

# Scattering Theory & QCD Spectroscopy

## Lecture 1 - Introduction

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William & Mary

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2024 Hampton University Graduate Studies (HUGS) Program

June 10th, 2024



U.S. DEPARTMENT OF  
**ENERGY**



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**WILLIAM & MARY**

CHARTERED 1693

# About Me

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## Andrew Jackura

- Assistant Professor, William & Mary, 2023 - present
- Postdoctoral Research Scholar, UC Berkeley, 2023
- Postdoctoral Research Fellow, Old Dominion University, 2019 - 2023
- Ph.D., Indiana University, 2019

## Research Interests

- Theoretical Hadron Spectroscopy
- Multi-particle reaction theory
- Finite-volume QFT
- Lattice QCD

⇒ *Three-body reactions and resonances*  
⇒ *Electroweak matrix elements of multi-hadron processes*

*If you want to chat, contact me*

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# About Me — 2014 HUGS Program





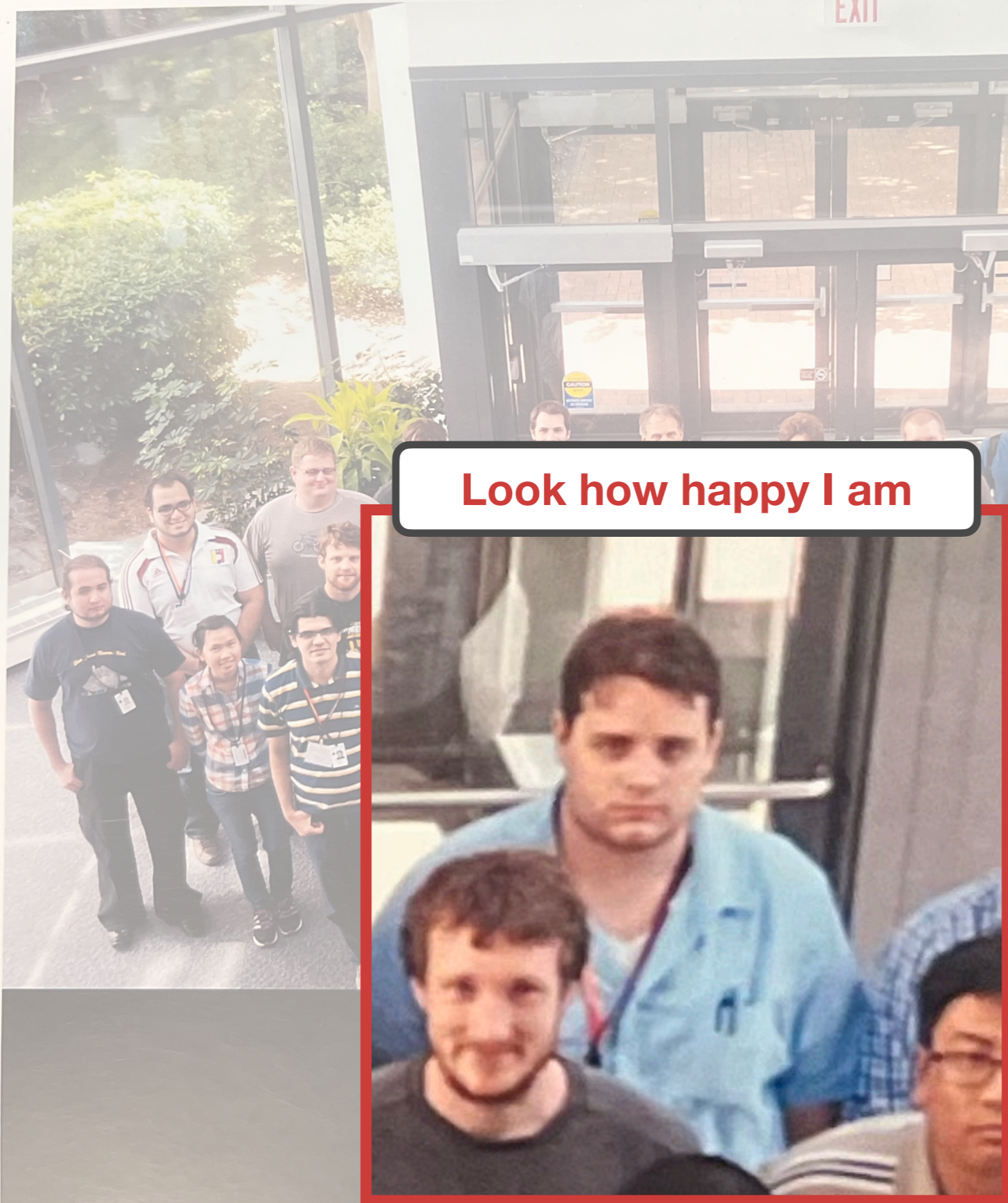
# About Me – 2014 HUGS Program



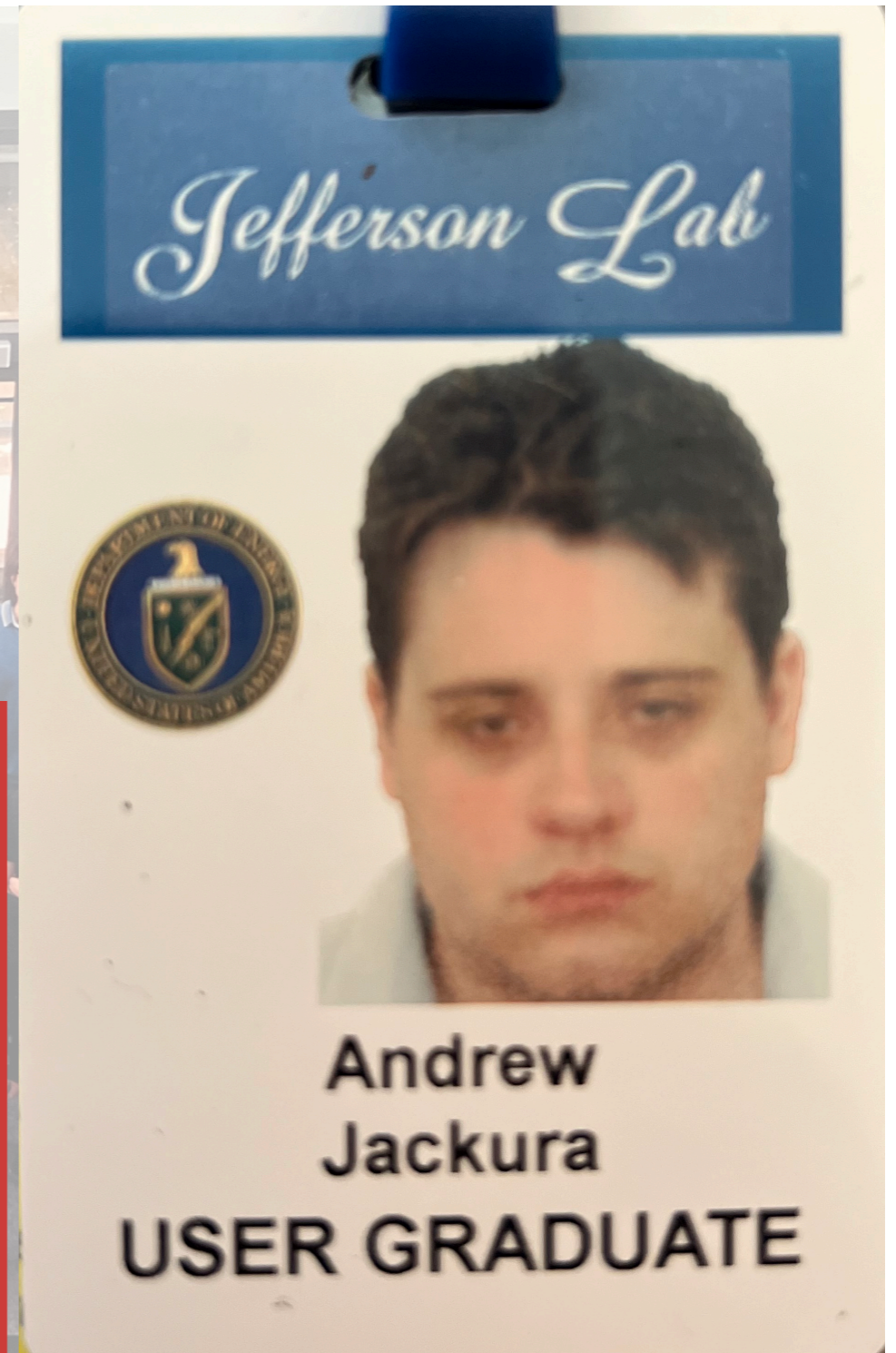
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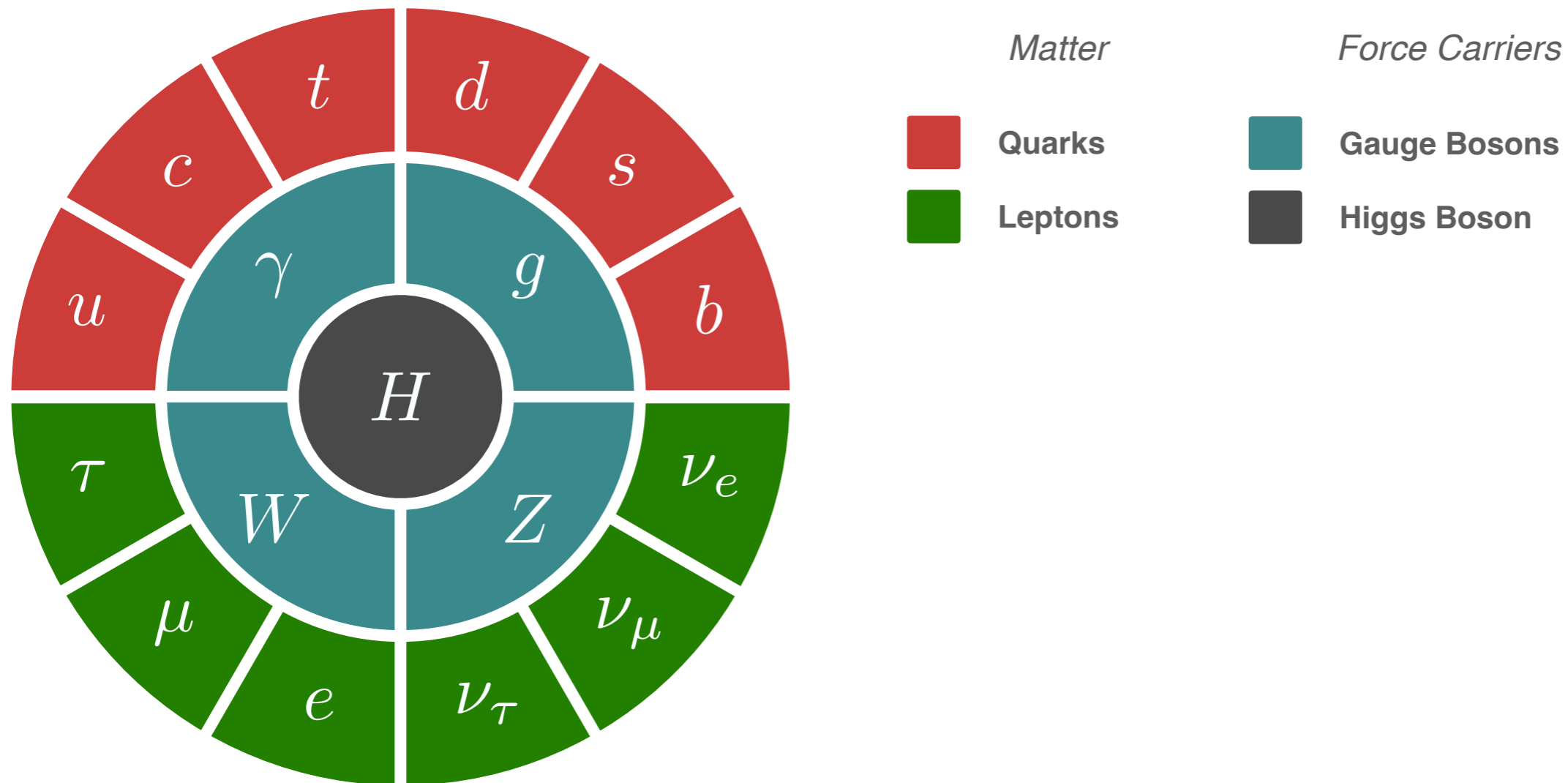
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# The Standard Model of Particle Physics

