Hypercentral Quark Model for Mass Spectra, Semileptonic Decays, and Regge Trajectories of Doubly Heavy Ξ Baryons

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Outline

Introduction

- A doubly heavy baryon combines two heavy quarks (b, c) with a light quark $((u, d)$ for Ξ baryons and s for Ω baryons).
- In 2002, the SELEX collaboration observed the Ξ_{cc}^{+} baryon with a measured mass of (3519 ± 1) MeV in the decay mode $\Xi_{cc}^{+} \to \Lambda_c^{+} K^{-} \pi^{+}$. The recent observation of Ξ_{cc}^{++} baryon with lifetime $\tau(\Xi_{cc}^{++}) = (0.256^{+0.024}_{-0.022} \pm 0.014)$ ps, demonstrates how really the LHC is a powerful discovery machine, and stimulates the theoretical studies of mass spectra of doubly heavy baryons.
- The doubly heavy Ξ baryons masses are experimentally unknown (except Ξ_{cc}^{+} and Ξ_{cc}^{++})
- For the doubly heavy baryons, no experimental semileptonic decays are reported, and only a limited number of theoretical calculations are available
- As there are more doubly heavy baryons that may be discovered in the future, proposing theoretical models for their structure is essential.

Introduction

In recent years, due availability of so many experimental facilities, the spectroscopy of heavy flavor hadrons has attracted considerable interest.

- **CLEO**
- **BaBar** and **Belle**
- **Selex**
- **CERN** : **LHCb**
- **Tevatron**
- Future experiments **PANDA**, **Belle-II**

The search for light resonances is the main focus of the baryon program at

- **JLab**
- **the Beijing Spectrometer (BES)**
- **the Electron Stretcher and Accelerator (ELSA) facility (the Crystal Barrel collaboration)**
- **the Two Arms Photon Spectrometer (TAPS)**
- **SAPHIR** and **CLAS**

New results are expected from analysis projects such as EBAC, Julich, SAID, and MAID.

Theoretical Framework

Since 1950's, the excited states of nucleons have been studied experimentally. Their study contributed to the discovery of the quark model by Gell-Mann and Zweig in 1964, and were critical for the discovery of "color" degrees of freedom as introduced by Greenberg.

Different Approaches

- relativistic quark model (Ebert et al.)
- variational approach (Roberts et al.)
- Lattice QCD (Padmanath et al., Brown et al., Paula et al.)
- Hamiltonian Model (Yoshida et al.)
- Chiral Quark Model (Li-Ye Xiao et al.)
- diquark picture (Qi-Fang LAij et al.)
- Sum rules (Azizi et al., Hua-Xing Chen et al., Aliev et al.)

In this study, we used the Hypercentral Constituent Quark Model, which was introduced in 1995 by M. Ferrariset et al.

The Hypercentral Constituent Quark Model (hCQM)

In hCQM, baryons are viewed as three-body systems made up of quarks. In the reference frame of the center of mass, the interquark motion is usually described by the so-called Jacobi coordinates (ρ, λ) given by

Jacobi coordinates

$$\vec{\rho} = \frac{1}{\sqrt{2}}(\vec{r}_1 - \vec{r}_2) \quad , \quad \vec{\lambda} = \sqrt{\frac{2}{3}}\left(\frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2} - \vec{r}_3\right), \tag{1}$$

with $r = \sqrt{\rho^2 + \lambda^2}$. In this reference frame, the reduced mass associated with each of the coordinate are given by

Reduced mass

$$m_\rho = \frac{2m_1m_2}{m_1 + m_2} \; , \; m_\lambda = \frac{2m_3(m_1^2 + m_2^2 + m_1m_2)}{(m_1 + m_2)(m_1 + m_2 + m_3)} \; , \; m = \frac{2m_\rho m_\lambda}{m_\lambda + m_\rho}. \tag{2}$$

The Hypercentral Constituent Quark Model (hCQM)

In the framework of 6-dimensional hypercentral constituent quark models, a three bodies baryonic system can be described by the Hamiltonian operator expressed as follows

Hamiltonian operator

$$\hat{H} = \frac{P_\rho^2}{2m_\rho} + \frac{P_\lambda^2}{2m_\lambda} + V(r) = -\frac{1}{2m}\left[\frac{d^2}{dr^2} + \frac{5}{r}\frac{d}{dr} - \frac{\vec{L}^2(\Omega_\rho, \Omega_\lambda, \xi)}{r^2}\right] + V(r), \quad (3$$

where $\Omega_\rho = (\theta_\rho, \varphi_\rho)$ and $\Omega_\lambda = (\theta_\lambda, \varphi_\lambda)$ are the hyperspherical coordinates. Thus, the eigenvalues equation of $\vec{L}^2(\Omega_\rho, \Omega_\lambda, \xi)$ can be written as

