GPDs from Single-Diffractive Hard Exclusive Processes (SDHEP)

Zhite Yu
(Jefferson Lab, Theory Center)

- ❑ Introduce SDHEP
- ❑ Rethink DVCS as an SDHEP
- ❑ Sensitivity to GPD x-dependence
- ❑ Briefly on factorization
- ❑ Go beyond $2 \to 3$ SDHEPs

In collaboration with
Jianwei Qiu and Nobuo Sato

JHEP 08 (2022) 103
PRD 107 (2023) 014007
PRL 131 (2023) 161902
PRD 109 (2024) 074023
arXiv: 2409.06882

Sep/24/2024
JLab CEBAF

Recap

❑ Why GPDs?

- Tomography
- Spin and mass decomposition → See Cédric's lecture
- Internal pressure, shear force, …

❑ How to obtain GPDs?

- Lattice QCD → Robert and Joe's lectures
- Model → Marija
- Factorization theorem → Christian and Jianwei

  \+ experimental exclusive processes → Charles and Francois-Xavier

Physical features for GPD processes

$$F^q(x,\xi,t) = \int \frac{\mathrm{d}z^-}{4\pi} e^{-ixP^+z^-} \langle p' \left| \bar{q}\left(z^-/2\right)\gamma^+ q\left(-z^-/2\right)\right| p\rangle$$
$$= \frac{1}{2P^+}\left[H^q(x,\xi,t)\,\bar{u}\left(p'\right)\gamma^+ u(p) - E^q(x,\xi,t)\,\bar{u}\left(p'\right)\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}u(p)\right]$$

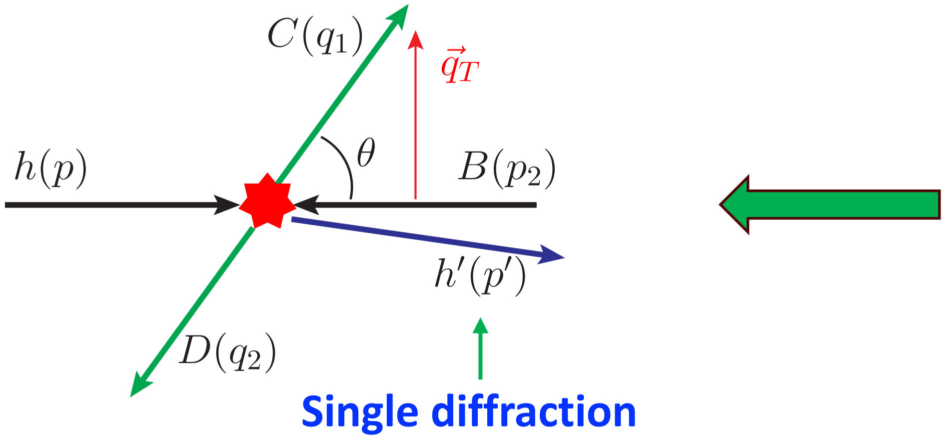
- **Amplitude nature** ⟶ **Hadron is unbroken** ⟶ ***Exclusive* process**
- **Collinear factorization property** $\mathcal{M} = \int_{-1}^{1} dx\, F(x,\xi,t)\, C(x,Q) + \mathcal{O}(\sqrt{-t}/Q)$
  - **The scale t is unconstrained in the GPD definition itself**
  - **But the GPD factorizability requires a hard scale $Q \gg \sqrt{-t}$**

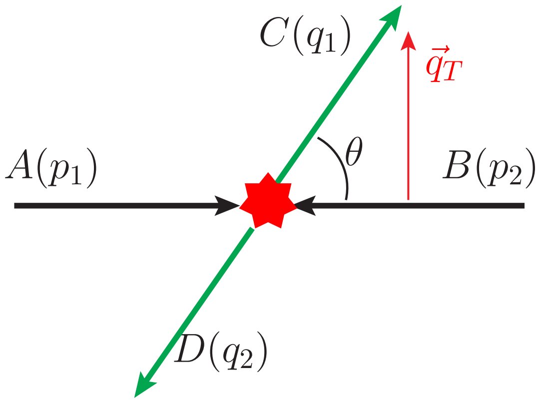
⇨ **A *hard* scattering with *diffractive* hadron**

Single-Diffractive Hard Exclusive Process (SDHEP)

$h(p) + B(p_2) \to h'(p') + C(q_1) + D(q_2)$

$A(p_1) + B(p_2) \to C(q_1) + D(q_2)$





Large-angle 2-to-2 exclusive scattering

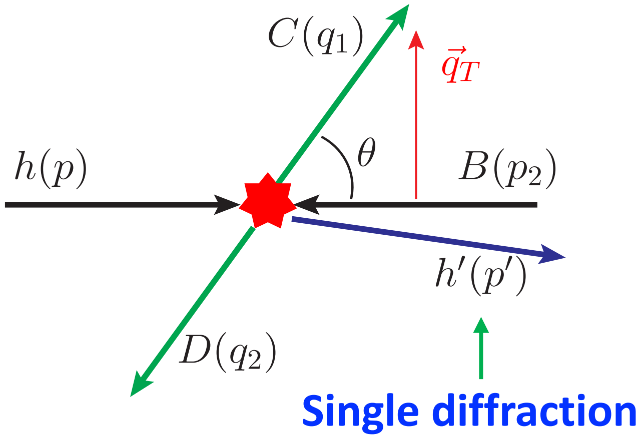
Two scales:

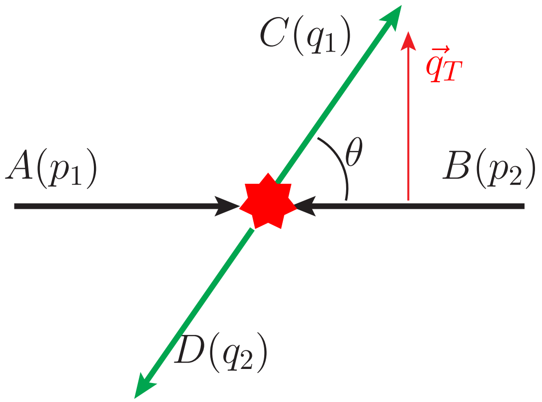
- **Hard q_T**
- **Soft t**

Single-Diffractive Hard Exclusive Process (SDHEP)

$h(p) + B(p_2) \to h'(p') + C(q_1) + D(q_2)$

$A(p_1) + B(p_2) \to C(q_1) + D(q_2)$





Large-angle 2-to-2 exclusive scattering

Two scales:

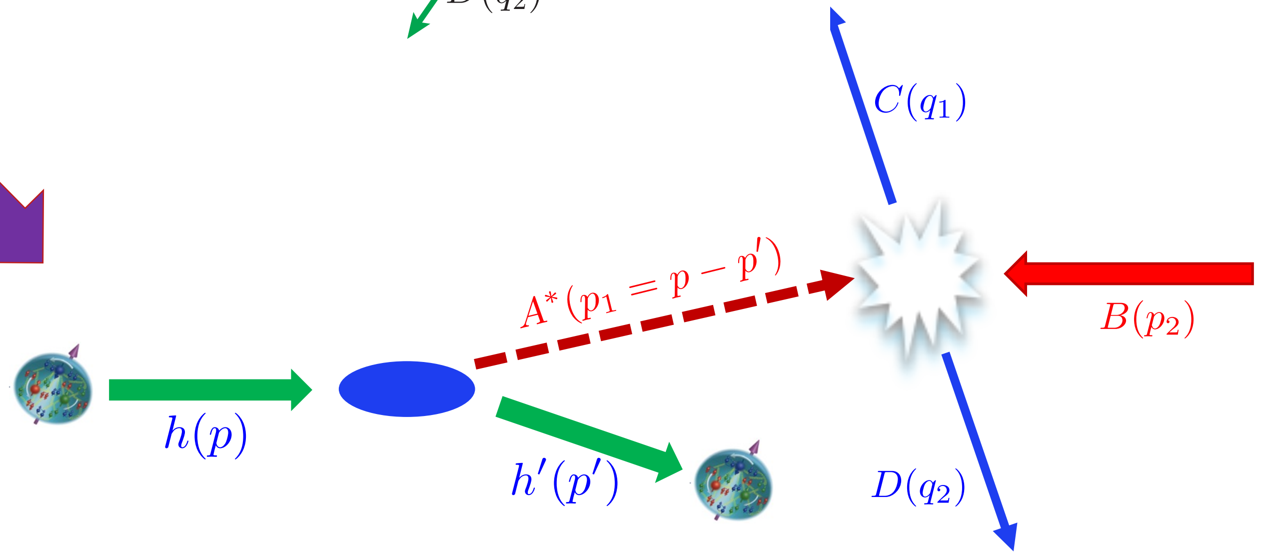
- **Hard q_T**
- **Soft t**

➢ **Two-stage paradigm**

$$N(p) \to N(p') + A^*(p_1 = p - p')$$

factorize

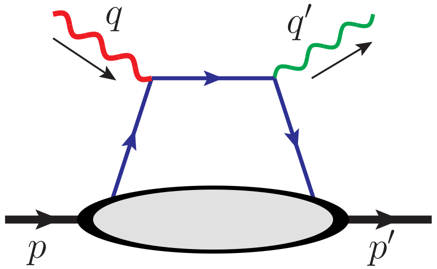
$$A^*(p_1) + e(p_2) \to e(q_1) + \gamma(q_2)$$



Necessary **for factorization:** $q_T \gg \sqrt{-t} \simeq \Lambda_{\rm QCD}$

“Golden” example: Rethinking DVCS as an SDHEP

DVCS



$$N(p) + \gamma^*(q) \to N(p') + \gamma(q')$$

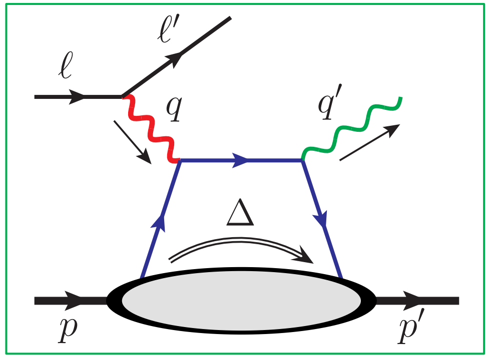
NOT a physical process!

Jefferson Lab

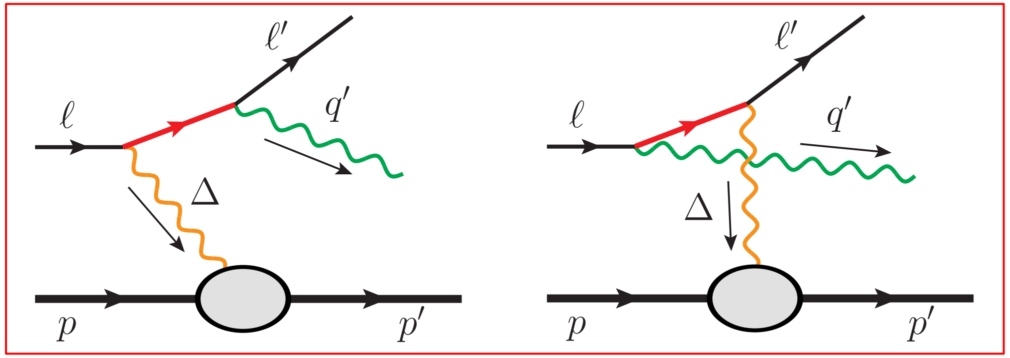
From DVCS to SDHEP (single-diffractive real photon electroproduction)

➢ **Physical process** $N(p) + e(\ell) \rightarrow N(p') + e(\ell') + \gamma(q')$

$\Delta = p' - p$



DVCS



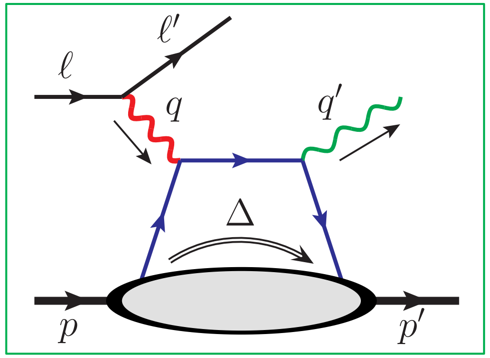
Bethe-Heitler (BH) process

Hard scale: Q

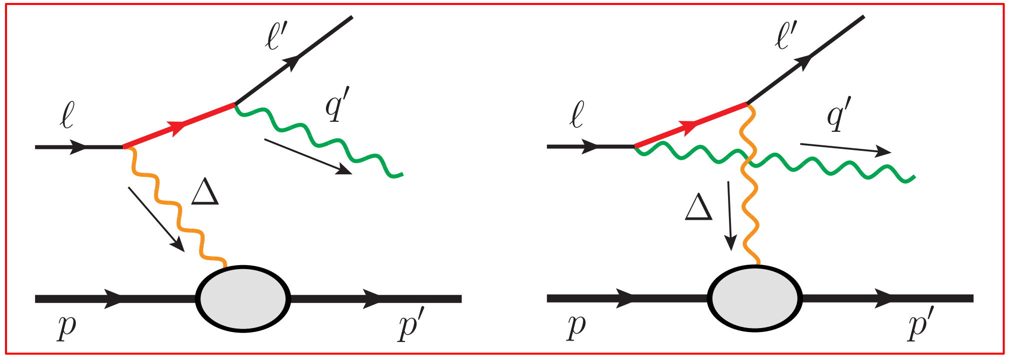
From DVCS to SDHEP (single-diffractive real photon electroproduction)

- **Physical process** $N(p) + e(\ell) \to N(p') + e(\ell') + \gamma(q')$

$\Delta = p' - p$



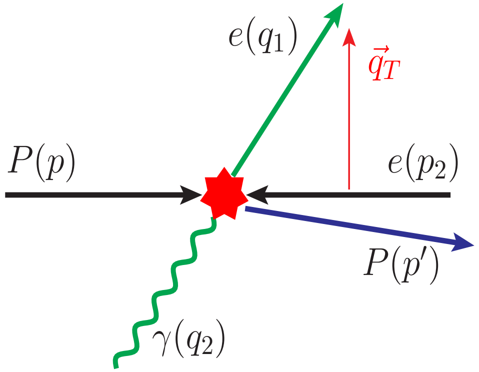
DVCS

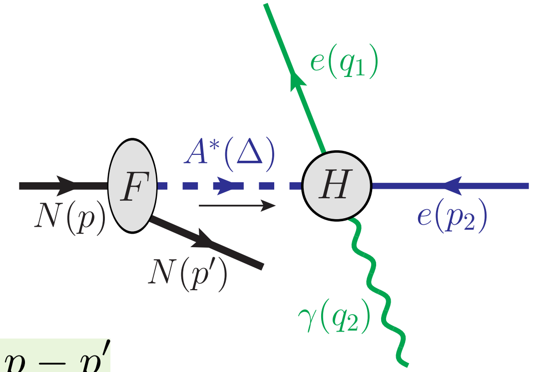


Bethe-Heitler (BH) process

Hard scale: *Q*

- **Switch to SDHEP "point of view"** $N(p) + e(p_2) \to N(p') + e(q_1) + \gamma(q_2)$

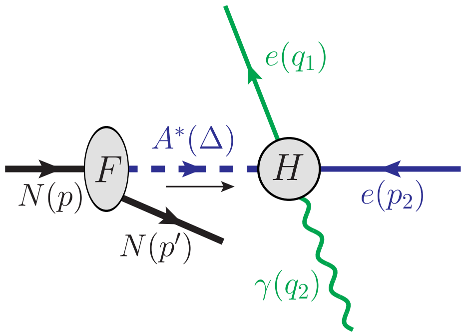




$\Delta = p_1 = p - p'$

Hard scale: q_T

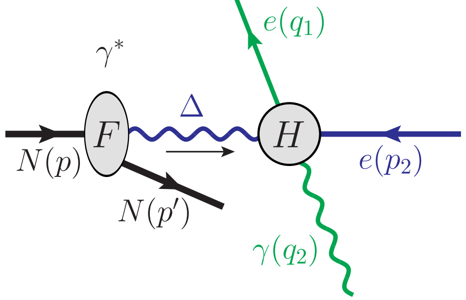
What is the A^*? --- channel expansion



$\Delta = p_1 = p - p'$

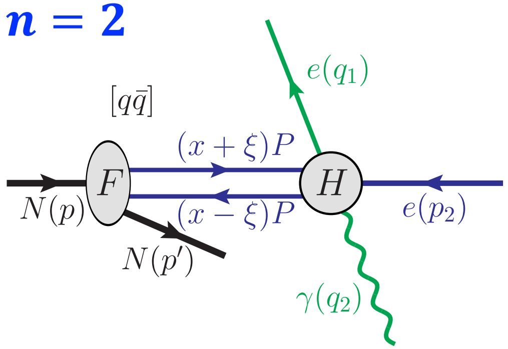
$=$

$n = 1$

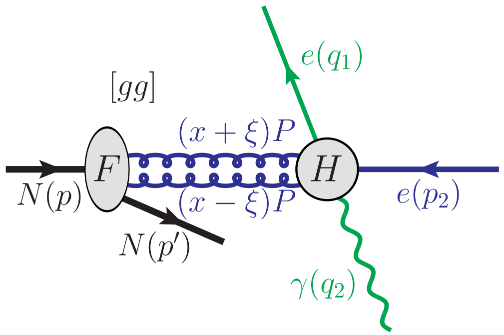


$+$

$n = 2$

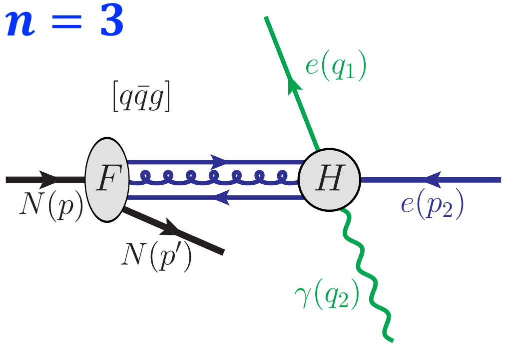


$+$

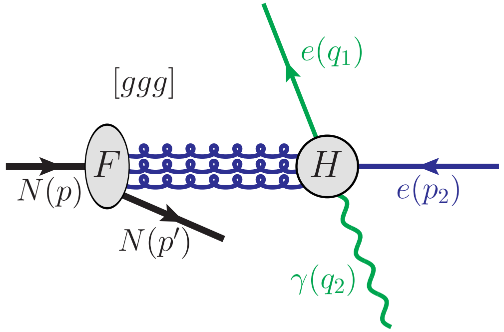


$n = 3$

$+$

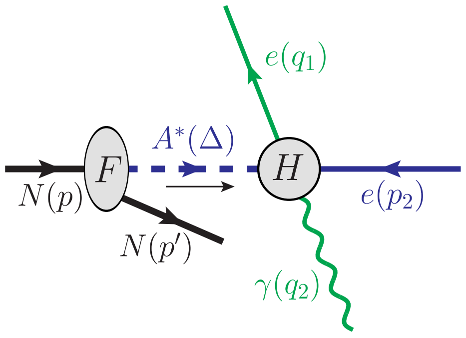


$+$



$+ \cdots \quad (n > 3)$

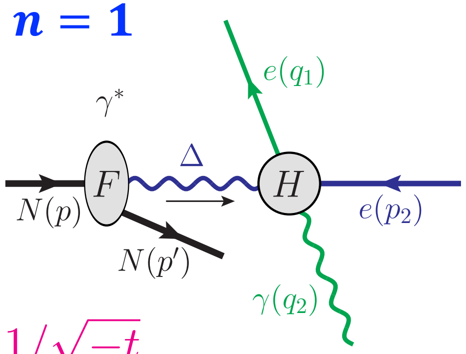
Channel expansion and power counting



$\Delta = p_1 = p - p'$

=

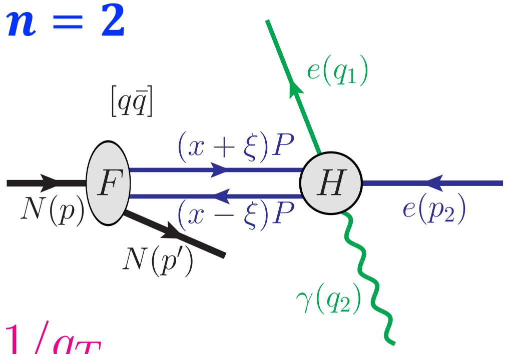
$n = 1$



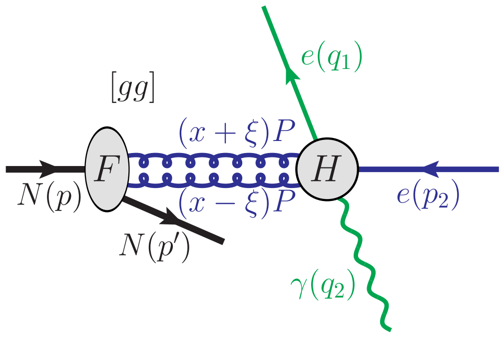
$1/\sqrt{-t}$

+

$n = 2$



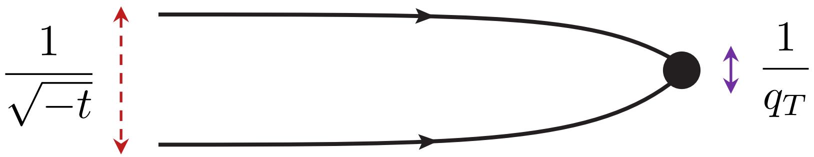
+



$1/q_T$

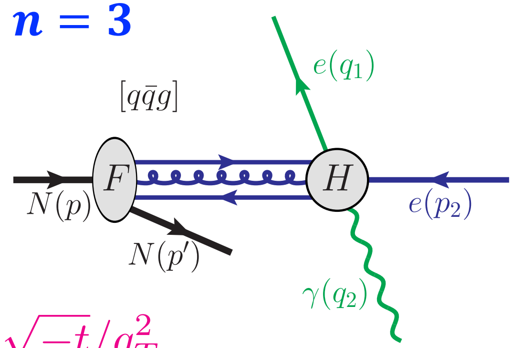
One more physically polarized parton in A^*

