = a^2 > 0, x^0 > 0$

- **S. J. Brodsky, G. F. de Teramond, H. G. Dosch and J. Erlich, Phys. Rept. 584, (2015)**

Why Light Front?

- It is an Ideal Framework to describe theoretically the hadronic structure in terms of quarks and gluons. It can overcome many obstacles and has many advantages:
  - Simple vacuum structure $\sim \langle 0|0\rangle = 0$
  - A dynamical system is characterized by ten fundamental quantities: energy, momentum, angular momentum and boost.
    $\sim$ seven out of which are kinematical. It allows unambiguous definition of the partonic content of a hadron, exact formulae for form factors, physics of angular momentum of constituents.
  - Dispersion Relation (for ON shell particles)

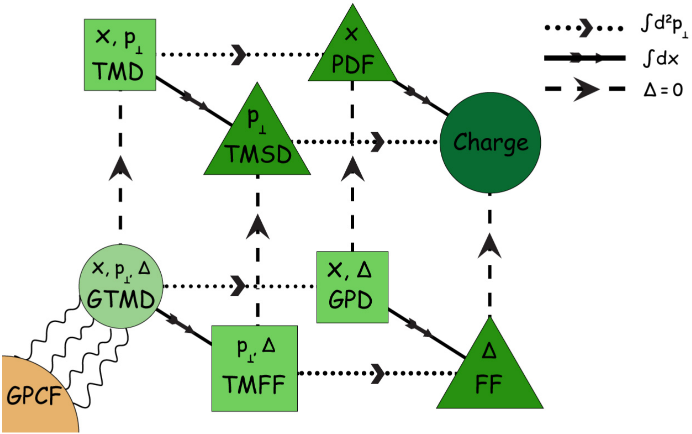
$$k^- = \frac{(k\perp)^2 + m^2}{k^+}$$

$\sim$ no square root factor.

Light-Front Coordinates

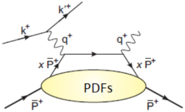
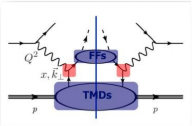
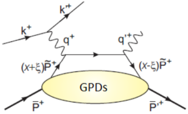
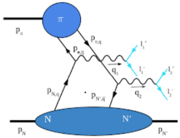
- A generic four Vector x^μ in light-cone coordinates is describe as $x^\mu = (x^-, x^+, x_\perp)$.
- $x^+ = x^0 + x^3$ is called as light-front time.
- $x^- = x^0 - x^3$ is called as light-front longitudinal space variable.
- $x^\perp = (x^1, x^2)$ is the transverse variable.
- Similarly, we can define the longitudinal momentum $k^+ = k^0 + k^3$ and light-front energy $k^- = k^0 - k^3$.

Distribution Functions

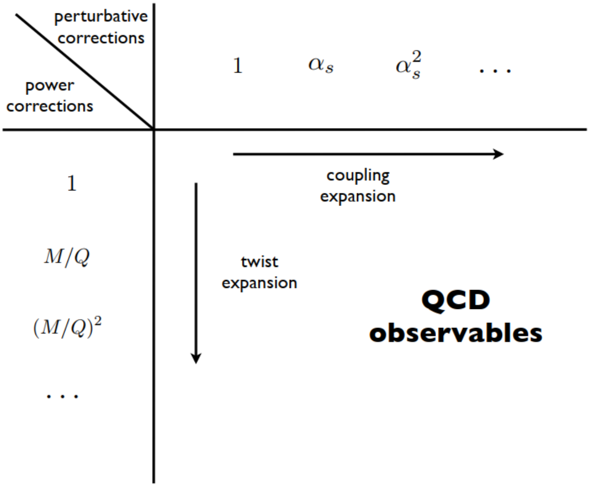


- S. Sharma and H. Dahiya, Eur. Phys. J. A 59, 235 (2023)

Experiments related with Distribution Functions

PDFs	TMDs	GPDs	GTMDs
1-D information	3-D information	3-D information	6-D information
			
DIS	SIDIS	DVCS	DDY

Twist?



- https://inspirehep.net/literature/1493030

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Light-Front Quark-Diquark Model

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Light-Front Quark-Diquark Model

- In this model the proton is described as an aggregate of an active quark and a diquark spectator of definite mass.

- The proton has spin-flavor $SU(4)$ structure and it has been expressed as a made up of isoscalar-scalar diquark singlet $|u\ S^0\rangle$, isoscalar-vector diquark $|u\ A^0\rangle$ and isovector-vector diquark $|d\ A^1\rangle$ states as

$$|P;\pm\rangle = C_S|u\ S^0\rangle^{\pm} + C_V|u\ A^0\rangle^{\pm} + C_{VV}|d\ A^1\rangle^{\pm}.$$

Here, the scalar and vector diquark has been denoted by S and A respectively. Their isospin has been represented by the superscripts on them.

- *T. Maji and D. Chakrabarti, Phys. Rev. D 94, 094020 (2016)*

Light-Front Quark-Diquark Model

- For the scalar $|\nu\ S\rangle^{\pm}$ and vector diquark $|\nu\ A\rangle^{\pm}$ case, the expansion of the Fock-state in two particles for $J^z = \pm 1/2$ can be specified as

$$|\nu\ S\rangle^{\pm} = \sum_{\lambda^q} \int \frac{dx\ d^2\mathbf{p}_\perp}{2(2\pi)^3\sqrt{x(1-x)}} \psi^{\pm(\nu)}_{\lambda^q}(x, \mathbf{p}_\perp) \Big| \lambda^q, \lambda^s; xP^+, \mathbf{p}_\perp \Big\rangle,$$

