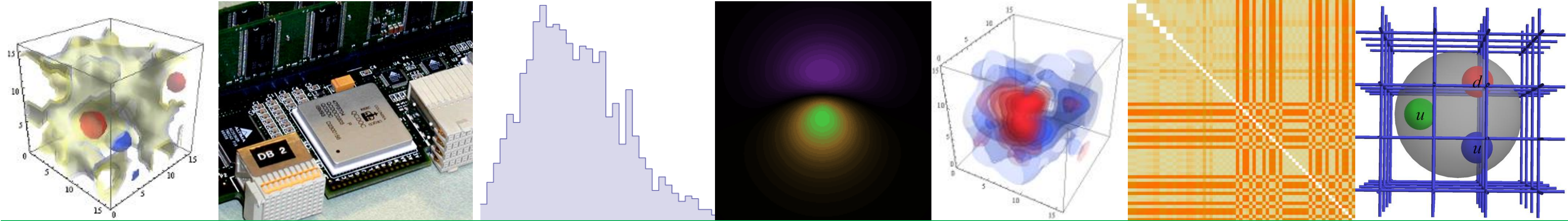

Lattice QCD for Hadronic Physics

Huey-Wen Lin

CTEQ

This work of HL is supported by the NSF under grant PHY 2209424 & 1653405, DOE under DE-SC0024053 and the Research Corporation for Science Advancement through the Cottrell Scholar Award

@LinQCD

NSF

U.S. DEPARTMENT OF ENERGY

RESEARCH CORPORATION for SCIENCE ADVANCEMENT

MICHIGAN STATE UNIVERSITY

Few Personal Facts

§ I am from a small town in Taiwan; PhD at Columbia U

Few Personal Facts

- I am from a small town in Taiwan; PhD at Columbia U
- Preferred pronoun: she/her
- Since NYC, I've lived in Virginia, Seattle, and the Bay Area
  - My biggest worry when I moved to the Midwest: snowstorms
- Like many women in physics, I married a physics PhD, and often find myself the only female in the room
  - I started a number of diversity-related activities (social events, surveys, codes of conduct, etc.)

Women in LatticeQCD

APS physics IDEA
Inclusion, Diversity, and Equity Alliance

Few Personal Facts

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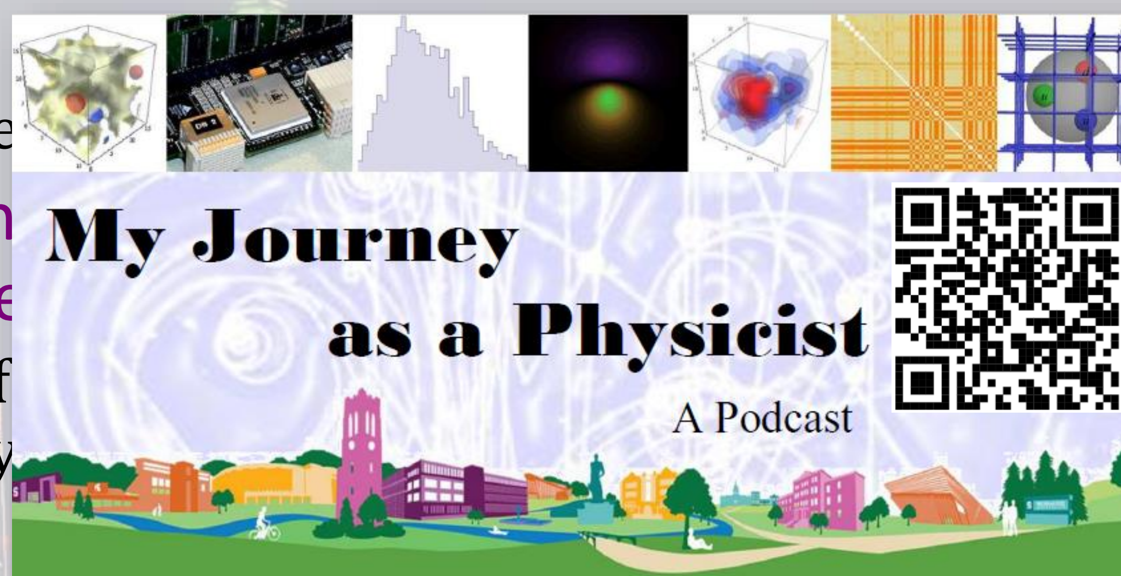


- My research is in **quantum chromodynamics (QCD)**
  - I use high-performance supercomputers to study the properties of the quarks and gluons in the hadrons (nucleon, pion, …)

My Journey as a Physicist

Home | Season 3 (LRP) | Season 2 (Snowmass) | Season 1 (LQCD) | The Team

My Journey as a Physicist

Season 3: Nuclear-Science Advisory Committee Long Range Plan (LRP)

This season's interviews were conducted by Bill Good and Kinza Hasan, and edited by Kiran Sakorikar and Esther Cohen-Lin.

Episode 1 (flyer) (transcript)

Prof. Gail Dodge (she/her)

Old Dominion University

Prof. Gail Dodge (she/her): Season 3 Episode 1

My Journey as a Physicist • By Bryan Stanley & Huey-Wen Lin • Oct 20, 2022

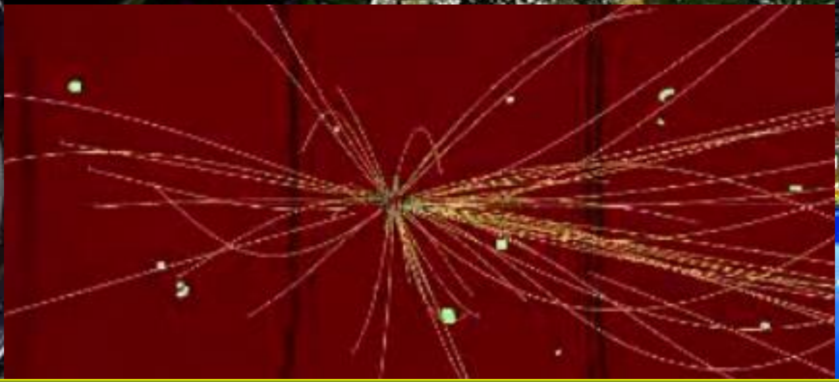
00:00 | Share | 28:01

Probing the Heart of Matter

§ Our natural instinct: Smash!

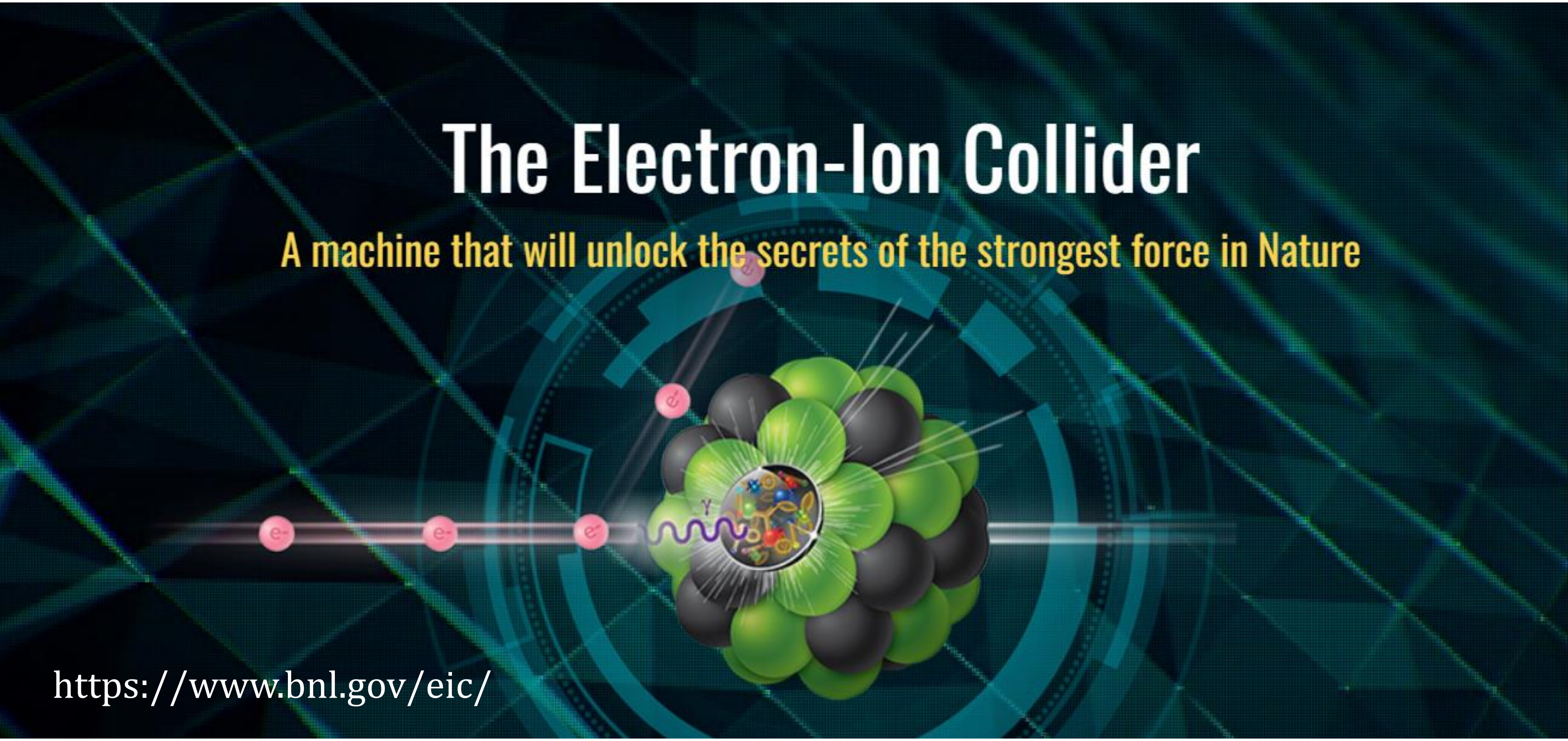
Probing the Heart of Matter

§ LHC strikes out onto the high-energy frontier (13 TeV)



Probing the Heart of Matter

§ The Electron-Ion Collider (EIC): The Ultimate QCD Microscope



EIC White Paper, 1212.1701; EIC Yellow Report (2103.05419);
The Present and Future of QCD (2303.02579)

Probing the Heart of Matter

§ Calculations done using world's largest supercomputers

- Many millions of CPU/GPU hours

3-Day Plan

§ Lecture Plan (Wed)

- Why lattice QCD?
- Anatomy of a lattice calculation
- Spectroscopy example

§ Tutorials (hands-on exercises)

- Work in small groups (4ish students)
- With Python Jupyter notebooks
  - Jackknife analysis
  - Calculating proton masses

3-Day Plan

§ Lecture Plan (Thur)

- Lattice calculation of hadron structure
  - Charges, moments and form factors
  - Proton spin/mass decomposition

§ Tutorials (hands-on exercises)

- Work in small groups (4-ish students)
- With Python Jupyter notebooks
  - Extracting nucleon charges

3-Day Plan

§ Lecture Plan (Fri)

- *x*-dependent parton distributions
  - Recent lattice PDFs progress
  - Applications to generalized parton distributions
  - Future prospects and challenges

From Thy Standard Model to Lattice QCD

The Standard Model and QCD

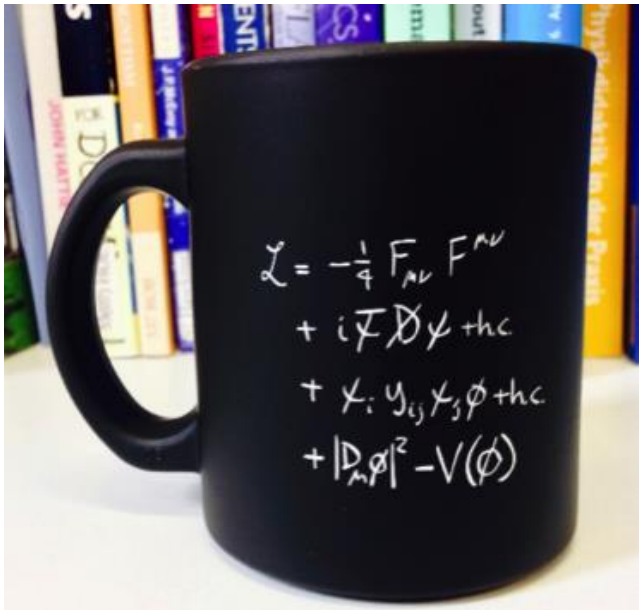


Image credit: CERN

- Quantum chromodynamics (QCD)
  - The strong interactions of quarks and gluons (SU(3) gauge)

Image source: https://www.particlezoo.net

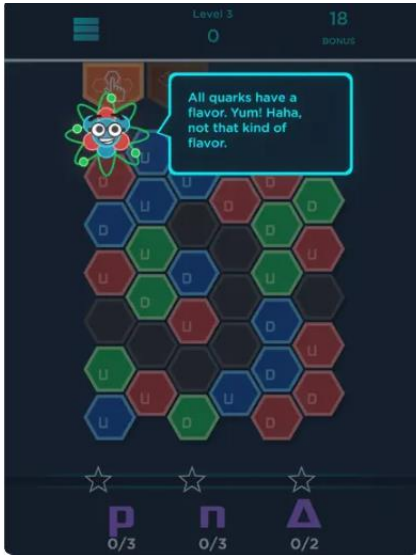
Learn QCD on Your Phone!

