What is a parton?

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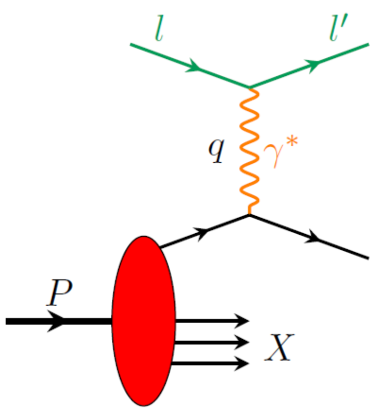
Outline

- DIS, partons and James D. Bjorken
- IMF and light-front quantization
- Standard QCD factorization and partons from light-front correlations
- Factorizations in longitudinal momentum distributions (quasi-PDF)
- What is a parton? parton as EFT and EFT for parton
- Conclusion

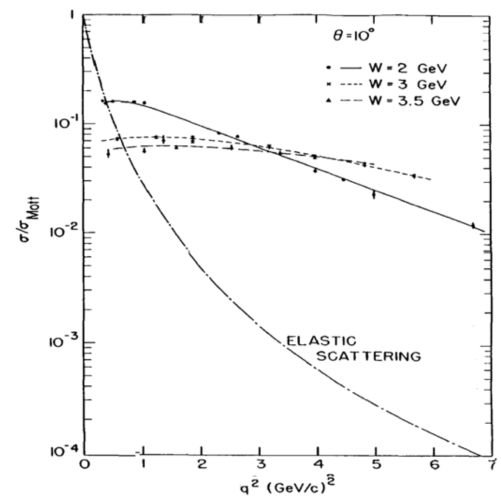
DIS, partons and James D Bjorken

Origin of partons

- High-energy electron-proton deep-inelastic scattering



$$\sigma_{\mathrm{DIS}} \sim \sigma_0 \left[W_2 + 2W_1 \tan^2\left(\frac{\theta}{2}\right) \right]$$



- Bjorken scaling of structure functions $W_i(Q^2, \nu)$

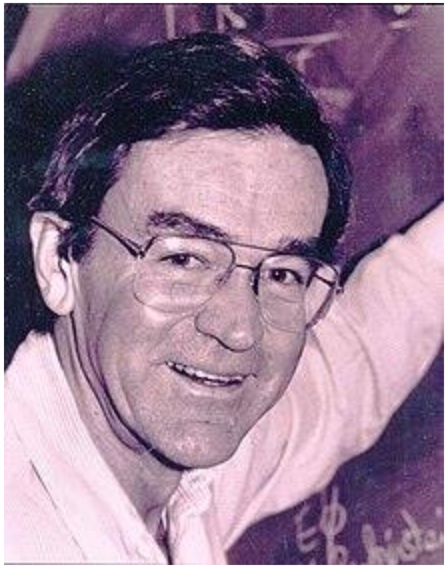
$x_B = Q^2/2M\nu$

Bjorken, Phys. Rev. 179, 1547 (1969)

$$\lim_{Q^2\to\infty,\ \nu/Q^2\ \text{fixed}} \nu W_2(Q^2,\nu) = MF_2(x)$$

$$\lim_{Q^2\to\infty,\ \nu/Q^2\ \text{fixed}} W_1(Q^2,\nu) = F_1(x)$$

J. D. Bjorken (BJ) June 22, 1934 – August 6, 2024



- B.Sc (MIT, 1956, Putnam fellow 1954)
- Ph.D (Stanford, 1959, with S. D. Drell)
- Dirac Medal (2004), Wolfe Prize (2015)

- Bjorken scaling and “partons”

Because increasing energy implies potentially improved spatial resolution, scaling implies independence of the absolute resolution scale, and hence effectively point-like substructure.

The first parton paper

J. D. Bjorken and E. A. Paschos, Phys. Rev. 185, 1975

Inelastic Electron-Proton and γ-Proton Scattering and the Structure of the Nucleon*

J. D. Bjorken and E. A. Paschos
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
(Received 10 April 1969)

A model for highly inelastic electron-nucleon scattering at high energies is studied and compared with existing data. This model envisages the proton to be composed of pointlike constituents ("partons") from which the electron scatters incoherently. We propose that the model be tested by observing γ rays scattered inelastically in a similar way from the nucleon. The magnitude of this inelastic Compton-scattering cross section can be predicted from existing electron-scattering data, indicating that the experiment is feasible, but difficult, at presently available energies.

