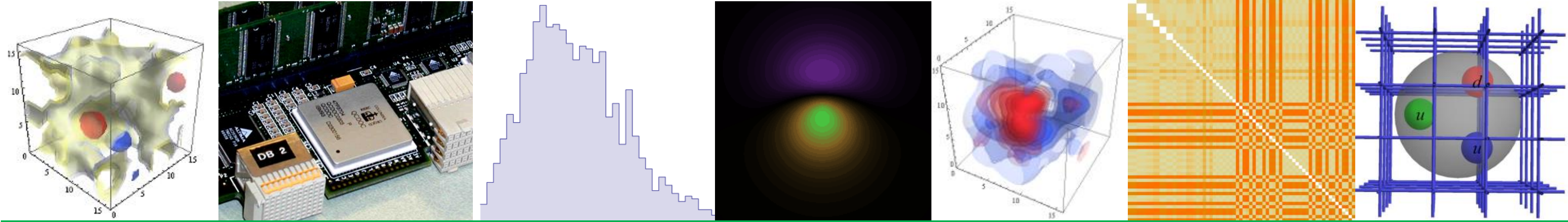


Lattice QCD for Hadronic Physics

Huey-Wen Lin

CTEQ

This work of HL is supported by the NSF under grant PHY 2209424 & 1653405, DOE under DE-SC0024053 and the Research Corporation for Science Advancement through the Cottrell Scholar Award

@LinQCD

NSF

U.S. DEPARTMENT OF ENERGY

RESEARCH CORPORATION for SCIENCE ADVANCEMENT

Last-Day Plan

§ Lecture Plan (Fri): 20 mins

- x-dependent parton distributions
  - Recent lattice PDFs progress
  - Applications to generalized parton distributions
  - Future prospects and challenges

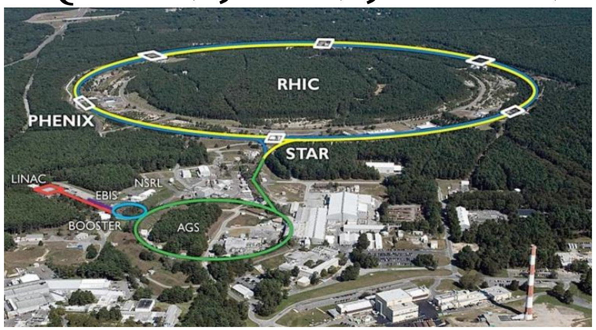
§ Hands-on exercises (40 mins)

- Work in small groups (4-ish students)
- With Python Jupyter notebooks
  - Three-point functions (PC) & extracting nucleon charges (IC; likely on your own)

Parton Distribution Functions

§ PDFs are universal quark/gluon distributions of nucleon

- Many ongoing/planned experiments
  (BNL, JLab, J-PARC, COMPASS, GSI, EIC, LHeC, …)

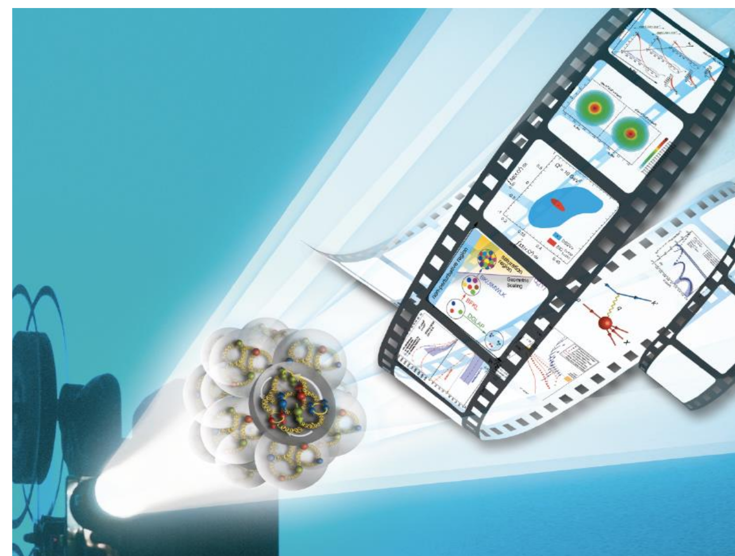


Electron Ion Collider:
The Next QCD Frontier

Imaging of the proton

How are the sea quarks and gluons, and their spins, distributed in space and momentum inside the nucleon?

EIC White Paper, 1212.1701



Global Analysis

§ Experiments cover diverse kinematics of parton variables

- Global analysis takes advantage of all data sets

Theory Input → **Global Analysis of PDFs** ← **Exp't Input**

§ Some choices made for the analysis

- Choice of data sets and kinematic cuts
- Strong coupling constant $\alpha_s(M_Z)$
- How to parametrize the distribution

$$xf(x,\mu_0) = a_0 x^{a_1}(1-x)^{a_2}P(x)$$

- Assumptions imposed

  SU(3) flavor symmetry, charge symmetry, strange and sea distributions

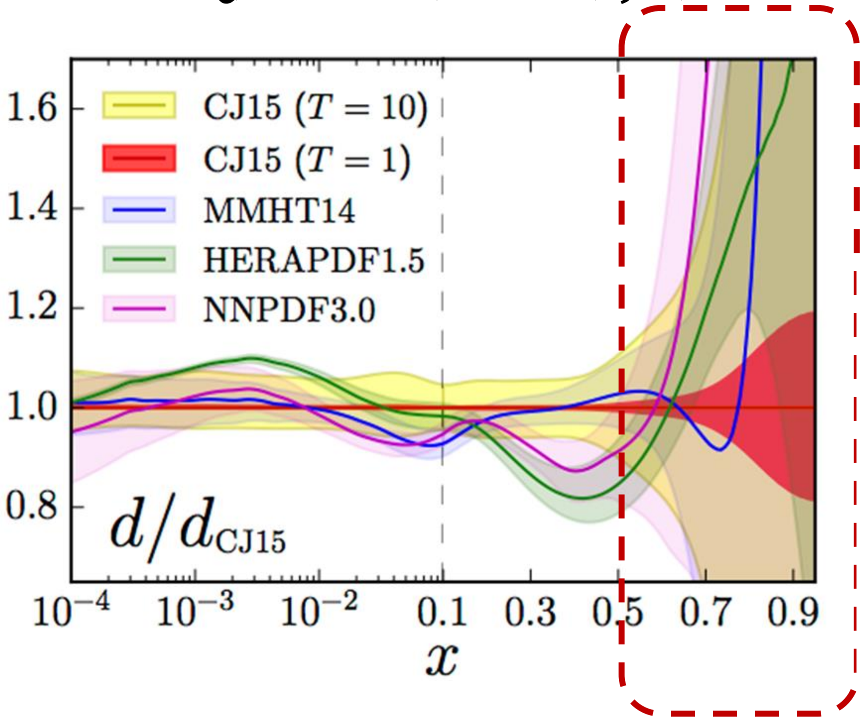
$$s = \bar{s} = \kappa(\bar{u} + \bar{d})$$

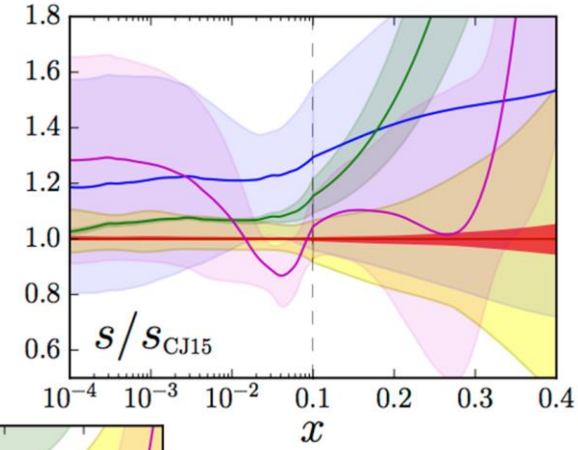
Global Analysis

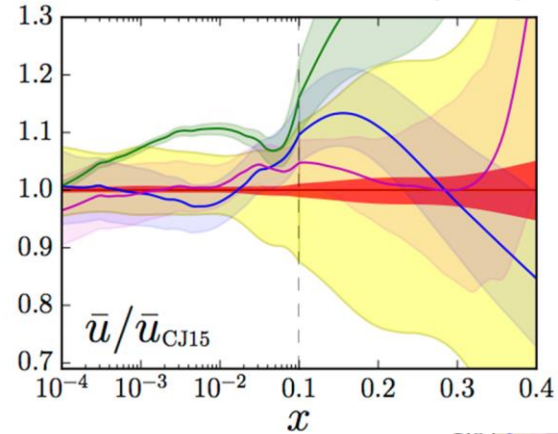
§ Discrepancies appear when data is scarce

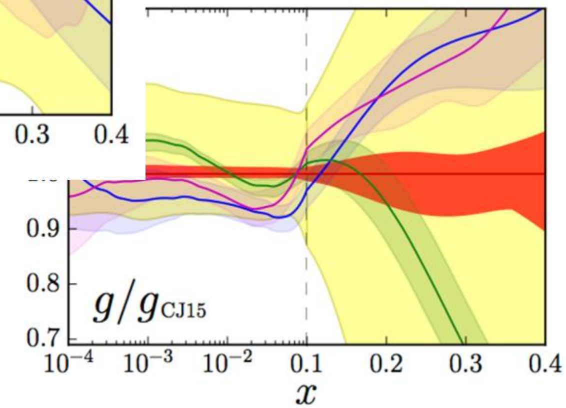
§ Many groups have tackled the analysis

- CTEQ, MSTW, ABM, JR, NNPDF, etc.







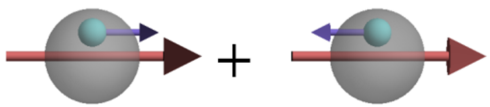


CTEQ-JLAB
https://www.jlab.org/theory/cj/

PDFs on the Lattice

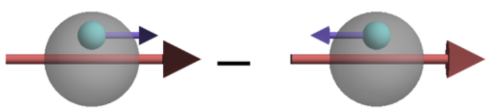
- Traditional lattice calculations rely on operator product expansion, only provide moments

most well known



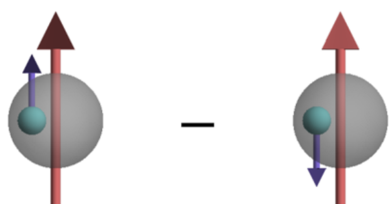
spin-averaged/unpolarized

$$\langle x^{n-1}\rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$$



spin-dependent
longitudinally polarized

$$\langle x^{n-1}\rangle_{\Delta q} = \int_{-1}^{1} dx\, x^{n-1} \Delta q(x)$$



spin-dependent
transversely polarized

$$\langle x^{n-1}\rangle_{\delta q} = \int_{-1}^{1} dx\, x^{n-1} \delta q(x)$$

very poorly known

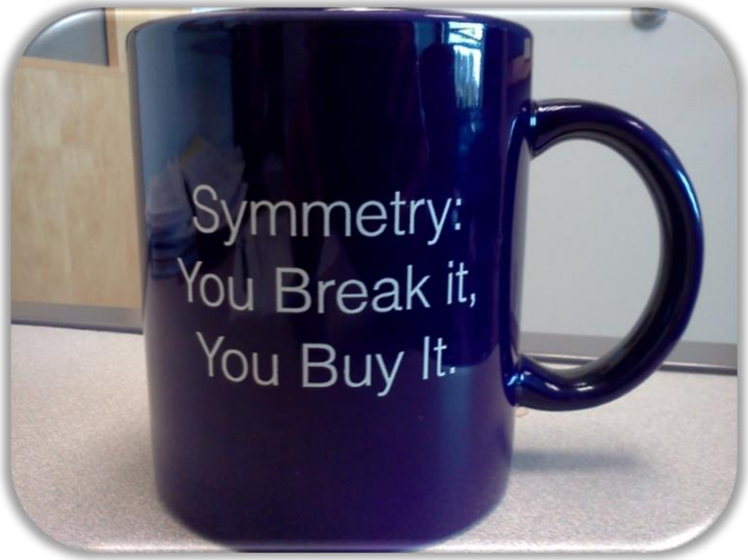
- True distribution can only be recovered with all moments

PDFs on the Lattice

- Limited to the lowest few moments
  - For higher moments, all ops mix with lower-dimension ops
  - Novel proposals to overcome this problem
- Relative error grows in higher moments
  - Calculation would be costly
  - Hard to separate valence contrib. from sea

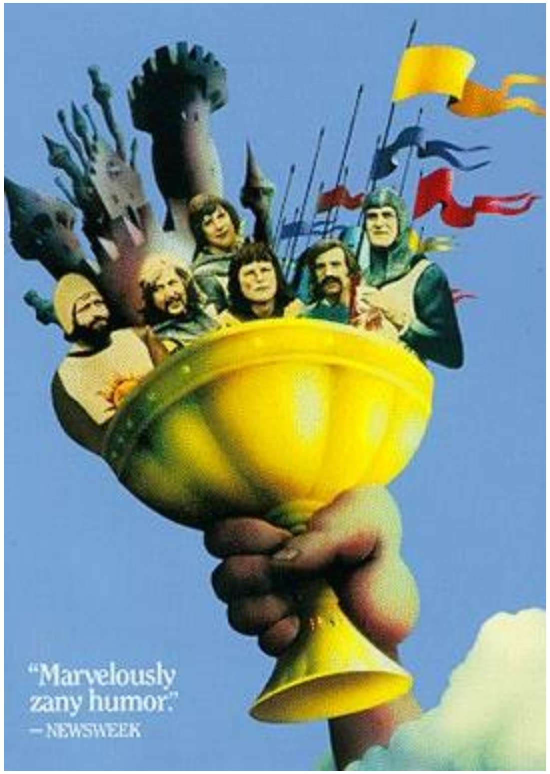
W. Detmold and C. Lin, Phys. Rev. D73 (2006) 014501

Z. Davoudi and M. J. Savage, Phys. Rev. D86 (2012) 054505



Beyond Traditional Moments?

- Longstanding obstacle!
- Holy grail of structure calculations
- Applies to many structure quantities:
  - Generalized parton distributions (GPDs)
  - Transverse-momentum distributions (TMD)
  - Meson distribution amplitudes…
  - Wigner distribution



A NEW HOPE

It is a period of war and economic uncertainty.

Turmoil has engulfed the galactic republics.

Basic truths at foundation of the human civilization are disputed by the dark forces of the evil empire.

A small group of QCD Knights from United Federation of Physicists has gathered in a remote location on the third planet of a star called Sol on the inner edge of the Orion–Cygnus arm of the galaxy.

The QCD Knights are the only ones who can tame the power of the Strong Force, responsible for holding atomic nuclei together, for giving mass and shape to matter in the Universe.

