Flavor asymmetry of light sea quarks in the proton

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Motivation: What is inside the proton?

Proton spin puzzle: Proton is a spin $\frac{1}{2}$ particle but how it is distributed ????

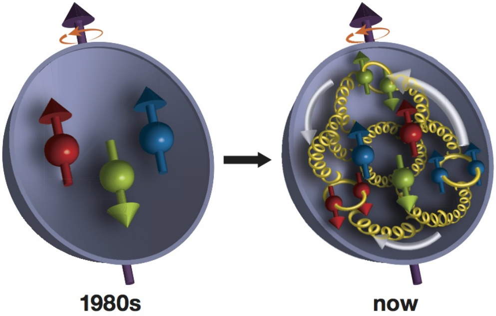


Figure: BNL

• The Valence quarks only contributes 30% to the proton spin. What about remaining 70% ?

• How much gluon and sea quark contribute to proton spin ?

Sea quarks

Fock states of the proton

$$|P\rangle = |uud\rangle, |uudg\rangle, |uudq\bar{q}\rangle + ...$$

• It is not possible to distinguish any individual down or up quark as a valence or sea quark.
• All the anti quarks in the proton belongs to sea quarks only.

$$u_{sea}(x) = \bar{u}_{sea}(x) = \bar{u}(x), \quad d_{sea}(x) = \bar{d}_{sea}(x) = \bar{d}(x)$$

Flavor symmetry: The sea are generated predominantly by gluons splitting into quark-antiquark pairs $g \rightarrow q\bar{q}$ and the splitting is flavor independent,

$$\bar{u}(x) = \bar{d}(x)$$

Flavor asymmetry in light sea quarks

- The Gottfried sum rule written in terms of proton and neutron structure functions:

$$
\begin{aligned}
S_G &= \int_0^1 \left[F_2^p(x) - F_2^n(x)\right] \frac{\mathrm{d}x}{x} \\
&= \frac{1}{3} \int_0^1 \left[u_v(x) - d_v(x)\right] \mathrm{d}x - \frac{2}{3} \int_0^1 \left[\bar{d}_p(x) - \bar{u}_p(x)\right] \mathrm{d}x \\
&= \frac{1}{3} - \frac{2}{3} \int_0^1 \left[\bar{d}_p(x) - \bar{u}_p(x)\right] \mathrm{d}x
\end{aligned}
$$

-K. Gottfried, Phys. Rev. Lett. 18 (1967) 1174.

- In light sea quark flavor symmetry: $\bar{d}_p(x) = \bar{u}_p(x)$ and $S_G = \frac{1}{3}$.

But NMC Collaboration reported a value of $S_G = 0.235 \pm 0.026$

$$\int_0^1 \left[\bar{d}_p(x) - \bar{u}_p(x)\right] \mathrm{d}x = 0.147 \pm 0.039,$$

-New Muon Collaboration,Phys.Rev.Lett. 66, 2712-2715 (1991)

$$\bar{d}_p(x) \neq \bar{u}_p(x)$$

Light front dynamics

Light-front dynamics describes how a relativistic system changes along a light-front direction.

• In light front,
LF time $x^+ = x^0 + x^3$
$x^- = x^0 - x^3$
$x^\perp = (x^1, x^2)$.
LF energy $p^- = p^0 - p^3$

• No square root in energy dispersion relation $k^2 = k^+ k^- - k_\perp^2 = m^2$.

$$H_{LF}|\psi\rangle = M^2|\psi\rangle$$

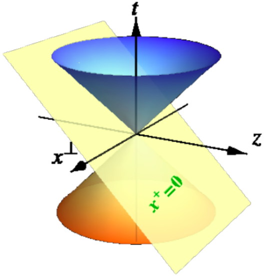


Figure: Leutwyler 1978

In LF, Solving nonperturbative QCD is equivalent to solving the Hamiltonian eigenvalue problem.

Light front wave functions (LFWFs)

- In LF, the hadron state $|\psi\rangle$ is expanded in multi-particle fock states $|n\rangle$ of free LF Hamiltonian $|\psi\rangle = \sum_n \psi_n |n\rangle$, where

$$|n\rangle = |uud\rangle, |uudg\rangle, |uudq\bar{q}\rangle$$

- LFWFs $\psi_n(x_i, k_{\perp,,i}, \lambda_i)$ depend only on the relative longitudinal, transverse momentum and spin of the parton.

- Momentum conservation $\sum_{i=1}^{n} x_i = 1, \sum_{i=1}^{n} k_{\perp,i} = 0$

- Overlap of LFWFs: PDFs, TMDs, GPDs...