I. INTRODUCTION

ONE of the most interesting results emerging from the study of inelastic lepton-hadron scattering at high energies and large momentum transfers is the possibility of obtaining detailed information about the structure, and about any fundamental constituents, of hadrons. We discuss here an intuitive but powerful model, in which the nucleon is built of fundamental pointlike constituents. The important feature of this model, as developed by Feynman, is its emphasis on the infinite-momentum frame of reference.

Infinite momentum frame (IMF)

It is argued that when the inelastic scattering process is viewed from this frame, the proper motion of the constituents of the proton is slowed down by the relativistic time dilatation, and the proton charge distribution is Lorentz-contracted as well. Then, under appropriate experimental conditions, the incident lepton scatters instantaneously and incoherently from the individual constituents of the proton, assuming such a concept makes sense.

J. D. Bjorken and E. A. Paschos, Phys. Rev. 185, 1975

Elements of parton model

1. Infinite momentum frame
   Proton is travelling with $p \to \infty, v = c$
2. Due to Lorentz time dilation, interaction between constituent partons disappear, and the proton is a collection of free particles (partons)
3. In DIS, the virtual photon scatters instantaneously and incoherently on the partons, which absorbs the photon and fly away freely.

Parton model and QCD

- The parton model for DIS provided the most important evidence that QCD is the correct theory of strong interactions.
- However, for DIS in QCD, parton model is only approximately true!
  1. Partons do not fly away freely, there are interactions which can be calculated in pQCD.
  2. Scattering is not completely incoherent. Coherent effects emerge as power corrections.
  3. Initial state interactions must exist to keep the parton falling apart, and more importantly determine PDFs.
  4. Partons do not exist in the real world, only an approximation.

Lorentz boost and interactions between constituents

- Relativistic energy-momentum relation

$$E = \sqrt{P^2 + M^2} = P + \frac{M^2}{2P} + \cdots$$

bound state mass effect is strongly suppressed

(twist-4, two-power suppressed).

- One must keep the sub-leading term to solve bound state

$$\hat{\mathrm{H}}_{\mathrm{eff}}|\mathrm{P}\rangle = \left(\hat{H} - \hat{P}\right)|P\rangle = (M^2/2P)|P\rangle$$

the interaction between partons is need to generate correct wave functions, mass, and PDF, even as $P \to \infty$.

IMF and light-front quantization

IMF

- First introduced by Fubini & Furlan, Physics, 1, 1965

  as a “slick math trick”

  to derive useful sum rules for scattering cross section on hadrons with a space-like momentum transfer.

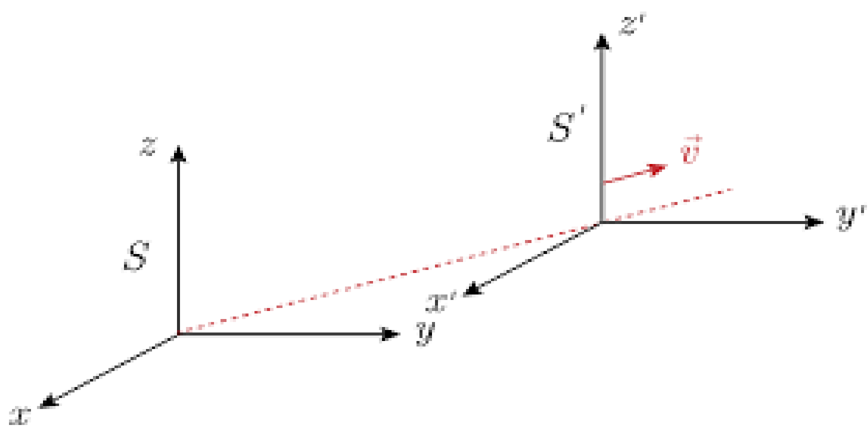
  Cabbibo-Radicati sum rule (photon scattering)

  Adler sum rule (neutrino scattering)

Field theory in IMF

- What a QFT (scalar) look like when boosted to IMF?

