Diffractive vector meson production at HERA using holographic light-front QCD

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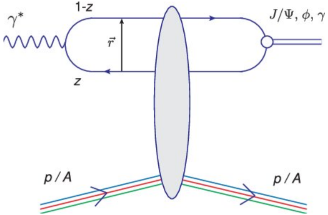
Outline

• Colour glass condensate (CGC) is a popular effective field theory for explaining physical phenomena in the proton saturation region.



• The forward scattering amplitude for the diffractive process : $\gamma^* p \to Vp$

$$\Im\mathrm{m}\,\mathcal{A}_\lambda^{\gamma^* p\to Vp}(s,t;Q^2) = \sum_{h,\bar{h}} \int \mathrm{d}^2\mathbf{r}\,\mathrm{d}x\; \Psi_{h,\bar{h}}^{\gamma^*,\lambda}(r,x;Q^2)\;\; \Psi_{h,\bar{h}}^{V,\lambda}(r,x)^*\; e^{-ix\mathbf{r}\cdot\mathbf{\Delta}}\, \mathcal{N}(x_\mathrm{m},\mathrm{r},\mathbf{\Delta})$$

• The high energy QCD dynamics of the dipole-proton interaction are all encoded in the scattering amplitude, $\mathcal{N}(x_m, r, \Delta)$.

• To compare with experiment, we compute the differential cross section :

$$\frac{d\sigma_\lambda^{\gamma^* p\to Vp}}{dt} = \frac{1}{16\pi}[\Im\mathrm{m}\mathcal{A}_\lambda^{\gamma^* p\to Vp}(s,t=0)]^2\;(1+\beta_\lambda^2)\exp(-B_D t)$$

— G. Watt and H. Kowalski, PRD 78, (2008), E. Iancu, et al. B 590, 199(2004)

Light-Front QCD

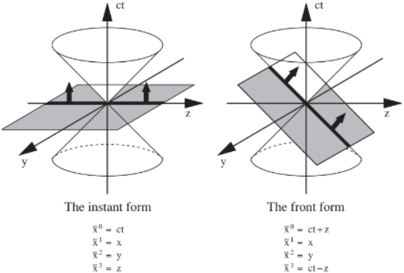
- Light Front QCD is an *ab initio* approach to study the strongly interacting system.
- Hamiltonian Formulation : $P^- P^+ |\psi\rangle = M^2 |\psi\rangle$, and has simplified Vacuum Structure.
- Light-Front coordinates : $x^\mu = (x^+, x^-, x^\perp)$; $k^\mu = (k^+, k^-, k^\perp)$;

Instant form

- All measurements are made at fixed t i.e. at $x^0 = 0$.
- Energy-momentum dispersion relation $p^0 = \sqrt{\vec{p}^2 + m^2}$.
- Vacuum is infinitely complex.

Front form

- All measurements are made at fixed light-cone time x^+ i.e. at $x^+ = x^0 + x^3 = 0$.
- Energy-momentum dispersion relation $p^- = \frac{\vec{p}_\perp^2 + m^2}{p^+}$.
- Vacuum is simple, as fluctuations are absent.



$$\bar{g}_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \qquad \bar{g}_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \frac{1}{2} & 0 & 0 & 0 \end{pmatrix}$$

- The choice of light-front gauge $A^+ = 0$, eliminate certain degrees of freedom.

— P.A.M. Dirac (RMP, 1949) ; A. Harindranath (1996) ; Stanley J. Brodsky (2008)

Photon LFWFs : LF QED

- The photon light-front wavefunctions can be computed perturbatively in QED.

$$\Psi^{\gamma,L}_{h,\bar{h}}(r,x;Q^2,m_f) = \sqrt{\frac{N_c}{4\pi}}\delta_{h,-\bar{h}}\, e\, e_f\, 2x(1-x)Q\frac{K_0(\epsilon r)}{2\pi}\,,$$

$$\Psi^{\gamma,T}_{h,\bar{h}}(r,x;Q^2,m_f) = \pm\sqrt{\frac{N_c}{2\pi}}\, e\, e_f\left[ie^{\pm i\theta_r}(x\delta_{h\pm,\bar{h}\mp} - (1-x)\delta_{h\mp,\bar{h}\pm})\partial_r + m_f\delta_{h\pm,\bar{h}\pm}\right]\frac{K_0(\epsilon r)}{2\pi}$$

Where $\epsilon^2 = x(1-x)Q^2 + m_f^2$

In $Q \to 0$ or $x \to (0,1)$ limit : m_f acts as infrared regulator.

