Scattering Theory & QCD Spectroscopy

Andrew W. Jackura
William & Mary

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Quarks, Gluons, & Hadrons

Our current view of the universe is we live in a 4-dimensional spacetime in which all phenomena can be described by a relatively small number of particles that interact via a few well-defined laws.

There are four known forces, Electromagnetism, Weak, Strong, and Gravitational. All forces except Gravitation are described by the Standard Model of Particle Physics.

The Standard Model of Particle Physics is an anomaly-free, renormalizable, relativistic Quantum Field Theory which is invariant under the local gauge group

$$SU(3)_c \times SU(2)_L \times U(1)_Y$$

which spontaneously breaks via a scalar field to $SU(3)_c \times U(1)_Q$

The sector of the SM which governs strong nuclear interactions is called Quantum Chromodynamics, or QCD.

quark fields

$$\mathcal{L}_{QCD} = \frac{i}{2} \sum_f \bar{q}_f \not{D} q_f + h.c.$$

$$- \frac{1}{2} tr(G_{\mu\nu} G^{\mu\nu})$$

where,

SU(3) generators

$$\not{D} = \gamma^\mu D_\mu = \gamma^\mu(\partial_\mu + i g_s A_\mu)$$

$$A_\mu = \sum_{a=1}^{8} A_\mu^a t^a$$

gluon fields (8 types)

strong coupling

and

$$G_{\mu\nu} = \frac{1}{i g_s} [D_\mu, D_\nu]$$

There are 6 types of quarks: $q_f = u, d, s, c, b, t$

All quarks are massive spin-$\frac{1}{2}$ fermions. The different quark types are called flavors. The quarks have another quantum number, color. There are three colors:

Red, Green, Blue (R, G, B)

Therefore, there are $3 \times 6 = 18$ quarks. The quarks are not directly observed. We only experimentally detect combinations of quarks called hadrons

Hadrons

Hadrons are color neutral bound state of quarks & gluons. Hadrons are observable in particle accelerators, and are our window into the nature of the strong interactions of QCD.

Classification

We observe different classes of hadrons

$\boxed{\text{Hadrons}}$

Baryons
- 3 valence q's
- spin-$\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots$
- $p, n, \Delta, \ldots$

Mesons
- valence $q + \bar{q}$
- spin-$0, 1, 2, \ldots$
- $\pi, K, \rho, \omega, \ldots$

Exotica*
- tetraquarks $(2q + 2\bar{q})$
- Pentaquarks $(4q + \bar{q})$
- Hybrids $(q\bar{q} + g)$
- Glueballs
- ...

* Exotica refers to Non-Quark model objects

Nuclei can be considered as baryonic molecules

Examples in the meson spectrum (pdg.lbl.gov)

	Mass	Lifetime*	Decay Channels
π^+	140 MeV	$\sim 3\times10^{-8}$ s	$\mu^+\nu_\mu$ ($\sim$100%)
π^0	135 MeV	$\sim 9\times10^{-17}$ s	2γ ($\sim$99%)
$f_0(500)/\sigma$	400 - 550 MeV	$\sim 10^{-24}$ s $\Gamma \sim$ 400 - 700 MeV	$\pi\pi$ ($\sim$100%)
$\rho(770)$	775 MeV	$\sim 10^{-23}$ s $\Gamma \sim$ 147 MeV	$\pi\pi$ ($\sim$100%)
$\omega(782)$	782 MeV	$\sim 10^{-22}$ s $\Gamma \sim$ 10 MeV	$\pi^+\pi^-\pi^0$ ($\sim$89%)

- Weak
- Electromagnetic
- Strong

* Recall: $\Gamma = \hbar/\tau$

There are 100's of known hadrons, all of which are in principle understood via QCD.

If we can "turn off" electromagnetic & weak interactions in the Standard Model, then we would find

<u>QCD Stable States</u>

π^+
π^0 $\quad \tau \to \infty \quad$ since $\quad \cancel{\pi^+ \to \mu^+ \nu_\mu} \quad \cancel{\pi^0 \to 2\gamma}$

<u>QCD Unstable States</u>

$$f_0(500)/\sigma \to \pi\pi$$
$$\rho \to \pi\pi$$
$$\omega \to \pi\pi\pi$$
$$f_0(980) \to \pi\pi/K\bar{K}$$
$$a_1(1260) \to \rho\pi/\sigma\pi \to \pi\pi\pi, \ldots$$
$$\vdots$$

Most hadrons are QCD unstable!