$\lambda^S = 0$ (singlet)

$$|\nu\ A\rangle^{\pm} = \sum_{\lambda^q} \sum_{\lambda^A} \int \frac{dx\ d^2\mathbf{p}_\perp}{2(2\pi)^3\sqrt{x(1-x)}} \psi^{\pm(\nu)}_{\lambda^q \lambda^A}(x, \mathbf{p}_\perp) \Big| \lambda^q, \lambda^A; xP^+, \mathbf{p}_\perp \Big\rangle.$$

$\lambda^A = \pm 1, 0$ (triplet)

Light-Front Quark-Diquark Model

	λ^q	λ^{Sp}	LFWFs for $J^z = +1/2$	LFWFs for $J^z = -1/2$
S	$+1/2$	0	$\psi_{+}^{+(\nu)} = N_S\ \varphi_1^{(\nu)}$	$\psi_{+}^{-(\nu)} = N_S\left(\frac{p^1-ip^2}{xM}\right)\varphi_2^{(\nu)}$
	$-1/2$	0	$\psi_{-}^{+(\nu)} = -\ N_S\left(\frac{p^1+ip^2}{xM}\right)\varphi_2^{(\nu)}$	$\psi_{-}^{-(\nu)} = N_S\ \varphi_1^{(\nu)}$
A	$+1/2$	$+1$	$\psi_{+\,+}^{+(\nu)} = N_1^{(\nu)}\sqrt{\frac{2}{3}}\left(\frac{p^1-ip^2}{xM}\right)\varphi_2^{(\nu)}$	$\psi_{+\,+}^{-(\nu)} = 0$
	$-1/2$	$+1$	$\psi_{-\,+}^{+(\nu)} = N_1^{(\nu)}\sqrt{\frac{2}{3}}\ \varphi_1^{(\nu)}$	$\psi_{-\,+}^{-(\nu)} = 0$
	$+1/2$	0	$\psi_{+\,0}^{+(\nu)} = -\ N_0^{(\nu)}\sqrt{\frac{1}{3}}\ \varphi_1^{(\nu)}$	$\psi_{+\,0}^{-(\nu)} = N_0^{(\nu)}\sqrt{\frac{1}{3}}\left(\frac{p^1-ip^2}{xM}\right)\varphi_2^{(\nu)}$
	$-1/2$	0	$\psi_{-\,0}^{+(\nu)} = N_0^{(\nu)}\sqrt{\frac{1}{3}}\left(\frac{p^1+ip^2}{xM}\right)\varphi_2^{(\nu)}$	$\psi_{-\,0}^{-(\nu)} = N_0^{(\nu)}\sqrt{\frac{1}{3}}\ \varphi_1^{(\nu)}$
	$+1/2$	-1	$\psi_{+\,-}^{+(\nu)} = 0$	$\psi_{+\,-}^{-(\nu)} = -\ N_1^{(\nu)}\sqrt{\frac{2}{3}}\ \varphi_1^{(\nu)}$
	$-1/2$	-1	$\psi_{-\,-}^{+(\nu)} = 0$	$\psi_{-\,-}^{-(\nu)} = N_1^{(\nu)}\sqrt{\frac{2}{3}}\left(\frac{p^1+ip^2}{xM}\right)\varphi_2^{(\nu)}$

Light-Front Quark-Diquark Model

- Generic ansatz of LFWFs $\varphi_i^{(\nu)}(x, \mathbf{p}_\perp)$ is being adopted from the soft-wall AdS/QCD prediction and the parameters a_i^ν, b_i^ν and δ^ν are established as

$$\varphi_i^{(\nu)}(x, \mathbf{p}_\perp) = \frac{4\pi}{\kappa}\sqrt{\frac{\log(1/x)}{1-x}}x^{a_i^\nu}(1-x)^{b_i^\nu}\exp\left[-\delta^\nu\frac{\mathbf{p}_\perp^2}{2\kappa^2}\frac{\log(1/x)}{(1-x)^2}\right].$$

Input Parameters

Parameter	u	d
C_S^2	1.3872	0
C_V^2	0.6128	0
C_{VV}^2	0	1
N_S	2.0191	0
N_0^ν	3.2050	5.9423
N_1^ν	0.9895	1.1616
a_1^ν	0.280 ± 0.001	0.5850 ± 0.0003
b_1^ν	0.1716 ± 0.0051	0.7000 ± 0.0002
a_2^ν	0.84 ± 0.02	$0.9434^{+0.0017}_{-0.0013}$
b_2^ν	0.2284 ± 0.0035	$0.64^{+0.0082}_{-0.0022}$
δ^ν	1	1
κ	0.4	0.4

Plan

TMD Correlator

- The unintegrated quark-quark correlator in the light-front formalism for SIDIS is defined as

$$\Phi^{\nu[\Gamma]}_{[\Lambda^{N_i}\Lambda^{N_f}]} = \frac{1}{2}\int \frac{dz^- d^2 z_T}{2(2\pi)^3} e^{ip.z} \langle P;\Lambda^{N_f}|\overline{\psi}^{\nu}(0)\Gamma \mathcal{W}_{[0,z]}\psi^{\nu}(z)|P;\Lambda^{N_i}\rangle\Big|_{z^+=0}.$$

- $|P;\Lambda^{N_i}\rangle$ and $|P;\Lambda^{N_f}\rangle$ are the initial and final states of the proton having momentum P with helicities Λ^{N_i} and Λ^{N_f}, respectively.
- The momentum of the proton (P), struck quark (p) and diquark (P_X) are

$$\begin{aligned} P &\equiv \left(P^+, \frac{M^2}{P^+}, \mathbf{0}\right), \\ p &\equiv \left(xP^+, \frac{p^2+|\mathbf{p}_\perp|^2}{xP^+}, \mathbf{p}_\perp\right), \\ P_X &\equiv \left((1-x)P^+, P_X^-, -\mathbf{p}_\perp\right). \end{aligned}$$