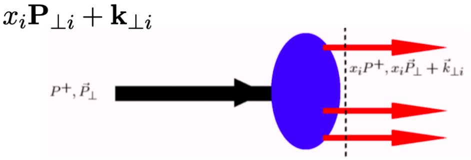
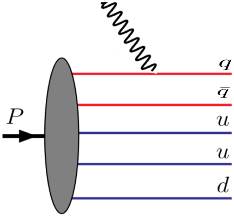


Figure: S Brodsky

LFWFs are as essential to hadron physics as DNA is to biology!
- Stanley Brodsky

Sea quarks spectator model

- Assume proton as an effective system of five particle fock state $|uudq\bar{q}\rangle$.
- Consider one of sea quark (spin-$\frac{1}{2}$) is active and rest of the system (spectator) is in spin-0 state.



$$|P;\uparrow(\downarrow)\rangle = \sum_{\bar{q}} \int \frac{\mathrm{d}x\, \mathrm{d}^2\mathbf{k}_\perp}{2(2\pi)^3\sqrt{x(1-x)}} \Big[\psi^{\uparrow(\downarrow)}_{\bar{q};+\frac{1}{2}}(x,\mathbf{k}_\perp) \big| +\frac{1}{2}, 0; xP^+, \mathbf{k}_\perp\big\rangle + \psi^{\uparrow(\downarrow)}_{\bar{q};-\frac{1}{2}}(x,\mathbf{k}_\perp) \big| -\frac{1}{2}, 0; xP^+, \mathbf{k}_\perp\big\rangle \Big].$$

- PC, D Chakrabarti ,C Mondal e-Print:**2312.01484**.

Model LFWFs

$$\psi^{\uparrow}_{\bar{q};+\frac{1}{2}}(x, \mathbf{k}_\perp) = \varphi(x, \mathbf{k}_\perp^2), \qquad \psi^{\uparrow}_{\bar{q};-\frac{1}{2}}(x, \mathbf{k}_\perp) = -\frac{k^1 + ik^2}{\kappa}\,\varphi(x, \mathbf{k}_\perp^2)\,,$$

$$\psi^{\downarrow}_{\bar{q};+\frac{1}{2}}(x, \mathbf{k}_\perp) = \frac{k^1 - ik^2}{\kappa}\,\varphi(x, \mathbf{k}_\perp^2)\,, \qquad \psi^{\downarrow}_{\bar{q};-\frac{1}{2}}(x, \mathbf{k}_\perp) = \varphi(x, \mathbf{k}_\perp^2)\,,$$

where $\varphi(x, \mathbf{k}_\perp^2)$ is the modified soft wall AdS/QCD two particle wave function solution

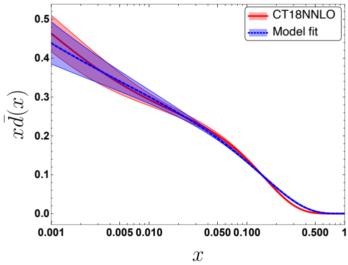
$$\varphi(x, \mathbf{k}_\perp) = \sqrt{A}\frac{4\pi}{\kappa}\sqrt{\frac{\log[1/(1-x)]}{x}}x^{\alpha}(1-x)^{3+\beta}\exp\left[-\frac{\log[1/(1-x)]}{2\kappa^2 x(1-x)}\mathbf{k}_\perp^2\right],$$

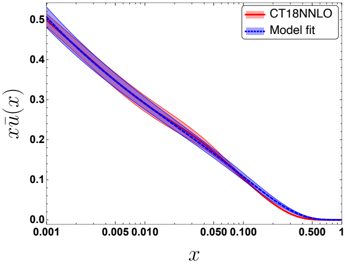
• The main idea is that at some initial scale, these functions are derived from the matching of the electromagnetic form factors of hadrons with arbitrary spin in the soft-wall AdS/QCD approach and in light-front QCD.

Fitting of model parameters

We fixed our model parameters by fitting the unpolarized PDF $f_1^{\bar{q}}(x)$ with the CTEQ18 NNLO global analysis data in $0.001 < x < 1$
Model initial scale $Q_0 = 2$ GeV.

