Electroweak
Two Nucleon Matrix Elements

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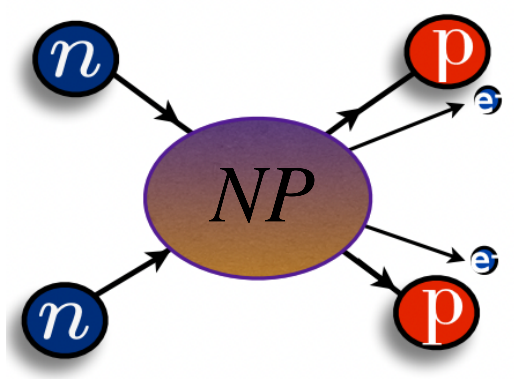
Nuclear Theory
New Physics



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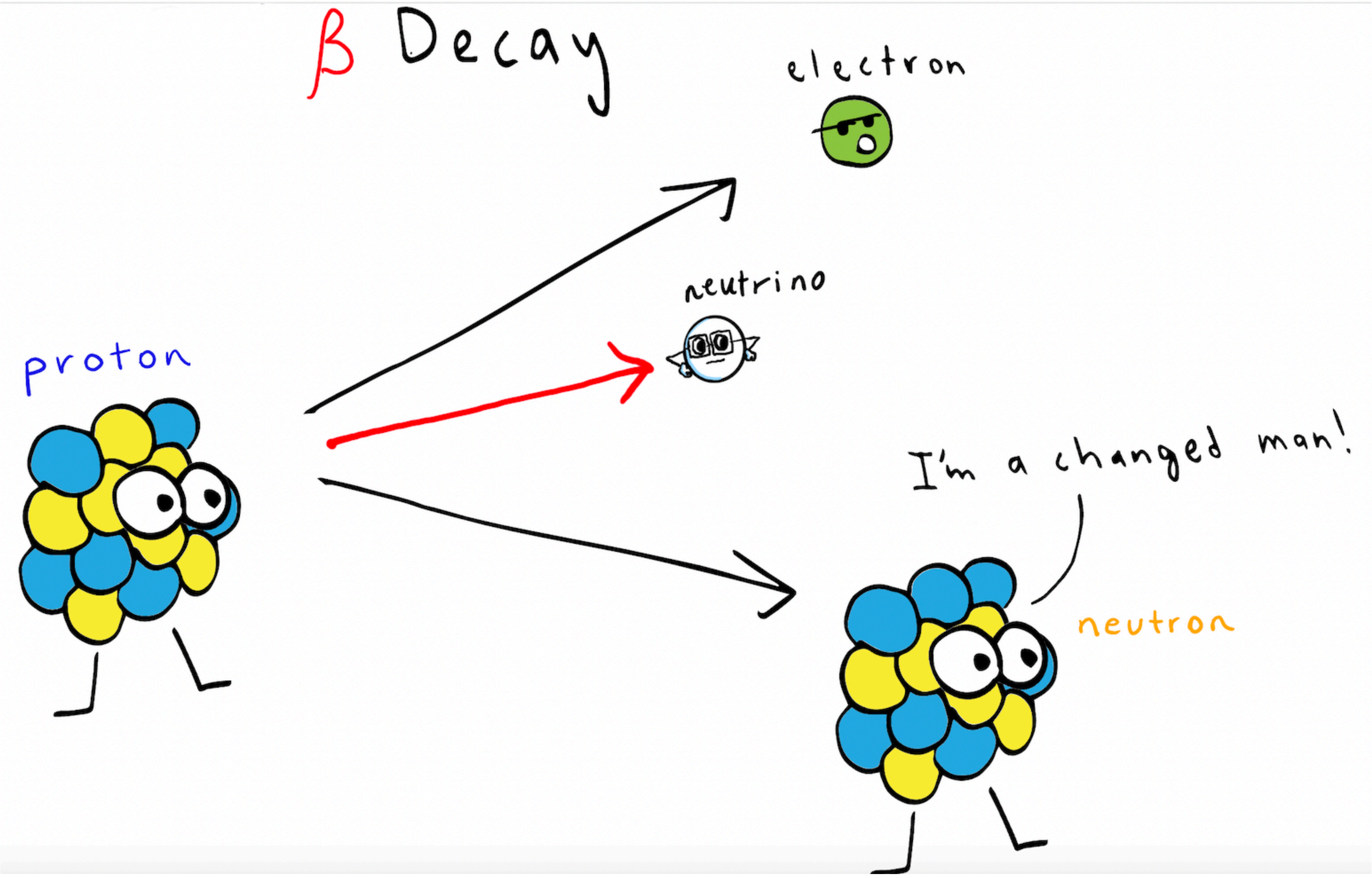
Graduate Research Fellowship Program

THE UNIVERSITY of NORTH CAROLINA at CHAPEL HILL

Introduction

- Quantitative predictions for electroweak interactions of hadronic system described by Standard Model (SM)



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A.Acar

Introduction

- Quantitative predictions for electroweak interactions of hadronic system described by Standard Model (SM)

Quarks

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Electroweak

γ

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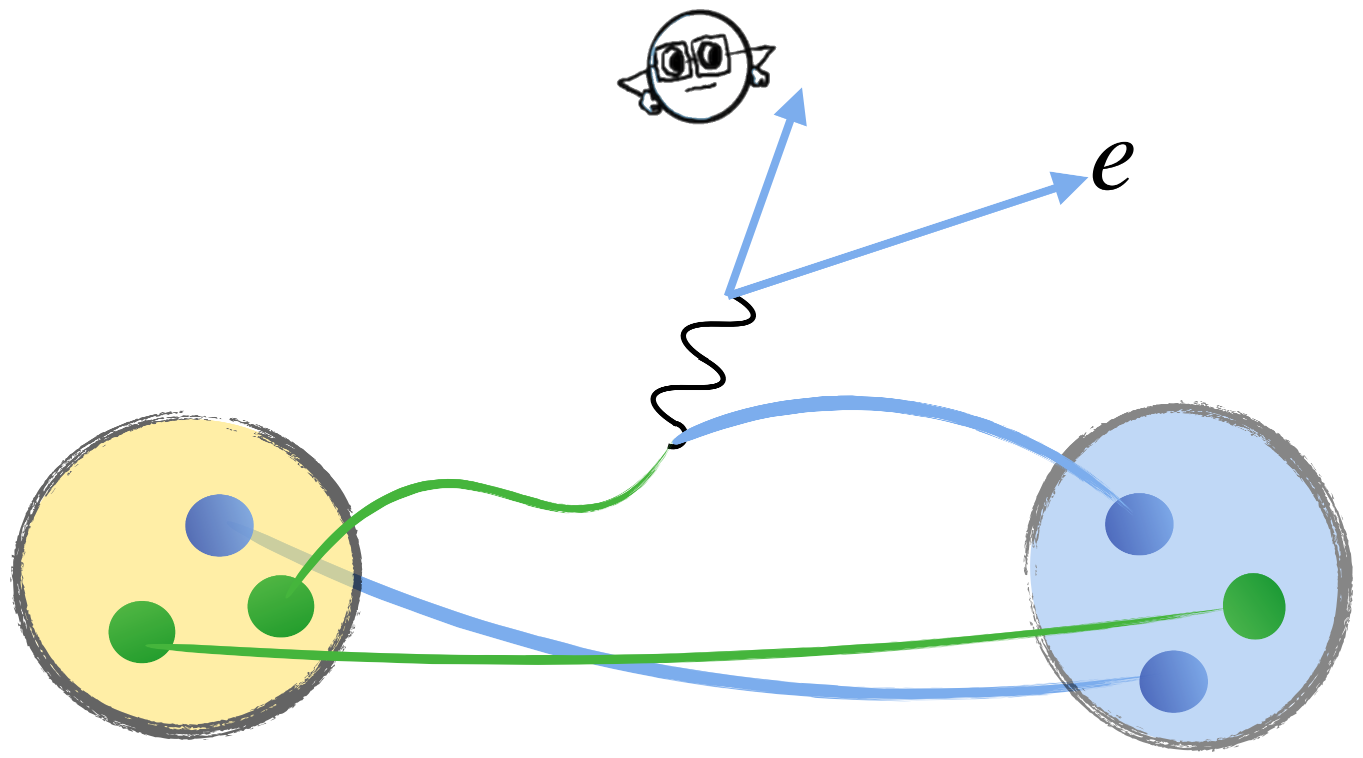
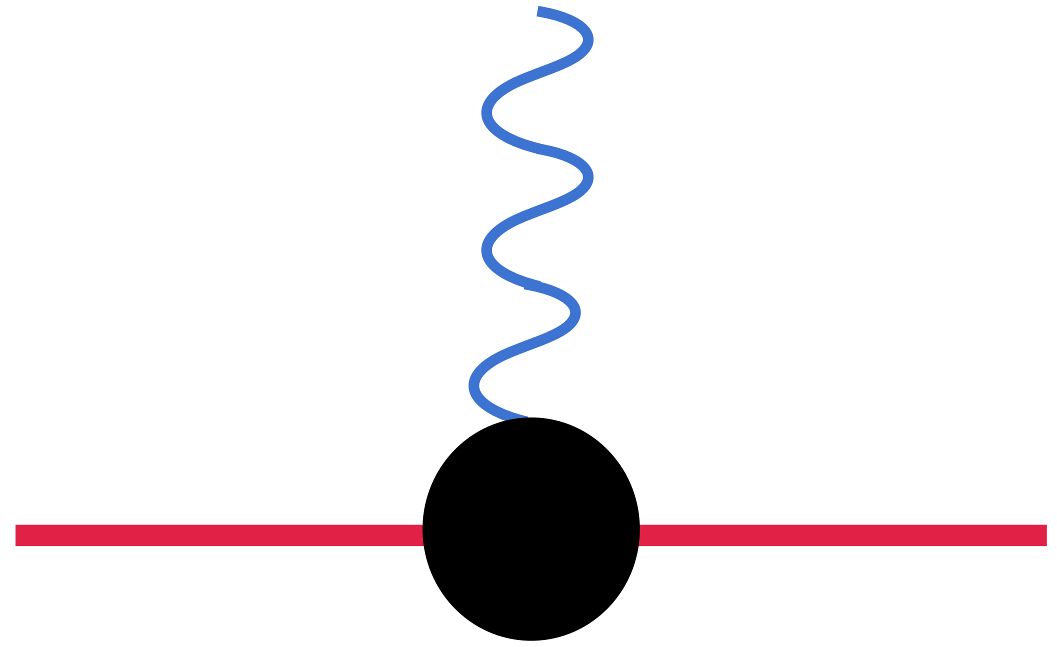
$$n \to pe\nu$$