one more suppression of $\sqrt{-t}/q_T$

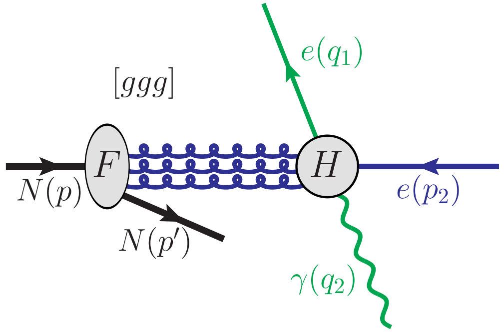


+

$n = 3$



+

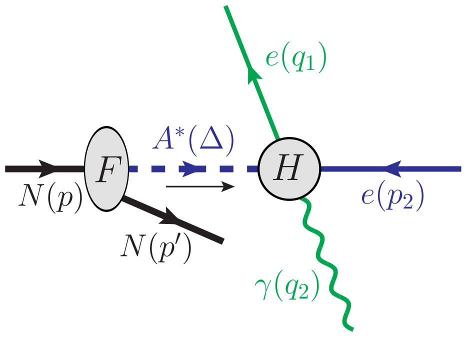


$\sqrt{-t}/q_T^2$

$+ \cdots$ $(n > 3)$ $\mathcal{O}\left(-t/q_T^3\right)$

Exercise: Show the γ^* channel scales as $1/\sqrt{-t}$.

Channel expansion and power counting

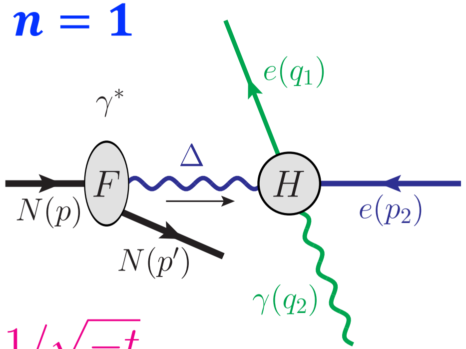


$\Delta = p_1 = p - p'$

$=$

Leading power (LP)

$n = 1$

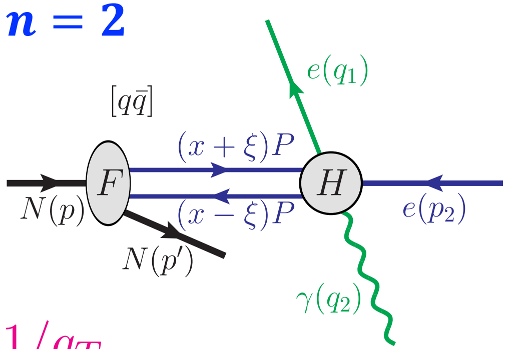


$1/\sqrt{-t}$

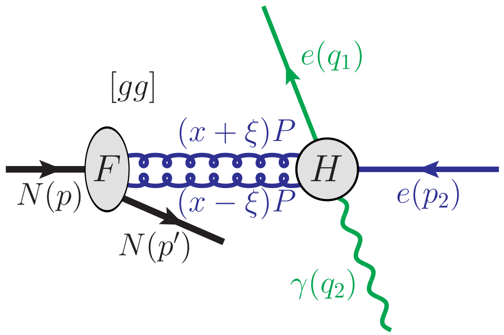
$+$

Next-to-leading power (NLP)

$n = 2$



$+$

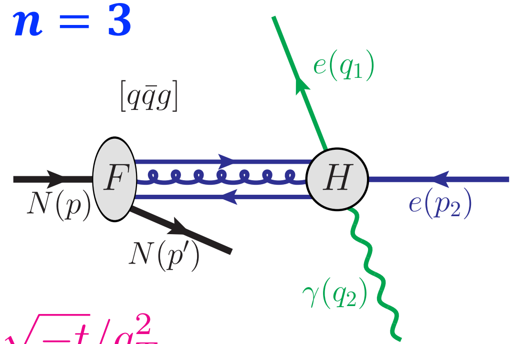


$1/q_T$

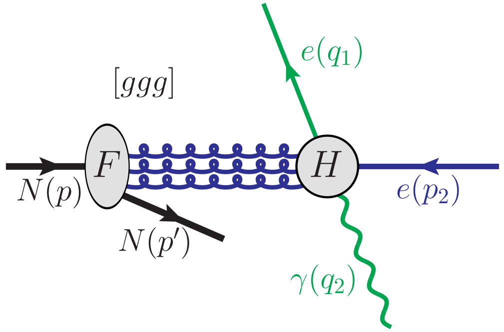
NNLP

$+$

$n = 3$



$+$



$\sqrt{-t}/q_T^2$

One more physically polarized parton in A^*

→ **one more suppression of $\sqrt{-t}/q_T$**

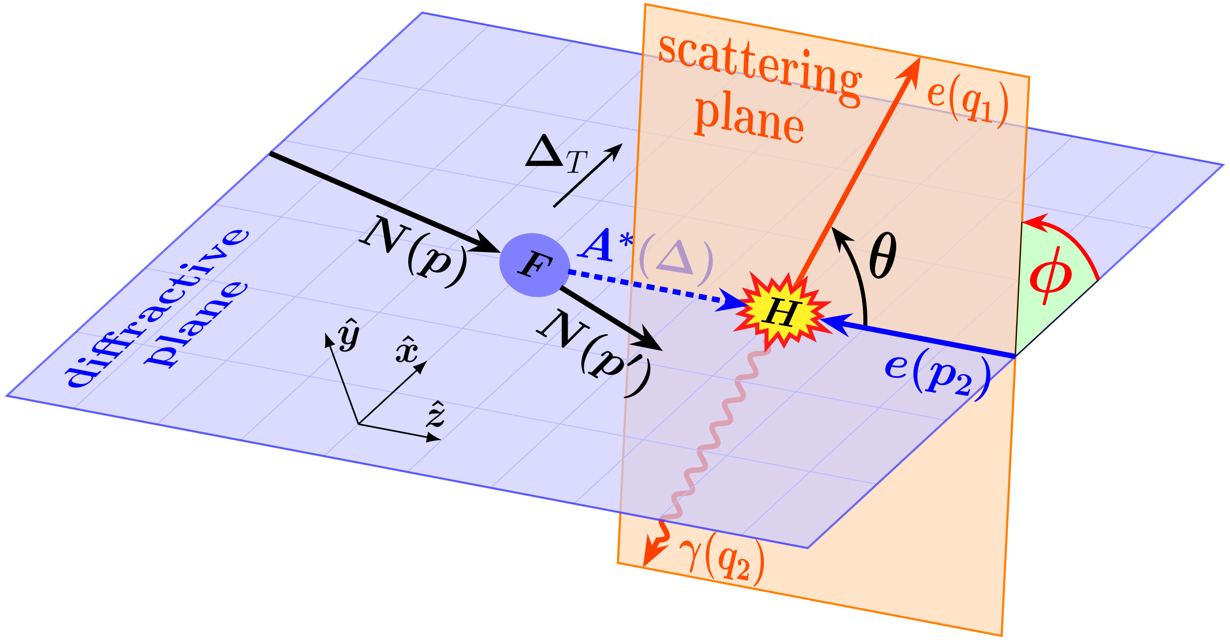
- **Consistent power counting**
- **Channel expansion = power expansion**

NNNLP…

$+ \cdots \ (n > 3) \quad \mathcal{O}\left(-t/q_T^3\right)$

Jefferson Lab

SDHEP frame

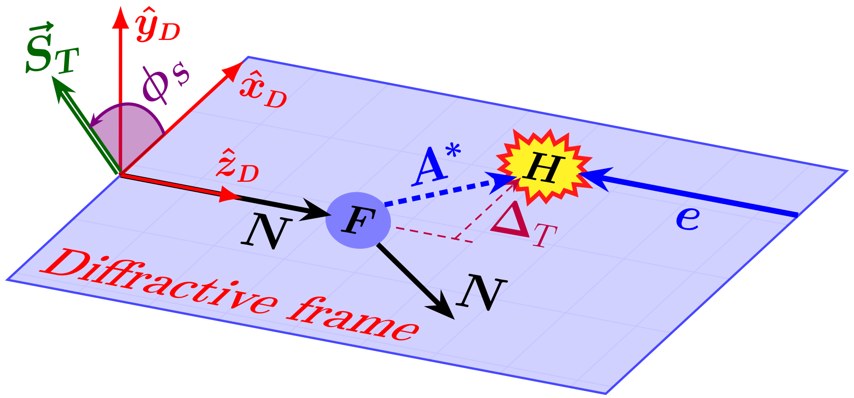


scattering plane
diffractive plane
Δ_T
$e(q_1)$
$N(p)$
F
$A^*(\Delta)$
H
θ
ϕ
$N(p')$
$e(p_2)$
$\hat{y}$
$\hat{x}$
$\hat{z}$
$\gamma(q_2)$

Two-stage kinematic description

- **Diffractive subprocess** $N(p) \to N(p') + A^*(\Delta = p - p')$

(Ne) c.m. frame



$\hat{x}_D$–$\hat{y}_D$–$\hat{z}_D$: **varying coordinate system**

- **Hard scattering** $A^*(\Delta) + e(p_2) \to e(q_1) + \gamma(q_2)$

Two-stage kinematic description

- **Diffractive subprocess** $N(p) \to N(p') + A^*(\Delta = p - p')$

  Describe in diffractive frame: $\hat{x}_D \parallel \vec{\Delta}_T$ **(varying event by event)**

  Trade azimuthal angle of the diffraction for ϕ_S (Jacobian = 1)

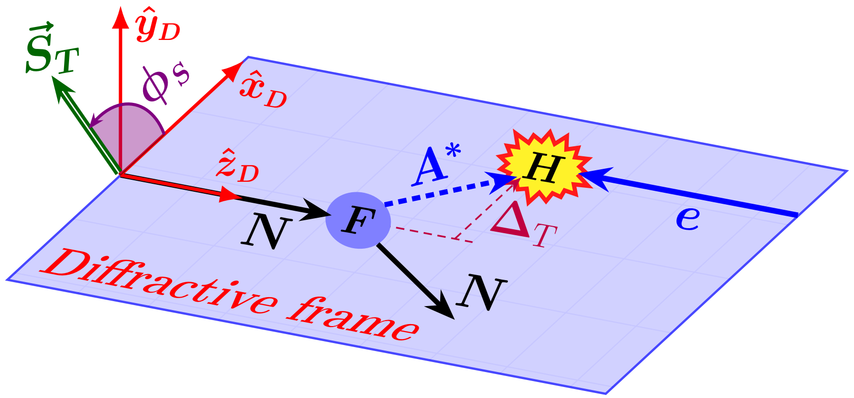
  Kinematic variables: $t = \Delta^2,\ \xi = \dfrac{(p - p')\cdot n}{(p + p')\cdot n},\ \phi_S$

  $n = (1, 0, 0, -1)/\sqrt{2}$

  ➡ **determine** $\hat{s} \simeq 2\xi s/(1+\xi)$ **of the hard scattering**

- **Hard scattering** $A^*(\Delta) + e(p_2) \to e(q_1) + \gamma(q_2)$

(Ne) c.m. frame



$\hat{x}_D$–$\hat{y}_D$–$\hat{z}_D$: **varying coordinate system**

Two-stage kinematic description

❑ Diffractive subprocess $N(p) \to N(p') + A^*(\Delta = p - p')$

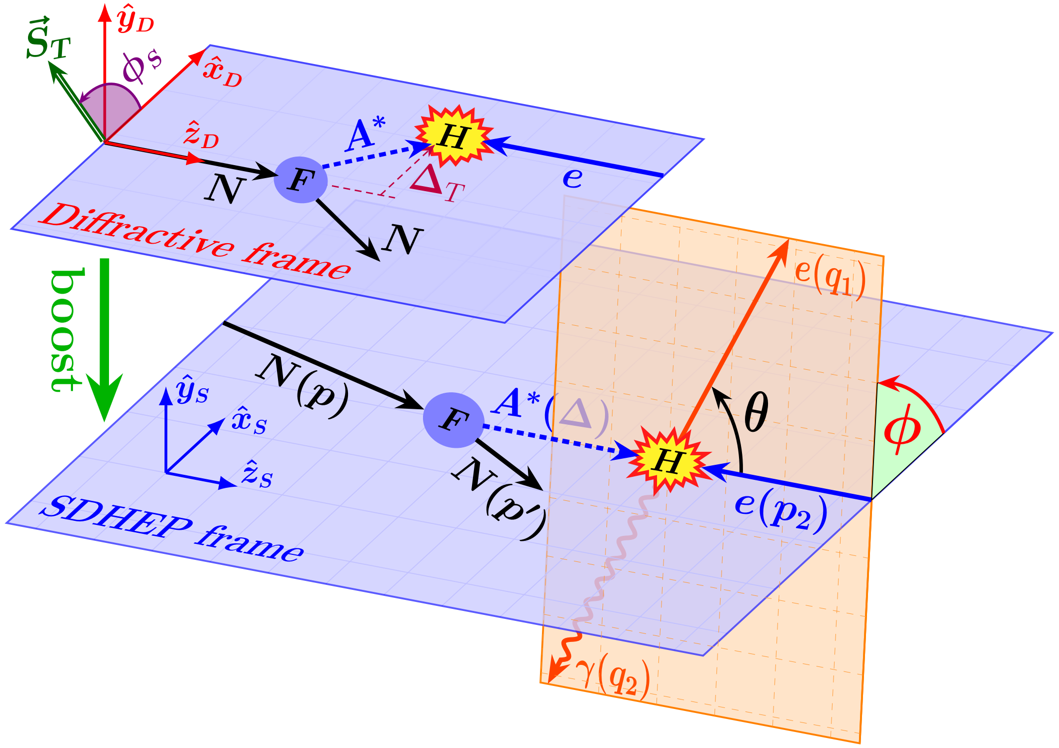
Describe in diffractive frame: $\hat{x}_D \parallel \vec{\Delta}_T$ **(varying event by event)**

Trade azimuthal angle of the diffraction for ϕ_S (Jacobian = 1)

Kinematic variables: $t = \Delta^2,\ \xi = \dfrac{(p - p')\cdot n}{(p + p')\cdot n},\ \phi_S$

$n = (1, 0, 0, -1)/\sqrt{2}$

➡ **determine** $\hat{s} \simeq 2\xi s/(1+\xi)$ **of the hard scattering**



$\hat{x}_S$–$\hat{y}_S$–$\hat{z}_S$ **: SDHEP frame coordinate system**

❑ Hard scattering $A^*(\Delta) + e(p_2) \to e(q_1) + \gamma(q_2)$

Describe in SDHEP frame --- $(A^* e)$ c.m. frame: $\hat{z}_S \parallel \vec{\Delta}$

Kinematic variables: $\theta,\ \phi$ $\quad \left[q_T = (\sqrt{\hat{s}}/2)\sin\theta \right]$

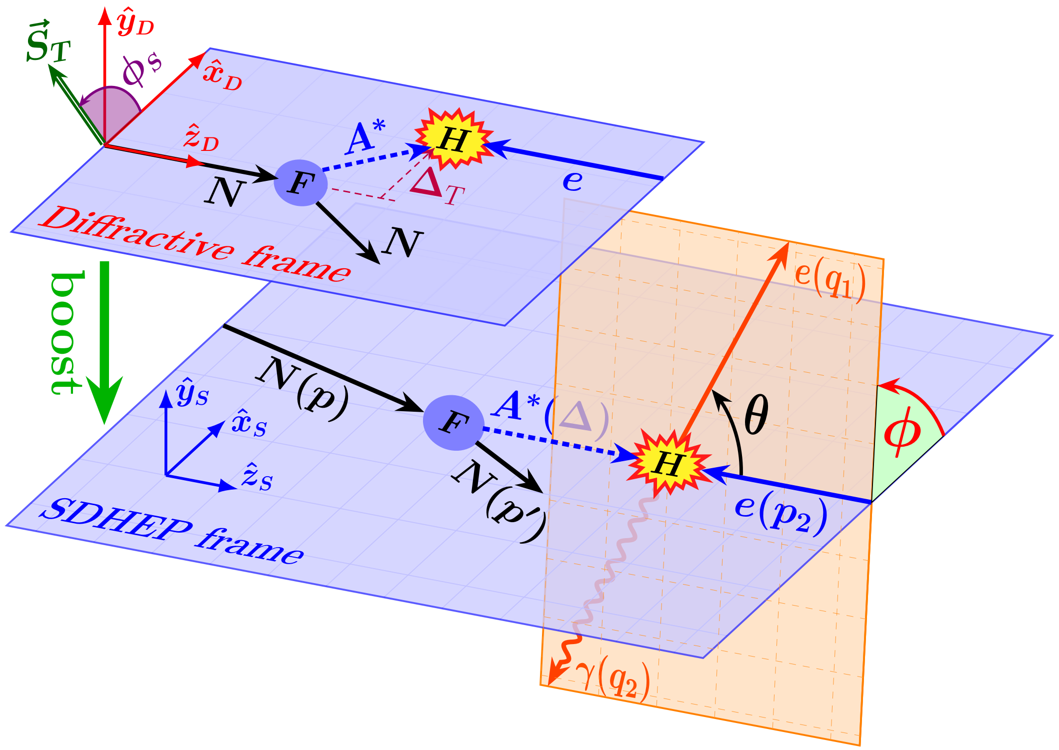
➡ $$\frac{d\sigma}{dt\, d\xi\, d\phi_S\, d\cos\theta\, d\phi}$$