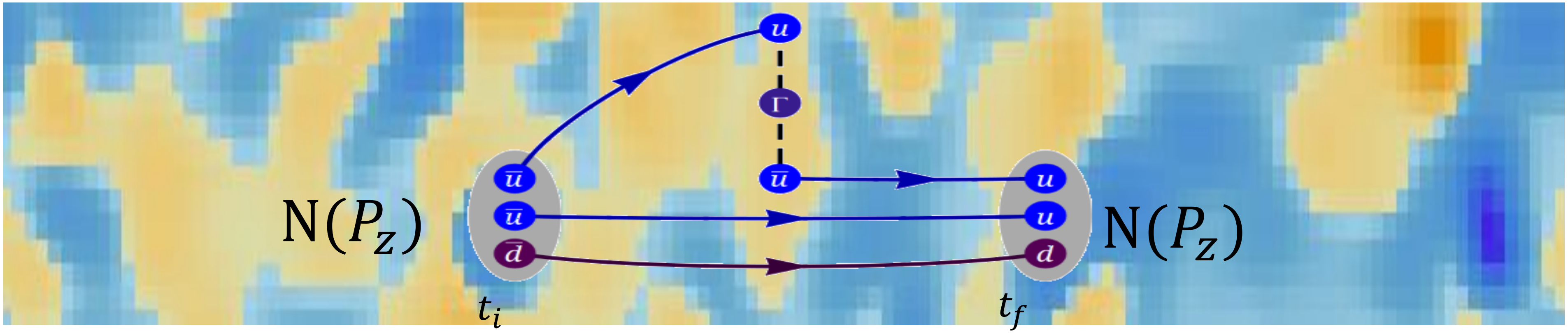
They carry secret plans to build the most powerful

Bjorken-x Dependent Hadron Structure

Biased selected results, highlighting work done by MSU students/postdocs

Lattice Parton Method

§ Large-momentum effective theory (LaMET)/quasi-PDF
(X. Ji, 2013; See 2004.03543 for review)



§ Compute quasi-distribution via

$$\tilde{q}(x,\mu,P_z) = \int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \left| \bar{\psi}(z)\Gamma \exp\left(-ig\int_0^z dz' A_z(z')\right)\psi(0) \right| P \right\rangle$$

§ Recover true distribution (take Pz → ∞ limit)

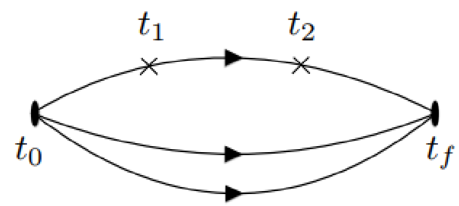
$$\tilde{q}(x,\mu,P_z) = \int_{-\infty}^{\infty} \frac{dy}{|y|} C\left(\frac{x}{y},\frac{\mu}{P_z}\right) \boldsymbol{q(y,\mu)} + \mathcal{O}\left(\frac{M_N^2}{P_z^2},\frac{\Lambda_{\text{QCD}}^2}{(xP_z)^2},\frac{\Lambda_{\text{QCD}}^2}{\left((1-x)P_z\right)^2}\right)$$

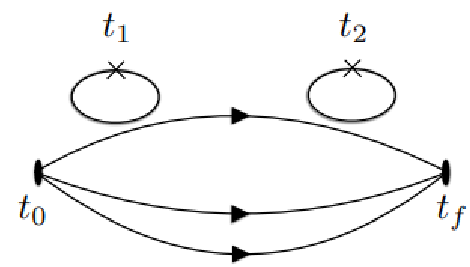
X. Xiong et al., 1310.7471; J.-W. Chen et al, 1603.06664

Lattice Parton Method

§ Short-distance factorization (SDF)

- **pseudo-PDF** method (A. Radyushkin, 2017)
- Hadronic tensor currents
  (Liu et al., hep-ph/9806491, … 1603.07352)
- Lattice cross-section method (LCS)
  (Y Ma and J. Qiu, 2014, 2017)
- Euclidean correlation functions
  (RQCD, 1709.04325)
- Compton amplitude approach (QCDSF, 1703.01153)



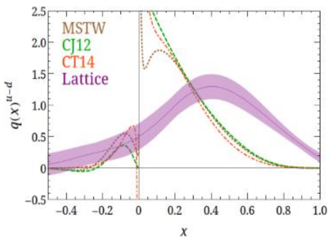


Quantities that can be calculated on the lattice today = $\sum$ Wanted PDFs, GPDs, etc. × pQCD-calculated kernel

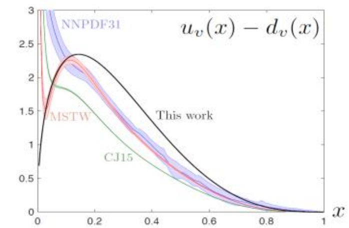
Lattice Parton Calculations

§ Rapid developments!

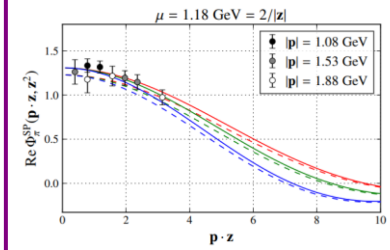
First unpol. PDF lattice calculation



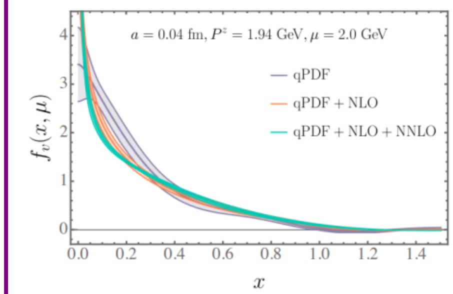
First lattice pseudo-PDFs



Euclidean correlation functions

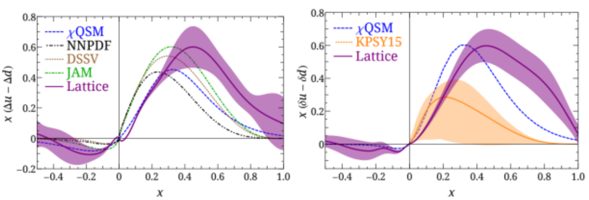


1st NNLO PDF

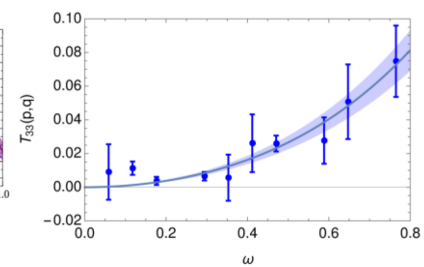


2013 2014 2015 2016 2017 2018 2019 2020 2021 2022 2023

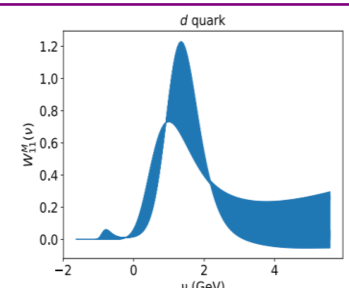
Pol. PDFs and mass corrections



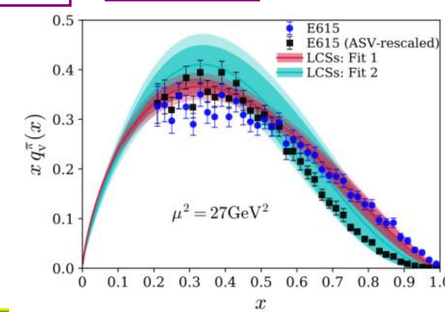
Compton amplitude



Hadronic tensor

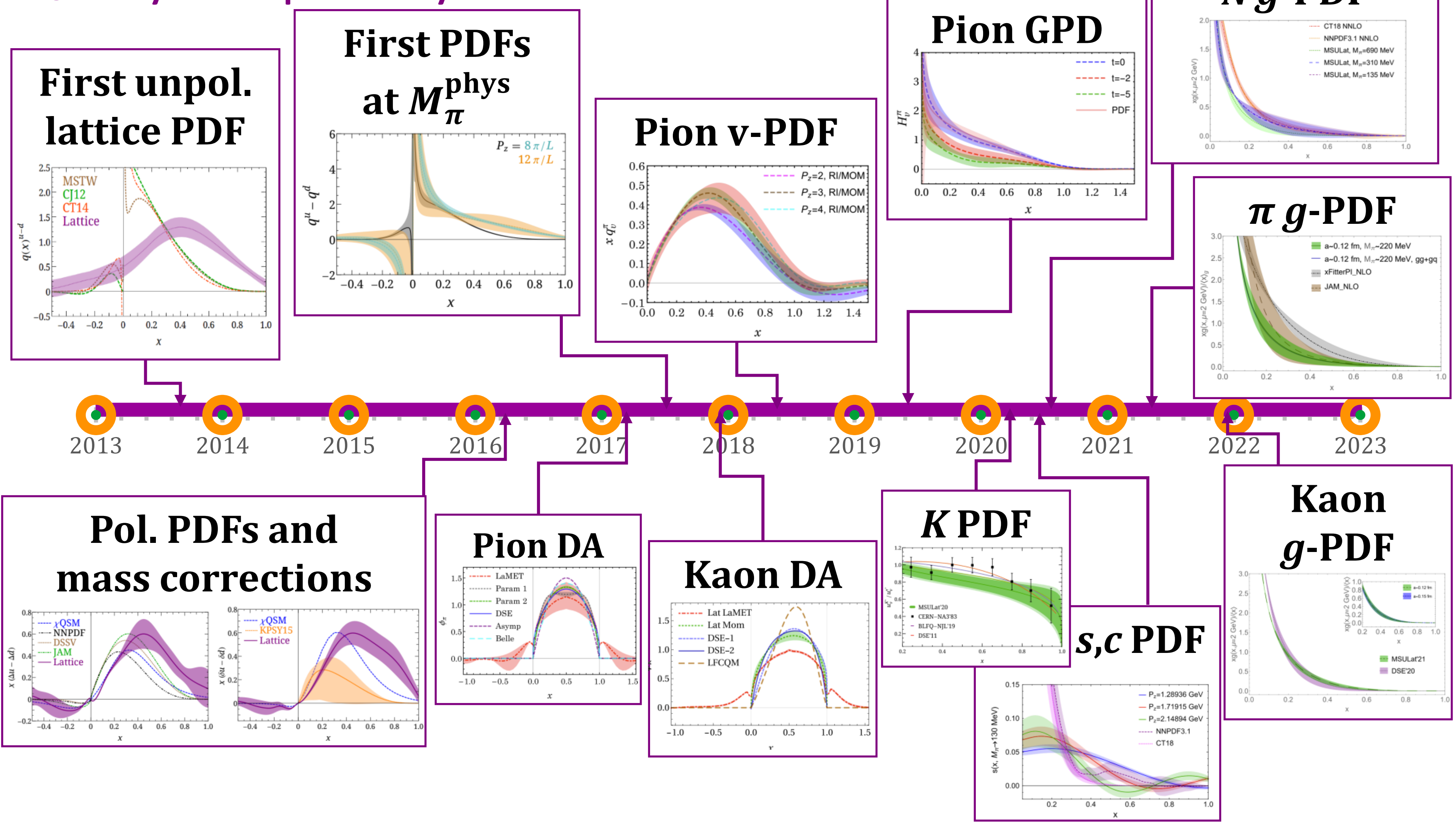


LCS



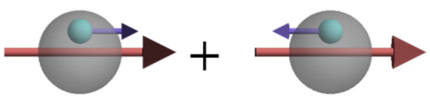
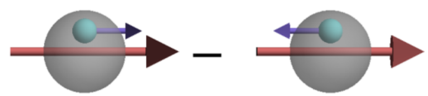
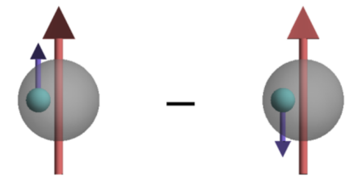
Lattice Parton Calculations

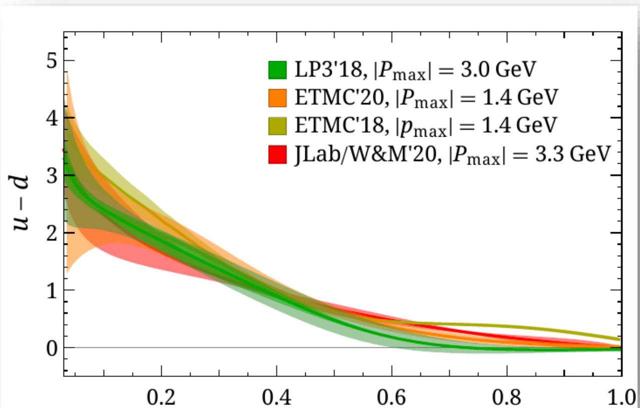
§ Physics quantity milestones

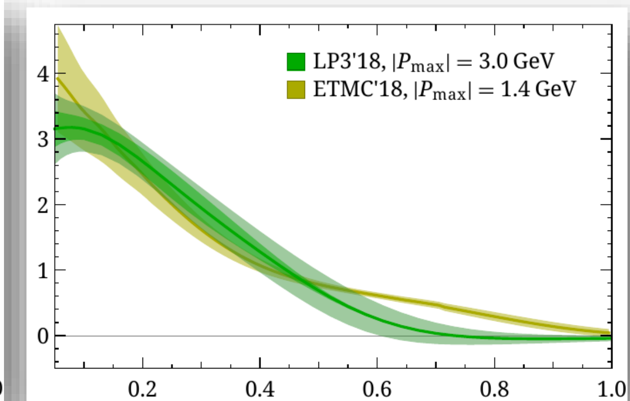


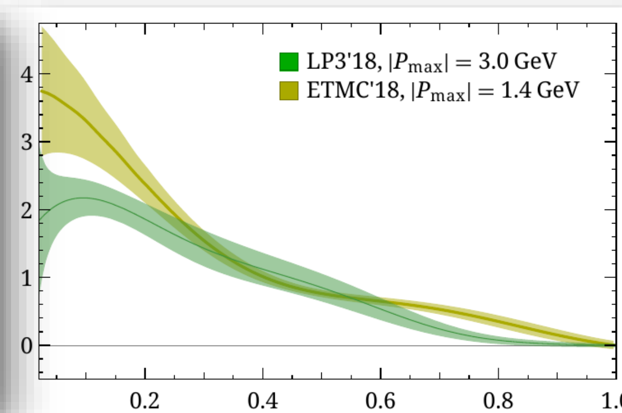
Lattice Example Results

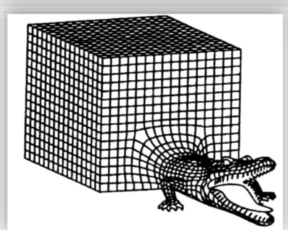
§ Summary of physical pion mass PDFs results

unpolarized	longitudinally polarized	transversely polarized
		
$u(x) - d(x)$	$\Delta u(x) - \Delta d(x)$	$\delta u(x) - \delta d(x)$









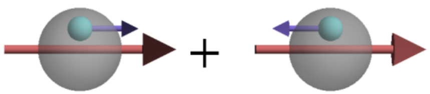
Finite volume,
Discretization,
…

2006.08636 (PDFLattice2019)