$$1 + J \to 1$$

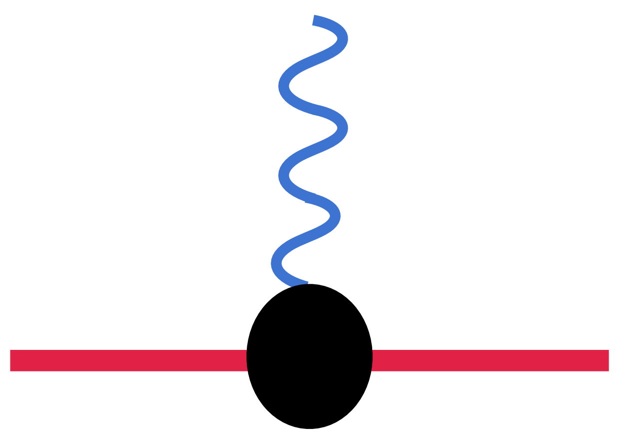


1 **QCD-stable Hadron** $|N\rangle, |\pi\rangle, \ldots$

J **Local, external current**

Introduction

- Quantitative predictions for electroweak interactions of hadronic system described by Standard Model (SM)
- Low-energy quantitative predictions LQCD
- Precision Experiments Reveal *New Physics*

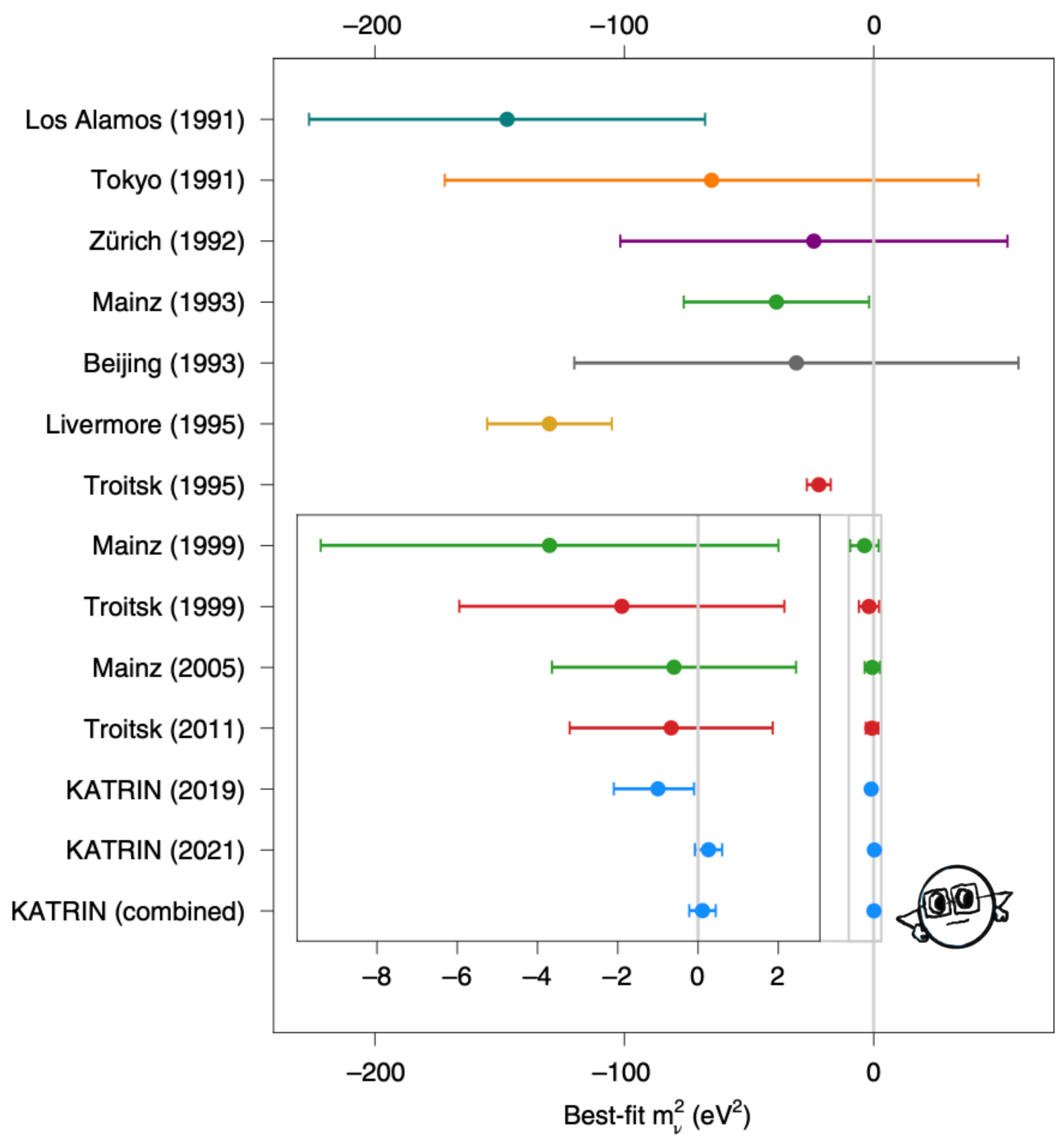


$1 + J \rightarrow 1$

$\nu\beta$

Beta decay

$n \rightarrow p e \nu$



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- Low-energy quantitative predictions   LQCD
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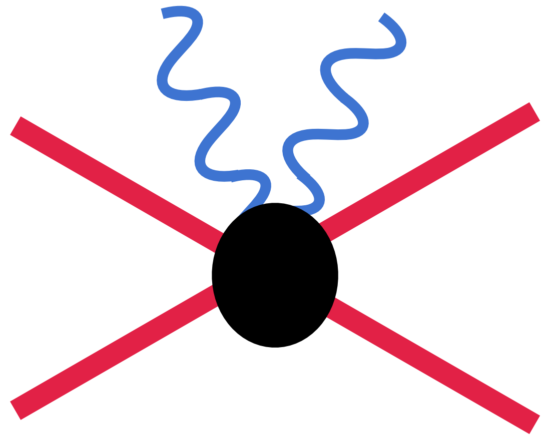

$2+J+J\rightarrow 2$

$2\nu\beta\beta$ Rare Nuclear Decay $nn \rightarrow ppee\nu\nu$

$0\nu\beta\beta$ Lepton Number Violation $nn \rightarrow ppee$

Introduction

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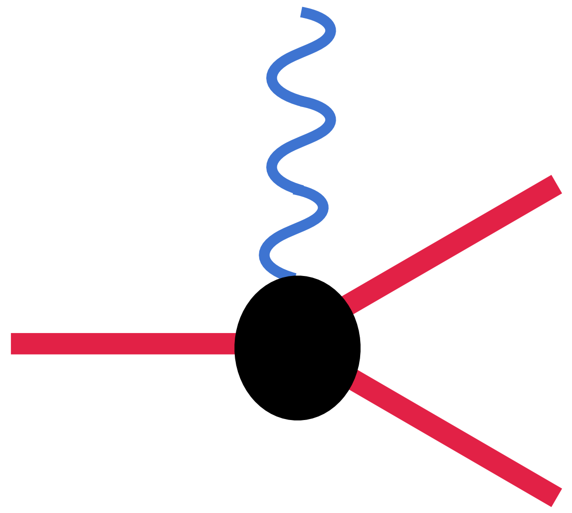
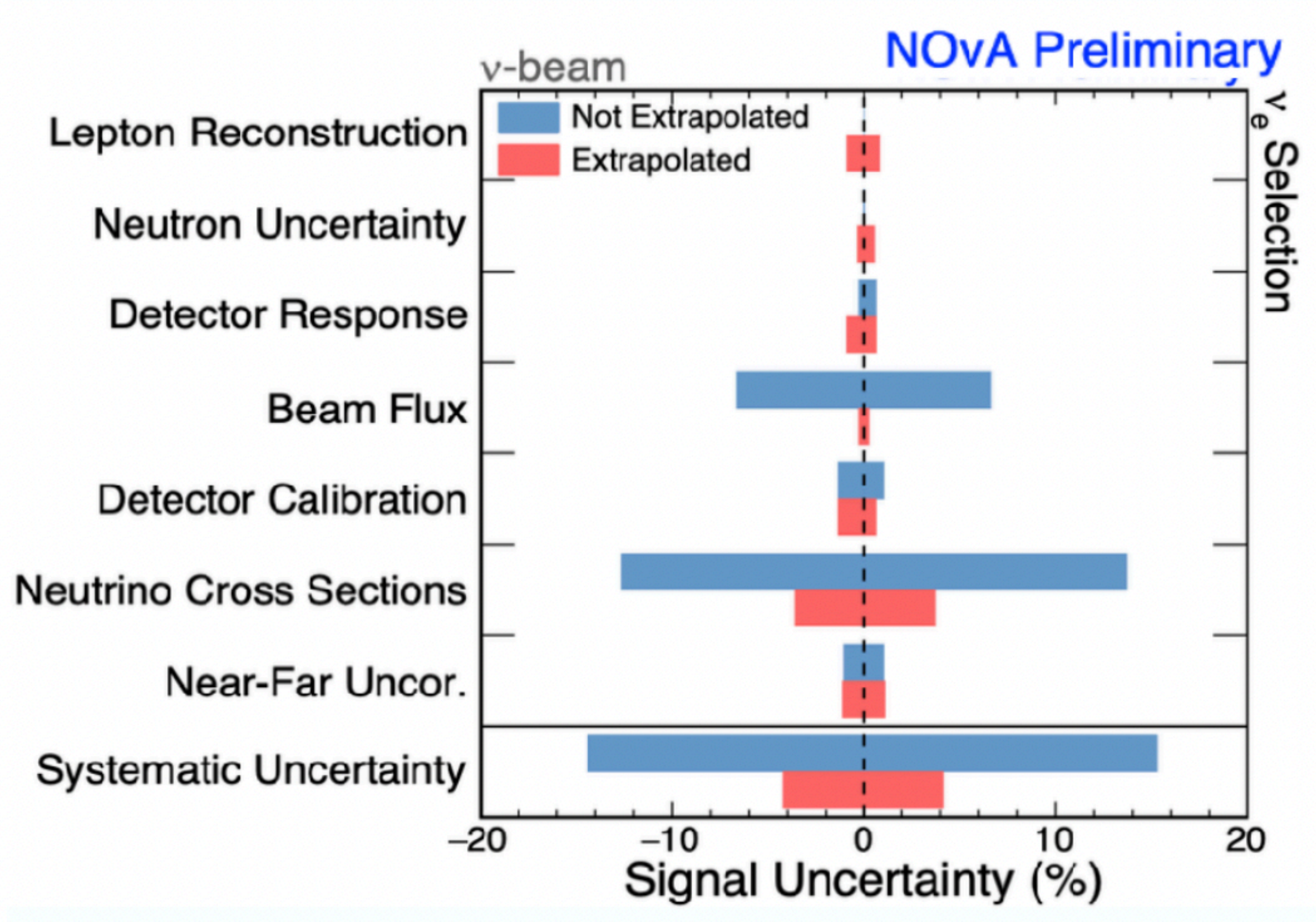