- *S. Meissner, A. Metz and M. Schlegel, JHEP 08, 056 (2009)*

TMD Parameterization for proton at twist-3

$$\Phi^{[1]}_{[\Lambda^{N_i}\Lambda^{N_f}]} = \frac{1}{2P^+}\,\bar{u}(P,\Lambda^{N_F})\left[e^{\nu}\left(x,\mathbf{p}_\perp^2\right) - \frac{i\sigma^{i+}p_T^i}{P^+}e_T^{\perp\nu}\left(x,\mathbf{p}_\perp^2\right)\right]u(P,\Lambda^{N_i}),$$

$$\Phi^{[\gamma_5]}_{[\Lambda^{N_i}\Lambda^{N_f}]} = \frac{1}{2P^+}\,\bar{u}(P,\Lambda^{N_F})\left[-\frac{i\sigma^{i+}\gamma_5 p_T^i}{P^+}e_T^{\nu}\left(x,\mathbf{p}_\perp^2\right) - i\sigma^{+-}\gamma_5 e_L^{\nu}\left(x,\mathbf{p}_\perp^2\right)\right] u(P,\Lambda^{N_i}),$$

$$\Phi^{[\gamma^j]}_{[\Lambda^{N_i}\Lambda^{N_f}]} = \frac{1}{2P^+}\,\bar{u}(P,\Lambda^{N_F})\left[\frac{p_T^j}{M}f^{\perp\nu}\left(x,\mathbf{p}_\perp^2\right) + \frac{M\,i\sigma^{j+}}{P^+}f_T^{\prime\nu}\left(x,\mathbf{p}_\perp^2\right) + \frac{p_T^j\,i\sigma^{k+}p_T^k}{M\,P^+}f_T^{\perp\nu}\left(x,\mathbf{p}_\perp^2\right) + \frac{i\sigma^{ij}p_T^i}{M}f_L^{\perp\nu}\left(x,\mathbf{p}_\perp^2\right)\right]u(P,\Lambda^{N_i}),$$

$$\Phi^{[\gamma^j\gamma_5]}_{[\Lambda^{N_i}\Lambda^{N_f}]} = \frac{1}{2P^+}\,\bar{u}(P,\Lambda^{N_F})\left[\frac{i\varepsilon_T^{ij}p_T^i}{M}g^{\perp\nu}\left(x,\mathbf{p}_\perp^2\right) + \frac{M\,i\sigma^{j+}\gamma_5}{P^+}g_T^{\prime\nu}\left(x,\mathbf{p}_\perp^2\right) + \frac{p_T^j\,i\sigma^{k+}\gamma_5 p_T^k}{M\,P^+}g_T^{\perp\nu}\left(x,\mathbf{p}_\perp^2\right) + \frac{p_T^j\,i\sigma^{+-}\gamma_5}{M}g_L^{\perp\nu}\left(x,\mathbf{p}_\perp^2\right)\right]u(P,\Lambda^{N_i}),$$

T-even TMDs
T-odd TMDs

TMD Parameterization for proton at twist-3

$$\Phi^{[i\sigma^{ij}\gamma_5]}_{[\Lambda^{N_i}\Lambda^{N_f}]} = -\frac{i\varepsilon_T^{ij}}{2P^+}\,\bar{u}(P,\Lambda^{N_F})\left[-\,h^{\nu}\left(x,\mathbf{p}_\perp^2\right) + \frac{i\sigma^{k+}p_T^k}{P^+}h_T^{\perp\nu}\left(x,\mathbf{p}_\perp^2\right)\right]u(P,\Lambda^{N_i})\,,$$

$$\Phi^{[i\sigma^{+-}\gamma_5]}_{[\Lambda^{N_i}\Lambda^{N_f}]} = \frac{1}{2P^+}\,\bar{u}(P,\Lambda^{N_F})\left[\frac{i\sigma^{i+}\gamma_5 p_T^i}{P^+}h_T^{\nu}\left(x,\mathbf{p}_\perp^2\right) + i\sigma^{+-}\gamma_5 h_L^{\nu}\left(x,\mathbf{p}_\perp^2\right)\right]u(P,\Lambda^{N_i})\,.$$

T-even TMDs
T-odd TMDs

TMD Parameterization for proton at twist-4

$$\Phi^{[\gamma^-]}_{[\Lambda^{N_i}\Lambda^{N_f}]} = \frac{M}{2(P^+)^2}\,\bar{u}(P,\Lambda^{N_F})\left[f_3^{\nu}\left(x,\mathbf{p}_\perp^2\right) - \frac{i\sigma^{i+}p_T^i}{P^+}\,f_{3T}^{\perp\nu}\left(x,\mathbf{p}_\perp^2\right)\right]u(P,\Lambda^{N_i})\,,$$

$$\Phi^{[\gamma^-\gamma_5]}_{[\Lambda^{N_i}\Lambda^{N_f}]} = \frac{M}{2(P^+)^2}\,\bar{u}(P,\Lambda^{N_F})\left[\frac{i\sigma^{i+}\gamma_5 p_T^i}{P^+}\,g_{3T}^{\nu}\left(x,\mathbf{p}_\perp^2\right) + i\sigma^{+-}\gamma_5\,g_{3L}^{\nu}\left(x,\mathbf{p}_\perp^2\right)\right] u(P,\Lambda^{N_i})\,,$$

$$\Phi^{[i\sigma^{j-}\gamma_5]}_{[\Lambda^{N_i}\Lambda^{N_f}]} = \frac{M}{2(P^+)^2}\,\bar{u}(P,\Lambda^{N_F})\left[\frac{i\varepsilon_T^{ij}p_T^i}{M}\,h_3^{\perp\nu}\left(x,\mathbf{p}_\perp^2\right) + \frac{M\,i\sigma^{j+}\gamma_5}{P^+}\,h_{3T}^{\nu}\left(x,\mathbf{p}_\perp^2\right) + \frac{p_T^j\,i\sigma^{k+}\gamma_5 p_T^k}{M\,P^+}\,h_{3T}^{\perp\nu}\left(x,\mathbf{p}_\perp^2\right) + \frac{p_T^j\,i\sigma^{+-}\gamma_5}{M}\,h_{3L}^{\perp\nu}\left(x,\mathbf{p}_\perp^2\right)\right]u(P,\Lambda^{N_i})\,.$$