— G. P. Lepage and S. J. Brodsky PRD22 (1980)

Meson LFWFs : LF Holographic QCD

- The longitudinal and transversely polarized vector meson light-front wave functions :

$$\Psi^{V,L}_{h,\bar{h}}(r,x) = \frac{1}{2}\delta_{h,-\bar{h}}\left[1 + \frac{m_f^2 - \nabla_r^2}{x(1-x)M_V^2}\right]\Psi_L(r,x)\,.$$

$$\Psi^{V,T}_{h,\bar{h}}(r,x) = \pm\left[ie^{\pm i\theta_r}(x\delta_{h\pm,\bar{h}\mp} - (1-x)\delta_{h\mp,\bar{h}\pm})\partial_r + m_f\delta_{h\pm,\bar{h}\pm}\right]\frac{\Psi_T(r,x)}{2x(1-x)}\,.$$

— J. R. Forshaw and R. Sandapen, PRL109 (2012)

The holographic vector meson LFWFs $\Psi_\lambda(r,x) = \mathcal{N}_\lambda \phi(\zeta) \times \chi(x)$

- Transverse mode : Can obtained by solving the LF Schrödinger equation :

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U_\perp(\zeta)\right)\phi(\zeta) = M_\perp^2 \phi(\zeta);$$

with $U_\perp(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(J-1)$ and $\zeta = \sqrt{x(1-x)}r_\perp$.

$$\Psi(x,\zeta) = \frac{\kappa}{\sqrt{\pi}}\sqrt{x(1-x)}\exp\left[-\frac{\kappa^2\zeta^2}{2}\right] .$$

- Longitudinal mode : 't Hooft equation which can be derived by using the QCD lagrangian in (1+1) dimension with large N_c approximations :

$$\left(\frac{m_q^2}{x} + \frac{m_{\bar{q}}^2}{1-x}\right)\chi(x) + \frac{g^2}{\pi}\mathcal{P}\int dy \frac{\chi(x)-\chi(y)}{(x-y)^2} = M_\parallel^2 \chi(x);$$

Using the matrix method ⇒ $\chi(x) \simeq x^{\beta_1}(1-x)^{\beta_2}$

- In the chiral limit ⇒ $\boxed{\beta_1 = (3m_q^2/\pi g^2)^{1/2}}$, and $\boxed{\beta_2 = (3m_{\bar{q}}^2/\pi g^2)^{1/2}}$

- Total Holographic vector meson light-front wavefunction :

$$\Psi_\lambda(x,\zeta) = \mathcal{N}_\lambda\sqrt{x(1-x)}x^{\beta_1}(1-x)^{\beta_2}\exp\left[-\frac{\kappa^2\zeta^2}{2}\right]$$

— Teramond, S.J. Brodsky PRD, 104 (2021), C. Mondal et al. PLB, 823 (2021), J.R. Hiller, Ann. Phys. 337 (2013)

Model predictions : Mass spectrum

- Using both the holographic Schrödinger Equation together with the 't Hooft Equation ρ and ϕ meson mass spectra :

$$M^2(n_\perp, n_\parallel, J, L) = 4\kappa^2\left(n_\perp + \frac{J+L}{2}\right) + M_\parallel^2(n_\parallel, m_q, m_{\bar{q}}, g) \ ,$$