$$1 + J \to 2$$

Resonance Production

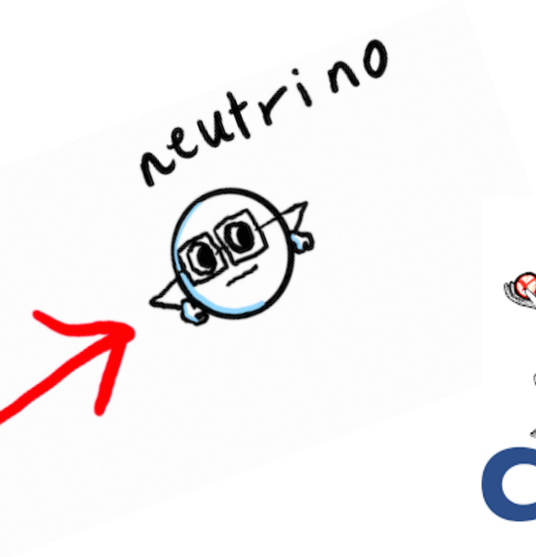
$$\nu N \to \Delta \to N\pi$$



Precision Frontier

- Standard Model **successful** **Not Complete**

- Low-Energy Precision Experiments Reveal *New Physics*



DUNE DEEP UNDERGROUND NEUTRINO EXPERIMENT — ^{40}Ar

Jefferson Lab — ^{40}Ar ^{48}Ti ^{2}H

PROJECT 8 — ^{3}H

PERKEO III — n

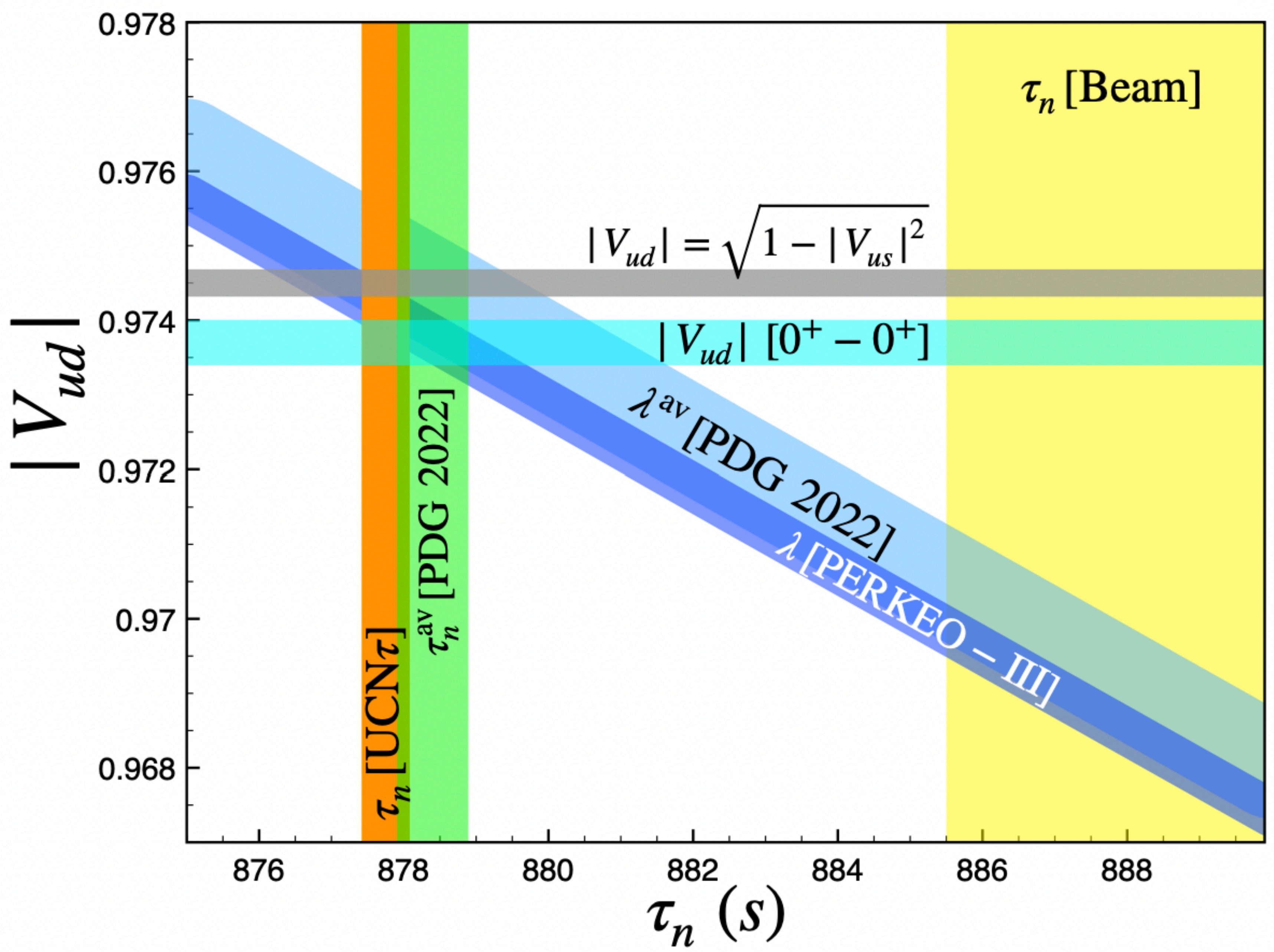
CUPID — ^{100}Mo

LEGEND — ^{76}Ge

KATRIN — ^{3}H

UCNτ — n

$$|\bar{V}_{ud}|^2 + |\bar{V}_{us}|^2 + |\bar{V}_{ub}|^2 = 1 + \Delta_{\rm CKM}(\epsilon_i)$$