Eigenvalues equation of $\vec{L}^2(\Omega_\rho, \Omega_\lambda, \xi)$

$$\vec{L}^2(\Omega_\rho, \Omega_\lambda, \xi) Y_{[\gamma], l_\rho, l_\lambda}(\Omega_\rho, \Omega_\lambda, \xi) = \gamma(\gamma + 4) Y_{[\gamma], l_\rho, l_\lambda}(\Omega_\rho, \Omega_\lambda, \xi), \qquad (4)$$

The Hypercentral Constituent Quark Model (hCQM)

Thus, the hyperradial Schrödinger equation reads

$$\frac{d^2\varphi_{\nu\gamma}(r)}{dr^2} + \left[2mE_{\nu\gamma} - 2mV(r) - \frac{(2\gamma+3)(2\gamma+5)}{4r^2}\right]\varphi_{\nu\gamma}(r) = 0. \quad (5)$$

The hypercentral interaction potential $V(r)$ in Eq.(5) has the following form

$$V(r) = V^{(0)}(r) + \left(\frac{1}{m_\rho} + \frac{1}{m_\lambda}\right) V^{(1)}(r) + V_{SD}(r), \quad (6)$$

$$V^{(0)}(r) = -\frac{b}{r}e^{-cr} + ar + dr^2, \quad (7)$$

$$V^{(1)}(r) = -C_F C_A \frac{\alpha_S^2}{4r^2}, \quad (8)$$

$$\alpha_S = \frac{\alpha_S(\mu_0)}{1 + (\frac{33-2n_f}{12\pi})\alpha_S(\mu_0)\ln(\frac{m_1+m_2+m_3}{\mu_0})}. \quad (9)$$

The spin-dependent forces V_{SD} are given in terms of the spin-spin interaction V_{SS}, spin-orbit interaction $V_{\gamma S}$ and the tensor term V_T as

$$\begin{aligned} V_{SD}(r) &= V_{SS}(r)\left[\vec{S}_\rho \cdot \vec{S}_\lambda\right] + V_{\gamma S}\left[\vec{\gamma}\cdot\vec{S}\right] + V_T(r)\left[\vec{S}^2.\right. \\ &- \left. 3\left(\vec{S}\cdot\frac{\vec{r}}{|\vec{r}|}\right)\left(\vec{S}\cdot\frac{\vec{r}}{|\vec{r}|}\right)\right], \end{aligned} \tag{10}$$

where V_{SS}, $V_{\gamma S}$ and V_T are given by

$$V_{SS}(r) = \frac{1}{3m_\lambda m_\rho}\nabla^2 V_V(r) \tag{11}$$

$$V_{\gamma S}(r) = \frac{1}{2m_\lambda m_\rho r}\left(3\frac{dV_V(r)}{dr} - \frac{dV_S(r)}{dr}\right) \tag{12}$$

$$V_T(r) = \frac{1}{6m_\lambda m_\rho}\left(3\frac{d^2V_V(r)}{dr^2} - \frac{1}{r}\frac{dV_V(r)}{dr}\right). \tag{13}$$

Mass equation of the doubly heavy Ξ baryons

$$M_B = \theta_1 + m_1 + m_2 + m_3 - \frac{1}{6m}\left[\frac{\theta_2}{2\nu + 1 + 2\sqrt{\theta_3 + \frac{1}{4}}}\right]^2, \quad (14)$$

$$\theta_1 = \frac{\mu(\vec{\gamma}\cdot\vec{S})d}{m_\lambda m_\rho} + \frac{10d}{3\delta^2} + \frac{2a}{\delta} - \frac{\delta\left(-\frac{1}{2}c^2\phi_1 + c\phi_2 - \phi_3\right)}{3} - \sum_{k=0}^{\infty}\frac{(-1)^k c^{k+1}(k+2)(k+3)}{6(k+1)!\delta^k}\left[\phi_3 - \frac{\phi_1 c^2}{(k+3)}\right] \quad (15)$$

$$\theta_2 = -\sum_{k=0}^{\infty}\frac{2m(-1)^k c^{k+1}(k+2)(k+4)}{2(k+1)!\delta^{k+1}}\left[\phi_3 - \frac{\phi_1 c^2}{(k+3)}\right] + \frac{48md}{\delta^3} + \frac{30ma}{\delta^2} - 3h_3 - \frac{12m\phi_1}{\delta^2} - \frac{8\epsilon_{\nu\gamma}}{\delta} \quad (16)$$

$$\theta_3 = -\sum_{k=0}^{\infty}\frac{2m(-1)^k c^{k+1}(k+3)(k+4)}{2(k+1)!\delta^{k+2}}\left[\phi_3 - \frac{\phi_1 c^2}{(k+3)}\right] + \frac{30md}{\delta^4} \quad (17)$$

