Exploration into the Baryon structure

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HAMPTON UNIVERSITY HUGS GRADUATE STUDIES PROGRAM

Jefferson Lab

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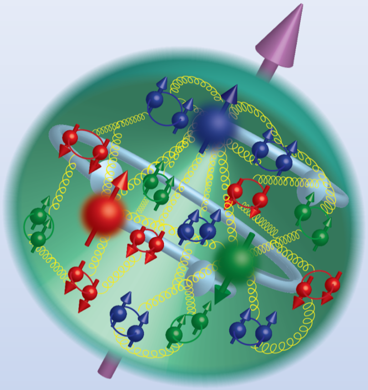
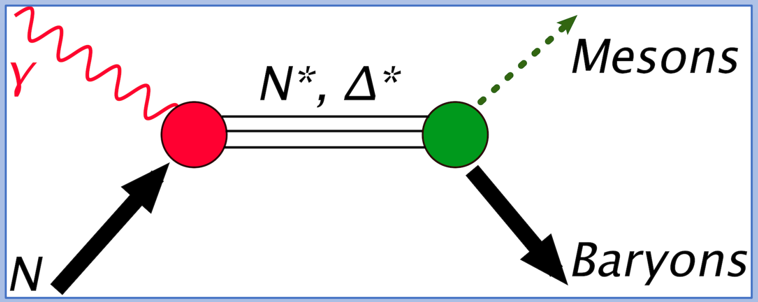
Introduction

- **Since the advent of QCD in 1970s we know that the nucleon is a bound state of three valence quarks along with a sea of gluons and quark-antiquark pairs.**



- **An implication of this understanding is that when energy is dumped into the nucleon ground states, they are excited, and can lose their energies only by emitting color singlet states, mostly mesons.**

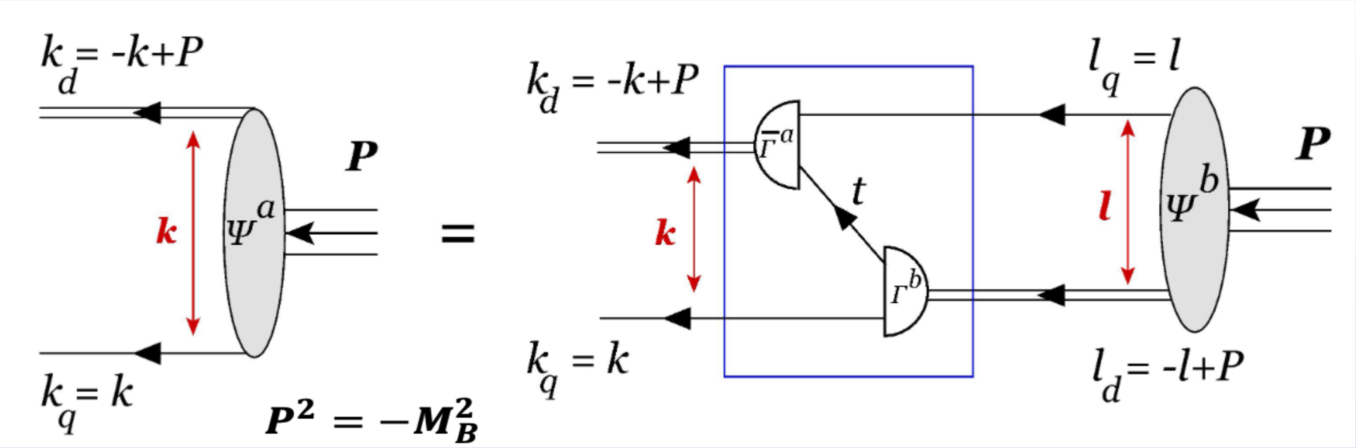


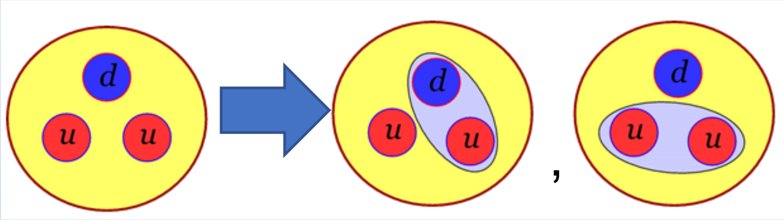
- The spectrum of these excited states.
  Refer to Volker Crede's lectures.

I	S	$J^P = \frac{1}{2}^+$	$\frac{3}{2}^+$	$\frac{5}{2}^+$	$\frac{1}{2}^-$	$\frac{3}{2}^-$	$\frac{5}{2}^-$
$\frac{1}{2}$	0	**N(940)** **N(1440)** N(1710) N(1880)	**N(1720)** N(1900)	**N(1680)** N(1860)	**N(1535)** **N(1650)** N(1895)	**N(1520)** N(1700) N(1875)	**N(1675)**
$\frac{3}{2}$	0	**Δ(1910)**	**Δ(1232)** Δ(1600) Δ(1920)	**Δ(1905)**	**Δ(1620)** Δ(1900)	**Δ(1700)** Δ(1940)	Δ(1930)
0	−1	**Λ(1115)** Λ(1600) Λ(1810)	**Λ(1890)**	**Λ(1820)**	**Λ(1405)** **Λ(1670)** Λ(1800)	**Λ(1520)** **Λ(1690)**	**Λ(1830)**
1	−1	**Σ(1190)** Σ(1660) Σ(1880)	**Σ(1385)**	**Σ(1915)**	Σ(1750)	**Σ(1670)** Σ(1940)	**Σ(1775)**
$\frac{1}{2}$	−2	**Ξ(1320)**	**Ξ(1530)**			Ξ(1820)	
0	−3		**Ω(1672)**				

Formalism

- The Faddeev amplitude Ψ for Baryons in a Bethe-Salpeter approach:





[Barabanov:2020jvn]

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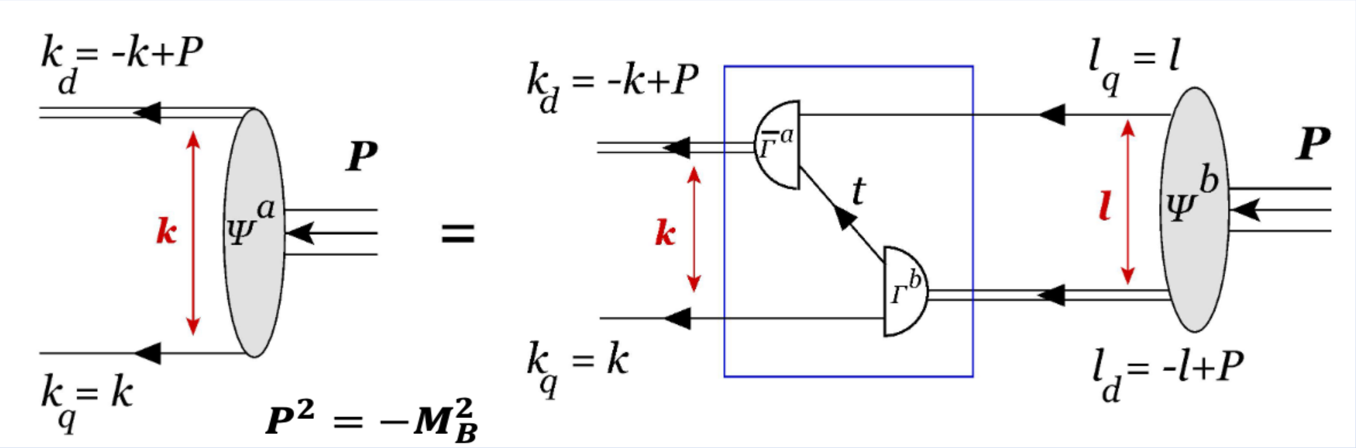

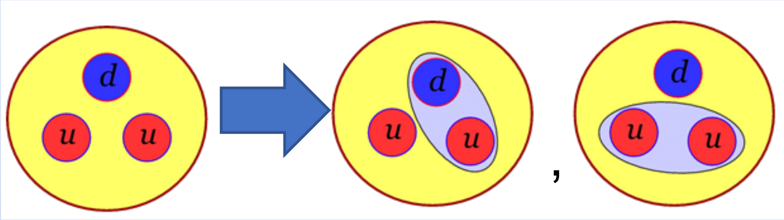

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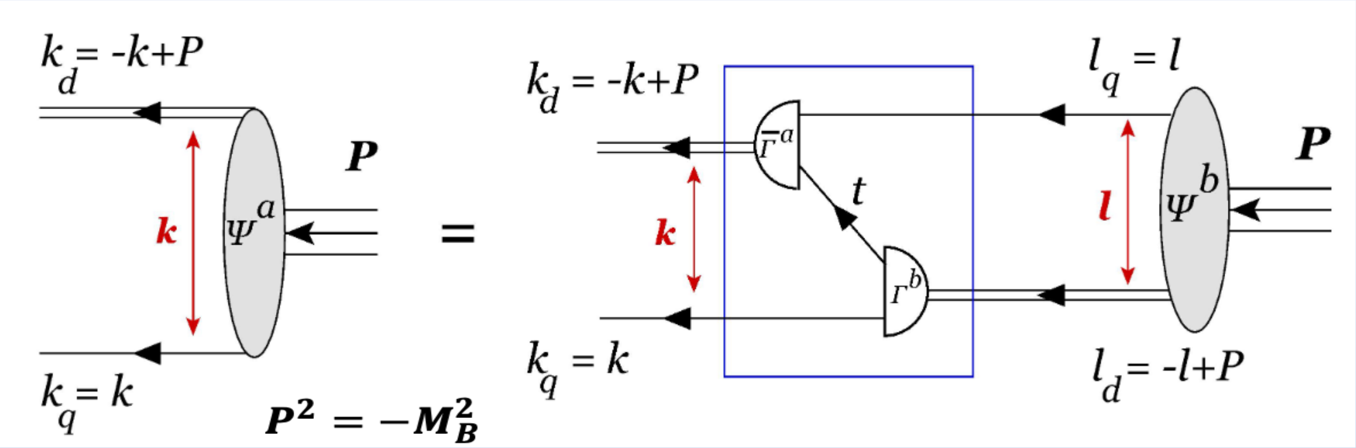

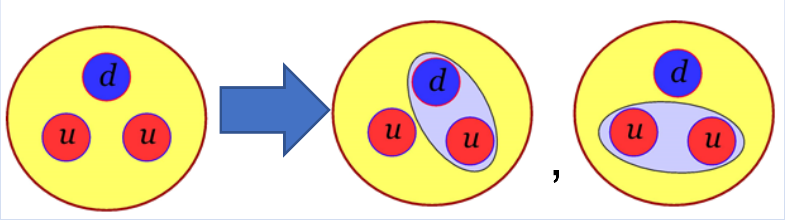

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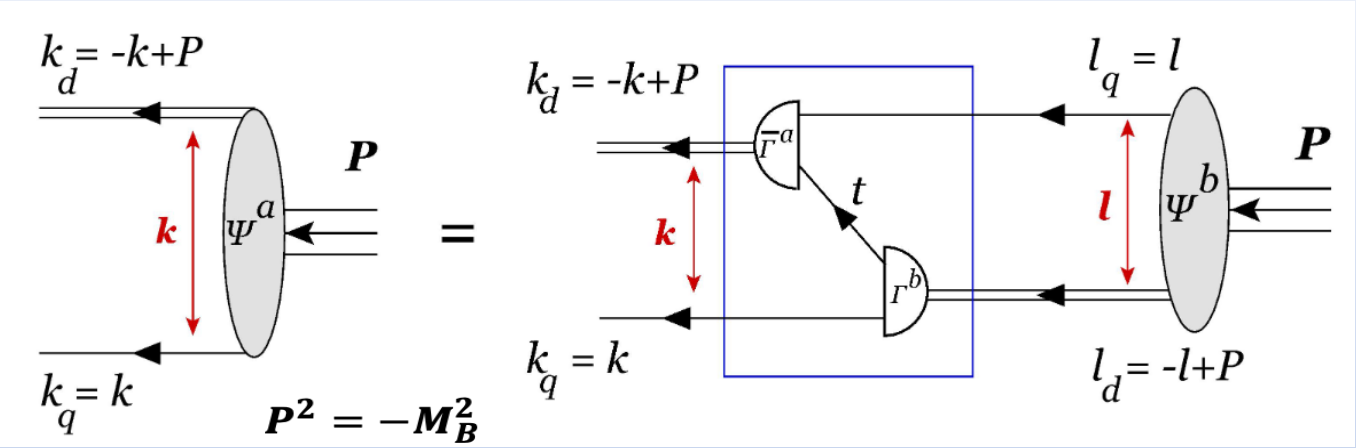