S. Weinberg, Dynamics at infinite momentum
Phys. Rev. 150 (1966) 1313



Weinberg's rules for IMF QFT

- All kinematic infinities (γ_∞-factor) can be removed from the calculations, resulting a "new" set of rules for Hamiltonian perturbation theory.
- Effective hamiltonian

  $E_{eff} = (k_\perp^2 + m^2)/2k_z$

  interactions, twist-4, non-local!
- The finding was confirmed by

  Susskind (1968), Bardakci, and Halpern (1968)

Weinberg's rules for QED

- S. J. Chang and S. K. Ma (1969)

  Feynman rules and quantum electrodynamics at infinite momentum,

  Phys.Rev. 180 (1969) 1506-1513

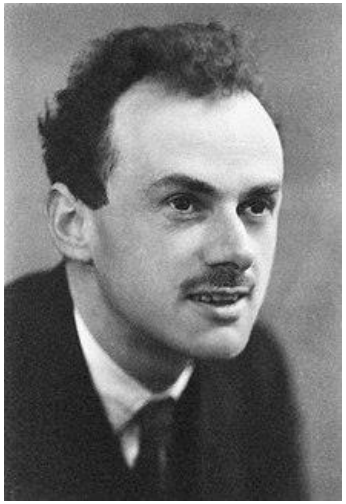
- J. Kogut and D. Soper

  Quantum Electrodynamics in the Infinite Momentum Frame,

  Phys.Rev.D 1 (1970) 2901-2913

Relativistic dynamics

- Three forms: (Dirac, 1949)
  - Instant: ordinary dynamics
  - Point: used in heavy-ion collision
  - Light-front: light-traveler dynamics?



REVIEWS OF MODERN PHYSICS VOLUME 21, NUMBER 3 JULY, 1949

Forms of Relativistic Dynamics

P. A. M. DIRAC
St. John's College, Cambridge, England

For the purposes of atomic theory it is necessary to combine the restricted principle of relativity with the Hamiltonian formulation of dynamics. This combination leads to the appearance of ten fundamental quantities for each dynamical system, namely the total energy, the total momentum and the 6-vector which has three components equal to the total angular momentum. The usual form of dynamics expresses everything in terms of dynamical variables at one instant of time, which results in specially simple expressions for six or these ten, namely the components of momentum and of angular momentum. There are other forms for relativistic dynamics in which others of the ten are specially simple, corresponding to various sub-groups of the inhomogeneous Lorentz group. These forms are investigated and applied to a system of particles in interaction and to the electromagnetic field.

Front-form (light-front) dynamics

- Front-form coordinates

$$x^+ = \frac{1}{\sqrt{2}}(x^0 + x^3)$$

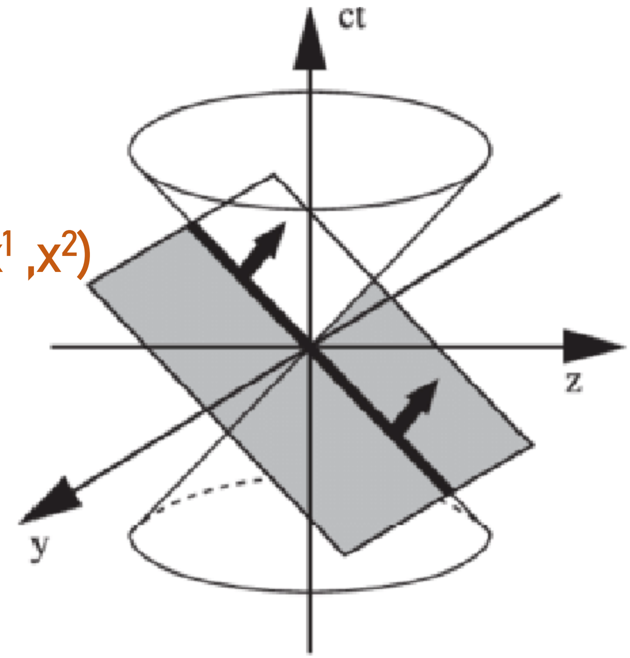
$$x^- = \frac{1}{\sqrt{2}}(x^0 - x^3)$$

$$x_\perp = (x^1, x^2)$$

- The front-front dynamics is determined by x^+ evolution through the "Light-front Hamiltonian"

$$P^- = \frac{1}{\sqrt{2}}(P^0 - P^3)$$

Light-front travelling in z(x^3) direction x^+=const



(x^-, x^1, x^2)

P.A.M. Dirac,
Rev. Mod. Phys. 21, 1949

Chang and Ma's discovery

- All Weinberg's rules in the P=∞ limit can be obtained by quantizing the theory with "new coordinates"

$$x^+ = \frac{x^0 + x^3}{\sqrt{2}}, \quad x^- = \frac{x^0 - x^3}{\sqrt{2}}$$

by treating x^+ as the new "time"

x^- as the new "space" dimension.