Magnetic moments, radiative transitions and semileptonic decays

- Magnetic moments of the doubly heavy Ξ baryons

The magnetic moments are fundamental properties of baryons that are essential for a complete description of their behavior in electromagnetic fields. Having the baryonic wave function at hand, one can easily calculate the magnetic moment of a doubly heavy Ξ baryon as follows:

$$\mu_B = \sum_i \langle \psi_{sf} |\hat{\mu}_i| \psi_{sf} \rangle = \sum_i \langle \psi_{sf} |\mu_i \sigma_i| \psi_{sf} \rangle , \quad (18)$$

where ψ_{sf} is the spin flavor wave function of the baryonic system, and $\hat{\mu}_i$ is the magnetic moment operator given by:

$$\hat{\mu}_i = \mu_i \sigma_i = \frac{e_i}{2m_i^{eff}} \sigma_i \quad , \quad m_i^{eff} = m_i \left(1 + \frac{\langle H \rangle}{\sum_i m_i} \right) . \quad (19)$$

In Eq.(19), e_i, m_i and σ_i stand for the charge, mass and spin of the $i-$th quark, and m_i^{eff} is the effective mass.

Magnetic moments, radiative transitions and semileptonic decays

- Radiative decay widths of the doubly heavy Ξ baryons

The M1 partial width of the decay $B^* \to \gamma B$ is given by:

$$\Gamma = \frac{1}{137}\frac{\omega^3}{M_P^2}\frac{2}{2J+1}\left(\frac{M_B}{M_{B^*}}\right)\mu^2(B^* \leftrightarrow B), \tag{20}$$

where M_P is the proton mass, M_{B^*} and J are the mass and spin of the decaying particle, M is the baryon's mass in its final state.

- transition magnetic moment

$$\mu(B^* \leftrightarrow B) = \sum_i \left\langle \psi_{sf}^{B} \middle| \frac{e_i}{2m_i^{eff}}\sigma_i \middle| \psi_{sf}^{B^*} \right\rangle \quad , \quad \omega = \frac{M_{B^*}^2 - M_B^2}{2M_B}, \tag{21}$$

ω is the photon momentum.

Magnetic moments, radiative transitions and semileptonic decays

- Semileptonic decay widths of the doubly heavy Ξ baryons

Close to zero recoil, the expressions for the semileptonic decay widths are simplified as follows:

$$\begin{aligned}
\Gamma_{\frac{1}{2}\to\frac{1}{2}} &= \frac{G_F^2}{12\pi^3} M_i^5 R^4 \left|V_{bc}\right|^2 \int_1^{\omega_{\max}} d\omega \sqrt{\omega^2-1}\left[I_1^+(\omega)\eta^2(\omega)\right. \\
&+ \left. I_1^-(\omega)\eta^2(\omega)\right], \qquad (22) \\
\Gamma_{\frac{3}{2}\to\frac{1}{2}} &= \frac{G_F^2}{24\pi^3} M_i^5 R^4 \left|V_{bc}\right|^2 \int_1^{\omega_{\max}} d\omega \sqrt{\omega^2-1} I_3(\omega)\eta^2(\omega), \qquad (23) \\
\Gamma_{\frac{3}{2}\to\frac{3}{2}} &= \frac{G_F^2}{24\pi^3} M_i^5 R^4 \left|V_{bc}\right|^2 \int_1^{\omega_{\max}} d\omega \sqrt{\omega^2-1}\left[I_4^+(\omega)\eta^2(\omega)\right. \\
&+ \left. I_4^-(\omega)\eta^2(\omega)\right], \qquad (24)
\end{aligned}$$

where $R = M_f/M_i$, V_{bc} is the CKM matrix element and $\omega_{\max} = (1+R^2)/2R$.

Numerical results

- For the six doubly heavy Ξ baryons, light quarks (u and d) are combined with heavy quarks. The mass difference between the light quarks is 12 MeV. Thus, it is obvious that when we move towards the calculation of the excited states the baryons masses would also have a very small mass difference.
- For sake of completeness, we calculated whole mass spectrum for all six doubly heavy Ξ baryons: Ξ_{cc}^{+}, Ξ_{cc}^{++}, Ξ_{bb}^{-}, Ξ_{bb}^{0}, Ξ_{bc}^{0} and Ξ_{bc}^{+}, and we noticed that it hardly differs less than $\approx$ 10 MeV.
- The Calculations have been implemented for the ground state (1S), the radial excited states (2S-5S) and the orbital excited states (1P-5P).
- We consider all isospin splittings and accordingly J^P values are determined.