$J^{P(C)}$	Name	$n_\perp$	$n_\parallel$	L	$M_\parallel$ [MeV]	$M_\perp$ [MeV]	M_{tot} [MeV] (This work)
1^{--}	$\rho(770)$	0	0	0	134	740	752
2^{++}	$a_2(1320)$	0	2	1	296	1281	1315
3^{--}	$\rho_3(1690)$	0	4	2	401	1654	1702
4^{++}	$a_4(1970)$	0	6	3	484	1957	2016
1^{--}	$\rho(1450)$	1	2	0	296	1281	1315
2^{++}	$a_2(1700)$	1	4	1	401	1654	1702
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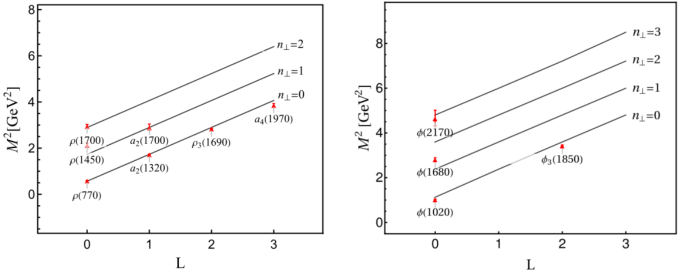
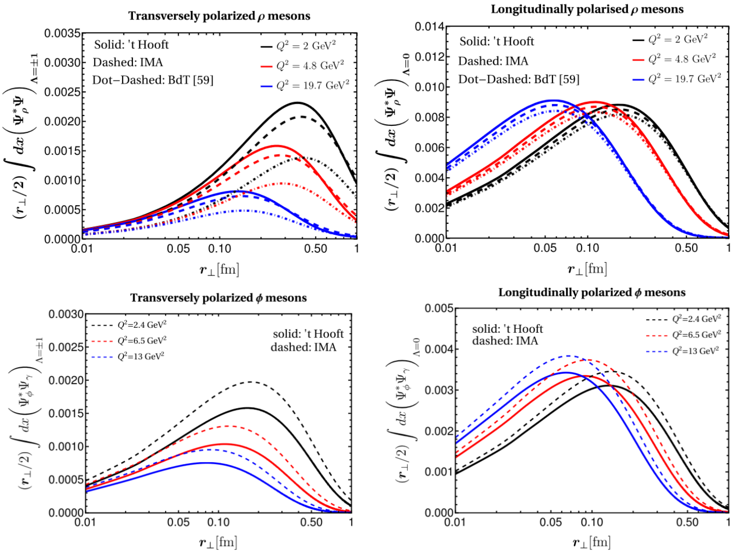


— BG, C. Mondal, S. Kaur PRD, 109 (2024), C. Mondal et al. PLB, 823 (2021)

LFWFs overlap

- Vector meson production scattering amplitude $\propto \Psi^{\gamma^*,\lambda}_{h,\bar{h}}(r,x;Q^2) \otimes \Psi^{V,\lambda}_{h,\bar{h}}(r,x)$



— BG, C. Mondal, S. Kaur PRD, 109 (2024)

Dipole cross section

- Dipole-proton scattering amplitude $\mathcal{N}(x_m, r, b)$ can be obtained by solving the Balitsky-Kovchegov (BK) equation.

- $\hat{\sigma}(x_{\mathsf{m}}, r) = \sigma_0 \mathcal{N}(x_{\mathsf{m}}, rQ_s, 0)$

$$\mathcal{N}(x_{\mathsf{m}}, rQ_s, 0) = \begin{cases} \mathcal{N}_0 \left(\frac{rQ_s}{2}\right)^{2\left[\gamma_s + \frac{\ln(2/rQ_s)}{\kappa\,\lambda\,\ln(1/x_{\mathsf{m}})}\right]} & \text{for} \quad rQ_s \leq 2 \\ 1 - \exp[-\mathcal{A}\ln^2(\mathcal{B}\,rQ_s)] & \text{for} \quad rQ_s > 2 \end{cases}$$

- Q_s is the saturation scale which is given as, $Q_s = (x_0/x_m)^{\lambda/2}$ GeV, $x_m = \frac{Q^2 + 4m_f^2}{W^2}$.

- CGC dipole model free parameters σ_0, λ, x_0 and γ_s fitted from recent H1 and ZEUS (2015) structure function F_2 data (with $x_Bj \leq 0.01$ and $Q^2 \in [0.045, 45]$ GeV^2) for $m_{u,d} \sim 0.046$.

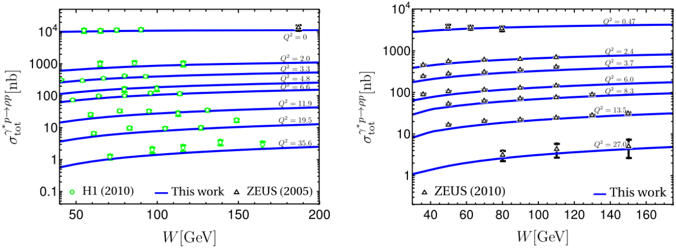
$$F_2(Q^2, x_{Bj}) = \frac{Q^2}{4\pi^2\alpha_{em}}\left(\sigma_L^{\gamma^* p}(Q^2, x_{Bj}) + \sigma_T^{\gamma^* p}(Q^2, x_{Bj})\right)$$