Azimuthal distribution

- ϕ_S **in diffraction** $N(p) \to N(p') + A^*(\Delta = p - p')$

$$F_{N\to NA^*}(t, \xi, \phi_S) \propto e^{-i\lambda_N \phi_S}$$

- ϕ **in hard scattering** $A^*(\Delta) + e(p_2) \to e(q_1) + \gamma(q_2)$



Azimuthal distribution

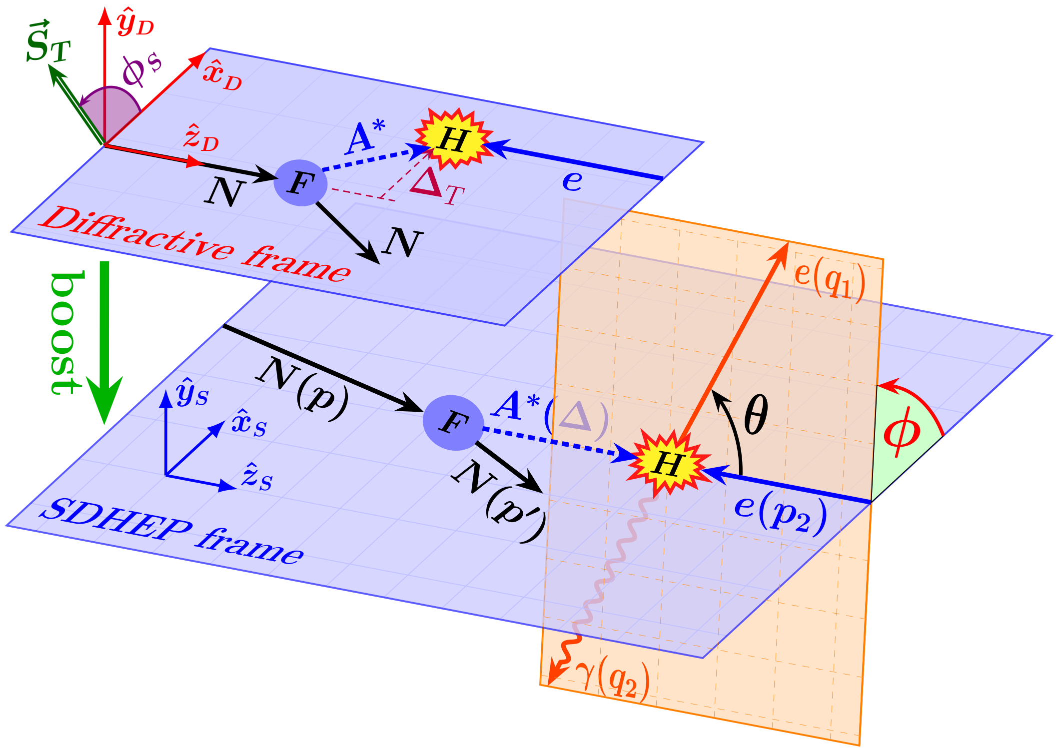
- ϕ_S **in diffraction** $N(p) \to N(p') + A^*(\Delta = p - p')$

$$F_{N\to NA^*}(t, \xi, \phi_S) \propto e^{-i\lambda_N \phi_S}$$

$\lambda_N = \pm 1/2$ **can interfere to give** $\cos\phi_S,\ \sin\phi_S$

transverse spin $s_T \neq 0$

- ϕ **in hard scattering** $A^*(\Delta) + e(p_2) \to e(q_1) + \gamma(q_2)$



Azimuthal distribution

- ϕ_S **in diffraction** $N(p) \to N(p') + A^*(\Delta = p - p')$

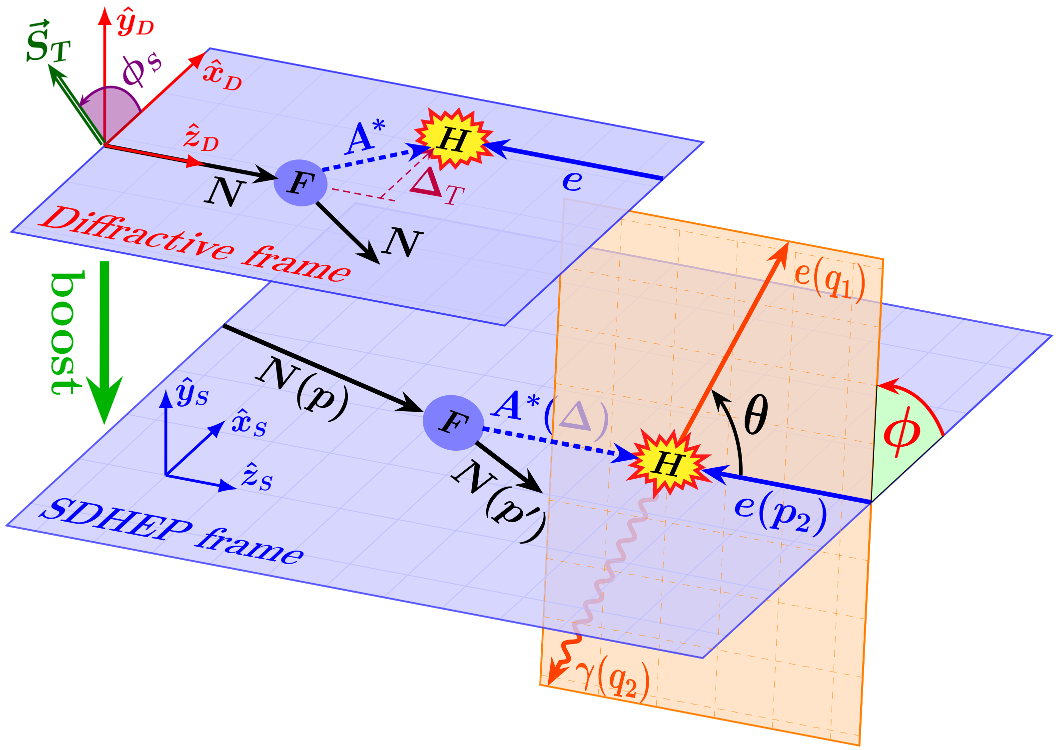
$$F_{N\to NA^*}(t,\xi,\phi_S) \propto e^{-i\lambda_N \phi_S}$$

$\lambda_N = \pm 1/2$ **can interfere to give** $\cos\phi_S,\ \sin\phi_S$

transverse spin $s_T \neq 0$

- ϕ **in hard scattering** $A^*(\Delta) + e(p_2) \to e(q_1) + \gamma(q_2)$

$$\mathcal{M}(t,\xi,\phi_S,\theta,\phi) = \sum_{A^*} F_{N\to NA^*}(t,\xi,\phi_S) \otimes G_{A^* e\to e\gamma}(\hat{s},\theta,\phi)$$
$$= \sum_{A^*} \left[e^{-i\lambda_N \phi_S} F_{N\to NA^*}(t,\xi)\right] \otimes \left[e^{i(\lambda_A-\lambda_e)\phi} G_{A^* e\to e\gamma}(\hat{s},\theta)\right]$$



Azimuthal distribution

- ϕ_S **in diffraction** $N(p) \to N(p') + A^*(\Delta = p - p')$

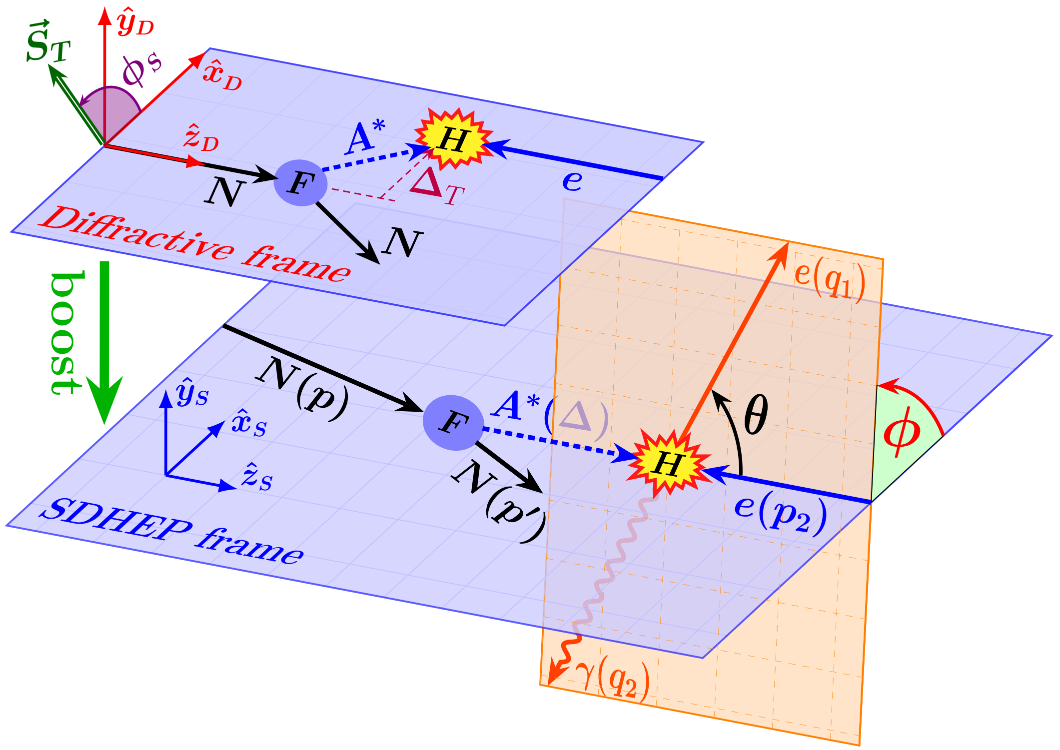
$$F_{N\to NA^*}(t, \xi, \phi_S) \propto e^{-i\lambda_N \phi_S}$$

$\lambda_N = \pm 1/2$ **can interfere to give** $\cos\phi_S,\ \sin\phi_S$

transverse spin $s_T \neq 0$

- ϕ **in hard scattering** $A^*(\Delta) + e(p_2) \to e(q_1) + \gamma(q_2)$

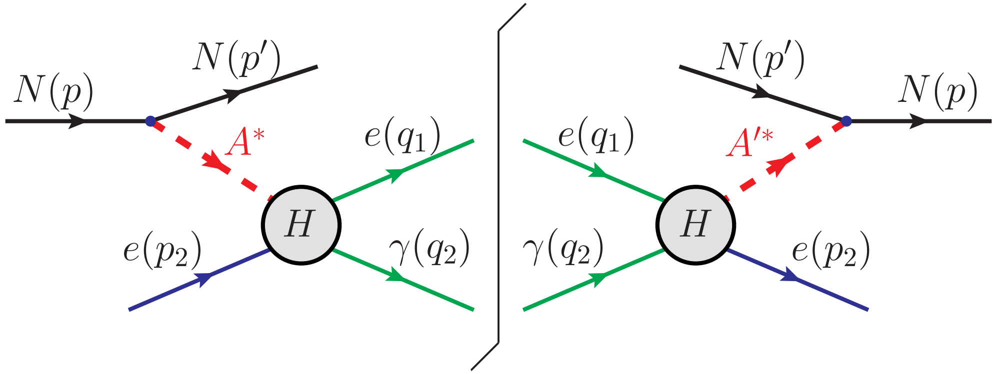
$$\mathcal{M}(t, \xi, \phi_S, \theta, \phi) = \sum_{A^*} F_{N\to NA^*}(t, \xi, \phi_S) \otimes G_{A^* e\to e\gamma}(\hat{s}, \theta, \phi)$$
$$= \sum_{A^*} \left[e^{-i\lambda_N \phi_S} F_{N\to NA^*}(t, \xi)\right] \otimes \left[e^{i(\lambda_A - \lambda_e)\phi} G_{A^* e\to e\gamma}(\hat{s}, \theta)\right]$$



Interference of $(\lambda_A,\ \lambda'_A)$ **channels**

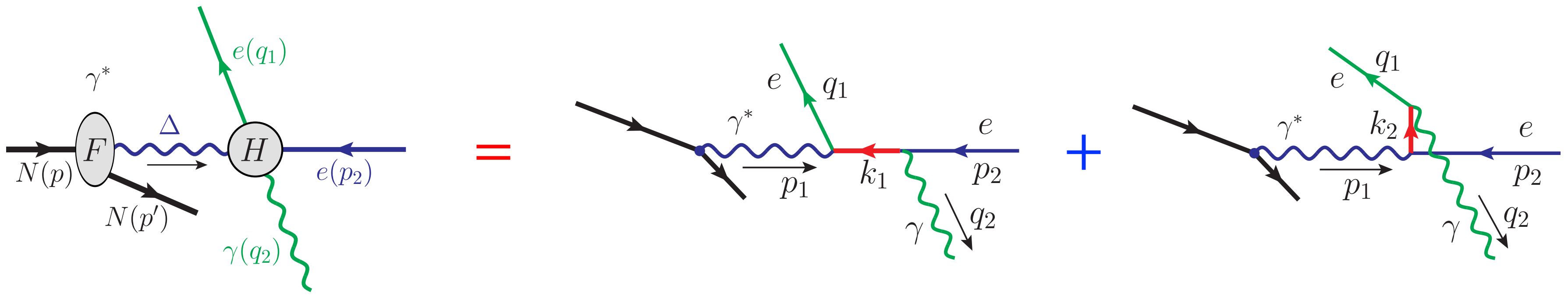
$$\Delta\lambda_A = \lambda_A - \lambda'_A$$

$\cos[(\Delta\lambda_A)\phi]$
$\sin[(\Delta\lambda_A)\phi]$



$n = 1$: γ^* channel --- BH subprocess

- **Advantage: the quasi-real state A^* has well-defined helicity for all $n = 1, 2, 3, \ldots$**



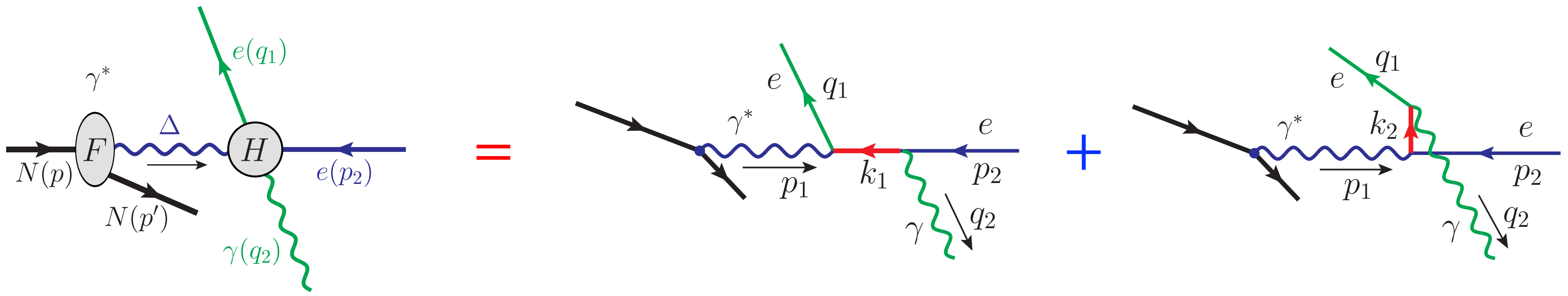
$$\mathcal{M}^{[1]} = \frac{-e}{t} F_N^\mu(p,p')\, G_\mu^\gamma(\Delta, p_2, q_1, q_2) = \frac{e}{t}\left[\sum_{\lambda=\pm 1} (F_N \cdot \epsilon_\lambda^*)(\epsilon_\lambda \cdot G^\gamma) - 2(F_N \cdot n)(\bar{n} \cdot G^\gamma)\right]$$

$$F_N^\mu(p,p') = \langle N(p')|J^\mu(0)|N(p)\rangle = \bar{u}(p',s')\left[F_1(t)\,\gamma^\mu - F_2(t)\,\frac{i\sigma^{\mu\nu}\Delta_\nu}{2m}\right] u(p,s)$$

Jefferson Lab

$n = 1$: γ^* channel --- BH subprocess

- Advantage: the quasi-real state A^* has well-defined helicity for all $n = 1, 2, 3, \ldots$



$$\mathcal{M}^{[1]} = \frac{-e}{t} F_N^\mu(p,p')\, G_\mu^\gamma(\Delta, p_2, q_1, q_2) = \frac{e}{t}\left[\sum_{\lambda=\pm 1} (F_N \cdot \epsilon_\lambda^*)(\epsilon_\lambda \cdot G^\gamma) - 2(F_N \cdot n)(\bar{n} \cdot G^\gamma)\right]$$

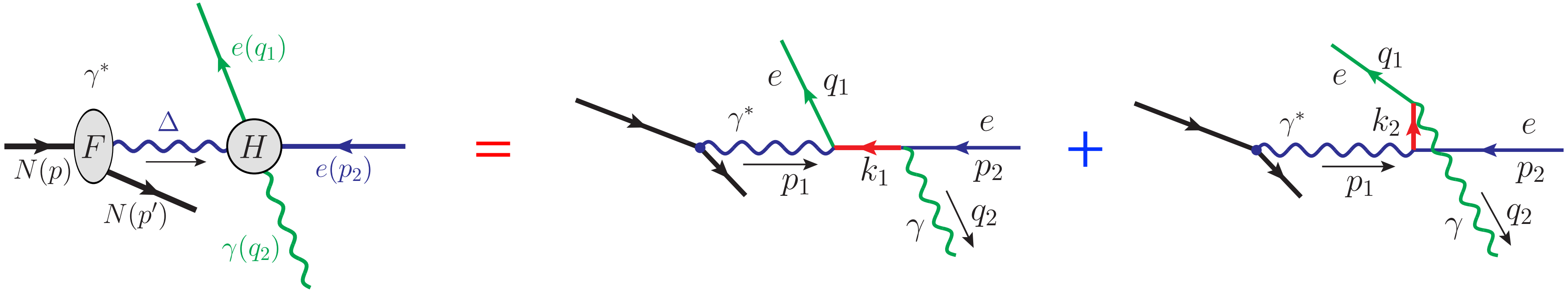
$\lambda_A^\gamma = \pm 1$ $\quad$ $\lambda_A^\gamma = 0$

$$F_N^\mu(p,p') = \langle N(p')|J^\mu(0)|N(p)\rangle = \bar{u}(p',s')\left[F_1(t)\,\gamma^\mu - F_2(t)\,\frac{i\sigma^{\mu\nu}\Delta_\nu}{2m}\right]u(p,s)$$

Jefferson Lab

$n = 1$: γ^* channel --- BH subprocess

❑ **Advantage: the quasi-real state A^* has well-defined helicity for all $n = 1, 2, 3, \ldots$**



$$\mathcal{M}^{[1]} = \frac{-e}{t} F_N^{\mu}(p, p')\, G_{\mu}^{\gamma}(\Delta, p_2, q_1, q_2) = \frac{e}{t}\left[\sum_{\lambda=\pm 1} (F_N \cdot \epsilon_\lambda^*)(\epsilon_\lambda \cdot G^\gamma) - 2(F_N \cdot n)(\bar{n} \cdot G^\gamma)\right]$$

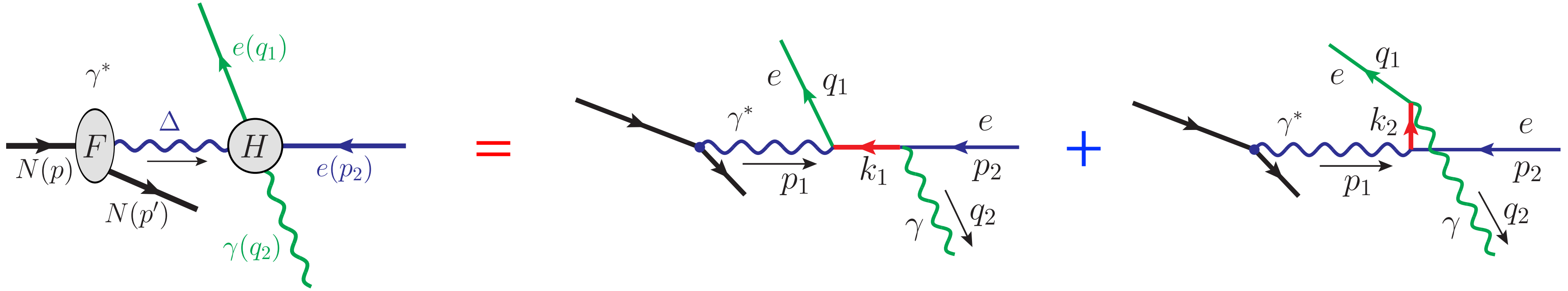
$\lambda_A^\gamma = \pm 1$ $\qquad$ $\lambda_A^\gamma = 0$

$$F_N^\mu(p, p') = \langle N(p')|J^\mu(0)|N(p)\rangle = \bar{u}(p', s')\left[F_1(t)\,\gamma^\mu - F_2(t)\,\frac{i\sigma^{\mu\nu}\Delta_\nu}{2m}\right] u(p, s)$$

- **Only the transverse polarization γ_T^* is at LP** $\qquad \mathcal{O}(1/\sqrt{-t})$
- **The longitudinal polarization γ_L^* is at NLP** $\qquad \mathcal{O}(1/q_T)$ ⟷ **Combine with $n = 2$ (DVCS)**

$n = 1$: γ^* channel --- BH subprocess

❑ **Advantage: the quasi-real state A^* has well-defined helicity for all $n = 1, 2, 3, \ldots$**



$$\mathcal{M}^{[1]} = \frac{-e}{t} F_N^\mu(p,p')\, G_\mu^\gamma(\Delta, p_2, q_1, q_2) = \frac{e}{t}\left[\sum_{\lambda=\pm1} (F_N \cdot \epsilon_\lambda^*)(\epsilon_\lambda \cdot G^\gamma) - 2(F_N \cdot n)(\bar{n} \cdot G^\gamma)\right]$$