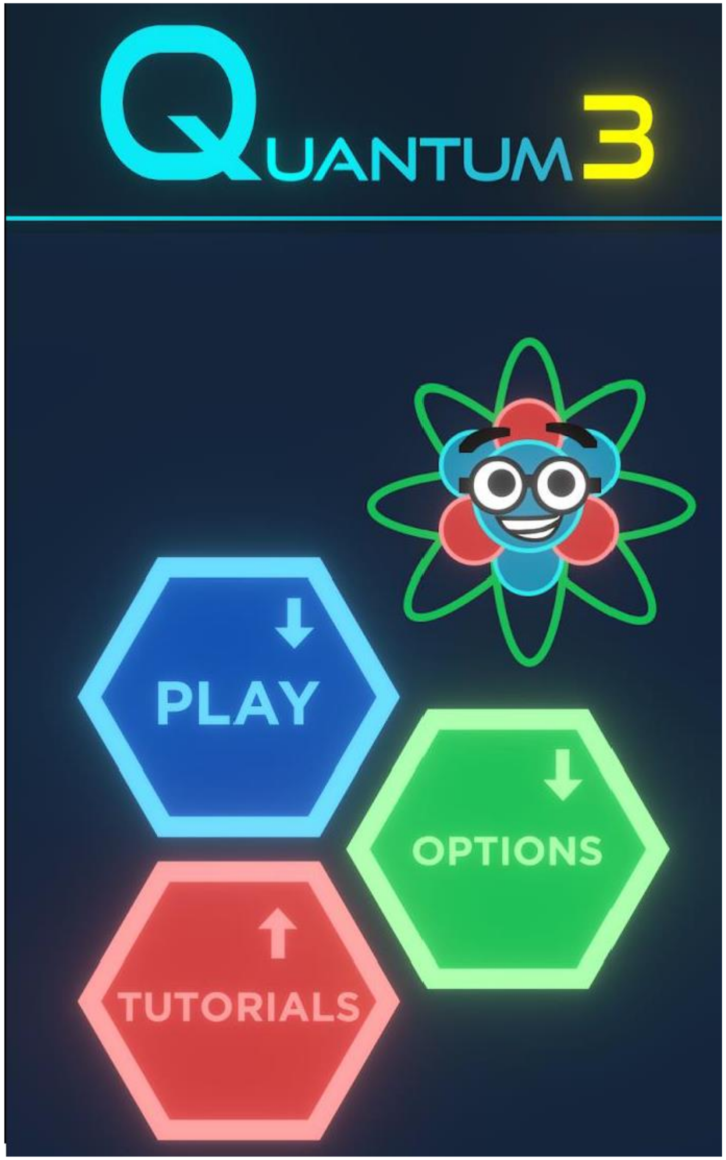
https://msutoday.msu.edu/news/2019/new-game-app-gets-young-girls-involved-in-stem/

Learn QCD on Your Phone!

- Learn QCD on your phone

[Google Play Store](#) [Apple Appstore](#)





Supported by the NSF under grant PHY 1653405

The Color Force

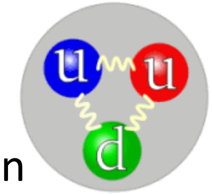
§ Quantum chromodynamics (QCD) describes strong interactions of *quarks* and *gluons* (SU(3) gauge)

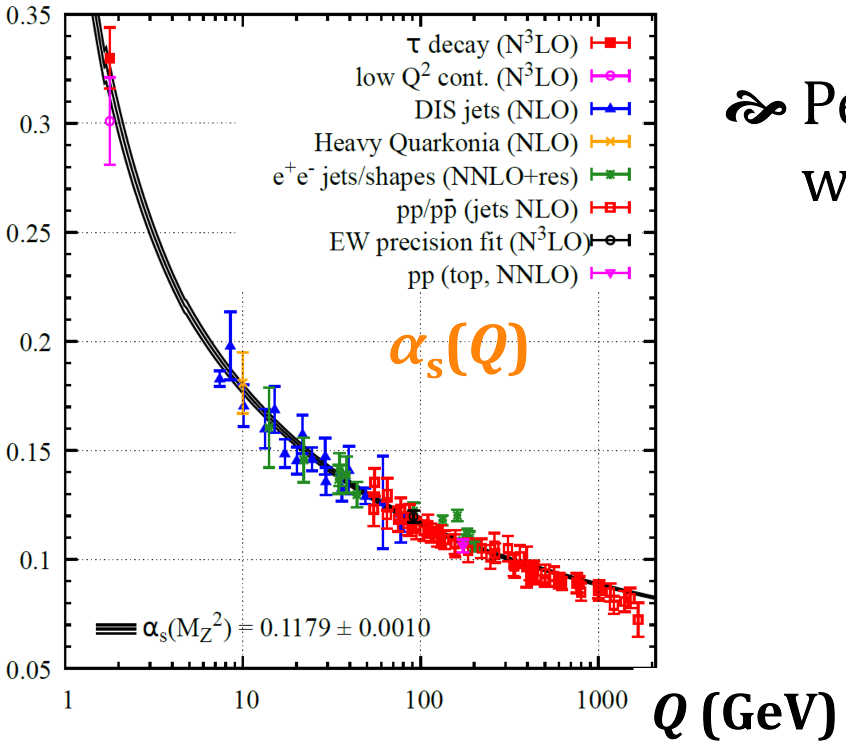
- "Confinement"
  no free quarks allowed

Hadrons

Pion

Proton



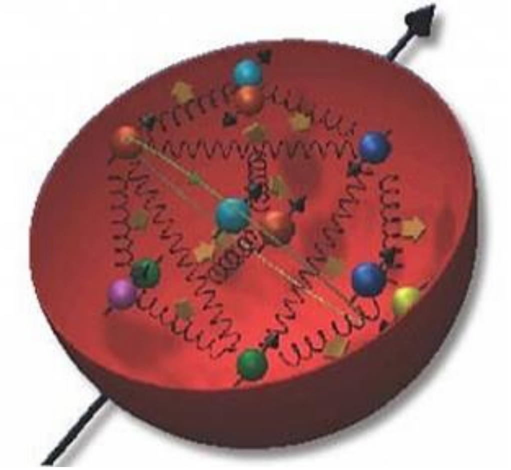


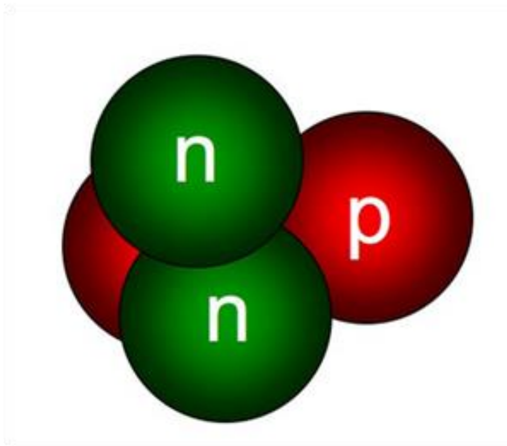
- Perturbation theory (like QED) works well at high energies

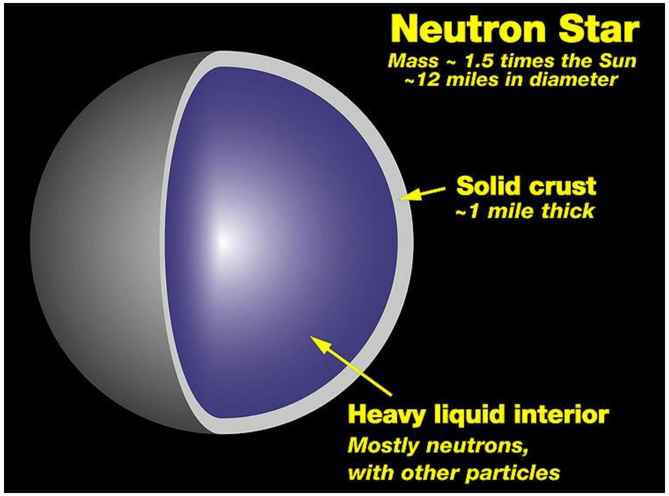
The Nobel Prize in Physics 2004

Wide-Scale Applications

§ What can we learn from it?







10^{-15} m

10^{4} m

Origin of proton mass and spin, imaging of proton, Parton distribution functions For new-physics searches, ...

Nuclei and why we exist

Neutron matter
How they evolve

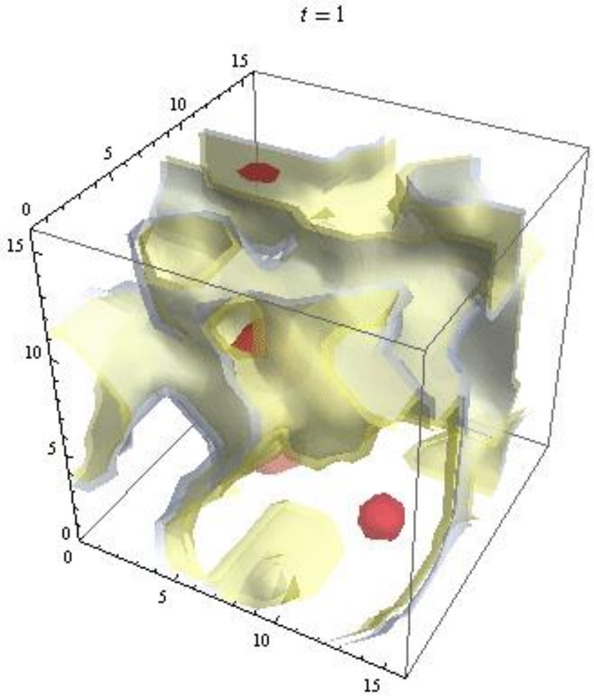
§ HOWEVER...

Difficulties at Low Energy

§ Even just the vacuum of QCD is complicated

Classical

QCD

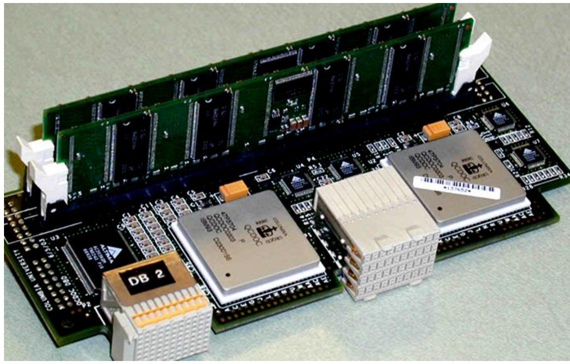


Difficulties at Low Energy

- Strong interactions make analytic calculation impossible

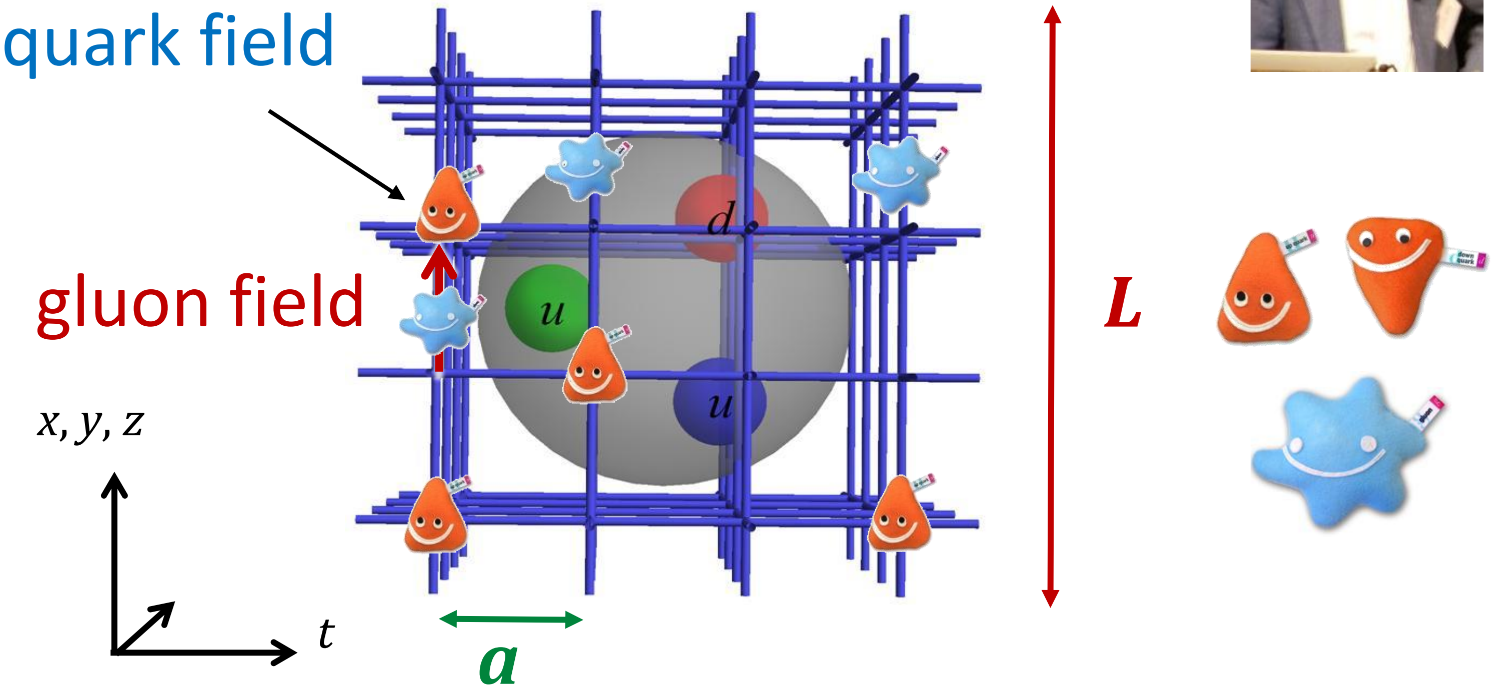
- Direct QCD calculation is desired
  → Lattice QCD

→ 0100101010 10111010... →



Wilson Comes to Rescue!

