Disconnected 3-Point Functions Using Wilson Loops on the Lattice

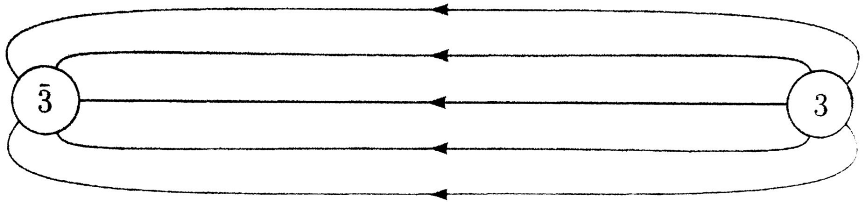
Jacob Peyton
New Mexico State University
for HUGS 2024

$$\mathcal{L}_{\mathrm{QCD}} = \bar{\psi}_i \left(i\gamma^\mu (D_\mu)_{ij} - m\,\delta_{ij} \right) \psi_j - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$$

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

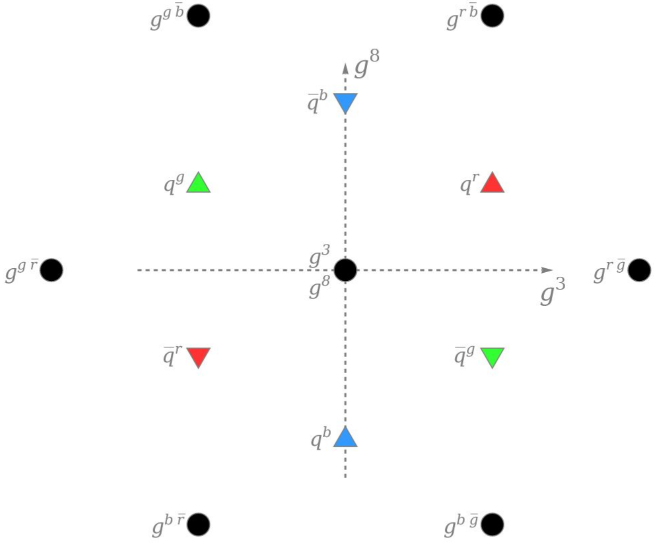
$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}$$

$$\lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$



c. Peskin & Schroeder

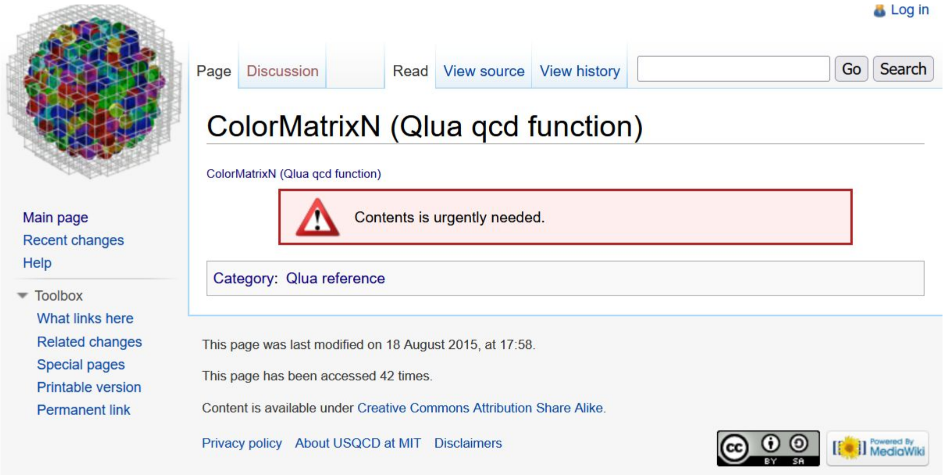
Quantum Chromodynamics
is *weird*…

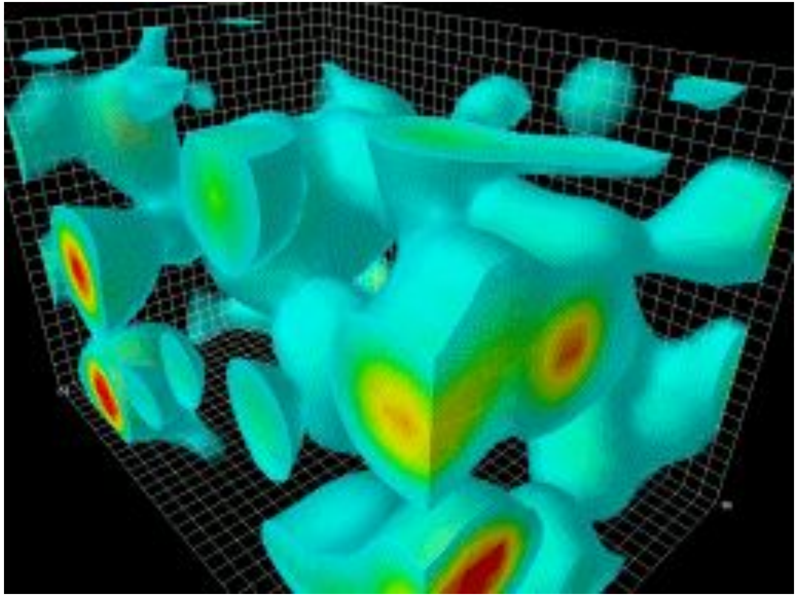


c. Wikipedia (r. 2024)

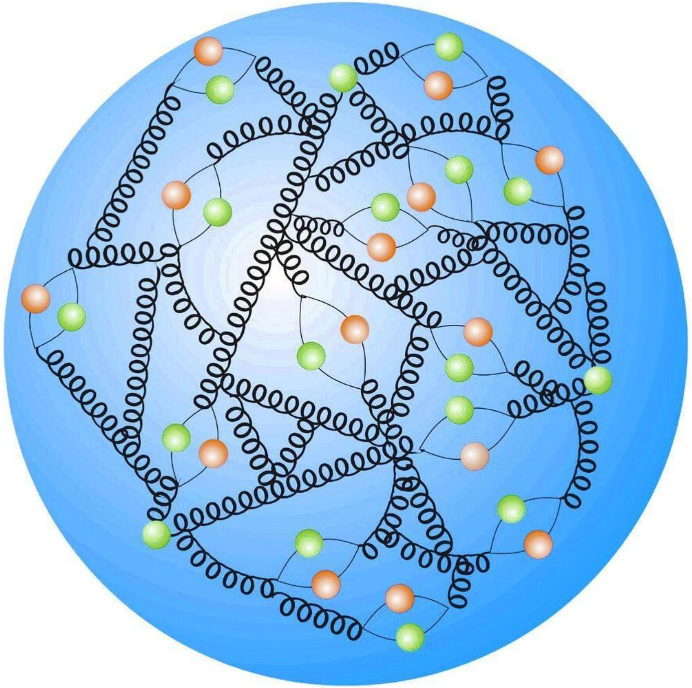
Introduction

…**Lattice**
Quantum Chromodynamics
is ***worse.***

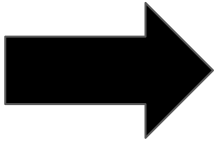


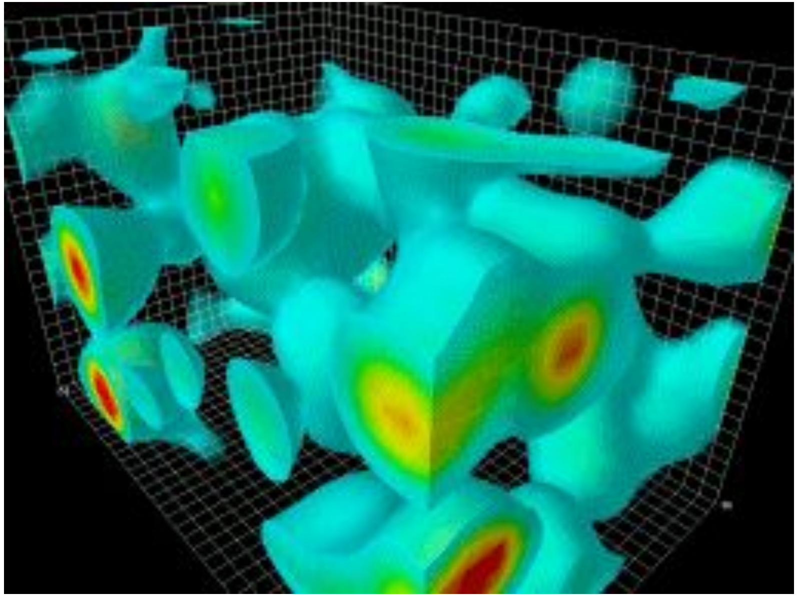


c. 2004, Derek Leinweber



c. Forbes (r. 2024)





c. 2004, Derek Leinweber

Creating a QCD Lattice in 3 Steps:

$$e^{iHt} \rightarrow e^{-Ht}$$

Creating a QCD Lattice in 3 Steps:

1. Use a Wick Rotation to go to Euclidean Time.

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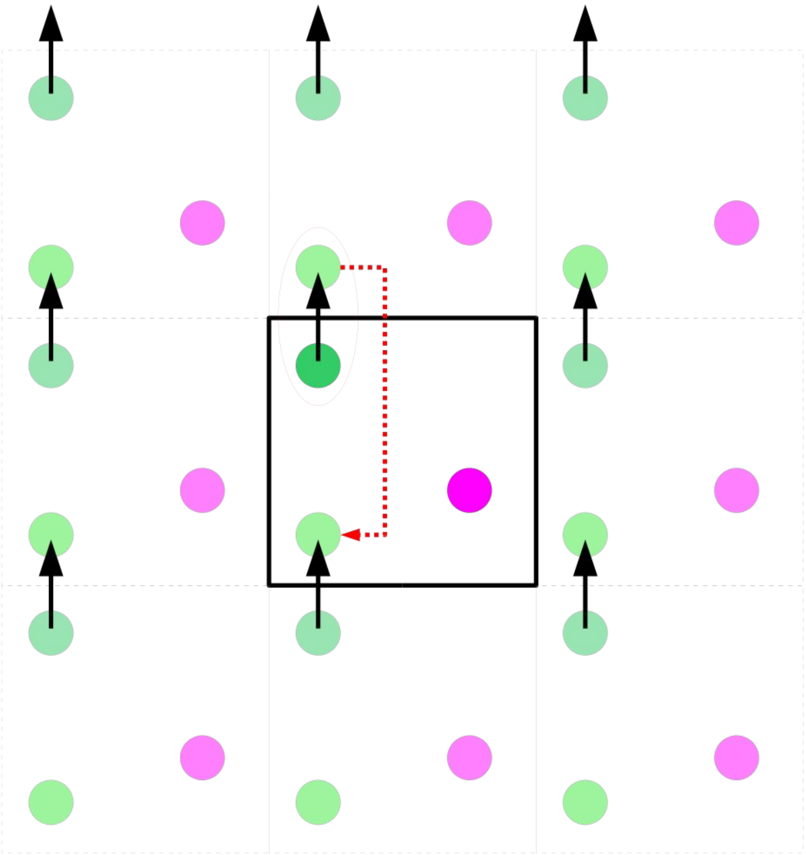

c. Wikipedia (r. 2024)

Creating a QCD Lattice in 3 Steps:

1. Use a Wick Rotation to go to Euclidean Time.
2. Implement Periodic Boundary Conditions to make Spacetime Finite but Continuous.