Lattice Example Results

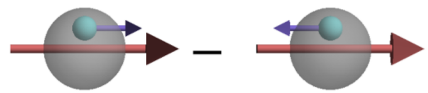
§ Summary of physical pion mass PDFs results

unpolarized



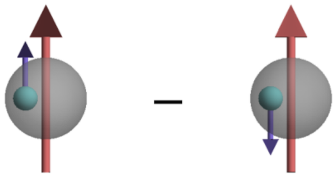
$u(x) - d(x)$

longitudinally polarized

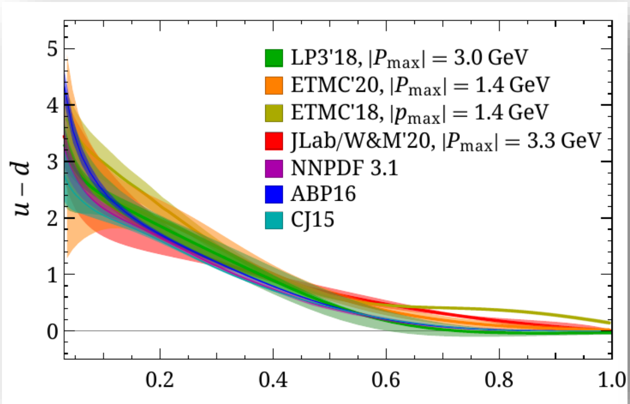


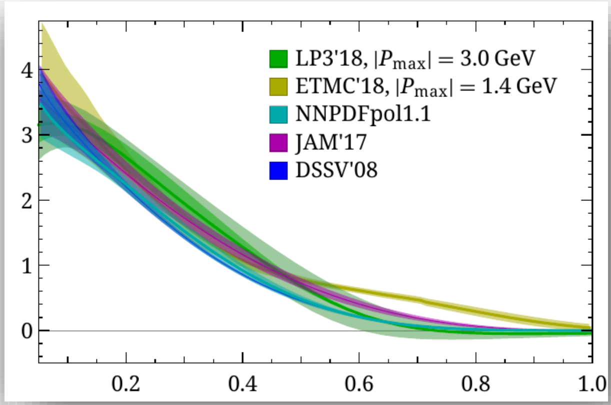
$\Delta u(x) - \Delta d(x)$

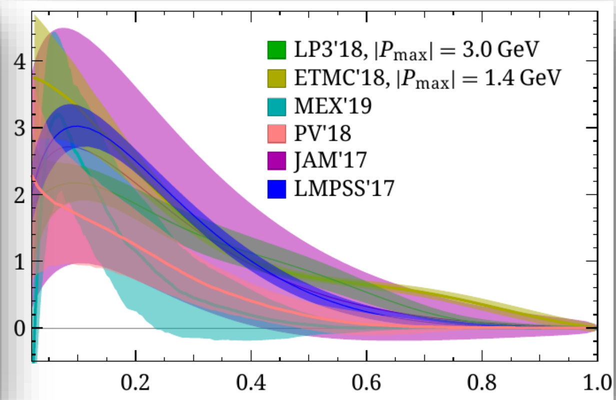
transversely polarized

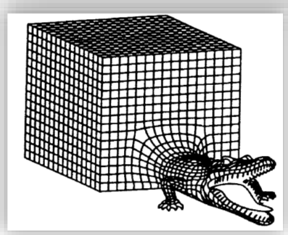


$\delta u(x) - \delta d(x)$





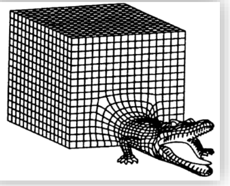




Finite volume,
Discretization,
…

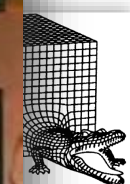
2006.08636 (PDFLattice2019)

Strange, Charm & Gluons PDFs



Glu … PDFs


Discretize QCD action
Use Monte Carlo to evaluate integrals
Collect 1 million measurements
Collect 1 million measurements
imgflip.com



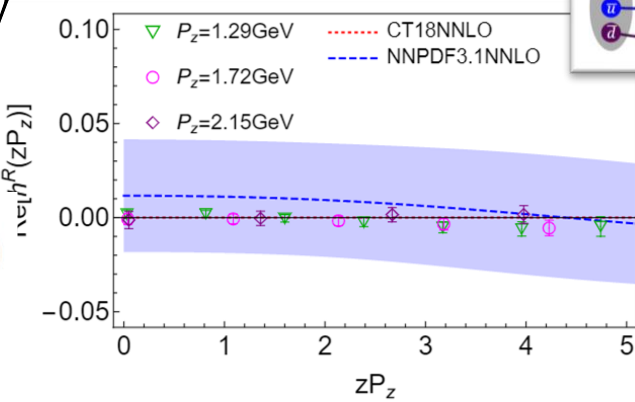
First Lattice Strange PDF

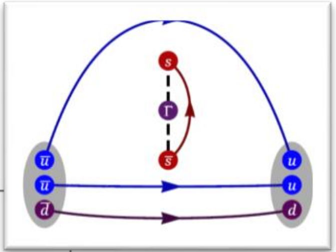
§ Results by MSULat/quasi-PDF method

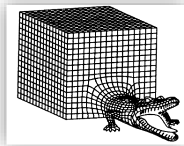
- Clover on 2+1+1 HISQ, 0.12-fm 310-MeV QCD vacuum
- Extrapolated to $M_\pi \approx 140$ MeV

2005.01124, R. Zhang et al (MSULat)

$$\mathrm{Re}[h(z)] \propto \int dx\,(s(x) - \bar{s}(x))\cos(xzP_z)$$







Quantities that can be calculated on the lattice

First Lattice Strange PDF

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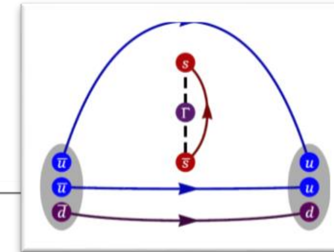

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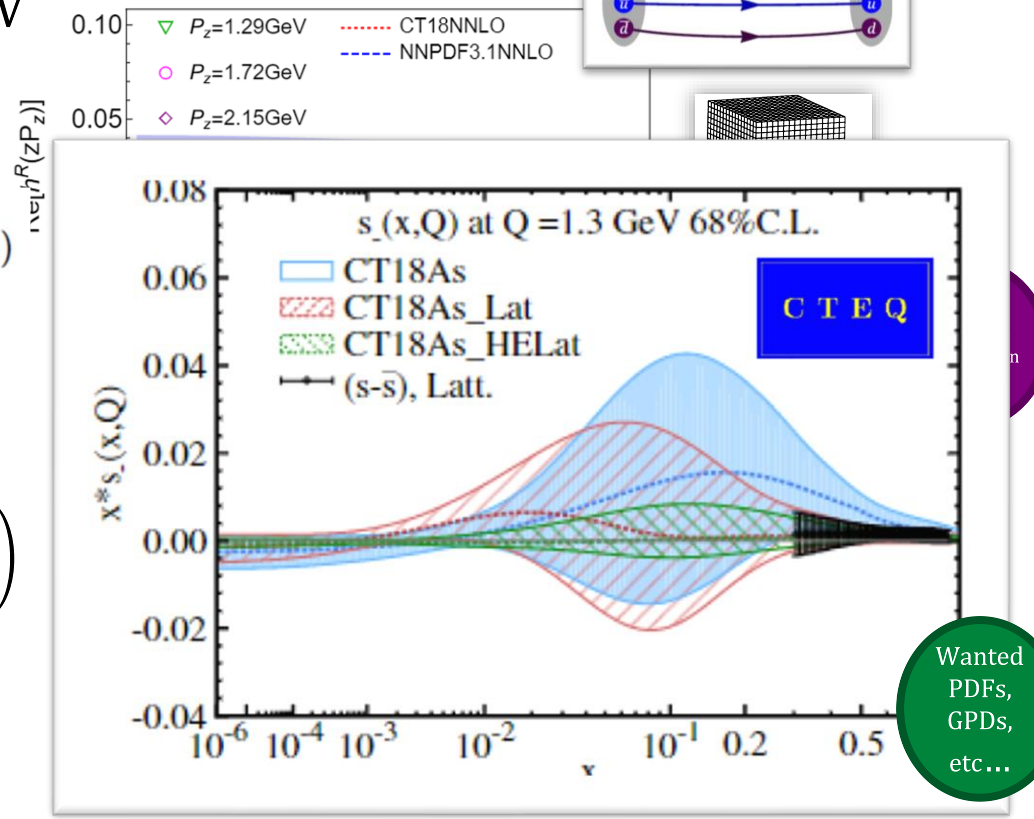


Wanted PDFs, GPDs, etc…

§ From quasi-PDF to PDF

$$\tilde{f}_q(x, P_z) = \int_{-1}^{1} \frac{dy}{|y|} f_q(y) C_{q/q}(x, y, P_z, \mu) + O\left(\frac{\Lambda_{\text{QCD}}^2}{x^2 P_z^2}, \frac{\Lambda_{\text{QCD}}^2}{(1-x)^2 P_z^2}\right)$$

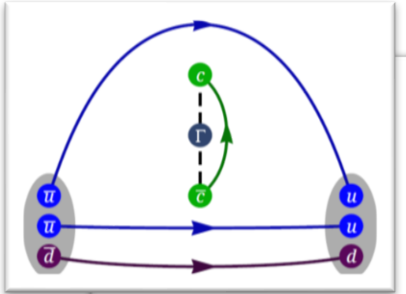
T. Hou, HL, M. Yan, C. Yuan, 2204.07944

§ The strangeness asymmetry $s(x,Q) - \bar{s}(x,Q)$ at $x > 0.2$ is difficult to measure, but can be predicted in lattice QCD

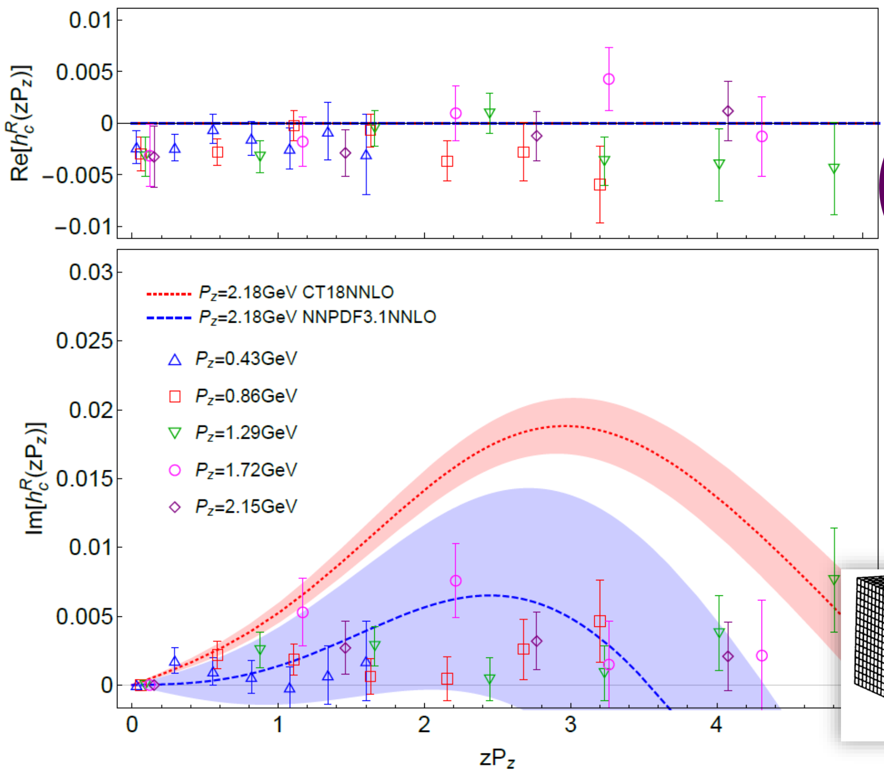
First Lattice Charm PDF

- Large uncertainties in global PDFs
- Results by MSULat/quasi-PDF method
  - Clover on 2+1+1 HISQ 0.12-fm 310-MeV QCD vacuum

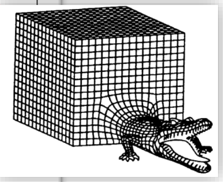
2005.01124, R. Zhang et al (MSULat)



- suggest a symmetric $c - \bar{c}$ distribution
- much smaller than strange PDF



Quantities that can be calculated on the lattice



First Lattice Charm PDF

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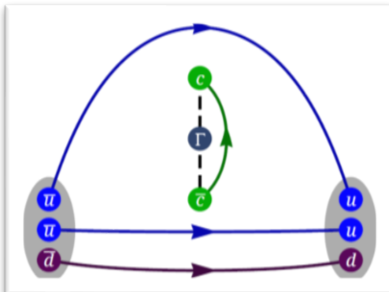
2005.01124, R. Zhang et al (MSULat)



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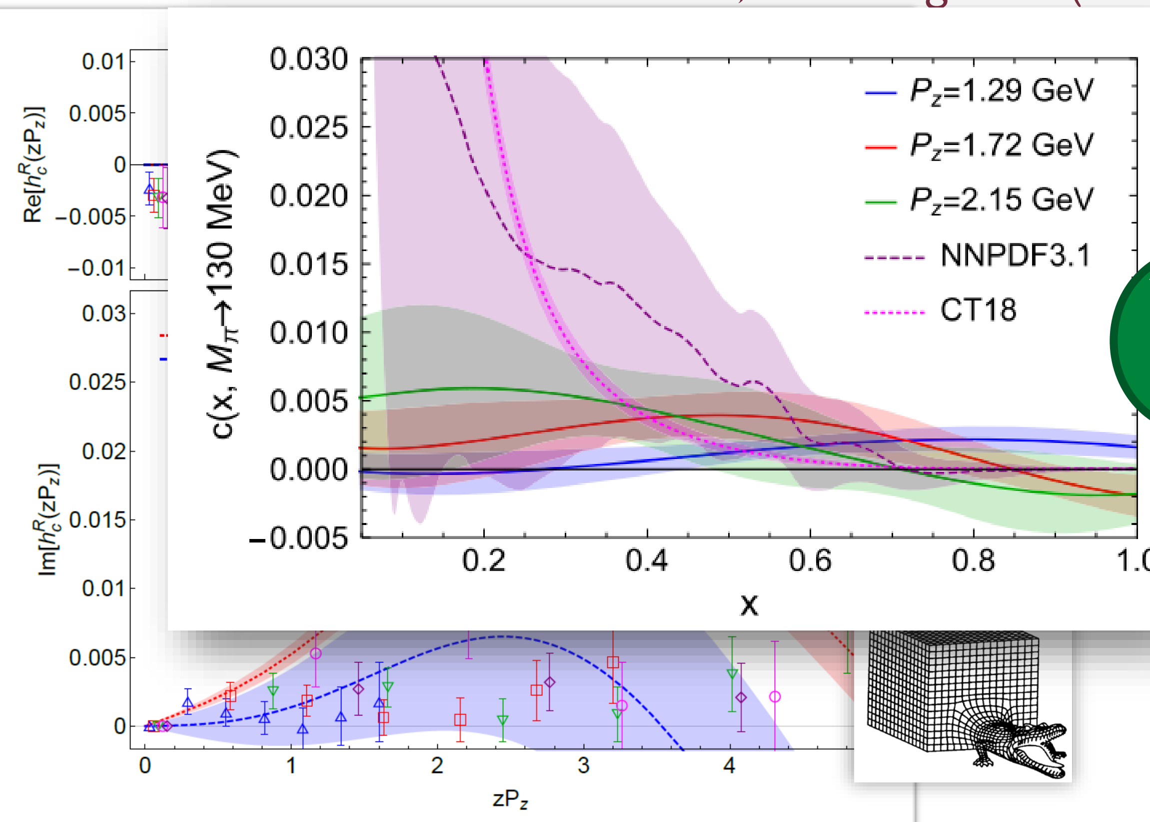
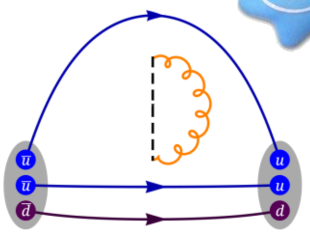