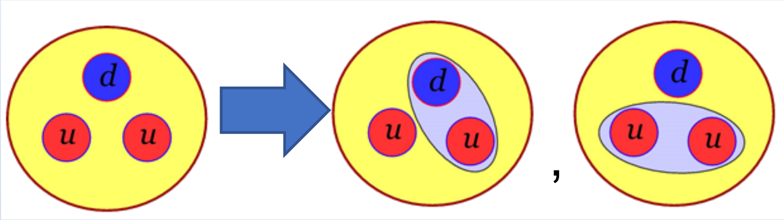

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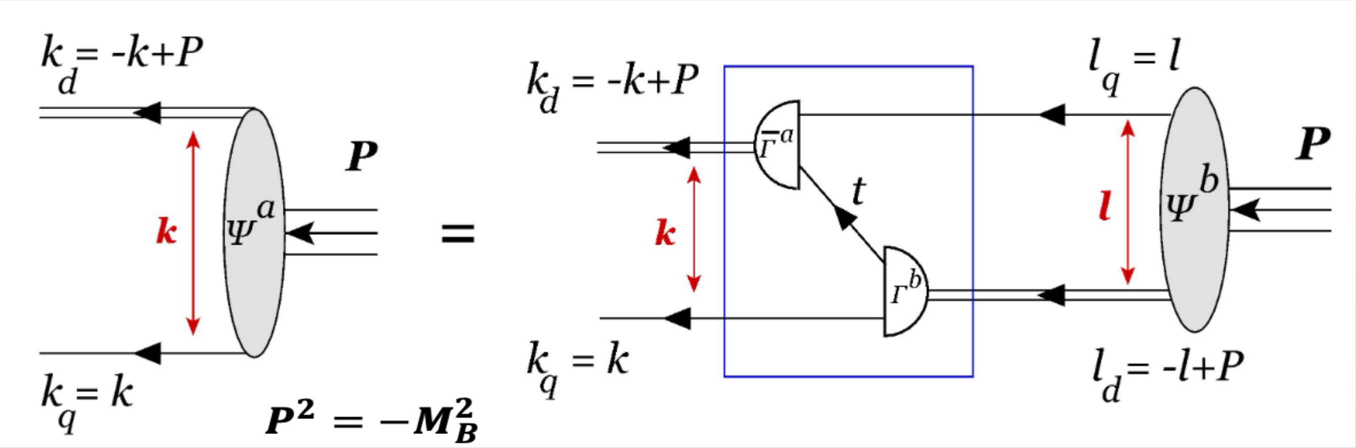

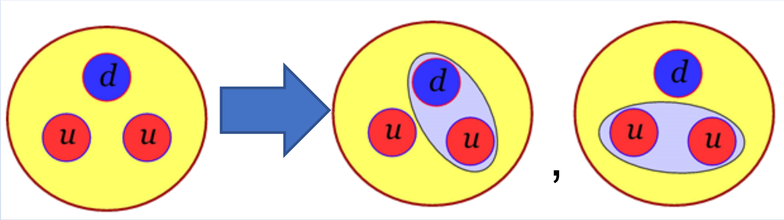

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$$\begin{aligned}\psi^P(k_i,\alpha_i,\sigma_i) &= [\Gamma^{0^+}(l;K)]^{\alpha_1\alpha_2}_{\sigma_1\sigma_2}\,\Delta^{0^+}(K)\,[\mathcal{S}^P_\rho(k;Q)u_\rho(Q)]^{\alpha_3}_{\sigma_3} \\ &+ [\mathsf{t}^j\Gamma^{1^+}_\mu]\,\Delta^{1^+}_{\mu\nu}\,[\mathcal{A}^{jP}_{\nu\rho}(k;Q)u_\rho(Q)] \\ &+ [\Gamma^{0^-}]\,\Delta^{0^-}\,[\mathcal{P}^P_\rho(k;Q)u_\rho(Q)] \\ &+ [\Gamma^{1^-}_\mu]\,\Delta^{1^-}_{\mu\nu}\,[\mathcal{V}^P_{\nu\rho}(k;Q)u_\rho(Q)]\,,\end{aligned}$$

$$\begin{aligned}\mathcal{S}^P_\rho(k;Q) &= \sum_{i=1}^{2} v^i_{0^+}(k;Q)\,\mathcal{G}^P\mathcal{X}^i_\rho(k;Q)\,, \\ \mathcal{A}^{jP}_{\nu\rho}(k;Q) &= \sum_{i=1}^{8} v^{ji}_{1^+}(k;Q)\,\mathcal{G}^P\mathcal{Y}^i_{\nu\rho}(k;Q)\,, \\ \mathcal{P}^P_\rho(k;Q) &= \sum_{i=1}^{2} v^i_{0^-}(k;Q)\,\mathcal{G}^{-P}\mathcal{X}^i_\rho(k;Q)\,, \\ \mathcal{V}^P_{\nu\rho}(k;Q) &= \sum_{i=1}^{8} v^i_{1^-}(k;Q)\,\mathcal{G}^{-P}\mathcal{Y}^i_{\nu\rho}(k;Q)\,,\end{aligned}$$

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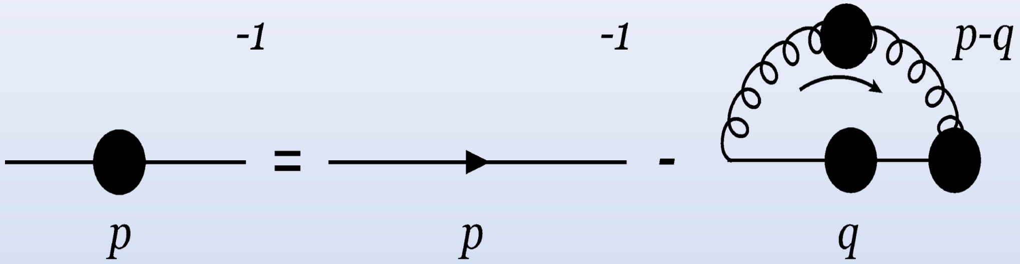
Schwinger – Dyson Equations

- Ideal for studying non-perturbative phenomena because nothing is assumed about the value of the coupling:



$$S^{-1}(p,\mu) = Z_{2F} S_0^{-1}(p) + Z_{1F} \int \frac{d^4 p}{(2\pi)^4} g^2 D_{\rho\nu}(p-q;\mu) \frac{\lambda^a}{2} \gamma_\rho S(q;\mu) \Gamma^a_\nu(q,p;\mu).$$

[Roberts:1994dr]

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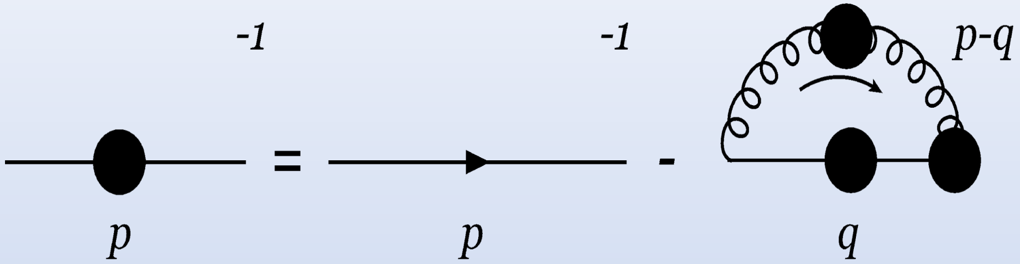

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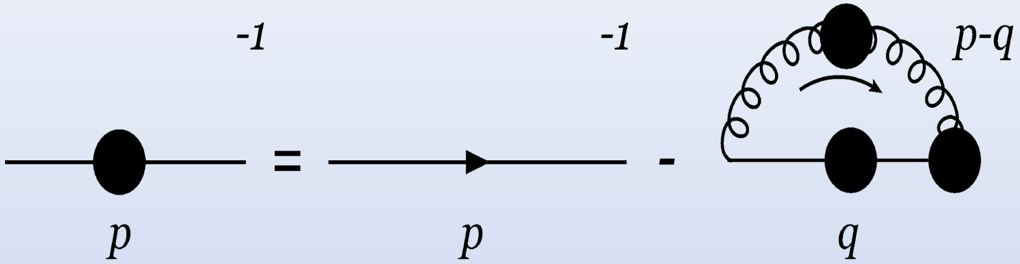

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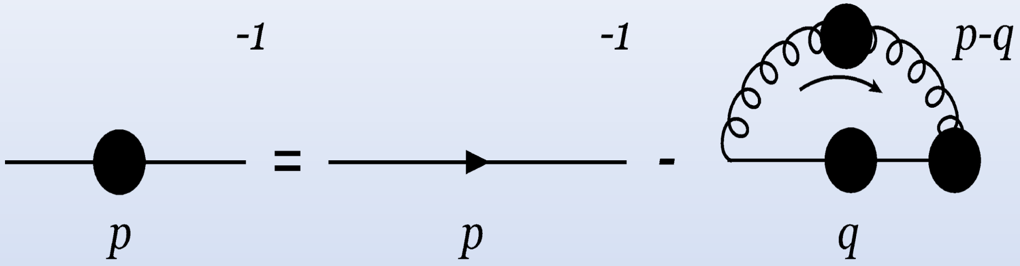

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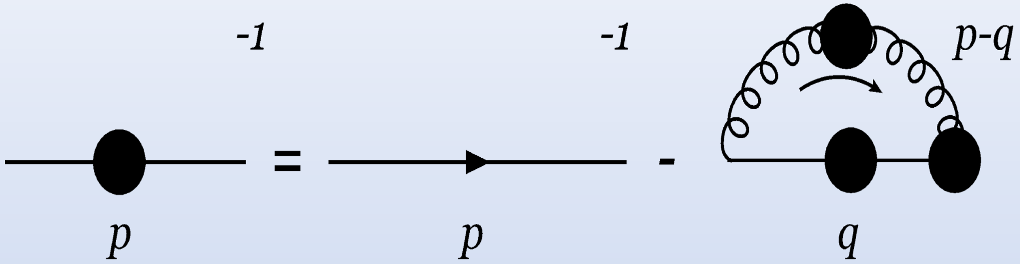