$\lambda_A^\gamma = \pm 1$ $\qquad$ $\lambda_A^\gamma = 0$

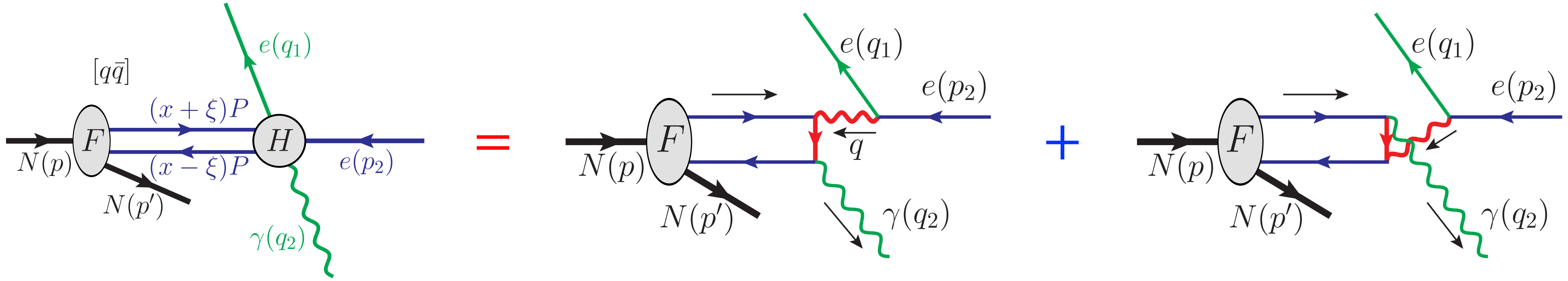
$$F_N^\mu(p,p') = \langle N(p')|J^\mu(0)|N(p)\rangle = \bar{u}(p',s')\left[F_1(t)\,\gamma^\mu - F_2(t)\,\frac{i\sigma^{\mu\nu}\Delta_\nu}{2m}\right] u(p,s)$$

- **Only the transverse polarization γ_T^* is at LP** $\quad \mathcal{O}(1/\sqrt{-t})$
- **The longitudinal polarization γ_L^* is at NLP** $\quad \mathcal{O}(1/q_T)$ ⟷ **Combine with $n = 2$ (DVCS)**

Difference from Breit frame: (1) Regular ϕ dependence; (2) γ^* goes from N to e (causality flip: space-like)

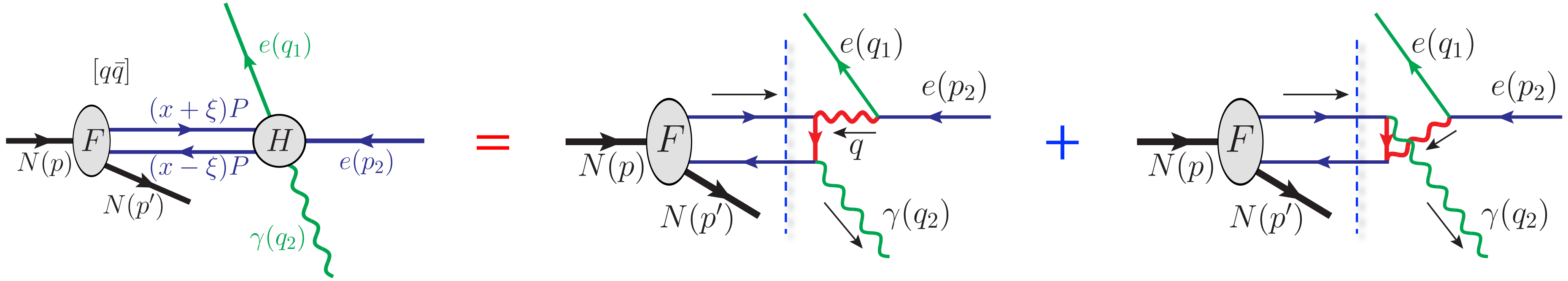
$n = 2$: $[q\bar{q}]$ channel --- DVCS (twist-2)

- Advantage: the quasi-real state A^* has well-defined helicity for all $n = 1, 2, 3, \dots$



$n = 2$: $[q\bar{q}]$ channel --- DVCS (twist-2)

- **Advantage: the quasi-real state A^* has well-defined helicity for all $n = 1, 2, 3, \ldots$**



$$\mathcal{M}^{[2]} \simeq \sum_q \int_{-1}^{1} dx \left[F^q(x,\xi,t)\, G^q(x,\xi;\hat{s},\theta,\phi) + \widetilde{F}^q(x,\xi,t)\, \widetilde{G}^q(x,\xi;\hat{s},\theta,\phi) \right] + \mathcal{O}\left(\sqrt{-t}/q_T^2\right)$$

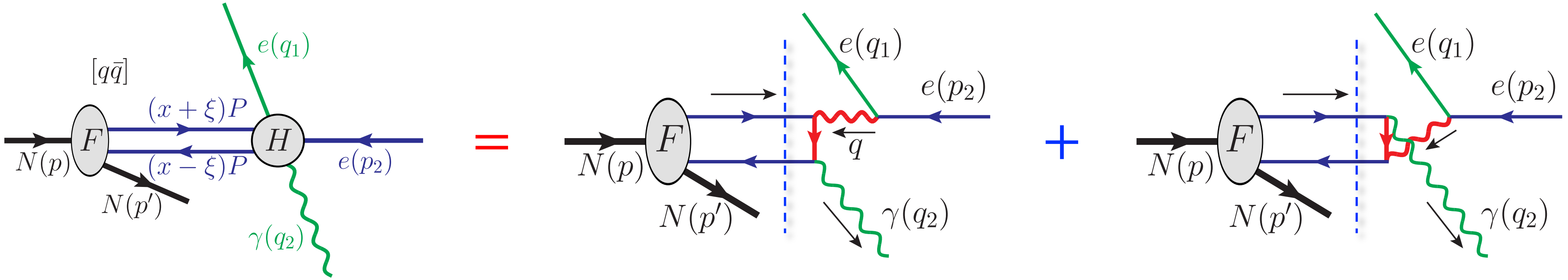
GPDs (H, E): Defined with γ^+.

GPDs $(\widetilde{H}, \widetilde{E})$: Defined with $\gamma^+\gamma_5$.

Both $(F, \widetilde{F})$ correspond to $[q\bar{q}]$ or $[gg]$ with total helicity λ_A^q or $\lambda_A^g = 0$.

$n = 2$: $[q\bar{q}]$ channel --- DVCS (twist-2)

- **Advantage: the quasi-real state A^* has well-defined helicity for all $n = 1, 2, 3, \ldots$**



$$\mathcal{M}^{[2]} \simeq \sum_q \int_{-1}^{1} dx \left[F^q(x,\xi,t)\, G^q(x,\xi;\hat{s},\theta,\phi) + \widetilde{F}^q(x,\xi,t)\, \widetilde{G}^q(x,\xi;\hat{s},\theta,\phi) \right] + \mathcal{O}\left(\sqrt{-t}/q_T^2\right)$$

GPDs (H, E): Defined with γ^+. **GPDs $(\widetilde{H}, \widetilde{E})$: Defined with $\gamma^+\gamma_5$.**

Both $(F, \widetilde{F})$ correspond to $[q\bar{q}]$ or $[gg]$ with total helicity λ_A^q or $\lambda_A^g = 0$.

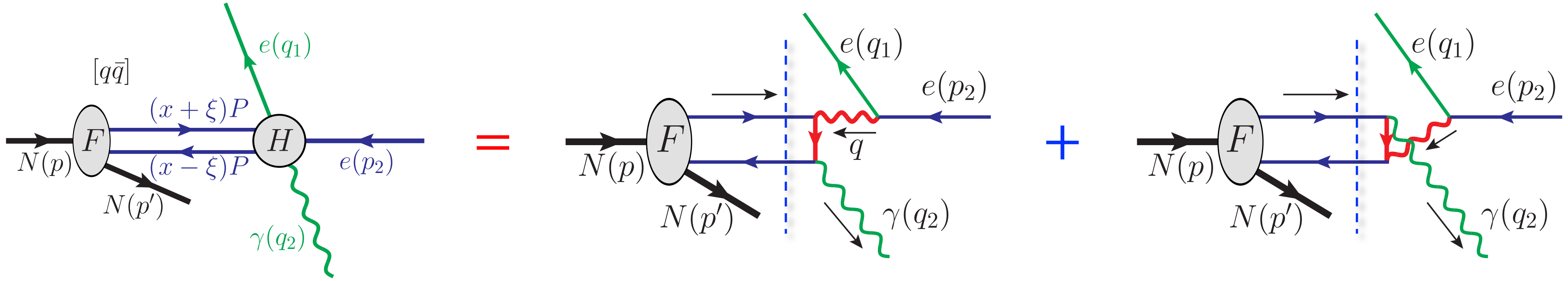
$$\{\mathcal{H}, \mathcal{E}\}(\xi,t) \equiv \sum_q e_q^2 \int_{-1}^{1} dx\, \{H^q, E^q\}(x,\xi,t) \left[\frac{1}{x-\xi+i\epsilon} + \frac{1}{x+\xi-i\epsilon} \right]$$

$$\{\widetilde{\mathcal{H}}, \widetilde{\mathcal{E}}\}(\xi,t) \equiv \sum_q e_q^2 \int_{-1}^{1} dx\, \{\widetilde{H}^q, \widetilde{E}^q\}(x,\xi,t) \left[\frac{1}{x-\xi+i\epsilon} - \frac{1}{x+\xi-i\epsilon} \right]$$

← **GPD moments**

$n = 2$: $[q\bar{q}]$ channel --- DVCS (twist-2)

❑ Advantage: the quasi-real state A^* has well-defined helicity for all $n = 1, 2, 3, \ldots$



$$\mathcal{M}^{[2]} \simeq \sum_q \int_{-1}^{1} dx \left[F^q(x,\xi,t)\, G^q(x,\xi;\hat{s},\theta,\phi) + \widetilde{F}^q(x,\xi,t)\, \widetilde{G}^q(x,\xi;\hat{s},\theta,\phi) \right] + \mathcal{O}\left(\sqrt{-t}/q_T^2\right)$$

GPDs (H, E): Defined with γ^+.

GPDs $(\widetilde{H}, \widetilde{E})$: Defined with $\gamma^+\gamma_5$.

Both $(F, \widetilde{F})$ correspond to $[q\bar{q}]$ or $[gg]$ with total helicity λ_A^q or $\lambda_A^g = 0$.

❑ Difference from Breit frame treatment

- Not separate at virtual photon $\gamma^*(q)$. Assign it to the hard part.
- In a coherent framework with BH --- “one higher twist” w.r.t. $A^* = \gamma^*$ channel
- Choose $n \propto p_2$

Jefferson Lab

Combine $n = 1$ and $n = 2$ channels

Amplitude level

LP $\mathcal{M}_{\rm I}: \quad A^* = \gamma_T^* \ (\lambda_A^\gamma = \pm 1)$

NLP $\mathcal{M}_{\rm II}: \quad (1)\ A^* = \gamma_L^* \ (\lambda_A^\gamma = 0); \quad (2)\ A^* = [q\bar{q}] \ (\lambda_A^q = 0)$ $+ [gg]$ **(high order)**

NNLP: …

Cross section level

$$|\mathcal{M}_{\rm I} + \mathcal{M}_{\rm II} + \cdots|^2 = \underbrace{|\mathcal{M}_{\rm I}|^2}_{\rm LP} + \underbrace{2\,{\rm Re}\,(\mathcal{M}_{\rm I}\mathcal{M}_{\rm II}^*)}_{\rm NLP} + \cdots$$

Combine $n = 1$ and $n = 2$ channels

❑ Amplitude level

LP $\mathcal{M}_\mathrm{I}: \quad A^* = \gamma_T^* \; (\lambda_A^\gamma = \pm 1)$

NLP $\mathcal{M}_\mathrm{II}: \quad (1)\; A^* = \gamma_L^* \; (\lambda_A^\gamma = 0); \quad (2)\; A^* = [q\bar{q}] \; (\lambda_A^q = 0)$ + $[gg]$ **(high order)**

NNLP: …

❑ Cross section level

$$|\mathcal{M}_\mathrm{I} + \mathcal{M}_\mathrm{II} + \cdots|^2 = \underbrace{|\mathcal{M}_\mathrm{I}|^2}_{\mathrm{LP}} + \underbrace{2\,\mathrm{Re}\,(\mathcal{M}_\mathrm{I}\mathcal{M}_\mathrm{II}^*)}_{\mathrm{NLP}} + \cdots$$

LP $|\mathcal{M}|^2_\mathrm{LP} = |\mathcal{M}_\mathrm{I}|^2$ **No ϕ modulation. $\lambda_A^\gamma = +1$ and $\lambda_A^\gamma = -1$ do NOT interfere until NNLP.**

Combine $n = 1$ and $n = 2$ channels

❑ Amplitude level

LP $\mathcal{M}_\mathrm{I}: \quad A^* = \gamma_T^* \ (\lambda_A^\gamma = \pm 1)$

NLP $\mathcal{M}_\mathrm{II}: \quad (1)\ A^* = \gamma_L^* \ (\lambda_A^\gamma = 0); \quad (2)\ A^* = [q\bar{q}] \ (\lambda_A^q = 0)$ $+ [gg]$ **(high order)**

NNLP: …

❑ Cross section level

$$|\mathcal{M}_\mathrm{I} + \mathcal{M}_\mathrm{II} + \cdots|^2 = \underbrace{|\mathcal{M}_\mathrm{I}|^2}_{\mathrm{LP}} + \underbrace{2\,\mathrm{Re}\,(\mathcal{M}_\mathrm{I}\mathcal{M}_\mathrm{II}^*)}_{\mathrm{NLP}} + \cdots$$

LP $|\mathcal{M}|^2_\mathrm{LP} = |\mathcal{M}_\mathrm{I}|^2$ **No ϕ modulation. $\lambda_A^\gamma = +1$ and $\lambda_A^\gamma = -1$ do NOT interfere until NNLP.**

NLP $|\mathcal{M}|^2_\mathrm{NLP} = 2\,\mathrm{Re}\,(\mathcal{M}_\mathrm{I}\mathcal{M}_\mathrm{II}^*)$ ➡ $\cos\phi$ **or** $\sin\phi$ **modulation.**

Combine $n = 1$ and $n = 2$ channels

❑ Amplitude level

LP $\mathcal{M}_{\rm I}$: $A^* = \gamma_T^*$ $(\lambda_A^\gamma = \pm 1)$

NLP $\mathcal{M}_{\rm II}$: (1) $A^* = \gamma_L^*$ $(\lambda_A^\gamma = 0)$; (2) $A^* = [q\bar{q}]$ $(\lambda_A^q = 0)$ + $[gg]$ **(high order)**

NNLP: …

❑ Cross section level

$$|\mathcal{M}_{\rm I} + \mathcal{M}_{\rm II} + \cdots|^2 = \underbrace{|\mathcal{M}_{\rm I}|^2}_{\rm LP} + \underbrace{2\,{\rm Re}\,(\mathcal{M}_{\rm I}\mathcal{M}_{\rm II}^*)}_{\rm NLP} + \cdots$$

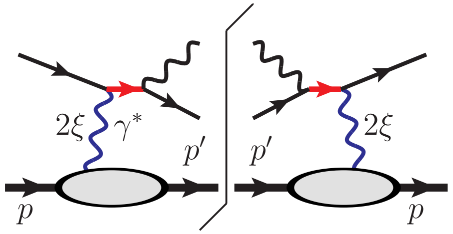
LP $|\mathcal{M}|^2_{\rm LP} = |\mathcal{M}_{\rm I}|^2$ **No ϕ modulation. $\lambda_A^\gamma = +1$ and $\lambda_A^\gamma = -1$ do NOT interfere until NNLP.**

NLP $|\mathcal{M}|^2_{\rm NLP} = 2\,{\rm Re}\,(\mathcal{M}_{\rm I}\mathcal{M}_{\rm II}^*)$ → **$\cos\phi$ or $\sin\phi$ modulation.**

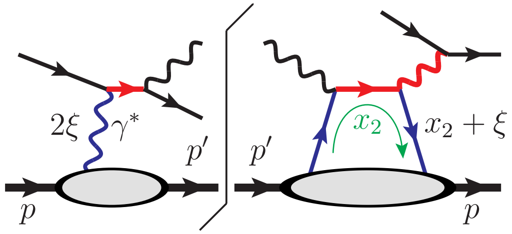
→ **Interference of different numbers of particles.**

↓

Unique feature to QFT, beyond non-rel. QM!



"twist-2"



"twist-3"

Cross section within NLP

$$\frac{d\sigma}{dt\, d\xi\, d\phi_S\, d\cos\theta\, d\phi} = \frac{1}{(2\pi)^2} \frac{d\sigma^{\rm unpol}}{dt\, d\xi\, d\cos\theta} \cdot \Big[1 + \lambda_e \lambda_N A_{LL}^{\rm LP} + \lambda_e s_T A_{TL}^{\rm LP} \cos\phi_S$$
$$+ \left(A_{UU}^{\rm NLP} + \lambda_e \lambda_N A_{LL}^{\rm NLP} \right) \cos\phi + \left(\lambda_e A_{UL}^{\rm NLP} + \lambda_N A_{LU}^{\rm NLP} \right) \sin\phi$$
$$+ s_T \left(A_{TU,1}^{\rm NLP} \cos\phi_S \sin\phi + A_{TU,2}^{\rm NLP} \sin\phi_S \cos\phi \right)$$
$$+ \lambda_e s_T \left(A_{TL,1}^{\rm NLP} \cos\phi_S \cos\phi + A_{TL,2}^{\rm NLP} \sin\phi_S \sin\phi \right) \Big]$$

In the experimental setting (fixed lab frame),

- **Nucleon spin vector** $\vec{s}_N = (s_T, 0, \lambda_N)$
- **Electron spin vector** $\vec{s}_e = (0, 0, \lambda_e)$

Subscripts: (nucleon, electron)

U = Unpolarized
L = Longitudinally polarized
T = Transversely polarized

Cross section within NLP

$$\frac{d\sigma}{dt\, d\xi\, d\phi_S\, d\cos\theta\, d\phi} = \frac{1}{(2\pi)^2} \frac{d\sigma^{\rm unpol}}{dt\, d\xi\, d\cos\theta} \cdot \Big[1 + \lambda_e \lambda_N A_{LL}^{\rm LP} + \lambda_e s_T A_{TL}^{\rm LP} \cos\phi_S$$
$$+ \left(A_{UU}^{\rm NLP} + \lambda_e \lambda_N A_{LL}^{\rm NLP}\right) \cos\phi + \left(\lambda_e A_{UL}^{\rm NLP} + \lambda_N A_{LU}^{\rm NLP}\right) \sin\phi$$
$$+ s_T \left(A_{TU,1}^{\rm NLP} \cos\phi_S \sin\phi + A_{TU,2}^{\rm NLP} \sin\phi_S \cos\phi\right)$$
$$+ \lambda_e s_T \left(A_{TL,1}^{\rm NLP} \cos\phi_S \cos\phi + A_{TL,2}^{\rm NLP} \sin\phi_S \sin\phi\right) \Big]$$

LP: from γ_T^* squared

- **Control the rate (unpolarized cross section). No ϕ modulation.**
- **Only a $\cos\phi_S$ modulation**
- **No single spin asymmetry, only double spin asymmetries**

Cross section within NLP

$$\frac{d\sigma}{dt\,d\xi\,d\phi_S\,d\cos\theta\,d\phi} = \frac{1}{(2\pi)^2}\frac{d\sigma^{\text{unpol}}}{dt\,d\xi\,d\cos\theta}\cdot\Big[1 + \lambda_e\lambda_N A_{LL}^{\text{LP}} + \lambda_e s_T A_{TL}^{\text{LP}}\cos\phi_S$$
$$+ \left(A_{UU}^{\text{NLP}} + \lambda_e\lambda_N A_{LL}^{\text{NLP}}\right)\cos\phi + \left(\lambda_e A_{UL}^{\text{NLP}} + \lambda_N A_{LU}^{\text{NLP}}\right)\sin\phi$$
$$+ s_T\left(A_{TU,1}^{\text{NLP}}\cos\phi_S\sin\phi + A_{TU,2}^{\text{NLP}}\sin\phi_S\cos\phi\right)$$
$$+ \lambda_e s_T\left(A_{TL,1}^{\text{NLP}}\cos\phi_S\cos\phi + A_{TL,2}^{\text{NLP}}\sin\phi_S\sin\phi\right)\Big]$$