T-even TMDs
T-odd TMDs

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TMD parameterization at higher twist

Γ	$\Phi^{[\Gamma]}_{++} + \Phi^{[\Gamma]}_{--}$	$\Phi^{[\Gamma]}_{++} - \Phi^{[\Gamma]}_{--}$	$\Phi^{[\Gamma]}_{-+} + \Phi^{[\Gamma]}_{+-}$	$\Phi^{[\Gamma]}_{-+} - \Phi^{[\Gamma]}_{+-}$
1	$\frac{2M}{P^+} e$	0	$\frac{-2i\mathbf{p}_y}{P^+} e_T^\perp$	$\frac{2\mathbf{p}_x}{P^+} e_T^\perp$
γ_5	0	$\frac{-4M}{P^+} e_L$	$\frac{-2\mathbf{p}_x}{P^+} e_T$	$\frac{2i\mathbf{p}_y}{P^+} e_T$
γ^1	$\frac{2\mathbf{p}_x}{P^+} f^\perp$	$\frac{2i\mathbf{p}_y}{P^+} f_L^\perp$	$\frac{2i\mathbf{p}_x\mathbf{p}_y}{MP^+} f_T^\perp$	$\frac{-2M}{P^+}\left[f_T' + \frac{\mathbf{p}_x^2}{M^2} f_T^\perp\right]$
γ^2	$\frac{2\mathbf{p}_y}{P^+} f^\perp$	$\frac{2i\mathbf{p}_x}{P^+} f_L^\perp$	$\frac{2iM}{P^+}\left[f_T' + \frac{\mathbf{p}_y^2}{M^2} f_T^\perp\right]$	$\frac{-2i\mathbf{p}_x\mathbf{p}_y}{MP^+} f_T^\perp$
$\gamma^1\gamma_5$	$\frac{-2i\mathbf{p}_y}{P^+} g^\perp$	$\frac{4\mathbf{p}_x}{P^+} g_L^\perp$	$\frac{2M}{P^+}\left[g_T' + \frac{\mathbf{p}_x^2}{M^2} g_T^\perp\right]$	$\frac{-2i\mathbf{p}_x\mathbf{p}_y}{MP^+} g_T^\perp$
$\gamma^2\gamma_5$	$\frac{2i\mathbf{p}_x}{P^+} g^\perp$	$\frac{4\mathbf{p}_y}{P^+} g_L^\perp$	$\frac{2\mathbf{p}_x\mathbf{p}_y}{MP^+} g_T^\perp$	$\frac{-2iM}{P^+}\left[g_T' + \frac{\mathbf{p}_y^2}{M^2} g_T^\perp\right]$
$i\sigma^{12}\gamma_5$	$\frac{2iM}{P^+} h$	0	$\frac{2\mathbf{p}_y}{P^+} h_T^\perp$	$\frac{2i\mathbf{p}_x}{P^+} h_T^\perp$
$i\sigma^{21}\gamma_5$	$\frac{-2iM}{P^+} h$	0	$\frac{-2\mathbf{p}_y}{P^+} h_T^\perp$	$\frac{-2i\mathbf{p}_x}{P^+} h_T^\perp$
$i\sigma^{+-}\gamma_5$	0	$\frac{4M}{(P)^2} h_L$	$\frac{2\mathbf{p}_x}{P^+} h_T$	$\frac{-2i\mathbf{p}_y}{P^+} h_T$
γ^-	$\frac{2M^2}{(P^+)^2} f_3$	0	$\frac{-2i\mathbf{p}_y M}{(P^+)^2} f_{3T}^\perp$	$\frac{2\mathbf{p}_x M}{(P^+)^2} f_{3T}^\perp$
$\gamma^-\gamma_5$	0	$\frac{4M^2}{(P^+)^2} g_{3L}$	$\frac{2\mathbf{p}_x M}{(P^+)^2} g_{3T}$	$\frac{-2i\mathbf{p}_y M}{(P^+)^2} g_{3T}$
$i\sigma^{1-}\gamma_5$	$\frac{-2i\mathbf{p}_y M}{(P^+)^2} h_3^\perp$	$\frac{4\mathbf{p}_x M}{(P^+)^2} h_{3L}^\perp$	$\frac{2M^2}{(P^+)^2}\left[h_{3T} + \frac{\mathbf{p}_x^2}{M^2} h_{3T}^\perp\right]$	$\frac{-2i\mathbf{p}_x\mathbf{p}_y}{(P^+)^2} h_{3T}^\perp$
$i\sigma^{2-}\gamma_5$	$\frac{2i\mathbf{p}_x M}{(P^+)^2} h_3^\perp$	$\frac{4\mathbf{p}_y M}{(P^+)^2} h_{3L}^\perp$	$\frac{2\mathbf{p}_x\mathbf{p}_y}{(P^+)^2} h_{3T}^\perp$	$\frac{-2iM^2}{(P^+)^2}\left[h_{3T} + \frac{\mathbf{p}_y^2}{M^2} h_{3T}^\perp\right]$

