THERMODYNAMICS OF FINITE VOLUME QUARK MATTER IN ANISOTROPIC MOMENTUM DISTRIBUTION

Nisha Chahal
(Supervised by Dr. Suneel Dutt and Dr. Arvind Kumar)

DR. B. R. AMBEDKAR NATIONAL INSTITUTE OF TECHNOLOGY, INDIA

THE 39th ANNUAL HAMPTON UNIVERSITY GRADUATE PROGRAM AT JEFFERSON LAB
In this presentation, we are going to discuss about,

- **Introduction**
  - The Big-Bang
  - QCD phase diagram
  - Theoretical Approaches

- **Polyakov quark meson model**
  - Lagrangian density
  - Thermodynamic potential
  - Polyakov loop

- **What do we need to study and why?**
  - Finite volume effects
  - Anisotropic momentum distribution

- **Results**

- **Conclusion**
The Big-Bang
Theoretical Approaches

- **Lattice QCD simulations**: It is a non-perturbative application of field theory based on the Feynman path integral technique.

- **MIT bag model**: Hadrons are considered to be composed of weakly interacting quarks confined within a finite region referred to as the "bag."

- **Nambu Jona Lasinio (NJL) and Polyakov NJL (PNJL) model**: focusing on the interaction between quarks and anti-quarks through a four-point interaction term in the Lagrangian density.

- **Chiral perturbation theory**: Includes the expansion in power of momentum and quark masses.

AND Many More.......
Quark meson model is an effective approach to studying the strong interactions between mesons and quarks.

The total Lagrangian of the model for $N_f$ flavors is given by

$$\mathcal{L} = \bar{\Psi} i \gamma^\mu \partial_\mu \Psi + \text{Tr} \left( \partial_\mu \varphi^\dagger \partial^\mu \varphi \right) - m^2 \text{Tr} \left( \varphi^\dagger \varphi \right) - \lambda_1 \left[ \text{Tr} \left( \varphi^\dagger \varphi \right) \right]^2 - \lambda_2 \left[ \text{Tr} \left( \varphi^\dagger \varphi \right)^2 \right] + c \left( \text{det}(\varphi) + \text{det} \left( \varphi^\dagger \right) \right) + \text{Tr} \left[ H \left( \varphi + \varphi^\dagger \right) \right]$$

$$+ \mathcal{L}_{qm} - \frac{1}{4} \text{Tr} \left( V_{\mu\nu} V^{\mu\nu} \right) + \frac{m_1^2}{2} V_{a\mu} V_a^\mu.$$
The $m_f^*$ represents the effective mass of constituent quarks given by

$$m_u^* = \frac{g_s}{2} \sigma_u, \quad m_d^* = \frac{g_s}{2} \sigma_d \quad \text{and} \quad m_s^* = \frac{g_s}{\sqrt{2}} \sigma_s. \quad (1)$$

The effective chemical potential of the quarks is modified as a consequence of vector-meson interactions and is defined in terms of quark chemical potential, $\mu_q$, isospin chemical potential, $\mu_I$ and strangeness chemical potential, $\mu_S$ as

$$\begin{align*}
\mu_u^* &= \mu_q + \mu_I - g_{\omega u} \omega - g_{\rho u} \rho \\
\mu_d^* &= \mu_q - \mu_I - g_{\omega d} \omega + g_{\rho d} \rho \\
\mu_s^* &= \mu_q - \mu_S - g_{\phi s} \phi.
\end{align*} \quad (2)$$
Polyakov Quark Meson model

- Polyakov loop

\[ \Phi(\tilde{x}) = (\text{Tr}_c L)/N_C, \]

and it's conjugate

\[ \bar{\Phi}(\tilde{x}) = (\text{Tr}_c L^\dagger)/N_C. \]