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Dressed quark-gluon vertex

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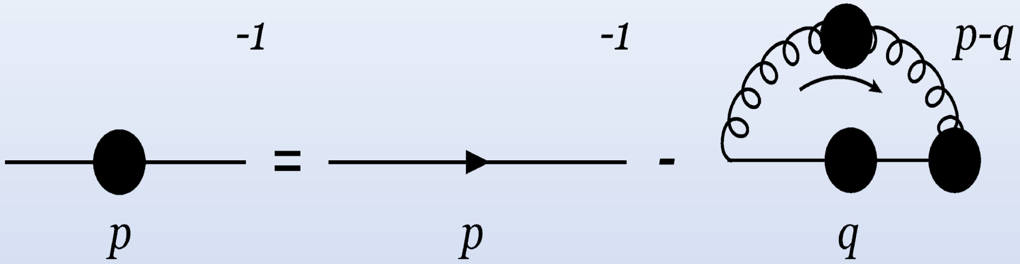

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Complete Propagator, dressed — Bare propagator, tree level — Dressed gluon propagator — Dressed quark-gluon vertex

- Truncation framework: Rainbow-Ladder

$$Z_{1F}\, g^2 D_{\rho\nu}(p-q;\mu)\frac{\lambda^a}{2}\gamma_\rho S(q;\mu)\Gamma^a_\nu(q,p;\mu) \rightarrow k^2\mathcal{G}(k^2)D^0_{\rho\nu}(k;\mu)\frac{\lambda^a}{2}\gamma_\rho S(q;\mu)\frac{\lambda^a}{2}\gamma_\nu$$

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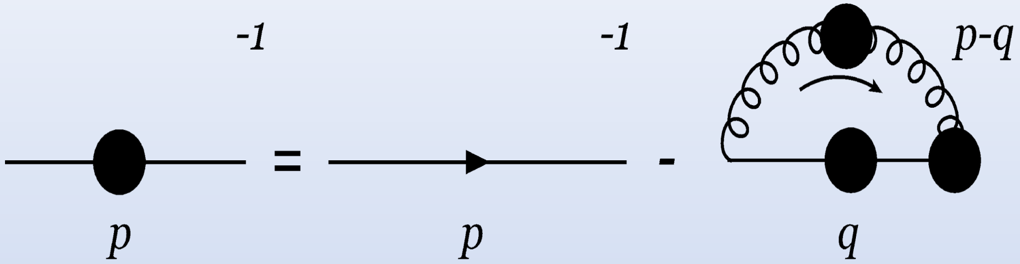

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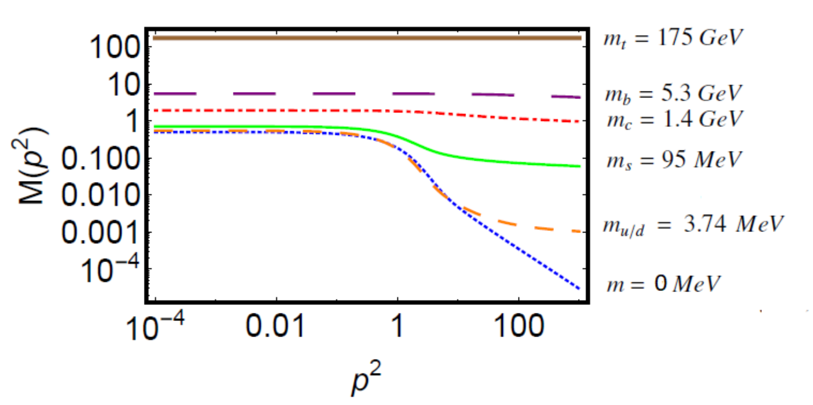

Constituent mass vs current mass

$$m_u + m_u + m_d \simeq 10\ MeV$$
$$m_{protón} \approx 1000\ MeV$$

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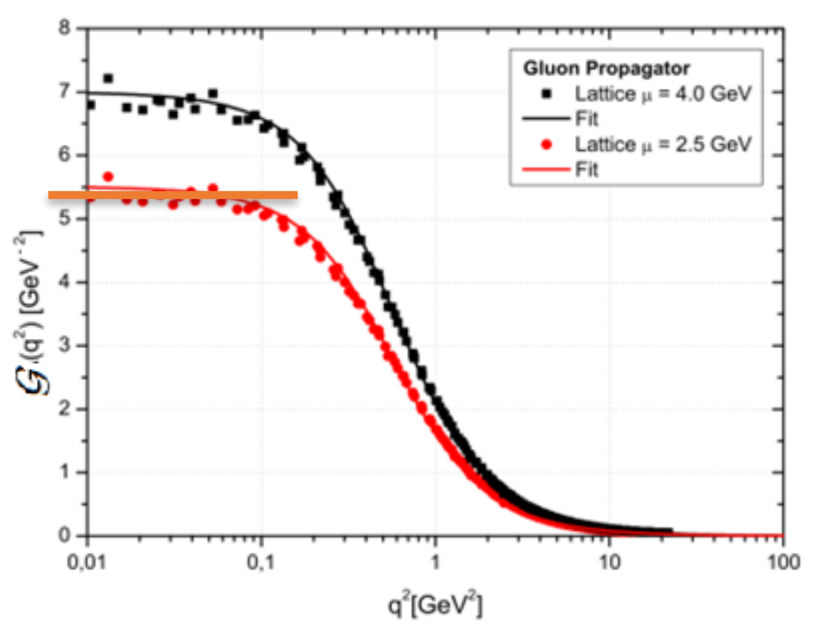

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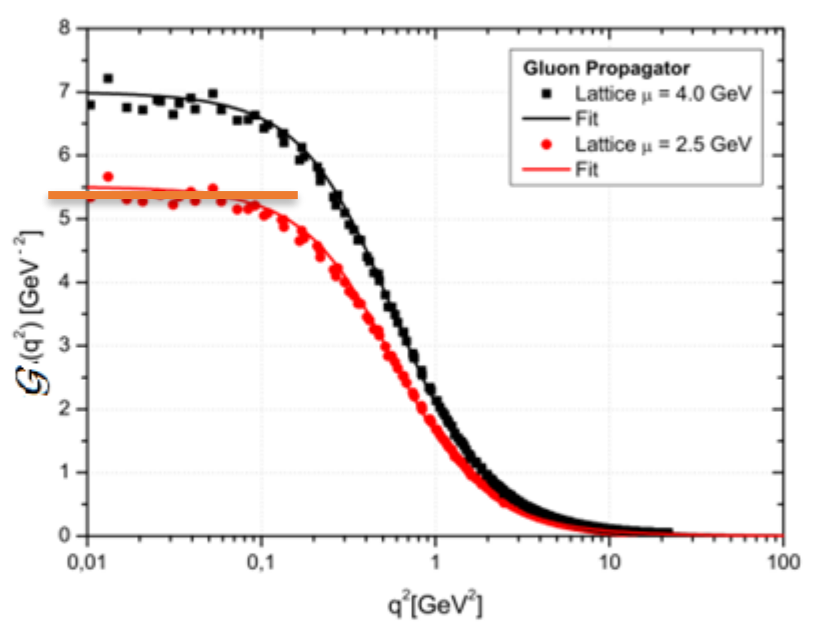

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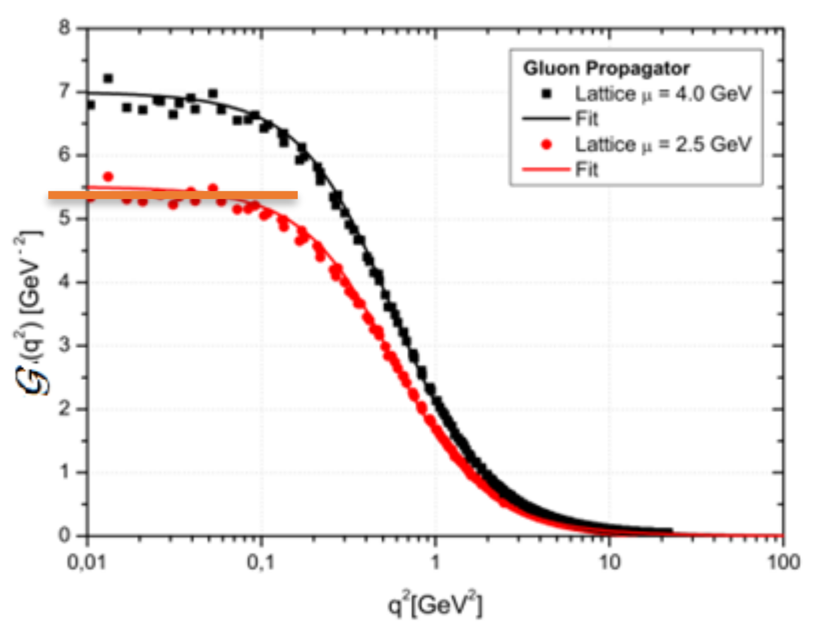

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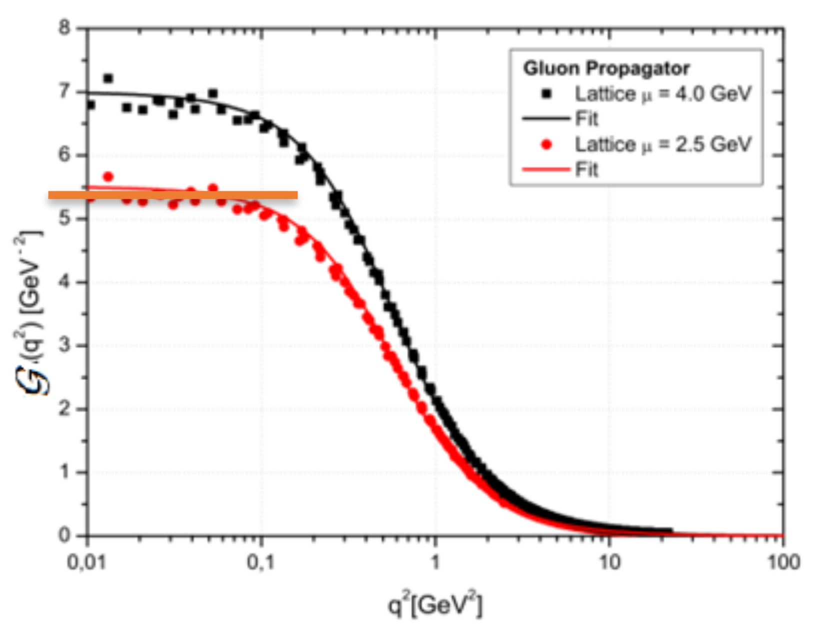

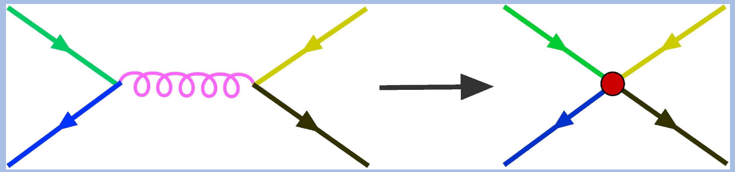

[Gutierrez-Guerrero:2010waf, Roberts:2010rn]

Bethe-Salpeter equation

- The full $q\bar{q}$ scattering matrix or t-matrix, contains poles for all $q\bar{q}$ bound states, that is, the physical mesons. [Salpeter:1951sz]

$$\left[\Gamma_H^{f_1\bar{f}_2}(k;P)\right]_{tu} = \int \frac{d^4q}{(2\pi)^4}\left[\chi_H^{f_1\bar{f}_2}(q;P)\right]_{sr} K_{tu}^{rs}(q,k;P),$$
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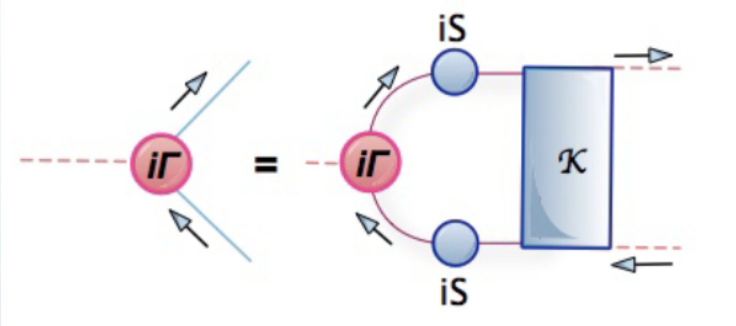