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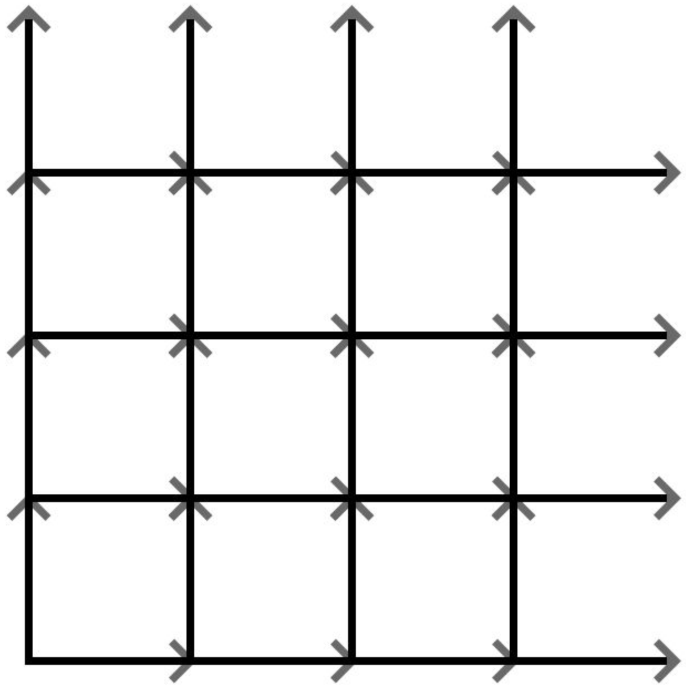

Creating a QCD Lattice in 3 Steps:

1. Use a Wick Rotation to go to Euclidean Time.
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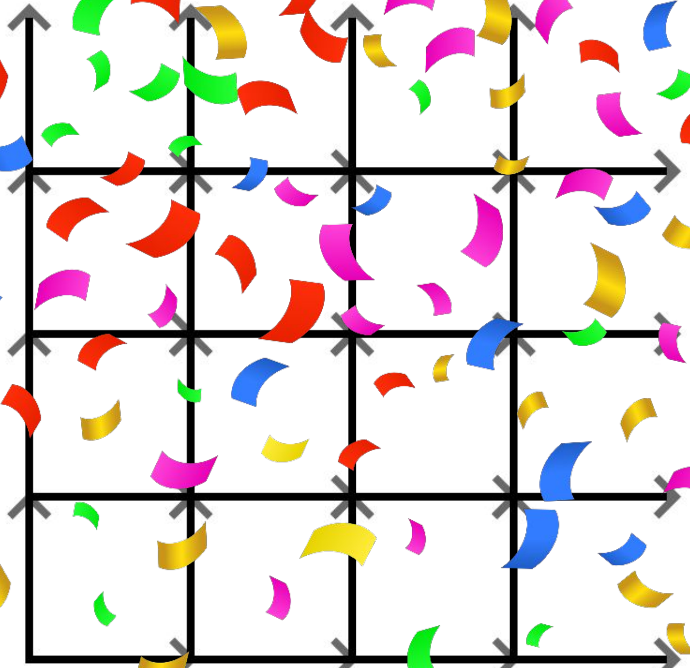
3. Discretize Spacetime.



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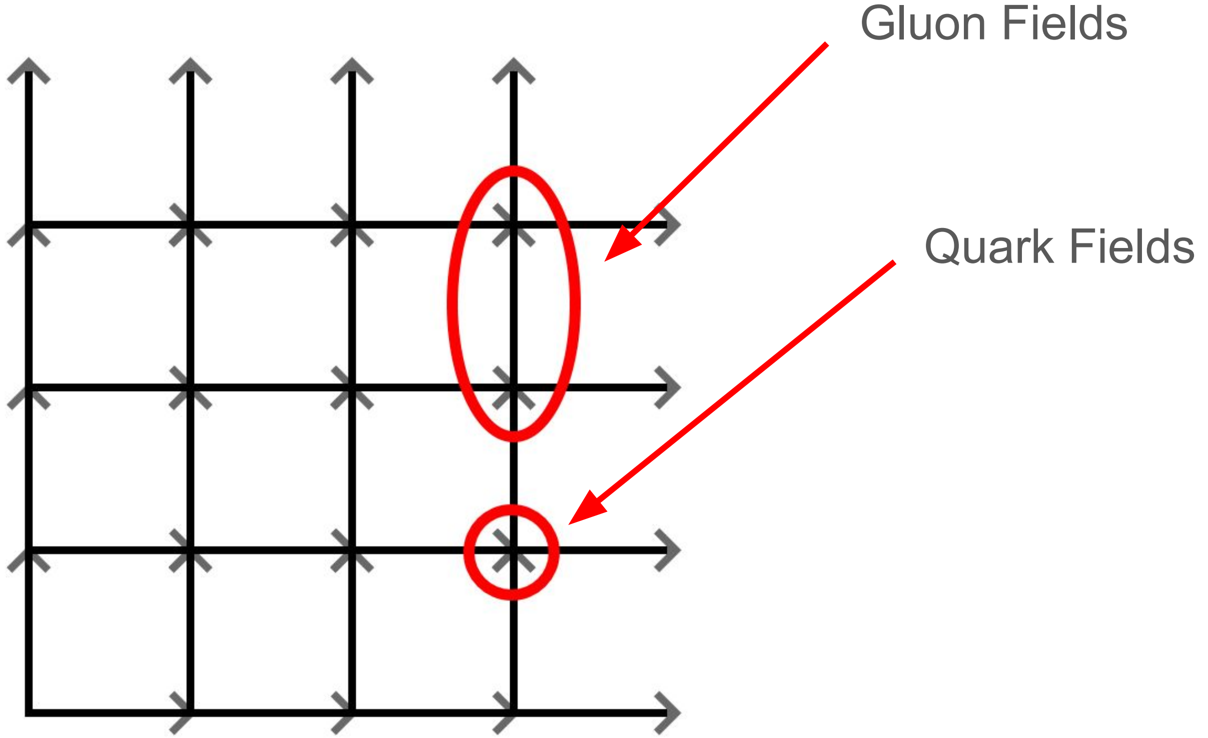
Gluon Fields
Quark Fields

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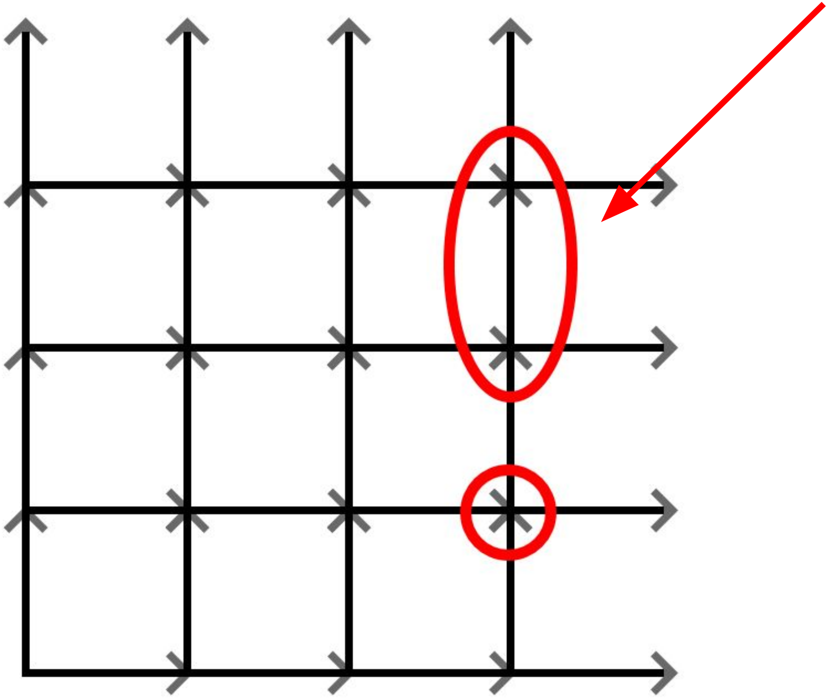
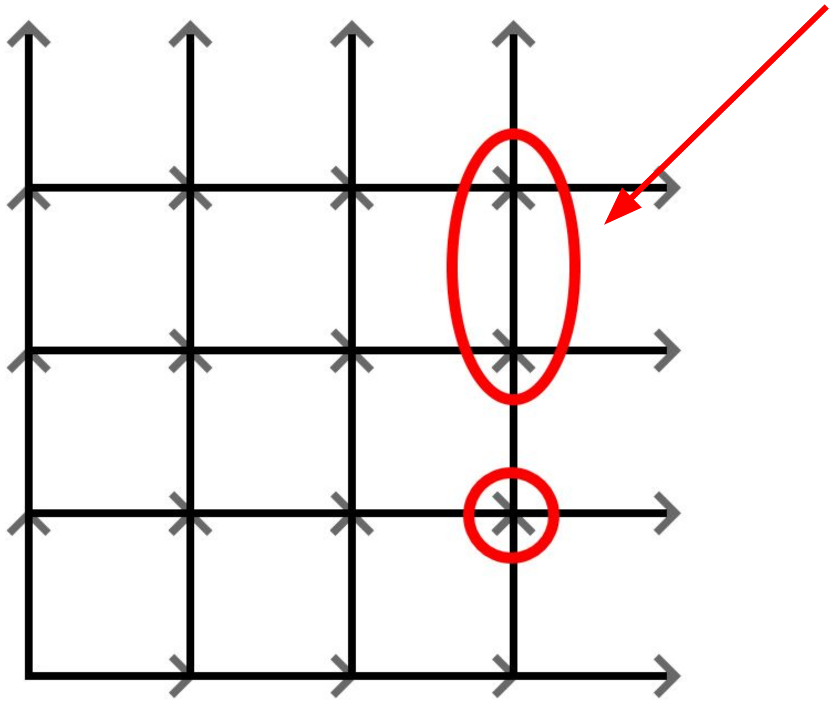