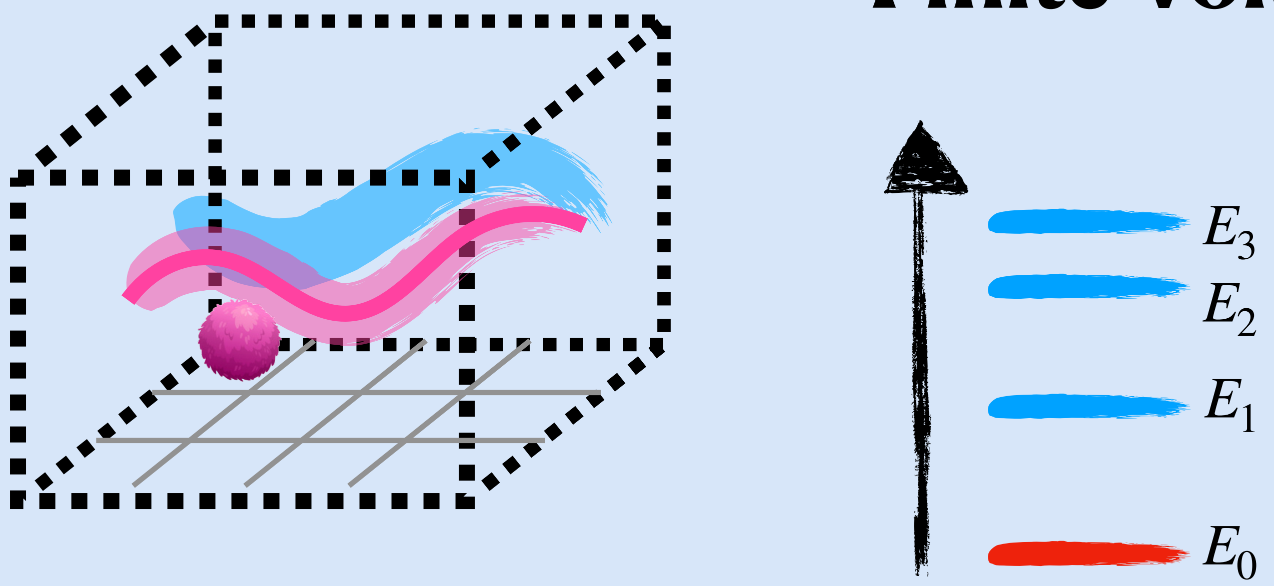


Nuclear Physics

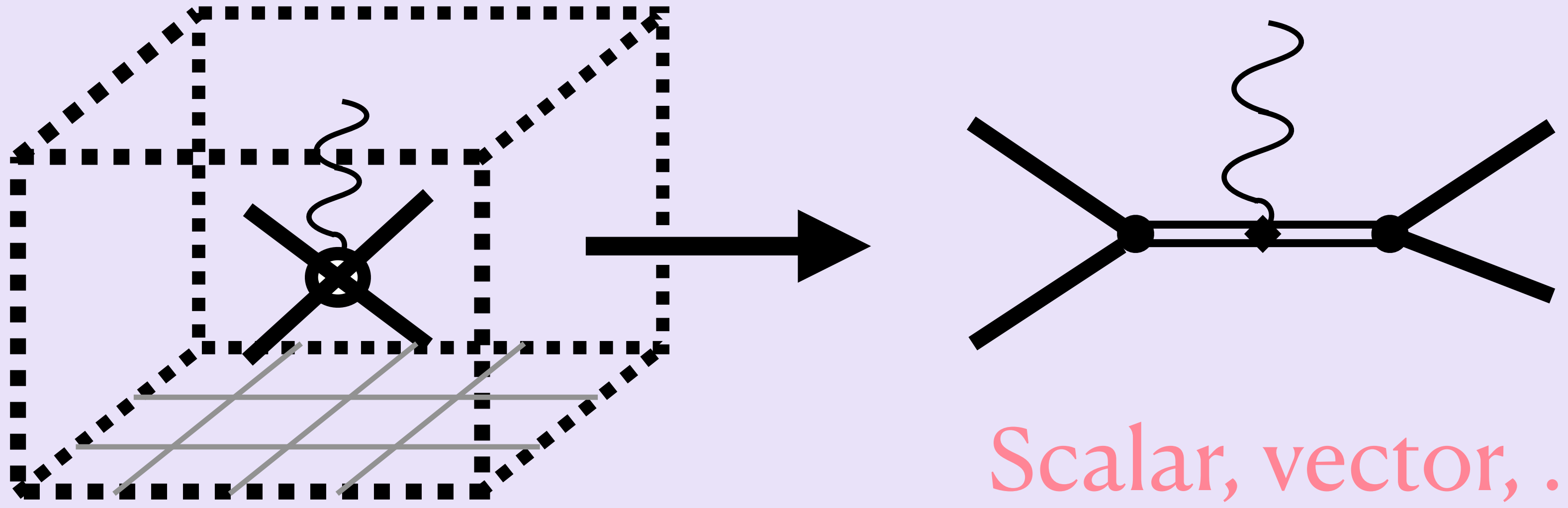
- **Quest:** state-of-the-art predictions with quantified uncertainties
- **Methods:** <u>Effective Field Theory, Lattice QCD</u>
- **Focus:** Electroweak Currents of Two Hadron Systems

<u>Calculations of NN systems</u>

Finite Volume



<u>Form Factors from Finite Volume</u>



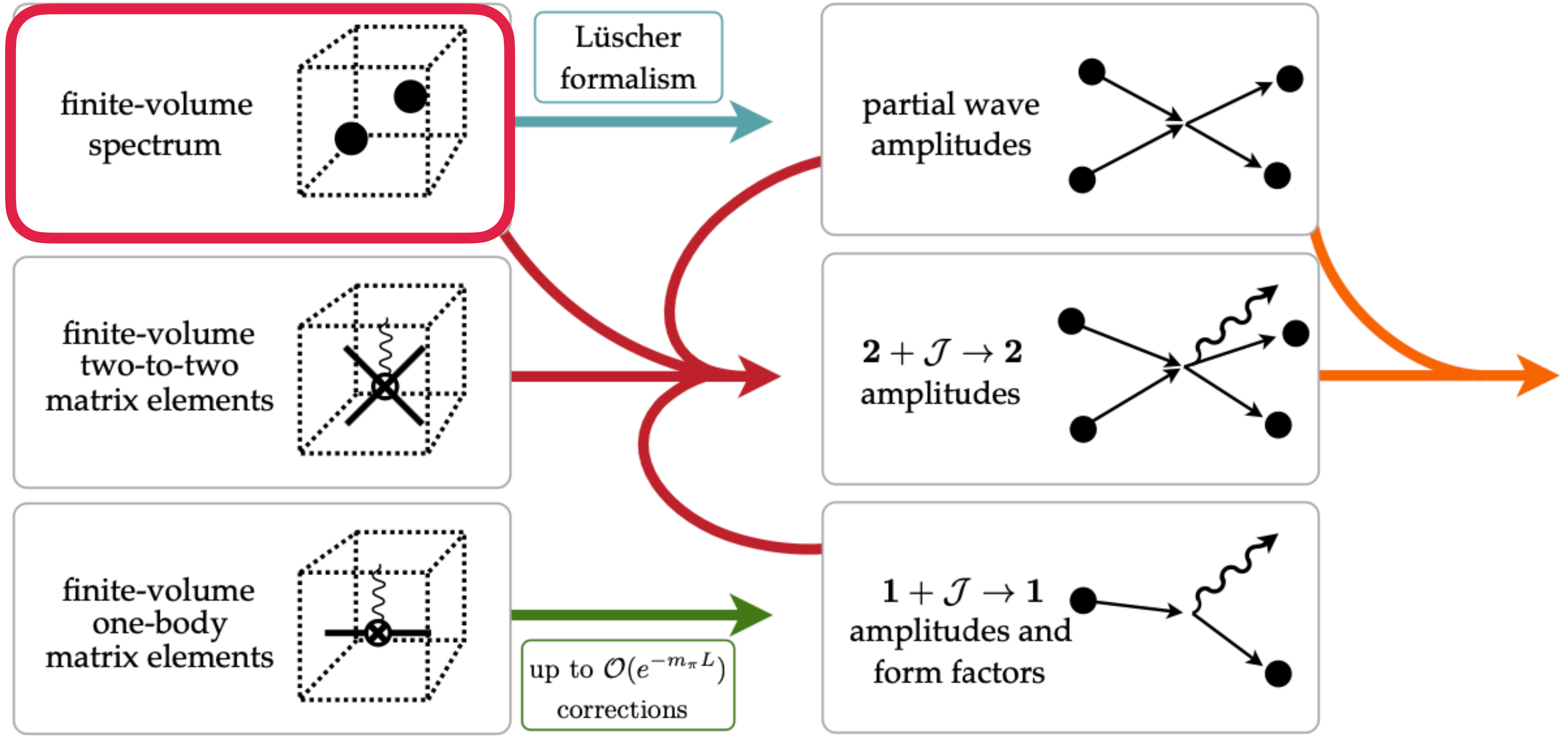
Scalar, vector, ...

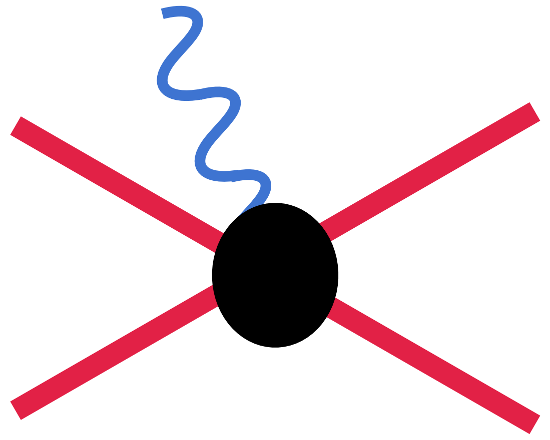
$2 + J \to 2$

$2 + J \to 2$ Amplitudes

- **Formalism to map FV to Physical Amplitudes**

$$|\langle \mathbf{2}|\mathcal{J}|\mathbf{2}\rangle_L| = \frac{1}{\sqrt{L^3}}\sqrt{\mathrm{Tr}\left[\mathcal{R}\,\mathcal{W}_{L,\mathrm{df}}\,\mathcal{R}\,\mathcal{W}_{L,\mathrm{df}}\right]}$$





[Baroni, Briceño, Hansen, Ortega-Gama (2018)]

Lattice Calculations

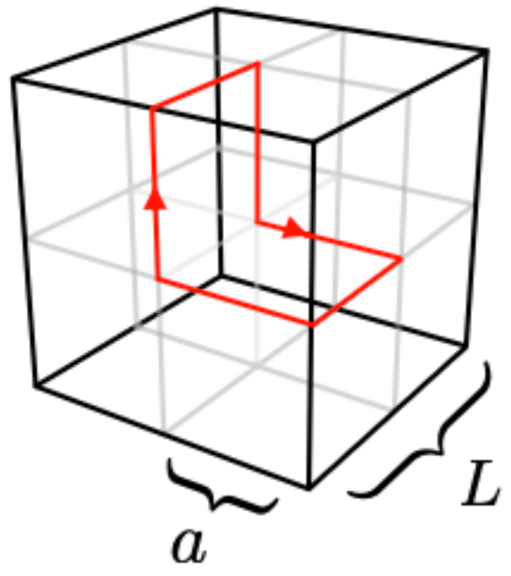
- Numerical Low-Energy Observables

$$\mathcal{L}_{\rm QCD} = \bar{\psi}_i \left(i\gamma^\mu (D_\mu)_{ij} - m\, \delta_{ij} \right) \psi_j - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$$

m_u

m_d g_{stronk}

m_s



FV

Finite, Euclidean Spacetime

$L \to \infty$

$a \to 0$

$m_\pi \to 139 MeV$

Lattice Calculation

- Numerical Low-Energy Observables
- FV Energies

Evaluate using Monte Carlo techniques:

$$C(x_0 - y_0, \mathbf{P}) = \frac{1}{Z_{Eucl.}} \int \mathcal{D}[U, q, \bar{q}] \mathcal{O}'_{\lambda'}(x_0, \mathbf{P}) \mathcal{O}^\dagger_\lambda(y_0, \mathbf{P}) \; e^{-S_{\text{Eucl.}}}$$