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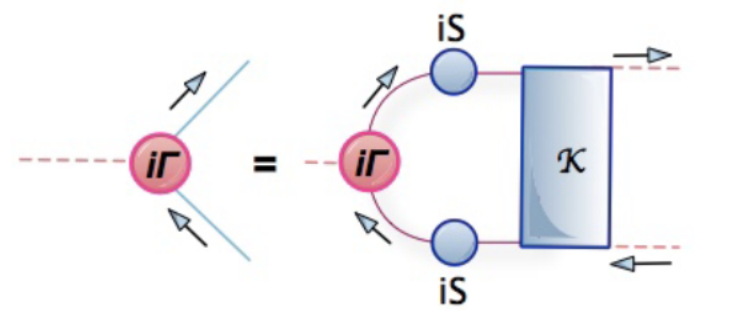

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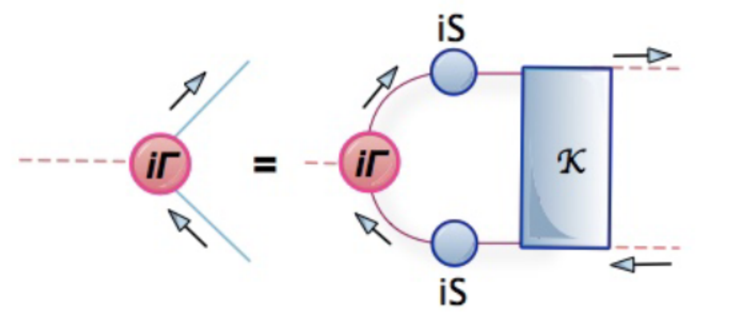

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- En el Modelo CI las ABS no depende de k, el momento relativo:

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\Gamma^{0^{++}}(P) &= \mathbb{1}E^{0^{++}}(P),\\
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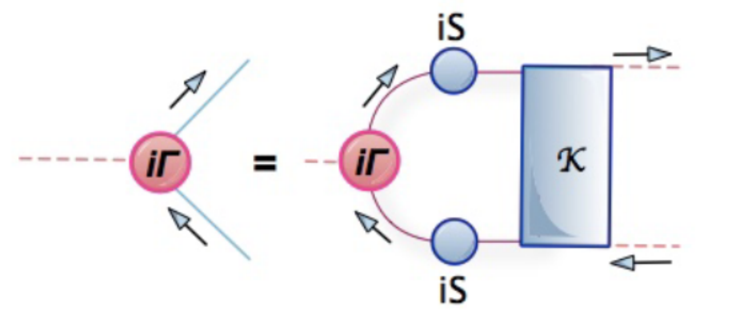

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\Gamma_\mu^{1^{++}}(P) &= \gamma_5\left[\gamma_\mu^T E^{1^{++}}(P) + \frac{1}{2M_R}\sigma_{\mu\nu}P_\nu F^{1^{++}}(P)\right],
\end{aligned}$$

$$P^2 = -M_H^2$$

Meson	Exp.	CI	Diquarks Mass
π	0.139	0.14	$(qq)_{0^+} = 0.78$
ρ	0.78	0.93	$(qq)_{1^+} = 1.06$
σ	1.2	1.22	$(qq)_{0^-} = 1.15$
a_1	1.260	1.37	$(qq)_{1^-} = 1.33$

[Roberts:2011cf, Roberts:2011wy, Yin:2019bxe, Chen:2012qr, Gutierrez-Guerrero:2019uwa, Gutierrez-Guerrero:2021rsx, Yin:2021uom]

Faddeev in CI and quark-diquark model

- The Faddeev equation in the CI dynamical quark-diquark picture:

$$\begin{bmatrix} \mathcal{S}^P(l;P) \\ \mathcal{A}_\mu^{P\,f}(l;P) \\ \mathcal{P}^P(l;P) \\ \mathcal{V}_\mu^P(l;P) \end{bmatrix} u_\rho = 4\int \frac{d^4 l}{(2\pi)^4} \mathcal{M}_{\mu\nu}^{fg}(k,l,P) \begin{bmatrix} \mathcal{S}^P(l;P) \\ \mathcal{A}_\nu^{P\,g}(l;P) \\ \mathcal{P}^P(l;P) \\ \mathcal{V}_\nu^P(l;P) \end{bmatrix} u_\rho,$$

$$\begin{aligned} \mathcal{S}^\pm &= (s^\pm \mathbb{1}_D)\mathcal{G}^\pm \\ i\,\mathcal{A}_\mu^{\pm f} &= \left(a_1^{\pm f}\,\gamma_5\,\gamma_\mu - i\,a_2^{\pm f}\gamma_5\,\hat{P}_\mu\right)\mathcal{G}^\pm \\ i\,\mathcal{P}^\pm &= (p^\pm\,\gamma_5)\mathcal{G}^\pm \\ i\,\mathcal{V}_\mu^\pm &= \left(v_1^\pm\,\gamma_\mu - i\,v_2^\pm \mathbb{1}_D \hat{P}_\mu\right)\mathcal{G}^\pm \end{aligned}$$

$$\Psi_N = \begin{bmatrix} r_1\ u[ud]_{0^+} \\ r_2\ d\{uu\}_{1^+} \\ r_3\ u\{ud\}_{1^+} \\ r_4\ u[ud]_{0^-} \\ r_5\ u[ud]_{1^-} \end{bmatrix}$$

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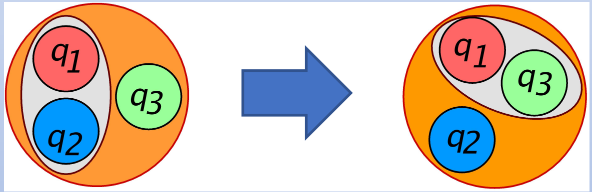
$$\mathcal{M}^{fg}_{[q_1q_3][q_1q_2]} = t^{q_1\,T} t^{[q_1q_2]} t^{[q_1q_3]\,T} t^{q_3}\left[g_{DB}^{P_BP_d}\,\Gamma^g_{[q_1q_2]}(l_{q_1q_2})\, S^T_{q_1}\, \bar{g}_{DB}^{P_BP_d}\,\bar{\Gamma}^f_{[q_1q_3]}(-k_{q_1q_3})\, S_{q_3}(l_{q_3})\, \Delta^g_{[q_1q_2]}(l_{q_1q_2})\right]$$



- Diquark breakup and recombination occurs via quark exchange.

Gustavo Paredes

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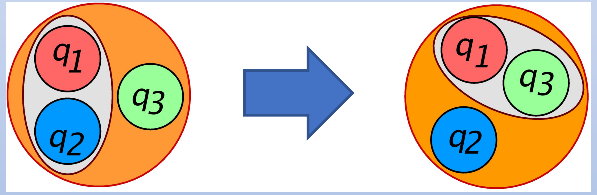
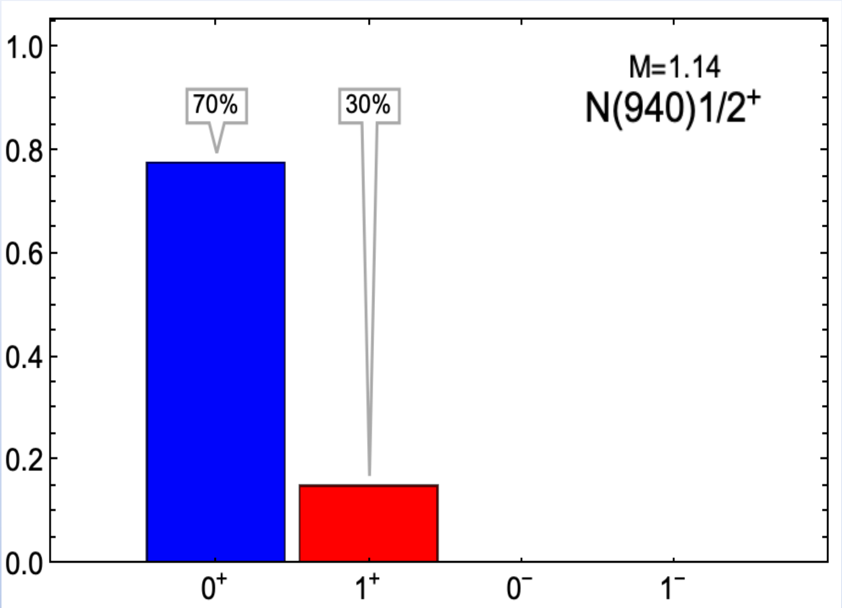


- Diquark breakup and recombination occurs via quark exchange.
- The kernel penalizes the contribution of diquarks whose parity is opposite to that of the baryon using a multiplicative factor $g^{P_BP_d}_{DB}$.

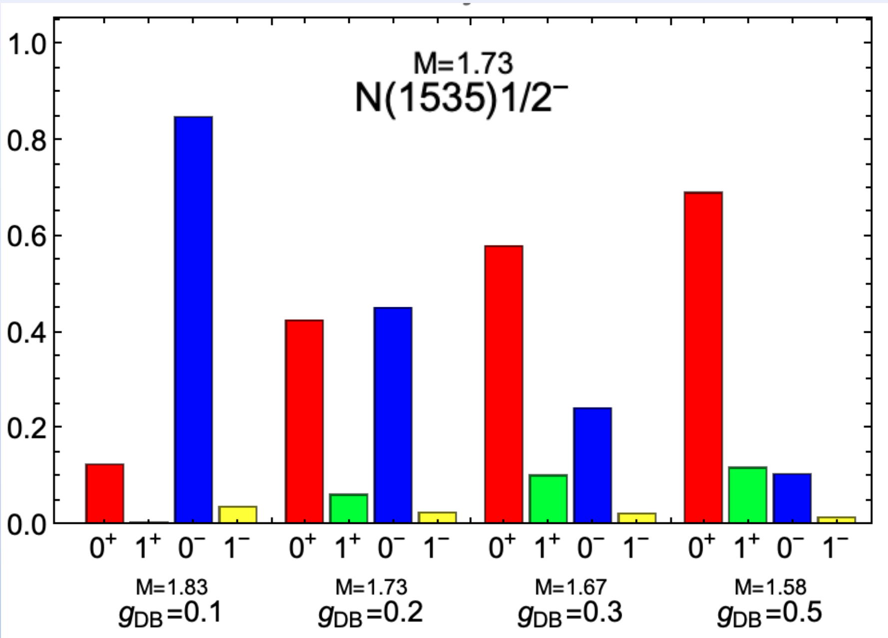
[Yin:2019bxe, Yin:2021oum, Gutierrez-Guerrero:2019uwa]

N(940) and N(1535)

- The produced masses and diquark content:



- The variation of $g_{DB} \rightarrow (1 \pm 0.5) g_{DB}$ produces:



[Lu:2017cln, Raya:2021pyr]