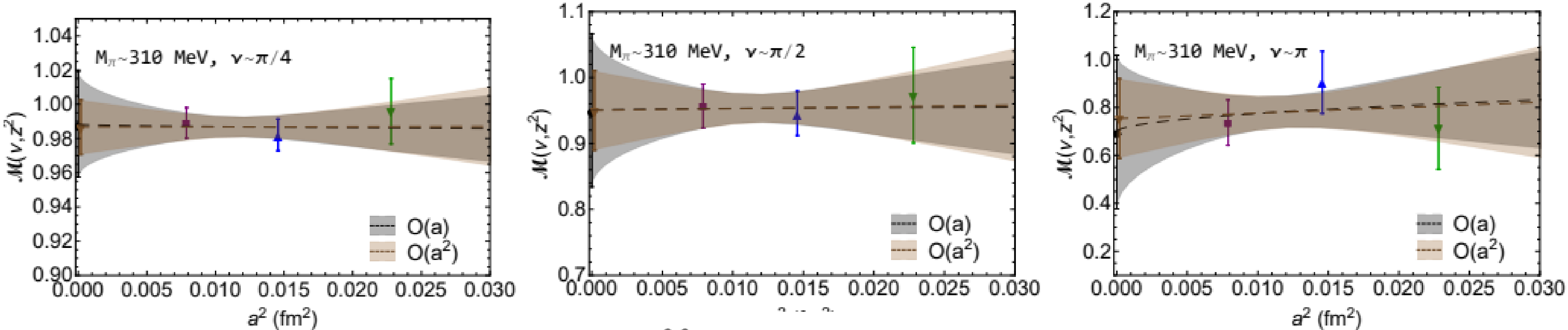
Gluon PDF in Nucleon

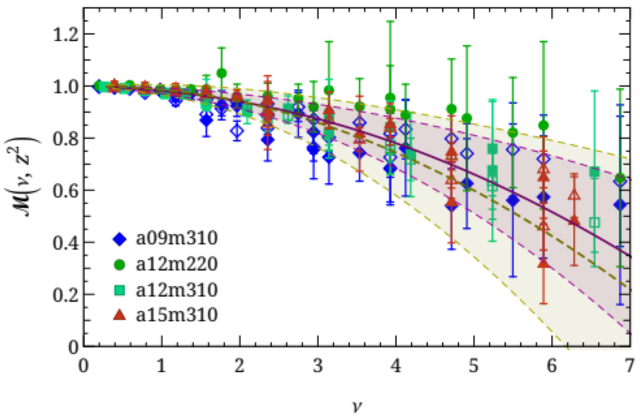
§ Continuum Gluon PDF w/ pseudo-PDF

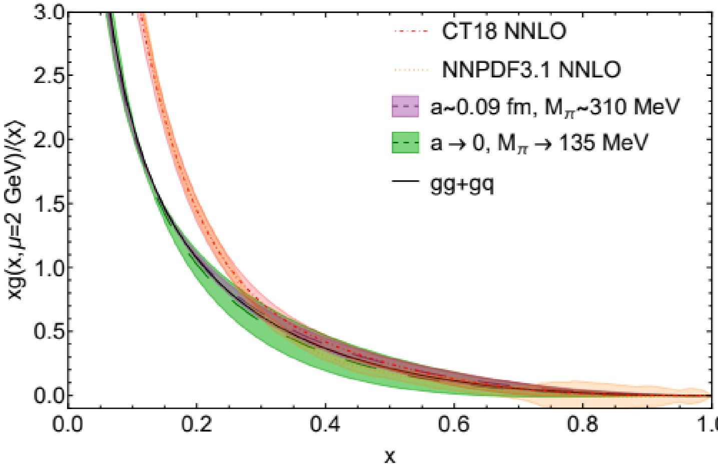


- 2+1+1 HISQ {0.09, 0.12, 0.15} fm, [220,310,700]-MeV pion, 10^5-10^6 statistics

arXiv:2210.09985



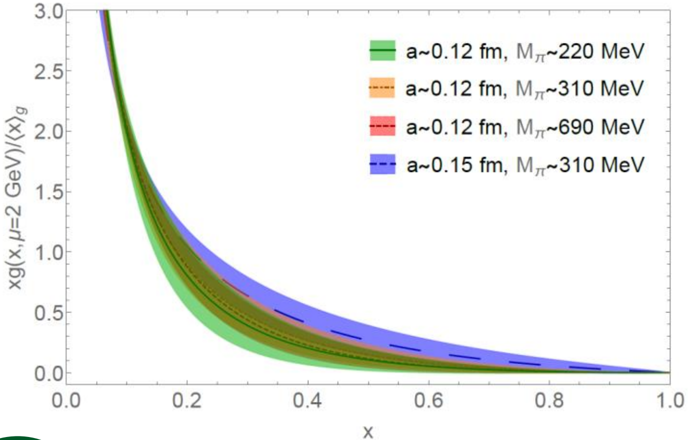


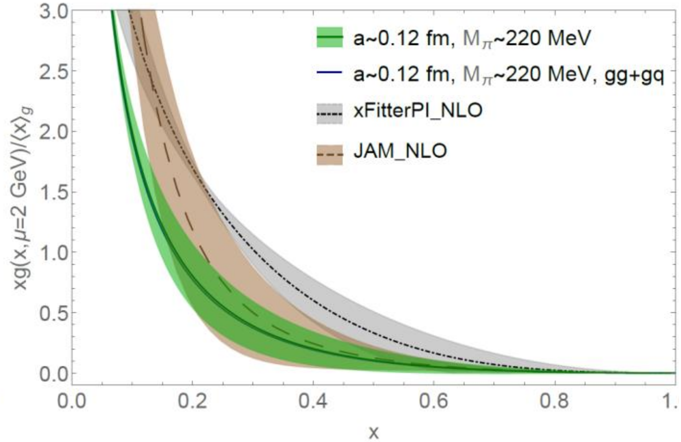


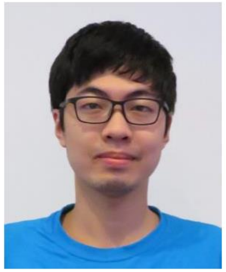
G: Bill Good

Meson Gluon PDFs

§ First pion and kaon gluon PDFs $g(x)/\langle x\rangle$ using pseudo-PDF



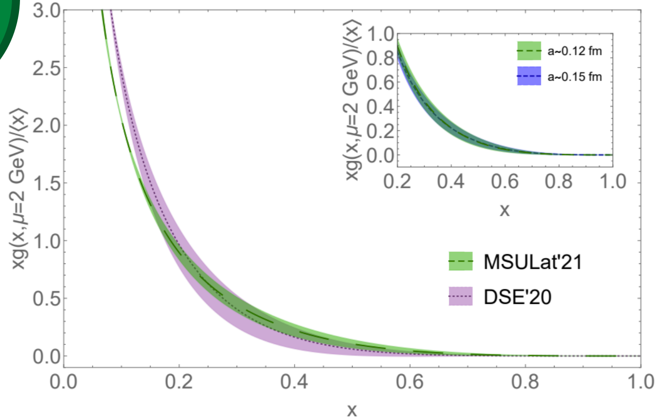




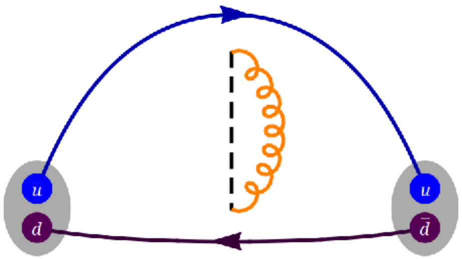
G: Zhouyou Fan

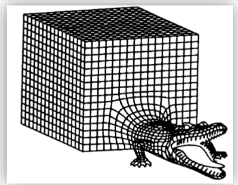
Wanted PDFs, GPDs, etc…

2104.06372, Fan et al. (MSULat); 2112.03124, Salas-Chavira et al. (MSULat)



G: Alejandro Salas-Chavira

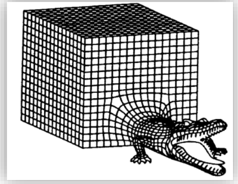




finite-volume, discretization, heavy quark mass, …

Generalized Parton Distributions

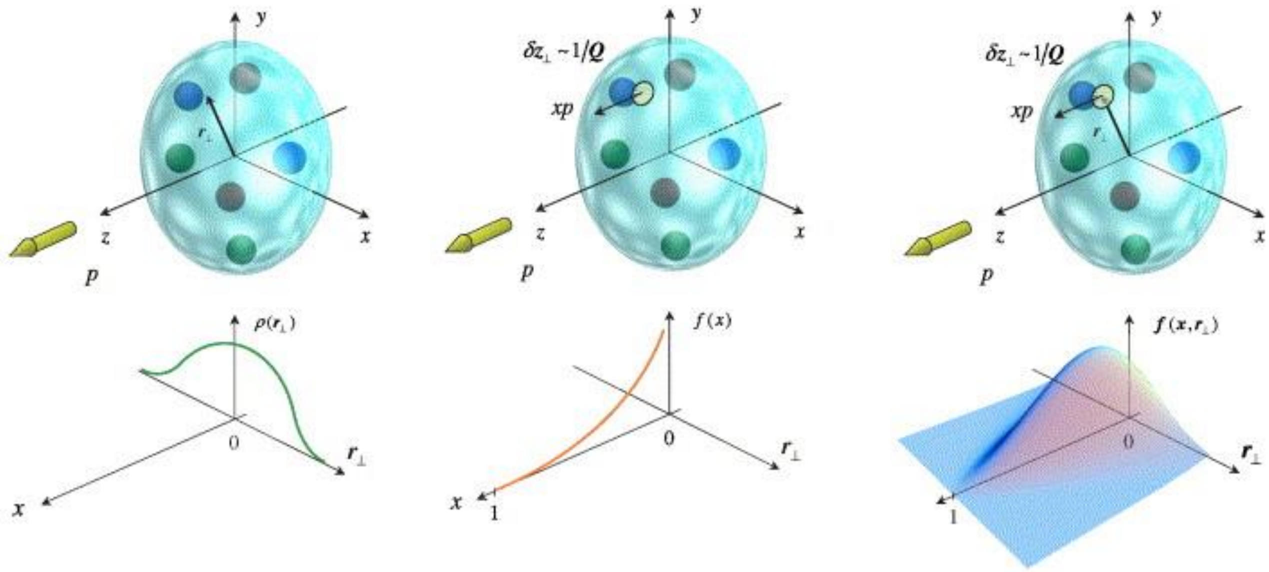
Single-ensemble result



finite-volume,
discretization,
heavy quark mass,
...

Biased selected/highlighted results

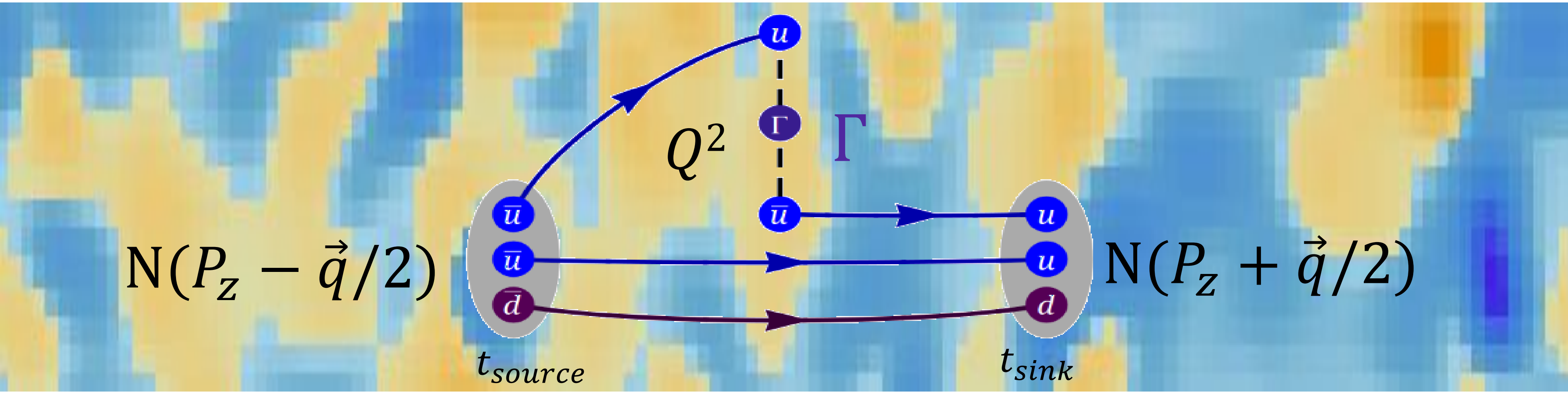
Bjorken-x Dependent GPDs



Picture from A. Belitskya and A Radyushkin, Physics Report, 416 (2015)

Generalized Parton Distributions

§ On the lattice, one needs to calculate the following (nucleon example)

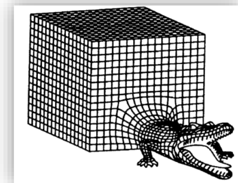


$$\tilde{F}(x, \tilde{\xi}, t, \bar{P}_z) = \frac{\bar{P}_z}{\bar{P}_0} \int \frac{dz}{4\pi} e^{ixz\bar{P}_z} \langle P' | \tilde{O}_{\gamma_0}(z) | P \rangle = \frac{\bar{u}(P')}{2\bar{P}^0} \left(H(x, \tilde{\xi}, t, \bar{P}_z)\gamma^0 + E(x, \tilde{\xi}, t, \bar{P}_z) \frac{i\sigma^{0\mu}\Delta_\mu}{2M} \right) u(P'')$$

$$p^\mu = \frac{p''^\mu + p'^\mu}{2}, \qquad \Delta^\mu = p''^\mu - p'^\mu, \qquad t = \Delta^2, \qquad \xi = \frac{p''^+ - p'^+}{p''^+ + p'^+}$$

Nucleon Tomography

§ Nucleon GPD using quasi-PDFs at physical pion mass



finite-volume,
discretization,

- Lattice details: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon quasi-GPD results

$$F^q(x,\xi,t) = \int \frac{\mathrm{d}z^-}{4\pi} e^{-ixP^+z^-} \langle p'|\bar{q}(z^-/2)\gamma^+ q(-z^-/2)|p\rangle$$

$$= \frac{1}{2P^+}\left[H^q(x,\xi,t)\,\bar{u}\,(p')\,\gamma^+ u(p) - E^q(x,\xi,t)\,\bar{u}\,(p')\,\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}u(p)\right]$$