Nature can be described by a remarkably *simple*\* theory

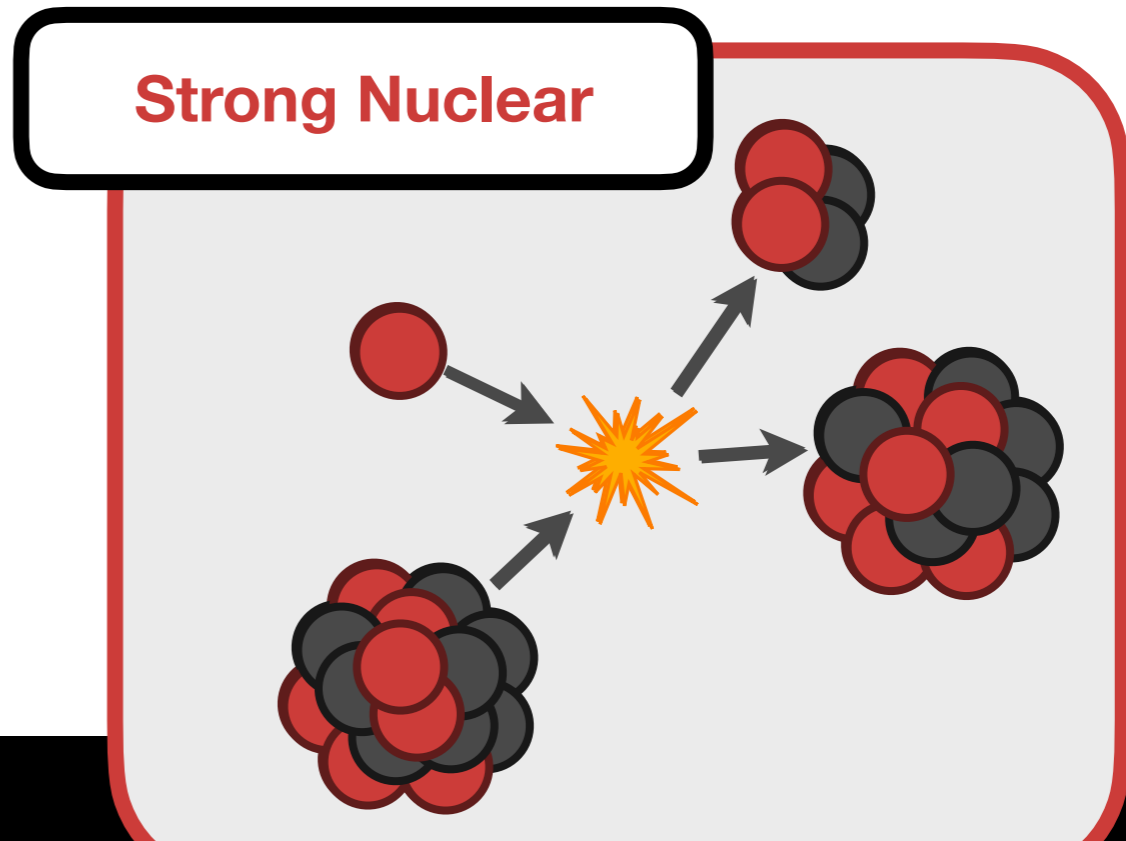
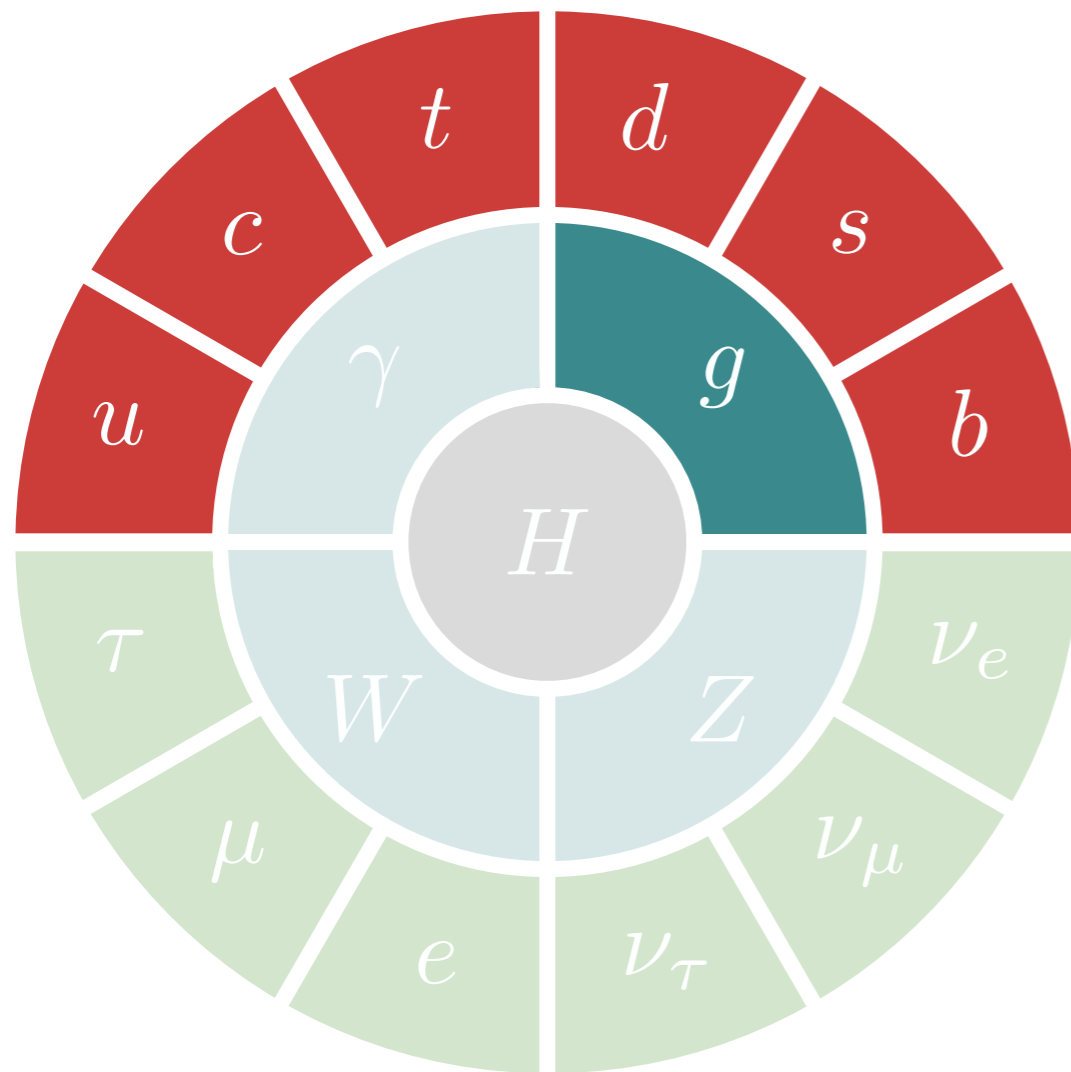