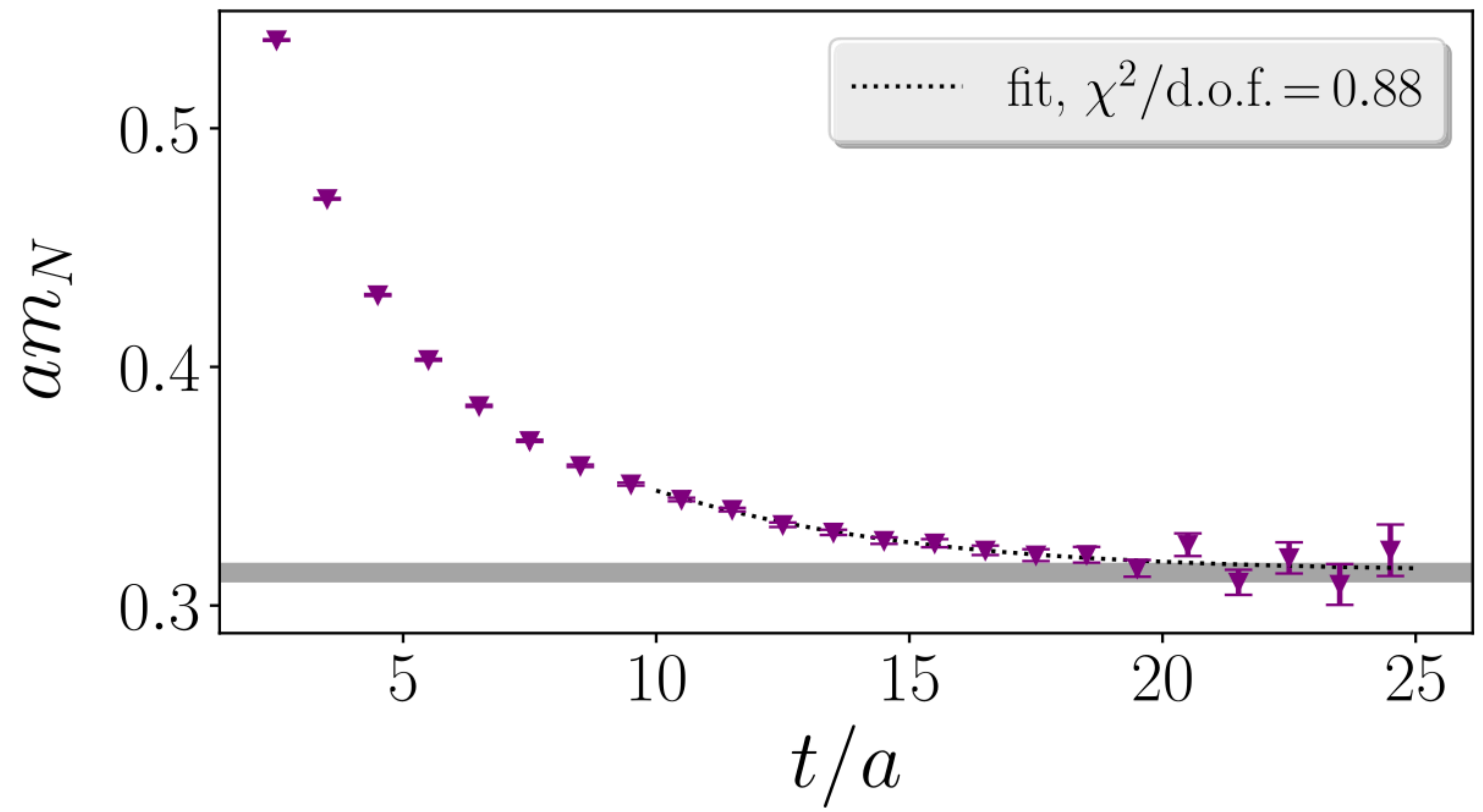
correlation function

$$C_L(t) = \langle O(t) O^\dagger(0) \rangle$$

$$\sum_n |n\rangle\langle n|$$

$$C_L(t) = \sum_{\mathbf{n}} Z_n Z_n^\dagger e^{-E_n t} \xrightarrow[t \to \infty]{} e^{-E_0 t}$$

[Bulava (2024)]



Finite Volume

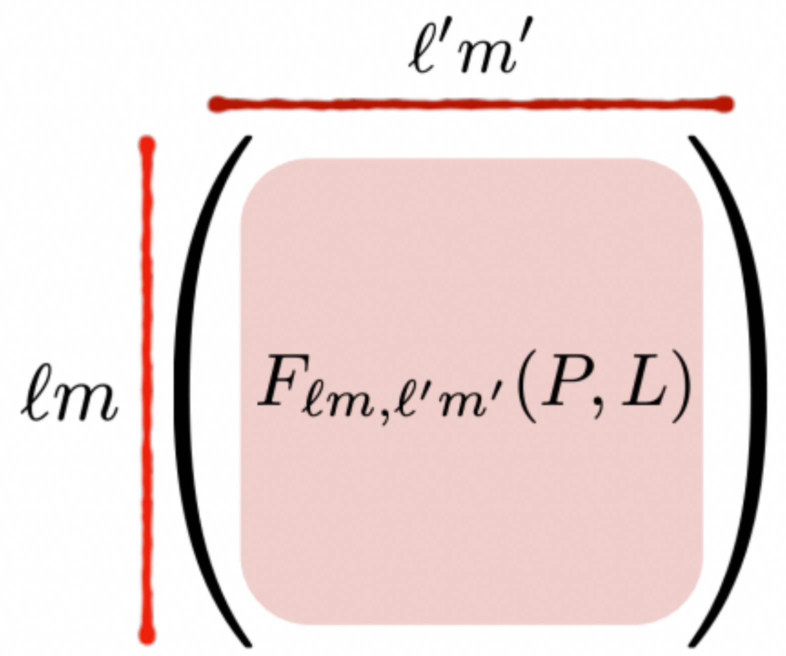
- Numerical Low-Energy Observables
- FV Energies
- Physical Observables   Lüscher Formalism

[Lüscher (1986)]
[Rummukainen and Gottlieb (1995)]
[Kim, Sachrajda, and. Sharpe (2005)]
[Leskovec and Prelovsek (2012)]
[Briceño and Davoudi (2012), (2013)]
[Briceño (2014)]

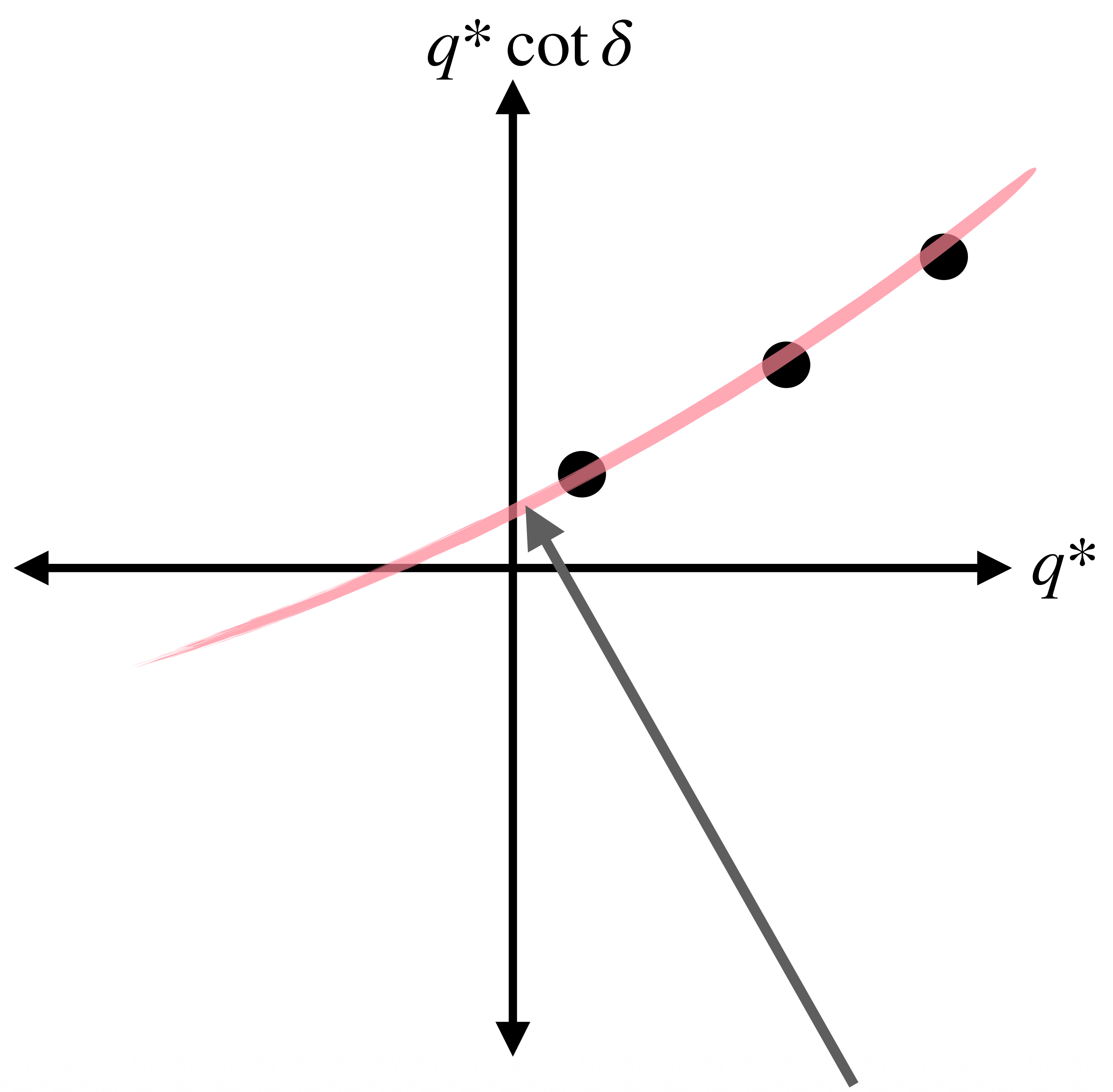
$$\det_{lm}\left[\mathcal{M}(s) + F^{-1}(P, L)\right] = 0 + O(e^{-m_\pi L})$$