Gluon Fields:

$$\mathcal{A}_\alpha = t_a \mathcal{A}_\alpha^a \equiv t_1 \mathcal{A}_\alpha^1 + t_2 \mathcal{A}_\alpha^2 + \cdots + t_8 \mathcal{A}_\alpha^8$$



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$$\rightarrow \begin{pmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{pmatrix}$$

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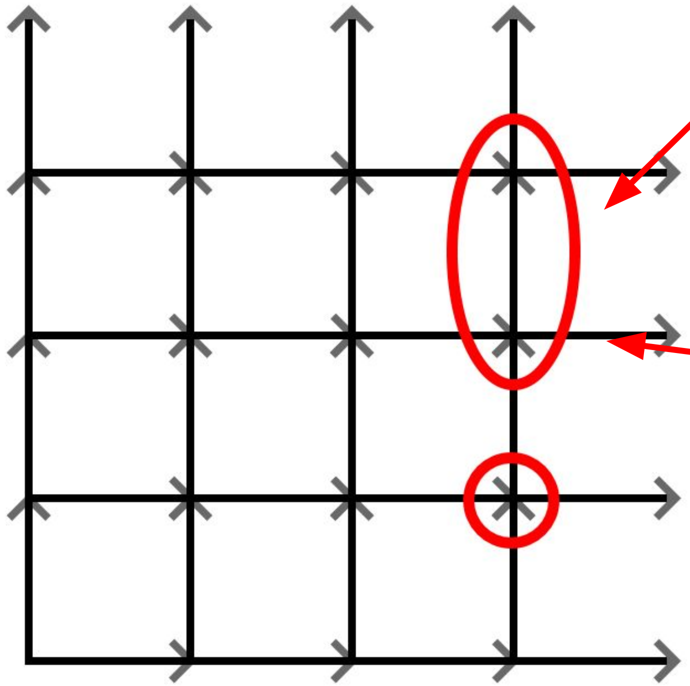

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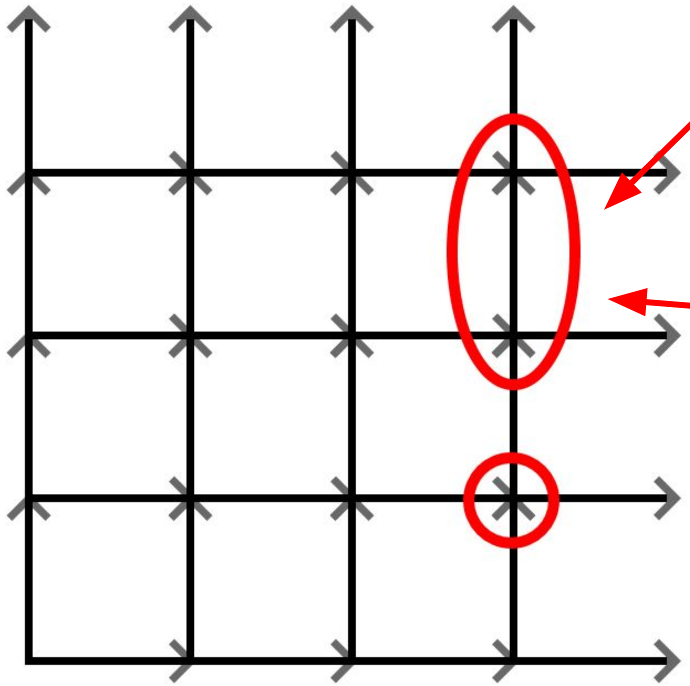
$$e^{i2g \begin{pmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{pmatrix}} \rightarrow \begin{pmatrix} c_1 & c_2 & c_3 \\ c_4 & c_5 & c_6 \\ c_7 & c_8 & c_9 \end{pmatrix}$$



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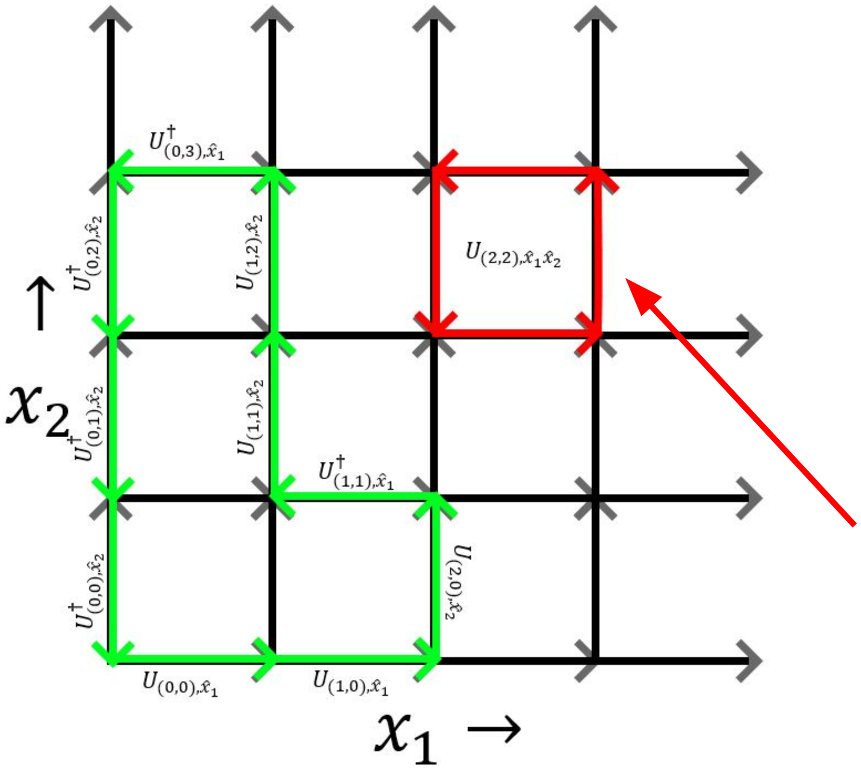
$$\mathcal{A}_\alpha = t_a \mathcal{A}^a_\alpha \equiv t_1 \mathcal{A}^1_\alpha + t_2 \mathcal{A}^2_\alpha + \cdots + t_8 \mathcal{A}^8_\alpha$$

$$U_{x,\hat{\mu}} = \begin{pmatrix} c_1 & c_2 & c_3 \\ c_4 & c_5 & c_6 \\ c_7 & c_8 & c_9 \end{pmatrix}$$



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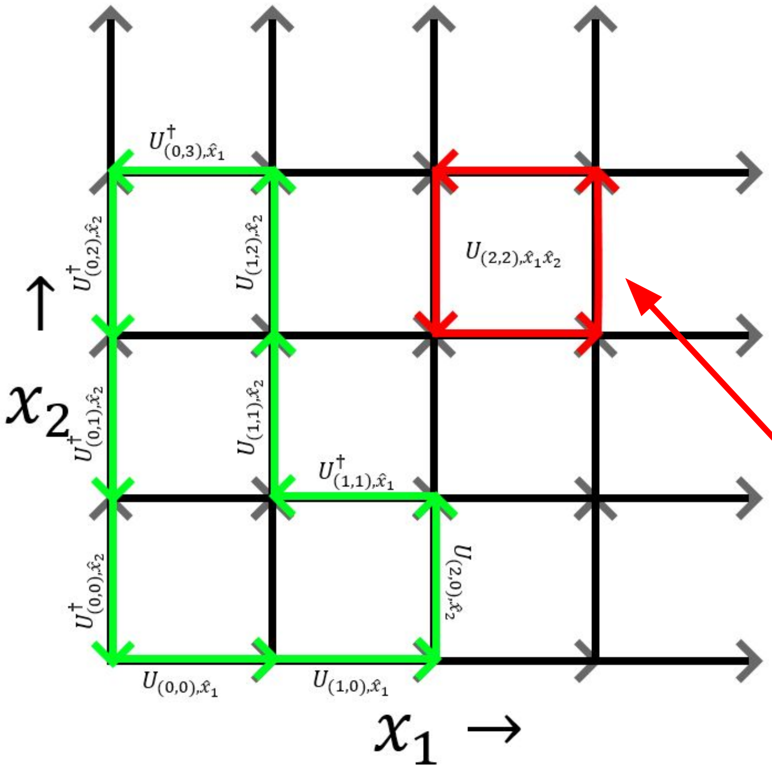
$$U_{x,\mu\nu} = \mathrm{Tr}\left(U_{x,\hat{\mu}} U_{x+\hat{\mu},\hat{\nu}} U^{\dagger}_{x+\hat{\nu},\hat{\mu}} U^{\dagger}_{x,\hat{\nu}}\right)$$



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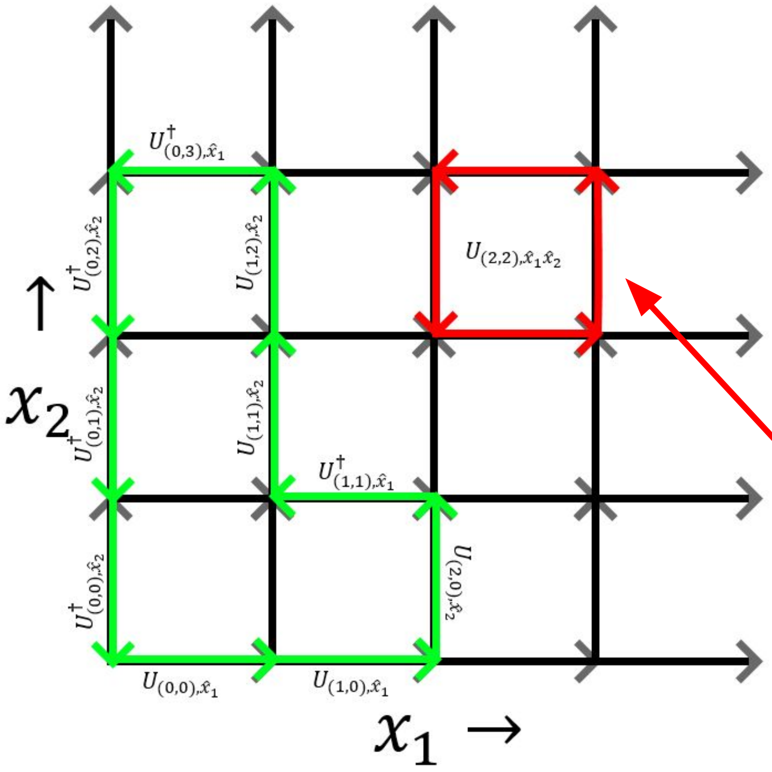
$$S_{G-SU(3)} = \frac{6}{g^2} \sum_x \sum_{\mu<\nu} \left(1 - \mathrm{Re}\left(U_{x,\mu\nu}\right)/3\right)$$

$$U_{x,\mu\nu} = \mathrm{Tr}\left(U_{x,\hat{\mu}} U_{x+\hat{\mu},\hat{\nu}} U^\dagger_{x+\hat{\nu},\hat{\mu}} U^\dagger_{x,\hat{\nu}}\right)$$