\* *simple* = An anomaly-free renormalizable relativistic quantum gauge field theory, invariant under the gauge group  $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$  which spontaneously breaks via a scalar field to  $SU(3)_C \otimes U(1)_Q$



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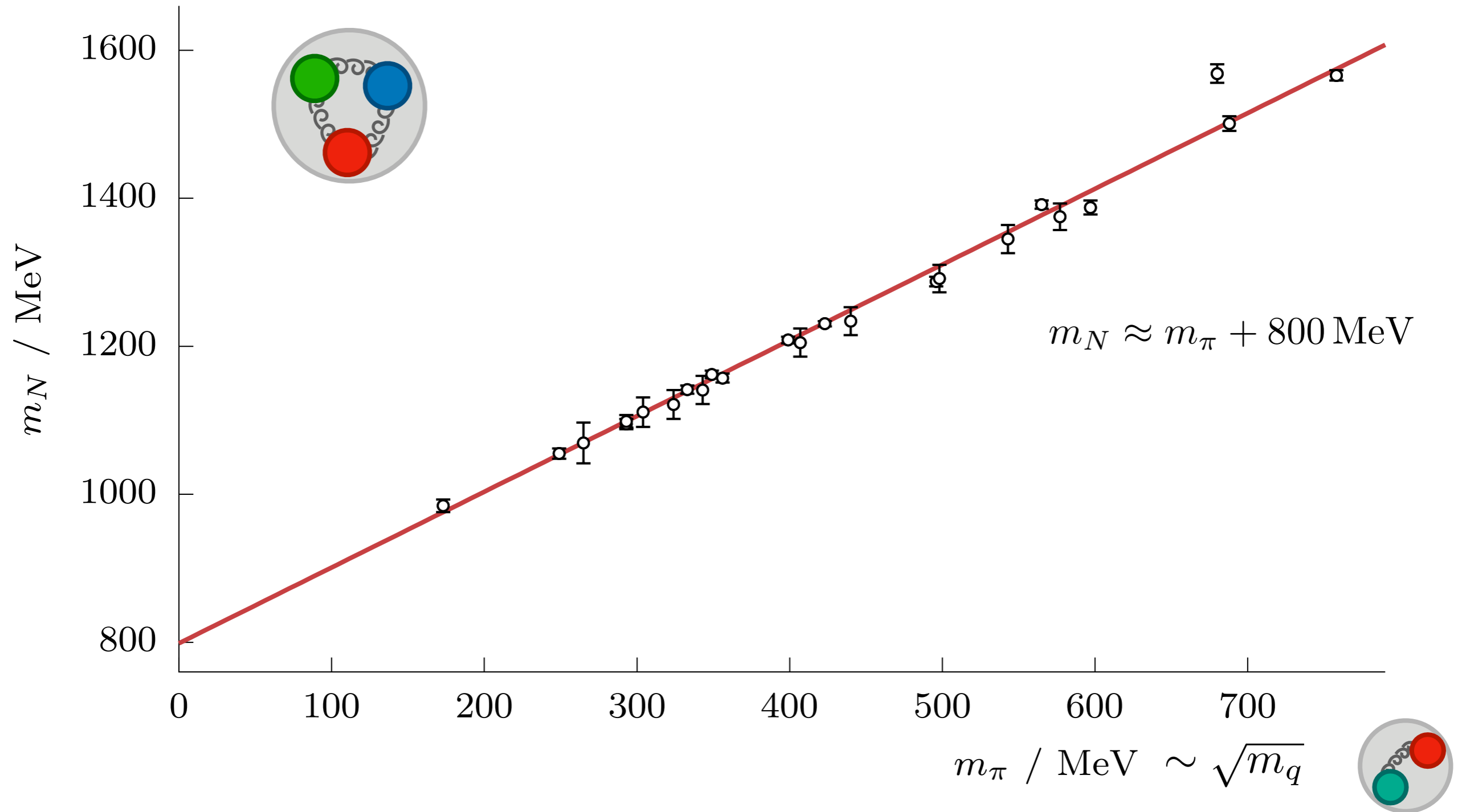
**Strong Nuclear**  
**Quantum ChromoDynamics (QCD)**

\* *simple* = An a  
invariant under the gauge group  $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$   
which spontaneously breaks via a scalar field to  $SU(3)_C \otimes U(1)_Q$



# Nuclear Forces

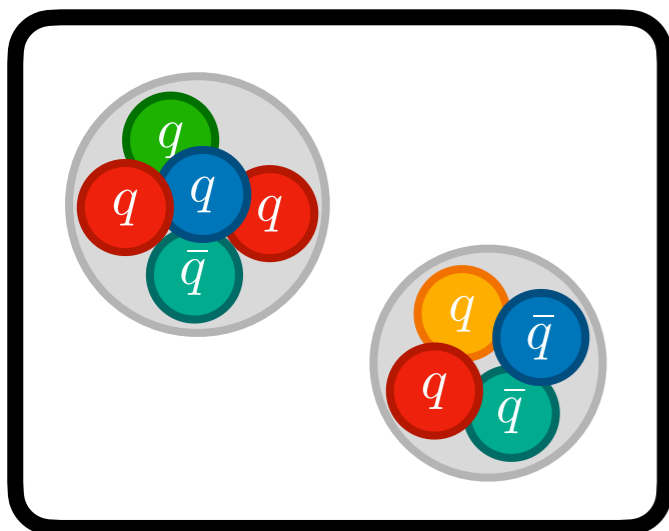
Most of our mass comes from nuclear forces!