T-even TMDs
T-odd TMDs

Explicit Expressions of TMDs

$$xe^{\nu}(x,\mathbf{p}_\perp^2) = \frac{1}{16\pi^3}\left(C_S^2 N_s^2 + C_A^2\left(\frac{2}{3}|N_1^\nu|^2 + \frac{1}{3}|N_0^\nu|^2\right)\right)\frac{m}{M}\left[|\varphi_1^\nu|^2 + \frac{p_\perp^2}{x^2M^2}|\varphi_2^\nu|^2\right],$$

$$xf^{\perp\nu}(x,\mathbf{p}_\perp^2) = \frac{1}{16\pi^3}\left(C_S^2 N_s^2 + C_A^2\left(\frac{2}{3}|N_1^\nu|^2 + \frac{1}{3}|N_0^\nu|^2\right)\right)\left[|\varphi_1^\nu|^2 + \frac{p_\perp^2}{x^2M^2}|\varphi_2^\nu|^2\right],$$

$$xg_L^{\perp\nu}(x,\mathbf{p}_\perp^2) = \frac{1}{32\pi^3}\left(C_S^2 N_s^2 + C_A^2\left(-\frac{2}{3}|N_1^\nu|^2 + \frac{1}{3}|N_0^\nu|^2\right)\right)\left[|\varphi_1^\nu|^2 - \frac{p_\perp^2}{x^2M^2}|\varphi_2^\nu|^2 - \frac{2m}{xM}|\varphi_1^\nu||\varphi_2^\nu|\right],$$

$$xg_T^{'\nu}(x,\mathbf{p}_\perp^2) = \frac{1}{16\pi^3}\left(C_S^2 N_s^2 - \frac{1}{3}C_A^2|N_0^\nu|^2\right)\frac{m}{M}\left[|\varphi_1^\nu|^2 + \frac{p_\perp^2}{x^2M^2}|\varphi_2^\nu|^2\right],$$

$$xg_T^{\perp\nu}(x,\mathbf{p}_\perp^2) = \frac{1}{8\pi^3}\left(C_S^2 N_s^2 - \frac{1}{3}C_A^2|N_0^\nu|^2)\right)\left[\frac{1}{x}|\varphi_1^\nu||\varphi_2^\nu| - \frac{m}{x^2M}|\varphi_2^\nu|^2\right],$$

Explicit Expressions of TMDs

$$xh_L^\nu(x,\mathbf{p}_\perp^2) = \frac{1}{16\pi^3}\left(C_S^2 N_s^2 + C_A^2\left(-\frac{2}{3}|N_1^\nu|^2 + \frac{1}{3}|N_0^\nu|^2\right)\right)\frac{1}{M}\left[m\left(|\varphi_1^\nu|^2 - \frac{p_\perp^2}{x^2M^2}|\varphi_2^\nu|^2\right) + \frac{2p_\perp^2}{xM}|\varphi_1^\nu||\varphi_2^\nu|\right],$$

$$xh_T^\nu(x,\mathbf{p}_\perp^2) = \frac{1}{8\pi^3}\left(-C_S^2 N_s^2 + \frac{1}{3}C_A^2|N_0^\nu|^2\right)\left[|\varphi_1^\nu|^2 - \frac{p_\perp^2}{x^2M^2}|\varphi_2^\nu|^2 - \frac{2m}{xM}|\varphi_1^\nu||\varphi_2^\nu|\right],$$

$$xh_T^{\perp\nu}(x,\mathbf{p}_\perp^2) = \frac{1}{16\pi^3}\left(C_S^2 N_s^2 - \frac{1}{3}C_A^2|N_0^\nu|^2\right)\left[|\varphi_1^\nu|^2 + \frac{p_\perp^2}{x^2M^2}|\varphi_2^\nu|^2\right].$$

- **S. Sharma**, *N. Kumar and H. Dahiya, Nucl. Phys. B (2023)*

Explicit Expressions of TMDs

$$x^2 f_3^{\nu}(x, \mathbf{p}_\perp^2) = \frac{1}{16\pi^3}\left(C_S^2 N_s^2 + C_A^2\left(\frac{2}{3}|N_1^{\nu}|^2 + \frac{1}{3}|N_0^{\nu}|^2\right)\right)\left(\frac{p_\perp^2 + m^2}{M^2}\right)\left[|\varphi_1^{\nu}|^2 + \frac{p_\perp^2}{x^2 M^2}|\varphi_2^{\nu}|^2\right],$$

$$x^2 g_{3L}^{\nu}(x, \mathbf{p}_\perp^2) = \frac{1}{32\pi^3}\left(C_S^2 N_s^2 + C_A^2\left(-\frac{2}{3}|N_1^{\nu}|^2 + \frac{1}{3}|N_0^{\nu}|^2\right)\right)\Bigg[(p_\perp^2 - m^2)\left[|\varphi_1^{\nu}|^2 - \frac{p_\perp^2}{x^2 M^2}|\varphi_2^{\nu}|^2\right] - \frac{4mp_\perp^2}{xM}|\varphi_1^{\nu}||\varphi_2^{\nu}|\Bigg],$$

$$x^2 h_{3L}^{\perp\nu}(x, \mathbf{p}_\perp^2) = \frac{1}{16\pi^3 M}\left(C_S^2 N_s^2 + C_A^2\left(-\frac{2}{3}|N_1^{\nu}|^2 + \frac{1}{3}|N_0^{\nu}|^2\right)\right)\Bigg[m\left[|\varphi_1^{\nu}|^2 - \frac{p_\perp^2}{x^2 M^2}|\varphi_2^{\nu}|^2\right] + \frac{(p_\perp^2 - m^2)}{xM}|\varphi_1^{\nu}||\varphi_2^{\nu}|\Bigg],$$