LP: from γ_T^* squared

$$\frac{d\sigma^{\text{unpol}}}{dt\,d\xi\,d\cos\theta} = \frac{\alpha_e^3}{(1+\xi)^2}\frac{m^2}{s\,t^2}\,\Sigma_{UU}^{\text{LP}}$$

$$\Sigma_{UU}^{\text{LP}} = \left[\frac{1}{\sin^2(\theta/2)} + \sin^2(\theta/2)\right]\left[\left(\frac{1-\xi^2}{2\xi^2}\frac{-t}{m^2} - 2\right)\left(F_1^2 - \frac{t}{4m^2}F_2^2\right) - \frac{t}{m^2}(F_1+F_2)^2\right]$$

$$A_{LL}^{\text{LP}} = \frac{1}{\Sigma_{UU}^{\text{LP}}}\cdot\left[\frac{1}{\sin^2(\theta/2)} - \sin^2(\theta/2)\right](F_1+F_2)\left[F_1\left(\frac{-t}{\xi m^2} - \frac{4\xi}{1+\xi}\right) - \frac{t}{m^2}F_2\right]$$

$$A_{TL}^{\text{LP}} = \frac{1}{\Sigma_{UU}^{\text{LP}}}\cdot\frac{\Delta_T}{2m}\left[\frac{1}{\sin^2(\theta/2)} - \sin^2(\theta/2)\right](F_1+F_2)\left[-4F_1 + \frac{1+\xi}{\xi}\frac{-t}{m^2}F_2\right]$$

Quadratic in (F_1, F_2)

Cross section within NLP

$$\frac{d\sigma}{dt\, d\xi\, d\phi_S\, d\cos\theta\, d\phi} = \frac{1}{(2\pi)^2}\frac{d\sigma^{\text{unpol}}}{dt\, d\xi\, d\cos\theta} \cdot \Big[1 + \lambda_e \lambda_N A_{LL}^{\text{LP}} + \lambda_e s_T A_{TL}^{\text{LP}} \cos\phi_S$$

$$+ \left(A_{UU}^{\text{NLP}} + \lambda_e \lambda_N A_{LL}^{\text{NLP}}\right)\cos\phi + \left(\lambda_e A_{UL}^{\text{NLP}} + \lambda_N A_{LU}^{\text{NLP}}\right)\sin\phi$$

$$+ s_T \left(A_{TU,1}^{\text{NLP}} \cos\phi_S \sin\phi + A_{TU,2}^{\text{NLP}} \sin\phi_S \cos\phi\right)$$

$$+ \lambda_e s_T \left(A_{TL,1}^{\text{NLP}} \cos\phi_S \cos\phi + A_{TL,2}^{\text{NLP}} \sin\phi_S \sin\phi\right)\Big]$$

NLP: from γ_T^*-γ_L^* and γ_T^*-$[q\bar{q}]$ interference

- **No contribution to the rate,**
  $\Longrightarrow$ only to azimuthal modulations $(\cos\phi, \sin\phi)$
- **Unpolarized part A_{UU}, SSA, and DSA**

Cross section within NLP

$$\frac{d\sigma}{dt\,d\xi\,d\phi_S\,d\cos\theta\,d\phi} = \frac{1}{(2\pi)^2}\frac{d\sigma^{\rm unpol}}{dt\,d\xi\,d\cos\theta}\cdot\Big[1+\lambda_e\lambda_N A_{LL}^{\rm LP}+\lambda_e s_T A_{TL}^{\rm LP}\cos\phi_S$$

$$+\left(A_{UU}^{\rm NLP}+\lambda_e\lambda_N A_{LL}^{\rm NLP}\right)\cos\phi+\left(\lambda_e A_{UL}^{\rm NLP}+\lambda_N A_{LU}^{\rm NLP}\right)\sin\phi$$

$$+s_T\left(A_{TU,1}^{\rm NLP}\cos\phi_S\sin\phi+A_{TU,2}^{\rm NLP}\sin\phi_S\cos\phi\right)$$

$$+\lambda_e s_T\left(A_{TL,1}^{\rm NLP}\cos\phi_S\cos\phi+A_{TL,2}^{\rm NLP}\sin\phi_S\sin\phi\right)\Big]$$

NLP: from γ_T^*-γ_L^* and γ_T^*-$[q\bar{q}]$ interference

$$A_{XX}^{\rm NLP}=\frac{1}{\Sigma_{UU}^{\rm LP}}\cdot\left(\frac{-t}{m\sqrt{\hat{s}}}\right)\Sigma_{XX}^{\rm NLP}$$

$$\Sigma_{UU}^{\rm NLP}=\frac{\Delta_T}{2m}\frac{1+\xi}{\xi}\left[\frac{2\sin\theta}{\xi}\left(F_1^2-\frac{t}{4m^2}F_2^2\right)-\frac{4+(1-\cos\theta)^2}{\sin\theta\cos^2(\theta/2)}(M_1\cdot{\rm Re}\,V_{\mathcal F})\right],$$

$$\Sigma_{LL}^{\rm NLP}=-\frac{\Delta_T}{m}\left[\sin\theta(F_1+F_2)\left(\frac{1+\xi}{\xi}F_1+F_2\right)+\frac{3-\cos\theta}{\sin\theta}(M_2\cdot{\rm Re}\,V_{\mathcal F})\right],$$

$$\Sigma_{TL,1}^{\rm NLP}=2\sin\theta\,(F_1+F_2)\left[F_1+\left(\frac{\xi}{1+\xi}+\frac{t}{4\xi m^2}\right)F_2\right]+\frac{2(3-\cos\theta)}{\sin\theta}(M_3\cdot{\rm Re}\,V_{\mathcal F}),$$

$$\Sigma_{TL,2}^{\rm NLP}=2\sin\theta\,(F_1+F_2)\left(F_1+\frac{t}{4m^2}F_2\right)-\frac{2(3-\cos\theta)}{\sin\theta}(M_4\cdot{\rm Re}\,V_{\mathcal F}),$$

$$\Sigma_{UL}^{\rm NLP}=-\frac{\Delta_T}{m}\frac{1+\xi}{\xi}\frac{3-\cos\theta}{\sin\theta}(M_1\cdot{\rm Im}\,V_{\mathcal F}),$$

$$\Sigma_{LU}^{\rm NLP}=-\frac{\Delta_T}{2m}\frac{4+(1-\cos\theta)^2}{\sin\theta\cos^2(\theta/2)}(M_2\cdot{\rm Im}\,V_{\mathcal F}),$$

$$\Sigma_{TU,1}^{\rm NLP}=\frac{4+(1-\cos\theta)^2}{\sin\theta\cos^2(\theta/2)}(M_3\cdot{\rm Im}\,V_{\mathcal F}),$$

$$\Sigma_{TU,2}^{\rm NLP}=\frac{4+(1-\cos\theta)^2}{\sin\theta\cos^2(\theta/2)}(M_4\cdot{\rm Im}\,V_{\mathcal F}).$$

- **Linear in GPD moments** $V_{\mathcal F}=(\mathcal{H},\mathcal{E},\widetilde{\mathcal{H}},\widetilde{\mathcal{E}})^T$
- **Controlled by the real matrix *M*, same for real and imaginary parts of GPD moments**

  $$M_i=(M_{i1},M_{i2},M_{i3},M_{i4})$$ **(see next slide)**
- **8 asymmetries ⇔ 8 (real) GPD moments**

Cross section within NLP

$$\frac{d\sigma}{dt\,d\xi\,d\phi_S\,d\cos\theta\,d\phi} = \frac{1}{(2\pi)^2}\frac{d\sigma^{\text{unpol}}}{dt\,d\xi\,d\cos\theta}\cdot\Big[1+\lambda_e\lambda_N A_{LL}^{\text{LP}} + \lambda_e s_T A_{TL}^{\text{LP}}\cos\phi_S$$

$$+\left(A_{UU}^{\text{NLP}}+\lambda_e\lambda_N A_{LL}^{\text{NLP}}\right)\cos\phi + \left(\lambda_e A_{UL}^{\text{NLP}}+\lambda_N A_{LU}^{\text{NLP}}\right)\sin\phi$$

$$+ s_T\left(A_{TU,1}^{\text{NLP}}\cos\phi_S\sin\phi + A_{TU,2}^{\text{NLP}}\sin\phi_S\cos\phi\right)$$

$$+\lambda_e s_T\left(A_{TL,1}^{\text{NLP}}\cos\phi_S\cos\phi + A_{TL,2}^{\text{NLP}}\sin\phi_S\sin\phi\right)\Big]$$

NLP: from γ_T^*-γ_L^* and γ_T^*-$[q\bar{q}]$ interference

$$M = \begin{bmatrix} F_1 & -\frac{t}{4m^2}F_2 & \xi(F_1+F_2) & 0 \\ (1+\xi)(F_1+F_2) & \xi(F_1+F_2) & \frac{1+\xi}{\xi}F_1 & -\xi F_1-(1+\xi)\frac{t}{4m^2}F_2 \\ \xi(F_1+F_2) & \left(\frac{\xi^2}{1+\xi}+\frac{t}{4m^2}\right)(F_1+F_2) & -\xi F_1+\frac{t}{4m^2}\frac{1-\xi^2}{\xi}F_2 & -\left(\frac{\xi^2}{1+\xi}+\frac{t}{4m^2}\right)F_1-\frac{\xi t}{4m^2}F_2 \\ \xi(F_1+F_2) & \frac{\xi t}{4m^2}(F_1+F_2) & -\xi F_1+\frac{t}{4m^2}\frac{1-\xi^2}{\xi}F_2 & -\left(\xi+\frac{t}{4\xi m^2}\right)F_1-\frac{\xi t}{4m^2}F_2 \end{bmatrix}$$

← M_1
← M_2
← M_3
← M_4

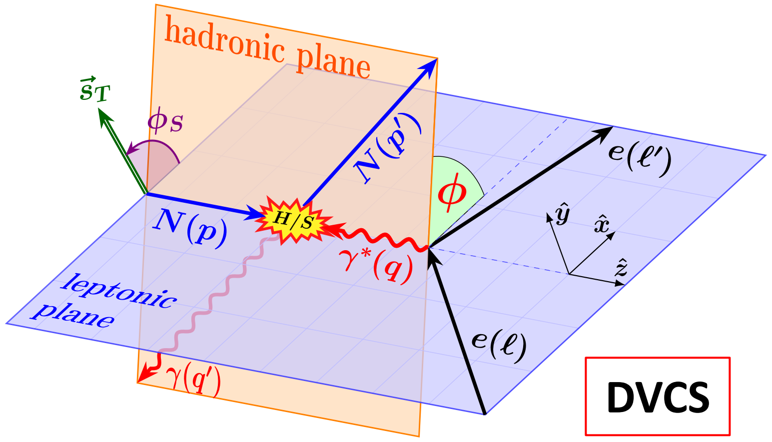
$$M\cdot\begin{bmatrix}\mathcal{H}\\ \mathcal{E}\\ \widetilde{\mathcal{H}}\\ \widetilde{\mathcal{E}}\end{bmatrix} = \begin{bmatrix}\hat{V}_1\\ \hat{V}_2\\ \hat{V}_3\\ \hat{V}_4\end{bmatrix}$$

← **Reconstructed from experiments (complex valued)**

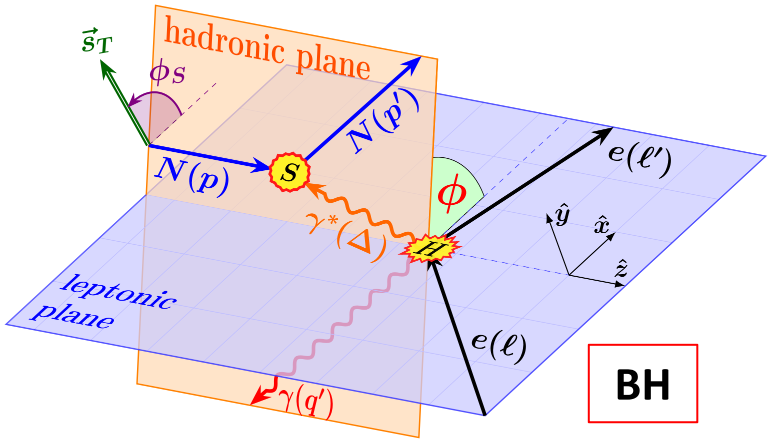
$\det M \neq 0$ → **Unique solution for GPD moments!**

Comparison between SDHEP frame and Breit frame

❑ Breit frame: centered around $\gamma^*(q)$



DVCS



BH

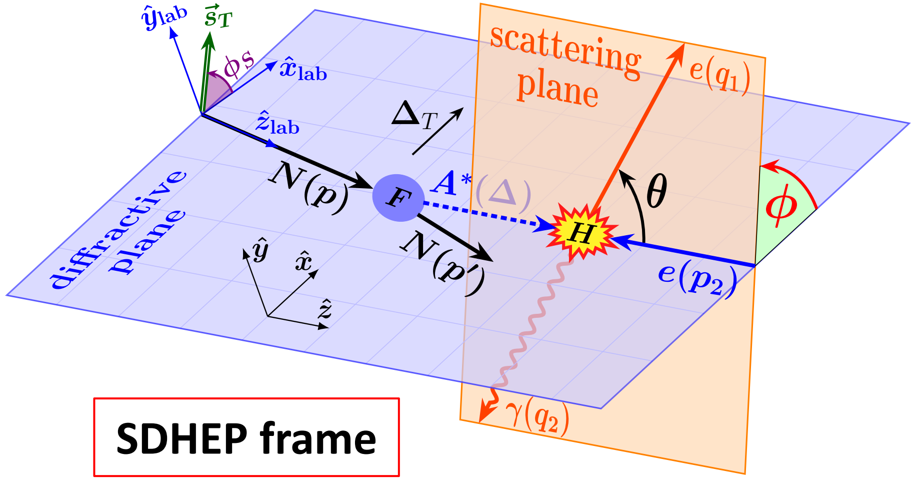
"Incoherent" treatments for DVCS and BH

⬇

Makes their interference calculation difficult

DVCS-square is in fact the least important!

❑ SDHEP frame: centered around $A^*(\Delta)$



SDHEP frame

- Clear physical picture: scale separation
- $A^* = \gamma^*, [q\bar{q}], [gg], [q\bar{q}g], [ggg], \ldots$
- Azimuthal distribution is *dynamical* when initial-state $\parallel z$
- Unique frame for a coherent azimuthal description

x-dependence

x-dependence problem: LO scaling

No matter which frame to work in, sensitivity to GPD is the same:

$$F_0^+(\xi,t) = \int_{-1}^{1} dx \frac{F^+(x,\xi,t)}{x-\xi+i\epsilon}$$

→ **"Scaling integral": independent of Q, q_T, or θ at leading order**

→ **Predictable θ shape. E.g.,** $\Sigma_{UU}^{\mathrm{NLP}} = \frac{\Delta_T}{2m}\frac{1+\xi}{\xi}\left[\frac{2\sin\theta}{\xi}\left(F_1^2 - \frac{t}{4m^2}F_2^2\right) - \frac{4+(1-\cos\theta)^2}{\sin\theta\cos^2(\theta/2)}(M_1\cdot \mathrm{Re}\,V_{\mathcal{F}})\right]$

Unknown but does not affect θ shape

- **Advantage:** **Helps to experimentally confirm parton-dominated dynamics (i.e., parton model)**
- **Disadvantage:** **Difficult to extract x-dependence of GPDs**

→ **Shadow GPD problem**

$$\int_{-1}^{1} dx \frac{S(x,\xi,t)}{x-\xi+i\epsilon} = 0$$

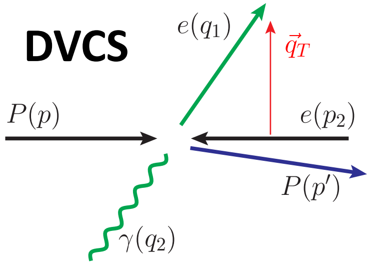
$$S(\pm\xi,\xi,t) = S(x,0,0) = 0$$

See Eric's lecture

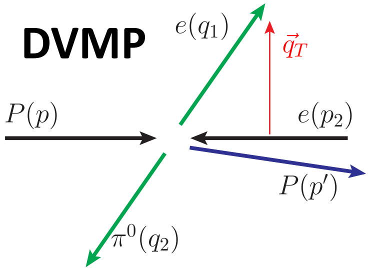
Classification of SDHEPs

❑ Electro-production (JLab, EIC, …)

DVCS

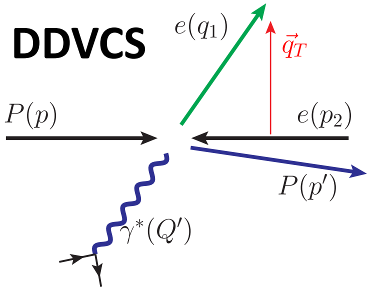


DVMP



add virtuality

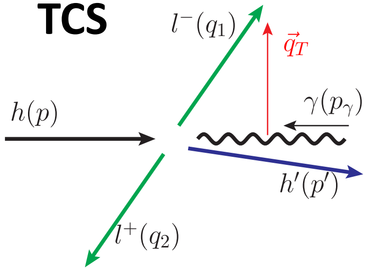
DDVCS

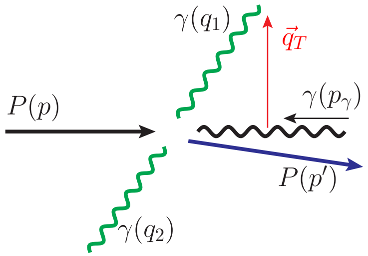


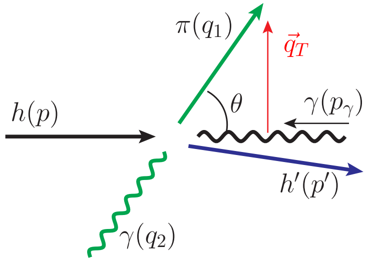
DHQP…

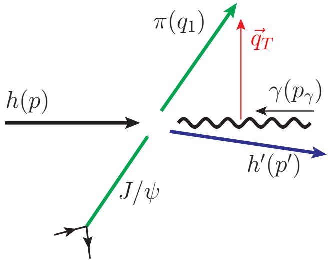
❑ Photo-production (JLab, EIC, …)

TCS





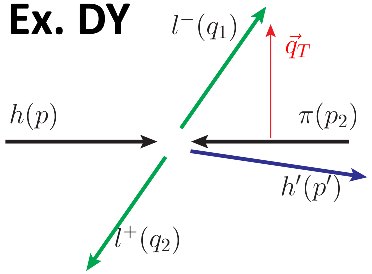


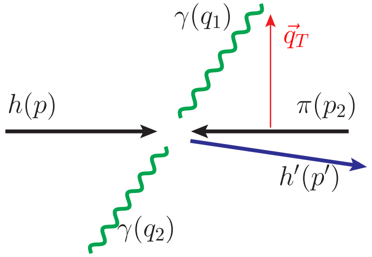


…

❑ Meso-production (AMBER, J-PARC, …)

Ex. DY





…

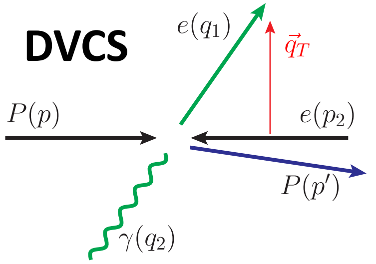
Generic discussion

[Qiu, Yu, PRD 107 (2023), 014007]

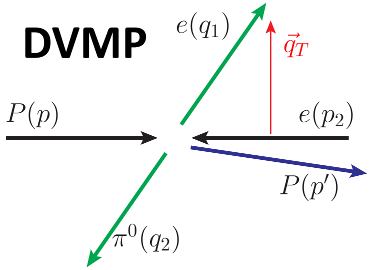
Classification of SDHEPs

❑ Electro-production (JLab, EIC, …)