- In the current work, we use the polynomial form of the Polyakov loop defined as\(^4\)

\[
\frac{U_{\text{poly}}(\Phi, \bar{\Phi})}{T^4} = - \frac{b_2(T)}{2} \bar{\Phi} \Phi - \frac{b_3}{6} (\Phi^3 + \bar{\Phi}^3) + \frac{b_4}{4} (\bar{\Phi} \Phi)^2,
\]

and the temperature-dependent coefficient \(b_2\) defined as

\[ b_2(T) = a_0 + a_1 \left( \frac{T_0}{T} \right) + a_2 \left( \frac{T_0}{T} \right)^2 + a_3 \left( \frac{T_0}{T} \right)^3. \]

A lower momentum cutoff, denoted as $p_{\text{min}}$ [MeV], equal to $\pi/R$ [MeV], where $R$ signifies the length of a cubic volume (designated as $\Lambda$) is introduced.

In the context of anisotropic quark matter, the modification of quasiparticle dispersion relations aligns with the anisotropic momentum distribution. In this case, the nontrivial dispersion relation for effective mass $m_f^*$ is characterized by

$$E_f^{*(\text{aniso})} = \sqrt{p^2 + \xi (p\cdot\hat{n})^2 + m_f^*}.$$  

(3)
Results

![Graphs showing results for different temperatures and radii with varying mu values.](image)

- **T = 30 MeV**
  - (a) $R = \infty$
  - (c) $R = 5 \text{ fm}$
  - (e) $R = 3 \text{ fm}$

- **T = 60 MeV**
  - (b) $R = \infty$
  - (d) $R = 5 \text{ fm}$

- **S**
  - $S = 200 \text{ MeV}$
  - $S = 0 \text{ MeV}$
  - $S = -100 \text{ MeV}$
  - $S = -160 \text{ MeV}$
Results

(a) Crossover \( S = 0, R = 5 \text{ fm} \)
Crossover \( R = 3 \text{ fm} \)
Crossover \( S = -100 \text{ MeV} \)
Crossover \( S = -160 \text{ MeV} \)
Crossover \( S = -200 \text{ MeV} \)
Crossover \( S = -250 \text{ MeV} \)

(b) Crossover \( S = 0, R = 5 \text{ fm} \)
Crossover \( R = 3 \text{ fm} \)
Crossover \( S = -100 \text{ MeV} \)
Crossover \( S = -160 \text{ MeV} \)
Crossover \( S = -200 \text{ MeV} \)
Crossover \( S = -250 \text{ MeV} \)
Results

(a)  
\[ R = \infty \quad R = 5 \text{ fm} \quad R = 3 \text{ fm} \]

(b)  
\[ \mu_s = 0 \text{ MeV} \quad \mu_s = -100 \text{ MeV} \quad \mu_s = -160 \text{ MeV} \]

(c)  
\[ \chi_4^q / \chi_2^q \]

(d)  
\[ \chi_4^q / \chi_2^q \]

Nisha Chahal (NITJ)  QUARK MATTER  June 13, 2024  12 / 16
Results

\( \mu_q = 0 \text{ MeV} \)

(a) \hspace{1cm} (b)

\( \mu_q = 300 \text{ MeV} \)

(c) \hspace{1cm} (d)

\( R = \infty \)

(e) \hspace{1cm} (f)

\( R = 3 \text{ fm} \)

(g) \hspace{1cm} (h)

\( q = 0 \text{ MeV} \)

\( q = 300 \text{ MeV} \)

Nisha Chahal (NITJ)
Results

crossover $\xi = -0.2$

crossover $\xi = 0$

crossover $\xi = 0.2$

crossover $\xi = 0.4$

deconfinement $\xi = -0.2$

deconfinement $\xi = 0$

deconfinement $\xi = 0.2$

deconfinement $\xi = 0.4$

CEP

1st order
The effects of strangeness chemical potential, anisotropic momentum distribution, and finite system size have been investigated.

The susceptibilities of conserved charges are enhanced in the transition region.

In future work, susceptibilities can be calculated at finite chemical potential values and compared with STAR data.

The model can be further improved by using the Functional Renormalization Approach (FRG).
thank you!