HL, Phys.Rev.Lett. 127 (2021) 18, 182001

Nucleon Tomography

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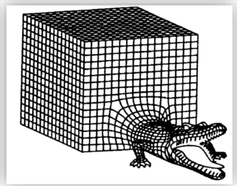
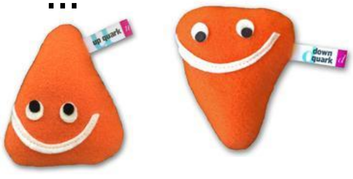
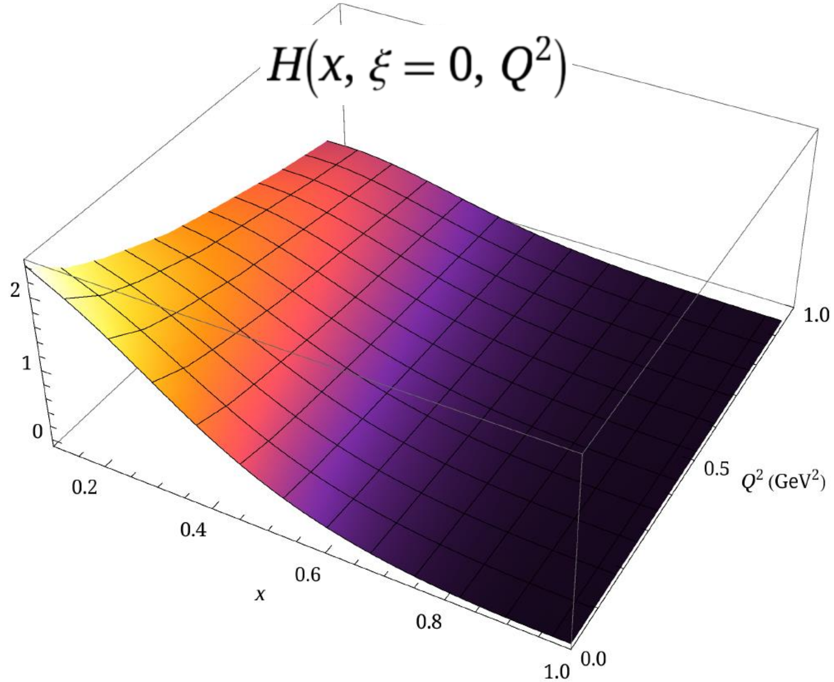
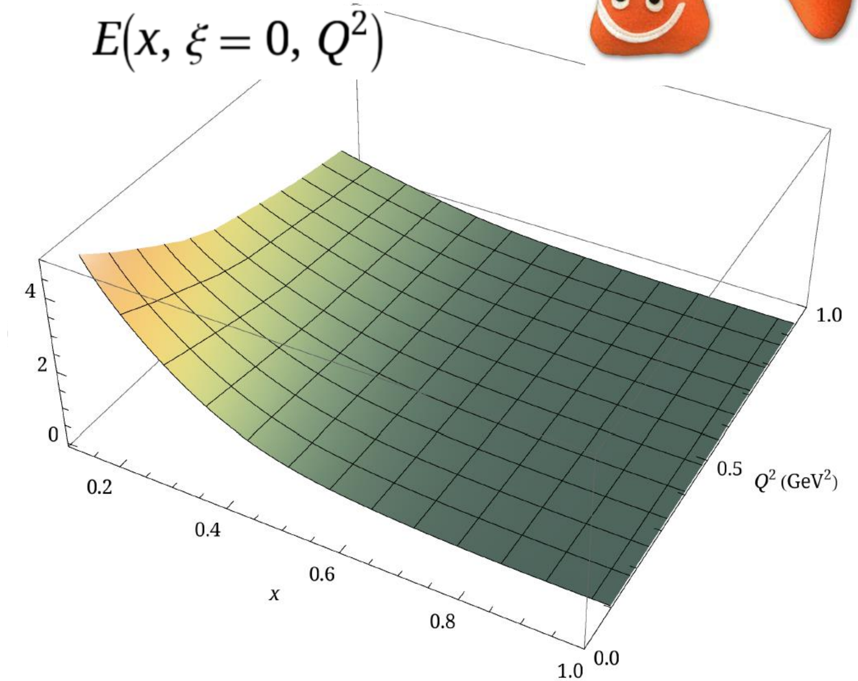


finite-volume,
discretization,
...







HL, Phys.Rev.Lett. 127 (2021) 18, 182001

Isovector Nucleon GPDs

§ Nucleon GPD using quasi-PDFs at physical pion mass

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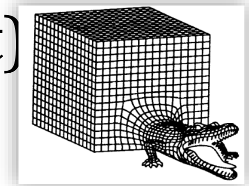
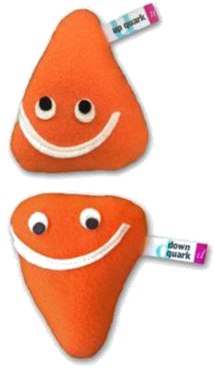
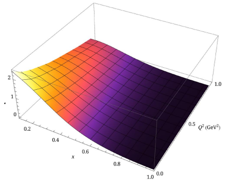
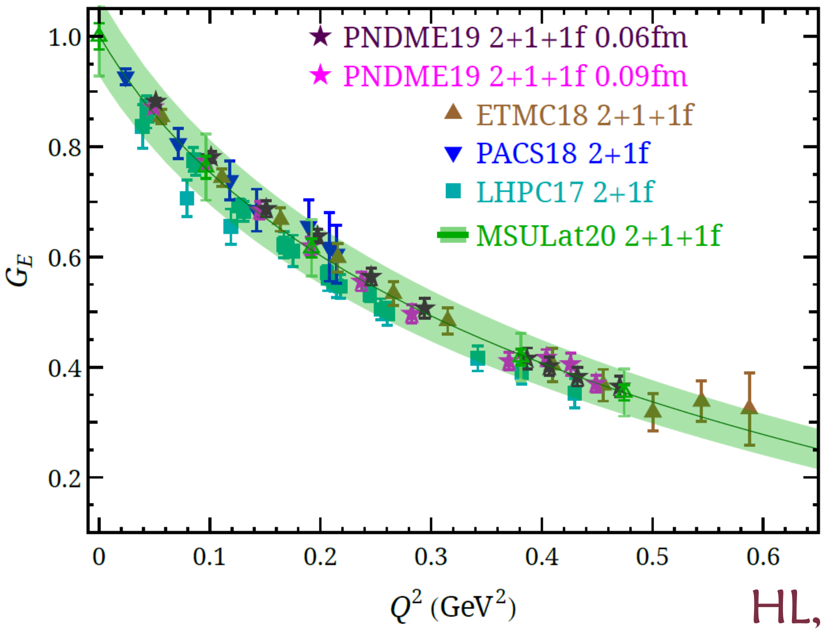


finite-volume,
discretization,
…



$$\int_{-1}^{+1} dx\, x^{n-1}$$  $$= \sum_{i=0,\text{even}}^{n-1} (-2\xi)^i A^q_{ni}(t) + (-2\xi)^n C^q_{n0}(t)\Big|_{n\ \text{even}}$$

$n = 1$

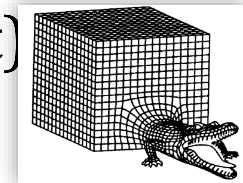


HL, Phys.Rev.Lett. 127 (2021) 18, 182001

Nucleon GPDs

§ Nucleon GPD using quasi-PDFs at physical pion mass

- Lattice details: clover/2+1+1 HISQ (MSULat)
  0.09 fm, **135-MeV** pion mass, $P_z \approx 2$ GeV



finite-volume, discretization, …

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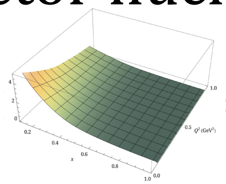
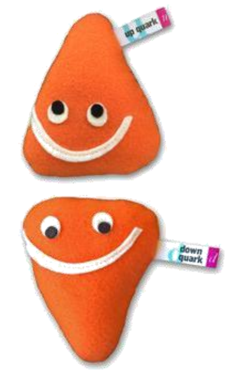
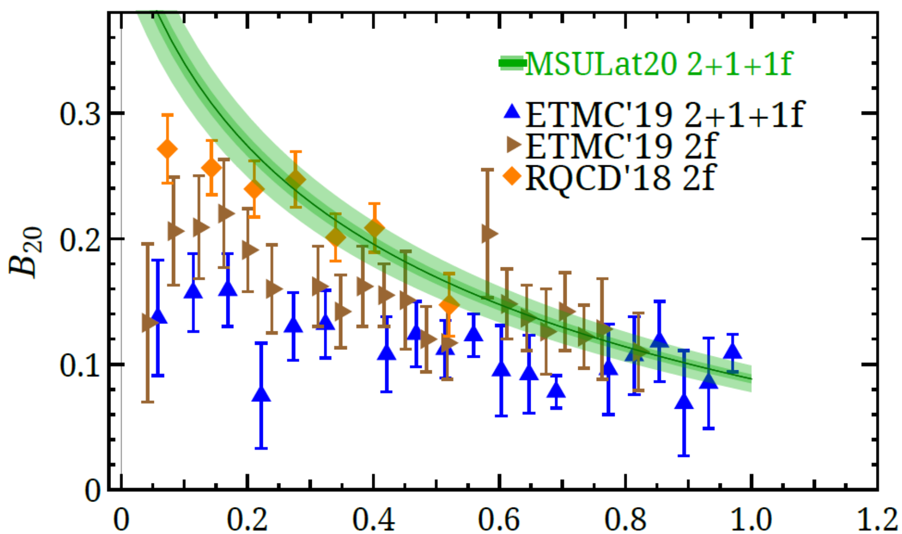
$$\int_{-1}^{+1} dx\, x^{n-1}$$  $$= \sum_{i=0,\text{even}}^{n-1} (-2\xi)^i B^q_{ni}(t) - (-2\xi)^n C^q_{n0}(t) \Big|_{n\ \text{even}}$$



$n = 2$



HL, Phys.Rev.Lett. 127 (2021) 18, 182001

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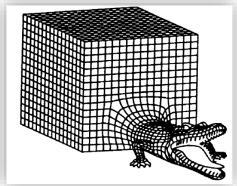
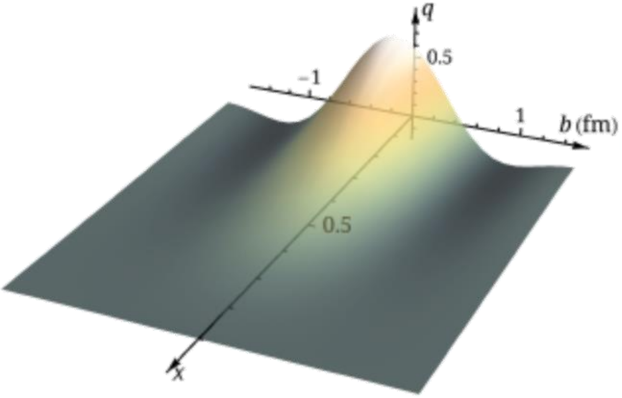
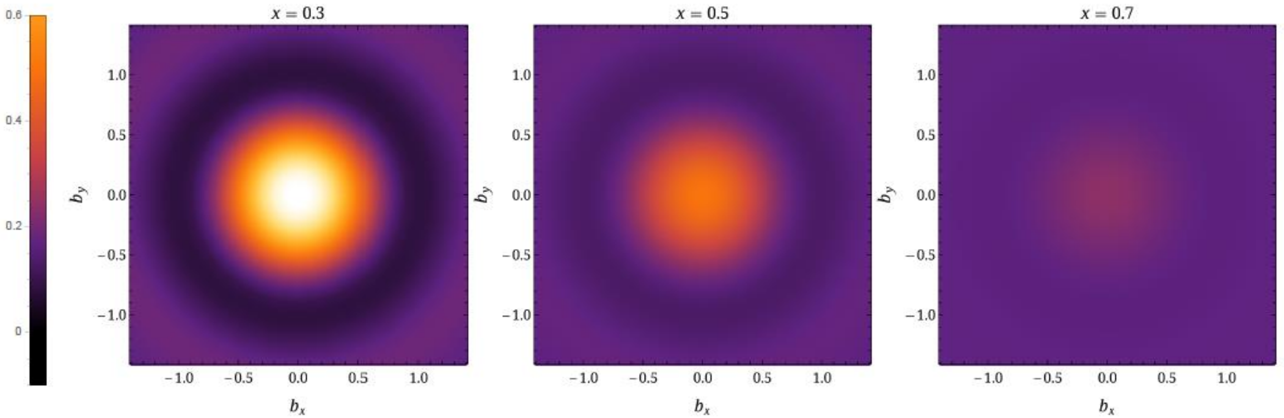


finite-volume, discretization,

$$q(x,b) = \int \frac{d\vec{q}}{(2\pi)^2} H(x, \xi = 0, t = -\vec{q}^2) e^{i\vec{q}\cdot\vec{b}}$$





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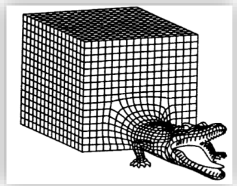

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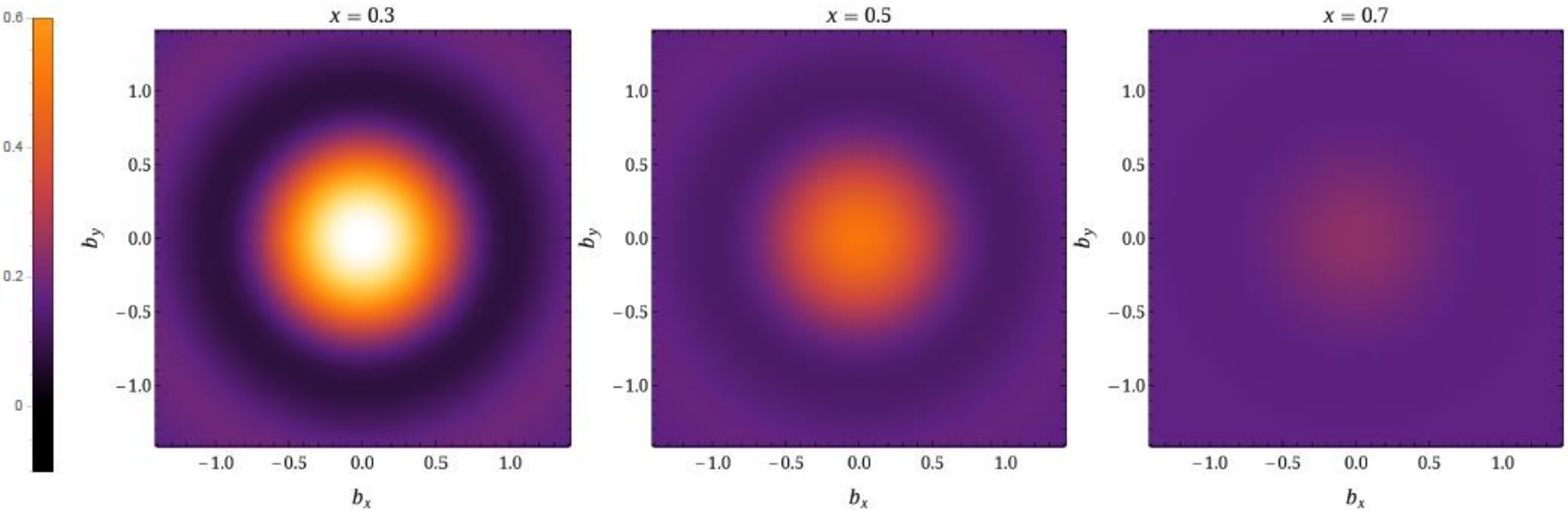


HL, Phys.Rev.Lett. 127 (2021) 18, 182001

Nucleon Tomography

- § Assuming we live in the Marvel Universe
  - The special quantum tunnel allows us to shrink to the size particle to sub-nucleon scale ($< 10^{-15}$m)
- § What would it look like to travel inside the nucleon?