- This light-front quantization is exactly Dirac's front form.

Parton structure

- Partons are obtained by solving the effective H in the infinite-momentum frame or equivalently light-front quantization.
- The effective Hamiltonian is twist-4 and non-local

  contains operators $\frac{1}{iD^+}$
- Power counting-> twist counting, because in the limit, we lost dimensional counting for the longitudinal direction (or scale).

Unrenormalized QCD Hamiltonian

$$H = P^- = H_0 + H_{int}, \tag{1.46}$$

where (Lepage and Brodsky 1980, Brodsky and Lepage 1989, Brodsky, Pauli, and Pinsky 1998)

$$H_0 = \frac{1}{2} \int dx^- \, d^2x_\perp \left(\bar{\tilde{q}} \, \gamma^+ \frac{m^2 - \nabla_\perp^2}{i\partial^+} \, \tilde{q} - \tilde{A}^a_\mu \, \nabla_\perp^2 \tilde{A}^{a\,\mu} \right) \tag{1.47}$$

is the free part of the Hamiltonian, while the interaction part is given by

$$\begin{aligned} H_{int} = \int dx^- d^2x_\perp \Big[& -2g \, \mathrm{tr} \left(i\partial^\mu \tilde{A}^\nu [\tilde{A}_\mu, \tilde{A}_\nu] \right) - \frac{g^2}{2} \mathrm{tr} \left([\tilde{A}^\mu, \tilde{A}^\nu][\tilde{A}_\mu, \tilde{A}_\nu] \right) \\ & - g\bar{\tilde{q}}\gamma^\mu A_\mu \tilde{q} + g^2 \, \mathrm{tr} \left([i\partial^+ \tilde{A}^\mu, \tilde{A}_\mu] \frac{1}{(i\partial^+)^2} [i\partial^+ \tilde{A}^\nu, \tilde{A}_\nu] \right) \\ & + g^2 \bar{\tilde{q}} \gamma^\mu A_\mu \gamma^+ \frac{1}{2i\partial^+} \gamma^\nu A_\nu \tilde{q} - g^2 \bar{\tilde{q}} \gamma^+ \left(\frac{1}{(i\partial^+)^2} [i\partial^+ \tilde{A}^\mu, \tilde{A}_\mu] \right) \tilde{q} \\ & + \frac{g^2}{2} \bar{q} \gamma^+ t^a q \frac{1}{(i\partial^+)^2} \bar{q} \gamma^+ t^a q \Big]. \end{aligned} \tag{1.48}$$

Renormalizing the Hamiltonian

- Renormalization in non-local theory
- Light-cone divergences

$$(x - y)^2 = 0$$

  multipole sources

  - Light-cone gauge, $A^+ = 0$
  - Time-ordered diagrams
  - Light-cone correlations
- Infinite number of renormalization constants! (all vacuum properties cannot be calculated)

Broken symmetries

- Light-front H formalism breaks many symmetries
    - Gauge symmetry
    - Rotational symmetry
- Restoring these symmetries is not simple.

consider H-atom, l=1, $m = 0, \pm 1$ must yield the same energy!

Still a non-perturbative problem

- Infinitely-large Fock space

  Convergent truncation scheme?

  K. Wilson et al, Phys. Rev. D 49 (1994)

  "Nonperturbative QCD: A Weak coupling treatment on the light front"

- Matrix diagonalization?