$$\mathcal{M}(s) \sim (p\cot\delta(s) - i\rho(s))^{-1}$$

$$p\cot\delta(s) = -\frac{1}{a} + \frac{1}{2}rp^2 \quad \text{ERE}$$



Encodes FV effects

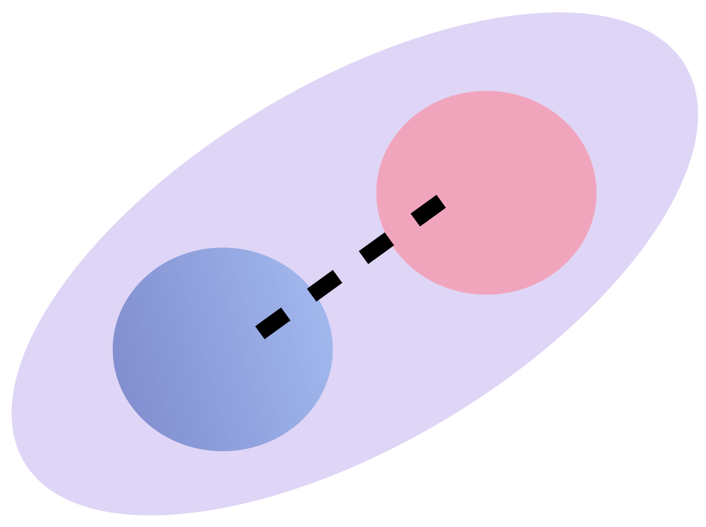


bound state : $\lim_{q\to 0} q\cot\delta < 0$

no bound state : $\lim_{q\to 0} q\cot\delta > 0$

NN System

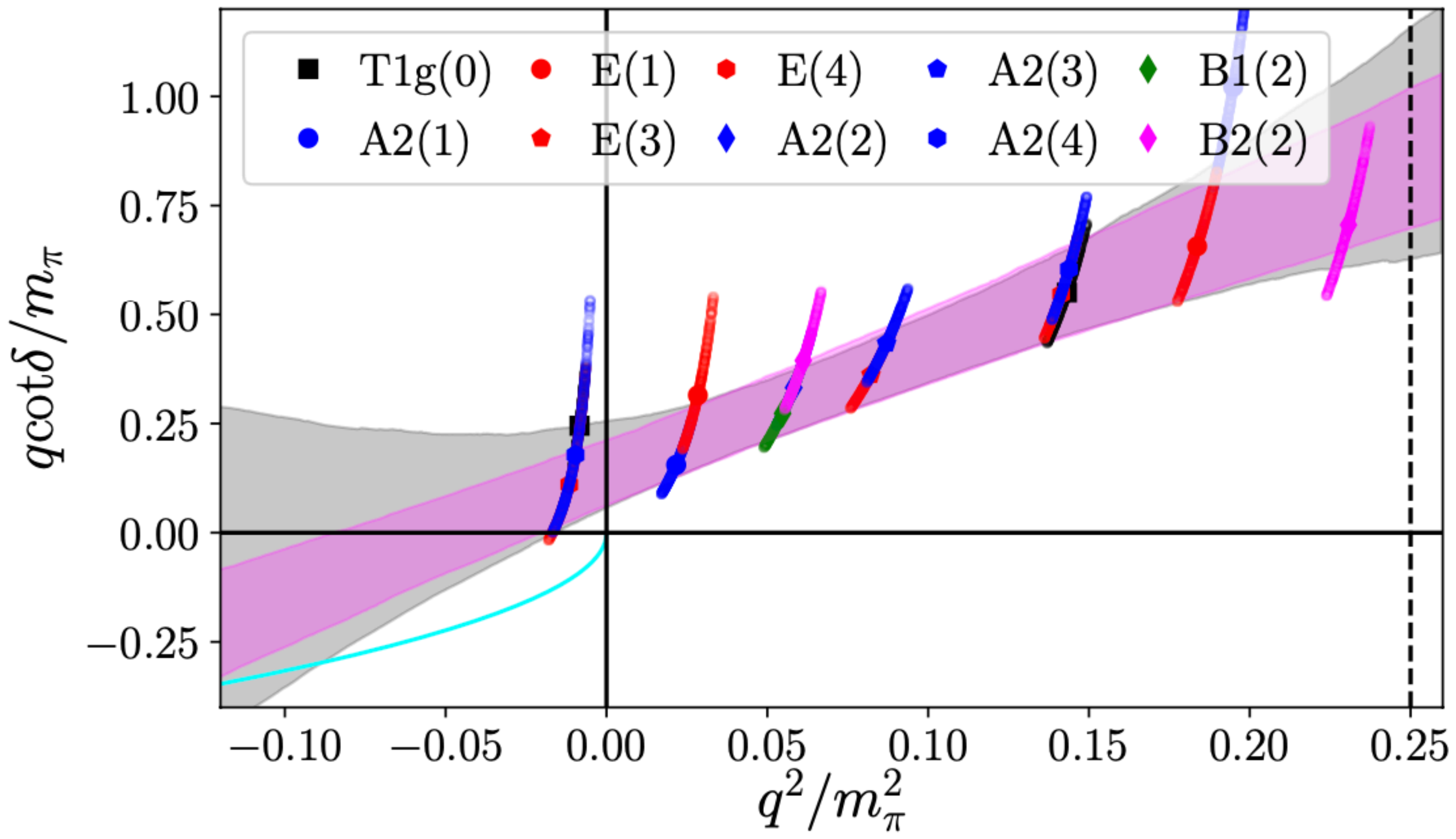
- **Benchmark System in Nuclear Physics**
- Challenge in LQCD $\frac{\text{Signal}}{\text{Noise}} \sim \sqrt{N} \exp\left[-A\left(m_N - \frac{3}{2}m_\pi\right)t\right]$
- **NN at SU(3) Symmetric Point**
- Precise NN spectrum



Deuteron

$B_d = 2.2$ MeV

[Hörz (2021)]



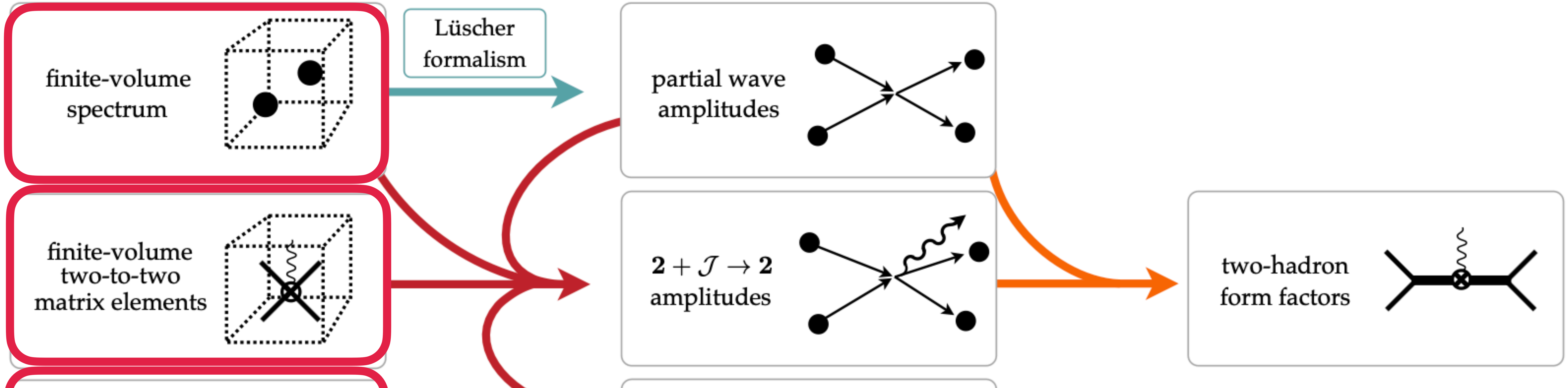
$m_\pi \sim 714$ MeV

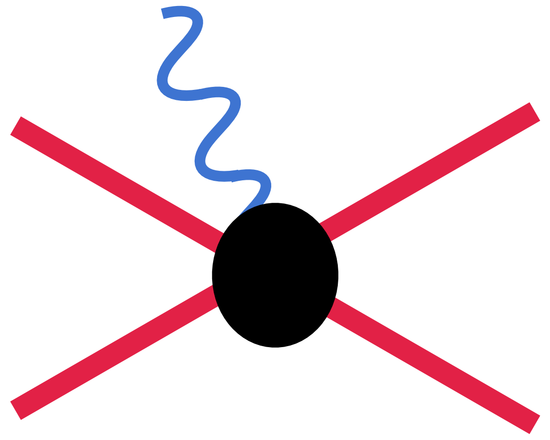
$m_u = m_d = m_s$

$2 + J \to 2$ Amplitudes

- **Formalism to map FV to Physical Amplitudes**

$$|\langle \mathbf{2}|\mathcal{J}|\mathbf{2}\rangle_L| = \frac{1}{\sqrt{L^3}}\sqrt{\mathrm{Tr}\left[\mathcal{R}\,\mathcal{W}_{L,\mathrm{df}}\;\mathcal{R}\,\mathcal{W}_{L,\mathrm{df}}\right]}$$