DVCS

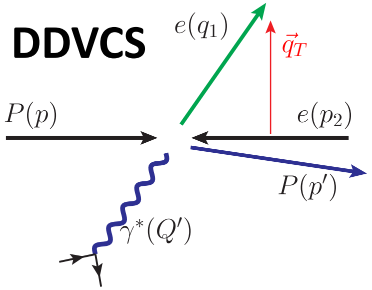


DVMP



add virtuality

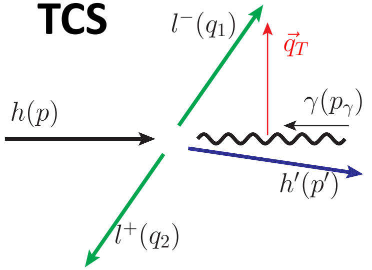
DDVCS

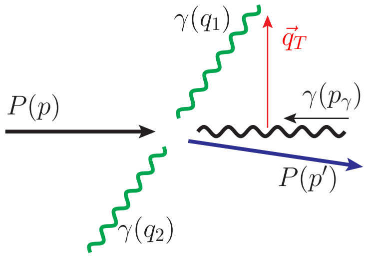


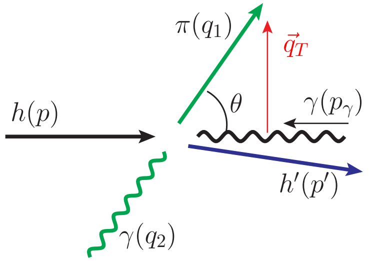
DHQP…

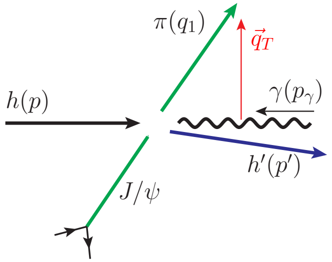
❑ Photo-production (JLab, EIC, …)

TCS





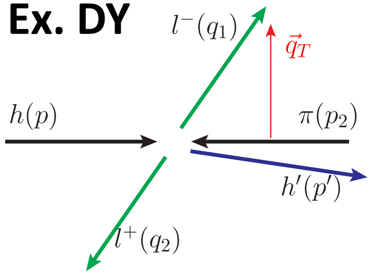


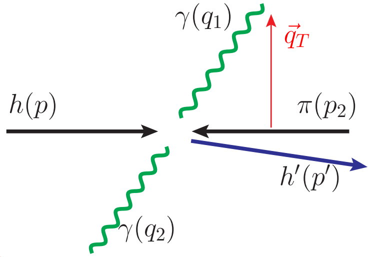


…

❑ Meso-production (AMBER, J-PARC, …)

Ex. DY



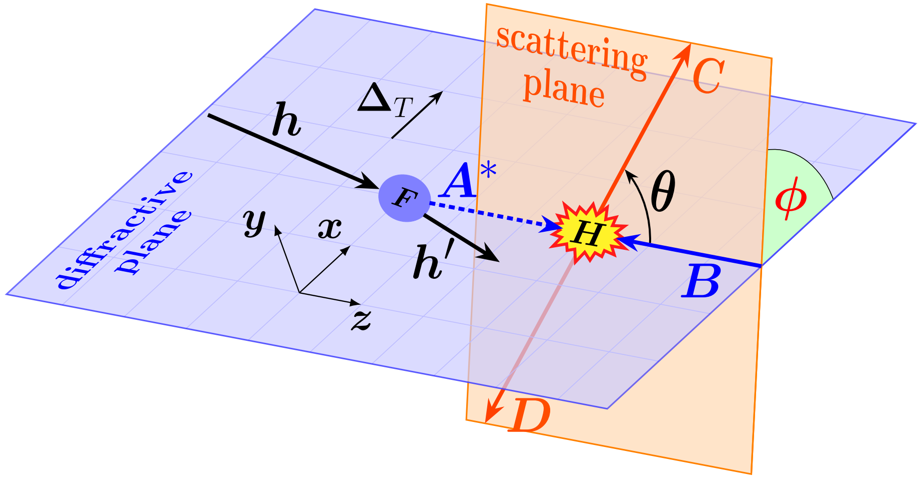


…

Generic discussion

[Qiu, Yu, PRD 107 (2023), 014007]

Where does the x-sensitivity come from?



❑ x-sensitivity ⇔ 2 → 2 hard scattering

Kinematics:

1. $\hat{s} = 2\,\xi\, s\,/\,(1+\xi)$ ⟵ ξ
2. θ or $q_T = (\sqrt{\hat{s}}/2)\sin\theta$ ⟷ x
3. ϕ ⟵ (A^*B) spin states

$$\mathcal{M}(Q,\phi) \simeq \sum_A e^{i(\lambda_A-\lambda_B)\phi} \cdot \int_{-1}^{1} dx\, F_A(x)\, C_A(x;Q) \qquad (Q = \theta \text{ or } q_T)$$

[suppressing t and ξ dependence]

- **Moment-type sensitivity** $C(x;Q) = G(x)\cdot T(Q)$ ⟹ $F_G = \int_{-1}^{1} dx\, G(x)\, F(x,\xi,t)$ **Independent of Q. Scaling for F_G.**

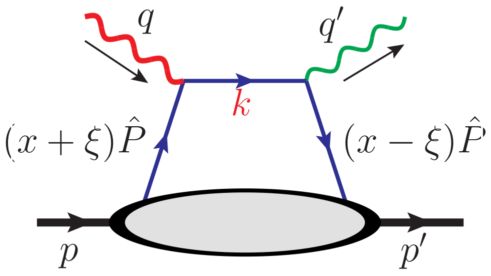
➡ **Inversion problem: *shadow GPD*** $S_G = \int_{-1}^{1} dx\, G(x)\, S(x,\xi) = 0$ [Bertone et al. PRD `21]

- **Enhanced sensitivity** $C(x;Q) \neq G(x)\cdot T(Q)$ ⟹ $d\sigma/dQ \sim \left|C(x;Q)\otimes_x F(x,\xi,t)\right|^2$

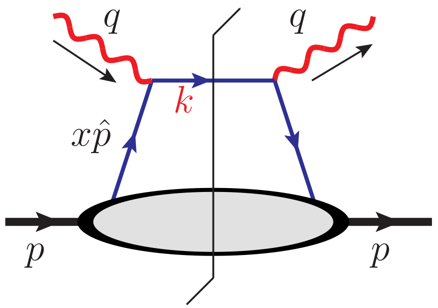
Scaling *breaking* at LO

Scaling kernels and moment sensitivity

Origin of scaling: **massless parton** **approximation + massless external states.**



DVCS



DIS

$q'^2 = 0$

$$k^2 = \left[q' + (x-\xi)\hat{P}\right]^2$$
$$= (x-\xi)\,(2\hat{P}\cdot q')$$

$$\Rightarrow \int_{-1}^{1} dx \frac{F(x,\xi,t)}{x-\xi+i\epsilon}$$

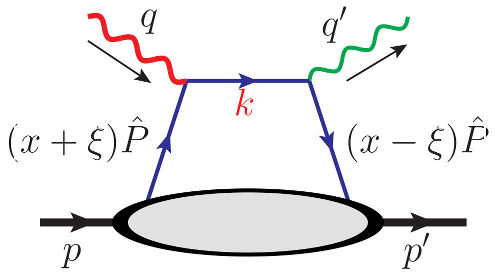
$$k^2 = [q + x\hat{p}]^2$$
$$= x\,(2\hat{p}\cdot q) - Q^2$$
$$= (2\hat{p}\cdot q)\,(x - x_B)$$

$$\Rightarrow \int dx f(x)\delta(x-\xi) = f(x_B)$$

Exercise: **Show for DVCS (at leading power)** $2\hat{P}\cdot q' = \frac{Q^2}{2\xi}$

Enhancing sensitivity by breaking the scaling

Origin of scaling: massless parton approximation + massless external states.



DVCS

$q'^2 = 0$

$$k^2 = \left[q' + (x-\xi)\hat{P}\right]^2 = (x-\xi)\,(2\hat{P}\cdot q')$$

$$\Rightarrow \int_{-1}^{1} dx \frac{F(x,\xi,t)}{x-\xi+i\epsilon}$$

$\Rightarrow$ **DDVCS** $q'^2 = Q'^2 > 0$

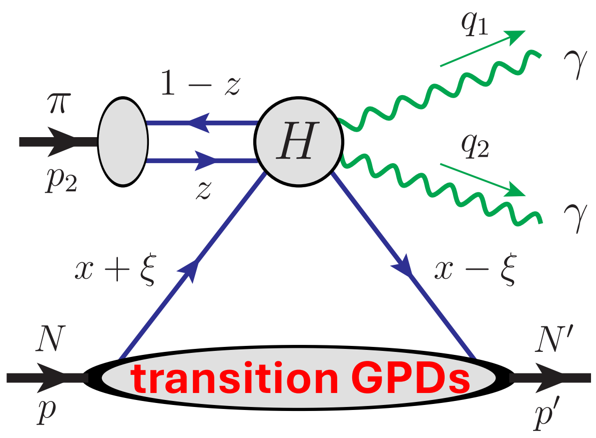
$$k^2 = \left[q' + (x-\xi)\hat{P}\right]^2 = (x-\xi)\,(2\hat{P}\cdot q') + Q'^2 = \frac{Q^2+Q'^2}{2\xi}\left[x - \xi\left(\frac{Q^2-Q'^2}{Q^2+Q'^2}\right)\right]$$

$$\Rightarrow \int_{-1}^{1} dx \frac{F(x,\xi,t)}{x-\xi\left(\frac{Q^2-Q'^2}{Q^2+Q'^2}\right)+i\epsilon}$$

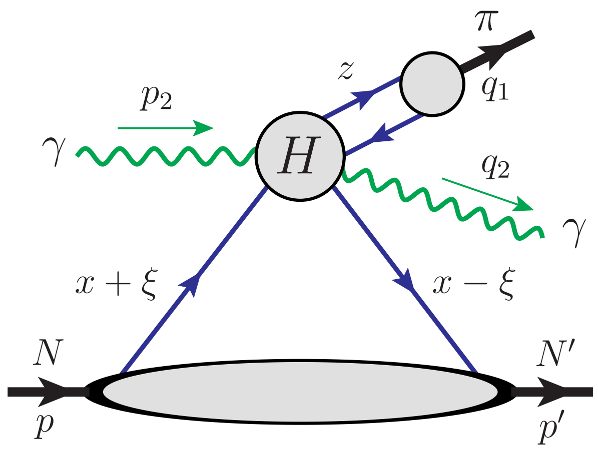
Scaling violation

Exercise: **Show for DDVCS (at leading power)** $2\hat{P}\cdot q' = \dfrac{Q^2+Q'^2}{2\xi}$

Two new example processes with enhanced x-sensitivity



J-PARC, AMBER



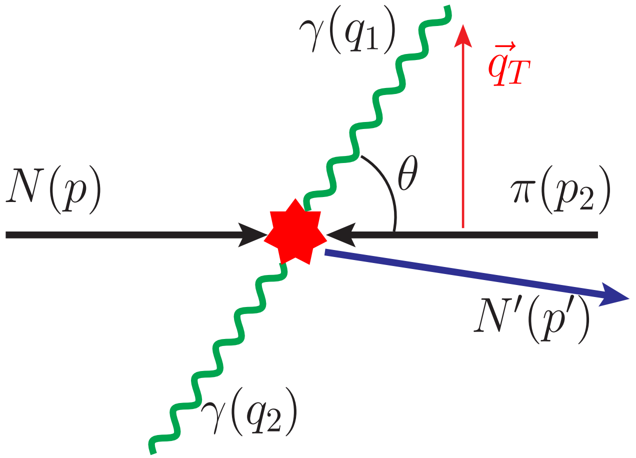
JLab Hall D

Qiu & Yu, JHEP 08 (2022) 103
Qiu & Yu, PRD 109 (2024) 074023

G. Duplancic et al., JHEP 11 (2018) 179
G. Duplancic et al., JHEP 03 (2023) 241
G. Duplancic et al., PRD 107 (2023), 094023
Qiu & Yu, PRD 107 (2023), 014007
Qiu & Yu, PRL 131 (2023), 161902

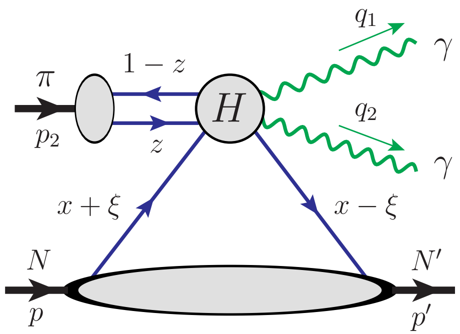
Enhanced x-sensitivity: (1) diphoton mesoproduction

[Qiu & Yu, JHEP 08 (2022) 103;
PRD 109 (2024) 074023]



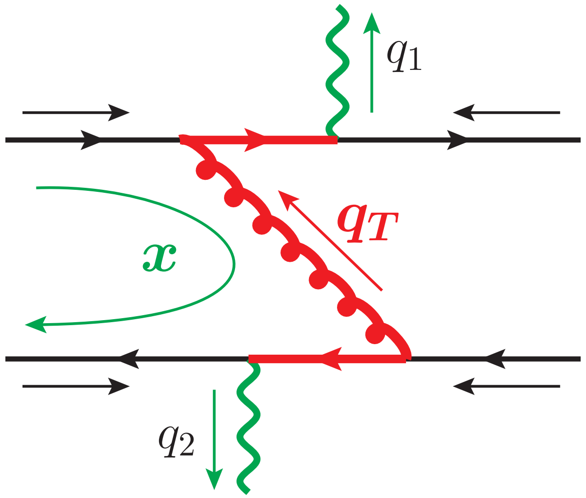
In addition to

$$F_0(\xi, t) = \int_{-1}^{1} \frac{dx\, F(x,\xi,t)}{x - \xi + i\epsilon}$$

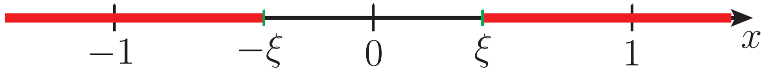


$i\mathcal{M}$ **also contains**

$$I(t, \xi; z, \theta) = \int_{-1}^{1} \frac{dx\, F(x,\xi,t)}{x - \rho(z;\theta) + i\epsilon\,\mathrm{sgn}\left[\cos^2(\theta/2) - z\right]}$$



$$\rho(z;\theta) = \xi \cdot \left[\frac{1 - z + \tan^2(\theta/2)\, z}{1 - z - \tan^2(\theta/2)\, z}\right] \in (-\infty, -\xi] \cup [\xi, \infty)$$



Enhanced x-sensitivity: (1) diphoton mesoproduction

[Qiu & Yu, PRD 109 (2024) 074023]

- **Diphoton process:** $N\pi \to N'\gamma\gamma$: (1) $p\pi^- \to n\gamma\gamma$; (2) $n\pi^+ \to p\gamma\gamma$

$$\frac{d\sigma}{d|t|\,d\xi\,d\cos\theta} = 2\pi\left(\alpha_e\alpha_s\frac{C_F}{N_c}\right)^2\frac{1}{\xi^2 s^3}\cdot\Bigg[(1-\xi^2)\sum_{\alpha=\pm}\left(|\mathcal{M}_\alpha^{[\widetilde{H}]}|^2+|\widetilde{\mathcal{M}}_\alpha^{[H]}|^2\right)-\left(\xi^2+\frac{t}{4m^2}\right)\sum_{\alpha=\pm}|\widetilde{\mathcal{M}}_\alpha^{[E]}|^2 - \frac{\xi^2 t}{4m^2}\sum_{\alpha=\pm}|\mathcal{M}_\alpha^{[\widetilde{E}]}|^2 - 2\xi^2\sum_{\alpha=\pm}\mathrm{Re}\left(\widetilde{\mathcal{M}}_\alpha^{[H]}\widetilde{\mathcal{M}}_\alpha^{[E]*}+\mathcal{M}_\alpha^{[\widetilde{H}]}\mathcal{M}_\alpha^{[\widetilde{E}]*}\right)\Bigg]$$

Nucleon transition GPDs

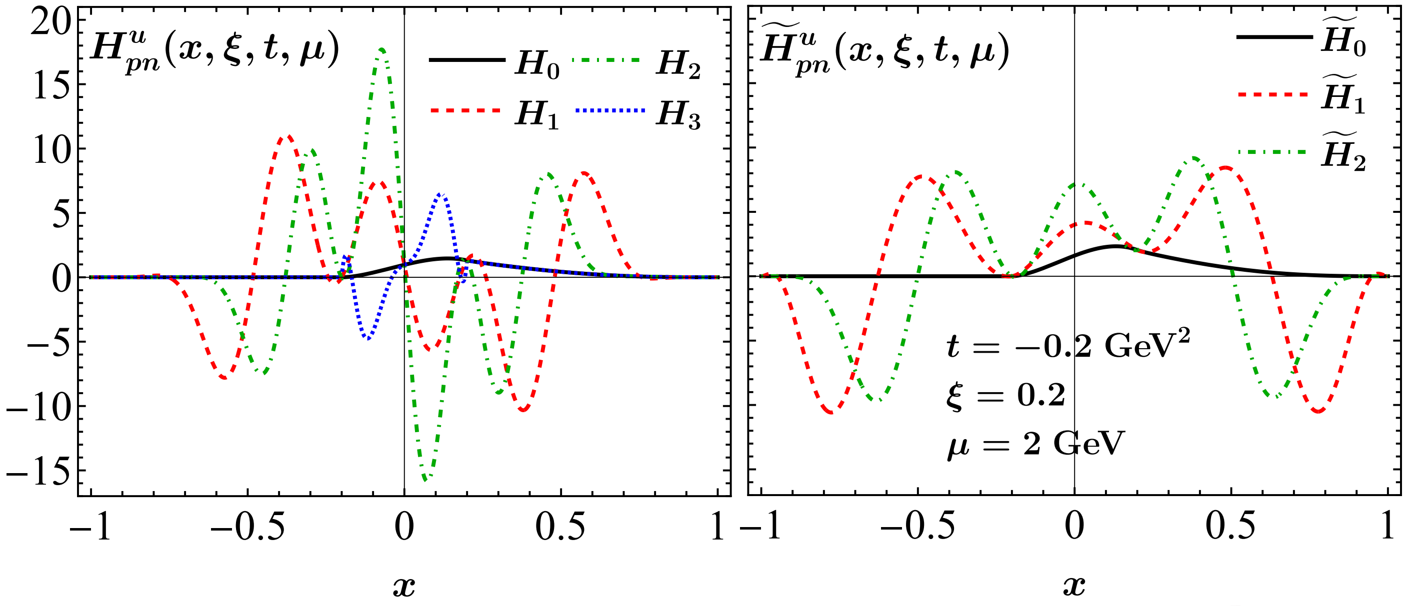
$$H^u_{pn} = H^u_p - H^d_p,\ \text{etc.}$$