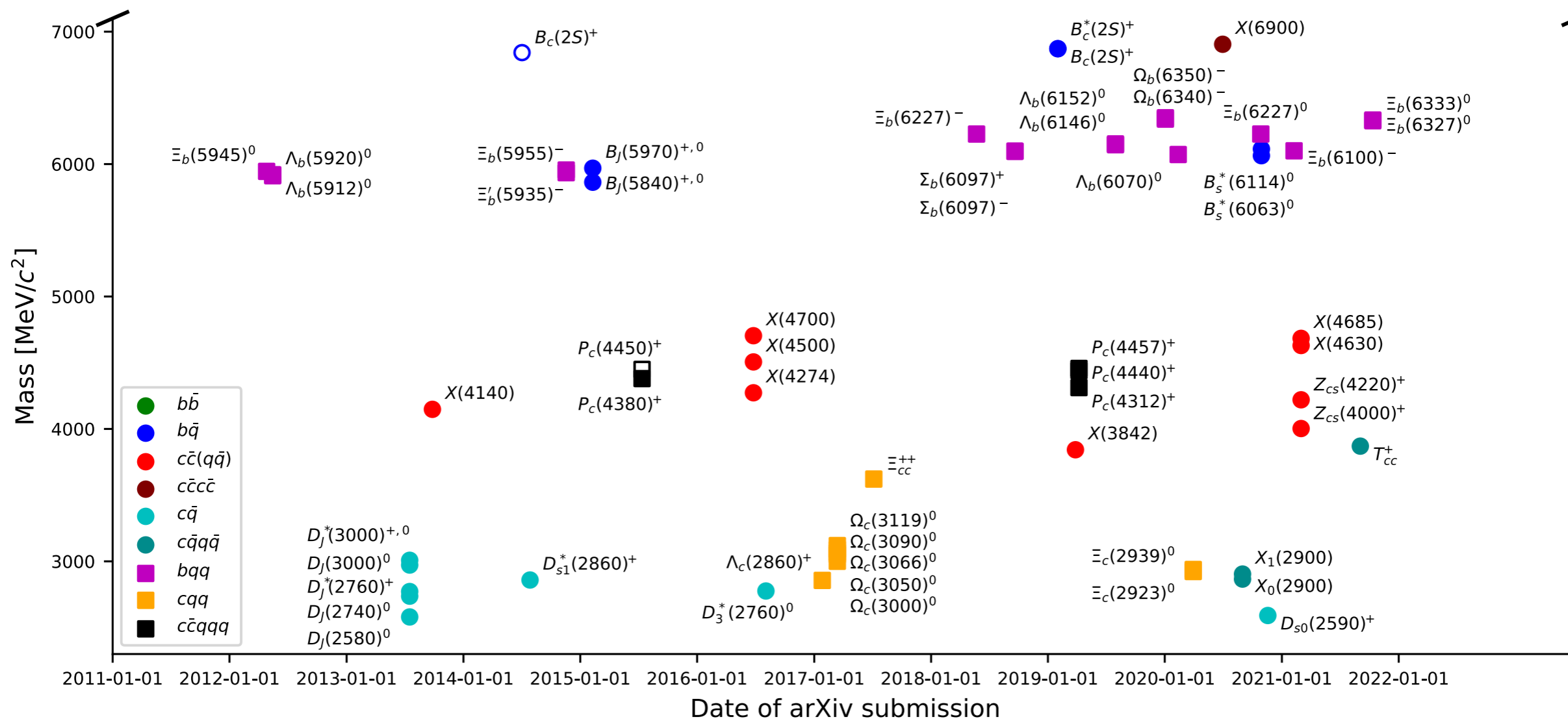


# New discoveries in Nuclear Physics

Zoo of *exotic* quark configurations discovered



62 new hadrons at the LHC





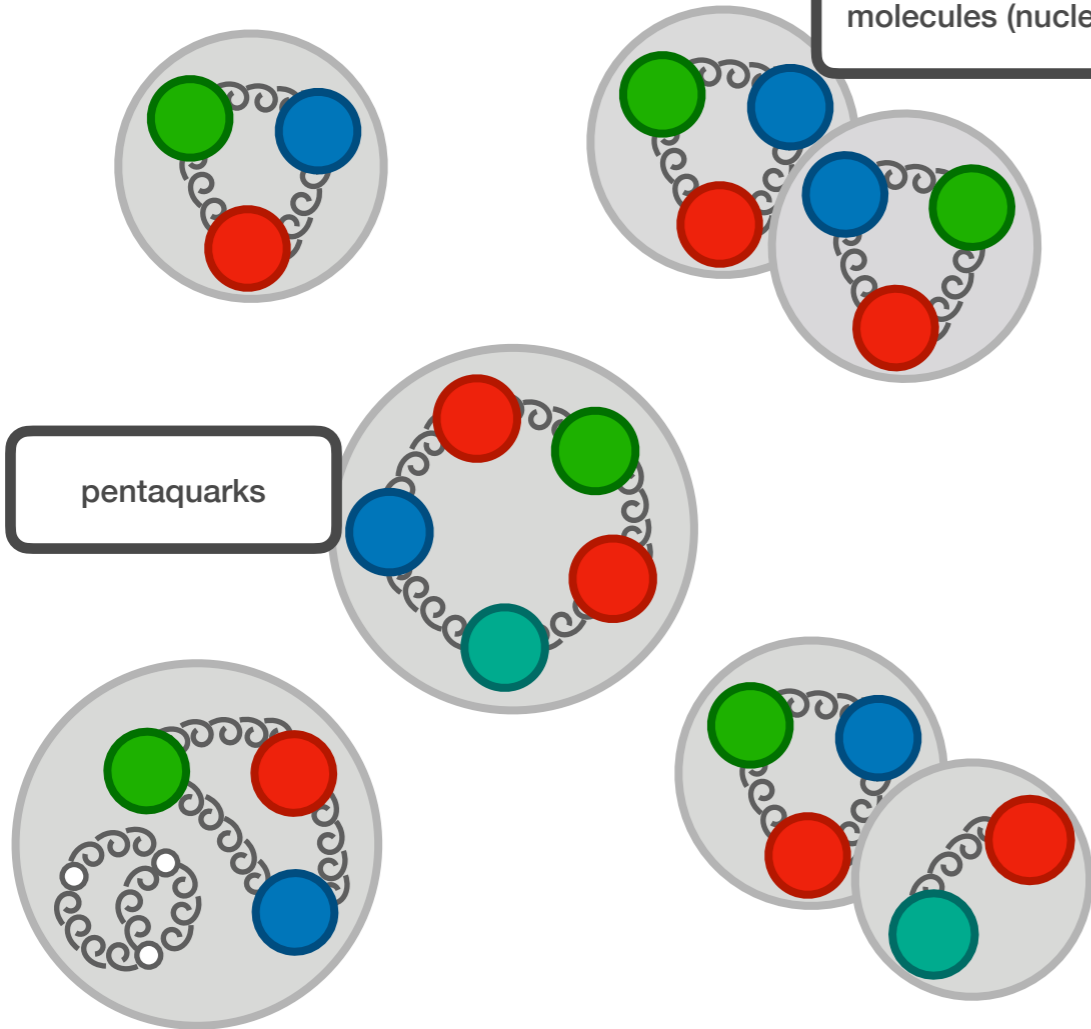
# Many types of nuclear particles

A zoo of particles — How to understand?

## Baryons (fermions)

molecules (nuclei)

pentaquarks

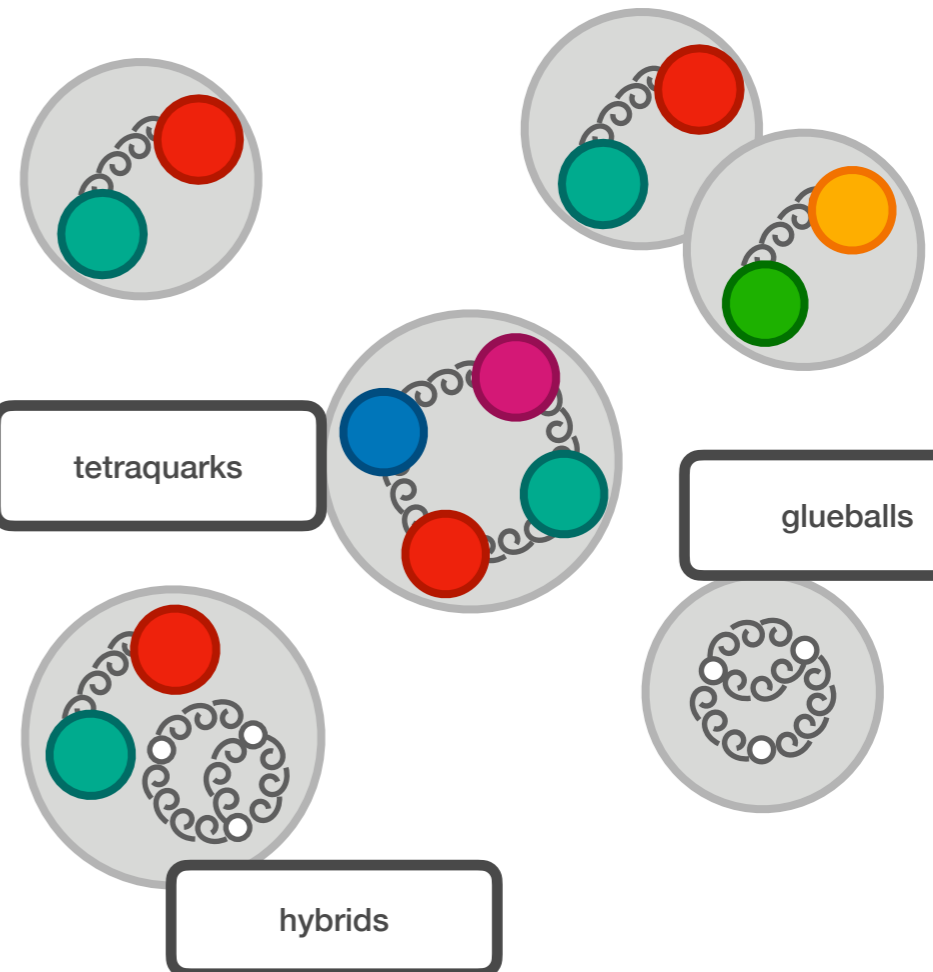


## Mesons (bosons)

tetraquarks

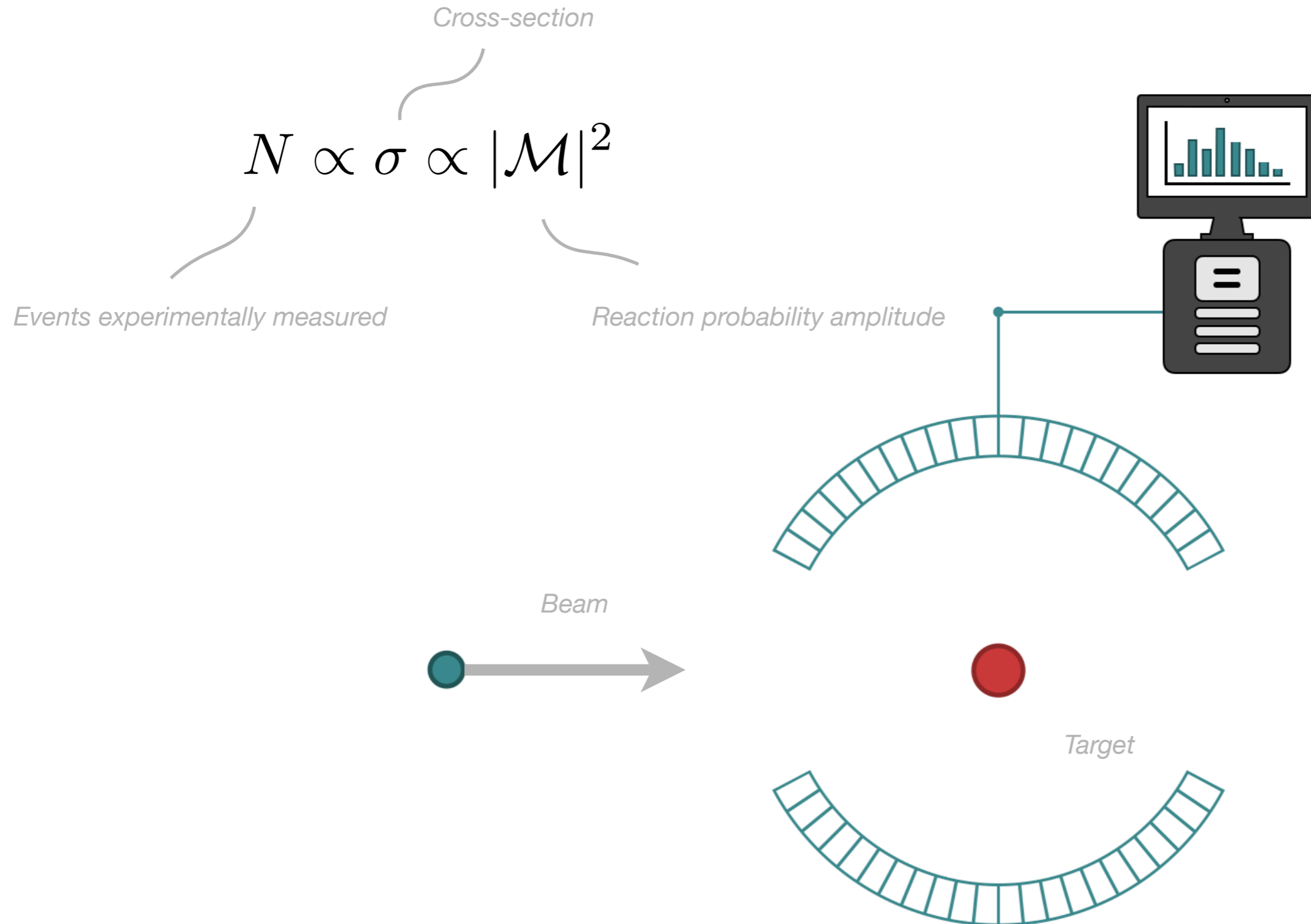
glueballs

hybrids



# Observing Hadrons

We observe strongly interacting hadrons through reactions in accelerators/colliders