$\Rightarrow$ The lifetime of QCD unstable states is of the order of strong interactions, $\tau \sim 10^{-23}$ s.

How do we observe such states?

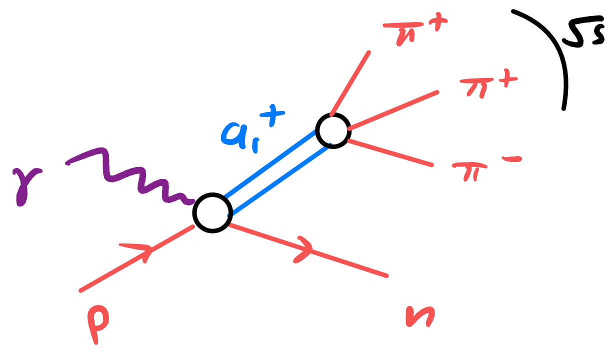
Unstable hadrons are observed as resonances in hadronic reactions

e.g., consider the $a_1(1260)$

photoproduction

@ GlueX

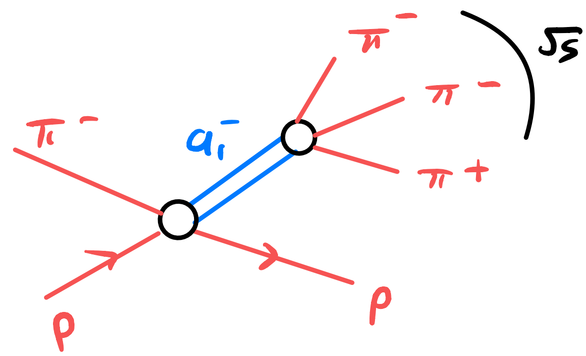
$\gamma p \to X N$



hadroproduction

@ COMPASS

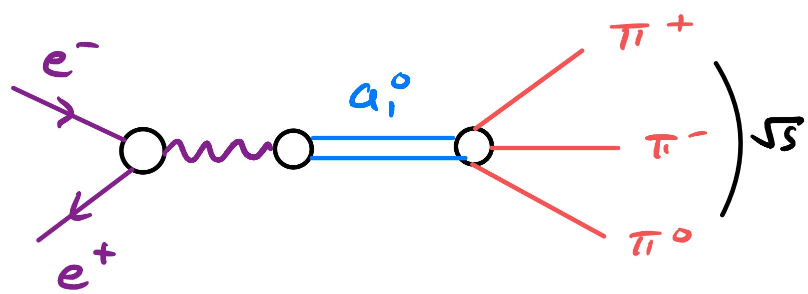
$\pi p \to X p$



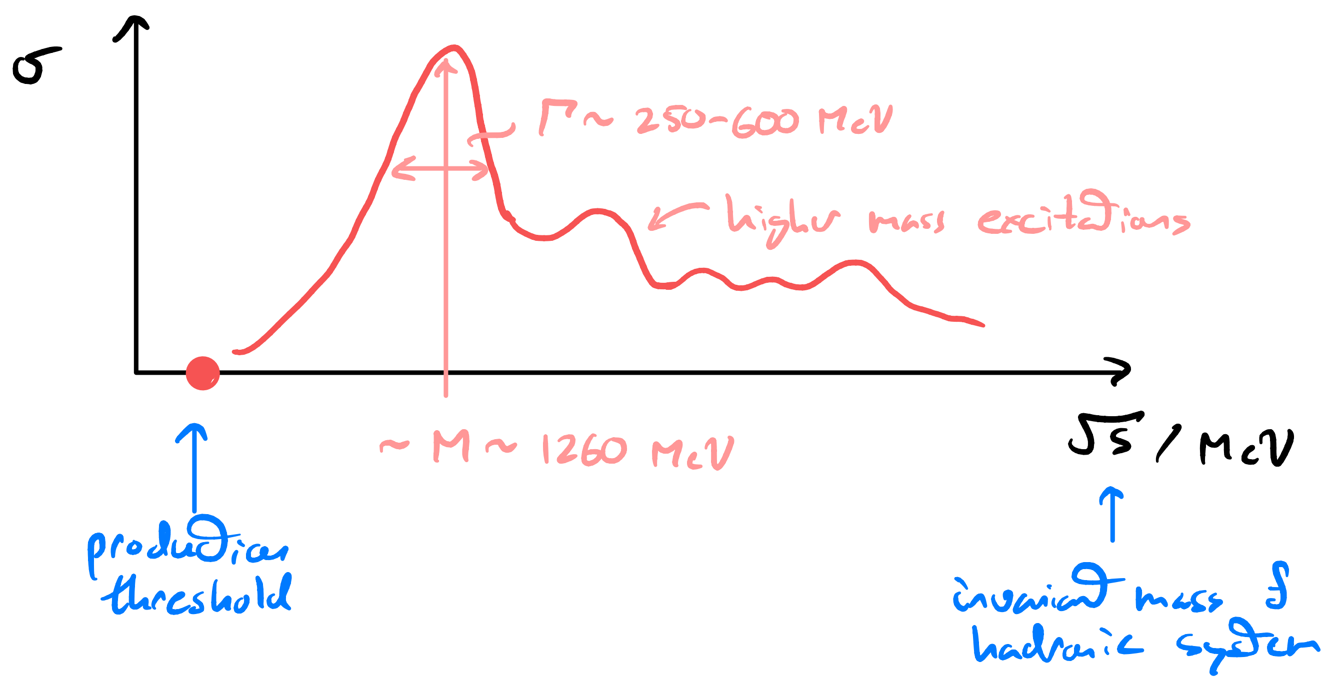
e^+e^--annihilation

@ BESIII

$e^+e^- \to X$



These resonances often* appear as enhancements in cross-sections



* This is subtle, especially with multiple scattering channels & overlapping resonances, cf. σ & $f_0(980)$ in $\pi\pi / K\bar{K}$.

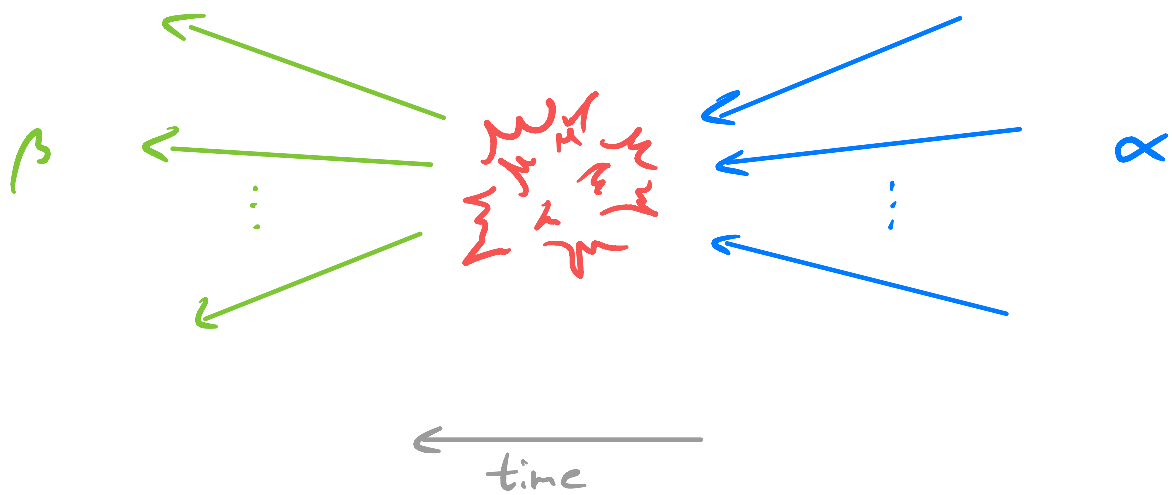
Resonances are universal $\Rightarrow$ Do not depend on the production mechanism. However, their presence may be more or less obvious in some experiments. For example, if the production coupling is small, we could miss them.

Resonance physics is inherently few-body physics. Resonances are rigorously defined as pole singularities in the complex energy-plane of scattering amplitudes. Thus, to understand their properties, we must have access to the scattering amplitude.

Hadron Spectroscopy is the study of the patterns & properties of hadrons, including their masses & lifetimes. QCD spectroscopy, is the theoretical study of the hadron spectrum & how it emerges from the quark & gluon dynamics of QCD. Studying the spectrum necessitates determining the relevant scattering amplitude for the hadronic resonance of interest.

Scattering Amplitudes

Consider the reaction of some initial state α to some final state β.