- The LHCb experiment: Ξ_{cc}^{++} with the mass $(3621.40 \pm 0.72 \pm 0.27 \pm 0.14)$ MeV and quark combination ccu.The decay mode of the experimental investigation is $\Xi_{cc}^{++} \to \Lambda_c^+ K^- \pi^+ \pi^-$.
- SELEX experiment: a ground state at 3520 MeV containing two charm quarks and a down quark in its decay mode $\Xi_{cc}^{+} \to p D^+ K^-$.
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In our model, due to the lack of experimental data, the values of the potential parameters are obtained from fitting our calculated masses with those reported in other works. The values of the constituent quark masses and the parameters of the problem are reported in Table 1.

Table 1: Parameters of the system under consideration.

Variables systems	m_q	α_S	C_F	C_A	a_V	a_S	b	c	d	δ
Unit	GeV	-	-	-	GeV^2	GeV^2	-	GeV	GeV^3	GeV
u quark	0.34	0.340	2/3	3	0.08655	0.2518	391.841	0.0802	0.0089	0.0691
d quark	0.35									
c quark	1.348									
b quark	4.750									

Ground state masses

The ground state masses of the doubly heavy Ξ baryons are listed in Table 2.

Table 2: The calculated ground state masses (in GeV) of Ξ baryons are listed with other relevant theoretical works.

Baryon	Ξ_{ccd}/Ξ_{ccu}		Ξ_{bbd}/Ξ_{bbu}		Ξ_{bcd}/Ξ_{bcu}	
J^P	$\frac{1}{2}^+$	$\frac{3}{2}^+$	$\frac{1}{2}^+$	$\frac{3}{2}^+$	$\frac{1}{2}^+$	$\frac{3}{2}^+$
Our	3.521	3.696	10.215	10.184	6.741	6.828
Ref.[1]	3.520/3.511	3.695/3.687	10.317/10.312	10.340/10.335	6.920/6.914	6.986/6.980
Ref.[72]	3.519					
Ref.[20]	3.685	3.754	10.314			
Ref.[22, 23]	3.520	3.695	10.199	10.316		
Ref.[14]	3.610	3.694				
Ref.[24]	3.610	3.692	10.143	10.178	6.943	6.985
Ref.[73]	3.561	3.642				
Ref.[28]	3.720		9.960		6.943	
Ref.[29]	3.687	3.752	10.322	10.352	7.014	7.064
Ref.[74]	3.676	3.753	10.340	10.367	7.011	7.074
Ref.[75]	3.547	3.719	10.185	10.216	6.904	6.936
Ref.[30]	3.579	3.565	10.189	10.218		
Ref.[31]	3.620	3.727	10.202	10.237	6.933	6.980
Ref.[32]	3.478	3.610	10.093	10.133	6.820	6.900
Ref.[76]	3.627	3.690	10.162	10.184	6.914	
Ref.[77]	3.519	3.620	9.800	9.980	6.650	6.690
Ref.[33]	3.612	3.706	10.197	10.136	6.919	6.986
Ref.[78]	3.510	3.548	10.130	10.144	6.792	6.827
Ref.[79]	3.570	3.610	10.170	10.220		

Ground state masses

Radial excited states masses of the doubly heavy Ξ baryons are listed in Table 3.

Table 3: The calculated radial excited state masses (in GeV) of Ξ baryons are listed with other relevant theoretical works.

Baryon	State	J^P	Our Calc	[1]	[1]	[20]	[74]	[75]	[30]	[31]	[29]
	2S		3.903	3.925	3.920	4.079	4.029	4.183	3.976	3.910	4.030
	3S		4.182	4.233	4.159	4.206		4.640		4.154	
	4S	$\frac{1}{2}^+$	4.436	4.502	4.501						
Ξ_{ccd}	5S		4.669	4.748	4.748						
and	2S		3.968	3.988	3.983	4.114	4.042	4.282	4.025	4.027	4.078
Ξ_{ccu}	3S		4.241	4.264	4.261	4.131		4.719			
	4S	$\frac{3}{2}^+$	4.482	4.520	4.519						
	5S		4.671	4.759	4.759						
	2S		10.062	10.612	10.609	10.571	10.576	10.751	10.482	10.441	10.551
	3S		10.411	10.862	10.862	10.612		11.170		10.630	
	4S	$\frac{1}{2}^+$	10.816	11.088	11.090					10.812	
Ξ_{bbd}	5S		10.826	11.297	11.301						
and	2S		9.998	10.619	10.617	10.592	10.578	10.770	10.501	10.482	10.574
Ξ_{bbu}	3S		10.485	10.855	10.866	10.593		11.184		10.673	
	4S	$\frac{3}{2}^+$	10.519	11.090	11.092					10.856	
	5S		10.812	11.298	11.302						
	2S		6.979	7.244	7.240			7.478			7.321
	3S		7.236	7.509	7.507			7.904			
	4S	$\frac{1}{2}^+$	7.429	7.746	7.744						
Ξ_{bcd}	5S		7.725	7.963	7.964						
and	2S		7.019	7.267	7.263			7.495			7.353
Ξ_{bcu}	3S		7.297	7.521	7.518			7.917			
	4S	$\frac{3}{2}^+$	7.474	7.752	7.752						
	5S		7.606	7.968	7.969						

Calculated masses of orbital excited states

Orbital excited states masses of Ξ_{cc}^{+} and Ξ_{cc}^{++} baryons are listed in Table 4.