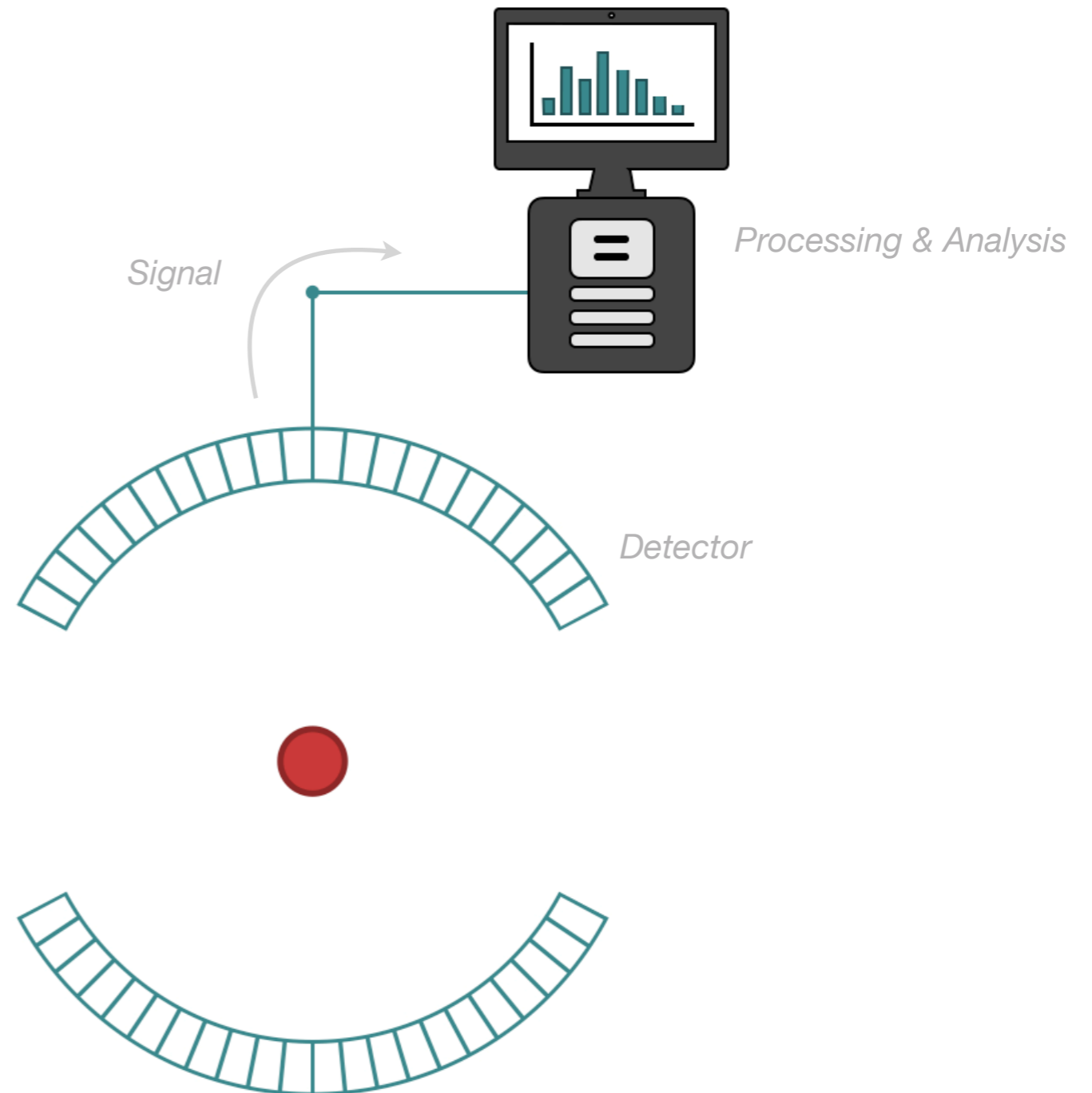




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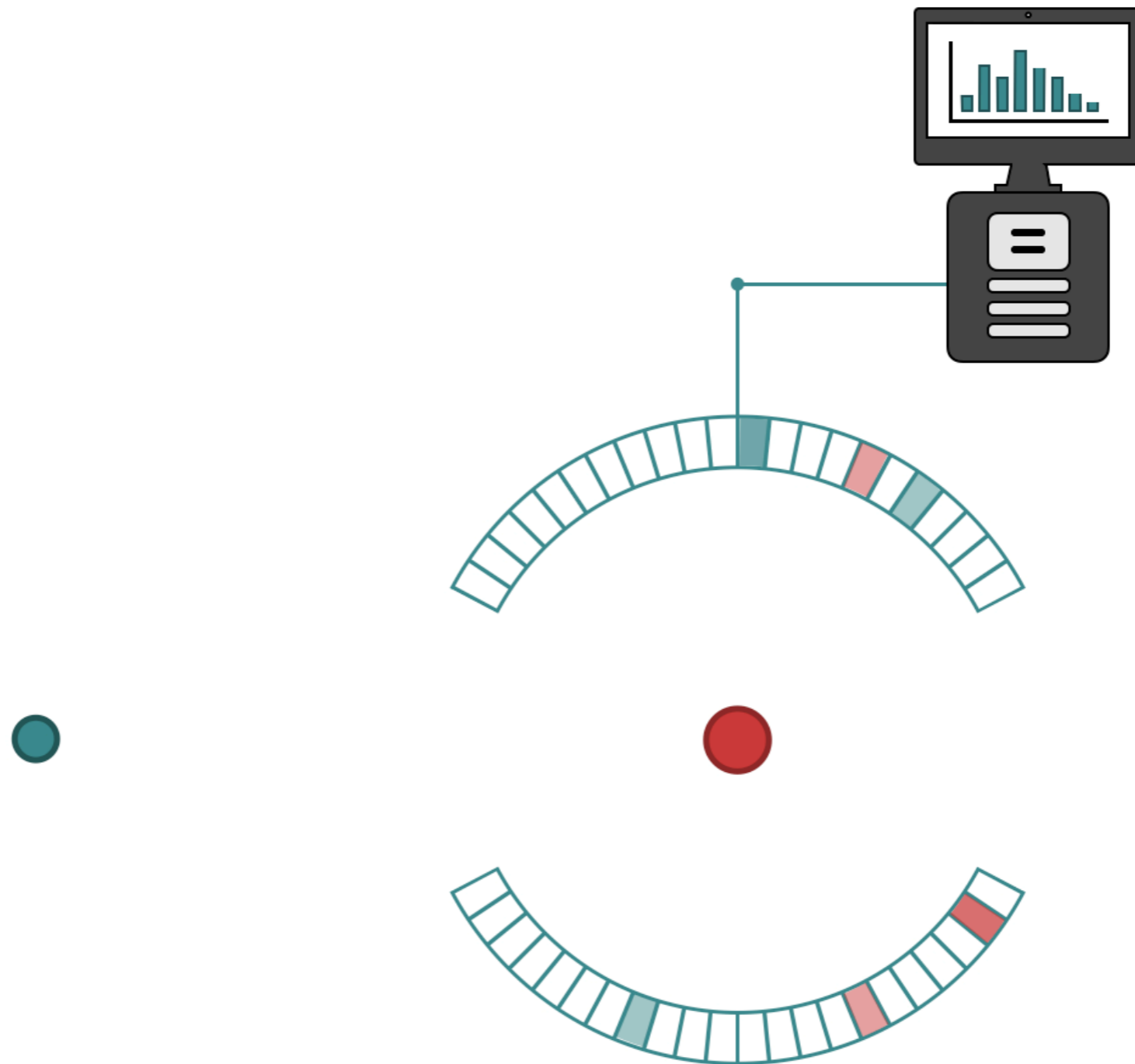
$$N \propto \sigma \propto |\mathcal{M}|^2$$



# Observing Hadrons

We observe strongly interacting hadrons through reactions in accelerators/colliders

- If the interaction is sufficiently attractive, particles can form a *resonance*





# Observing Hadrons

We observe strongly interacting hadrons through reactions in accelerators/colliders