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$$e^{-\beta \Delta S} > \mathrm{rand}(0,1)$$

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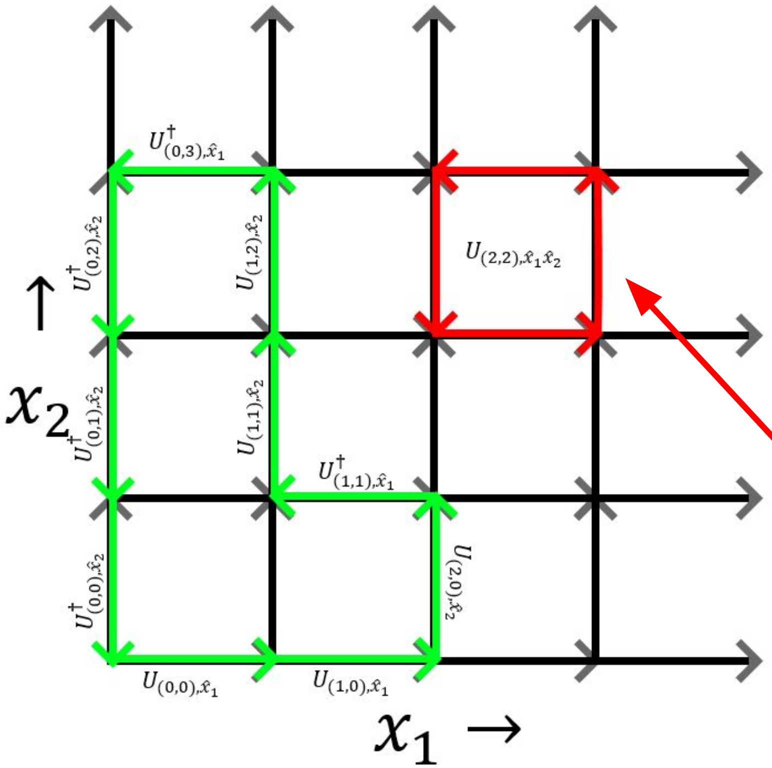

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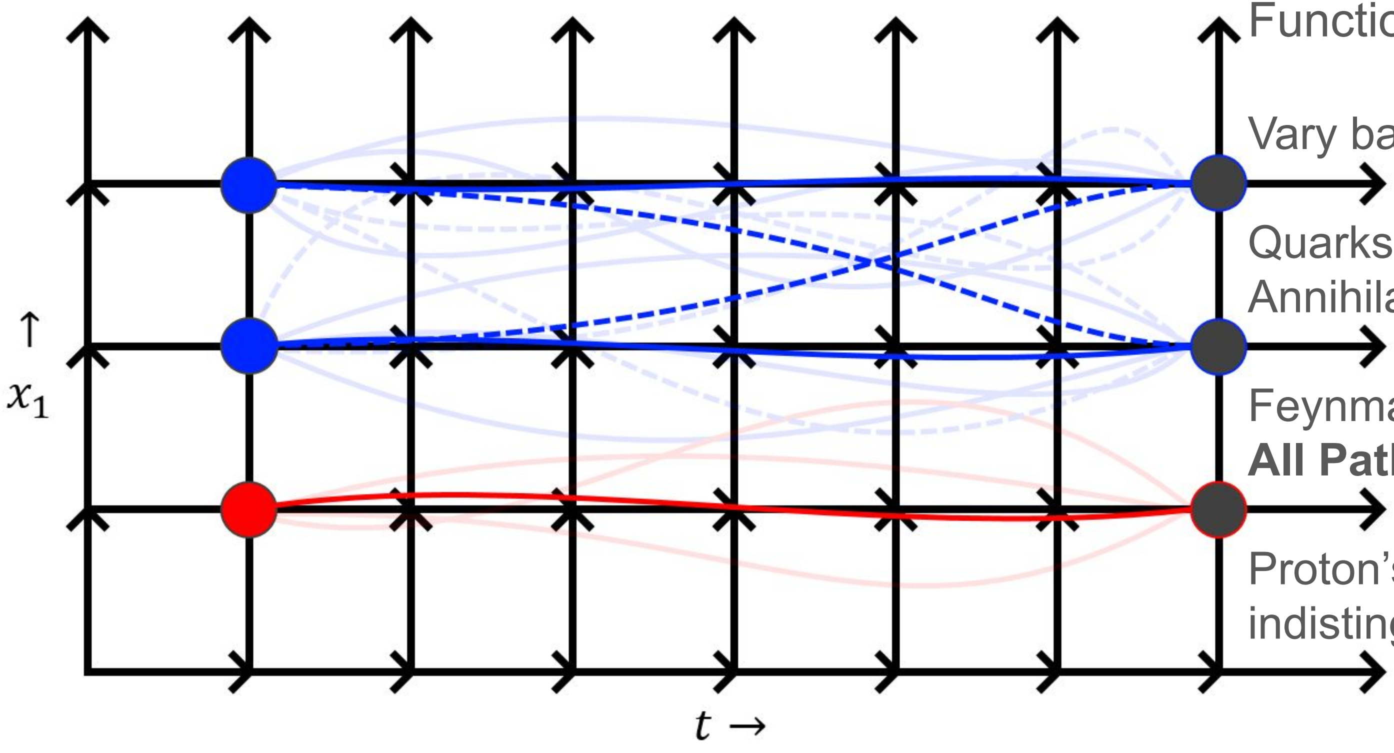
$$C(\vec{x},t) = \langle \mathcal{O}(\vec{x},t)\mathcal{O}^{\dagger}(\vec{0},0)\rangle$$



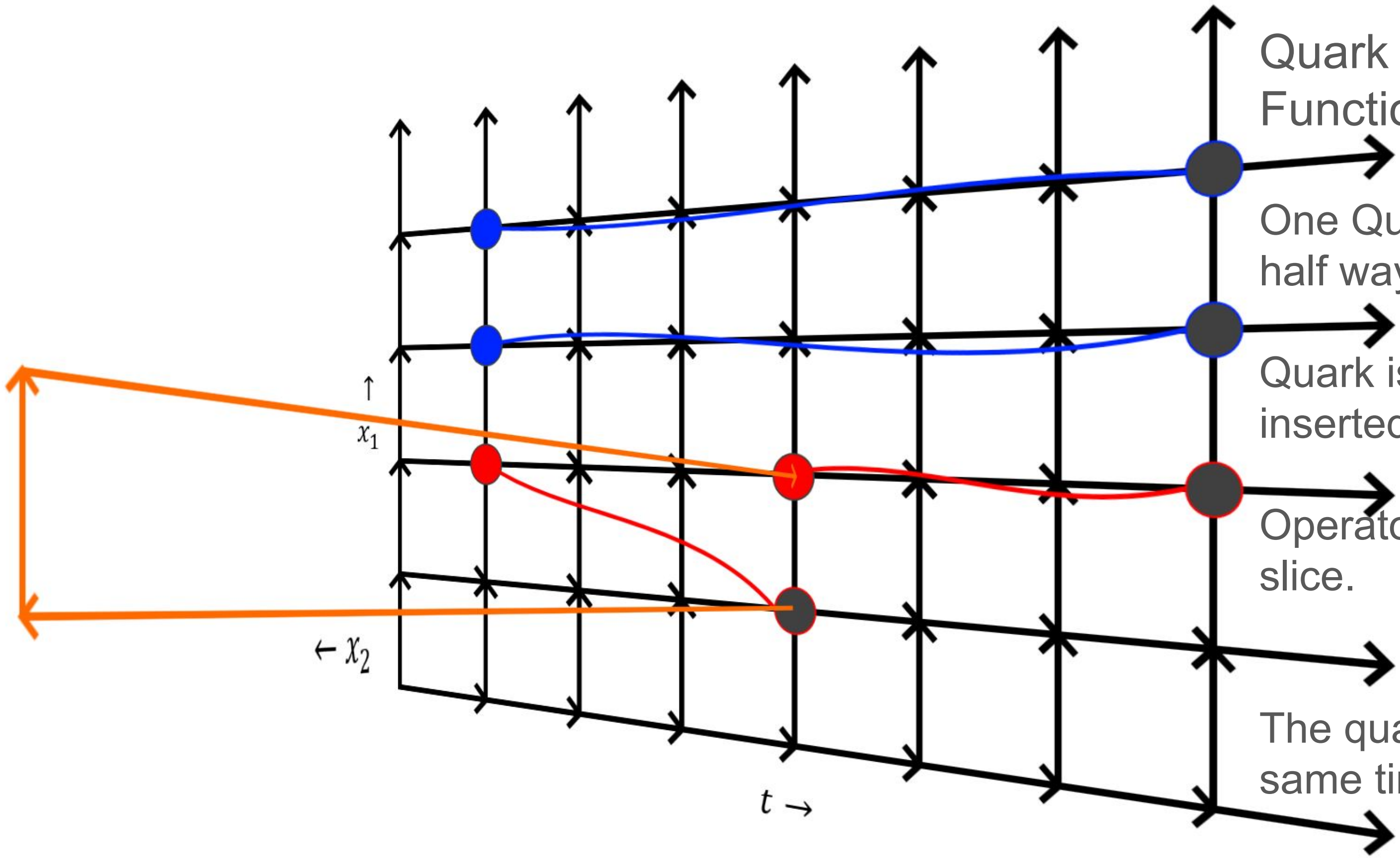
Quark Fields in 2pt Functions:

Vary based on the simulation

Quarks created at one point, Annihilated at another.

Feynman Path Integral
All Paths Used

Proton's Up Quarks are indistinguishable.