Explicit Expressions of TMDs

$$x^2 g_{3T}^{\nu}(x, \mathbf{p}_\perp^2) = \frac{1}{8\pi^3 M}\left(C_S^2 N_s^2 - \frac{1}{3} C_A^2 |N_0^\nu|^2\right)\Bigg[m\left[|\varphi_1^\nu|^2 - \frac{p_\perp^2}{x^2 M^2}|\varphi_2^\nu|^2\right] + \frac{(p_\perp^2 - m^2)}{xM}|\varphi_1^\nu||\varphi_2^\nu|\Bigg],$$

$$x^2 h_{3T}^{\nu}(x, \mathbf{p}_\perp^2) = \frac{1}{16\pi^3}\left(C_S^2 N_s^2 - \frac{1}{3} C_A^2 |N_0^\nu|^2\right)\left(\frac{p_\perp^2 + m^2}{M^2}\right)\left[|\varphi_1^\nu|^2 + \frac{p_\perp^2}{x^2 M^2}|\varphi_2^\nu|^2\right],$$

$$x^2 h_{3T}^{\perp\nu}(x, \mathbf{p}_\perp^2) = -\frac{1}{8\pi^3}\left(C_S^2 N_s^2 - \frac{1}{3} C_A^2 |N_0^\nu|^2\right)\left[|\varphi_1^\nu|^2 + \frac{m^2}{x^2 M^2}|\varphi_2^\nu|^2 - \frac{2m}{xM}|\varphi_1^\nu||\varphi_2^\nu|\right].$$

- **S. Sharma** *and H. Dahiya, Int. J. Mod. Phys. A 37, 2250205 (2022)*

Relations between twist-2 and twist-3

$$xe^q(x,\mathbf{p}_\perp^2) = x\tilde{e}^q(x,\mathbf{p}_\perp^2) + \frac{m}{M}\, f_1^q(x,\mathbf{p}_\perp^2),$$

$$xf^{\perp q}(x,\mathbf{p}_\perp^2) = x\tilde{f}^{\perp q}(x,\mathbf{p}_\perp^2) + f_1^q(x,\mathbf{p}_\perp^2),$$

$$xg_L^{\perp q}(x,\mathbf{p}_\perp^2) = x\tilde{g}_L^{\perp q}(x,\mathbf{p}_\perp^2) + g_1^q(x,\mathbf{p}_\perp^2) + \frac{m}{M}\, h_{1L}^{\perp q}(x,\mathbf{p}_\perp^2),$$

$$xg_T^{\prime q}(x,\mathbf{p}_\perp^2) = x\tilde{g}_T^{\prime q}(x,\mathbf{p}_\perp^2) + \frac{m}{M} h_1^q(x,\mathbf{p}_\perp^2) - \frac{m}{M}\,\frac{p_\perp^2}{M^2}\, h_{1T}^{\perp q}(x,\mathbf{p}_\perp^2),$$

$$xg_T^{\perp q}(x,\mathbf{p}_\perp^2) = x\tilde{g}_T^{\perp q}(x,\mathbf{p}_\perp^2) + g_{1T}^{\perp q}(x,\mathbf{p}_\perp^2) + \frac{m}{M}\, h_{1T}^{\perp q}(x,\mathbf{p}_\perp^2),$$

$$\frac{xh_L^q(x,\mathbf{p}_\perp^2)}{2} = x\tilde{h}_L^q(x,\mathbf{p}_\perp^2) - \frac{p_\perp^2}{M^2}\, h_{1L}^{\perp q}(x,\mathbf{p}_\perp^2) + \frac{m}{M}\, g_1^q(x,\mathbf{p}_\perp^2),$$

$$\frac{xh_T^q(x,\mathbf{p}_\perp^2)}{2} = x\tilde{h}_T^q(x,\mathbf{p}_\perp^2) - h_1^q(x,\mathbf{p}_\perp^2) - \frac{p_\perp^2}{2M^2}\, h_{1T}^{\perp q}(x,\mathbf{p}_\perp^2) + \frac{m}{2M}\, g_{1T}^{\perp q}(x,\mathbf{p}_\perp^2),$$

$$xh_T^{\perp q}(x,\mathbf{p}_\perp^2) = x\tilde{h}_T^{\perp q}(x,\mathbf{p}_\perp^2) + h_1^q(x,\mathbf{p}_\perp^2) - \frac{p_\perp^2}{2M^2}\, h_{1T}^{\perp q}(x,\mathbf{p}_\perp^2).$$

Relations between twist-2 and twist-4

$$x^2\ f_3(x, \mathbf{p}_\perp^2) = \left(\frac{p_\perp^2 + m^2}{M^2} \right) f_1^\nu(x, \mathbf{p}_\perp^2),$$

$$x^2\ h_{3T}^\nu(x, \mathbf{p}_\perp^2) = \left(\frac{p_\perp^2 + m^2}{M^2} \right) h_{1T}^\nu(x, \mathbf{p}_\perp^2),$$

$$x^2\ h_{3L}^{\nu\perp}(x, \mathbf{p}_\perp^2) = -\left(\frac{p_\perp^2 - m^2}{M^2} \right) h_{1L}^{\nu\perp}(x, \mathbf{p}_\perp^2) + \frac{2m}{M} g_{1L}^\nu(x, \mathbf{p}_\perp^2),$$

$$x^2\ g_{3L}^\nu(x, \mathbf{p}_\perp^2) = \left(\frac{p_\perp^2 - m^2}{M^2} \right) g_{1L}^\nu(x, \mathbf{p}_\perp^2) + \frac{2mp_\perp^2}{M^3} h_{1L}^{\nu\perp}(x, \mathbf{p}_\perp^2),$$