- Lattice gauge theory was proposed in the 1970s by Wilson



- Recover physical limit

$$m_q \to m_q^{\text{phys}}, \boldsymbol{a} \to \mathbf{0}, \boldsymbol{L} \to \infty$$

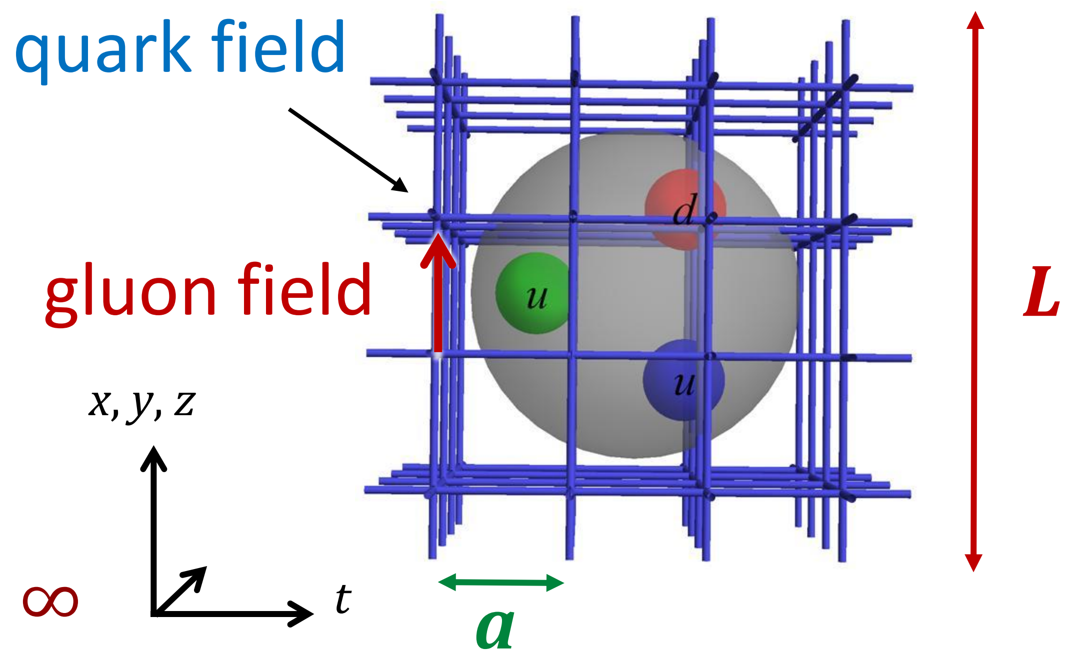
Introducing the Lattice

- Lattice QCD is an ideal theoretical tool for investigating strong-coupling regime of quantum field theories
- Physical observables are calculated from the path integral

$$\langle 0|O(\bar{\psi},\psi,A)|0\rangle = \frac{1}{Z}\int \mathcal{D}A\,\mathcal{D}\bar{\psi}\,\mathcal{D}\psi\, e^{iS(\bar{\psi},\psi,A)} O(\bar{\psi},\psi,A)$$

in **Euclidian** space

- Quark mass parameter (described by m_π)
- Impose a UV cutoff
  discretize spacetime
- Impose an infrared cutoff
  finite volume



- Recover physical limit

$$m_\pi \to m_\pi^{\text{phys}},\ a \to 0,\ L \to \infty$$

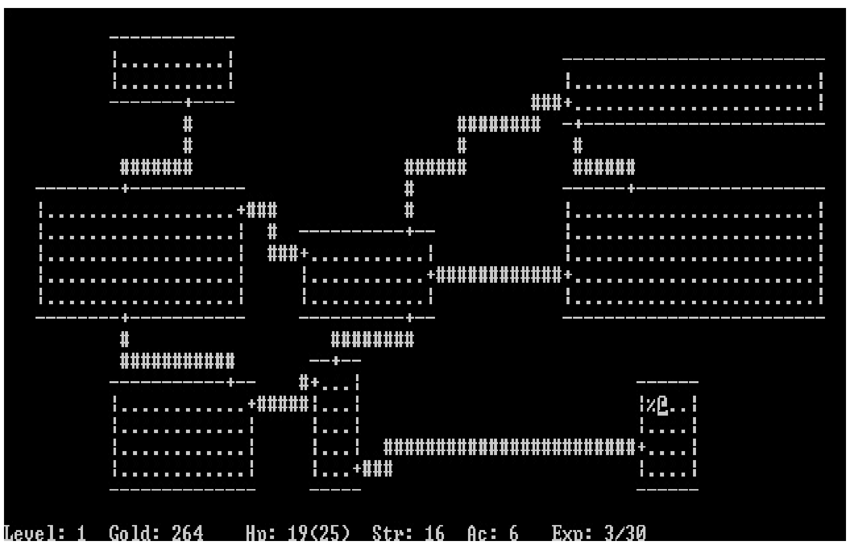
Are We There Yet?

§ Lattice gauge theory was proposed in the 1970s by Wilson

- Why haven't we solved QCD yet?

§ Progress is limited by computational resources

1980s



Today

§ Greatly assisted by advances in algorithms

- Physical pion-mass ensembles are not uncommon!

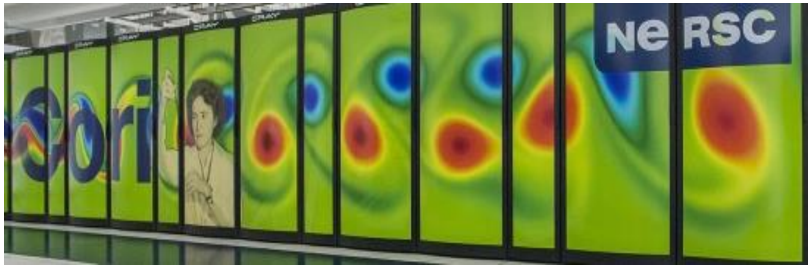
Anatomy of a Lattice Calculation

1. Hardware (computational resources) and Software (Code)

If you are young and energetic…

FNAL pi0 Cluster (USQCD SciDAC)

Stampede2@TACC (NSF ACCESS)



Cori@NERSC (DOE ERCAP, ALCC)

Anatomy of a Lattice Calculation

1. Hardware (computational resources) and Software (Code)

US Lattice Quantum Chromodynamics

USQCD has developed a suite of software enabling lattice QCD computations to be peformed with high performance across a variety of architectures, including both custom facilities and commodity clusters. This software is made up of software library modules that can be re-used by higher level applications. The approximate organisation of the packages into layers is depicted below (we omit here 3rd party packages or packages auxiliary to another package). Please click on the plaquettes below to find project web pages of the individual software modules, as well as to complete lattice QCD packages which use them.

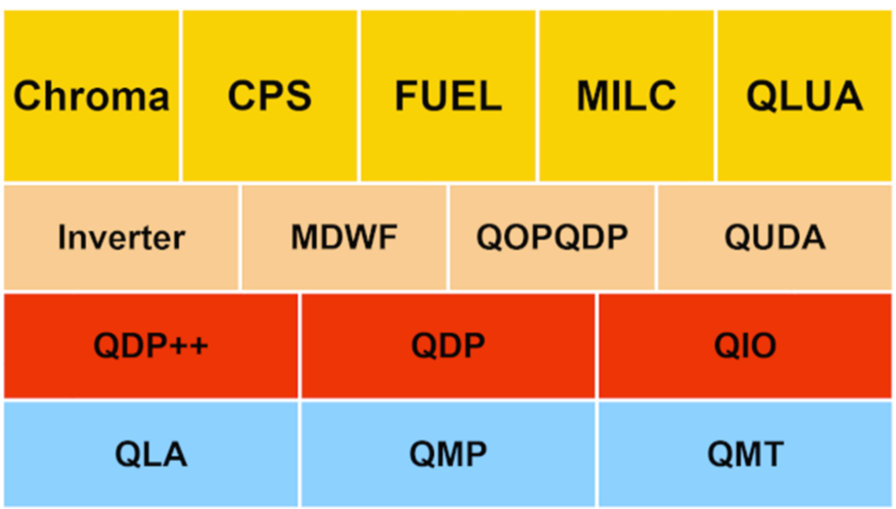


Figure 1: The SciDAC Layers and the software module architecture.

- The lattice community shares much software
  - for example, many in the US lattice community use code from http://usqcd-software.github.io

Anatomy of a Lattice Calculation

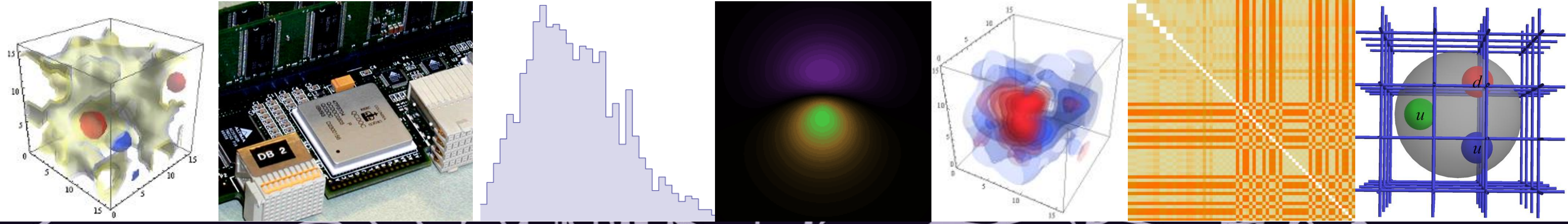
1. Hardware (computational resources) and Software (Code)

Online tutorials available:

http://www.int.washington.edu/PROGRAMS/12-2c/

Week 3 (Aug. 20–24, 2012)

- "Hadron Structure", James Zanotti (University of Adelaide)
- "Lattice QCD+QED", Taku Izubuchi (BNL)
- "Computational Lattice QCD", Balint Joo (Thomas Jefferson Lab)
  - Exercises: seattle_tut.tar.gz (2012 Aug 24)
  - Code: package-int.tar.gz (110 MB)
  - Lecture 1: Slides Video
  - Lecture 2: Slides Video
  - Lecture 3: Slides Video
  - Lecture 4: Slides Video
  - Lecture 5: Slides Video
- "Extreme Computing Trilogy: Nuclear Physics", Martin Savage (University of Washington)
- "Cold Atoms and Unitary Fermi Gas", Michael M. Forbes (INT)
- "Introduction to GPU Computing", Mike Clark (NVIDIA)
- "Introduction to QUDA — GPU Computing for LQCD", Mike Clark (NVIDIA)
- "Extreme Computing Trilogy: Infrastructure", Kenneth Roche (PNNL)



Toturials @ INT Summer School on Problem Solving in Lattice QCD

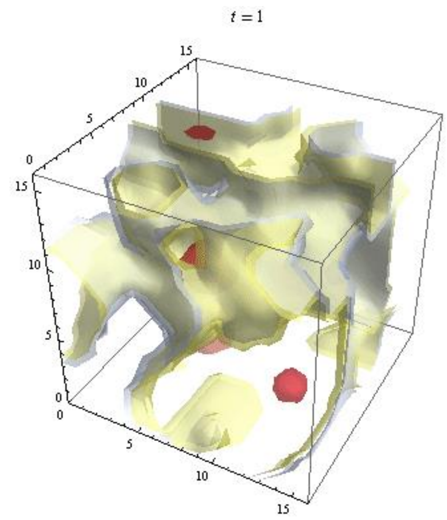
Lecture playlist:

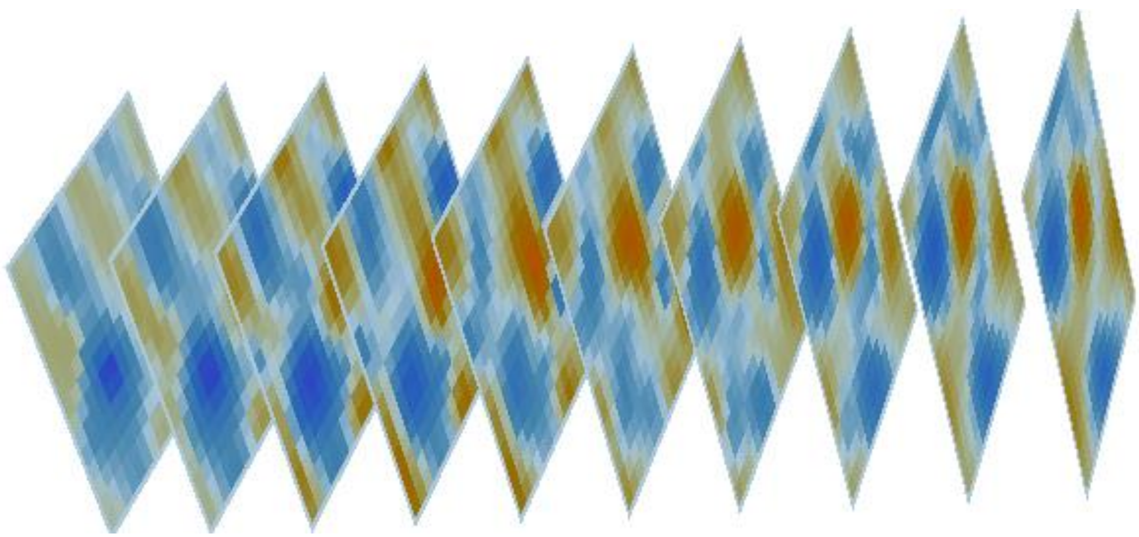
https://www.youtube.com/watch?v=FsN34Sm6Ldo&list=PLDi14w7i5C3Bm3U1IQ4n596UZQhOpr1Cx