GPD models = GK model + shadow GPDs

Goloskokov & Kroll, \`05, \`07, \`09

Bertone et al. \`21
Moffat et al. \`23

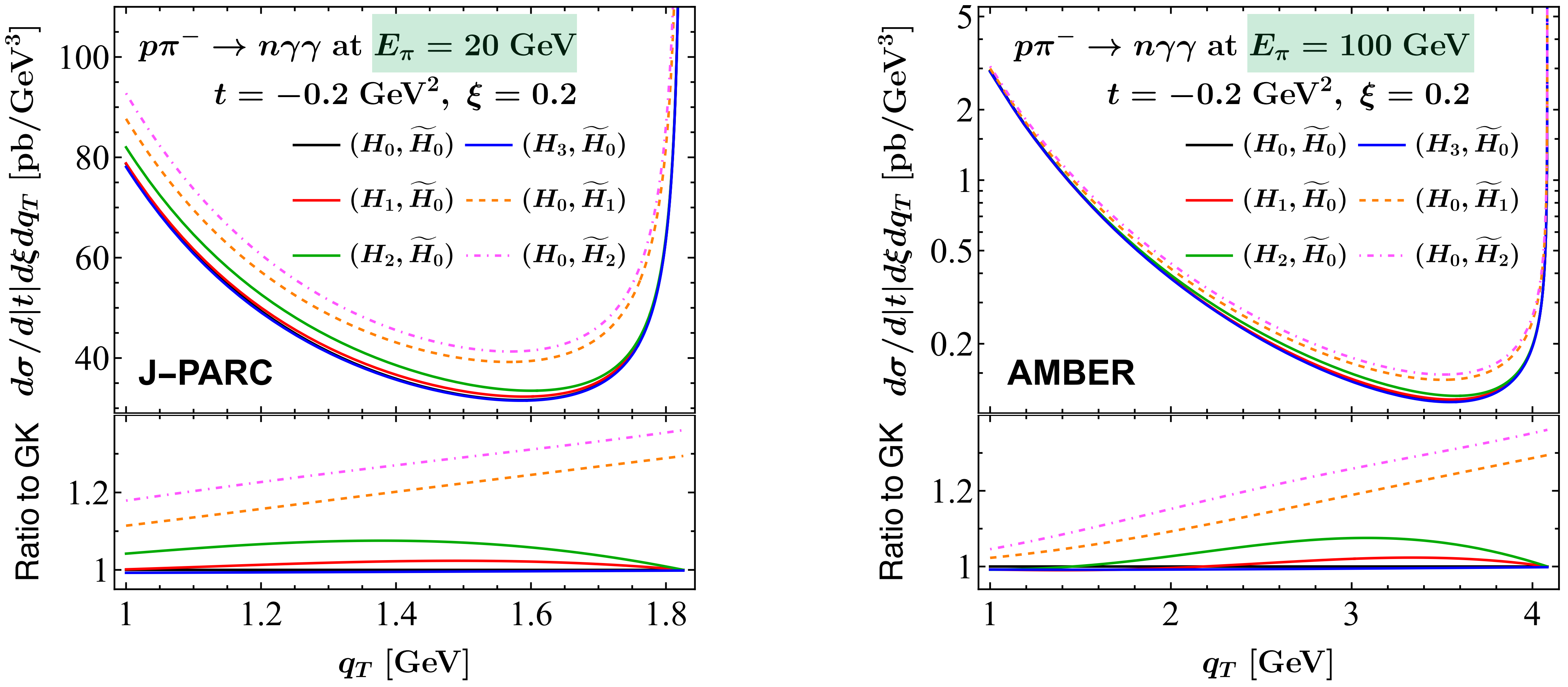
$$\int_{-1}^{1}\frac{dx\,S(x,\xi)}{x-\xi\pm i\epsilon} = 0$$



Jefferson Lab

Enhanced x-sensitivity: (1) diphoton mesoproduction (at J-PARC or AMBER)

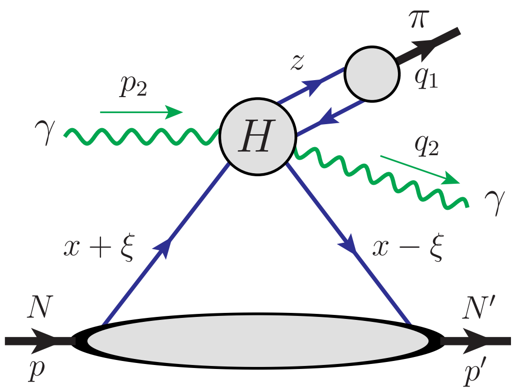
[Qiu & Yu, PRD 109 (2024) 074023]

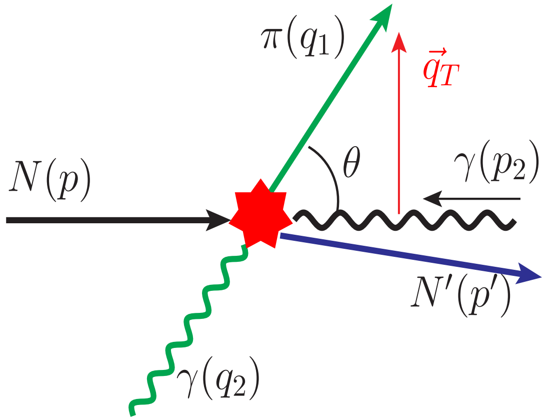


Jefferson Lab

Enhanced x-sensitivity: (2) γ-π pair photoproduction

[Qiu & Yu, PRL 131 (2023) 161902]

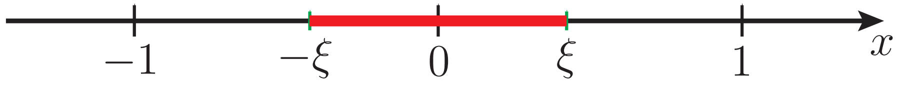




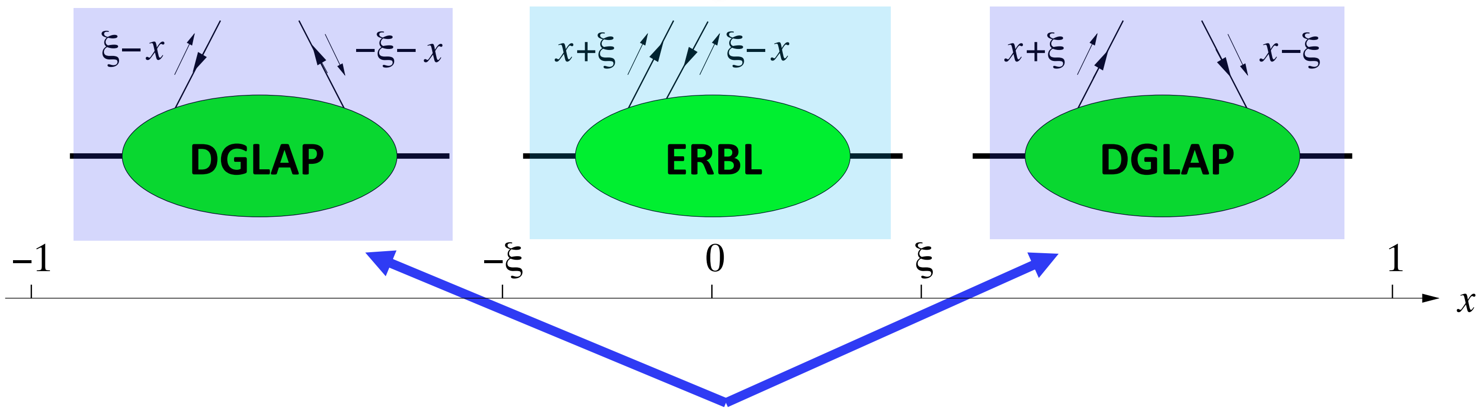
$i\mathcal{M}$ **also contains the special integral**

$$I'(t,\xi;z,\theta) = \int_{-1}^{1} \frac{dx\, F(x,\xi,t)}{x - \rho'(z;\theta) + i\epsilon}$$

$$\rho'(z;\theta) = \xi \cdot \left[\frac{\cos^2(\theta/2)\,(1-z) - z}{\cos^2(\theta/2)\,(1-z) + z}\right] \in [-\xi, \xi]$$



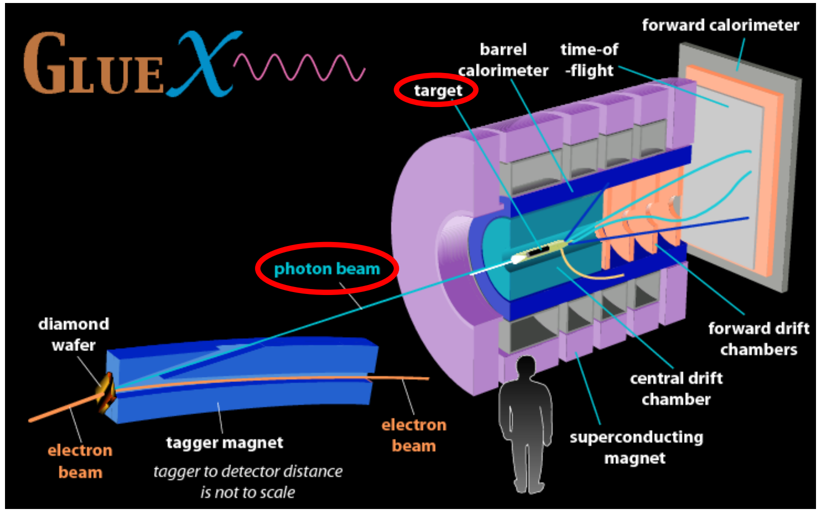
Complementary sensitivity

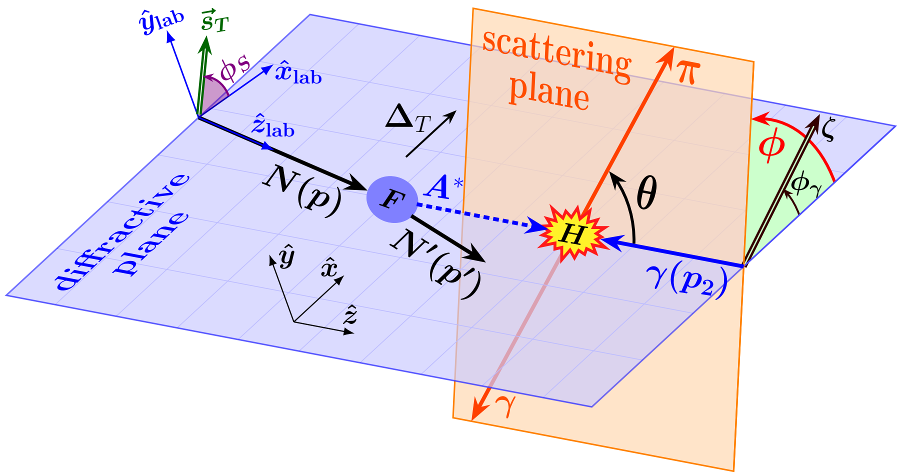


$N\,\pi \to N'\,\gamma\,\gamma$

Enhanced x-sensitivity: (2) γ-π pair photoproduction (at JLab Hall D)

[Qiu & Yu, PRL 131 (2023) 161902]





Polarization asymmetries

$$\frac{d\sigma}{d|t|\,d\xi\,d\cos\theta\,d\phi} = \frac{1}{2\pi}\frac{d\sigma}{d|t|d\xi d\cos\theta}\cdot\left[1+\lambda_N\lambda_\gamma\,A_{LL} + \zeta\,A_{UT}\cos 2\left(\phi-\phi_\gamma\right)+\lambda_N\zeta\,A_{LT}\sin 2\left(\phi-\phi_\gamma\right)\right]$$

$$\frac{d\sigma}{d|t|\,d\xi\,d\cos\theta} = \pi\left(\alpha_e\alpha_s\right)^2\left(\frac{C_F}{N_c}\right)^2\frac{1-\xi^2}{\xi^2 s^3}\Sigma_{UU}$$

$$\Sigma_{UU} = |\mathcal{M}_+^{[\widetilde{H}]}|^2+|\mathcal{M}_-^{[\widetilde{H}]}|^2+|\widetilde{\mathcal{M}}_+^{[H]}|^2+|\widetilde{\mathcal{M}}_-^{[H]}|^2,$$

$$A_{LL} = 2\,\Sigma_{UU}^{-1}\,\mathrm{Re}\left[\mathcal{M}_+^{[\widetilde{H}]}\,\widetilde{\mathcal{M}}_+^{[H]*}+\mathcal{M}_-^{[\widetilde{H}]}\,\widetilde{\mathcal{M}}_-^{[H]*}\right],$$

$$A_{UT} = 2\,\Sigma_{UU}^{-1}\,\mathrm{Re}\left[\widetilde{\mathcal{M}}_+^{[H]}\,\widetilde{\mathcal{M}}_-^{[H]*}-\mathcal{M}_+^{[\widetilde{H}]}\,\mathcal{M}_-^{[\widetilde{H}]*}\right],$$

$$A_{LT} = 2\,\Sigma_{UU}^{-1}\,\mathrm{Im}\left[\mathcal{M}_+^{[\widetilde{H}]}\,\widetilde{\mathcal{M}}_-^{[H]*}+\mathcal{M}_-^{[\widetilde{H}]}\,\widetilde{\mathcal{M}}_+^{[H]*}\right].$$

Neglecting: (1) E and $\widetilde{E}$; (2) gluon channel

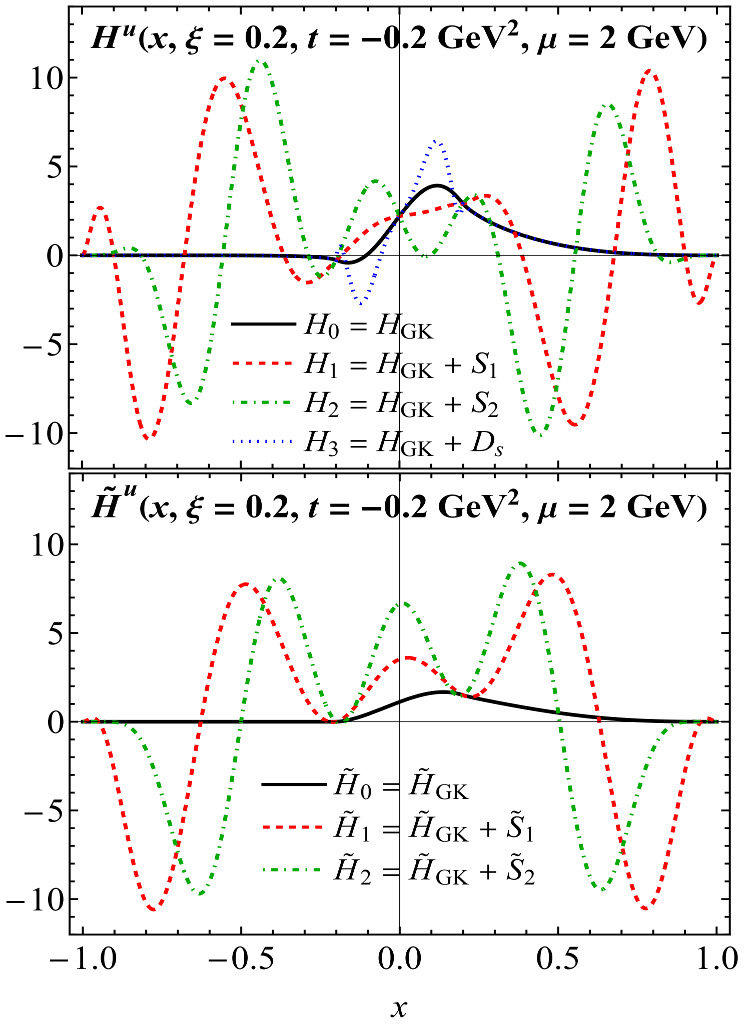
Jefferson Lab

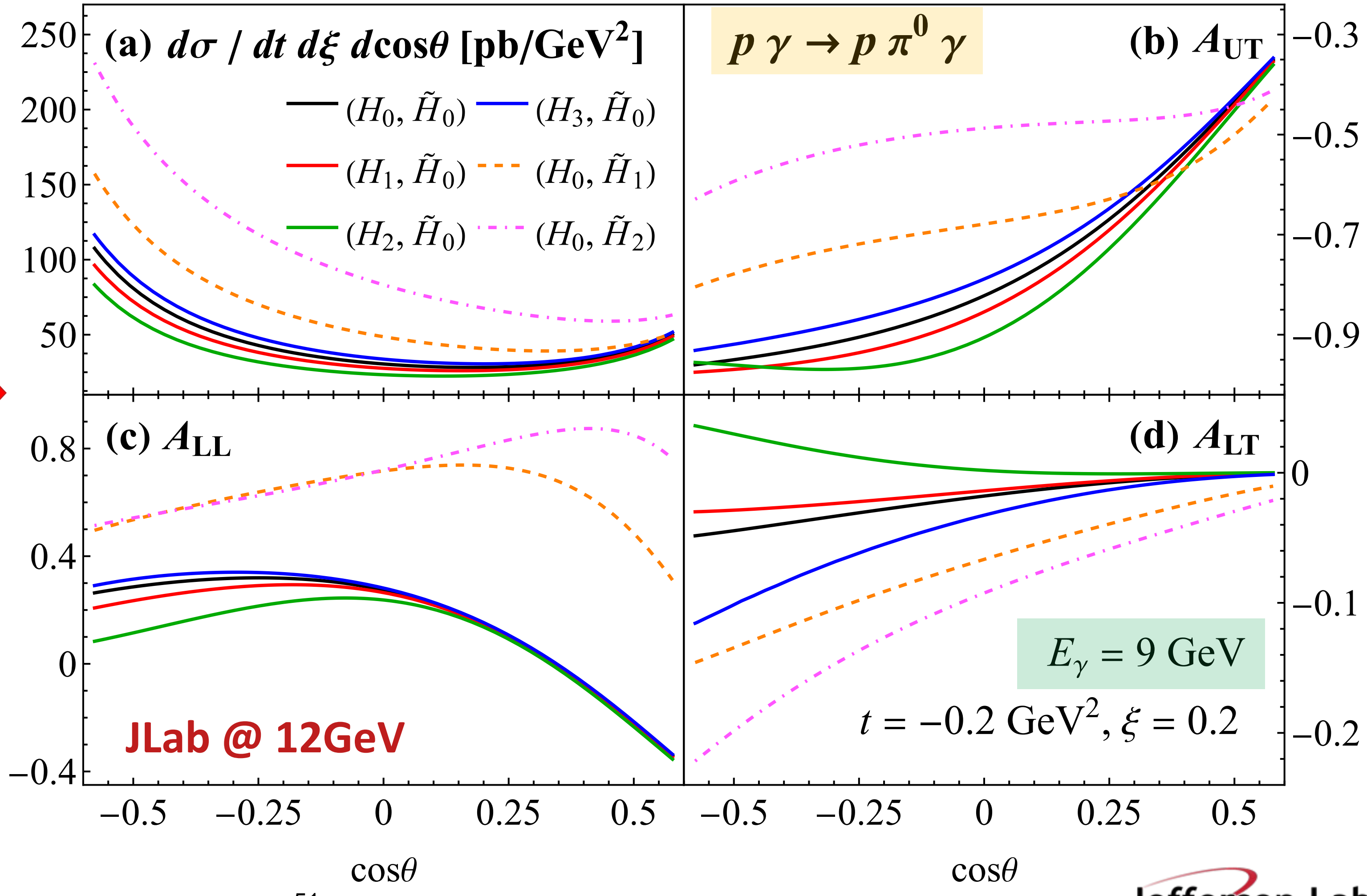
Enhanced x-sensitivity: (2) γ-π pair photoproduction (at JLab Hall D)