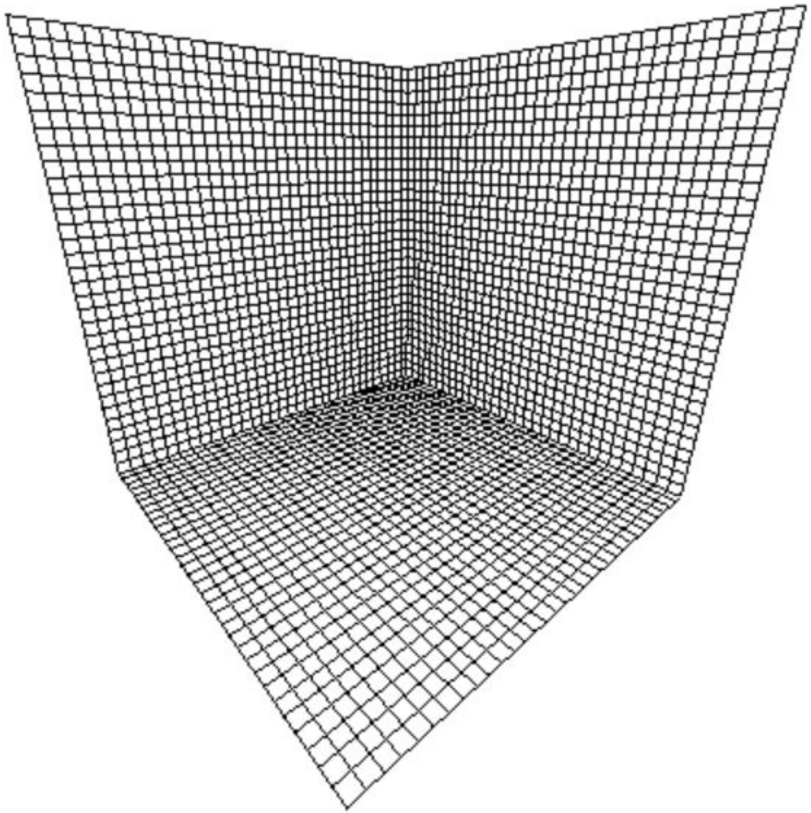
Quark Fields in 3pt Functions:

One Quark's path is 'broken' half way through.

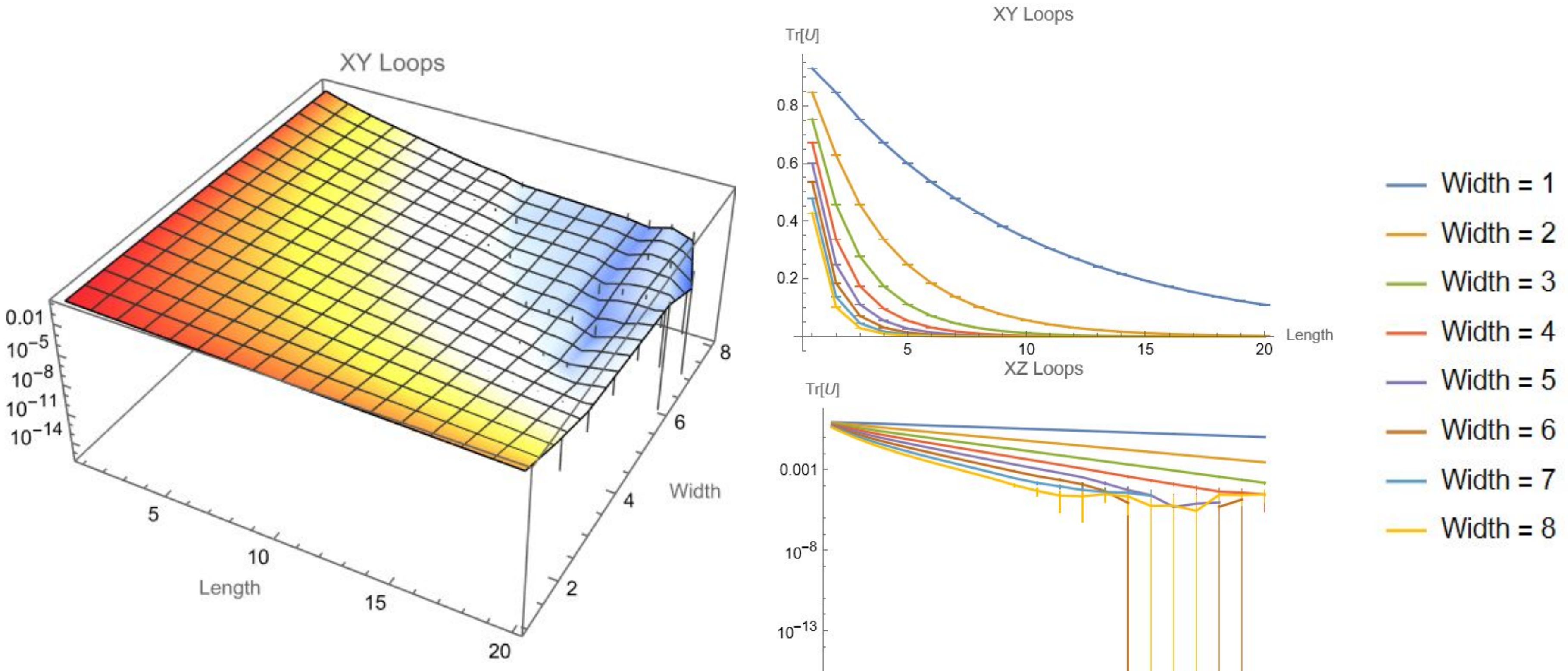
Quark is annihilated, operator inserted.

Operator exists in a single time slice.

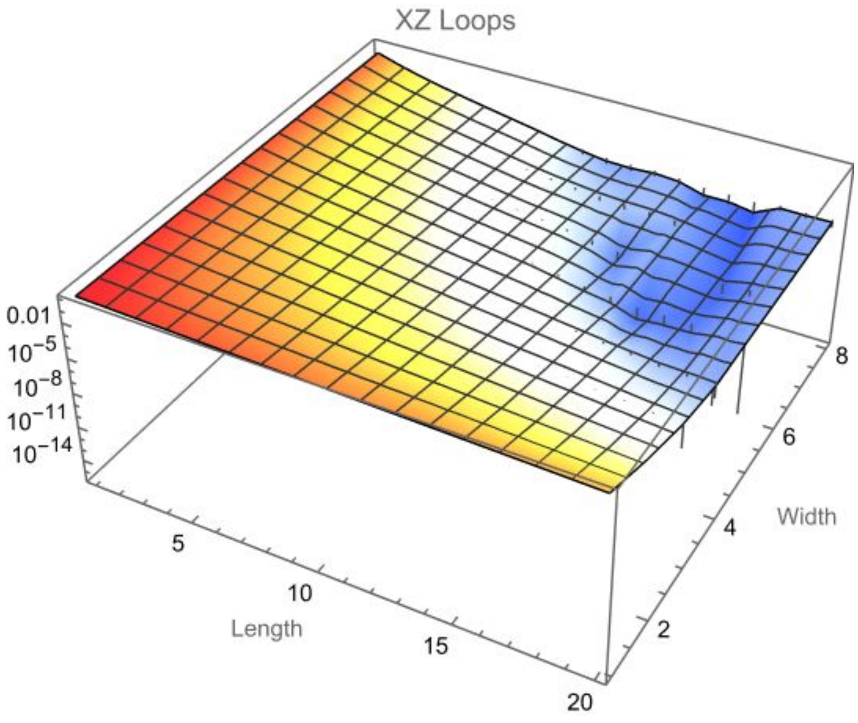
The quark is recreated at the same time but displaced.



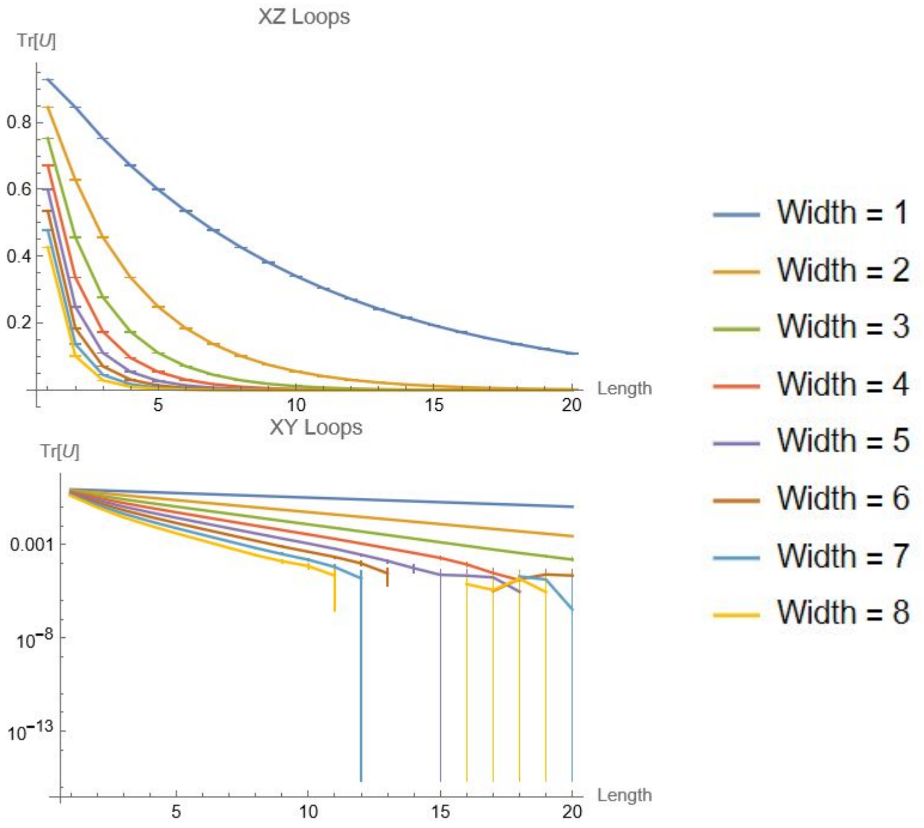
- Size: $32^3 \times 96$
- 0.114 Fermi Lattice Spacing
- 3.65 Fermi on a Side
- Pion Mass: 317 MeV
- Temperature: 55.5 KeV



XY Loops
0.01
10^{-5}
10^{-8}
10^{-11}
10^{-14}
5
10
15
20
Length
2
4
6
8
Width
XY Loops
Tr[U]
0.8
0.6
0.4
0.2
5
10
15
20
Length
XZ Loops
Tr[U]
0.001
10^{-8}
10^{-13}
5
10
15
20
Length
Width = 1
Width = 2
Width = 3
Width = 4
Width = 5
Width = 6
Width = 7
Width = 8



XZ Loops
0.01
10^{-5}
10^{-8}
10^{-11}
10^{-14}
5
10
Length
15
20
2
4
6
8
Width



XZ Loops
Tr[U]
0.8
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0.4
0.2
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Length
XY Loops
Tr[U]
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A Database of 2-Point Functions