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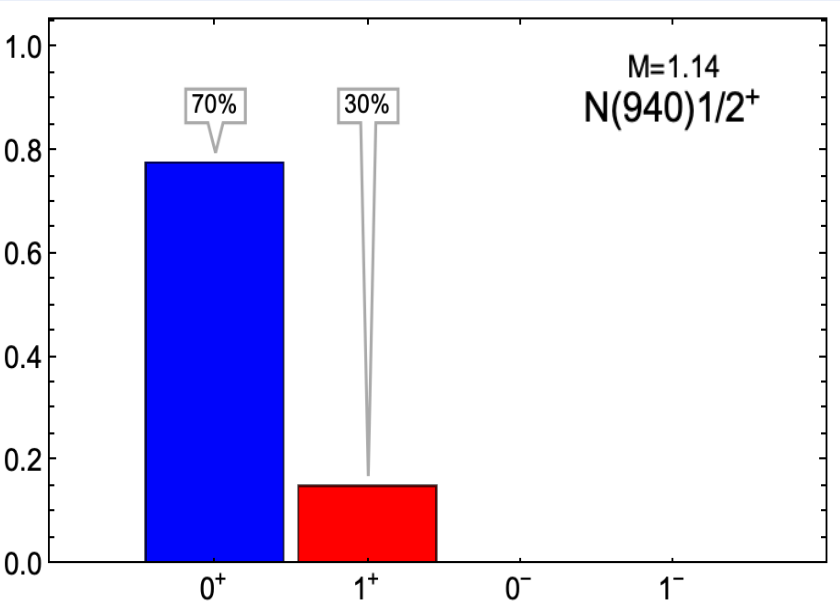

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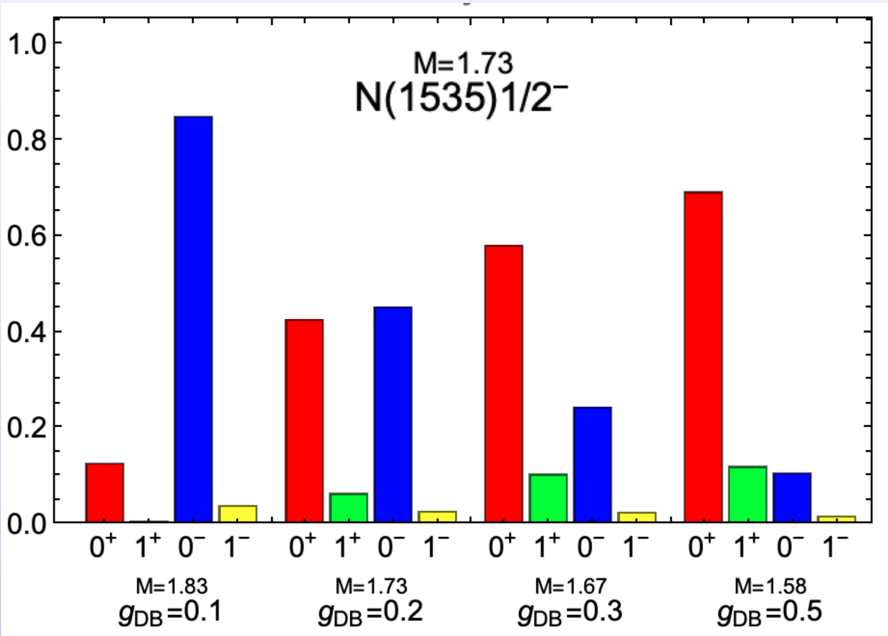

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[Lu:2017cln, Raya:2021pyr]

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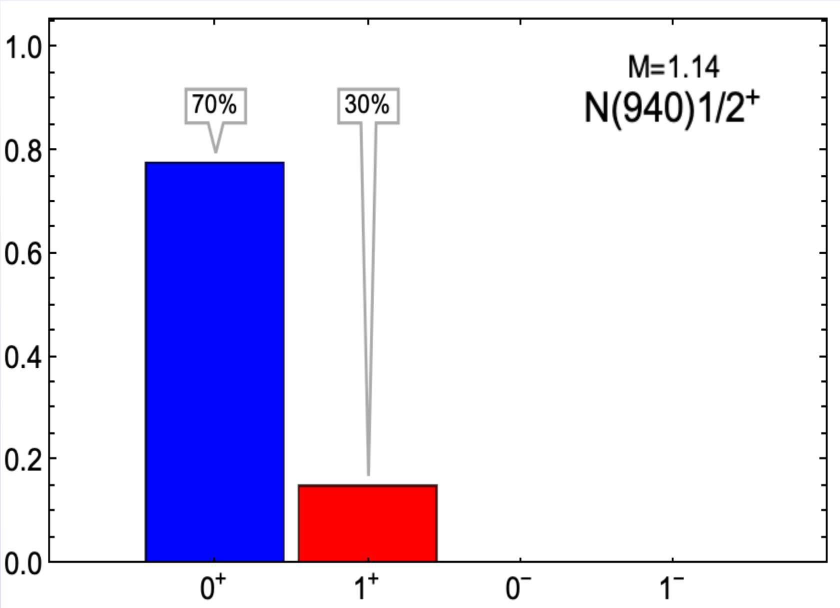

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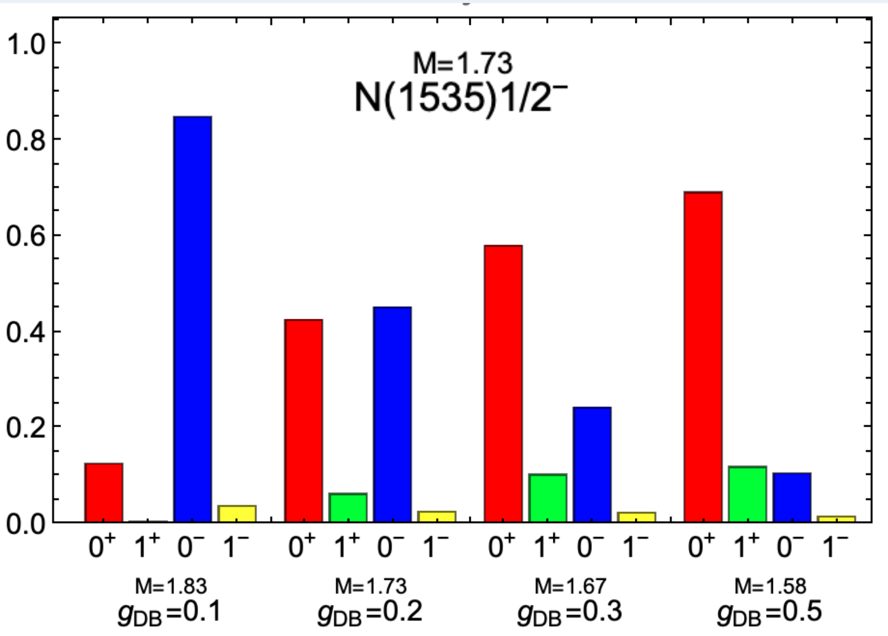

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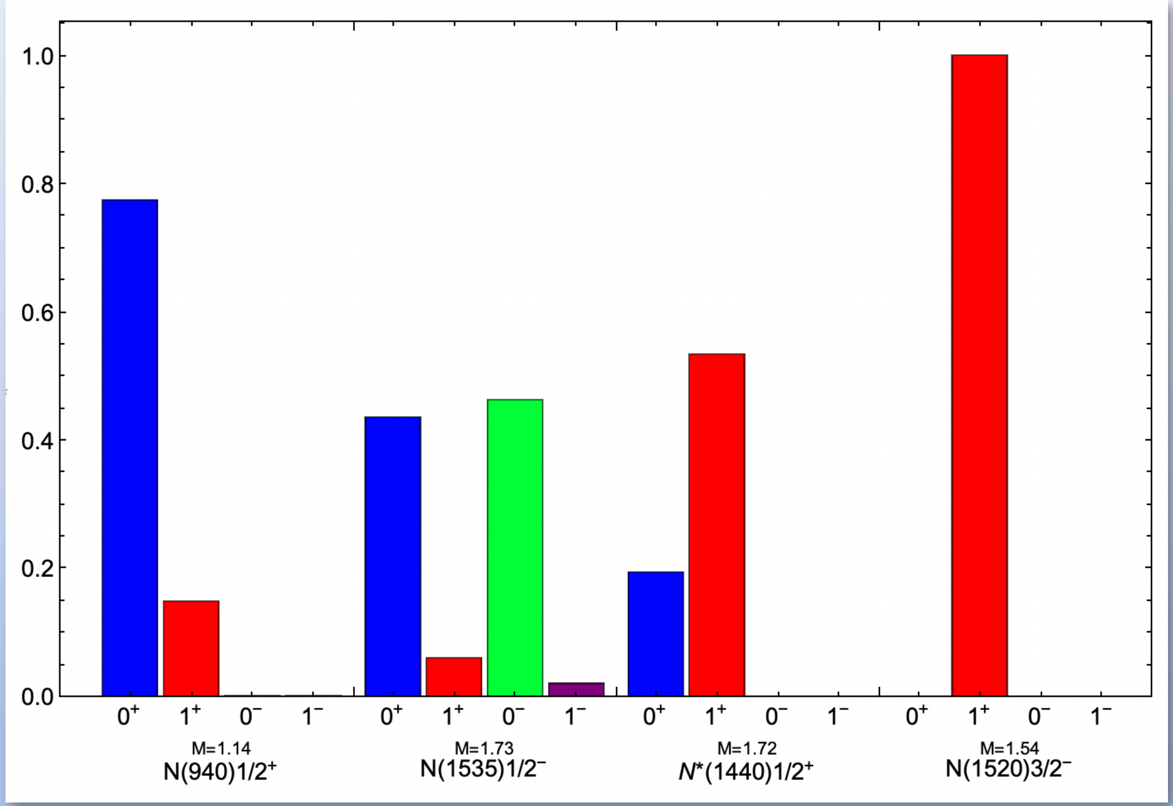
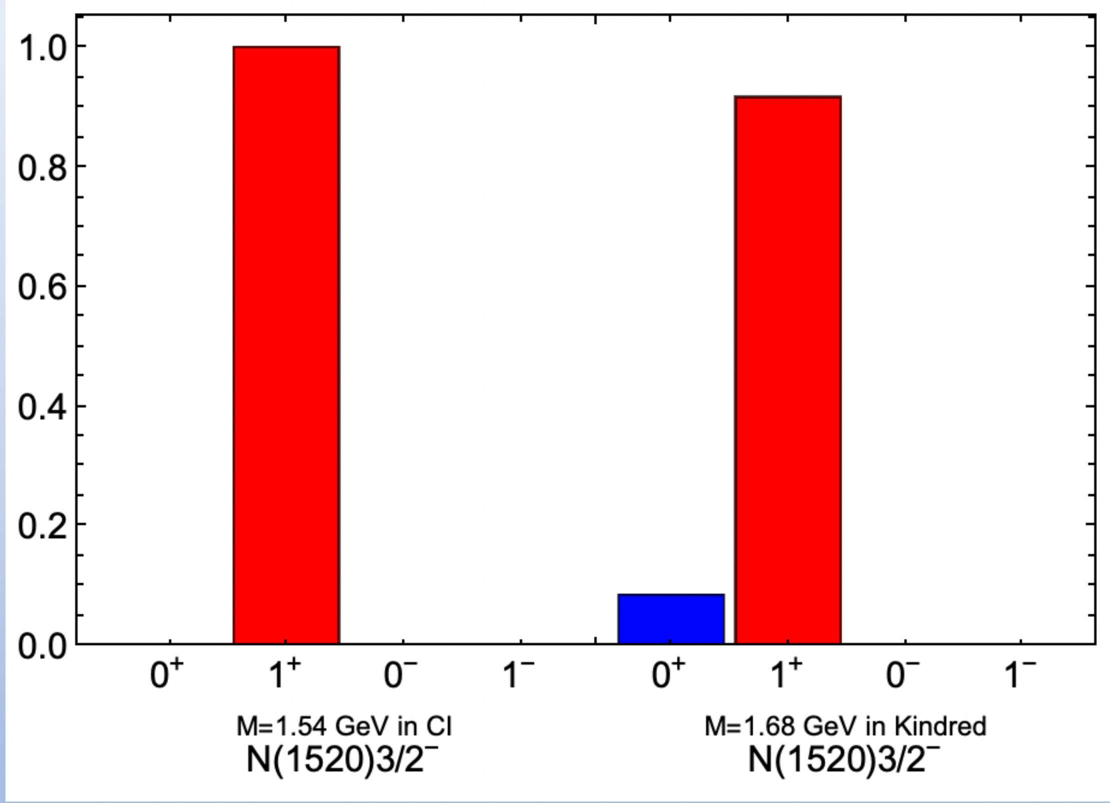
- The nucleon $N(1535)$ shows a similar contribution from $0^+ | 0^-$ diquarks for $g_{DB} = 0.2$.