Table 4: Orbitaly excited states masses for Ξ_{cc} baryon (in GeV).

State	Our Calc Ξ_{cc}^{+}	Our Calc Ξ_{cc}^{++}	[1] Ξ_{cc}^{+}	[1] Ξ_{cc}^{++}	[20]	[74]	[30]	[31]	[22]	[32]	[29]	[14]
$(1^2P_{1/2})$	3.850	3.848	3.865	3.861	3.947	3.910	3.880	3.838			4.073	3.892
$(1^2P_{3/2})$	3.838	3.833	3.847	3.842	3.949	3.921		3.959	3.786	3.834	4.079	3.989
$(1^4P_{1/2})$	3.867	3.861	3.875	3.871								
$(1^4P_{3/2})$	3.849	3.843	3.856	3.851								
$(1^4P_{5/2})$	3.882	3.875	3.890	3.888	4.163	4.092		4.155	3.949	4.047	4.089	
$(2^2P_{1/2})$	4.125	4.118	4.161	4.140	4.135	4.074	4.018	4.085				
$(2^2P_{3/2})$	4.112	4.102	4.144	4.140	4.137	4.078	4.197					
$(2^4P_{1/2})$	4.133	4.129	4.169	4.167								
$(2^4P_{3/2})$	4.131	4.120	4.152	4.149								
$(2^4P_{5/2})$	4.157	4.146	4.183	4.181	4.488							
$(3^2P_{1/2})$	4.382	4.373	4.426	4.409	4.149							
$(3^2P_{3/2})$	4.365	4.358	4.411	4.409	4.159							
$(3^4P_{1/2})$	4.378	4.371	4.433	4.432								
$(3^4P_{3/2})$	4.368	4.359	4.419	4.417								
$(3^4P_{5/2})$	4.376	4.372	4.399	4.396	4.534							
$(4^2P_{1/2})$	4.612	4.603	4.671	4.671								
$(4^2P_{3/2})$	4.617	4.616	4.658	4.657								
$(4^4P_{1/2})$	4.624	4.619	4.678	4.678								
$(4^4P_{3/2})$	4.639	4.632	4.664	4.664								
$(4^4P_{5/2})$	4.648	4.639	4.646	4.646								
$(5^2P_{1/2})$	4.814	4.809	4.901	4.902								
$(5^2P_{3/2})$	4.793	4.785	4.889	4.889								
$(5^4P_{1/2})$	4.820	4.818	4.908	4.909								
$(5^4P_{3/2})$	4.745	4.741	4.895	4.895								
$(5^4P_{5/2})$	4.800	4.798	4.878	4.879								

Calculated masses of orbital excited states

Orbital excited states masses of Ξ^-_{bb} and Ξ^0_{bb} baryons are listed in Table 5.

Table 5: Orbitaly excited states masses for Ξ_{bb} baryon (in GeV).

State	Our Calc Ξ_{bb}	Our Calc Ξ^0_{bb}	[1] Ξ_{bb}	[1] Ξ^0_{bb}	[20]	[74]	[31]	[22]	[29]	Others
$(1^2P_{1/2})$	10.015	10.011	10.514	10.511	10.476	10.493	10.368		10.691	10.406[30]
$(1^2P_{3/2})$	10.123	10.119	10.509	10.506	10.476	10.495	10.408	10.474	10.692	10.390[79]
$(1^4P_{1/2})$	10.026	10.021	10.517	10.514						
$(1^4P_{3/2})$	9.912	9.904	10.512	10.509						10.430 [28]
$(1^4P_{5/2})$	10.046	10.041	10.521	10.518	10.759			10.588	10.695	
$(2^2P_{1/2})$	10.146	10.144	10.770	10.770	10.703	10.710	10.563			10.612 [30]
$(2^2P_{3/2})$	10.151	10.148	10.766	10.762	10.704	10.713	10.607			
$(2^4P_{1/2})$	10.164	10.163	10.772	10.772						
$(2^4P_{3/2})$	10.156	10.155	10.768	10.767						
$(2^4P_{5/2})$	10.148	10.151	10.773	10.776	10.973	10.713				
$(3^2P_{1/2})$	10.347	10.349	11.001	11.002	10.740		10.744			
$(3^2P_{3/2})$	10.341	10.342	10.997	10.998	10.742		10.788			
$(3^4P_{1/2})$	10.341	10.342	11.003	11.004						
$(3^4P_{3/2})$	10.338	10.339	10.999	11.000						
$(3^4P_{5/2})$	10.329	10.335	10.994	11.007	11.004					
$(4^2P_{1/2})$	10.577	10.581	11.214	11.217			10.900			
$(4^2P_{3/2})$	10.584	10.590	11.210	11.213						
$(4^4P_{1/2})$	10.547	10.549	11.216	11.219						
$(4^4P_{3/2})$	10.571	10.573	11.212	11.215						
$(4^4P_{5/2})$	10.566	10.574	11.208	11.222						
$(5^2P_{1/2})$	10.727	10.732	11.413	11.418						
$(5^2P_{3/2})$	10.711	10.715	11.410	11.415						
$(5^4P_{1/2})$	10.719	10.725	11.415	11.420						
$(5^4P_{3/2})$	10.732	10.737	11.412	11.417						
$(5^4P_{5/2})$	10.711	10.718	11.407	11.423						