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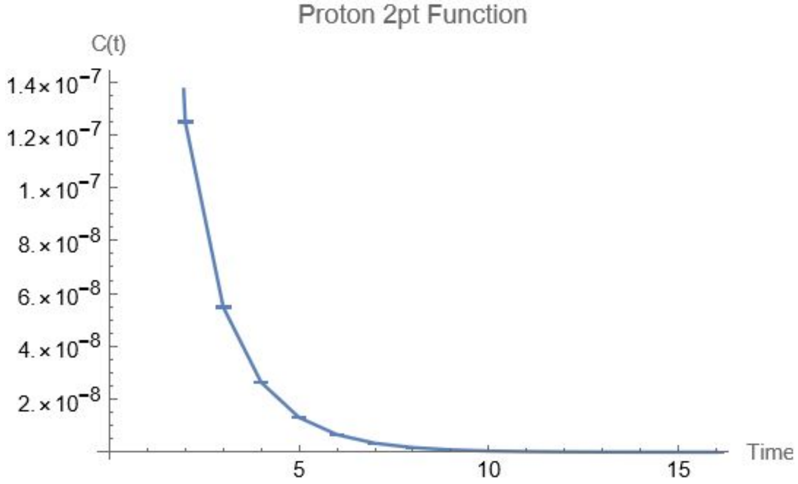
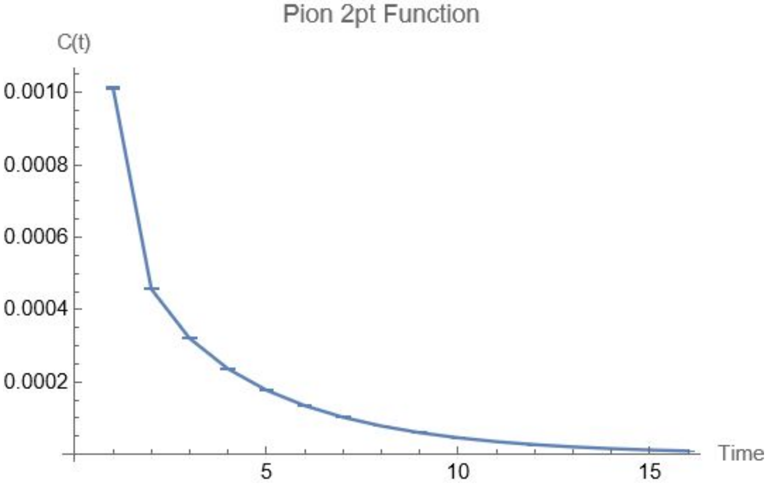
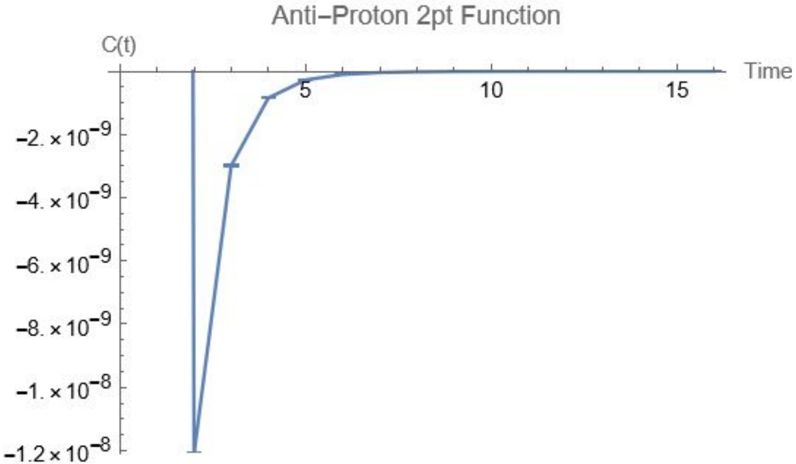
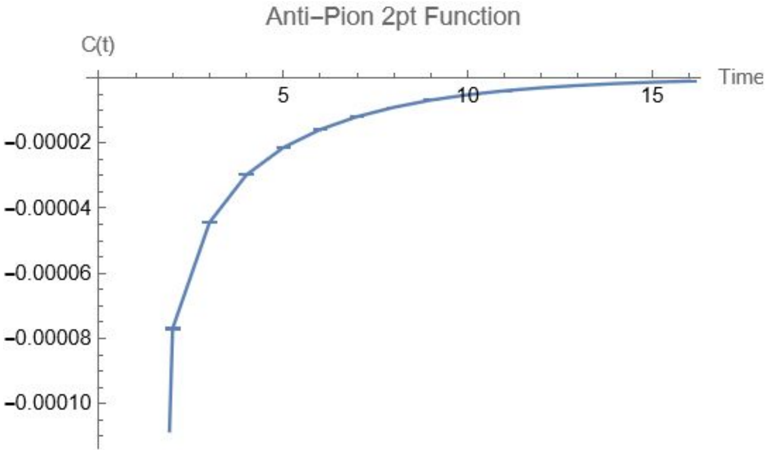
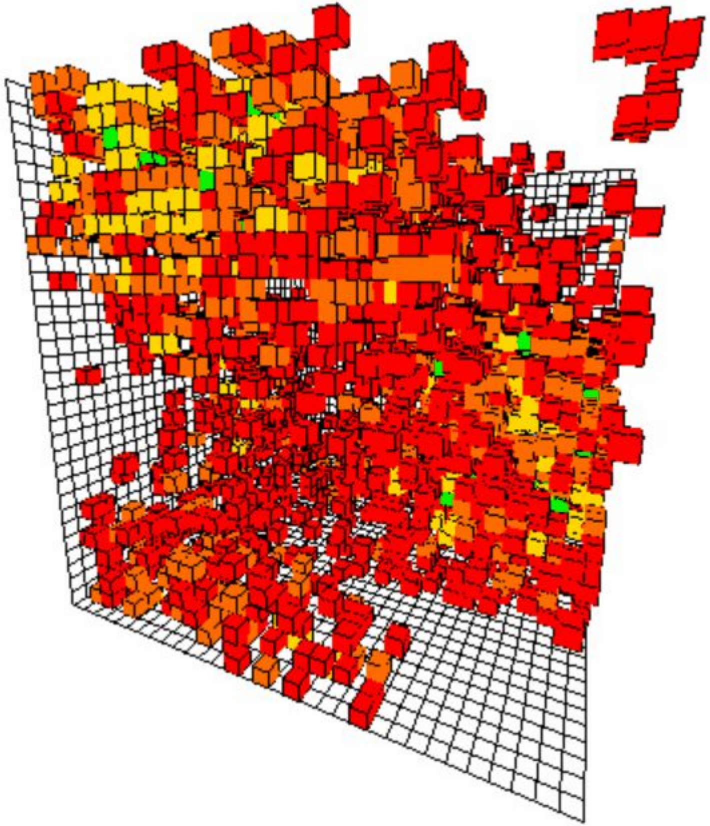






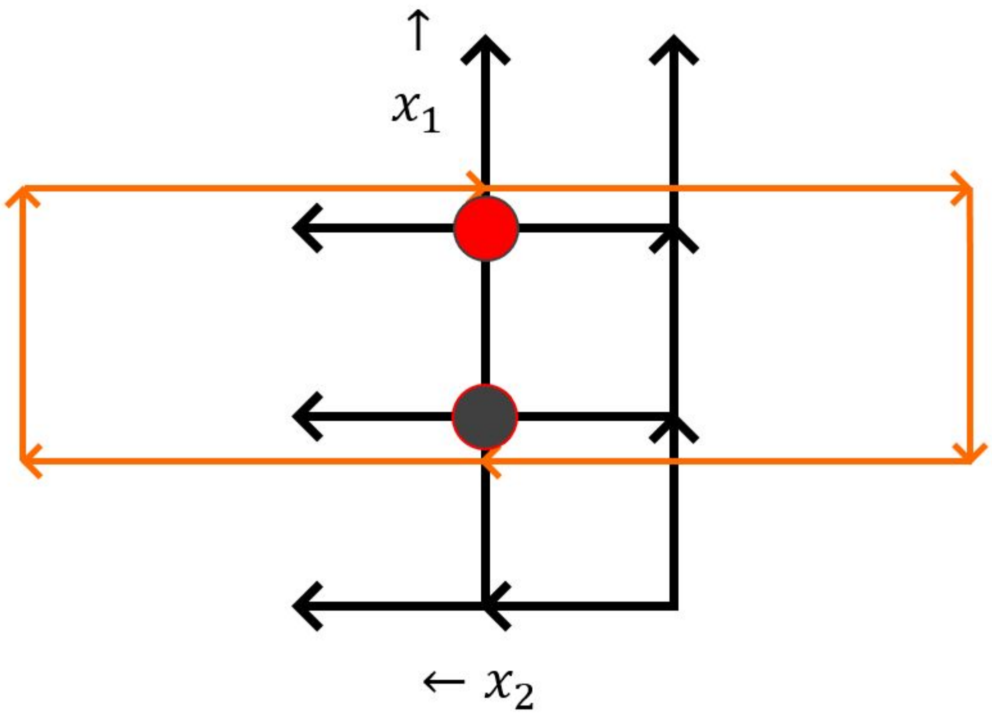




Correlation Fraction

- Contribution to 3-Point Function (but least interesting)
- Dipole Approximation to Distribution Functions at small Longitudinal Momentum Fraction x

$$\frac{\langle C(t_s)U(t_i)\rangle}{\langle C(t_s)\rangle\langle U(t_i)\rangle}$$



So Much Jackknifing (Data Analysis)

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So Much Jackknifing (Data Analysis)

1. Average over 12 2pt Sources for all 4 Particles
2. Offset Wilson Loops based on Source Times and Average
3. Combine 2pt Source Data and Offset Wilson Loops for all 4 Particles and all 12 Sources per Configuration and then Average
4. Perform Jackknife Analysis on Wilson Loops over 968 Configurations
5. Perform Jackknife Analysis on 2pt Functions over 968 Configurations
6. Perform Jackknife Analysis on Combined Wilson Loops and 2pt Functions over 968 Configurations
7. Calculate Correlation Fraction
8. Perform Jackknife Analysis on Correlation Fraction over 968 Configurations