Anatomy of a Lattice Calculation

1. Hardware (computational resources) and Software (Code)

2. Some QCD Vacuum (gauge configurations)





Anatomy of a Lattice Calculation

1. Hardware (computational resources) and Software (Code)

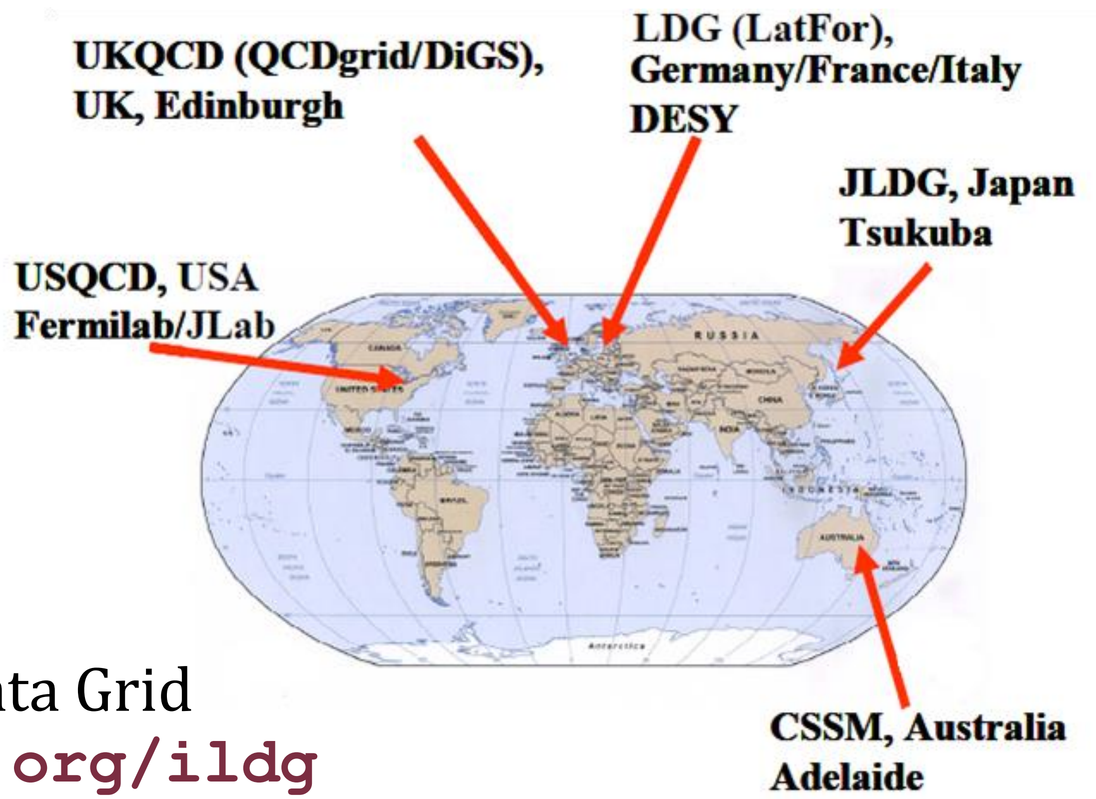
2. Some QCD Vacuum (gauge configurations)

- WWW (ILDG)

ILDG

International Lattice Data Grid

http://www.usqcd.org/ildg

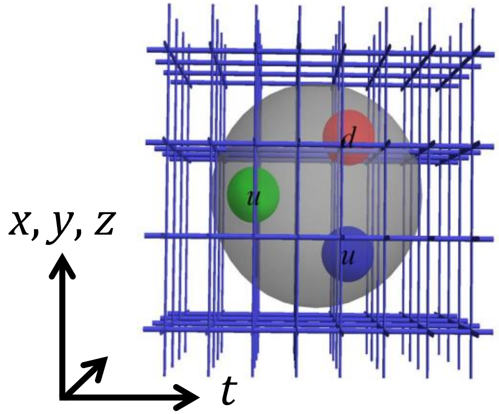


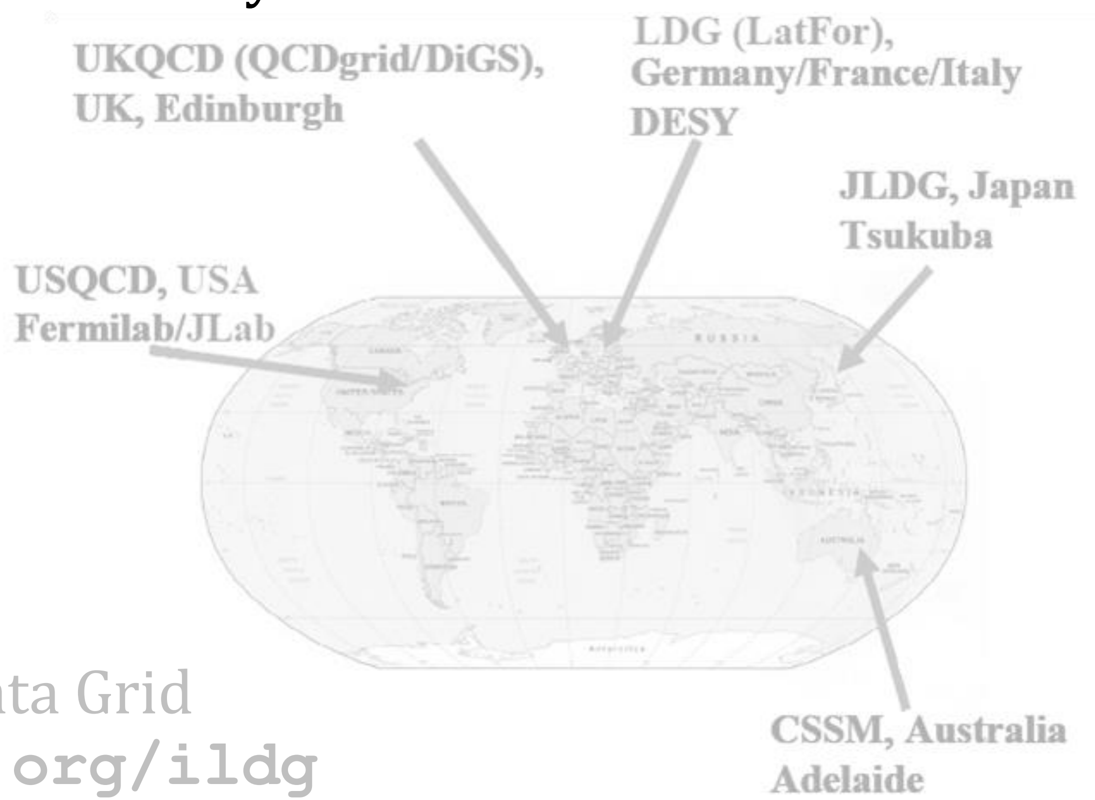
Anatomy of a Lattice Calculation

1. Hardware (computational resources) and Software (Code)

2. Some QCD Vacuum (gauge configurations)

- WWW (ILDG) or generate it yourself 0810.3588 350+





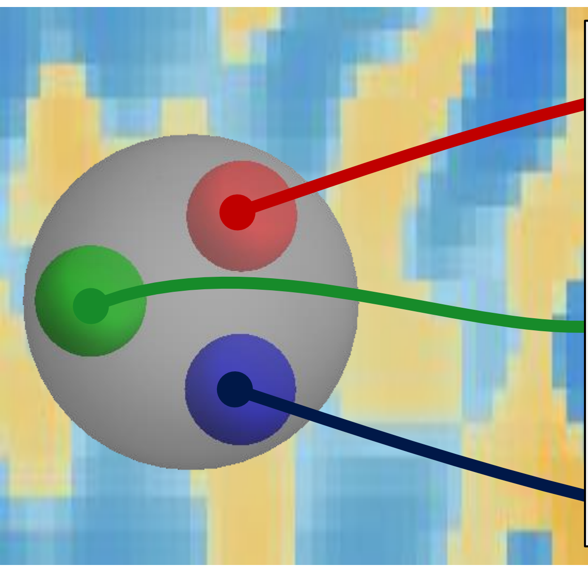
ILDG

International Lattice Data Grid

http://www.usqcd.org/ildg

Anatomy of a Lattice Calculation

1. Hardware (computational resources) and Software (Code)
2. Some QCD Vacuum (gauge configurations)
3. Correlators (hadronic observables)
   - Invert Dirac operator matrix (rank 10^{12})
   - Combine using color, spin and momentum into hadrons



§ Simple hadron operators (not unique)

- $\pi^+(x) = \bar{u}^a(x)\gamma_5 d^a(x)$
- $\rho^+(x) = \bar{u}^a(x)\gamma_\mu d^a(x)$
- $p_\delta(x) = \left(\bar{u}^a(x)C\gamma_5 d^b(x)\right) u_\delta^c \epsilon_{abc}$
- $\Delta^{++}_{\mu,\delta}(x) = \left(\bar{u}^a(x)C\gamma_\mu u^b(x)\right) u_\delta^c \epsilon_{abc}$

Time

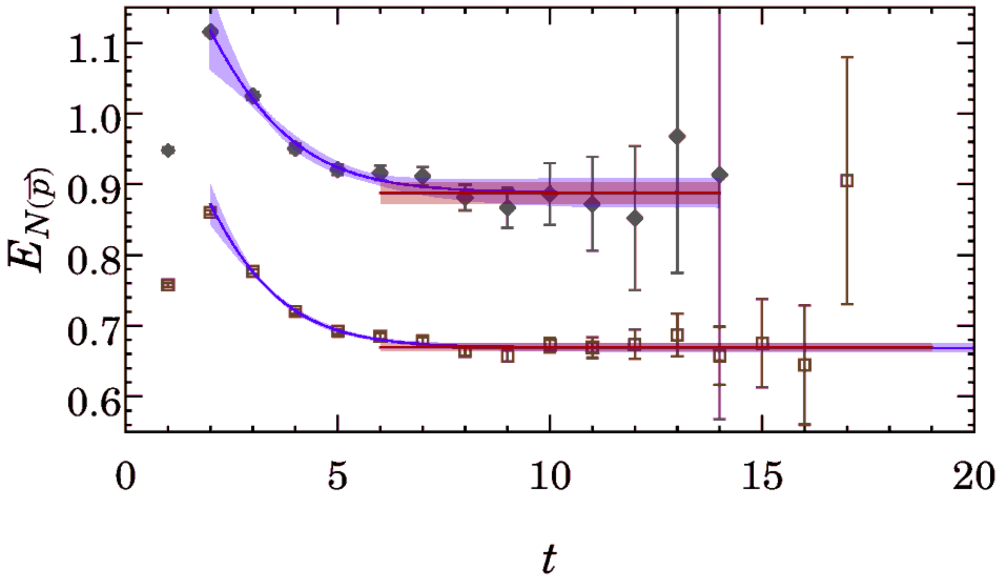
Anatomy of a Lattice Calculation

Exercise: 1) Derive the time-dependent correlator formula. How does it differ between the pion and nucleon?
2) Are there other ways to define M_{eff}? Can you find an M_{eff} that gives the mass of the first radially excited state?

4. Analysis (extraction of masses or couplings)

$$\langle J_N J_N \rangle \propto \sum_n \langle J_N | n \rangle \langle n | J_N \rangle e^{-E_n t}$$

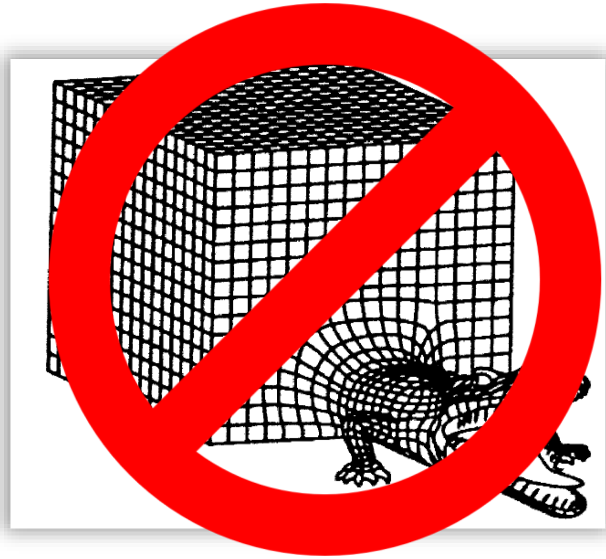
$$M_{\text{eff}} = -\log\left(\frac{\langle J_N J_N \rangle (t+1)}{\langle J_N J_N \rangle (t)}\right)$$



Anatomy of a Lattice Calculation

1. Hardware (computational resources) and Software (Code)
2. Some QCD Vacuum (gauge configurations)
3. Correlators (hadronic observables)
4. Analysis (extraction of masses or couplings)
5. Systematic Uncertainty (nonzero *a*, finite *L*, etc.)