[Lu:2017cln, Raya:2021pyr]

Results for N*

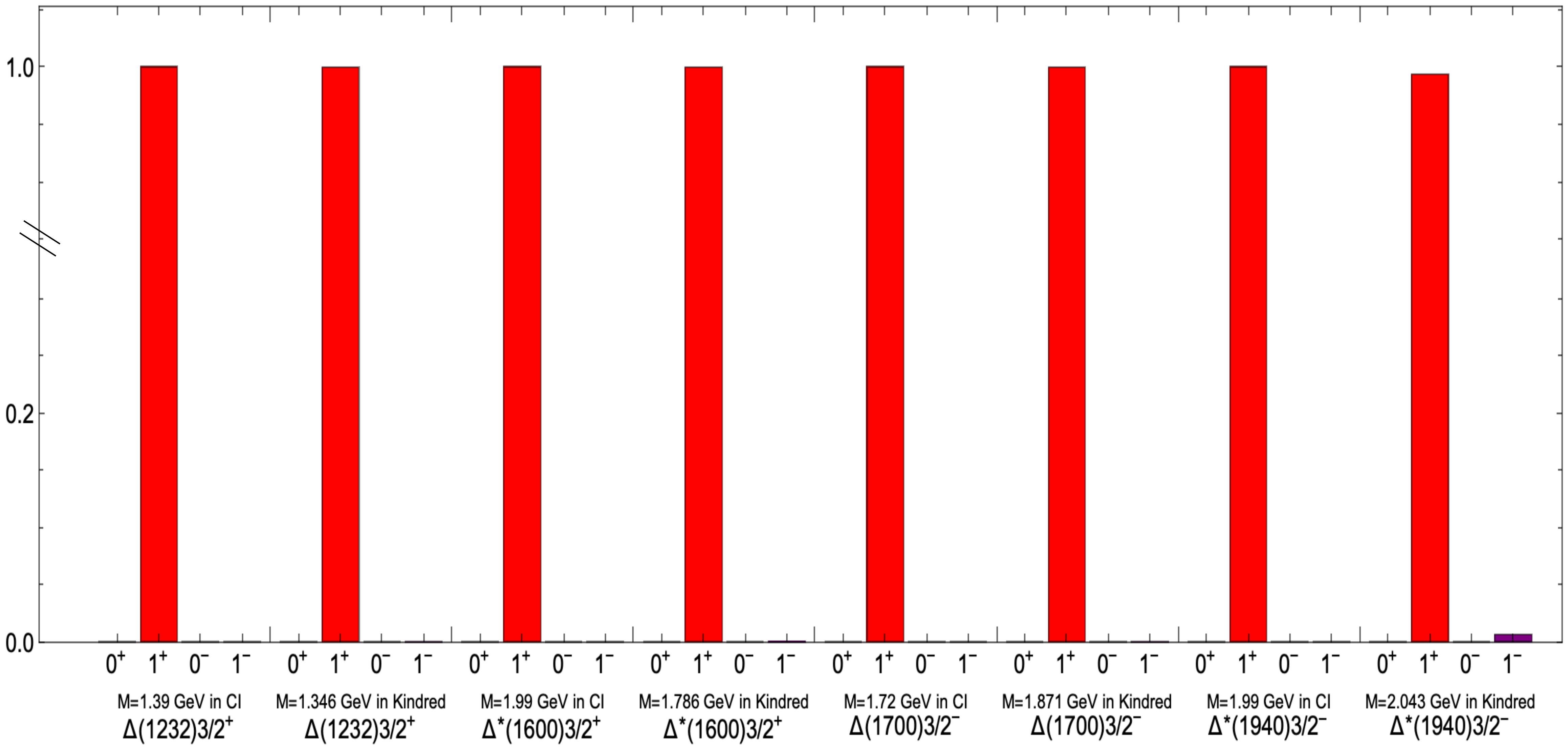
- In collaboration with K. Raya





[Liu:2022nku]

Results for Δ^*



[Liu:2022ndb]

Gustavo Paredes

Summary and Scope 1

- A description of $N(940)1/2^+$, $\Delta(1232)3/2^+$, $N(1440)1/2^+$, $N(1535)1/2^-$, $\Delta(1600)3/2^+$ in CI and QCD kindred models is already available in the literature.

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- Thank you.

$\gamma^* p \to N, \Delta\left(\frac{3}{2}^{\pm}\right)$ Transition Form Factor

- The electromagnetic current:

[Segovia:2014aza, Nicmorus:2010sd]

$$J^{\mu\lambda}(K,Q) = \Lambda_+(P_f)\, R^{\lambda\alpha}(P_f)\, i\,\gamma_5\, \Gamma^{\alpha\mu}(K,Q)\, \Lambda_+(P_i)$$

- The vertex with G_M^* magnetic dipole, G_E^* electric quadrupole and G_C^* Coulomb quadrupole:

$$\Gamma^{\alpha\mu} = b\left[\frac{i\omega}{2\lambda_+}\left(G_M^\star - G_E^\star\right)\gamma_5\, \varepsilon^{\alpha\mu\gamma\delta} K^\gamma \widehat{Q}^\delta \; - G_E^\star\, T_Q^{\alpha\gamma}\, T_K^{\gamma\mu} - \frac{i\tau}{\omega}\, G_C^\star\, \widehat{Q}^\alpha K^\mu\right],$$

$$P_T^\mu = T_Q^{\mu\nu} P^\nu = P^\mu - (P\cdot\widehat{Q})\,\widehat{Q}^\mu\,,$$

$$T_P^{\mu\nu} = \delta^{\mu\nu} - \hat{P}^\mu \hat{P}^\nu$$

$$\gamma_T^\mu = T_P^{\mu\nu}\gamma^\nu$$

$$\Lambda_+(P) = \frac{1}{2M_N}\sum_{r=\pm} u(P,r)\bar{u}(P,r) = \frac{1}{2M_N}\left(M_N - i\gamma\cdot P\right)$$

$$\tau := \frac{Q^2}{2\,(M_\Delta^2 + M_N^2)}\,, \quad \lambda_\pm := \frac{(M_\Delta \pm M_N)^2 + Q^2}{2\,(M_\Delta^2 + M_N^2)}$$

$$\frac{1}{2M_B}\sum_{r=-3/2}^{3/2} u_\rho(P,r)\bar{u}_\mu(P,r) = \Lambda_+(P) R_{\rho\mu}(P),$$

$$R_{\mu\nu}(P) = \delta_{\mu\nu}\mathbb{1}_D - \frac{1}{3}\gamma_\mu\gamma_\nu + \frac{2}{3}\hat{P}_\mu\hat{P}_\nu\mathbb{1}_D - \frac{i}{3}\left[\hat{P}_\mu\gamma_\nu - \hat{P}_\nu\gamma_\mu\right]$$

$$\omega := \sqrt{\lambda_+\lambda_-} \text{ and } b := \sqrt{\tfrac{3}{2}}\,(1 + M_\Delta/M_N).$$

Current using diquarks

- In general, the electromagnetic urrent is:

$$\mathcal{J}_{\mu,x}(P_f, P_i) = \Lambda^{\mathcal{P}_f}_{+,x}(P_f)\left(i\, e\, \Gamma_{\mu,x}(P_f, P_i)\right)\Lambda^{\mathcal{P}_i}_{+,x}(P_i)$$

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- In the quark-diquark model, the electromagnetic current is described considering the interaction diagrams of the photon with the diquarks inside baryon.

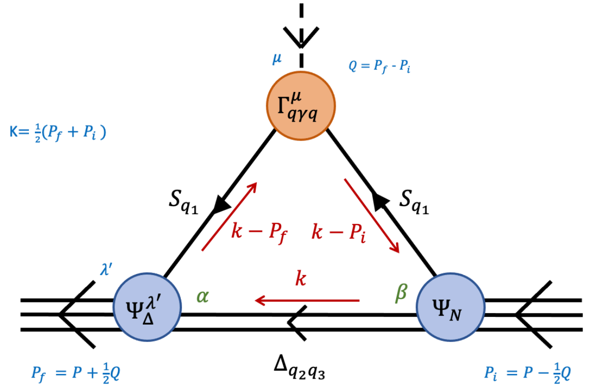
$$\begin{aligned}\mathcal{J}_{\mu,x}(P_f,P_i) &= \sum_{I=Diagramas}\int_l \Lambda^{\mathcal{P}_f}_{+,x}(P_f)\left(\Gamma^{I}_{\mu,x}(l;P_f,P_i)\right)\Lambda^{\mathcal{P}_i}_{+,x}(P_i)\\ &= \Lambda^{\mathcal{P}_f}_{+,x}(P_f)\left[\sum_d \Pi^d(l;P_f,P_i) + \sum_{d_1,d_2}\Pi^{(d_1,d_2)}(l;P_f,P_i)\right]\Lambda^{\mathcal{P}_i}_{+,x}(P_i)\end{aligned}$$

[Raya:2023wye]

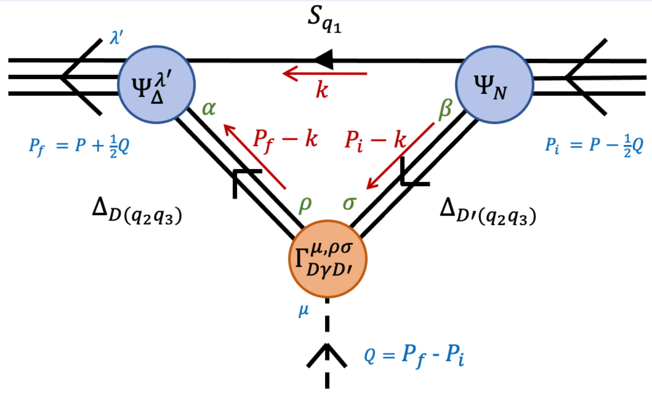
- Where Π^d represents the diagrams where the photon hits the quark and $\Pi^{(d_1,d_2)}$ represents the diagrams where the photon hits the diquark.