Calculated masses of orbital excited states

Orbital excited states masses of Ξ^0_{bc} and Ξ^+_{bc} baryons are listed in Table 6.

Table 6: Orbitaly excited states masses for Ξ_{bc} baryon (in GeV).

State	Our Calc Ξ^0_{bc}	Our Calc Ξ^+_{bc}	[1] Ξ^0_{bc}	[1] Ξ^+_{bc}	[29]
$(1^2P_{1/2})$	6.852	6.847	7.160	7.156	7.390
$(1^2P_{3/2})$	6.832	6.827	7.149	7.144	7.394
$(1^4P_{1/2})$	6.862	6.854	7.166	7.141	7.399
$(1^4P_{3/2})$	6.848	6.842	7.155	7.150	
$(1^4P_{5/2})$	6.572	6.573	7.175	7.171	
$(2^2P_{1/2})$	7.112	7.108	7.425	7.422	
$(2^2P_{3/2})$	7.158	7.155	7.415	7.412	
$(2^4P_{1/2})$	7.182	7.181	7.430	7.426	
$(2^4P_{3/2})$	7.196	7.194	7.420	7.417	
$(2^4P_{5/2})$	7.201	7.203	7.408	7.434	
$(3^2P_{1/2})$	7.310	7.308	7.664	7.662	
$(3^2P_{3/2})$	7.301	7.299	7.655	7.654	
$(3^4P_{1/2})$	7.309	7.307	7.668	7.666	
$(3^4P_{3/2})$	7.295	7.296	7.659	7.658	
$(3^4P_{5/2})$	7.301	7.304	7.648	7.673	
$(4^2P_{1/2})$	7.512	7.531	7.884	8.015	
$(4^2P_{3/2})$	7.489	7.490	7.876	7.877	
$(4^4P_{1/2})$	7.513	7.512	7.888	7.888	
$(4^4P_{3/2})$	7.503	7.504	7.880	7.880	
$(4^4P_{5/2})$	7.494	7.510	7.870	7.895	
$(5^2P_{1/2})$	7.697	7.699	8.091	8.092	
$(5^2P_{3/2})$	7.695	7.694	8.084	8.085	
$(5^4P_{1/2})$	7.703	7.711	8.094	8.096	
$(5^4P_{3/2})$	7.708	7.709	8.087	8.088	
$(5^4P_{5/2})$	7.691	7.692	8.078	8.079	

The magnetic moments $\hat{\mu}_i$ of double heavy Ξ baryons

Magnetic moments $\hat{\mu}_i$ of double heavy Ξ baryons with positive parity $J^P = \frac{1}{2}^+$ and $\frac{3}{2}^+$ are listed in Table 7 in terms of the nuclear magnaton μ_N.

Table 7: The magnetic moments of the doubly heavy Ξ baryons in unit of nuclear magnaton μ_N.