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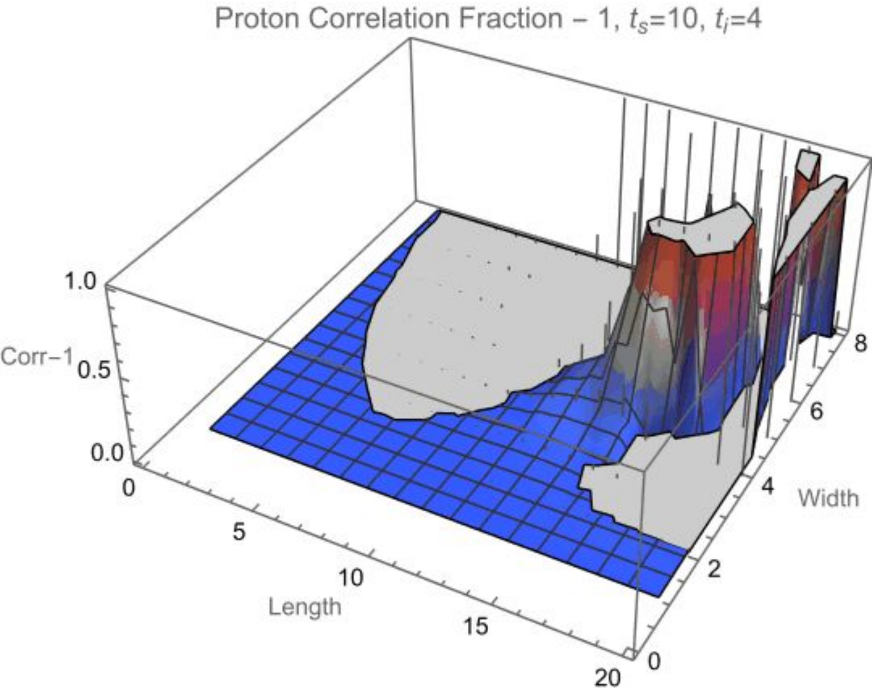
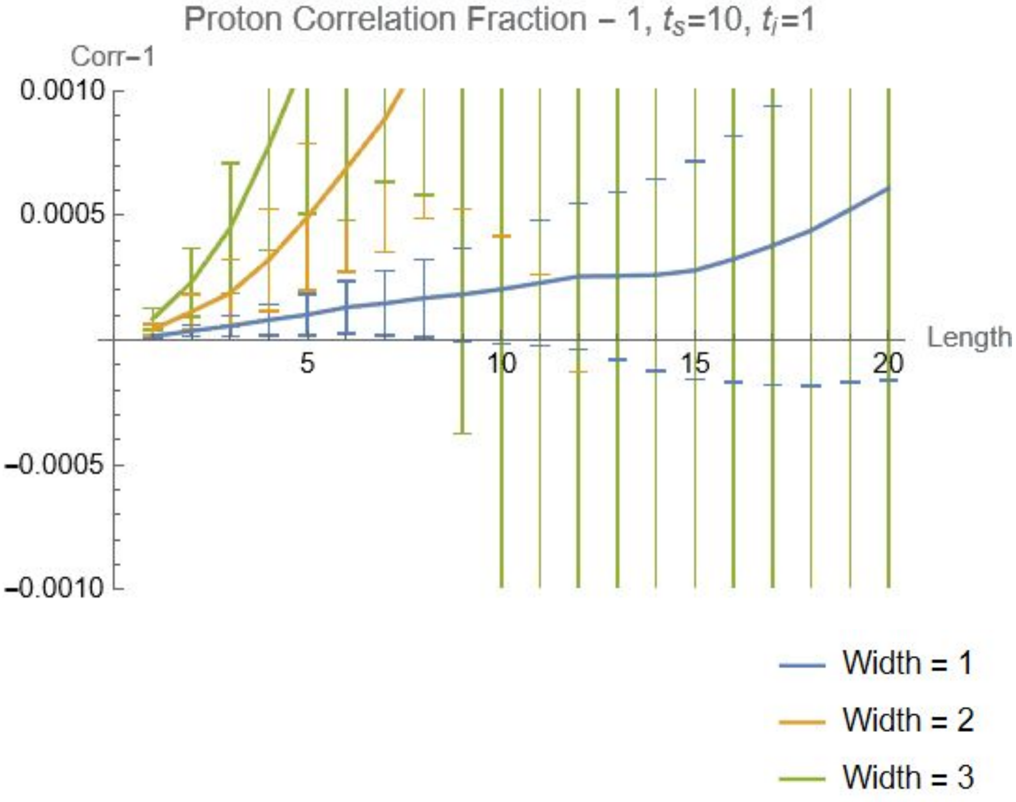
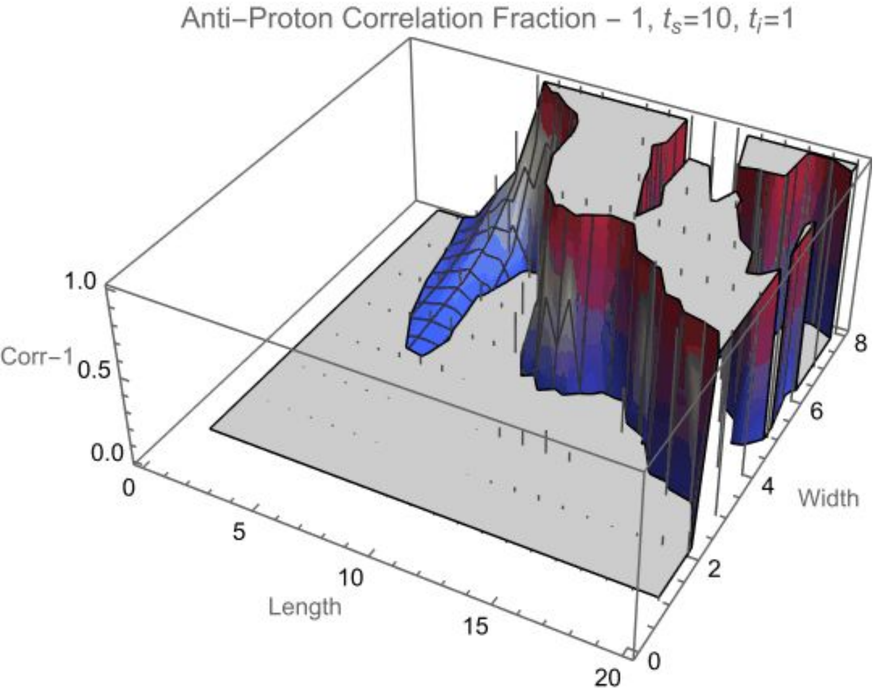
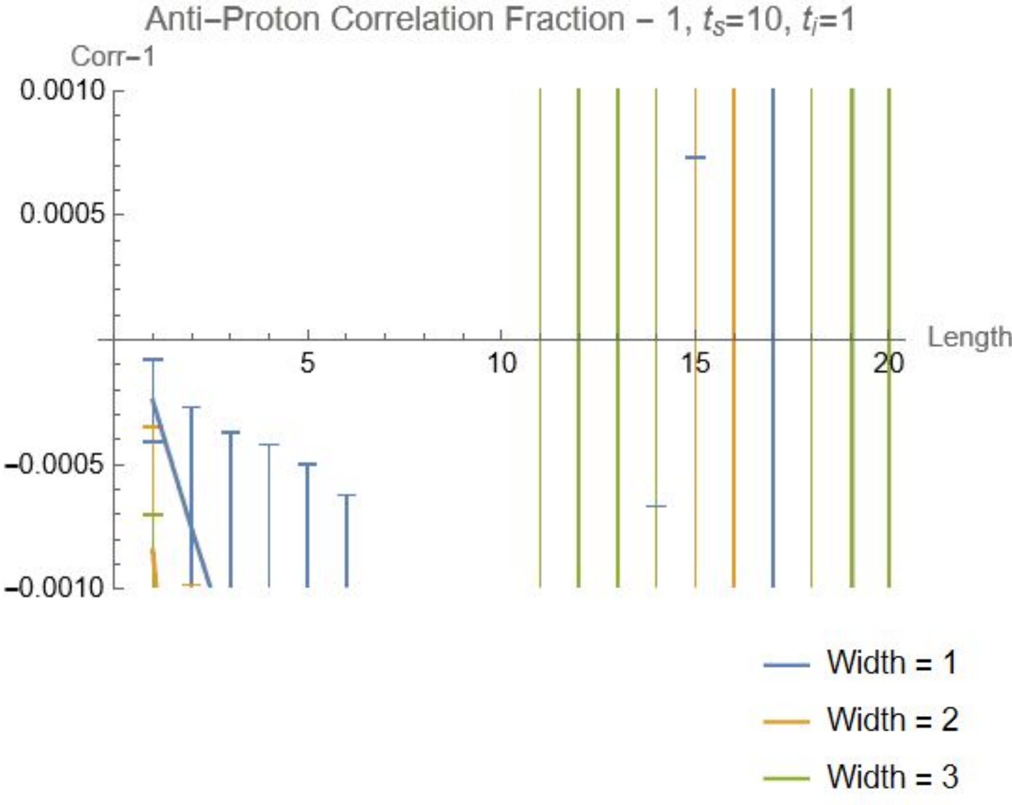