- The fitted parameters are $\sigma_0 = 26.3$ mb, $\gamma_s = 0.741, \lambda = 0.219, x_0 = 1.81 \times 10^{-5}$

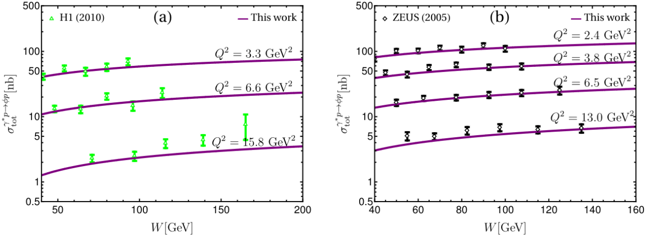
with a χ^2/d.o.f $= 1.03$.

— E. Iancu, et al. B 590, 199 (2004), Ahmady, Sandapen, and Sharma PRD 94, (2016)

- ρ meson cross-section as a function of COM energy W in different Q^2 bins :

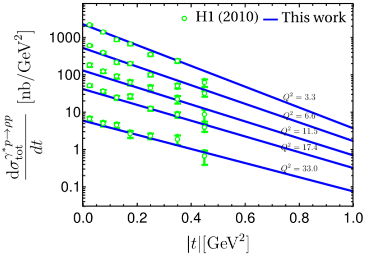


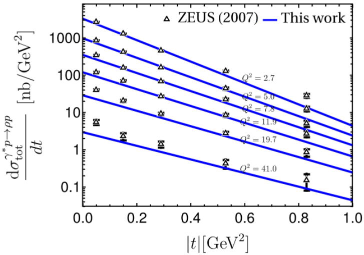
- ϕ meson cross-section as a function of COM energy W in different Q^2 bins :



— BG, C. Mondal et al. PRD(2024) ; H1 Collaboration : NPB 463 (1996), JHEP (2010) ; ZEUS Collaboration : NPB 718 (2005)

- ρ vector meson production differential cross-section with $|t|$:





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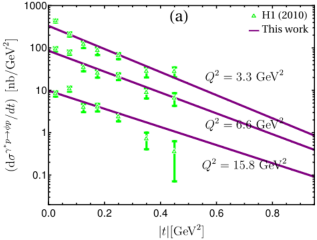
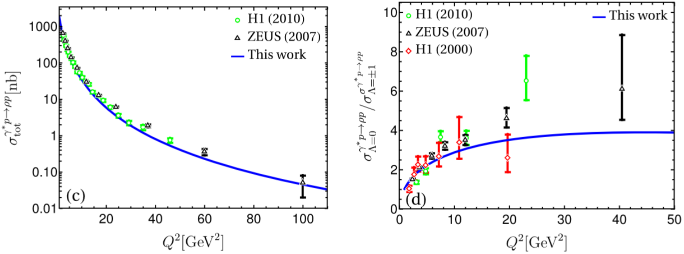


— BG, C. Mondal et al. PRD(2024) ; H1 Collaboration : JHEP (2010) ; ZEUS Collaboration (2007)

Dependence on Q^2

- ρ vector meson production total cros-section as a function of Q^2 at fixed W :



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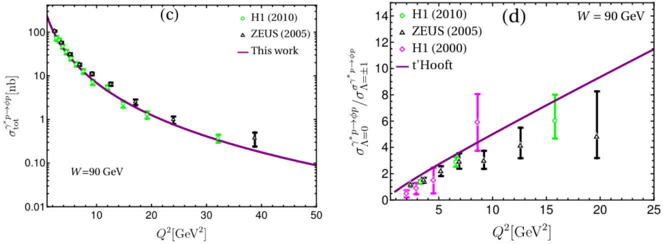
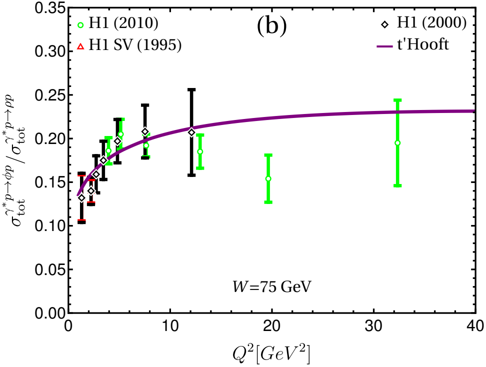