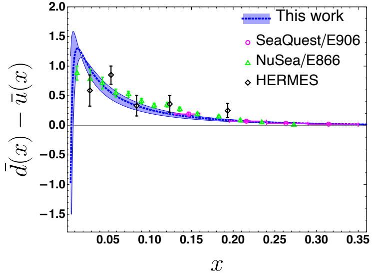
$$f_1^{\bar{q}}(x) = x\bar{q}(x) = \int \frac{\mathrm{d}^2\mathbf{k}_\perp}{16\pi^3} \left[|\psi^{\uparrow}_{\bar{q};+\frac{1}{2}}(x,\mathbf{k}_\perp)|^2 + |\psi^{\uparrow}_{\bar{q};-\frac{1}{2}}(x,\mathbf{k}_\perp)|^2 \right]$$

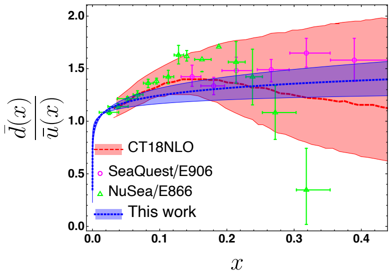




	A	α	β
$\bar{u}$	$0.055^{0.003}_{-0.002}$	$-0.611^{0.001}_{0.001}$	$-1.33^{-0.131}_{-0.181}$
$\bar{d}$	$0.082^{-0.006}_{0.007}$	$-0.572^{-0.014}_{0.016}$	$-1.33^{-0.132}_{0.163}$

Flavor asymmetries of light sea quarks



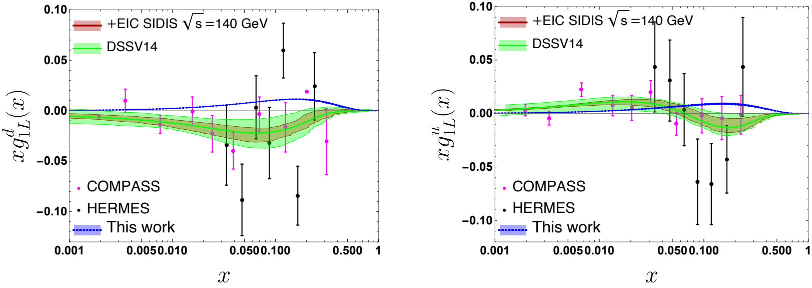


Model/Experiments	x-range	$\int \mathrm{d}x \left[\bar{d}(x) - \bar{u}(x)\right]$
This work	$0.13 < x < 0.45$	0.015 ± 0.004
SeaQuest/E906	$0.13 < x < 0.45$	0.015 ± 0.003
This work	$0.015 < x < 0.35$	0.069 ± 0.015
NuSea/E866	$0.015 < x < 0.35$	0.0803 ± 0.011
NMC	$0.004 < x < 0.80$	0.148 ± 0.039
HERMES	$0.020 < x < 0.30$	0.16 ± 0.03

- PC, D Chakrabarti ,C Mondal e-Print:**2312.01484**.

Sea quarks helicity

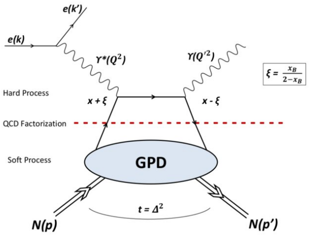
$$g_{1L}^{\bar{q}}(x) = \int \frac{\mathrm{d}^2\mathbf{k}_\perp}{16\pi^3} \left[|\psi^{\uparrow}_{\bar{q};+\frac{1}{2}}(x,\mathbf{k}_\perp)|^2 - |\psi^{\uparrow}_{\bar{q};-\frac{1}{2}}(x,\mathbf{k}_\perp)|^2 \right]$$



$$\Delta\Sigma^{\bar{d}} = \int_{0.001}^{1} \mathrm{d}x g_{1L}^{\bar{d}}(x) = 0.031 \pm 0.001\,, \quad \Delta\Sigma^{\bar{u}} = \int_{0.001}^{1} \mathrm{d}x g_{1L}^{\bar{u}}(x) = 0.026 \pm 0.002\,.$$

GPDs in terms of LFWFs

- GPDs contain important information about the nucleon mass, angular momentum and mechanical properties of the proton.



$$H^{\bar{q}}(x,-\Delta_\perp^2)=\int\frac{d^2\mathbf{k}_\perp}{16\pi^3}\left[\psi^{\dagger\uparrow}_{\bar{q};+\frac{1}{2}}(x,\mathbf{k}''_\perp)\,\psi^{\uparrow}_{\bar{q};+\frac{1}{2}}(x,\mathbf{k}'_\perp)+\psi^{\dagger\uparrow}_{\bar{q};-\frac{1}{2}}(x,\mathbf{k}''_\perp)\,\psi^{\uparrow}_{\bar{q};-\frac{1}{2}}(x,\mathbf{k}'_\perp)\right]$$