Baryon	Quark content	J^P	Function	Our Calc	ref.[83]	Others
Ξ_{cc}^{+}	(ccd)	$\frac{1}{2}^+$	$\frac{4}{3}\mu_c - \frac{1}{3}\mu_d$	0.803	0.784	0.43 ± 0.09 [84]
Ξ_{cc}^{++}	(ccu)	$\frac{1}{2}^+$	$\frac{4}{3}\mu_c - \frac{1}{3}\mu_u$	-0.121	0.031	-0.23 ± 0.05 [84]
Ξ_{bb}^{-}	(bbd)	$\frac{1}{2}^+$	$\frac{4}{3}\mu_b - \frac{1}{3}\mu_d$	0.205	0.196	0.28 ± 0.04 [84]
Ξ_{bb}^{0}	(bbu)	$\frac{1}{2}^+$	$\frac{4}{3}\mu_b - \frac{1}{3}\mu_u$	-0.594	-0.663	-0.51 ± 0.09 [84]
Ξ_{bc}^{0}	(bcu)	$\frac{1}{2}^+$	$\frac{2}{3}\mu_b + \frac{2}{3}\mu_c - \frac{1}{3}\mu_u$	0.517	0.527	0.560 [85]
Ξ_{bc}^{+}	(bcd)	$\frac{1}{2}^+$	$\frac{2}{3}\mu_b + \frac{2}{3}\mu_c - \frac{1}{3}\mu_u$	-0.441	-0.304	-0.540 [85]
Ξ_{cc}^{*+}	(ccd)	$\frac{3}{2}^+$	$2\mu_c + \mu_d$	0.083	0.068	$-$
Ξ_{cc}^{*++}	(ccu)	$\frac{3}{2}^+$	$2\mu_c + \mu_u$	2.311	2.218	
Ξ_{bb}^{*-}	(bbd)	$\frac{3}{2}^+$	$2\mu_b + \mu_d$	-2.005	0.196	
Ξ_{bb}^{*0}	(bbu)	$\frac{3}{2}^+$	$2\mu_b + \mu_u$	-1.497	-1.607	
Ξ_{bc}^{*0}	(bcu)	$\frac{3}{2}^+$	$\mu_b + \mu_c + \mu_u$	-0.524	-0.448	
Ξ_{bc}^{*+}	(bcd)	$\frac{3}{2}^+$	$\mu_b + \mu_c + \mu_d$	1.846	2.107	

Radiative and semileptonic decay widths

The radiative and semileptonic decay widths of double heavy Ξ baryons in Tables 8 and 9 respectively.

Table 8: Radiative decay width of the doubly heavy Ξ baryons in unit of KeV.

Decay	Our Calc	ref.[86]	Others
$\Gamma\left(\Xi_{cc}^{*++} \to \Xi_{cc}^{++}\gamma\right)$	1.792	2.22 ± 0.098	7.210 [48]
			1.430 [45]
$\Gamma\left(\Xi_{bb}^{*0} \to \Xi_{bb}^{0}\gamma\right)$	0.396	0.40 ± 0.044	0.310 [46]
			0.126 [45]
$\Gamma\left(\Xi_{bc}^{*+} \to \Xi_{bc}^{+}\gamma\right)$	0.243	0.205 ± 0.009	0.209 [87]

Table 9: Semileptonic decay width of the doubly heavy Ξ baryons in unit of 10^{-14} GeV.

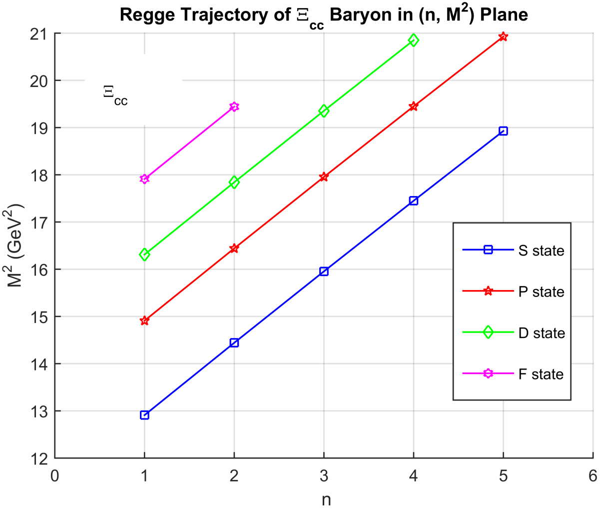
Decay	Our Calc	ref.[88]	ref.[86]	Decay	Our Calc	ref.[88]	ref.[86]
$\Gamma_{\frac{1}{2}\to\frac{1}{2}}\left(\Xi_{bb} \to \Xi_{bc}^{'} l\bar{\nu}_l\right)$	0.670	$1.06^{+0.13}_{-0.03}$	1.04 ± 0.08	$\Gamma_{\frac{1}{2}\to\frac{3}{2}}\left(\Xi_{bc} \to \Xi_{cc}^{*} l\bar{\nu}_l\right)$	0.936	$0.75^{+0.06}$	1.01 ± 0.11
$\Gamma_{\frac{1}{2}\to\frac{1}{2}}\left(\Xi_{bb} \to \Xi_{bc} l\bar{\nu}_l\right)$	1.727	$1.92^{+0.25}_{-0.05}$	1.84 ± 0.14	$\Gamma_{\frac{3}{2}\to\frac{1}{2}}\left(\Xi_{bb}^{*} \to \Xi_{bc}^{*} l\bar{\nu}_l\right)$	0.438	$0.35^{+0.03}$	0.32 ± 0.01
$\Gamma_{\frac{1}{2}\to\frac{1}{2}}\left(\Xi_{bc} \to \Xi_{cc} l\bar{\nu}_l\right)$	1.910	$2.57^{+0.26}_{-0.03}$	1.54 ± 0.21	$\Gamma_{\frac{3}{2}\to\frac{1}{2}}\left(\Xi_{bb}^{*} \to \Xi_{bc}^{'} l\bar{\nu}_l\right)$	0.891	$1.04^{+0.06}$	1.02 ± 0.08
$\Gamma_{\frac{1}{2}\to\frac{1}{2}}\left(\Xi_{bc}^{'} \to \Xi_{cc} l\bar{\nu}_l\right)$	1.401	$1.36^{+0.10}_{-0.03}$	1.30 ± 0.13	$\Gamma_{\frac{3}{2}\to\frac{1}{2}}\left(\Xi_{bc}^{*} \to \Xi_{cc} l\bar{\nu}_l\right)$	0.442	$0.43^{+0.06}$	0.38 ± 0.09
$\Gamma_{\frac{1}{2}\to\frac{3}{2}}\left(\Xi_{bb} \to \Xi_{bc}^{*} l\bar{\nu}_l\right)$	0.701	$0.61^{+0.04}$	0.64 ± 0.07	$\Gamma_{\frac{3}{2}\to\frac{3}{2}}\left(\Xi_{bb}^{*} \to \Xi_{bc}^{*} l\bar{\nu}_l\right)$	2.066	$2.09^{+0.16}$	2.04 ± 0.22
$\Gamma_{\frac{1}{2}\to\frac{3}{2}}\left(\Xi_{bc}^{'} \to \Xi_{cc}^{*} l\bar{\nu}_l\right)$	2.385	$2.33^{+0.16}$	2.21 ± 0.19	$\Gamma_{\frac{3}{2}\to\frac{3}{2}}\left(\Xi_{bc}^{*} \to \Xi_{cc}^{*} l\bar{\nu}_l\right)$	2.413	$2.63^{+0.40}$	1.52 ± 0.15