— BG, C. Mondal et al. PRD(2024) ; H1 Collaboration JHEP (2010) ; ZEUS Collaboration : NPB 718 (2005),

- ϕ to ρ cross-section ratio is simply given by the squared ratio of the effective electric charges of the quark-antiquark coupling to the photon :

$$\lim_{Q^2\to\infty} \frac{\sigma_\phi}{\sigma_\rho} = \frac{e_s^2}{e_{u/d}^2} = \left(\frac{1/3}{1/\sqrt{2}}\right)^2 = 0.22$$

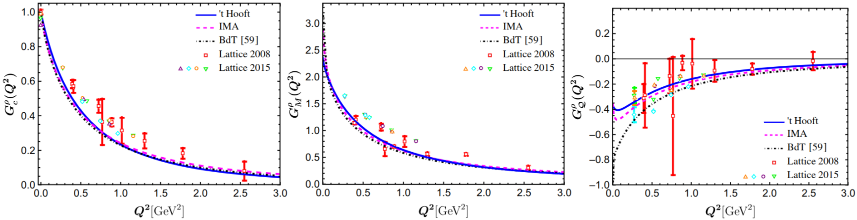


- H1 : $\frac{\sigma(\phi)}{\sigma(\rho)} = 0.191 \pm 0.007$ (stat.)$^{+0.008}_{-0.006}$ (syst.) for $\left(Q^2 + M_V^2 \geq 2\ \text{GeV}^2\right)$.

— BG, C. Mondal, S. Kaur (manuscript in prepration) ; H1 Collaboration JHEP (2010) ; H1 Collaboration : Physics Letters B 483 (2000)

- The EM FFs : plus component of the electromagnetic current, $J^+(0)$:

$$I^{+}_{\Lambda',\Lambda}(Q^2) \triangleq \langle V(P',\Lambda')|\frac{J^+(0)}{2P^+}|V(P,\Lambda)\rangle = \sum_{h,\bar{h}} \int_0^1 \int_0^\infty \frac{\mathrm{d}x\mathrm{d}^2\mathbf{k}_\perp}{16\pi^3} \Psi^{\Lambda'*}_{h\bar{h}}(x,\mathbf{k}_\perp+(1-x)\mathbf{q}_\perp)\Psi^{\Lambda}_{h\bar{h}}(x,\mathbf{k}_\perp)$$



- The static properties : rms charge radius, magnetic moment, and quadrupole moment :

$$\langle r_\rho^2\rangle = -\frac{6}{G_C(0)}\frac{\partial G_C(Q^2)}{\partial Q^2}\Big|_{Q^2\to 0}, \quad \boxed{eG_C(0) = e\,, \quad eG_M(0) = 2M_V\,\mu\,, \quad -eG_Q = M_V^2\,Q}.$$

	This work			BLFQ [111]	BSE [118]	Lattice QCD [119]	Lattice QCD [121]	LFQM [108]	NJL model [120]
	't Hooft	IMA	BdT						
$\sqrt{\langle r_\rho^2\rangle}$	0.75	0.94	0.91	0.44	0.73	0.819(42)	0.55(5)	0.52	0.82
μ_ρ	2.40	2.90	3.30	2.15	2.01	2.067(76)	2.17(10)	1.92	2.48
$\mathcal{Q}_\rho$	−0.027	−0.023	−0.066	−0.063	−0.026	−0.0452(61)	−0.035	−0.028	−0.070

— S. J. Brodsky and J. R. Hiller, PRD(1992) ; H.M. Choi and C.R. Ji, PRD(2004) ; BG et al. PRD(2024).

Conclusion

- Color glass condensate (CGC) is a theoretical framework widely used to explain physical phenomena occurring within proton saturation region.
- Light-Front Holographic AdS/QCD predictions along with 't Hooft for diffractive ρ and ϕ production are in good agreement with the HERA data.
- The ϕ to ρ total cross section ratios are found to be independent of $(Q^2 + M_V^2)$ and consistent with ratio expected from quark charge counting, $\phi : \rho = 2 : 9$.
- Electromagnetic Form factors and static properties are also consistent with the available predictions in litreture.
- The study of VM production at HERA thus provides new insights for the understanding of QCD.

Thanks for your attentation ! !