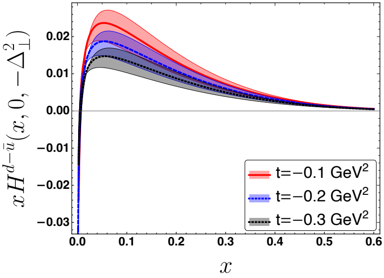
$$\tilde{H}^{\bar{q}}(x,-\Delta_\perp^2)=\int\frac{d^2\mathbf{k}_\perp}{16\pi^3}\left[\psi^{\dagger\uparrow}_{\bar{q};+\frac{1}{2}}(x,\mathbf{k}''_\perp)\,\psi^{\uparrow}_{\bar{q};+\frac{1}{2}}(x,\mathbf{k}'_\perp)-\psi^{\dagger\uparrow}_{\bar{q};-\frac{1}{2}}(x,\mathbf{k}''_\perp)\,\psi^{\uparrow}_{\bar{q};-\frac{1}{2}}(x,\mathbf{k}'_\perp)\right]$$

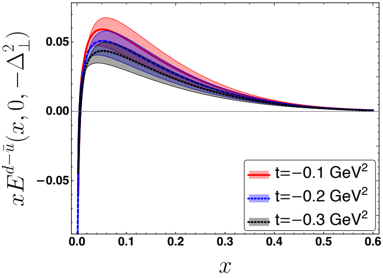
$$E^{\bar{q}}(x,-\Delta_\perp^2)=\frac{-2M_N}{\Delta_\perp^1-i\Delta_\perp^2}\int\frac{d^2\mathbf{k}_\perp}{16\pi^3}\left[\psi^{\dagger\uparrow}_{\bar{q};+\frac{1}{2}}(x,\mathbf{k}''_\perp)\psi^{\downarrow}_{\bar{q};+\frac{1}{2}}(x,\mathbf{k}'_\perp)+\psi^{\dagger\uparrow}_{\bar{q};-\frac{1}{2}}(x,\mathbf{k}''_\perp)\psi^{\downarrow}_{\bar{q};-\frac{1}{2}}(x,\mathbf{k}'_\perp)\right]$$

where, transverse momentum of the the final and initial struck quark

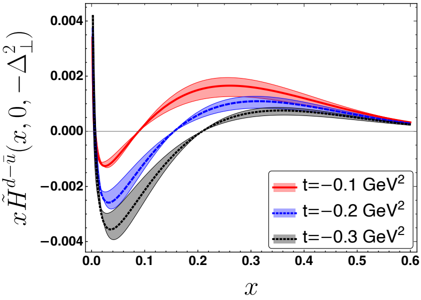
$$\mathbf{k}''_\perp=\mathbf{k}_\perp+(1-x)\frac{\Delta_\perp}{2},\qquad \mathbf{k}'_\perp=\mathbf{k}_\perp-(1-x)\frac{\Delta_\perp}{2}\,.$$

Flavor asymmetry at non-zero momentum transfer





- Electric and Magnetic asymmetries are negative in $0.001 < x < 0.005$ independent of choice of $\Delta_\perp^2$.
- The helicity distribution in our model changes its sign with x and $\Delta_\perp^2$.



Transverse momentum distributions (TMDs)

TMDs describe correlations between the transverse momentum of partons and the polarization of the partons and/or parent nucleon.

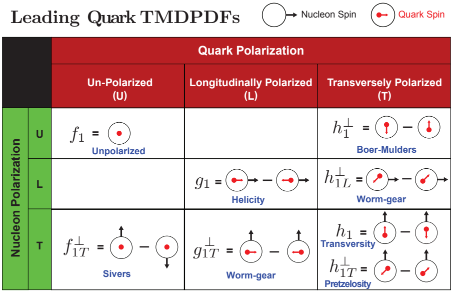


Figure: TMD handbook 2023

TMDs in terms of LFWFs:

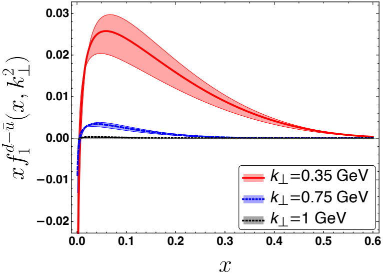
$$f_1^{\bar{q}}(x, \mathbf{k}_\perp) = \frac{1}{16\pi^3}\left[|\psi^{\uparrow}_{\bar{q};+\frac{1}{2}}(x, \mathbf{k}_\perp)|^2 + |\psi^{\uparrow}_{\bar{q};-\frac{1}{2}}(x, \mathbf{k}_\perp)|^2\right],$$