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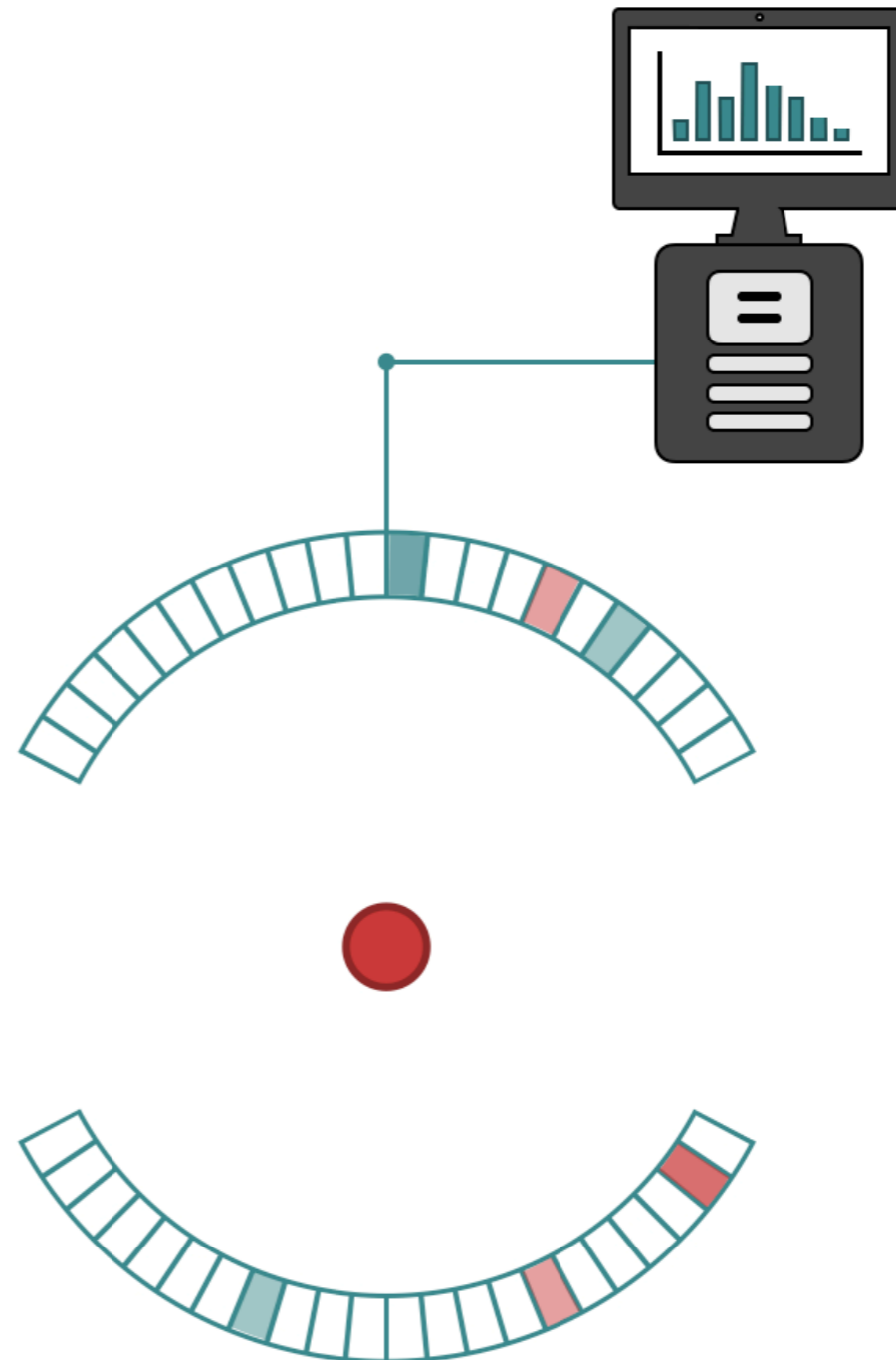
## Resonance

Unstable hadron which decays via strong nuclear interaction

Has a mass *and* finite lifetime

$$\tau \sim 10^{-23} \text{ sec}$$

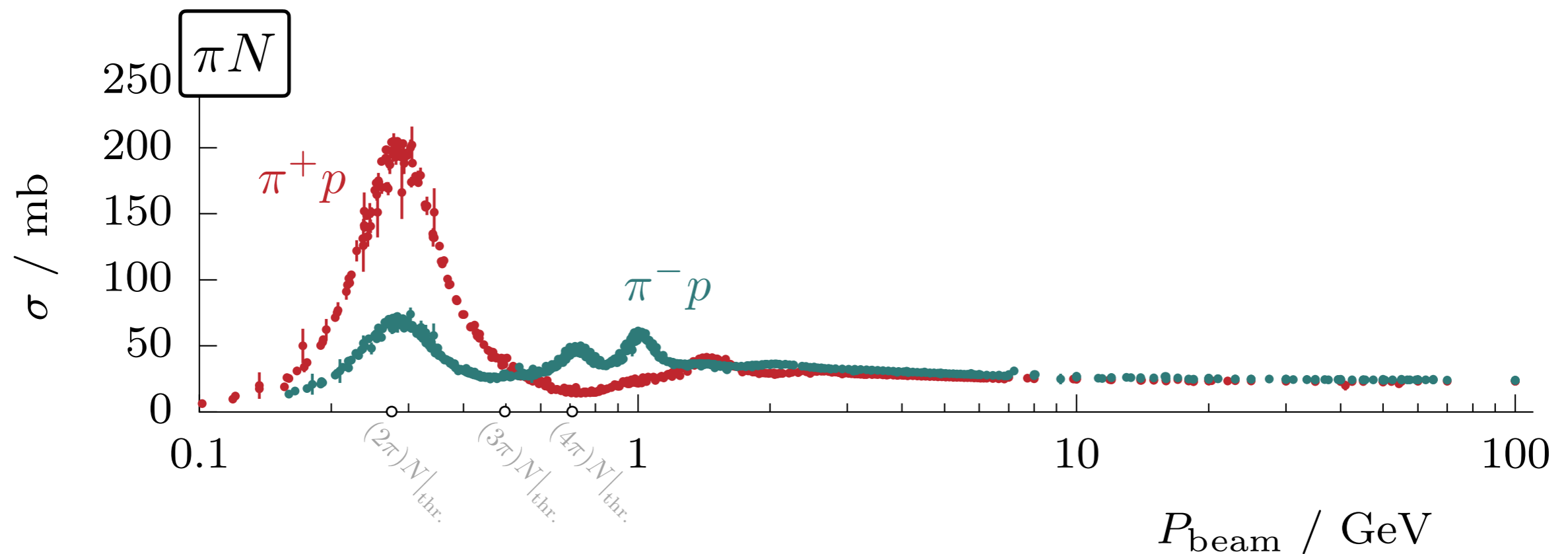
***Most hadrons are resonances***



# Observing Hadrons

We observe strongly interacting hadrons through reactions in accelerators/colliders

- If the interaction is sufficiently attractive, particles can form a **resonance**
- Resonances can appear as enhancements in the cross-section / amplitude