- Extrapolation to the continuum limit
  $(m_\pi \to m_\pi^{\text{phys}}, L \to \infty, a \to 0)$



Anatomy of a Lattice Calculation

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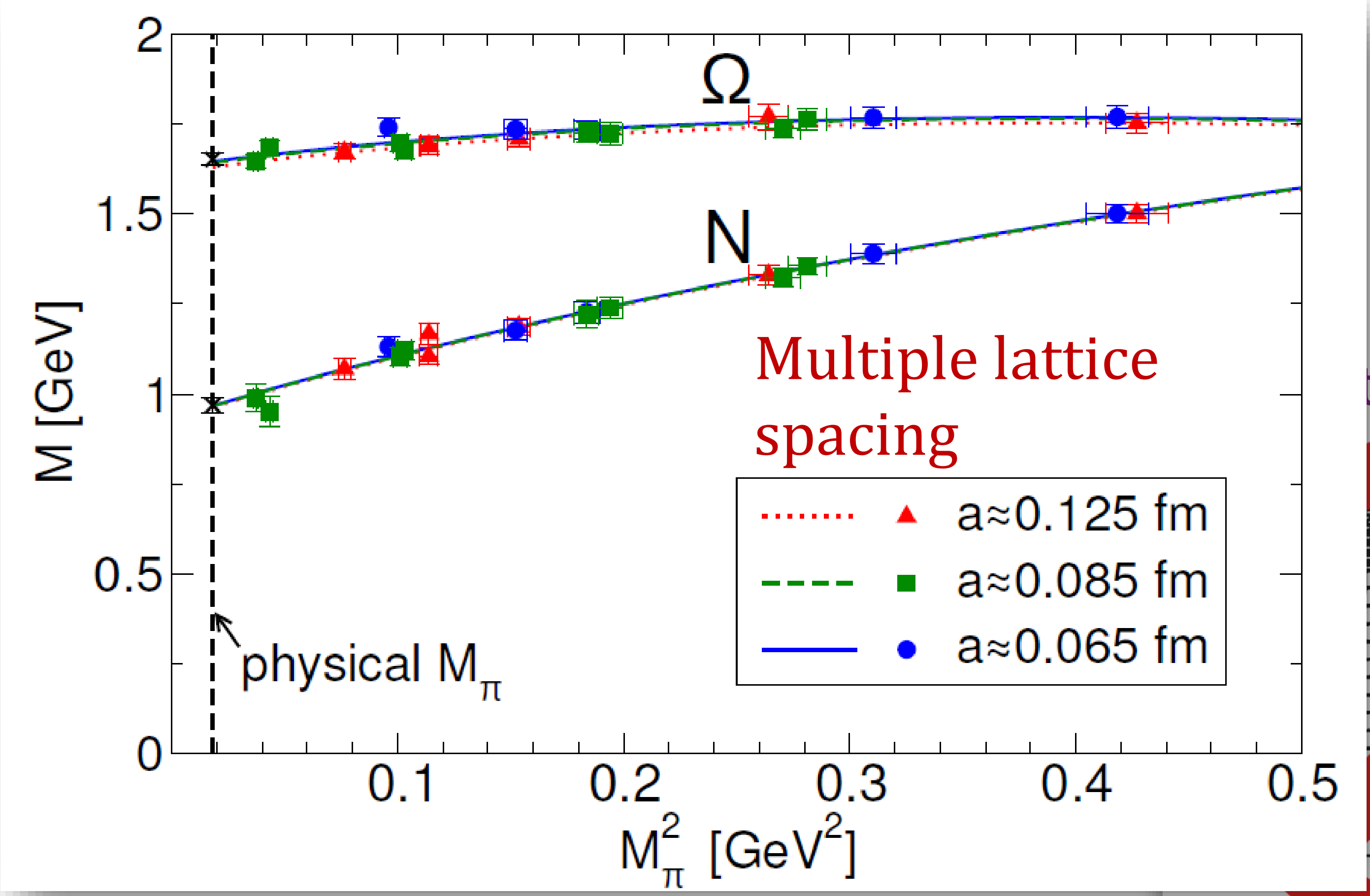
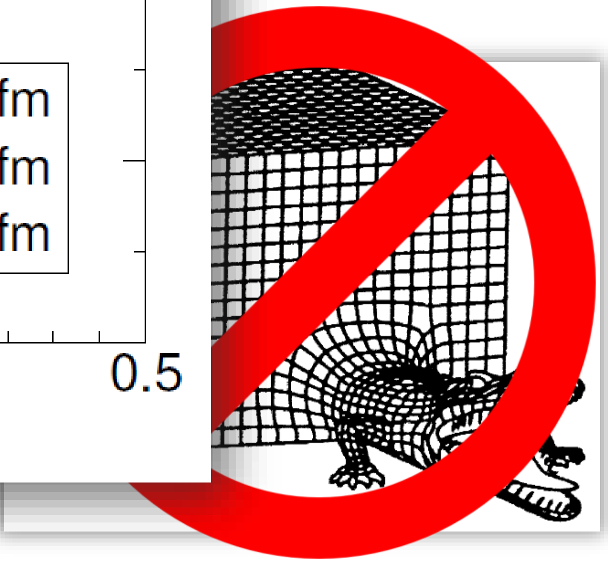


tc.)

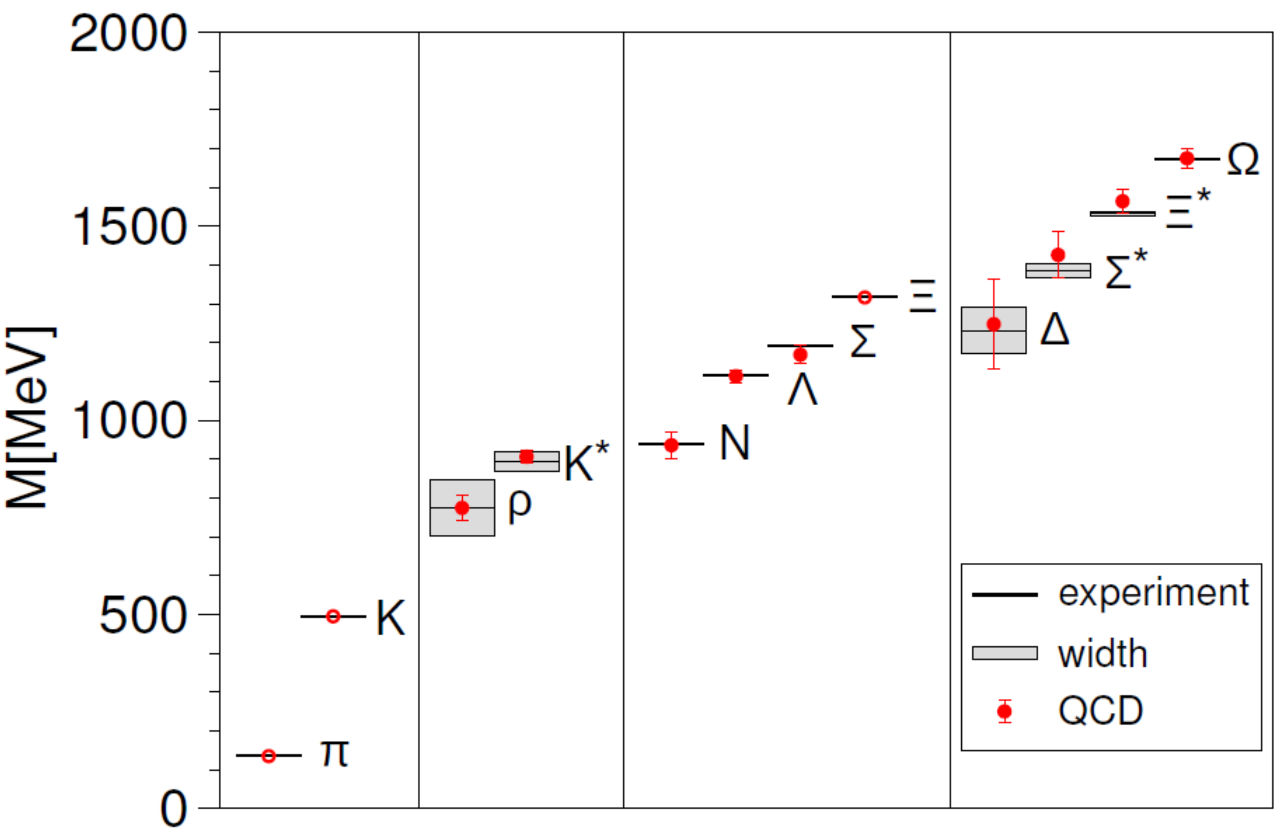


BMW Collaboration, Science (2008)

Lattice in the News

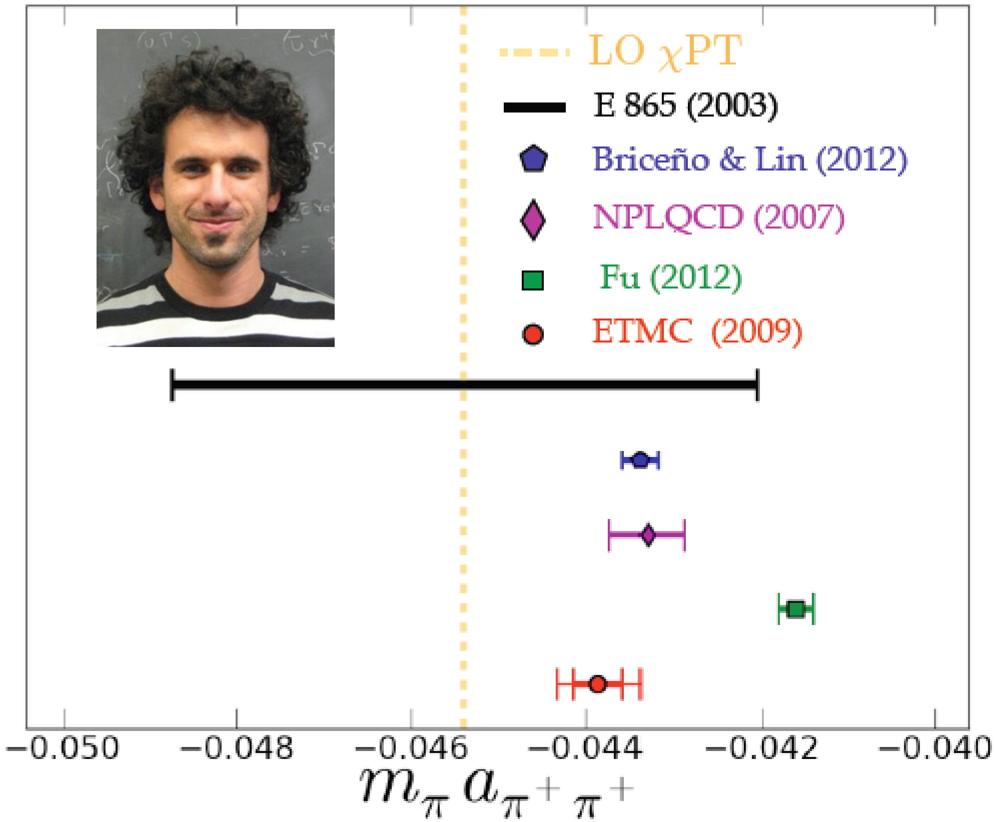
§ Post-dictions of well known quantities

- Example: BMW Collaboration, *Science* 2008



Successful Examples

- § Provide higher precision for known quantities
- § Make a lot of mass predictions