$$x^2\ h_{3T}^{\nu\perp}(x, \mathbf{p}_\perp^2) = \frac{m^2}{M^2} h_{1T}^{\nu\perp}(x, \mathbf{p}_\perp^2) - 2\ h_1^\nu(x, \mathbf{p}_\perp^2) + \frac{2m}{M}\ g_{1T}^\nu(x, \mathbf{p}_\perp^2),$$

$$x^2\ g_{3T}^\nu(x, \mathbf{p}_\perp^2) = \left(\frac{p_\perp^2 - m^2}{M^2} \right) g_{1T}^\nu(x, \mathbf{p}_\perp^2) + \frac{mp_\perp^2}{M^3} h_{1T}^{\nu\perp}(x, \mathbf{p}_\perp^2) + \frac{2m}{M}\ h_1^\nu(x, \mathbf{p}_\perp^2).$$

TMD Amplitude matrix for scalar diquark

- **S. Sharma** *et al.* , *arXiv:2405.13727 [hep-ph] (2024)*

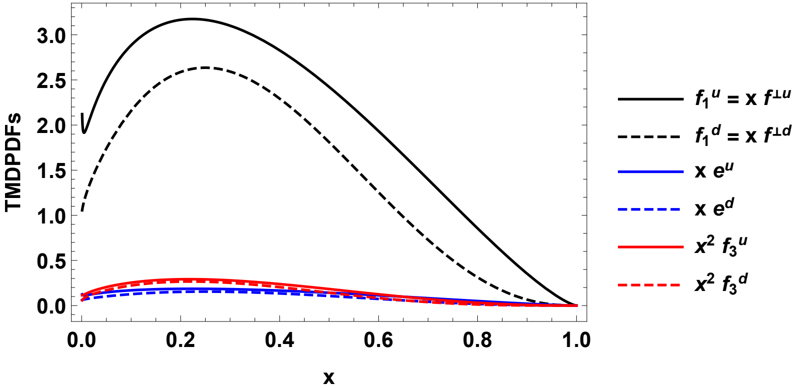
$$A^{\lambda_{Sp}} = \begin{bmatrix} A^{\lambda_{Sp}}_{++,++} & A^{\lambda_{Sp}}_{++,+-} & A^{\lambda_{Sp}}_{++,-+} & A^{\lambda_{Sp}}_{++,--} \\ A^{\lambda_{Sp}}_{+-,++} & A^{\lambda_{Sp}}_{+-,+-} & A^{\lambda_{Sp}}_{+-,-+} & A^{\lambda_{Sp}}_{+-,--} \\ A^{\lambda_{Sp}}_{-+,++} & A^{\lambda_{Sp}}_{-+,+-} & A^{\lambda_{Sp}}_{-+,-+} & A^{\lambda_{Sp}}_{-+,--} \\ A^{\lambda_{Sp}}_{--,++} & A^{\lambda_{Sp}}_{--,+-} & A^{\lambda_{Sp}}_{--,-+} & A^{\lambda_{Sp}}_{--,--} \end{bmatrix}$$

$$A^{\lambda_S} = \frac{1}{\frac{C_S^2}{16\pi^3}} \begin{bmatrix} h_1 & -\frac{(\mathbf{p}_x+\iota\mathbf{p}_y)}{2M}\varrho & \frac{(\mathbf{p}_x-\iota\mathbf{p}_y)}{2M}\varrho & h_1 \\ -\frac{(\mathbf{p}_x-\iota\mathbf{p}_y)}{2M}\varrho & -\frac{p_\perp^2}{2M^2}h_{1T}^\perp & \frac{(\mathbf{p}_x-\iota\mathbf{p}_y)^2}{2M^2}h_{1T}^\perp & -\frac{(\mathbf{p}_x-\iota\mathbf{p}_y)}{2M}\varrho \\ \frac{(\mathbf{p}_x+\iota\mathbf{p}_y)}{2M}\varrho & \frac{(\mathbf{p}_x+\iota\mathbf{p}_y)^2}{2M^2}h_{1T}^\perp & -\frac{p_\perp^2}{2M^2}h_{1T}^\perp & \frac{(\mathbf{p}_x+\iota\mathbf{p}_y)}{2M}\varrho \\ h_1 & -\frac{(\mathbf{p}_x+\iota\mathbf{p}_y)}{2M}\varrho & \frac{(\mathbf{p}_x-\iota\mathbf{p}_y)}{2M}\varrho & h_1 \end{bmatrix}$$

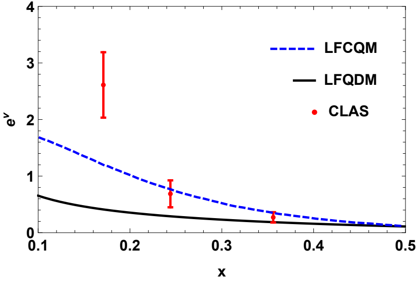
$$A^{\lambda^{Sp}}_{\Lambda^{N_f}\lambda^{q_f},\ \Lambda^{N_i}\lambda^{q_i}} = \psi^{\Lambda^{N_f}\dagger}_{\lambda^{q_f}\lambda^{Sp}}(x,\mathbf{p}_\perp)\psi^{\Lambda^{N_i}}_{\lambda^{q_i}\lambda^{Sp}}(x,\mathbf{p}_\perp)$$

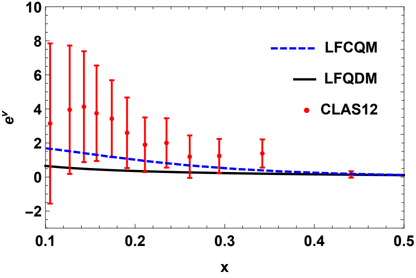
$$\varrho(say) = g_{1T} = -2h_{1L}^\perp$$