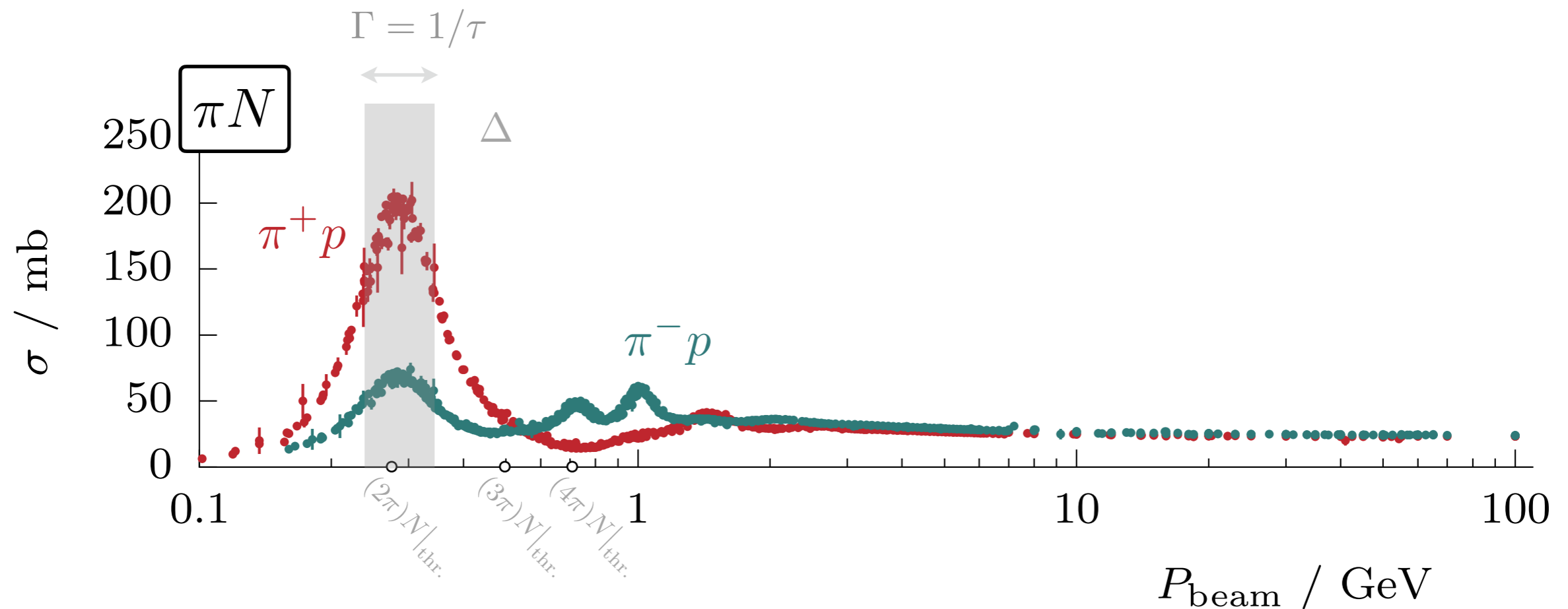




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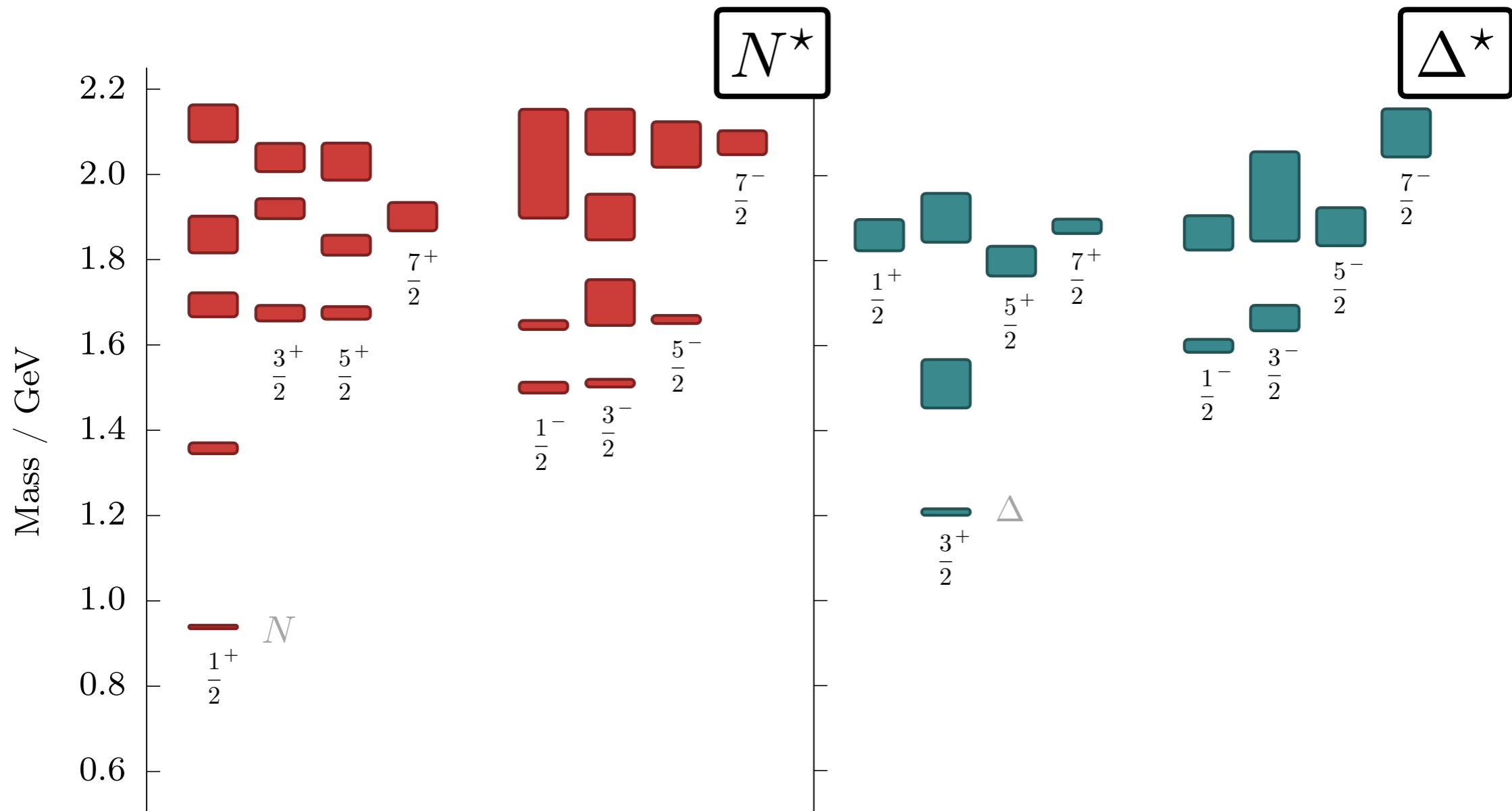
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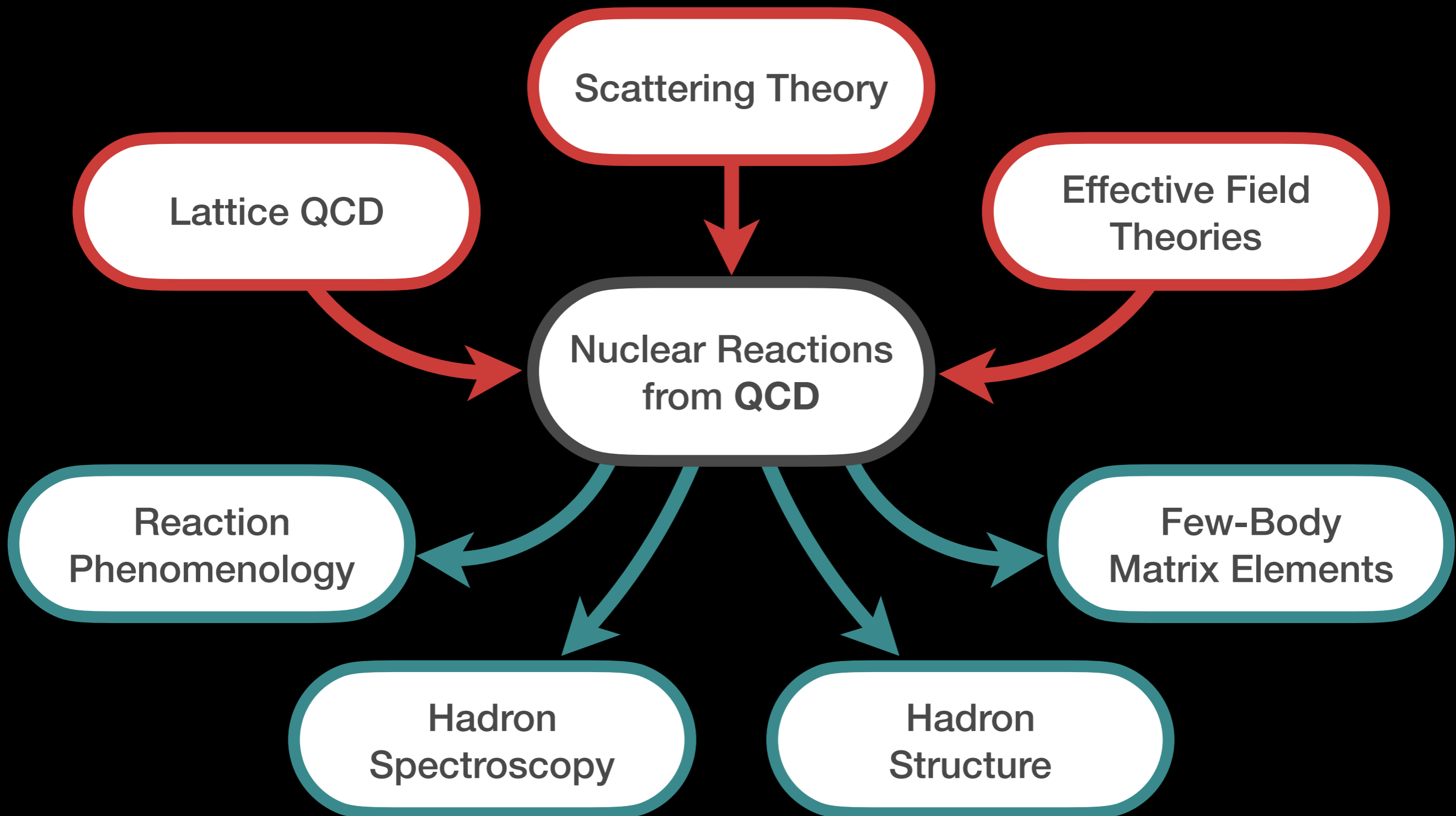




# Modern Nuclear Physics

Connect low-energy hadron & nuclear physics to **QCD**

- Quantitatively describe few-body reactions and their impact on observables



# Outline

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## I. Scattering Theory

- Amplitudes and the  $S$  matrix
- Partial Wave Analysis
- Resonances & Bound States

## II. Connecting QCD to Scattering Amplitudes

- Overview of Lattice QCD
- Correlators and Finite-Volume Spectra
- Lüscher Quantization Condition

**I will assume you have had *some* exposure to...**

particle physics (e.g., you know what a quark is)

quantum field theory (e.g., you know what a Feynman diagram is)

complex analysis (e.g., you know what a contour integral is)