The S-matrix encodes all dynamics for such a process

$$\boxed{S_{\beta\alpha} = \langle \beta | \hat{S} | \alpha \rangle}$$

operator defined to take $\hat{S}|\text{in}\rangle = |\text{out}\rangle$

Here, $|\alpha\rangle$ & $|\beta\rangle$ are some generic multiparticle state.

The usual approach to determining S-matrix elements is to compute them via perturbation theory within some particular QFT.

The Lehmann-Symanzik-Zimmermann (LSZ) reduction formula allows one to non-perturbatively relate scattering amplitudes to QFT correlation functions

$$S_{\beta\alpha} = \prod_{j\in\alpha} \prod_{k\in\beta} \frac{(p_k^2 - m_k^2)}{i\sqrt{Z}} \, C_{\beta\alpha} \, \frac{(p_j^2 - m_j^2)}{i\sqrt{Z}}$$

wavefunction renormalization

time ordered correlation function (in momentum space)

Fourier Transform

$$C_{\beta\alpha} = \mathcal{FT}\{ \langle \mathcal{O}_\beta \mathcal{O}_\alpha^+ \rangle \}$$

field operators annihilating final state

field operators creating initial state

Aside: Reminder of Relativistic kinematics

$p = (p^0, \vec{p})$ is 4-momentum

the on-shell condition $p^2 = m^2$, $p^0 \geq 0$

$$\Rightarrow \quad p^{0\,2} = \boxed{E_p^2 = m^2 + \vec{p}^{\,2}}$$

relativistic dispersion relation

under a Lorentz transformation with velocity $\vec{\beta}$,

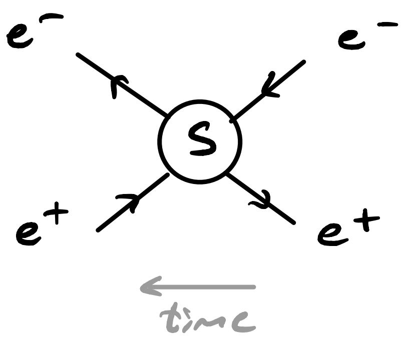
$$\boxed{\begin{aligned} \vec{p}_{\parallel}{}' &= \gamma(\vec{p}_{\parallel} + E_p \vec{\beta}) \\ \vec{p}_{\perp}{}' &= \vec{p}_{\perp} \\ E_p' &= \gamma(E_p + \vec{p}\cdot\vec{\beta}) \end{aligned}}$$

active transformations

with $\vec{p}_{\parallel} = \frac{(\vec{p}\cdot\vec{\beta})}{\beta} \frac{\vec{\beta}}{\beta}$, $\vec{p}_{\perp} = \vec{p} - \vec{p}_{\parallel}$, $\gamma = \sqrt{\frac{1}{1-\beta^2}}$

The LSZ formula allows us to generate a perturbation series in the coupling. We usually express these in terms of Feynman diagrams

For example, $e^-e^+ \to e^-e^+$



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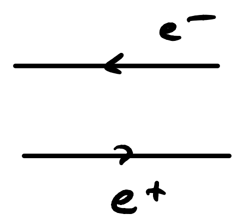
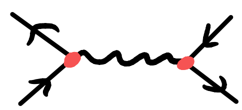
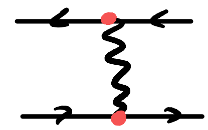


$\mathcal{O}(\alpha^0)$

$+$



$+$



$\mathcal{O}(\alpha)$

$+ \ \mathcal{O}(\alpha^2)$

Recall that $\alpha = \frac{e^2}{4\pi} \approx \frac{1}{137}$, fine-structure constant

In trying to apply this to QCD, we run into an issue in that the asymptotic states are hadrons, but the fields of QCD are in terms of quarks & gluons. Connecting these two regimes is incredibly difficult. What makes this task hard is the fact for low-energy physics, $\alpha_s \equiv \frac{g_s^2}{4\pi} \sim 1$, so every term in the perturbation series contributes.

We can make progress by using general principles of physics to construct non-perturbative representations for the scattering amplitude. These principles include:

- Poincaré (Lorentz + Translational) Invariance
- Probability conservation
- Causality
- Crossing

These can be used effectively to construct analytic representations for amplitudes, which can be used to phenomenologically study processes using experimental data, or as we will see, in theoretical studies with Lattice QCD.