Successful Examples

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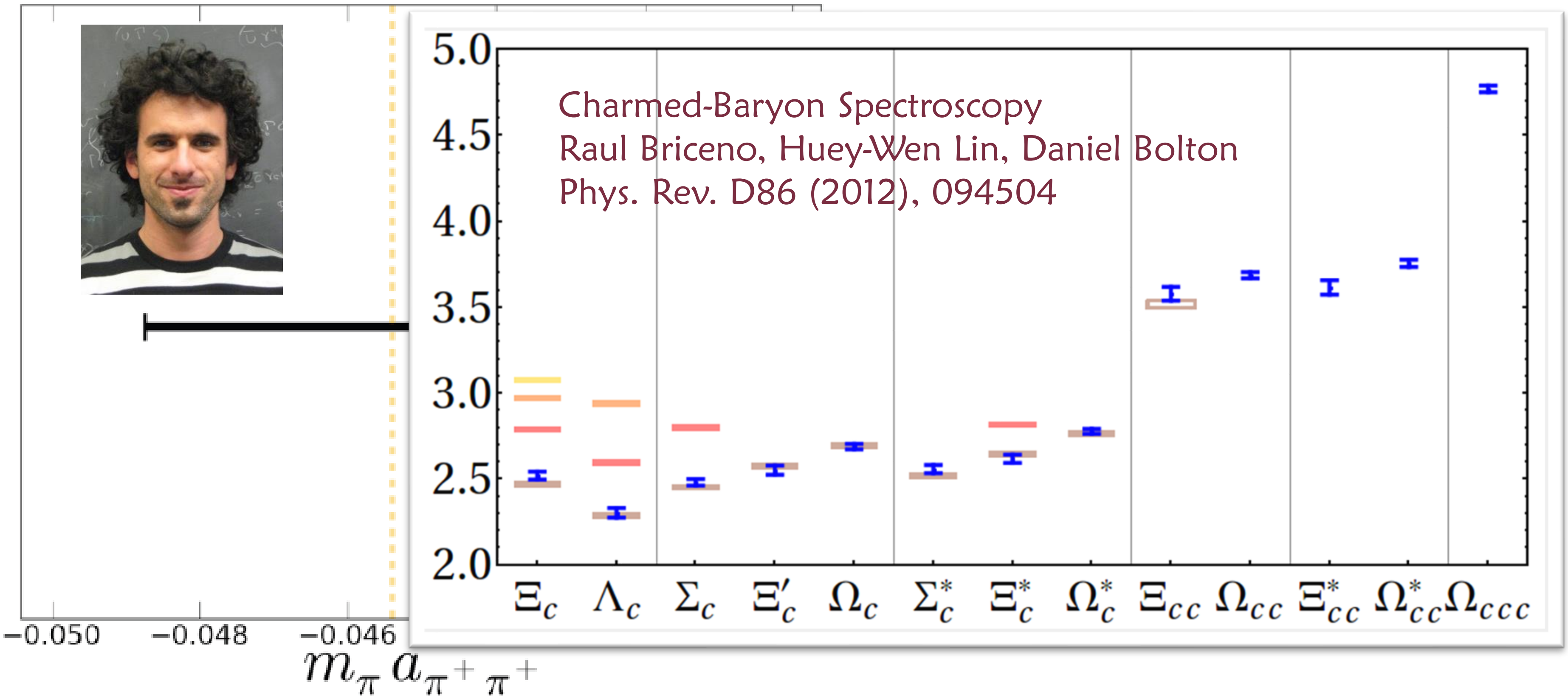
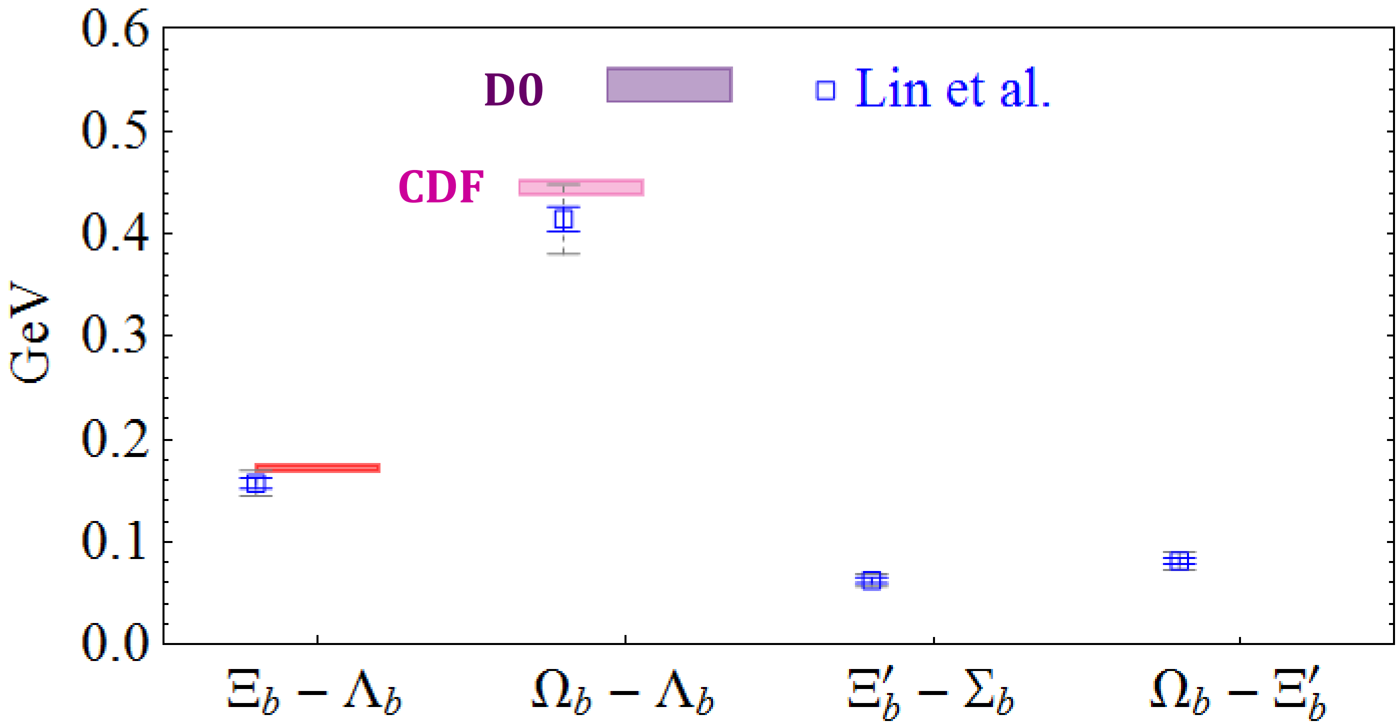


Bottom Baryons

§ Inconsistency in the CDF and DØ results for Ω_b mass

- Our Ω_b agrees with the CDF result



HWL, S. D. Cohen, N. Mathur, K. Orginos, Phys. Rev. D80, 054027 (2009)

Bottom Baryons

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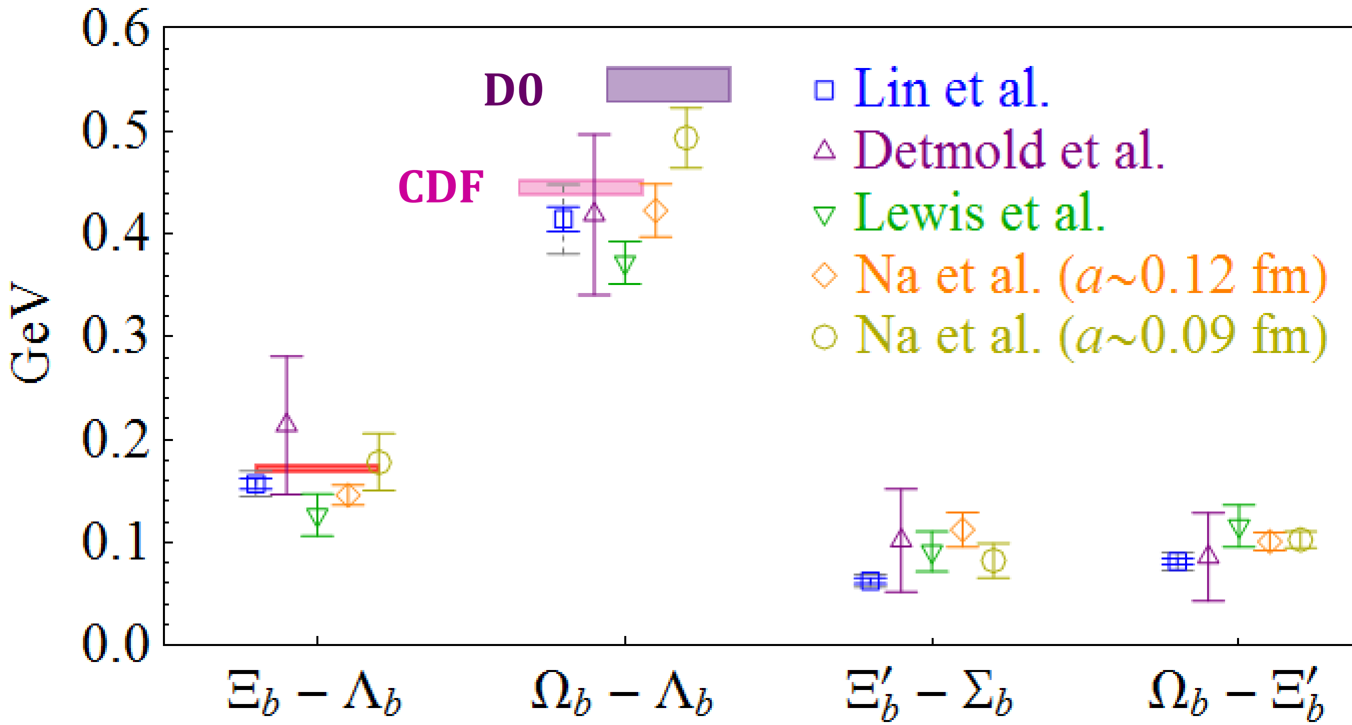
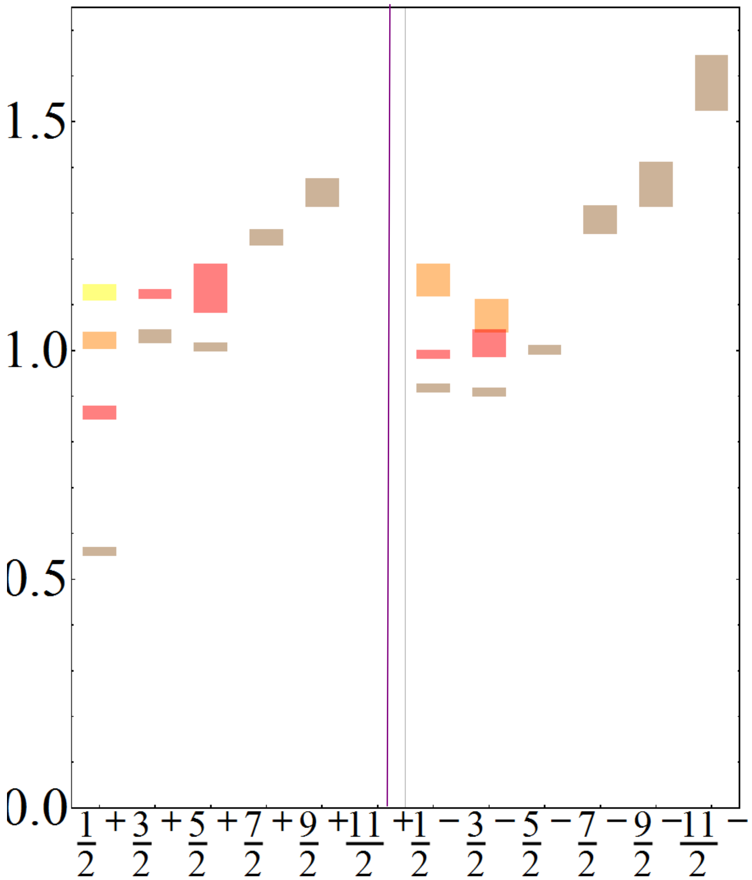


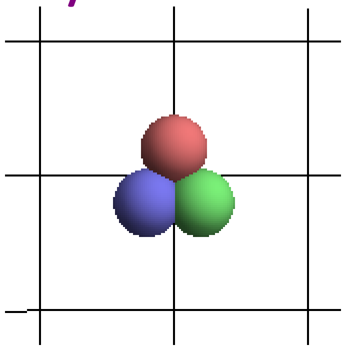
HWL, S. D. Cohen, N. Mathur, K. Orginos, Phys. Rev. D80, 054027 (2009);
W. Detmold et al., (2008), 0812.2583[hep-lat]; R. Lewis et al., PRD79, 014502 (2009);
H. Na et al., PoS LATTICE2008, 119 (2008).

Nucleon

- All baryon spin states wanted: |J| = 1/2, 3/2, 5/2, …
- List of 4-star states



- Only 3 distinguished states with J = 1/2



$$\epsilon^{abc}(u^{Ta}(x)\,C\gamma_5\,d^b(x))\,u^c(x)$$

$$\epsilon^{abc}(u^{Ta}(x)\,C\,d^b(x))\,\gamma_5\,u^c(x)$$

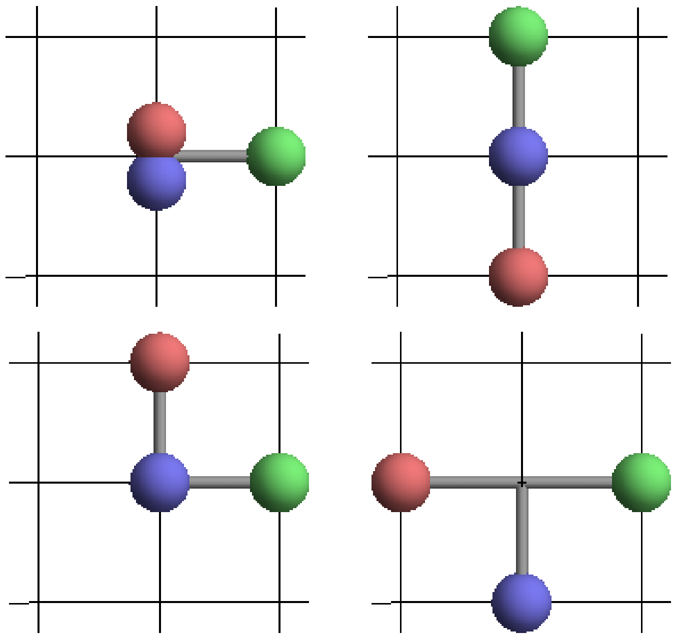
$$\epsilon^{abc}(u^{Ta}(x)\,C\gamma_5\gamma_4\,d^b(x))\,u^c(x)$$