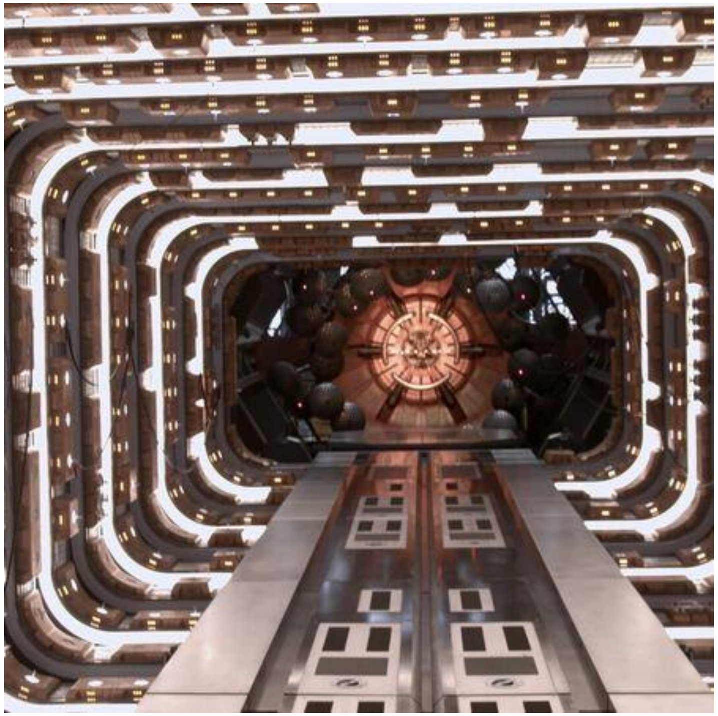
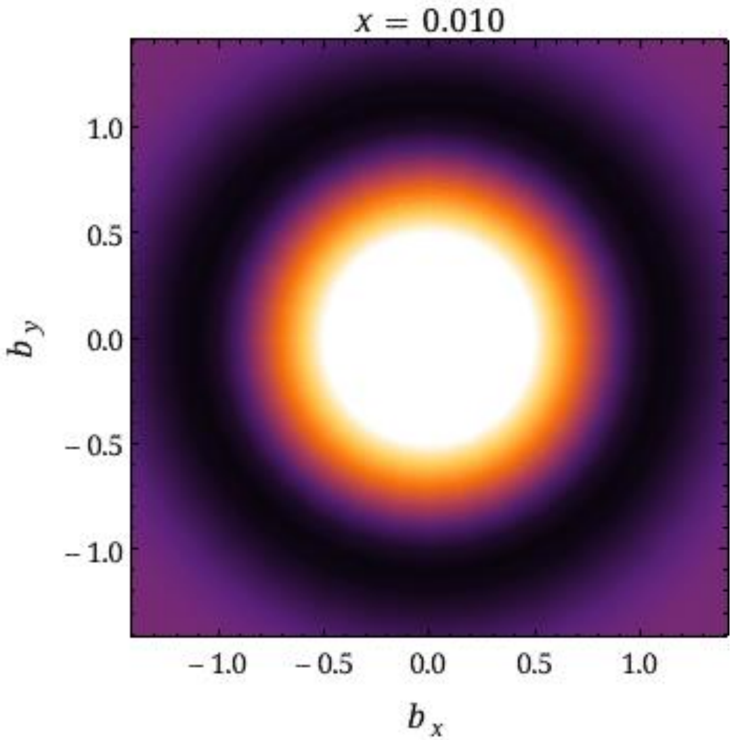


Image credit: Marvel Studios



Thanks to Cottrell Scholar Award from RCSA

Challenges

- Large momentum is essential
  - With sufficient statistics nucleons may reach 5 GeV
- Renormalization of linear divergence
  - Wilson-line ops have linear divergences that must be subtracted
- Methods for signal-to-noise improvement
  - Gluonic observables, new ideas for large momentum
- Inverse problems PDF extraction in SDF
  - Remove the model/preconditioner-choice dependence
- Reaching long-range correlations in LaMET
  - For small-x physics, new methods for calculating longer-range correlations must be developed

Whitepaper: Lattice QCD Calculations of Parton Physics, 2202.07193

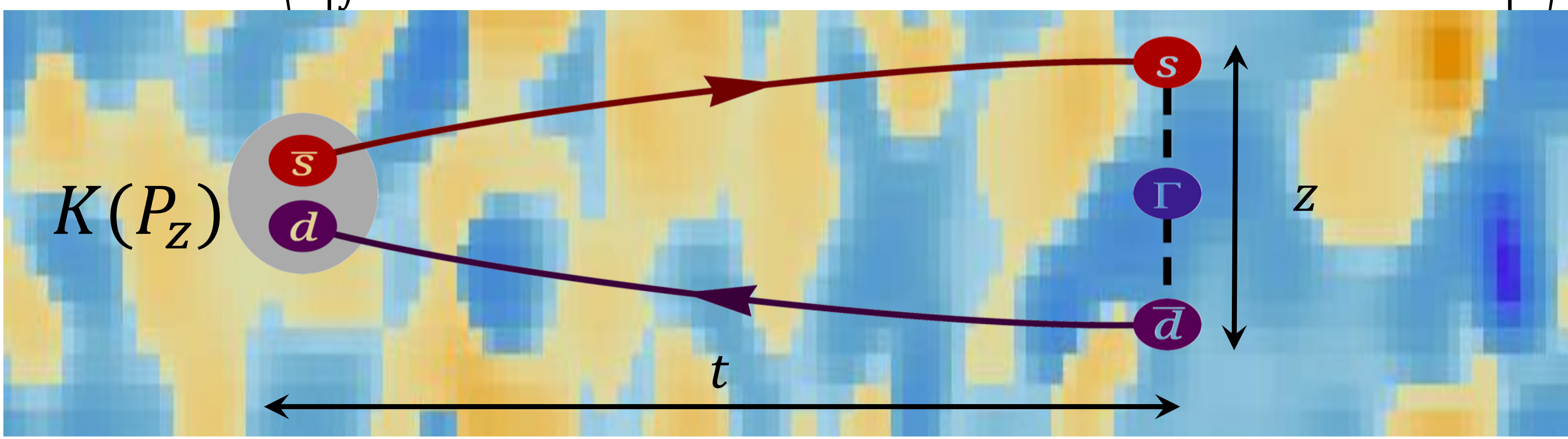
Application on Inverse Problem

Quantities that can be calculated on the lattice today = $\sum$ Wanted PDFs, GPDs, etc. × pQCD-calculated kernel

R. Zhang, C. Honkala, et al. (MSULat), 2005.13955

Example: Pion/Kaon Distribution Amplitude

$$C_M^{\mathrm{DA}}(z,P,t) = \left\langle 0 \left| \int d^3y\, e^{i\vec{P}\cdot\vec{y}} \bar{\psi}_1(\vec{y},t)\gamma_z\gamma_5 U(\vec{y},\vec{y}+z\,\hat{z})\psi_2(\vec{y}+z\,\hat{z},t)\bar{\psi}_2(0,0)\gamma_5\psi_1(0,0) \right| 0 \right\rangle$$



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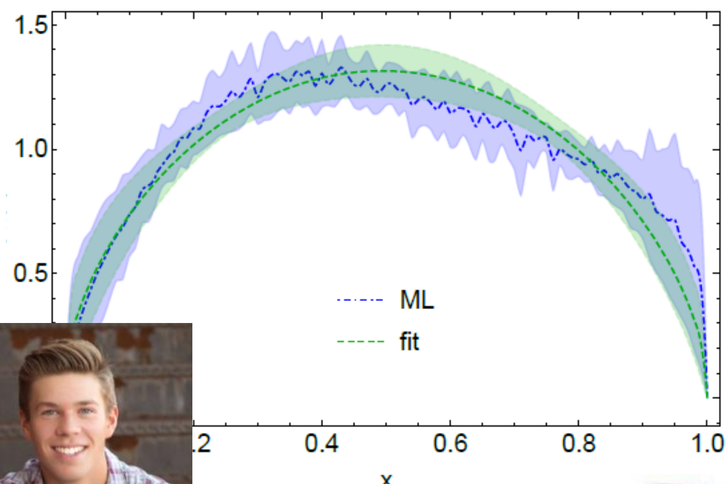
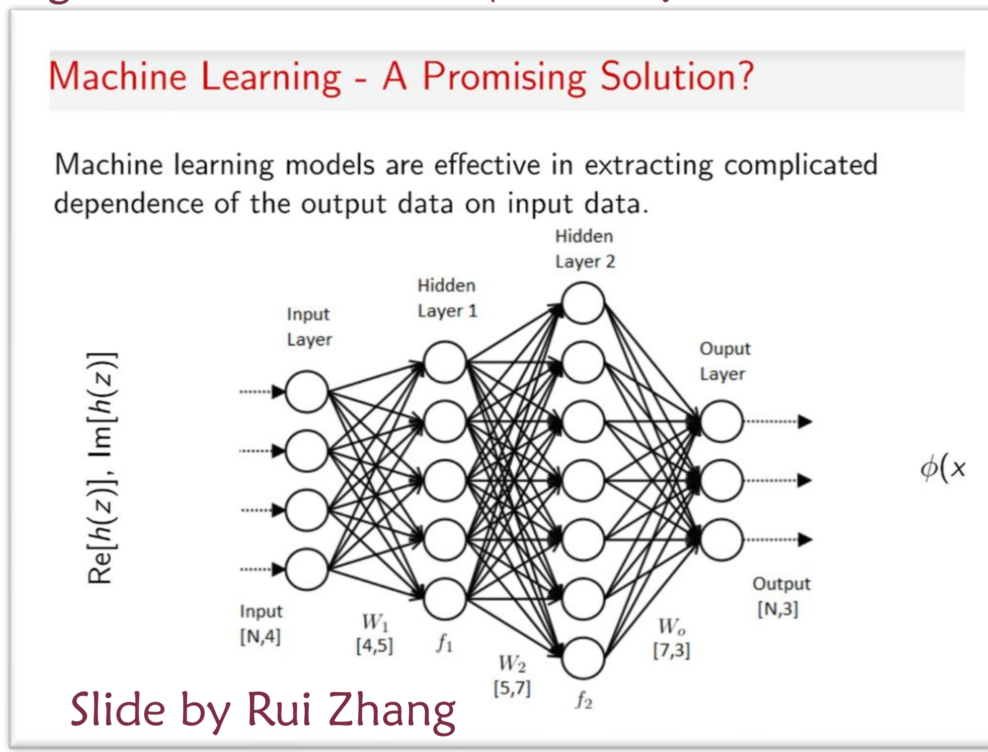
Pion Distribution Amplitude



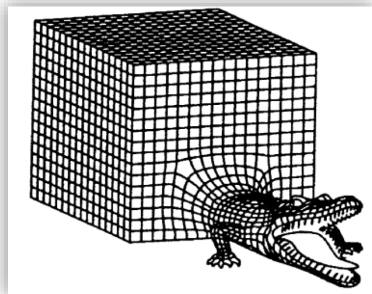
UG: Carson Honkala



Slide by Rui Zhang

Take Aways?

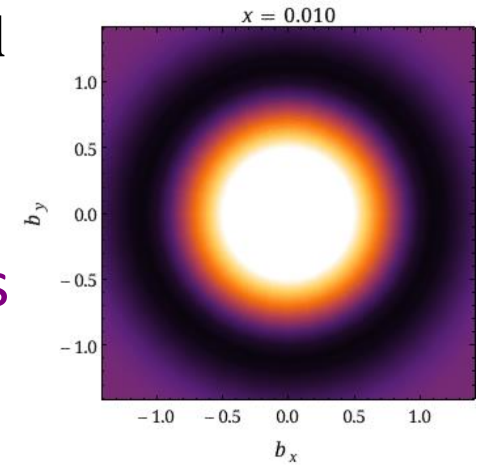
- Exciting era using LQCD to study hadron properties
  - Many interesting quantities; some at precision level
- Get an overall picture of lattice calculations
  - Few examples on spectroscopy and structure (limited time)
- Lattice calculations can be wrong
  - If systematics are not examined carefully
  - You cannot just say "Lattice says so"; which one? What has been done?



- There are limitations
  - Some quantities are harder to do on the lattice

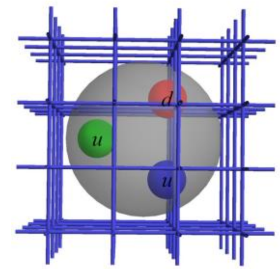
Take Aways?

- § Exciting era using LQCD to study hadron properties
  - Many interesting quantities; some at precision level
- § Get an overall picture of lattice calculations
  - Few examples on structure (limited time)
- § Precision and progress are limited on resources
  - Challenges = new opportunities quantities
- § In the future



Theory Input → **Global Analysis of PDFs/GPDs** ← **Exp't Input**

+



EXCLAIM

Thanks to MILC collaboration for sharing their 2+1+1 HISQ lattices & USQCD/NSF/DOE for computational resources

This work is partially sponsored by grants NSF PHY 1653405 & 1653405, DOE DE-SC0024053 & RCSA Cottrell Scholar Award

Learning by Doing!

Time for Tutorials

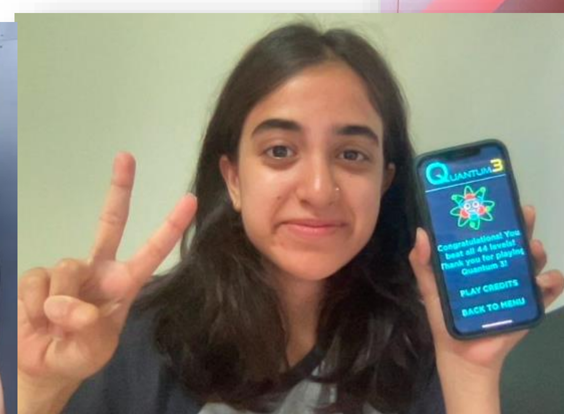
- Form groups of about 4 students
  - Preferably forming different groups from yesterday's
  - Rearrange the chairs so your group can face each other
- Introduce yourself to the other students in your group (5 mins)
  - Name and preferred pronoun
  - Where are you from and tell us a few things about the place
  - Tell us a weird fact you happen to know for no reason or what you plan to do this Sunday?
  - **Make sure you listen to each other!**

Want to Win a Free T-shirt?