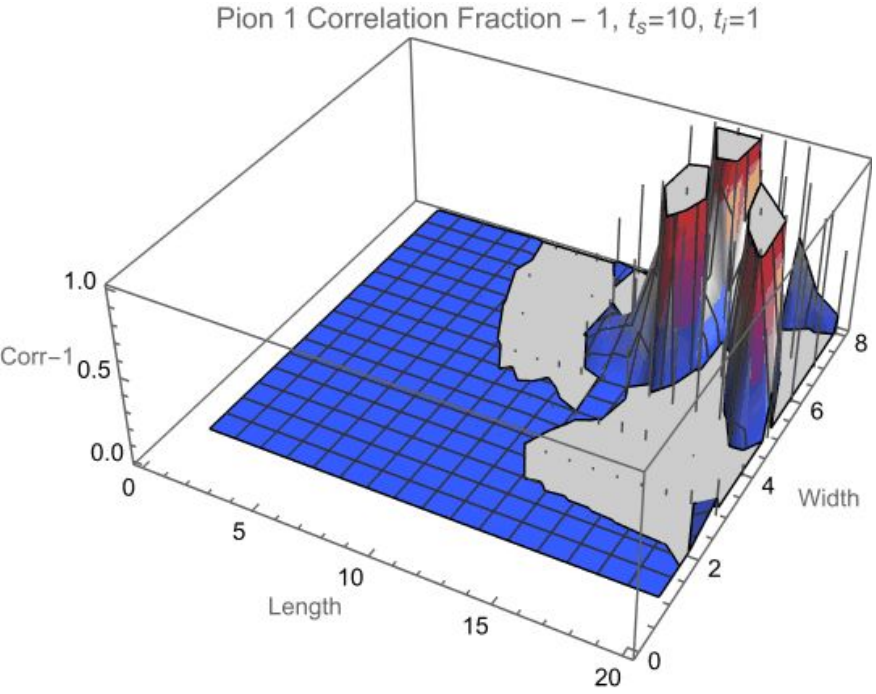


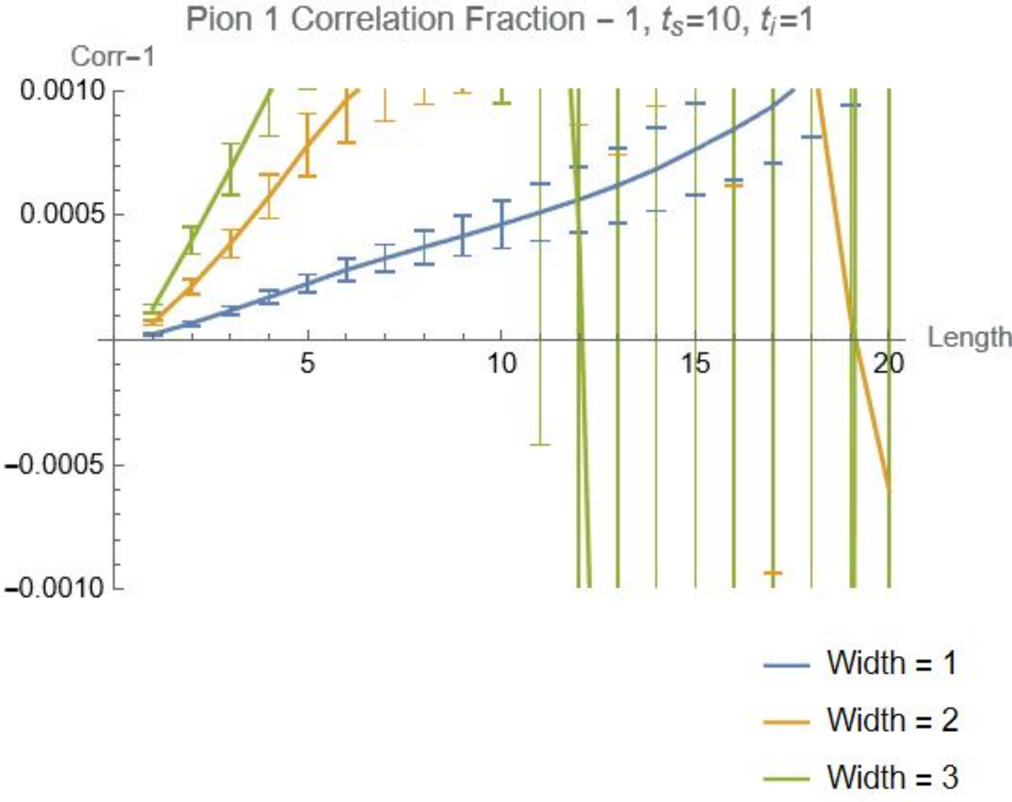
Correlator > 1



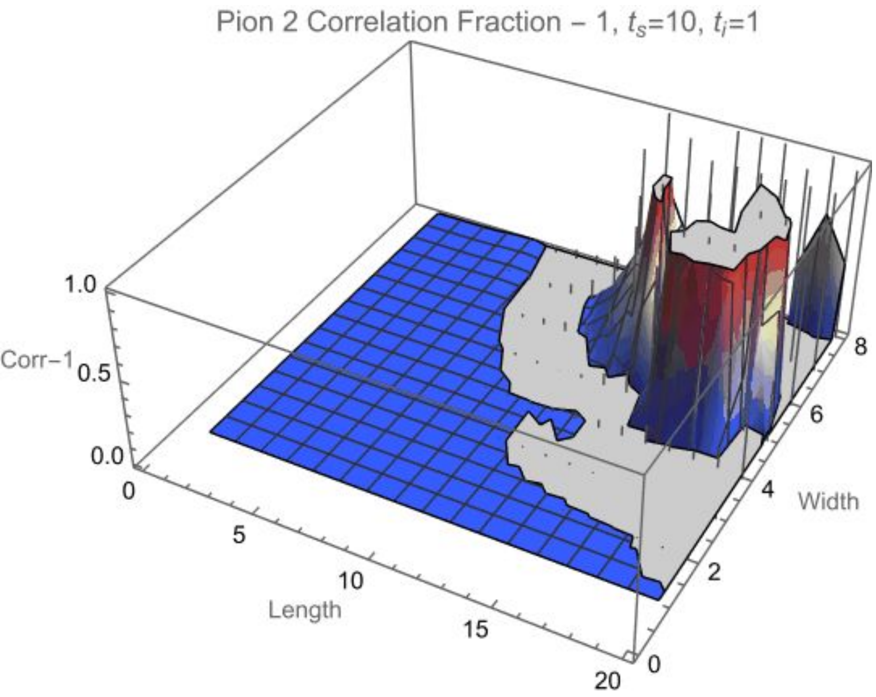


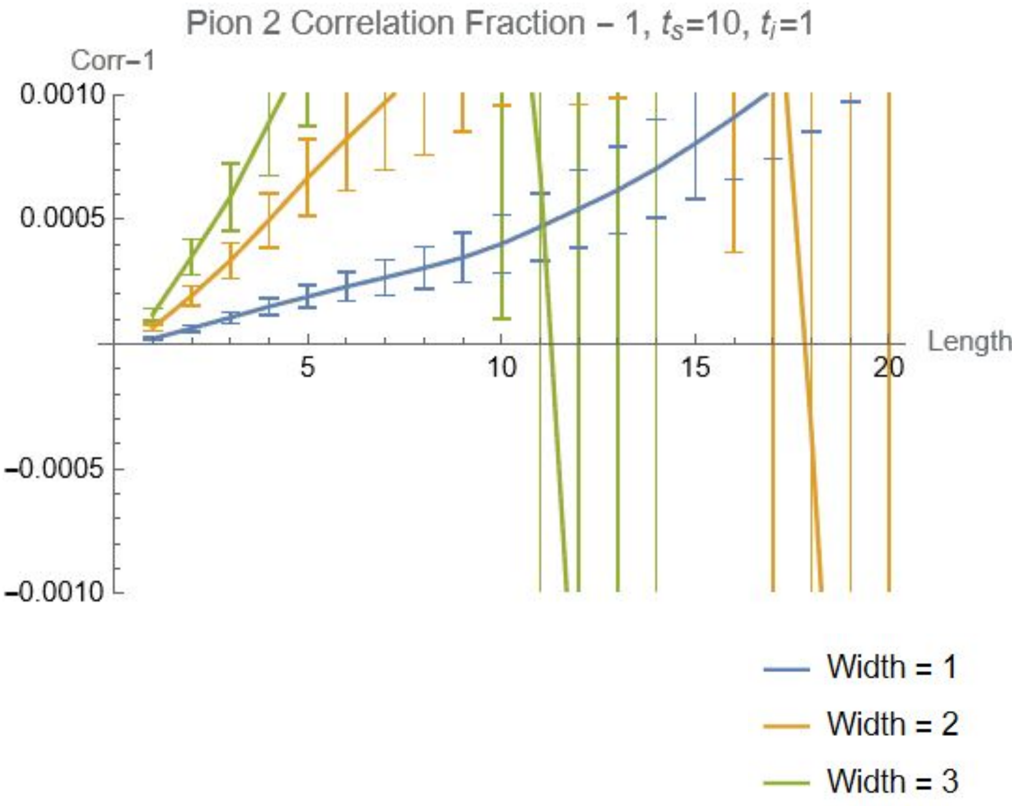
Correlator > 1





Correlator > 1

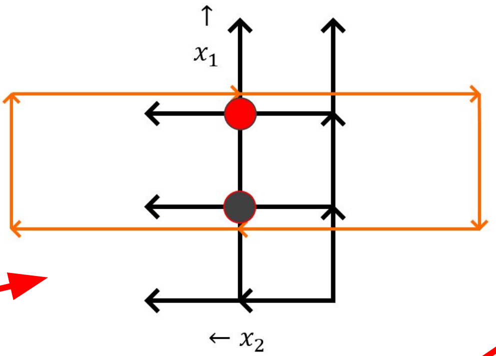


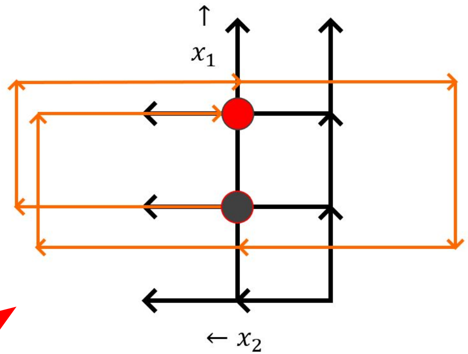


Thank You!

Bonus: 3pt Functions and TMDs

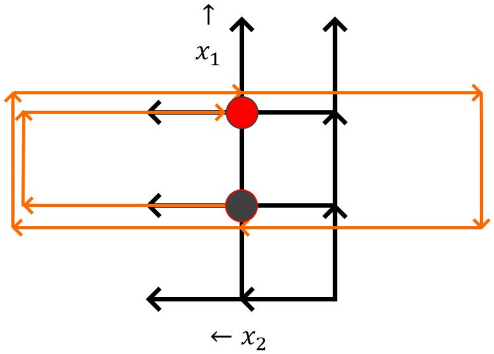
Operator analysis of pT -widths of TMDs
Boer et al, 2015
arXiv 1503.03760v2





$$f_1^{[+]}(x,p_T^2) = f_1^{[-]}(x,p_T^2) = f_1(x,p_T^2) + \delta f_1^{(B1)}(x,p_T^2)$$

$$f_1^{[\square+]}(x,p_T^2) = f_1(x,p_T^2) + 9\delta f_1^{(B1)}(x,p_T^2)$$



$$f_1^{[(\square)+]}(x,p_T^2) = f_1(x,p_T^2) + \delta f_1^{(B1)}(x,p_T^2) + \delta f_1^{(B2)}(x,p_T^2)$$

Parton distribution function for quarks in an s-channel approach

Hautmann & Soper

arXiv 0702077v1

$$x f_{q/p}(x,\mu) = \frac{N_c}{3\pi^4} \int d\boldsymbol{b} \int d\boldsymbol{\Delta}\ \theta(\Delta^2\mu^2 > a^2)\ \frac{\Xi(\boldsymbol{b},\boldsymbol{\Delta})}{\Delta^4}.$$

$1/Px \propto$

Fourier Conjugate Relation