Form Factors using diquarks

- Each diagram has the following form:



$$\int \frac{d^4k}{(2\pi)^4} \mathcal{D}_{\lambda'\alpha}(P_f) S_q(P_f - k)\Gamma^{\mu}(P_f - k, P_i - k) S_q(P_i - k) \mathcal{A}^i_{\beta}(-P_i)\Delta_{\alpha\beta}(-k)$$



$$\int \frac{d^4k}{(2\pi)^4} S_q(k)\mathcal{D}_{\lambda'\alpha}(P_f)\Delta_{\alpha\rho}(P_f - k)\Gamma^{\mu,\rho\sigma}(P_f - k, P_i - k)\Delta_{\sigma\beta}(P_i - k)\mathcal{A}^i_{\beta}(P_i)$$

Diagrams courtesy of Luis Albino

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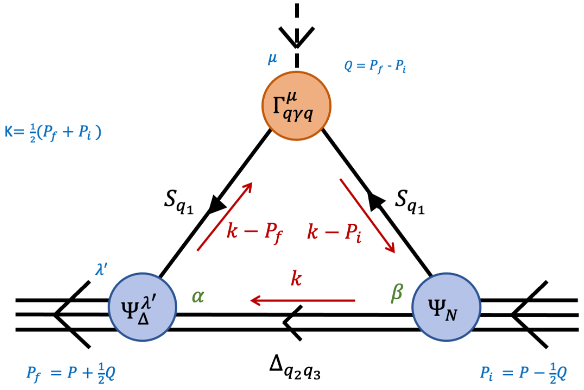

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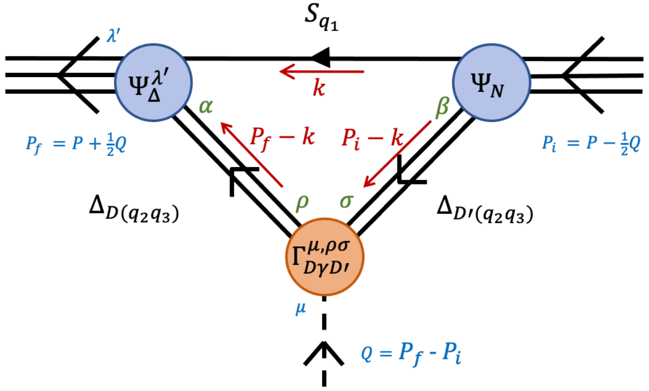

$$\int \frac{d^4k}{(2\pi)^4} S_q(k) \mathcal{D}_{\lambda'\alpha}(P_f) \Delta_{\alpha\rho}(P_f - k) \Gamma^{\mu,\rho\sigma}(P_f - k, P_i - k) \Delta_{\sigma\beta}(P_i - k) \mathcal{A}^i_\beta(P_i)$$

Diagrams courtesy of Luis Albino

- We need to consider whether the photon hits the quark and the diquarks are spectators (4 contributions), and whether the photon hits the diquarks (16 contributions):

Ini/Fin	0^+	0^-	1^+	1^-
0^+	$0^+ \to 0^+$	$0^+ \to 0^-$	$0^+ \to 1^+$	$0^+ \to 1^-$
0^-	$0^- \to 0^+$	$0^- \to 0^-$	$0^- \to 1^+$	$0^- \to 1^-$
1^+	$1^+ \to 0^+$	$1^+ \to 0^-$	$1^+ \to 1^+$	$1^+ \to 1^-$
1^-	$1^- \to 0^+$	$1^- \to 0^-$	$1^- \to 1^+$	$1^- \to 1^-$

[Raya:2021pyr]

- **Similar results for $\gamma^* p \rightarrow N(1535)\,1/2^-$ and $\gamma^* p \rightarrow N(1520)\,3/2^-$ are not yet available.**

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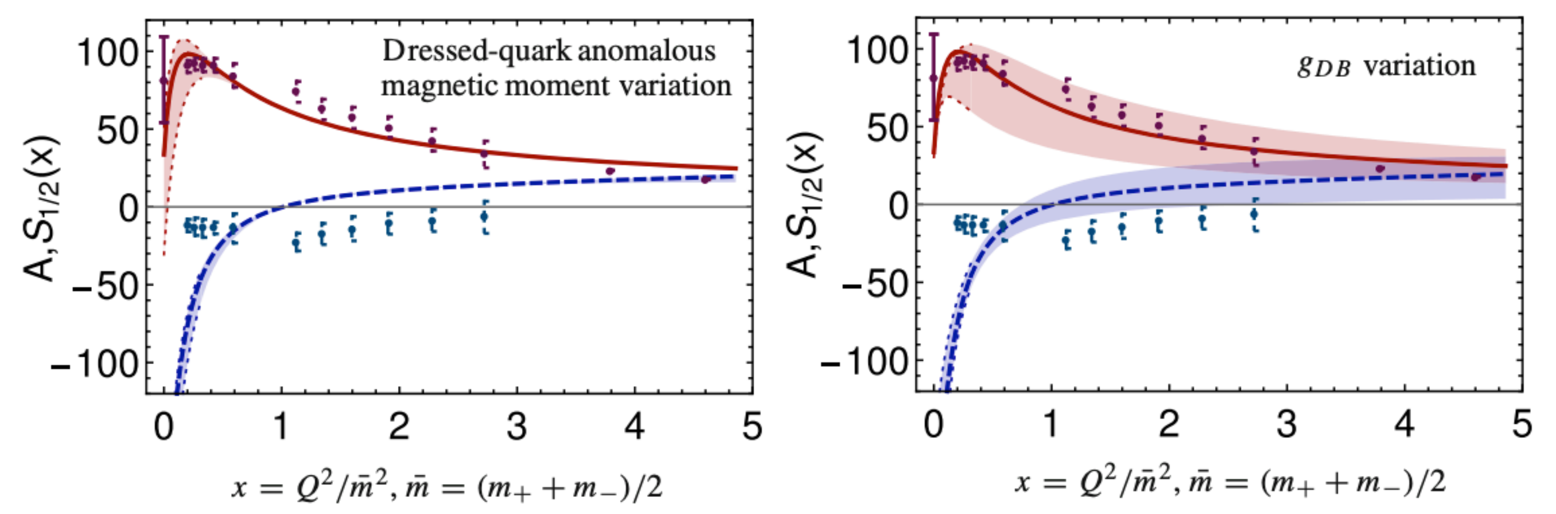
- **An insightful starting point can be provided by the contact interaction.**
- **A contact interaction treatment of $\gamma^* p \rightarrow N(1535)1/2^-$ transition amplitudes and form factors provides first results providing us insight into its relative diquark content.**

$$A_{1/2} = 2\mathcal{K}\left(F_1^* + \frac{m_{N^*} - m_N}{m_{N^*} + m_N} F_2^*\right)$$

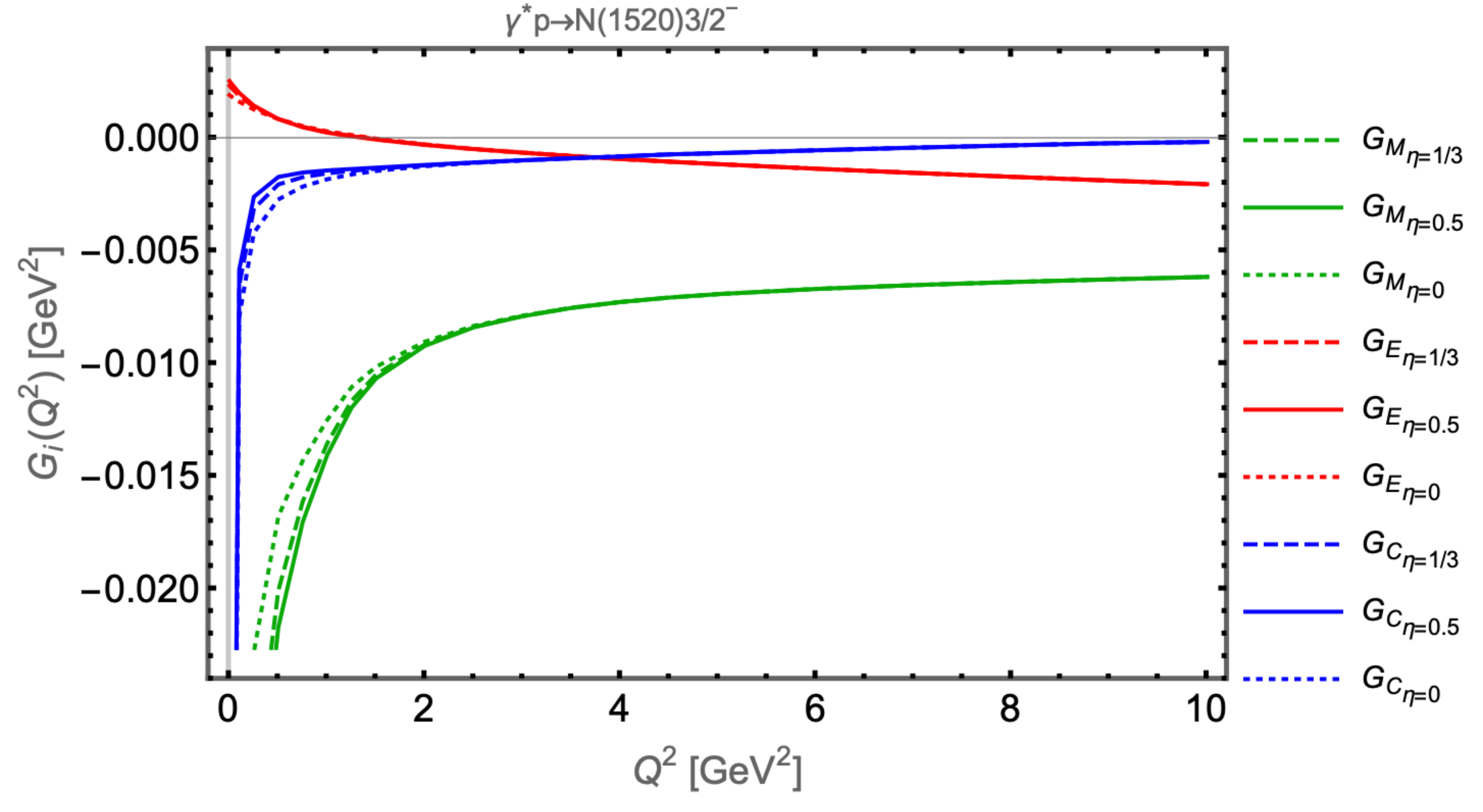
$$S_{1/2} = -\sqrt{2}\mathcal{K}(m_{N^*} + m_N)\frac{|\mathbf{q}|}{Q^2} \times \left(\frac{m_{N^*} - m_N}{m_{N^*} + m_N} F_1^* - \tau F_2^*\right)$$



K. Raya, L.X. Gutiérrez, AB, L. Chang, Z-F. Cui, Y. Lu, C.D. Roberts, J. Segovia, Eur. Phys. J. A 57 (2021) 9, 266

Working on

- G_M^* magnetic dipole, G_E^* electric quadrupole, G_C^* Coulomb quadrupole.
- In collaboration with L. Albino, K. Raya and J. Segovia.



Summary and Scope

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- Thank you.