$N = 4$  supersymmetric Yang-Mills Theory



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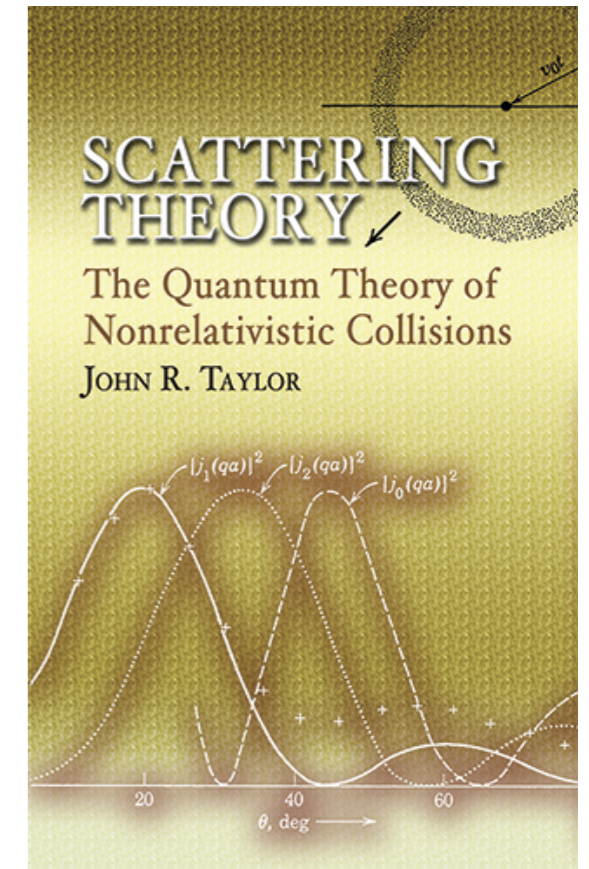
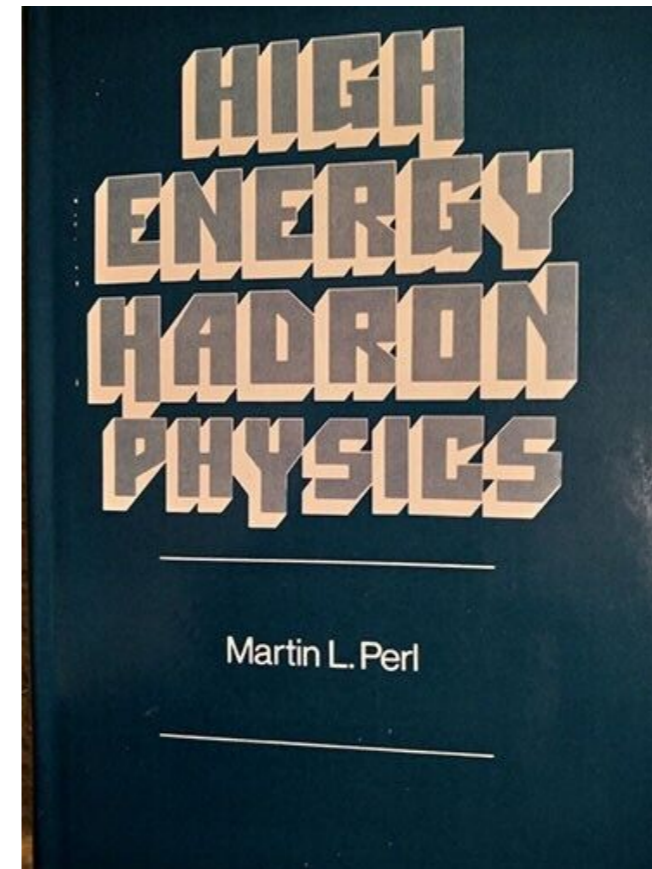
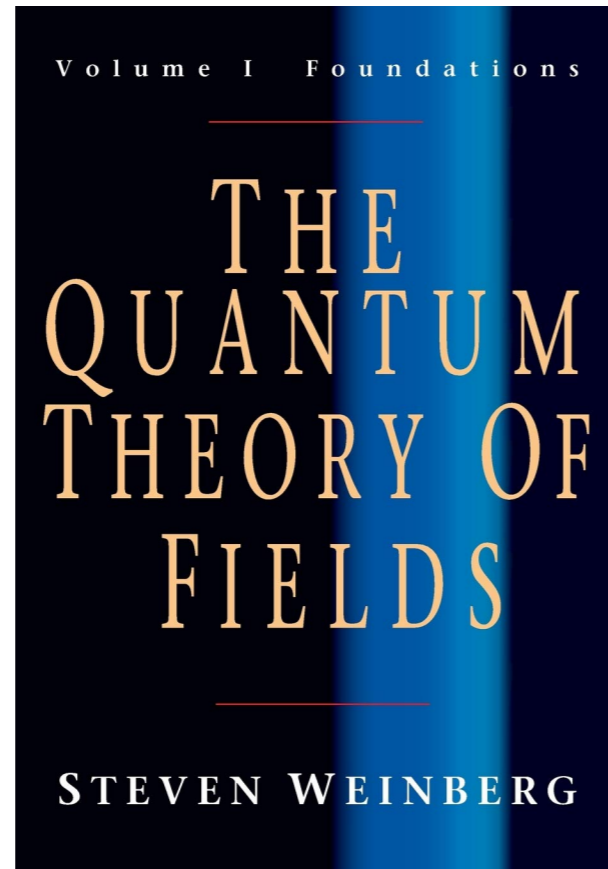
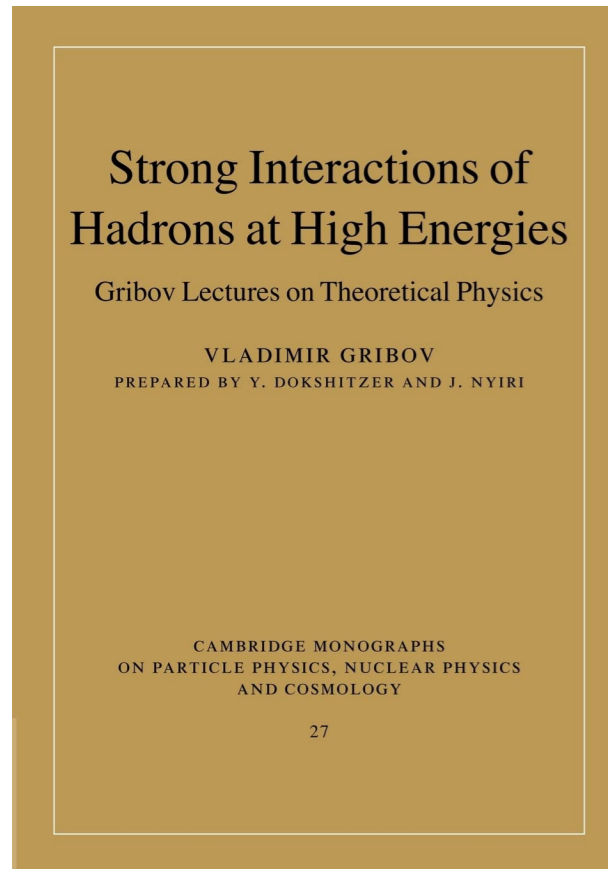
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# Some References (that I like)

Also see JPAC summer schools <http://jpac.nucleares.unam.mx/schools.html>





# Some References (that I like)

arXiv:1706.06223 (2017)

## Scattering processes and resonances from lattice QCD

Raúl A. Briceño,<sup>1,\*</sup> Jozef J. Dudek,<sup>1,2,†</sup> and Ross D. Young<sup>3,‡</sup>

<sup>1</sup>*Thomas Jefferson National Accelerator Facility, 12000 Jefferson Avenue, Newport News, Virginia 23606, USA*

<sup>2</sup>*Department of Physics, College of William and Mary, Williamsburg, Virginia 23187, USA*

<sup>3</sup>*Special Research Center for the Subatomic Structure of Matter (CSSM), Department of Physics, University of Adelaide, Adelaide 5005, Australia*

(Dated: June 21, 2017)

The vast majority of hadrons observed in nature are not stable under the strong interaction, rather they are *resonances* whose existence is deduced from enhancements in the energy dependence of scattering amplitudes. The study of hadron resonances offers a window into the workings of quantum chromodynamics (QCD) in the low-energy non-perturbative region, and in addition, many probes of the limits of the electroweak sector of the Standard Model consider processes which feature hadron resonances. From a theoretical standpoint, this is a challenging field: the same dynamics that binds quarks and gluons into hadron resonances also controls their decay into lighter hadrons, so a complete approach to QCD is required. Presently, lattice QCD is the only available tool that provides the required non-perturbative evaluation of hadron observables. In this article, we review progress in the study of few-hadron reactions in which resonances and bound-states appear using lattice QCD techniques. We describe the leading approach which takes advantage of the periodic finite spatial volume used in lattice QCD calculations to extract scattering amplitudes from the discrete spectrum of QCD eigenstates in a box. We explain how from explicit lattice QCD calculations, one can rigorously garner information about a variety of resonance properties, including their masses, widths, decay couplings, and form factors. The challenges which currently limit the field are discussed along with the steps being taken to resolve them.

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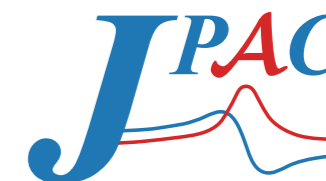
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The vast majority of hadrons observed in nature are resonances, rather than bound states. Resonances are particles whose existence is revealed by the energy dependence of scattering amplitudes. Lattice QCD provides a window into the workings of quantum chromodynamics in the non-perturbative region, and in addition, many processes in the sector of the Standard Model consider processes with resonances. From a theoretical standpoint, this is a challenging field: understanding how quarks and gluons into hadron resonances also controls the complete approach to QCD is required. Presently, lattice QCD provides that provides the required non-perturbative evaluation. In this article, we review progress in the study of few-hadron bound-states appear using lattice QCD techniques, which takes advantage of the periodic finite spatial lattice to extract scattering amplitudes from the discretized system in a box. We explain how from explicit lattice QCD calculations we can obtain information about a variety of resonance properties: decay couplings, and form factors. The challenge is to understand the physics discussed along with the steps being taken to resolve these issues.

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  - A. Variational analysis of correlation matrices
  - B. Operator construction

arXiv:2112.13436 (2021)



## Novel approaches in Hadron Spectroscopy

Miguel Albaladejo<sup>a,b</sup>, Lukasz Bibrzycki<sup>c</sup>, Sebastian M. Dawid<sup>d,e</sup>, César Fernández-Ramírez<sup>f,g</sup>, Sergi González-Solís<sup>d,e,h</sup>, Astrid N. Hiller Blin<sup>a</sup>, Andrew W. Jackura<sup>a,i</sup>, Vincent Mathieu<sup>j,k</sup>, Mikhail Mikhasenko<sup>l,m</sup>, Victor I. Mokeev<sup>a</sup>, Emilie Passemar<sup>a,d,e</sup>, Alessandro Pilloni<sup>n,o,\*</sup>, Arkaitz Rodas<sup>a,p</sup>, Jorge A. Silva-Castro<sup>f</sup>, Wyatt A. Smith<sup>d</sup>, Adam P. Szczepaniak<sup>a,d,e</sup>, Daniel Winney<sup>d,e,q,r</sup>,

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