Nucleon

- Rotation symmetry is reduced
  rotation SO(3) ⇒ octahedral O_h group

- Include more quark orientations

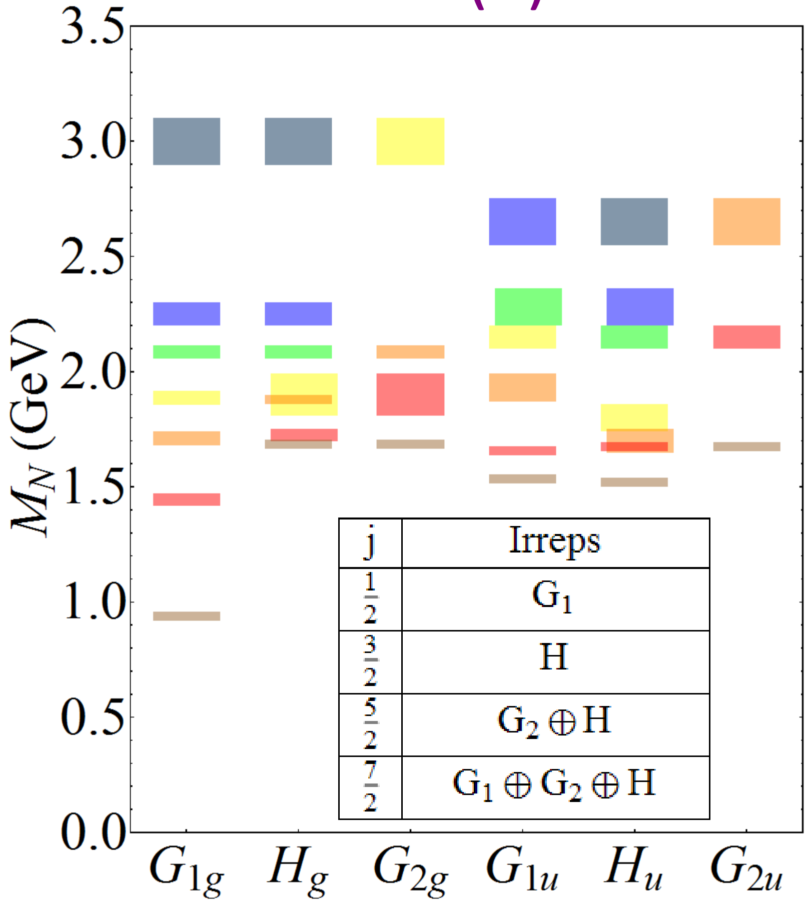
j	Irreps
$\frac{1}{2}$	G_1
$\frac{3}{2}$	H
$\frac{5}{2}$	$G_2 \oplus H$
$\frac{7}{2}$	$G_1 \oplus G_2 \oplus H$



S. Basak et al., Phys. Rev. D72, 094506 (2005)

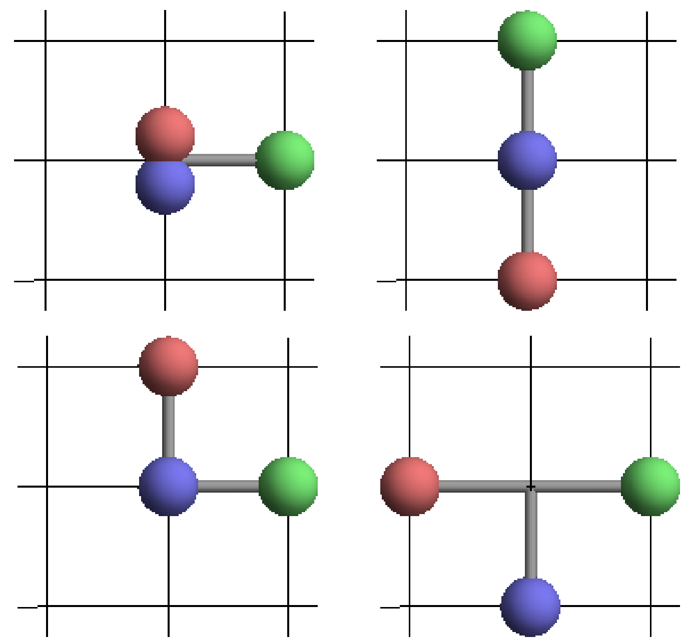
Nucleon

- Rotation symmetry is reduced
  rotation SO(3) ⇒ octahedral O_h group



j	Irreps
$\frac{1}{2}$	G_1
$\frac{3}{2}$	H
$\frac{5}{2}$	$G_2 \oplus H$
$\frac{7}{2}$	$G_1 \oplus G_2 \oplus H$

- Include more quark orientations



- More details on operators: check out this YouTube video

Variational Method

§ Recall: still a coupling problem

lowest-mass state dominates $C(t) = \sum_n \langle O|n\rangle\langle n|O\rangle e^{-E_n t}$

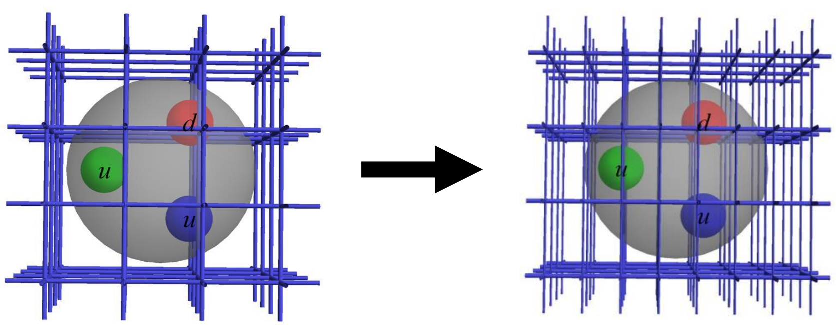
§ Decouple them:

1. Construct correlator matrix $C_{i\ j}(t) = \langle 0 \mid \mathcal{O}_i(t)^\dagger \mathcal{O}_j(0) \mid 0\rangle$
2. Solve the gen. eigensystem $C(t_0)^{-1/2} C(t) C(t_0)^{-1/2} \psi = \lambda(t, t_0)\psi$
3. Simple analysis of eigenvalues (t-dependence) to get each excited state $\lambda_n(t, t_0) = e^{-(t-t_0)E_n}(1 + \mathcal{O}(e^{-|\delta E|(t-t_0)}))$

C. Michael, Nucl. Phys. B 259, 58 (1985)

M. Lüscher and U. Wolff, Nucl. Phys. B 339, 222 (1990)

§ Higher resolution (at least in time direction)



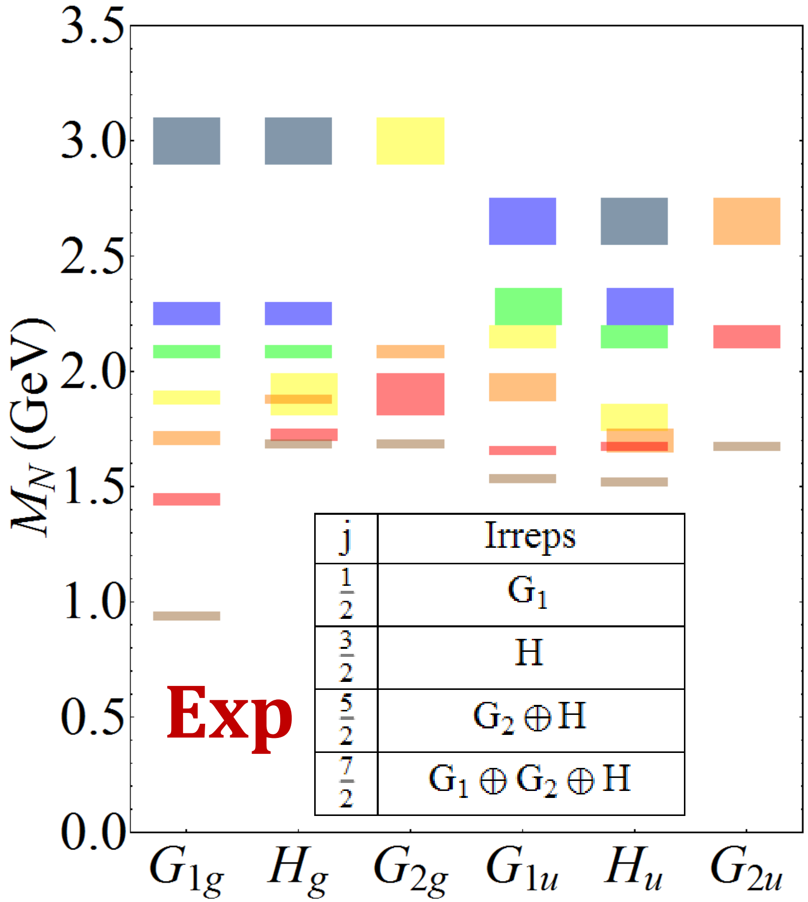
R. Edwards, B. Joo, HWL, Phys. Rev. D 78, 014505 (2008);
HWL et al., Phys. Rev. D 79, 034502 (2009)

N_f = 2+1 Study: Nucleon

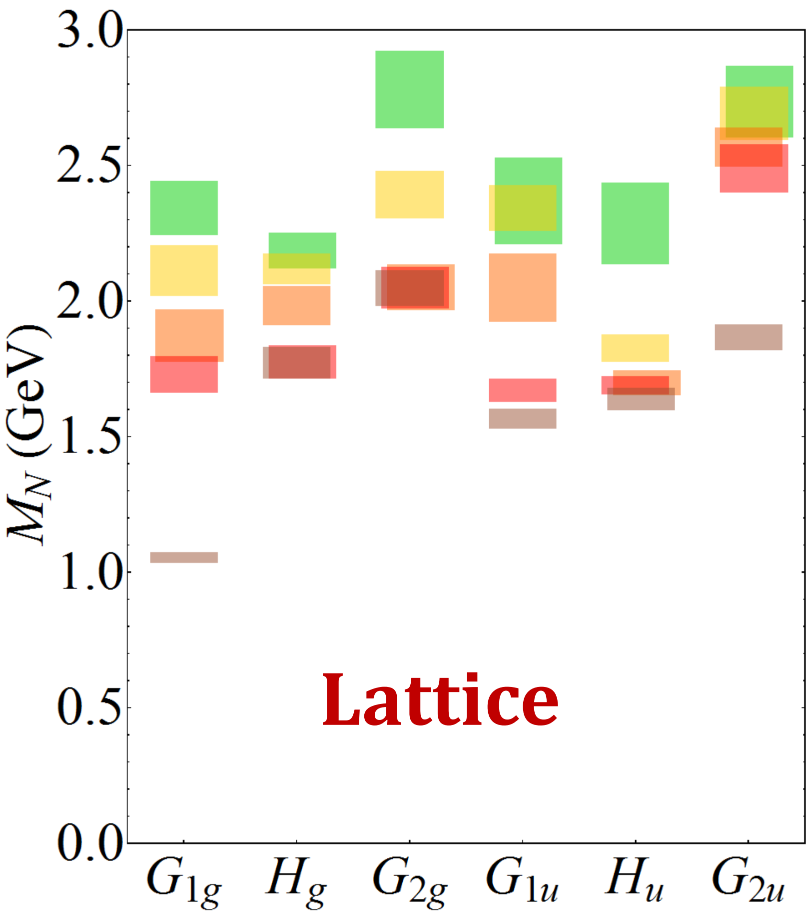
§ $N_f = 2+1$, anisotropic clover action

HSC, 1004.5072[hep-lat]

$V = 16^3{\times}128$, $a_s \approx 0.12$ fm, $a_s/a_t \approx 3.5$, $M_\pi \approx 390$ MeV



j	Irreps
$\frac{1}{2}$	G_1
$\frac{3}{2}$	H
$\frac{5}{2}$	$G_2 \oplus H$
$\frac{7}{2}$	$G_1 \oplus G_2 \oplus H$



Less Known Case

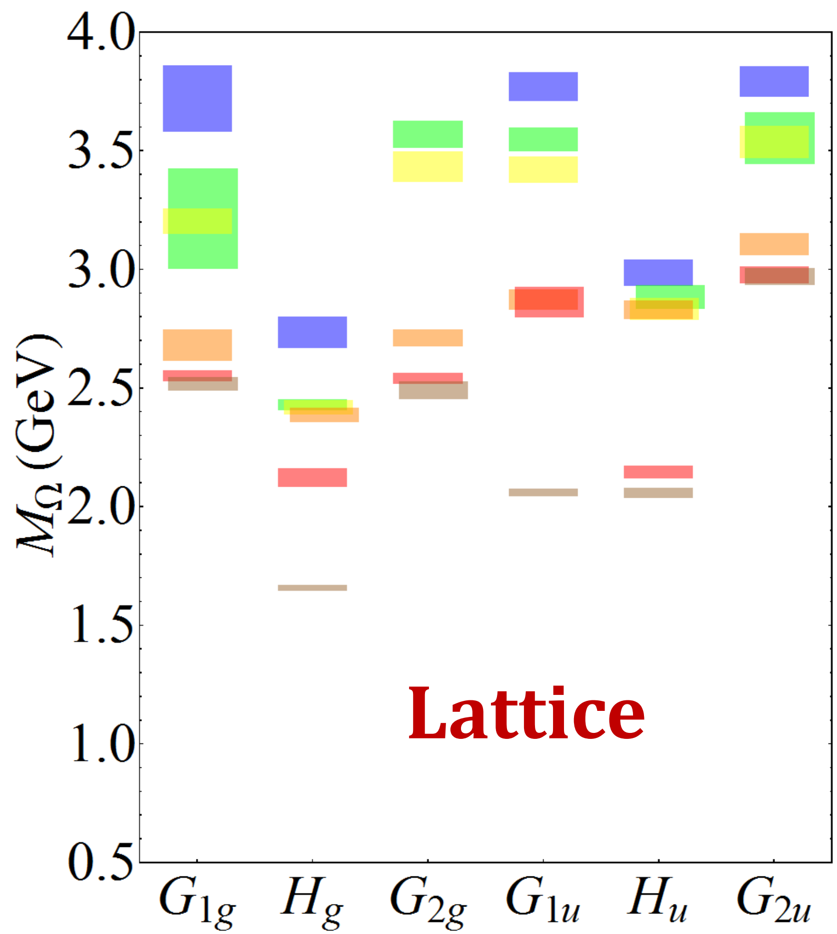
§ $N_f = 2{+}1$, anisotropic clover action HSC, 1004.5072[hep-lat]

$V = 16^3{\times}128$, $a_s \approx 0.12$ fm, $a_s/a_t \approx 3.5$, $M_\pi \approx 390$ MeV

Ω BARYONS $(S = -3, I = 0)$

Ω^-	$0(3/2^+)$	****
$\Omega(2250)^-$	$0(?^?)$	***
$\Omega(2380)^-$		**
$\Omega(2470)^-$		**

Exp



§ Predictive power

Learning by Doing!