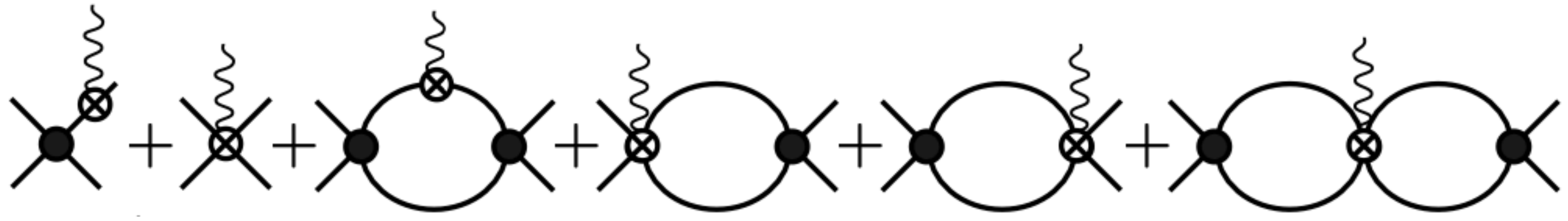


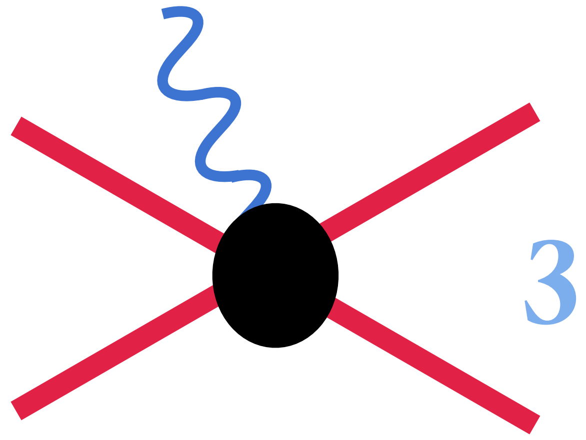
[Baroni, Briceño, Hansen, Ortega-Gama (2018)]

2 Hadron Form Factors

- **Formalism developed**

$$\left|\langle \mathbf{2}|\mathcal{J}|\mathbf{2}\rangle_L\right| = \frac{1}{\sqrt{L^3}}\sqrt{\mathrm{Tr}\left[\mathcal{R}\,\mathcal{W}_{L,\mathrm{df}}\;\mathcal{R}\,\mathcal{W}_{L,\mathrm{df}}\right]}$$

$\mathcal{W}_{\mu_1\cdots\mu_n} =$ 



- **Test on a Low energy EFT**

$$_\infty\langle P_f|\,\mathcal{J}^\mu(0)\,|P_i\rangle_\infty = (P_f + P_i)^\mu F(Q^2)$$

FV State Correction

$$|E,\mathbf{P}\rangle_\infty = \mathcal{R}\,|E,\mathbf{P}\rangle_L$$

Transition Amplitude

$$\mathcal{W}_{L,\mathrm{df}}$$

$$\langle E,\mathbf{P}'|\,J\,|E,\mathbf{P}\rangle_\infty$$

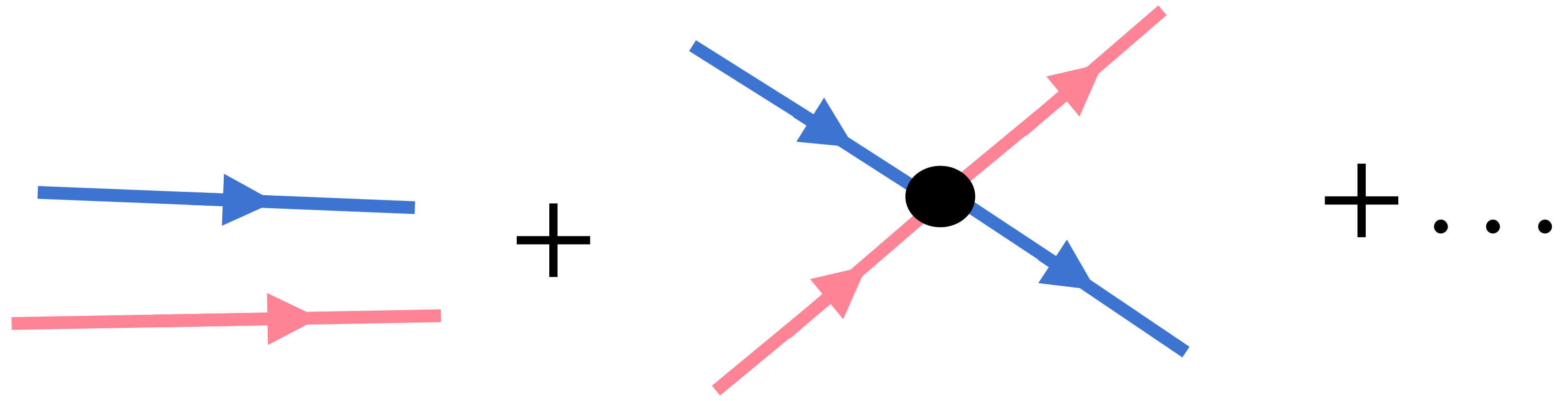
Nuclear EFT

- **Low energy Nuclear EFT**
  2 point-like non-relativistic nucleons
  interacting with contact interaction

Energies << m_π (same condition as Lüscher)

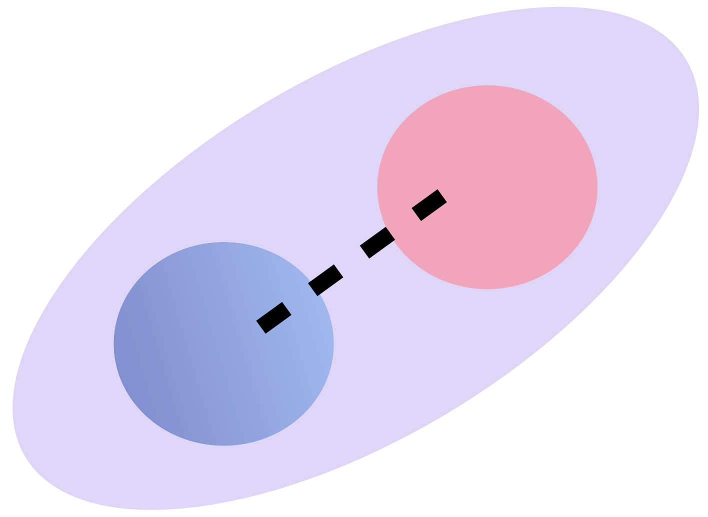
Endres, Kaplan, Lee, Nicholson (2011)

$$\mathcal{L}_{\text{eff}} = \psi^\dagger \left(i\partial_\tau + \frac{\nabla^2}{2M} \right) \psi + g_0 \left(\psi^\dagger \psi\right)^2 + \ldots$$



- **Tunable g**
  Test interactions with deep or shallow bound states

- 



Deuteron

$B_d = 2.2$ MeV

FV Spectrum

- **Low energy Nuclear EFT**
  2 point-like non-relativistic nucleons interacting with contact interaction

- **Discretize LEFT**

$$C(\tau) = \langle \Psi_{\text{snk},2} | e^{-H\tau} | \Psi_{\text{src},2} \rangle = \langle \Psi_{\text{snk},2} | \left[e^{-H} \right]^{\tau} | \Psi_{\text{src},2} \rangle$$

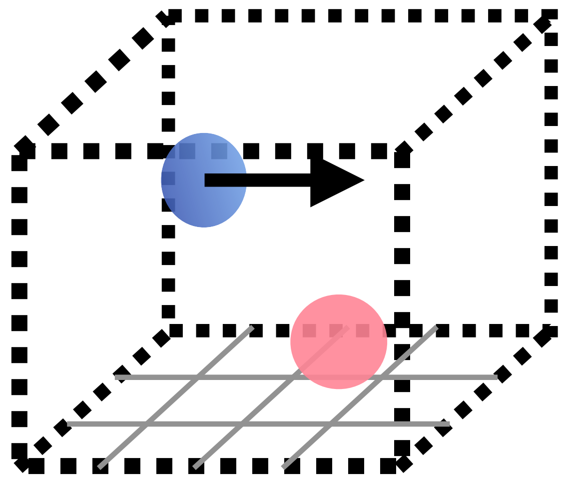
Tunable **L, g, M, P**

Transfer matrix

$$\langle pq | \mathcal{T} | p'q' \rangle$$

Eigenvalues ~ L^6

$$E_0 = -\ln \lambda(g_0)$$



FV Spectrum
[Nicholson (2016)]

FV Spectrum

- **Low energy Nuclear EFT**
  2 point-like non-relativistic nucleons interacting with contact interaction