GPD models = GK model + shadow GPDs

$$\int_{-1}^{1} \frac{dx\, S(x,\xi)}{x-\xi \pm i\epsilon} = 0$$

Goloskokov, Kroll, `05, `07, `09
Bertone et al. `21
Moffat et al. `23





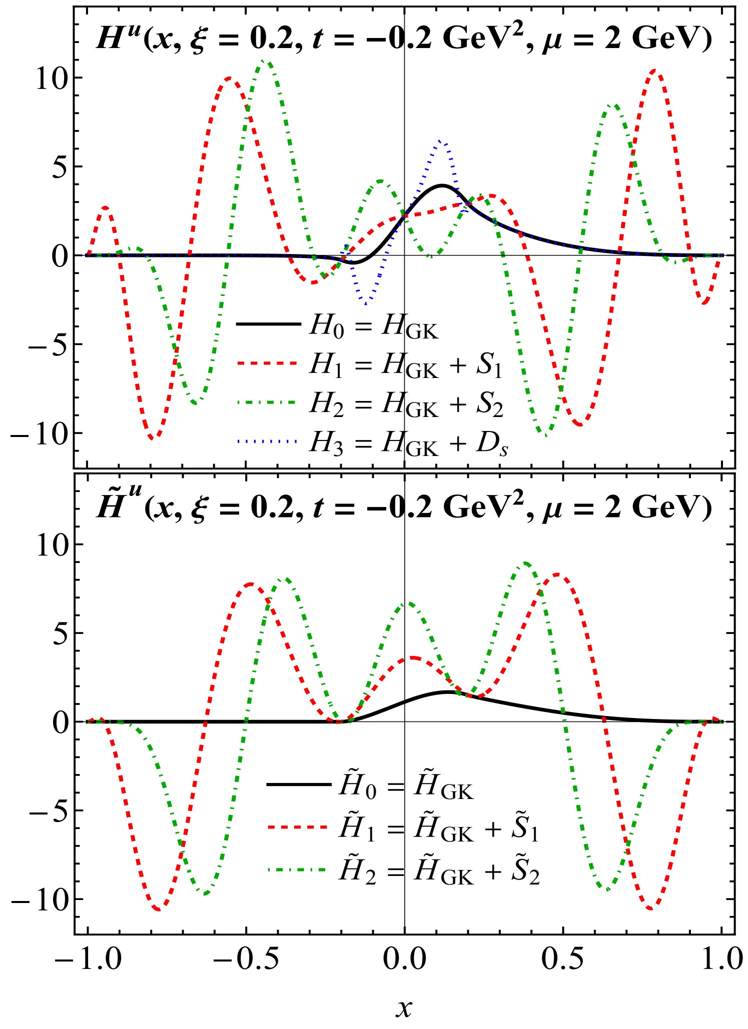
Jefferson Lab

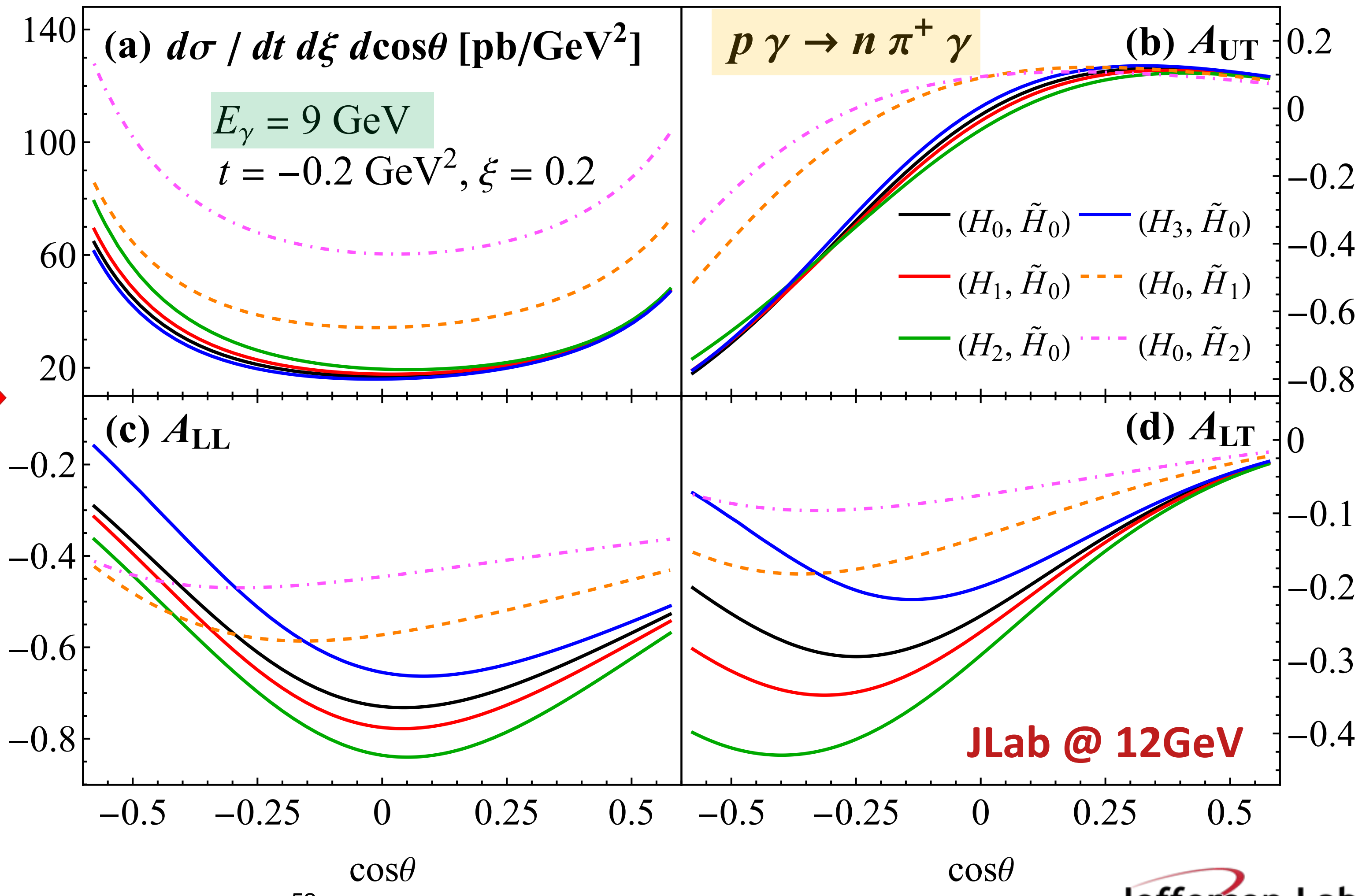
Enhanced x-sensitivity: (2) γ-π pair photoproduction (at JLab Hall D)

GPD models = GK model + shadow GPDs

$$\int_{-1}^{1} \frac{dx\, S(x,\xi)}{x - \xi \pm i\epsilon} = 0$$

Goloskokov, Kroll, `05, `07, `09
Bertone et al. `21
Moffat et al. `23



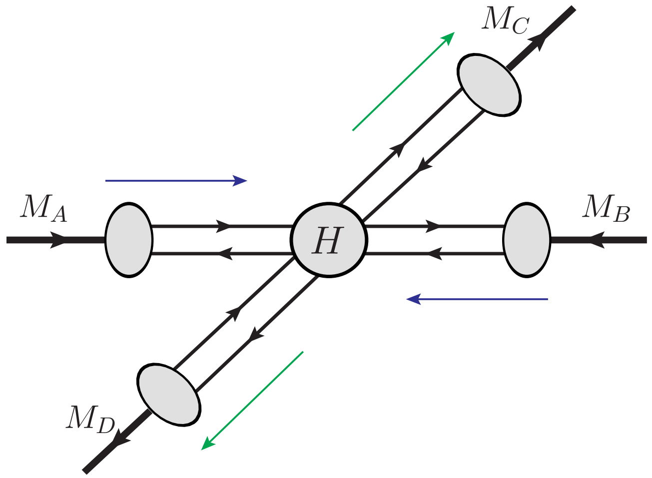


Jefferson Lab

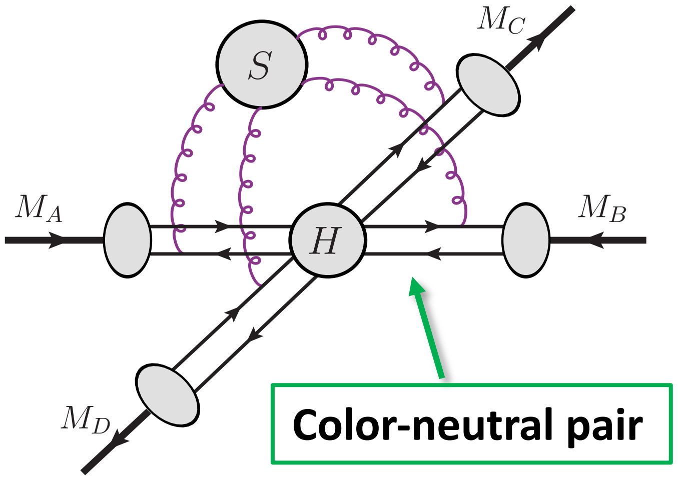
Brief mention of factorization

Exclusive factorization: large-angle 2 → 2 scattering

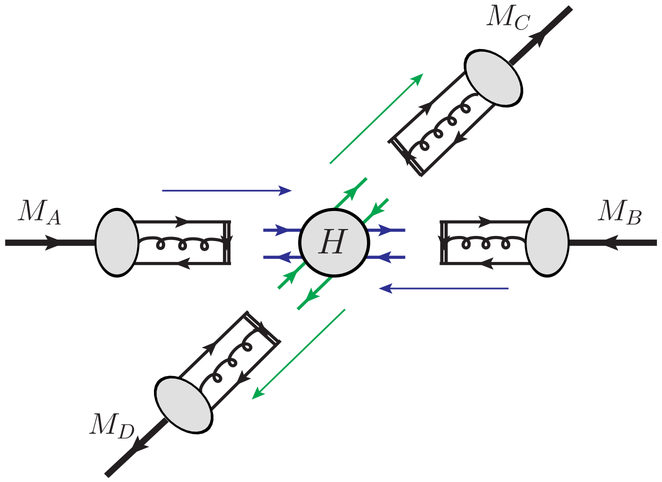
Born level



+ soft connection

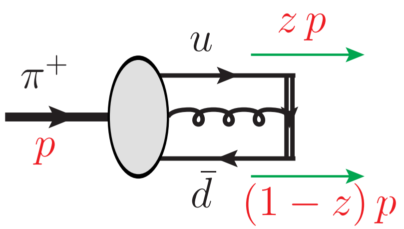


Factorization



Meson distribution amplitude (DA)

[Lepage & Brodsky, PRD 1980;
Adv. Ser. Direct. High Energy Phys. 5, 93 (1989)]

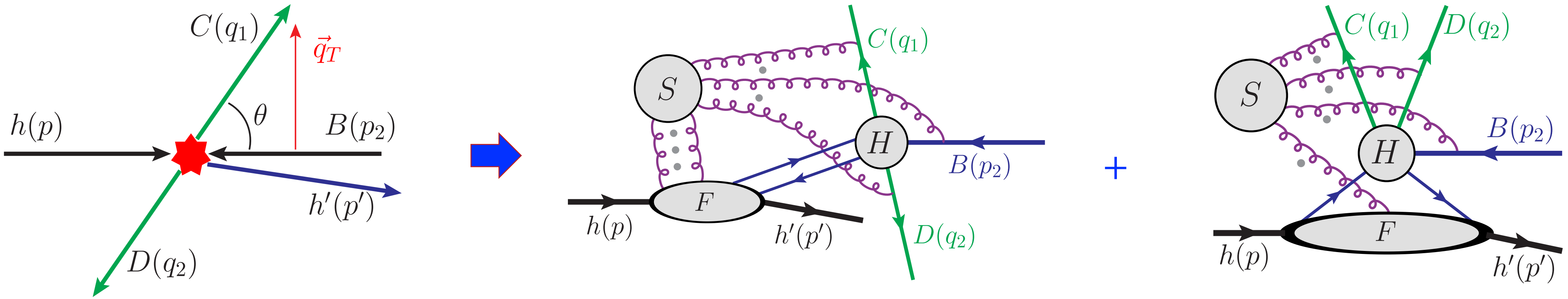


$$= D_{u/\pi^+}(z) = \int_{-\infty}^{\infty} \frac{dy^-}{4\pi}\, e^{izp^+y^-} \langle 0|\bar{d}(0)\gamma^+\gamma_5 W_n(0,y^-)u(y^-)|\pi^+(p)\rangle$$

"Lightcone wavefunction"

SDHEP factorization: generic consideration

[Qiu & Yu, PRD 107 (2023), 014007]



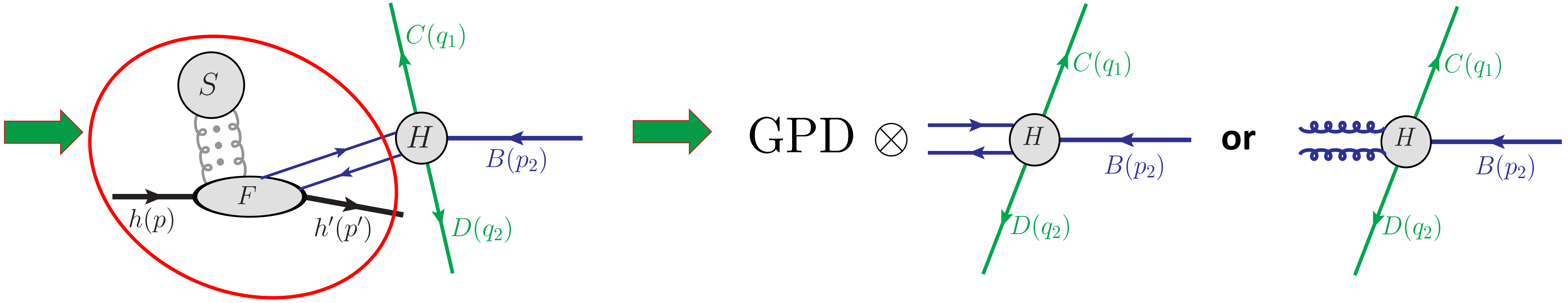
ERBL region

$|x| < \xi$

DGLAP region

$\xi < |x| < 1$

Soft gluons cancel when coupling to mesons!



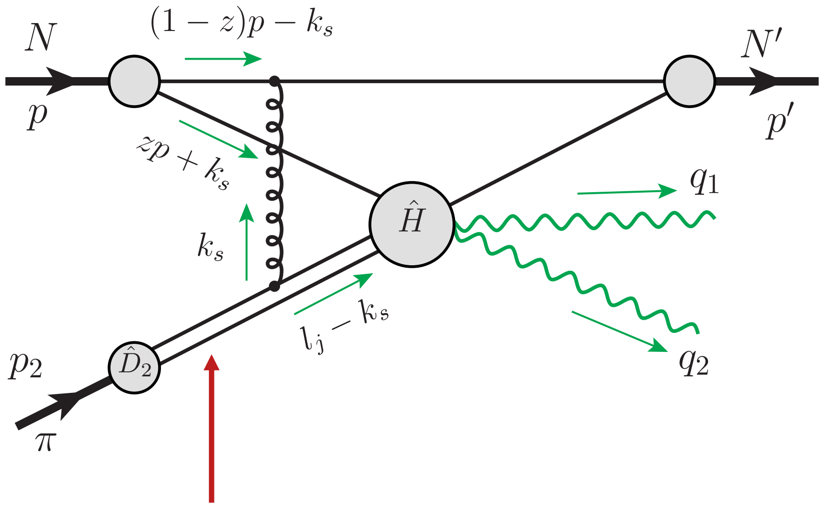
GPD

SDHEP factorization: why single diffractive?

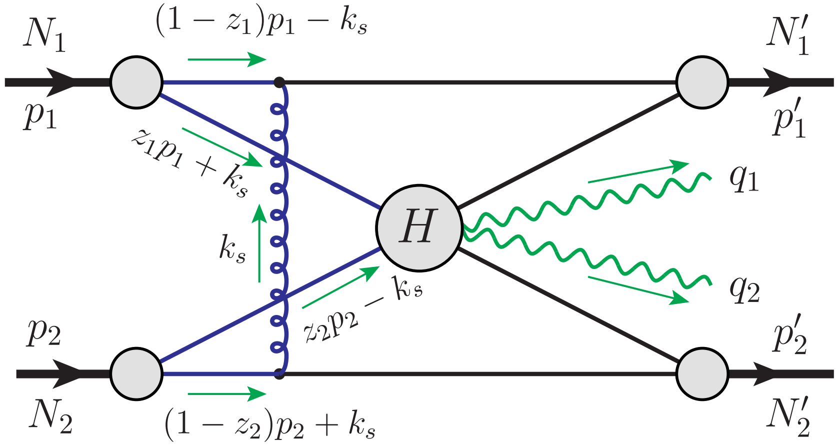
[Qiu & Yu, PRD 107 (2023), 014007]

❑ From single-diffractive to double-diffractive process?

Glauber pinch for diffractive scattering



Factorizable thanks to pion



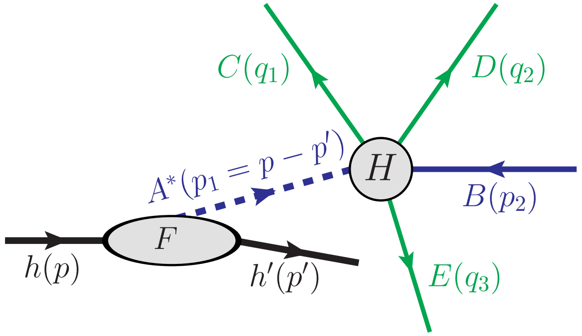
Non-factorizable **even with hard scale**

Both k_s^+ and k_s^- are pinched in Glauber region!

❑ How to generalize? --- Beyond two to three!

$$h(p) + B(p_2) \to h'(p') + C(q_1) + D(q_2) + E(q_3)$$

Examples: DDVCS, diphoton electroproduction, …

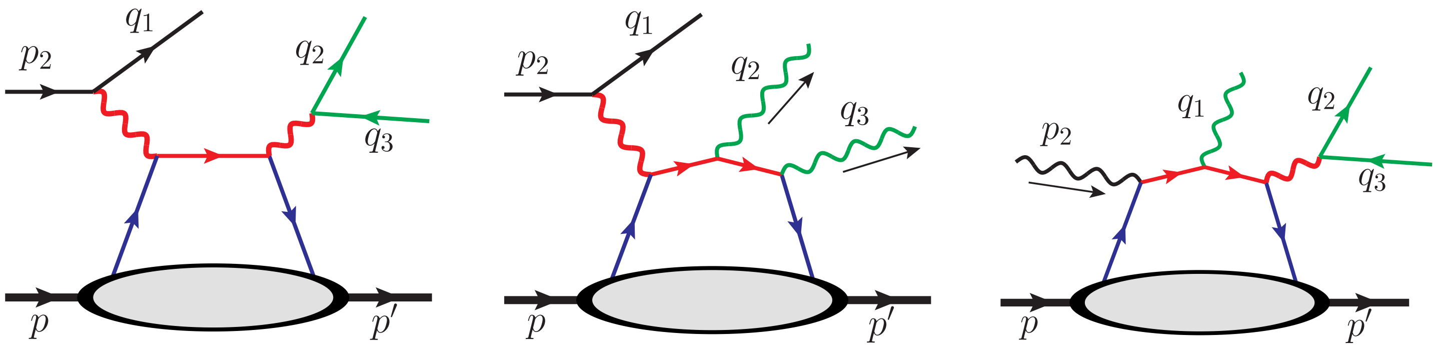


Comments and suggested exercises

❑ $2 \to 3$ vs. $2 \to 4$

- $2 \to 3$:
  - simpler kinematics
  - straightforward formulation of factorization
  - mostly scaling propagators
- $2 \to 4$:
  - more intricate kinematics
  - more likely to have enhanced sensitivity
  - lower rate

❑ Some $2 \to 4$ processes



[hep-ph/0208275 hep-ph/0210313 hep-ph/0307369 2303.13668]

[2003.03263]

New

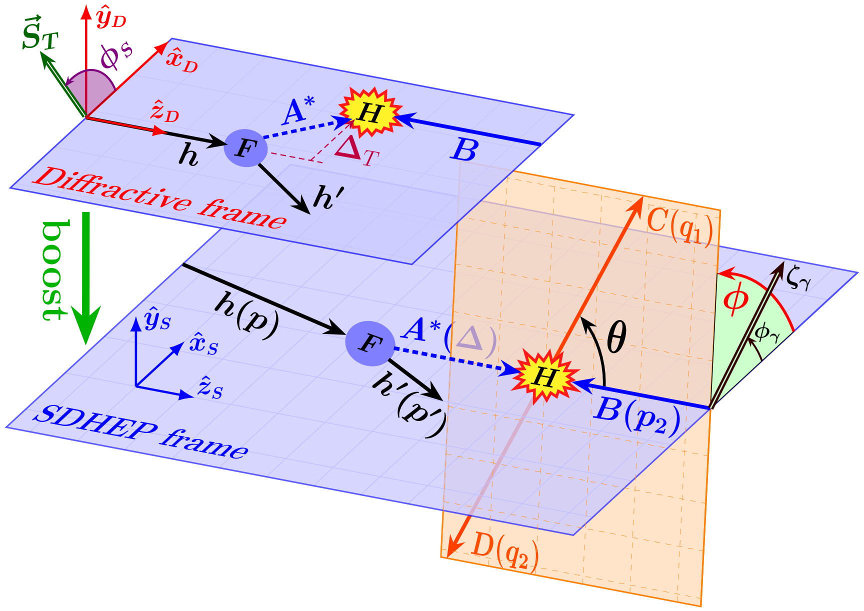
Exercises:

1. Do they carry enhanced sensitivity?
2. Where is it?
3. Do they have Bethe-Heitler channels?
4. What are the QCD and QED couplings of GPD and BH channels?
5. How do the amplitudes scale?
6. How to formulate the phase space?

Reference: Qiu and Yu, PRD 2023, Sec. VI F.

Summary: towards a global fit

❑ Single-diffractive hard exclusive process (SDHEP)



- Generic kinematic description
- Encompasses all GPD-related processes
- Clear factorization structure
- Straightforward to generalize

❑ Towards a global fit

- Sensitivity to the x-dependence ⟶ moment type vs. enhanced type
- Separation of flavor dependence ⟶ multiple processes
- Separation of GPD spin structure ⟶ azimuthal modulations
- Extending ξ and t coverage ⟶ various experiment energies

Thank you!