Unpolarized TMDPDFs



Comparison with Phenomenology





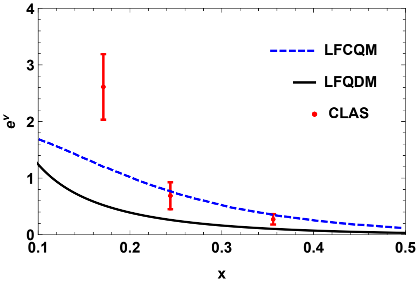
LFQDM - $0.09 GeV^2$ **S. Sharma**, *N. Kumar and H. Dahiya, Nucl. Phys. B (2023)*

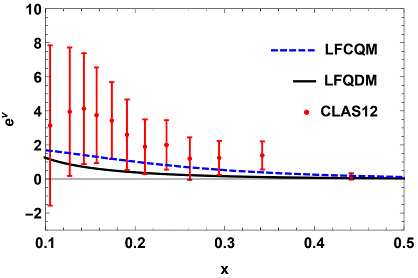
LFCQM - $1 GeV^2$ *S. Rodini and B. Pasquini, Nuovo Cim. C 42, 112 (2019)*

CLAS - $1 GeV^2$ *A. Courtoy, arXiv:1405.7659 [hep-ph] (2014)*

CLAS12 - $1 GeV^2$ *A. Courtoy et al. , Phys. Rev. D 106, 014027 (2022)*

Comparison with Phenomenology





LFQDM - $1 GeV^2$ **S. Sharma**, *N. Kumar and H. Dahiya, Nucl. Phys. B (2023)*
LFCQM - $1 GeV^2$ *S. Rodini and B. Pasquini, Nuovo Cim. C 42, 112 (2019)*
CLAS - $1 GeV^2$ *A. Courtoy, arXiv:1405.7659 [hep-ph] (2014)*
CLAS12 - $1 GeV^2$ *A. Courtoy et al. , Phys. Rev. D 106, 014027 (2022)*


thank you!

Average Transverse Momenta

- The average transverse momenta ($r = 1$) and the average square transverse momenta ($r = 2$) for TMD $\Upsilon^{\nu}(x, \mathbf{p}_{\perp}^2)$ in LFQDM is defined as

$$\langle \mathbf{p}_{\perp}^{r}(\Upsilon) \rangle^{\nu} = \frac{\int dx \int d^2 p_{\perp} p_{\perp}^{r} \Upsilon^{\nu}(x, \mathbf{p}_{\perp}^2)}{\int dx \int d^2 p_{\perp} \Upsilon^{\nu}(x, \mathbf{p}_{\perp}^2)},$$

TMD w	ν	e^{ν}	$f^{\perp\nu}$	$g_L^{\perp\nu}$	$g_T^{'\nu}$	$g_T^{\perp\nu}$	h_L^{ν}	h_T^{ν}	$h_T^{\perp\nu}$	g_T^{ν}
LFQDM	$\langle p_{\perp} \rangle^{u}$	0.99	0.99	1.25	0.99	0.88	1.23	1.25	0.99	1.17
LFQDM	$\langle p_{\perp} \rangle^{d}$	1.00	1.00	1.13	1.00	1.07	1.25	1.13	1.00	1.18
LFCQM	$\langle p_{\perp} \rangle^{\nu}$	0.92	0.92	....	....	....	....	....	....	....

LFQDM - ***S. Sharma****, N. Kumar and H. Dahiya, Nucl. Phys. B (2023)*

LFCQM - C. Lorcé, B. Pasquini and P. Schweitzer, JHEP 01, 103 (2015)

Average Transverse Momenta

TMD w	ν	e^{ν}	$f^{\perp\nu}$	$g_L^{\perp\nu}$	$g_T^{'\nu}$	$g_T^{\perp\nu}$	h_L^{ν}	h_T^{ν}	$h_T^{\perp\nu}$	g_T^{ν}
LFQDM	$\langle p_\perp^2 \rangle^u$	0.94	0.94	1.30	0.94	0.77	1.39	1.30	0.94	1.28
LFQDM	$\langle p_\perp^2 \rangle^d$	0.96	0.96	1.15	0.96	1.06	1.43	1.15	0.96	1.30
LFCQM	$\langle p_\perp^2 \rangle^{\nu}$	0.86	0.86	....	....	....	....	....	....	....
Bag Model	$\langle p_\perp^2(x_\nu)\rangle_{Gauss}$	0.68	0.94	1.11	....	1.11	1.01	1.11	0.94	0.84

LFQDM - ***S. Sharma****, N. Kumar and H. Dahiya, Nucl. Phys. B (2023)*

LFCQM - C. Lorcé, B. Pasquini and P. Schweitzer, JHEP 01, 103 (2015)

Bag Model - H. Avakian, A. Efremov, P. Schweitzer and F. Yuan, Phys. Rev. D 81, 074035 (2010)

Average Transverse Momenta

MODEL	f_3^{ν} (LFQDM)	f_3^{ν} (LFCQM)
$\langle p_\perp \rangle^u$	0.26	0.28
$\langle p_\perp \rangle^d$	0.27	0.28
$\langle p_\perp^2 \rangle^u$	0.073	0.11
$\langle p_\perp^2 \rangle^d$	0.078	0.11

LFQDM - **S. Sharma** *and H. Dahiya, Int. J. Mod. Phys. A 37, 2250205 (2022)*

LFCQM - C. Lorcé, B. Pasquini and P. Schweitzer, JHEP 01, 103 (2015)