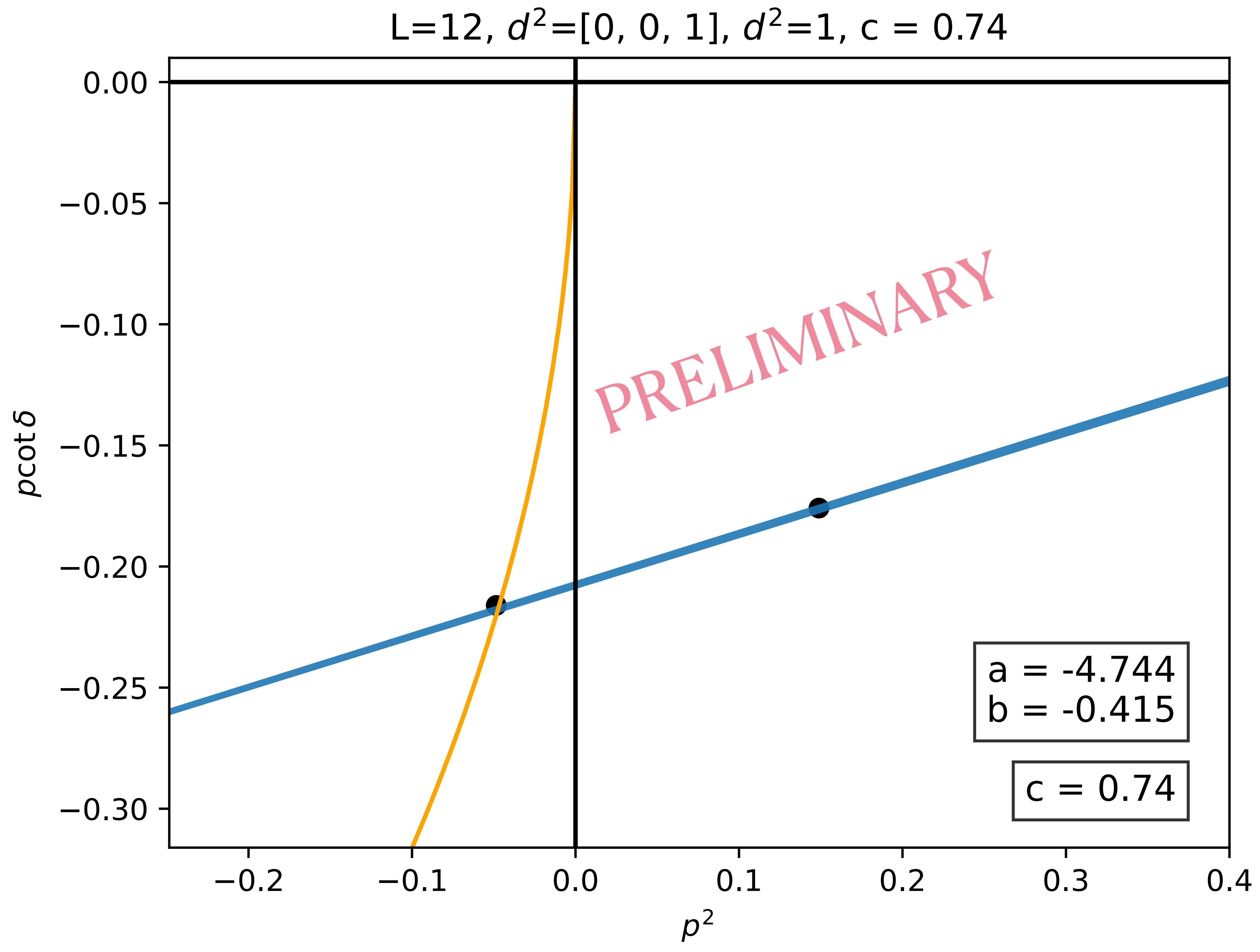
- **Discretize LEFT**

- **Non-Relativistic Lüscher (s-wave)**

$$E^*_{NR} \longrightarrow \det_{lm}\left[\mathcal{M}(s) + F^{-1}(P,L)\right] = 0$$

$$\downarrow$$

$$p\cot\delta + \frac{2}{\pi L}\sum_n \frac{1}{(\frac{p^*L}{2\pi}) - n^2}$$

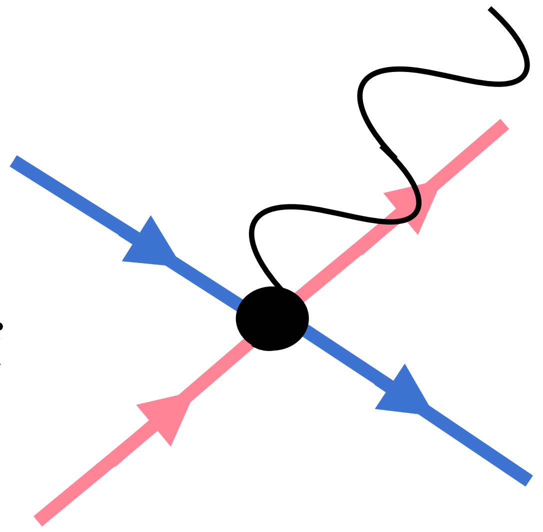


$$p\cot\delta = -\frac{1}{a} + \frac{1}{2}rk^2$$

Matrix Elements

- $2 + J^\mu \to 2$ **Matrix Elements**
  vector (EM) current

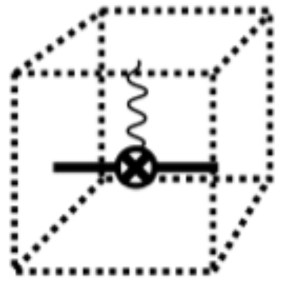
FV Spectrum $\mathbf{P}_i$



$\mathbf{P}_f$

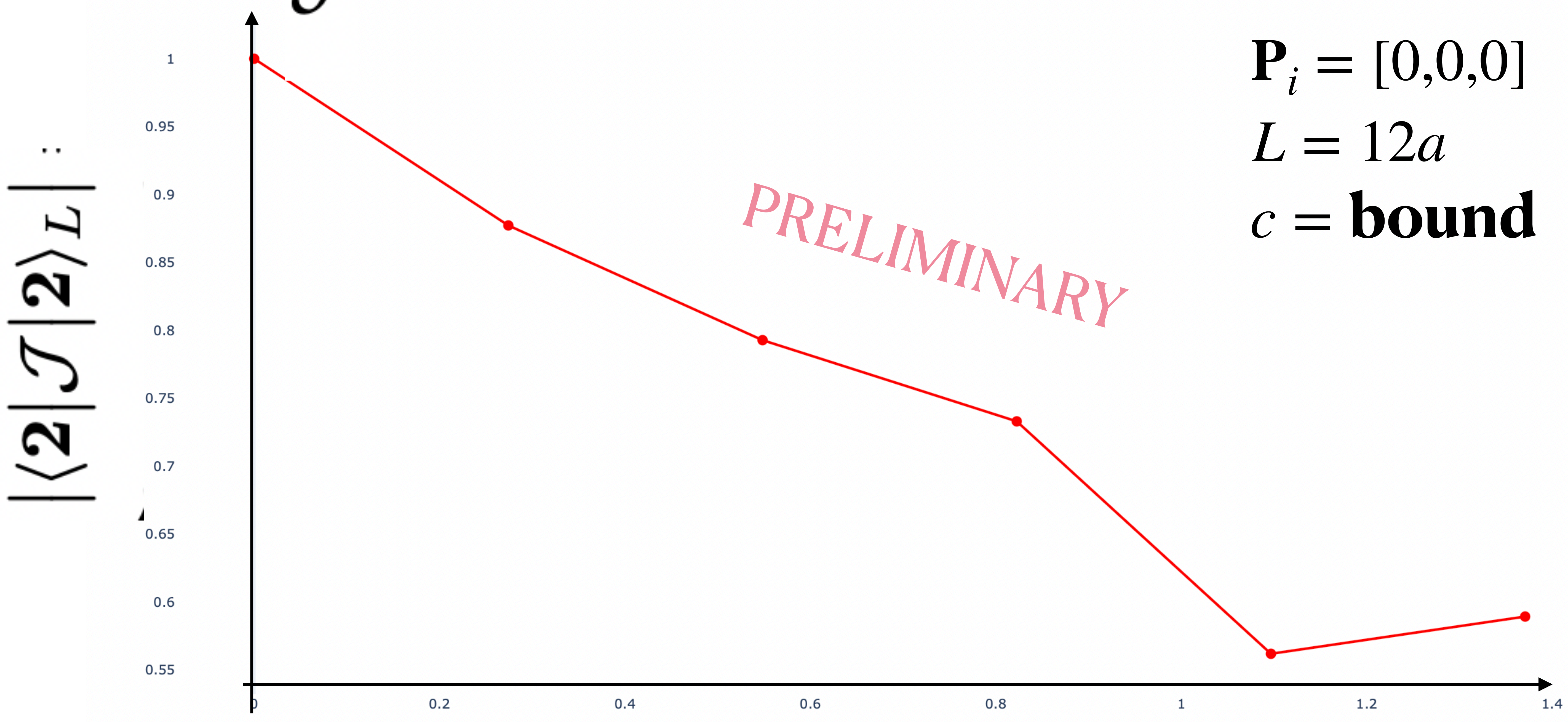
finite-volume two-to-two matrix elements

finite-volume one-body matrix elements



$$|\langle \mathbf{2}|\mathcal{J}|\mathbf{2}\rangle_L| = \frac{1}{\sqrt{L^3}}\sqrt{\mathrm{Tr}\left[\mathcal{R}\,\mathcal{W}_{L,\mathrm{df}}\;\mathcal{R}\,\mathcal{W}_{L,\mathrm{df}}\right]}$$

$\mathcal{J}^0$



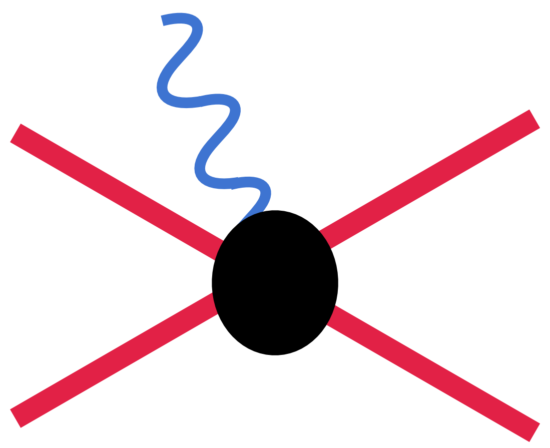
$\mathbf{P}_i = [0,0,0]$
$L = 12a$
$c =$ **bound**

$Q^2 = (\mathbf{P}_f - \mathbf{P}_i)^2$

- Non-relativistic version of Formalism

Conclusion

- Quantitative predictions for electroweak interactions of hadronic system require advancements in LQCD calculations and FV Formalisms
- NN systems must resolve energy spectrum before reliable form factors
- Testing of FV Formalism with Low-energy LEFT



$2 + J^{\mu} \rightarrow 2$