- Complete all levels of Quantum 3 game before 11:59AM June 4th (Tue)

  - GET IT ON Google Play · Download on the App Store

  - Answer the questions in the [Google Form]



Students Wanted

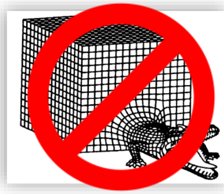
LGT4HEP website: https://lgt4hep.github.io/

Backup Slides

First Continuum PDF

§ Nucleon PDFs using quasi-PDFs in the continuum limit

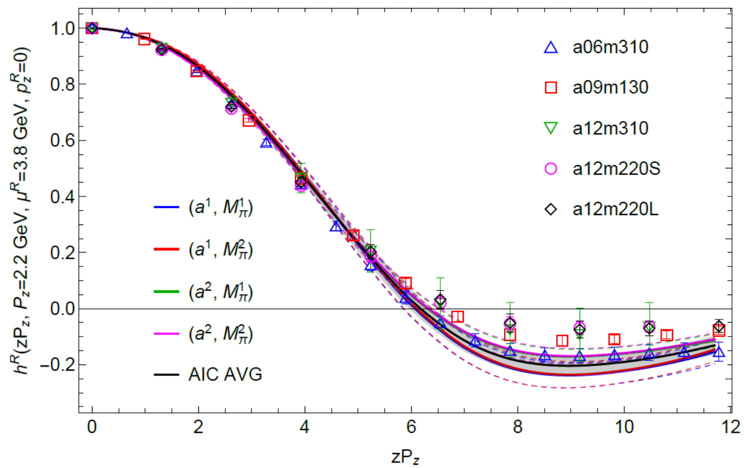
- Lattice details: clover/2+1+1 HISQ (MSULat)
  - $a \approx \{0.06, 0.09, 0.12\}$ fm,
  - $M_\pi \in \{135, 220, 310\}$-MeV pion,
  - $M_\pi L \in \{3.3, 5.5\}$.
  - $P_z \approx 2$ GeV

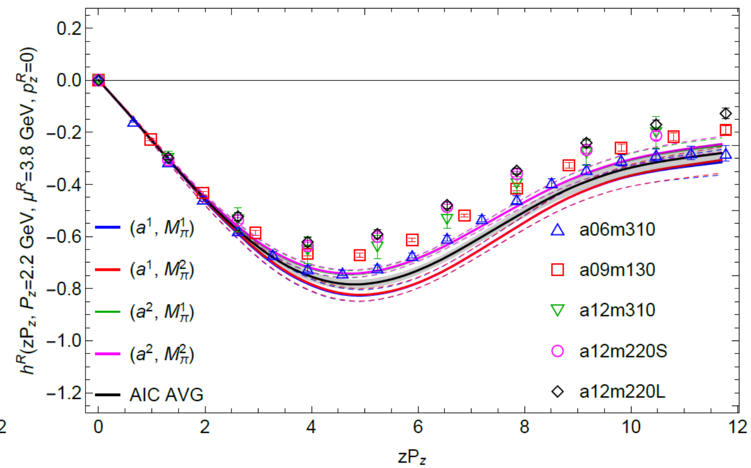


2011.14971, HL et al (MSULat)

- Naïve extrapolation to physical-continuum limit

Quantities that can be calculated on the lattice





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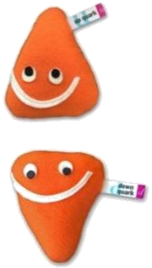

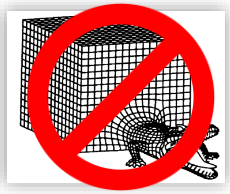

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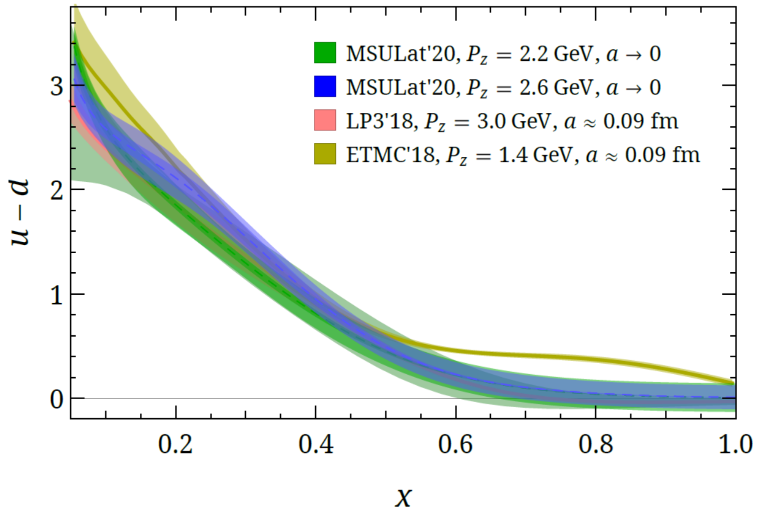
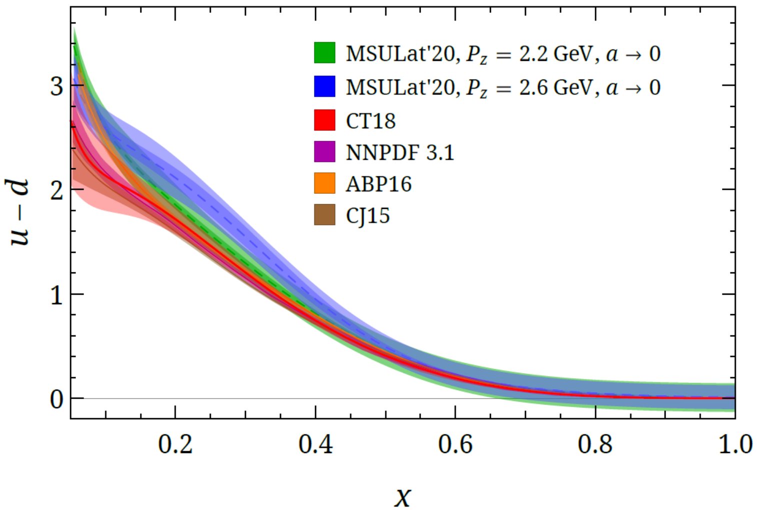
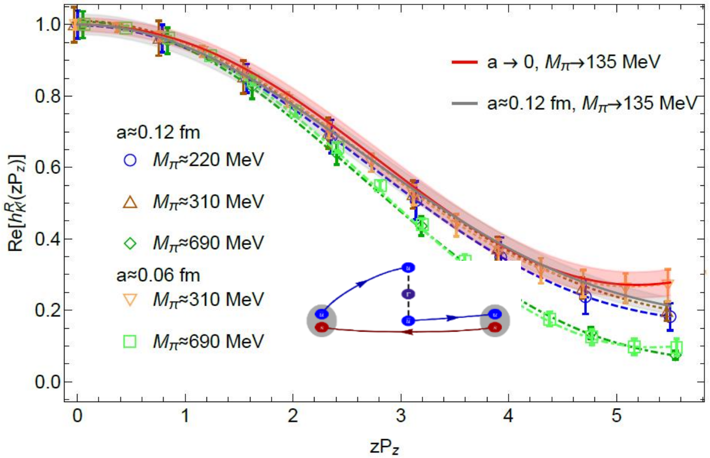
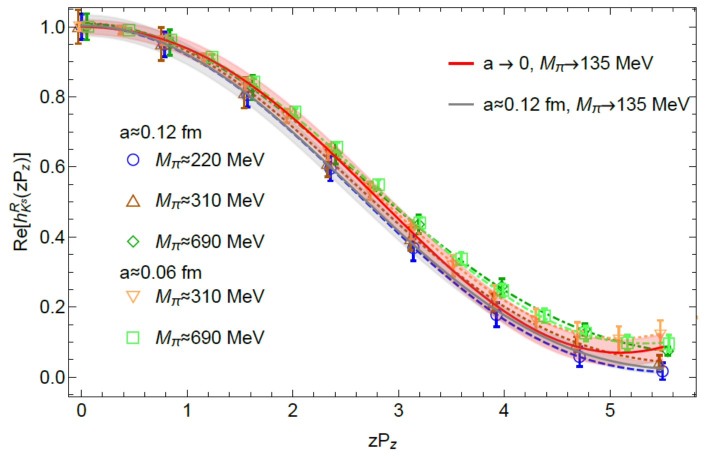




Meson Valence-quark PDFs

§ Pion/Kaon PDFs using quasi-PDF in the continuum limit





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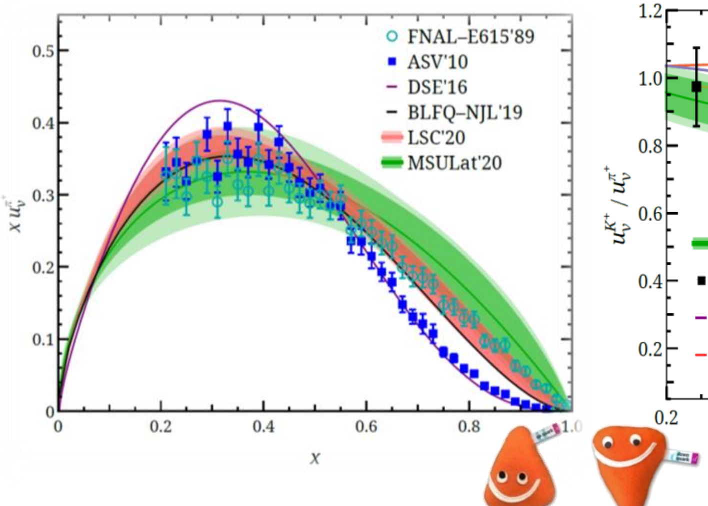
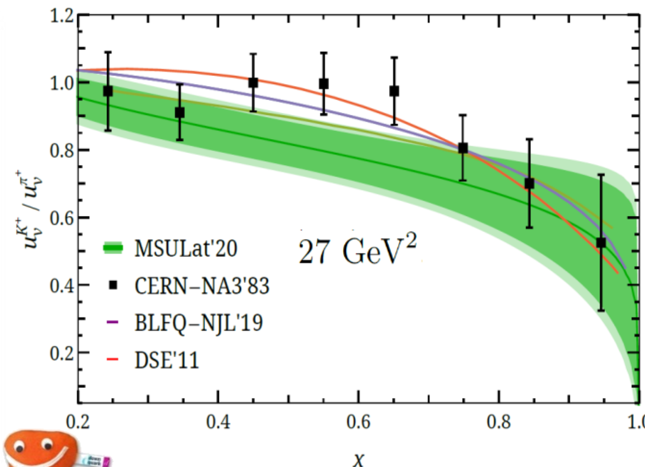
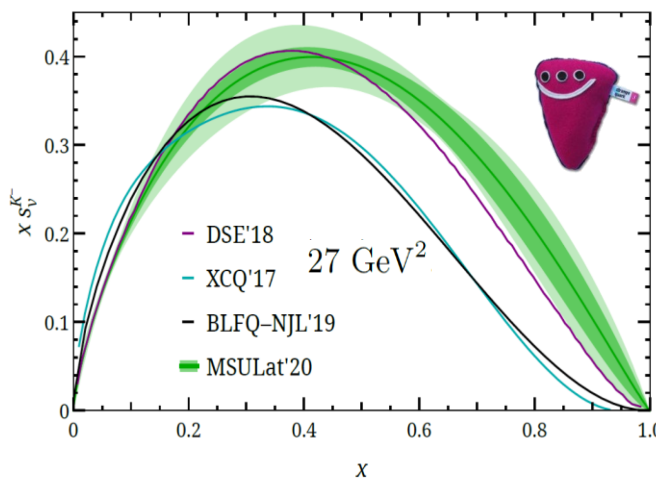
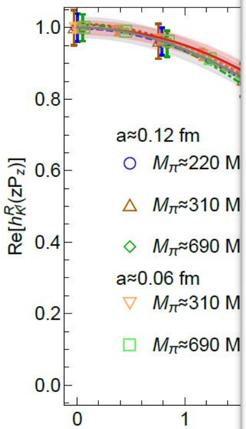






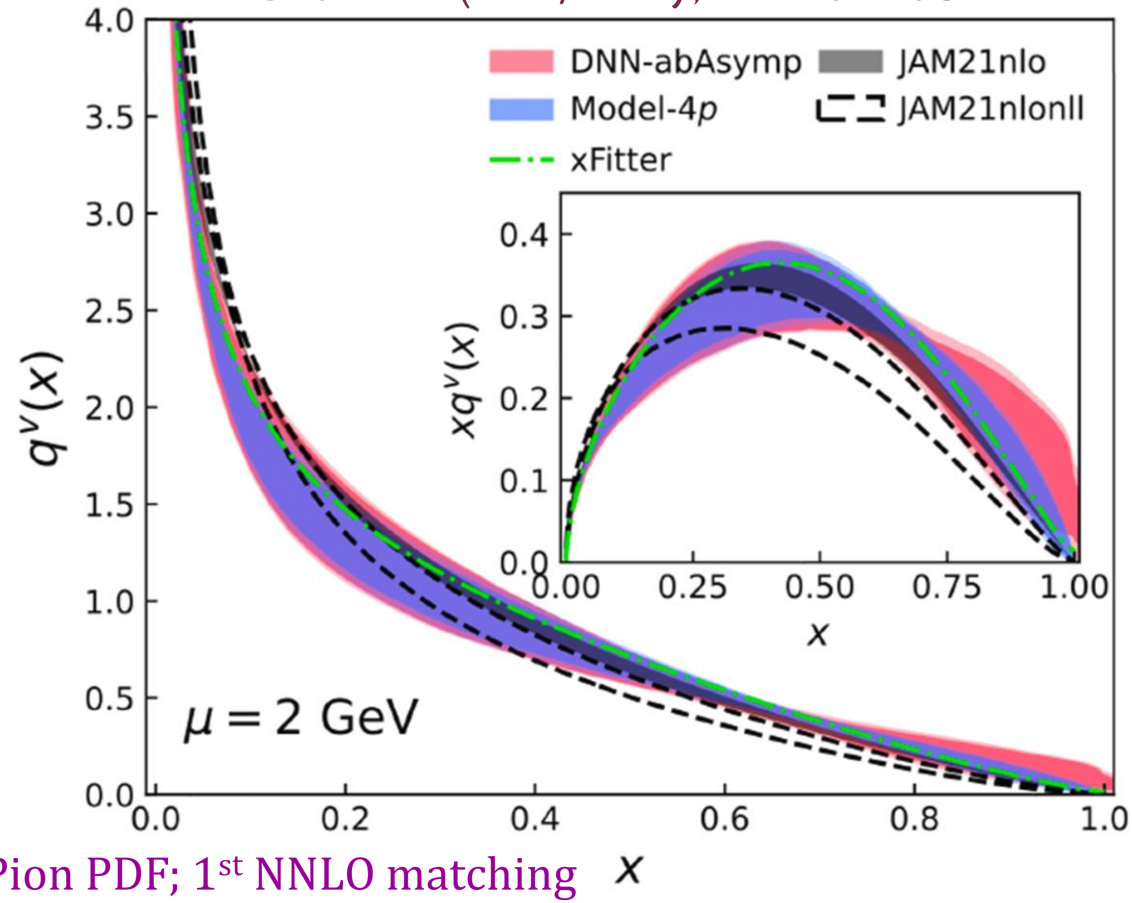
Meson Valence-quark PDFs

§ Pion/Ka… …um limit



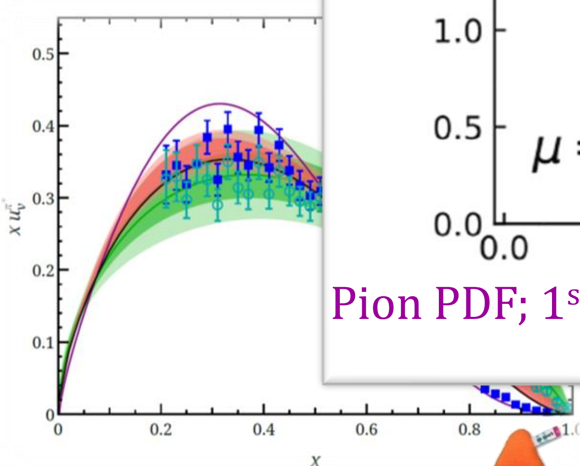
Quantities that can be calculated on the lattice