Time for Tutorials

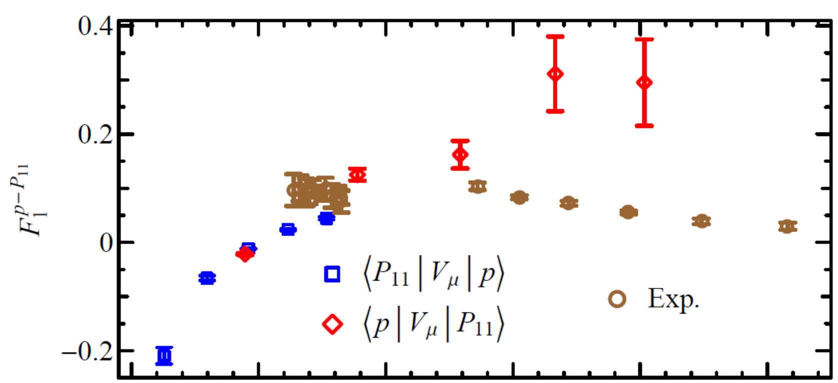
- § Prior Python with Jupyter notebook experience required
  - How many of you have not used these before?
- § Form groups of about 4 students
  - Rearrange the tables so your group is sitting together
- § Introduce yourself to the other students in your group (5-10 mins)
  - Name and preferred pronoun
  - Where are you from and tell us a few things about the place
  - If you could hang out with any cartoon character, who would you choose and why?
  - **Make sure you listen to each other!**

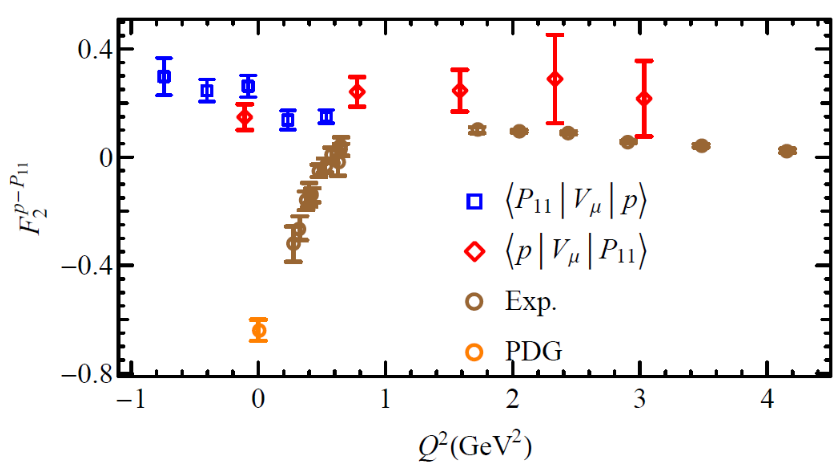
Backup Slides

Transition Form Factors

§ Challenge: Resonances and multiple-particle final states

- Neutrino scattering above the pion-production threshold, where the scattered nucleon is excited into resonances
- Inputs: need knowledge on $N \to \Delta$ and $\boldsymbol{N} \to \boldsymbol{N}^*$
- N-P_{11} transition EM form factors
- $\langle N_2 | V_\mu | N_1 \rangle_\mu (q) = \bar{u}_{N_2}(p') \left[F_1(q^2) \left(\gamma_\mu - \frac{q_\mu}{q^2} \not{q} \right) + \sigma_{\mu\nu} q_\nu \frac{F_2(q^2)}{M_{N_1} + M_{N_2}} \right] u_{N_1}(p) e^{-iq\cdot x}$



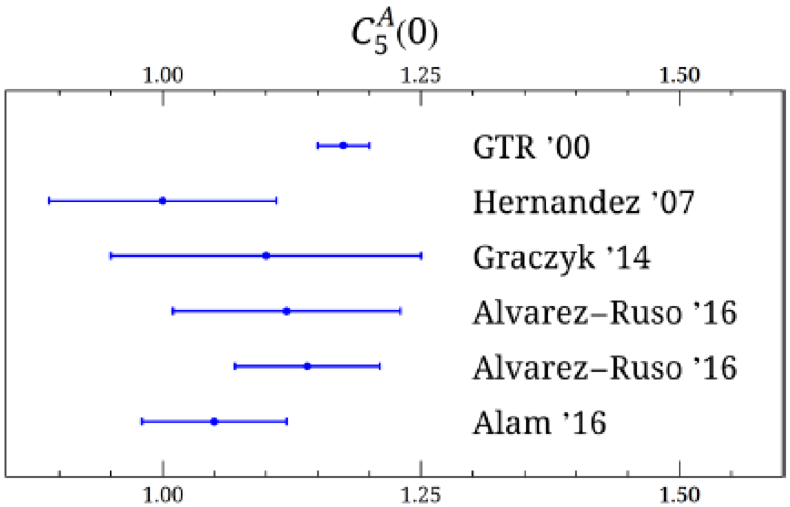


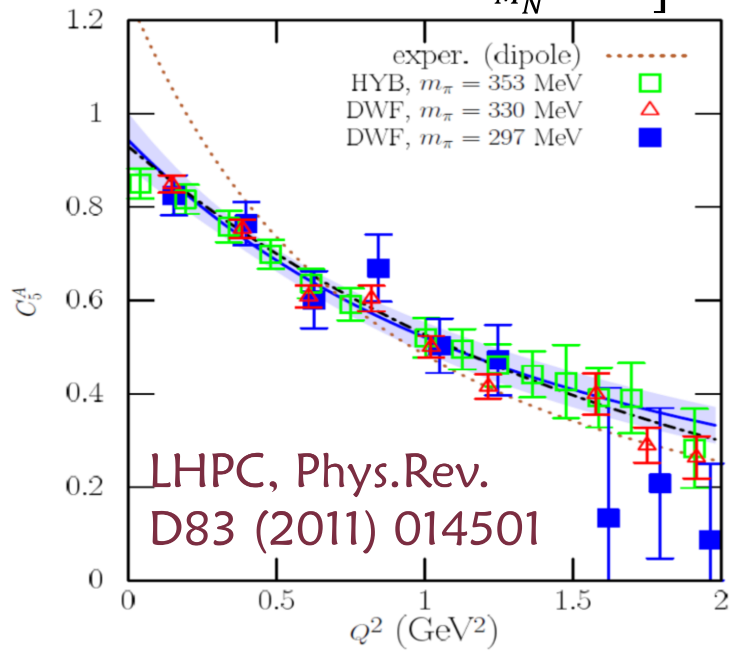
Lin et al, Phys.Rev.D78 (2008), 114508

Transition Form Factors

§ Challenge: Resonances and multiple-particle final states

- Neutrino scattering above the pion-production threshold, where the scattered nucleon is excited into resonances
- Inputs: need knowledge on $N \to \Delta$ and $N \to N^*$
- $N\Delta$ axial form factors C_{3-6}^A in the transition axial current are
- $-\mathcal{A}_{N\Delta}^{\mu} = \bar{u}_\alpha(p')\left[\frac{C_3^A}{M_N}(g^{\alpha\mu}\not{q} - q^\alpha\gamma^\mu) + \frac{C_4^A}{M_N^2}(g^{\alpha\mu}q \cdot p' - q^\alpha p'^\mu) + \boldsymbol{C_5^A} g^{\alpha\mu} + \frac{C_6^A}{M_N^2}q^\alpha q^\mu\right]\gamma_5 u(p)$



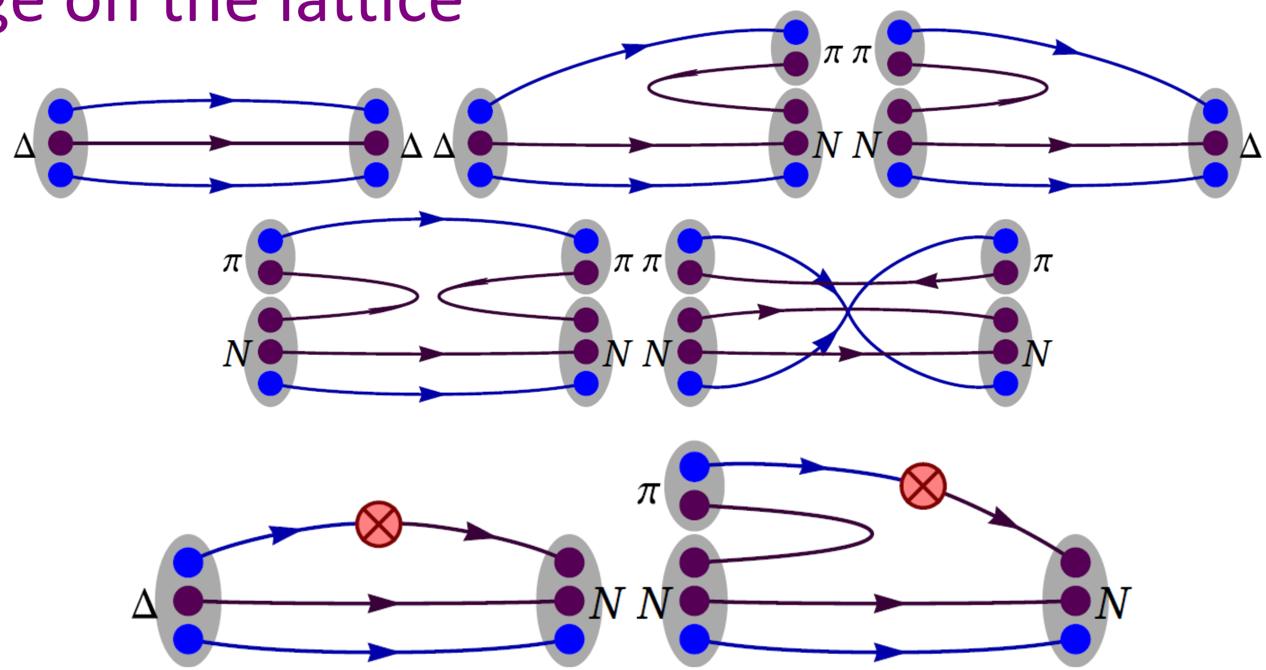


Transition Form Factors

§ Challenge: Resonances and multiple-particle final states

- Neutrino scattering above the pion-production threshold, where the scattered nucleon is excited into resonances
- Inputs: need knowledge on $N \to \Delta$ and $N \to N^*$
- CAVER: Past lattice calculations have used $E_\pi + E_N > E_{\Delta,N^*}$

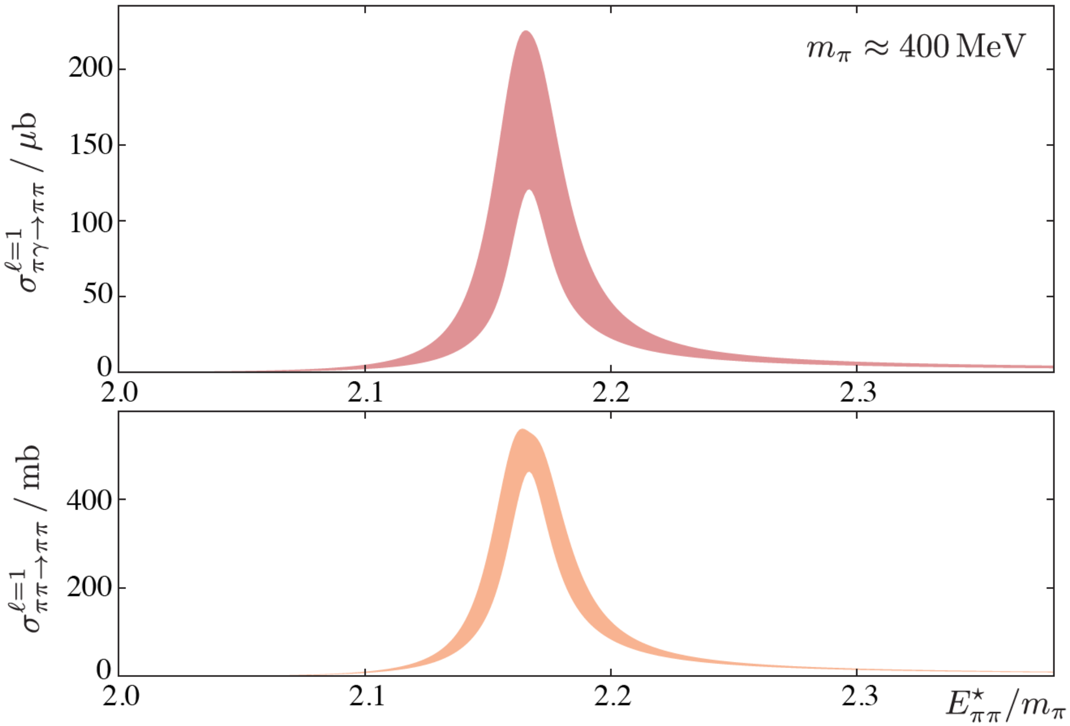
§ Challenge on the lattice



Transition Form Factors

§ Challenge: Resonances and multiple-particle final states

- Simpler cases show promising results from lattice



R. Briceno et al, Phys. Rev. D92, 074509 (2015)