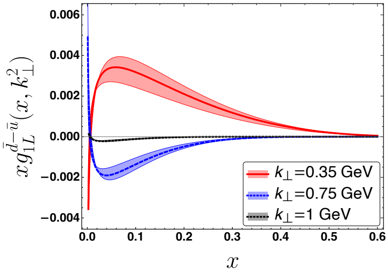
$$g_{1L}^{\bar{q}}(x, \mathbf{k}_\perp) = \frac{1}{16\pi^3}\left[|\psi^{\uparrow}_{\bar{q};+\frac{1}{2}}(x, \mathbf{k}_\perp)|^2 - |\psi^{\uparrow}_{\bar{q};-\frac{1}{2}}(x, \mathbf{k}_\perp)|^2\right],$$

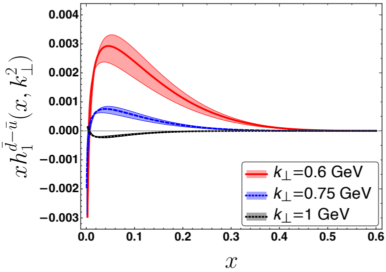
$$h_1^{\bar{q}}(x, \mathbf{k}_\perp^2) = \frac{1}{2(2\pi)^3}\psi^{\uparrow\dagger}_{\bar{q},+\frac{1}{2}}(x, \mathbf{k}_\perp)\psi^{\downarrow}_{\bar{q},-\frac{1}{2}}(x, \mathbf{k}_\perp),$$

Flavor asymmetries in terms of TMDs

- In the range $0.001 < x < 0.005$, $\bar{u} > \bar{d}$ independent of $k_\perp$.
- The sign of helicity changes with $k_\perp$.



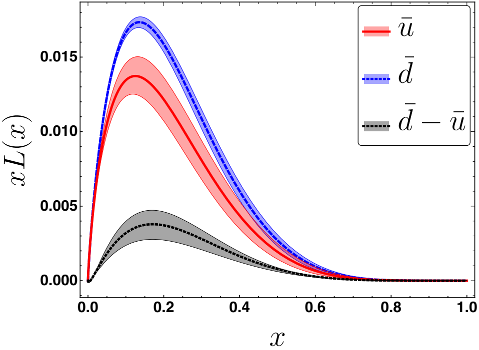




Orbital angular momentum

$$L^{\bar{q}}(x) = \frac{1}{2}\left\{x\left[H^{\bar{q}/P}(x,0,0) + E^{\bar{q}/P}(x,0,0)\right] - \widetilde{H}^{\bar{q}/P}(x,0,0)\right\}.$$

	$L^{\bar{u}}$	$L^{\bar{d}}$	$\Delta\Sigma^{\bar{u}}/2$	$\Delta\Sigma^{\bar{d}}/2$
Model-I	0.040 ± 0.003	0.048 ± 0.002	0.013 ± 0.001	0.015 ± 0.001
LC model	0.025	0.046	0.00	0.00



LC model- X Luan, Zhun Lu, Eur.Phys.J.C 83 (2023) 6, 504

Summary

- We constructed a scalar spectator model for light sea quarks of the proton and fitted the model parameters using the CTEQ18 NNLO data.
- We predicted flavor asymmetry in the form of $\bar{d} - \bar{u}$, $\frac{\bar{d}}{\bar{u}}$ and compared our results with the latest SeaQuest, NuSea and HERMES results.
- We also interpreted the flavor asymmetry at non-zero transverse momentum and non-zero momentum transfer.
- We limit our analysis to scalar spectator states only, but there is room for expansion to include vector spectator configurations.

References

- An Introduction to light front dynamics for pedestrians
  A. Harindranath.
- QCD on the Light-Front – A Systematic Approach to Hadron Physics
  S Brodsky, Guy F. de Téramond, Hans Günter Dosch
- Light front quark-diquark model for the nucleons
  T Maji et al Phys.Rev.D 94 (2016) 9, 094020
- Gluon distributions in the proton in a light-front spectator model
  D Chakrabarti et al Phys.Rev.D 108 (2023) 1, 014009

Questions?

Light front AdS/QCD inspired models

- The field theory, which is defined on the boundary of the AdS, should be conformal invariant in order to have a correspondence.
- QCD is not a conformally invariant theory, but it shows strong coupling at low energy, which needs a non- perturbative description.
- There are two ways to break the conformal symmetry: Soft wall and Hard wall AdS/QCD.
- The main idea is that at some initial scale, these functions are derived from the matching of the electromagnetic form factors of hadrons with arbitrary spin in the soft-wall AdS/QCD approach and in light-front QCD.