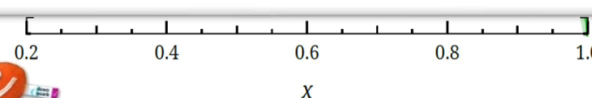
X. Gao et al (BNL/ANL), 2112.02208

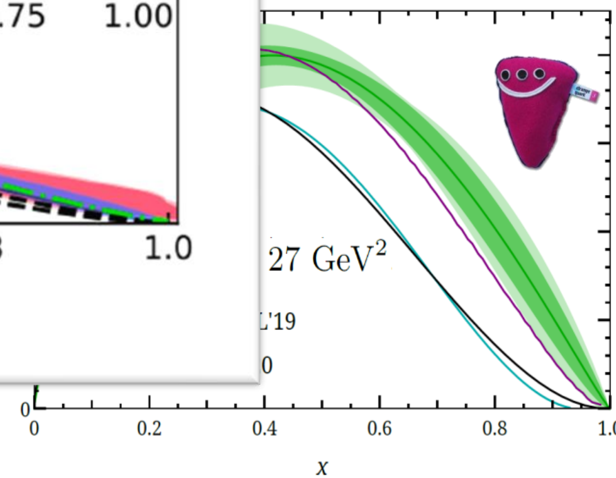


Pion PDF; 1st NNLO matching

Wanted PDFs, GPDs, etc…







Nucleon Polarized GPDs

§ Helicity GPD ($\widetilde{H}$)using quasi-PDFs at **physical pion mass**

- MSULat: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 2$ GeV

$$\widetilde{F}^q(x,\xi,t) = \int \frac{dz^-}{4\pi} e^{-ixP^+z^-} \langle p' | \bar{q}(z^-/2)\gamma^+\gamma_5 q(-z^-/2) | p \rangle$$

$$= \frac{1}{2P^+} \left[\widetilde{H}^q(x,\xi,t)\, \bar{u}(p')\,\gamma^+\gamma_5 u(p) - \widetilde{E}^q(x,\xi,t)\, \bar{u}(p')\, \frac{\gamma_5 \Delta^+}{2m} u(p) \right]$$

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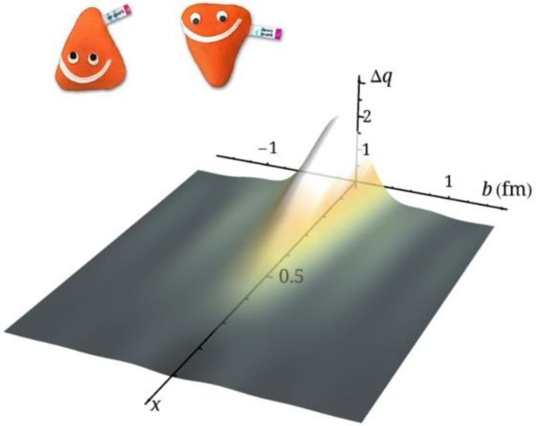
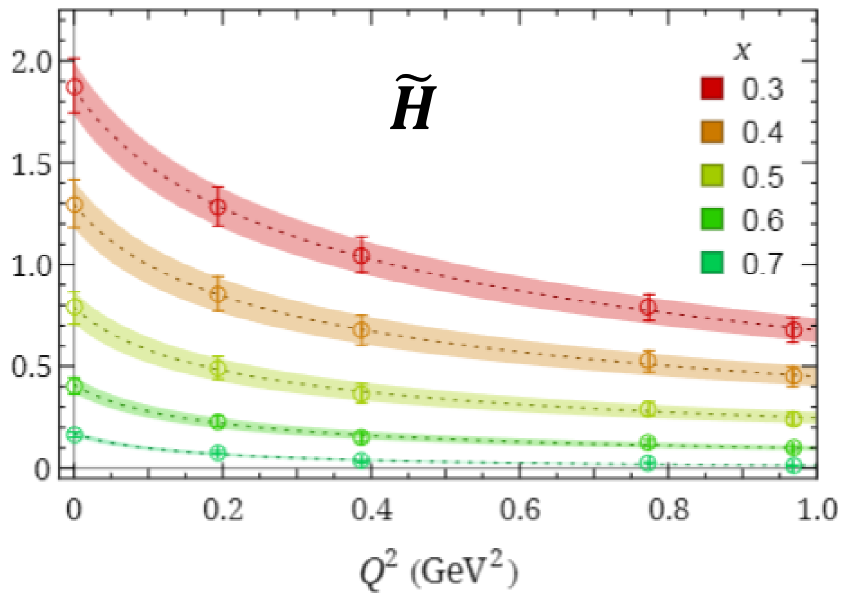
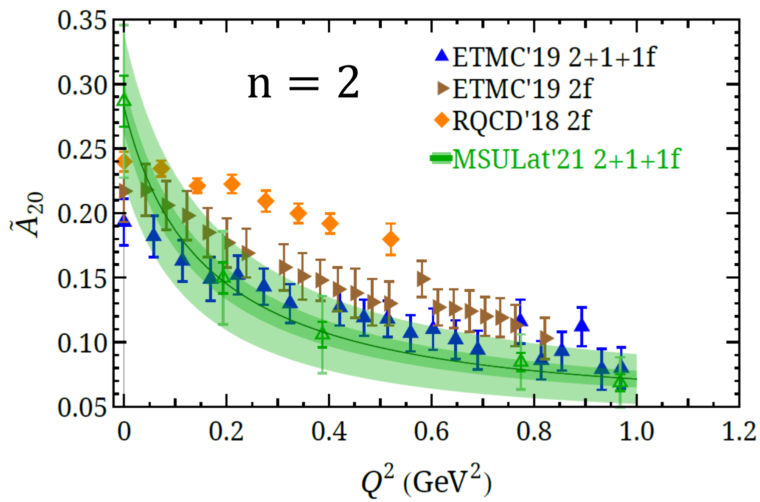
HL (MSULat), Phys.Lett.B 824 (2022) 136821





- Take the integral to form moments



Valence-Quark Pion GPD

§ Pion GPD (H^π) using quasi-PDFs at physical pion mass

- Lattice details: clover/2+1+1 HISQ
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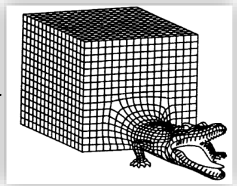
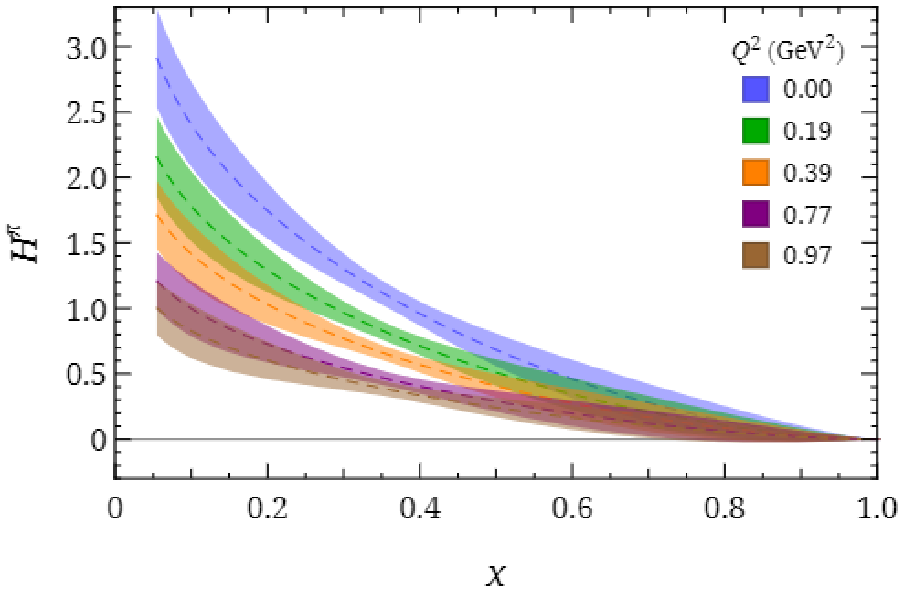
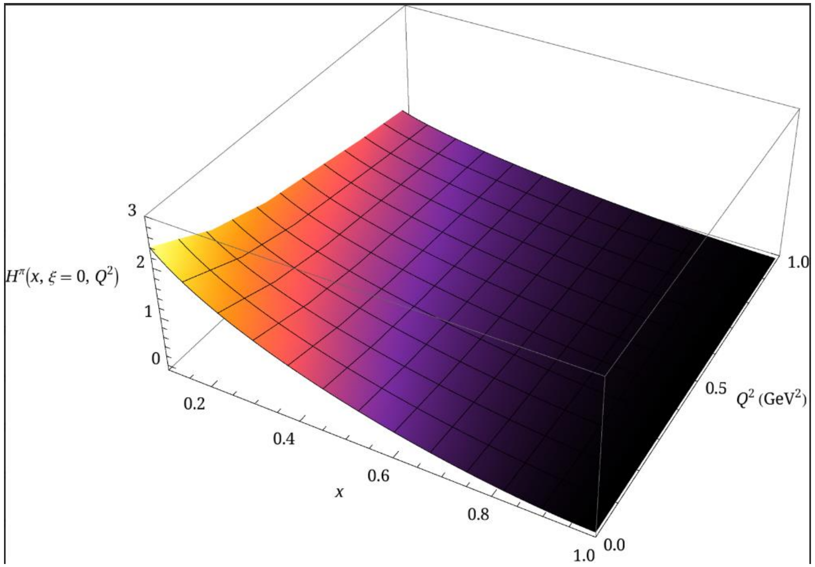


finite-volume, discretization,

MSULat, Preliminary





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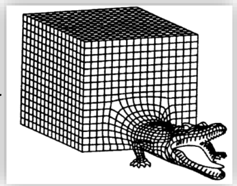
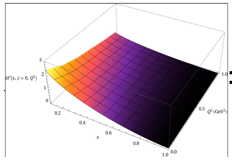


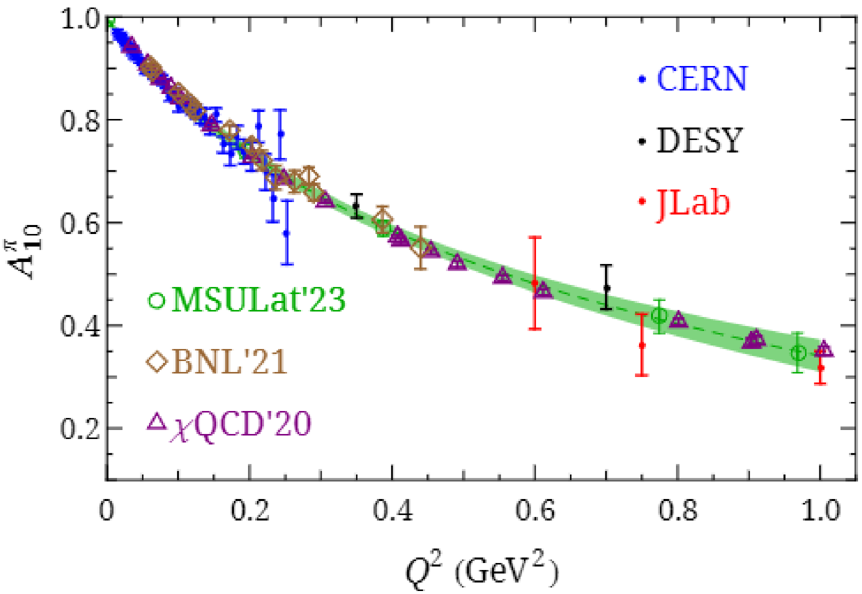
finite-volume, discretization,

- $\xi = 0$ valence-quark Pion GPD results

$$\int_{-1}^{+1} dx\, x^{n-1}$$



$$= A^{\pi}_{ni}(t)$$



MSULat, Preliminary

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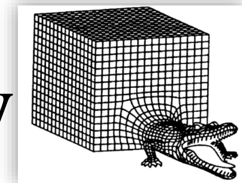

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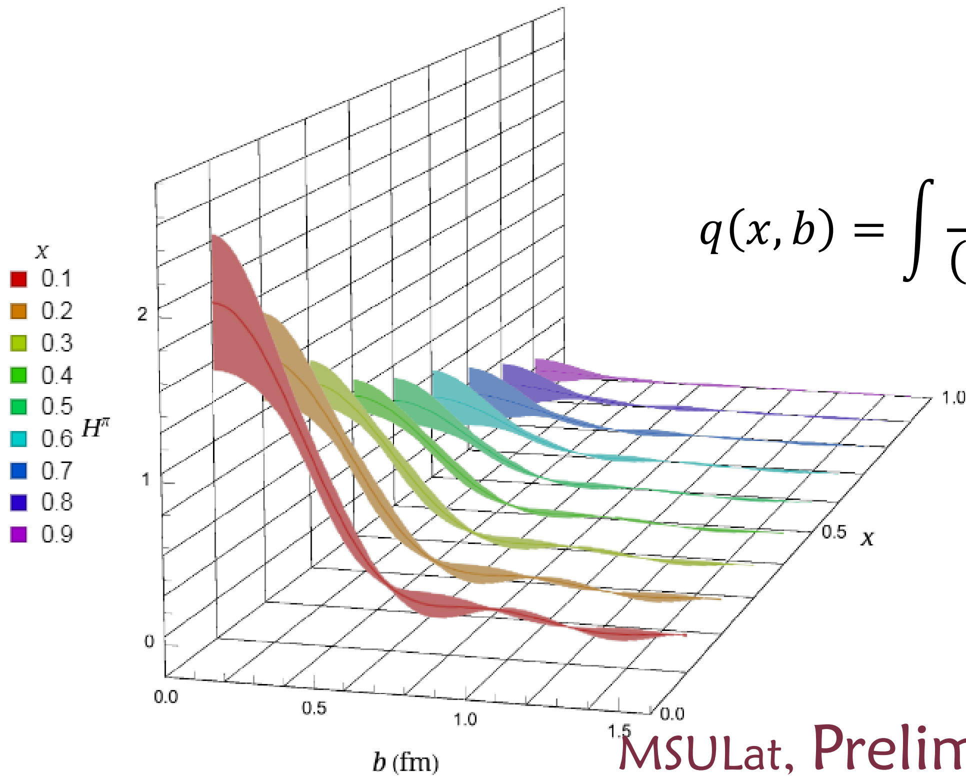
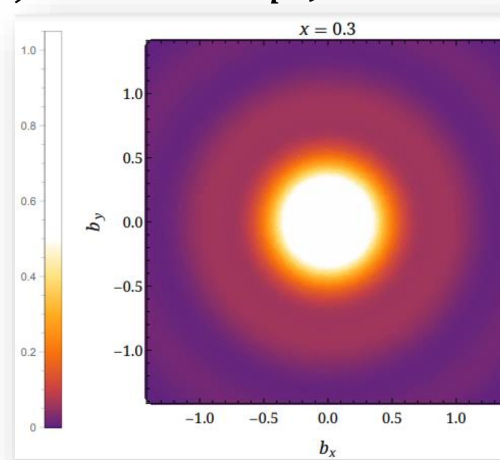


$$q(x,b) = \int \frac{d\vec{q}}{(2\pi)^2} H(x, \xi = 0, t = -\vec{q}^2) e^{i\vec{q}\cdot\vec{b}}$$



MSULat, Preliminary