We will only focus on a few aspects of this approach.

Poincaré invariance is assumed as we work within a relativistic framework.

Consider a single particle state, $|\vec{p}, \sigma\rangle$, which is stable & has a mass m.

therefore,

$$E_p = \sqrt{m^2 + \vec{p}^{\,2}}.$$

so,

$$\boxed{\begin{aligned} \hat{P}^2 |\vec{p}, \sigma\rangle &= m^2 |\vec{p}, \sigma\rangle \\ \hat{P}^\mu |\vec{p}, \sigma\rangle &= p^\mu |\vec{p}, \sigma\rangle \end{aligned}}$$

$p^\mu = (E_p, \vec{p})$

The particle has a spin j, & a projection σ,

$$\boxed{\begin{aligned} \hat{J}^2 |\vec{p}, \sigma\rangle &= j(j+1) |\vec{p}, \sigma\rangle \\ \hat{J}_z |\vec{p}, \sigma\rangle &= \sigma |\vec{p}, \sigma\rangle \end{aligned}}$$

Under translations, $U(a)|\vec{p}, \sigma\rangle = e^{-ip\cdot a} |\vec{p}, \sigma\rangle$

A general multiparticle state,

$$|\alpha\rangle \equiv |\vec{p}_1 \sigma_1\rangle \otimes |\vec{p}_2 \sigma_2\rangle \otimes \cdots \otimes |\vec{p}_{n_\alpha} \sigma_{n_\alpha}\rangle$$

thus transforms as

$$\boxed{U(a)|\alpha\rangle = e^{-i\sum_{j=1}^{n_\alpha} p_j \cdot a} |\alpha\rangle}$$

Here, $\sum_{j=1}^{n_\alpha} p_j \equiv P_\alpha$ total momentum of α

So, for the S matrix element for $\alpha \to \beta$,

$$\langle \beta | \hat{S} | \alpha \rangle \xrightarrow[\text{translations}]{} e^{-i(P_\beta - P_\alpha)\cdot a} \langle \beta | \hat{S} | \alpha \rangle$$

So, if $P_\alpha \neq P_\beta \Rightarrow S_{\beta\alpha} = 0$ identically.

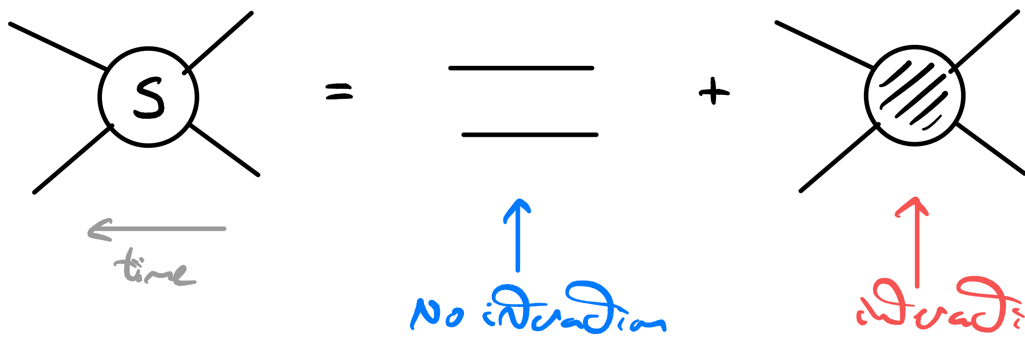
Using symmetries allows one to enforce constraints on $S_{\beta\alpha}$. Lorentz transformations impose further restrictions. If the system has other internal quantum numbers, e.g., flavor, isospin, ..., then these will also impose restrictions.

e.g., $\langle I_\beta | \hat{S} | I_\alpha \rangle = \delta_{I_\beta I_\alpha} S^{I}_{\beta\alpha}$

↳ isospin

The S-matrix encodes all dynamics, including no interactions. It is useful to remove the case where the particles never interact,

e.g., for $2 \to 2$ scattering



We introduce the transfer of T-matrix via

$$\hat{S} = \hat{1} + i\hat{T}$$

convention

So, S-matrix element is

$$\boxed{S_{\beta\alpha} = \delta_{\beta\alpha} + (2\pi)^4 \delta^{(4)}(P_\beta - P_\alpha)\, i\mathcal{M}_{\beta\alpha}}$$

momentum conservation

covariant amplitude
or
scattering amplitude