Backups

where $\mathcal{G}^{+(-)} = \mathbb{I}_{\rm D}(i\gamma_5)$ and, with $T_{\mu\nu} = \delta_{\mu\nu} + \hat{Q}_\mu \hat{Q}_\nu$ $\gamma^\perp_\mu = T_{\mu\nu}\gamma_\nu$, $k^\perp_\mu = T_{\mu\nu}k_\nu$, $\hat{k}^\perp_\mu \hat{k}^\perp_\mu = 1$,

$$\mathcal{X}^1_\rho(k;Q) = i\sqrt{3}\,\hat{k}^\perp_\rho \gamma_5\,,$$
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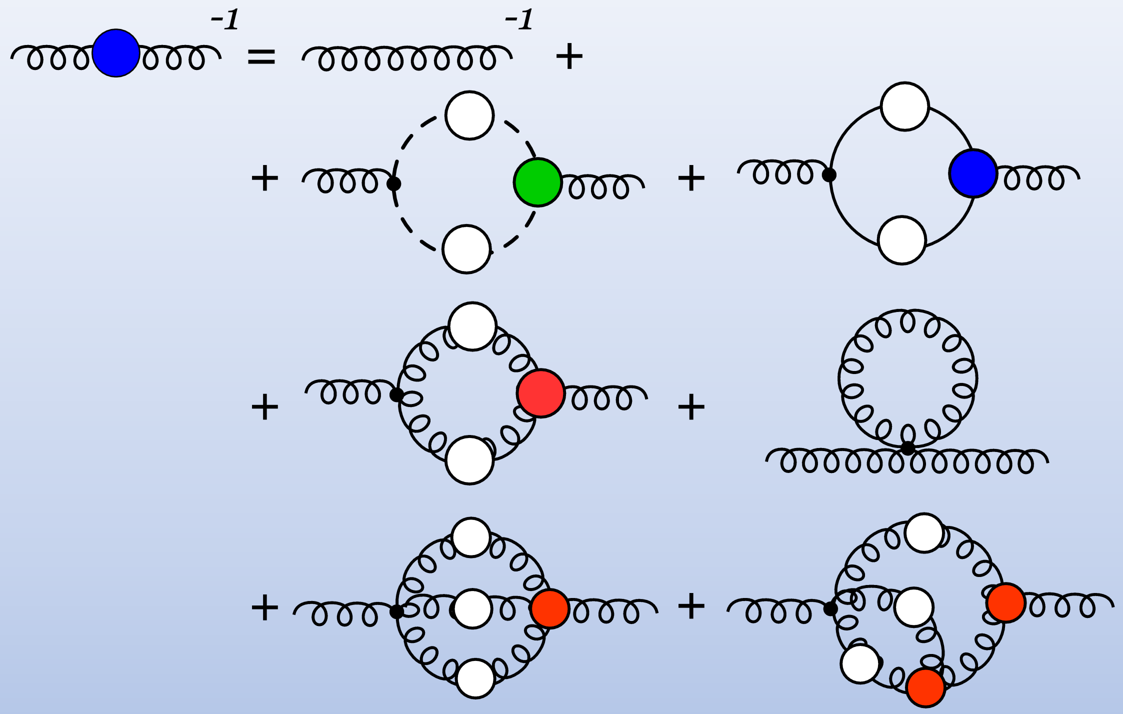
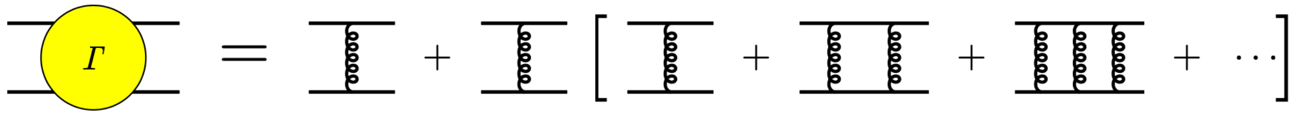
[Liu:2022nku]





Backups

- General Faddeev amplitudes for Baryons

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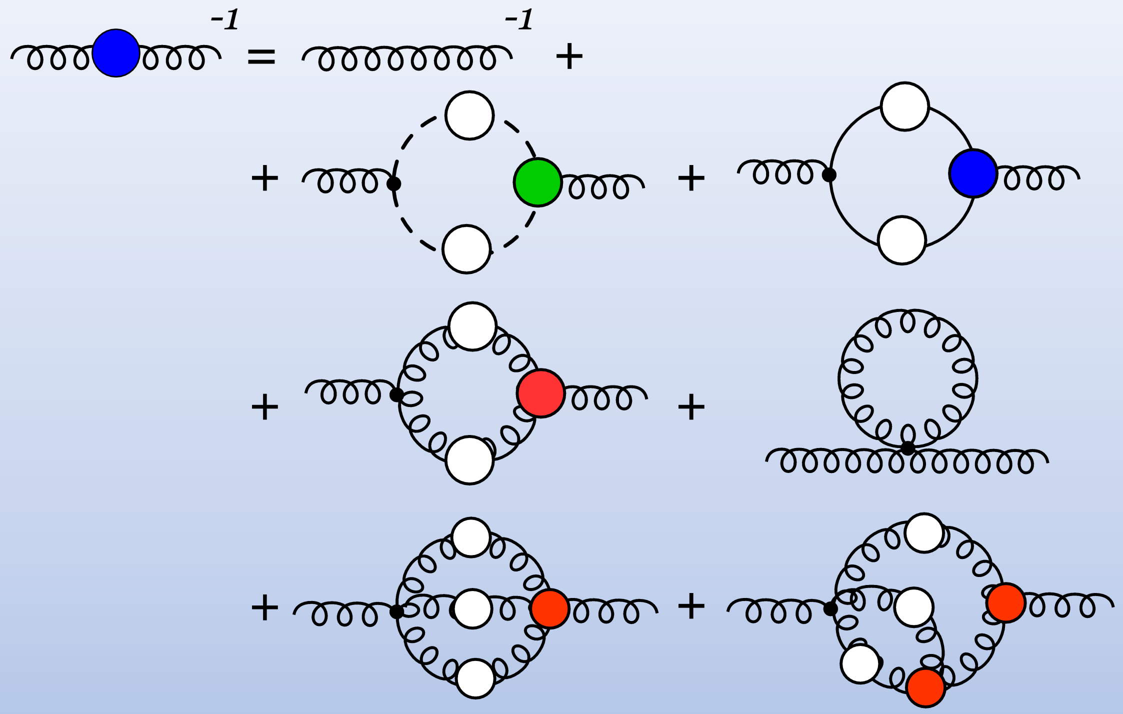

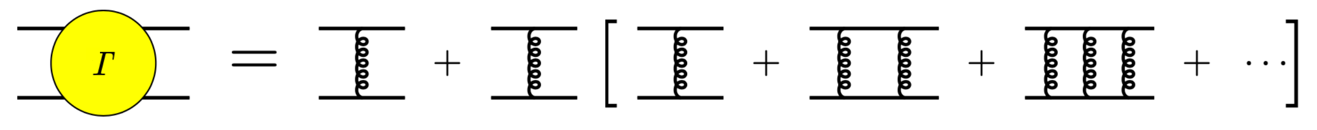

Backups N^*, Δ^* Elastic Form Factor

- We need to normalize the Faddeev amplitudes in the transition diagram and for this we need the properties of the elastic form factors.

- The current for N EFF:

$$J_\mu(K,Q) = ie\,\Lambda_+(P_f)\,\Gamma_\mu(K,Q)\,\Lambda_+(P_i)$$

- The current for Δ EFF:

[Segovia:2014aza, Nicmorus:2010sd]

$$J_{\mu,\lambda\omega}(K,Q) = \Lambda_+(P_f)\,R_{\lambda\alpha}(P_f)\,\Gamma_{\mu,\alpha\beta}(K,Q)\,\Lambda_+(P_i)\,R_{\beta\omega}(P_i)$$

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$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4m_N^2}F_2(Q^2)$$

$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

$$\tau_B = \frac{Q^2}{4m_B^2}$$

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- The G_{E0} form factor in terms of the F_i:

$$G_{E0}(Q^2) = \left(1 + \frac{2\tau_\Delta}{3}\right)\left(F_1^* - \tau_\Delta F_2^*\right) - \frac{\tau_\Delta}{3}\left(1+\tau_\Delta\right)\left(F_3^* - \tau_\Delta F_4^*\right),$$

Backups Photon hits the quark

[Raya:2023wye]

- With the general structure:

$$\Pi^r(l; P_f, P_i) = q_r \int_l \bar{\psi}^{f(r)}\, S(l_f^+)\, \Gamma_\mu^{q\gamma}(Q)\, S(l_i^+)\, \psi^{i(r)}\, \Delta^r(-l)$$

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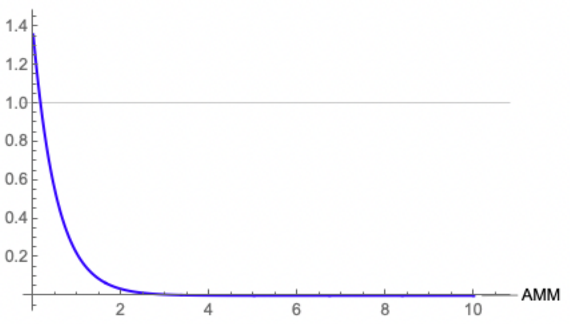

- Where η is the contribution of the anomalous magnetic moment of the quark (AMM), the origin of this contribution is related to DCSB.

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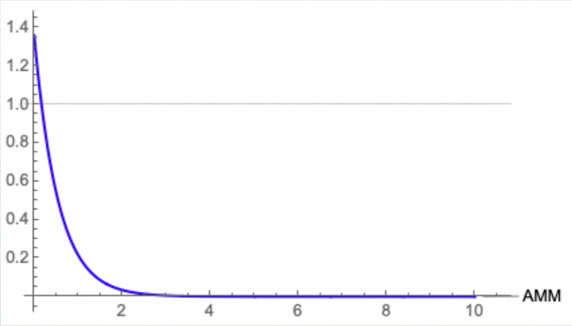

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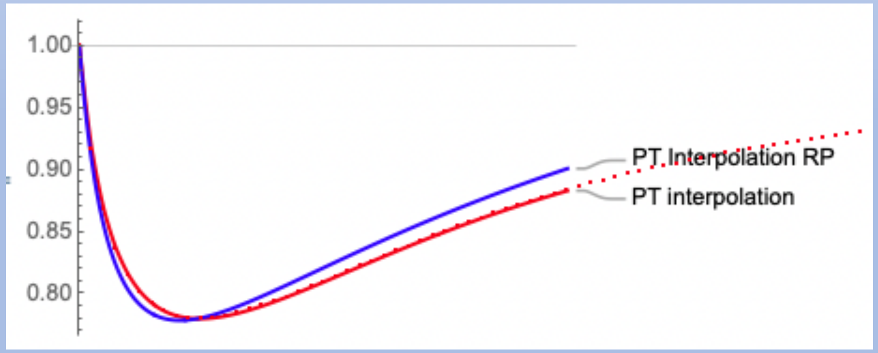

- Where P_T is a dressing function of the inhomogeneous BSE and its behavior recovers the tree-level vertex.

Backups Photon hits the diquark

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$$\Pi^{(d_1,d_2)}(l;P_f,P_i) = q_{d_2|d_1} \int_l \psi^{f(d_2)}\, S(l)\, \bar{\psi}^{i(d_1)}\, \Delta^r(-l)\, \Gamma^{(d_1,d_2)\gamma}_{\mu,x}(l_f^-, l_i^-)\, \Delta^{d_1}(l_i^-)$$

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[Wilson:2011aa, Raya:2017ggu]

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$$\Gamma^{(qq,1^+)}_{\mu,\rho\sigma}(k_f = K + Q/2, k_i = K - Q/2) = \sum_{j=1}^{3} T^j_{\mu,\rho\sigma}(K,Q) F_j^{(qq,1^+)}(Q^2),$$

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- Transition:

$$\Gamma^{10}_{\rho\mu}(k_2,k_1) = \Gamma^{01}_{\rho\mu}(-k_2,k_1) = \Gamma^{01}_{\mu\rho}(k_1,k_2),$$

$$\Gamma^{01}_{\mu\rho}(k_1,k_2) = \frac{g_{01}}{m_{(qq,1^+)}} \epsilon_{\mu\rho\alpha\beta} k_{1\alpha} k_{2\beta} G^{01}(Q^2).$$

[Wilson:2011aa, Raya:2017ggu]