Regge trajectories of doubly heavy Ξ baryons

- In the 1960's, Tullio Regge introduced the concept of Regge trajectories in hadron physics.
- Regge theory is a successful fundamental theory of strong interactions at very high energies.
- One of the most distinctive feature of Regge theory are the Regge trajectories. Regge trajectories are directly related to mass spectra of hadrons.
- Using hadron masses, the trajectories can be generated in (n, M^2) and (J, M^2) planes:
- $J = \beta M^2 + \beta_0$ and $J = \alpha M^2 + \alpha_0$
- In the Regge trajectories, the fundamental point is that they can predict the masses of unobserved states. The most important properties of Regge trajectories are linearity, divergence and parallelism. In the present work, we found that all the Regge trajectories show linear behavior.

Regge Trajectories Plot for the Ξ_{cc}^{++} baryon

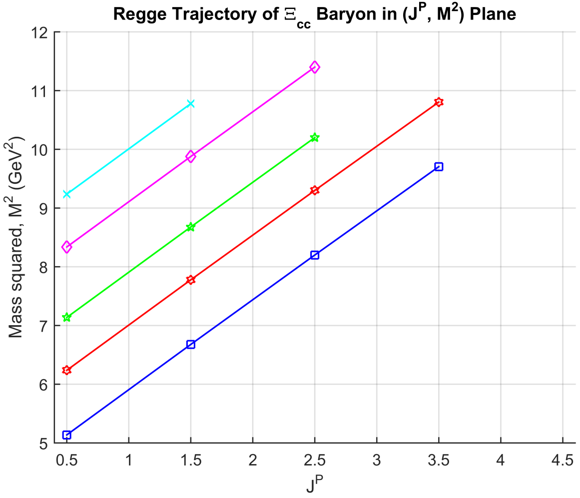
straight lines were obtained by the linear fitting in all Figures.



Regge trajectory $(M^2 \rightarrow n)$

Regge Trajectories Plot for the Ξ_{cc}^{++} baryon

We used the natural $\left(J^P = \frac{1}{2}^+, J^P = \frac{3}{2}^-, J^P = \frac{5}{2}^+, J^P = \frac{7}{2}^-\right)$ parity masses



Regge trajectory $(M^2 \to J)$

Conclusion

- The baryons containing two heavy quarks, namely charm-charm, bottom-bottom and bottom-charm with a light quarks (u or d) are reviewed.
- The mass spectra of the doubly heavy Ξ baryons are determined using the hypercentral constituent quark model.
- The magnetic moments are also calculated as well as radiative decay widths and semileptonic decay widths of the doubly heavy Ξ baryons.
- The Regge trajectories are useful to determine the unknown states.
- This study will definitely be useful for the identification of baryonic states from resonances in future experiments.

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