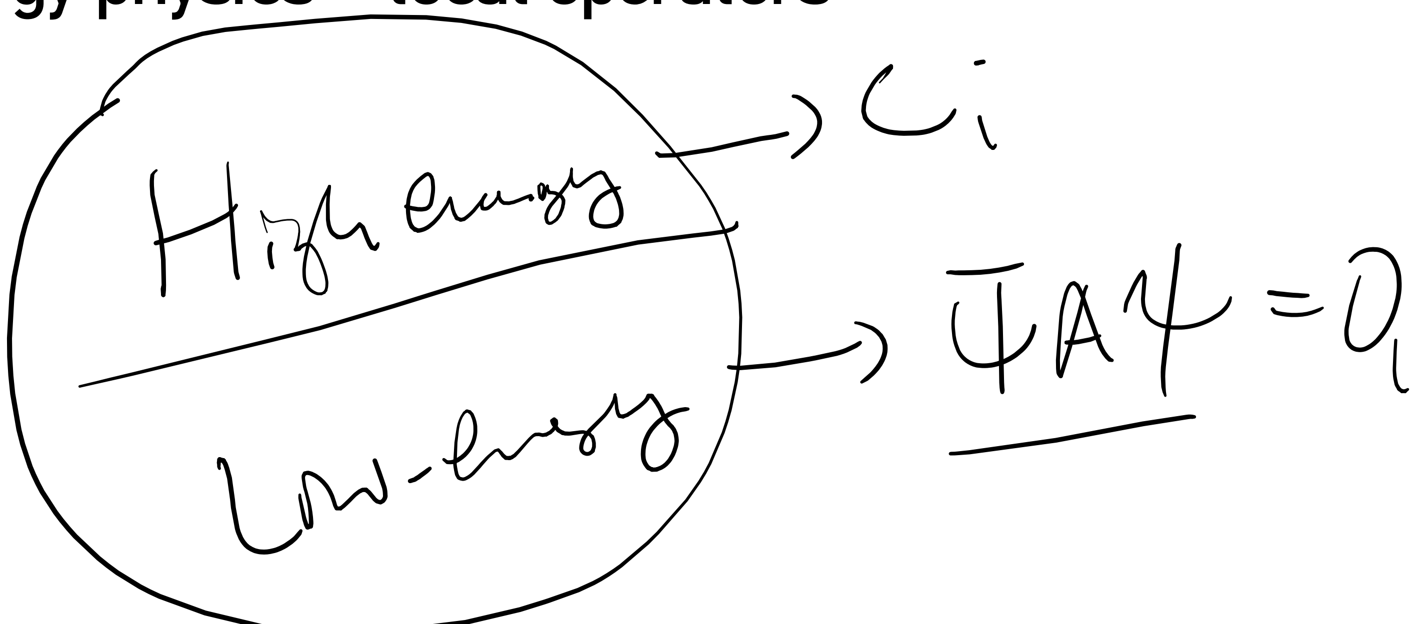
  light-front quark models

- Solving light-front QCD has not made a fundamental progress in the last 50 years!

Standard QCD factorization and parton from light-front correlations

QCD factorization

- For QCD with multiple scales (hard and soft), we have Wilson's OPE or factorization
- High-energy physics-> coefficient function
- Low-energy physics-> local operators



Schematic view

- We can write fields in terms of low-energy and high-energy part

$$\psi = \psi_{high} + \psi_{low}$$

$$A = A_{high} + A_{low}$$

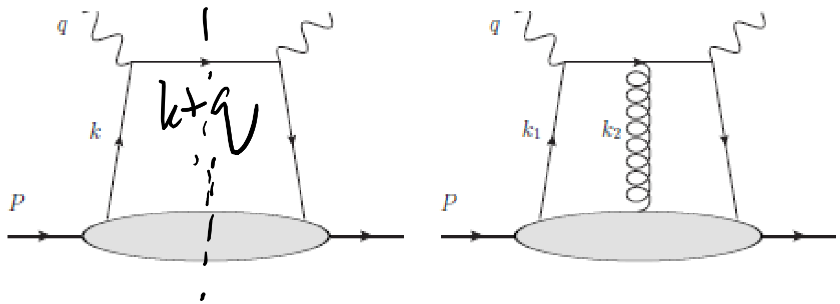
- For HEP interested in pQCD, we have an EFT with ψ_{high} plus "high-energy constants"

$$\langle \bar{\psi}_{low} .. A_{low} \dots \psi_{low} \rangle$$

which parametrizes the low-energy physics.

- These low-energy DOF can be described by soft and collinear modes (SCET)

QCD factorization



IG. 1: Tree-level diagrams for DIS process.

Fig. 1, in which the hadron tensor is,

$$W^{\mu\nu}(x_B, Q^2) = \frac{1}{2\pi}\mathrm{Im}\int i\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left[\gamma^\mu S(k+q)\gamma^\nu M(k)\right] + \text{crossing} \qquad (3)$$

where $S(k)$ is the single quark propagator of four-momentum k^μ, and $M(k)$ is the single quark Green's function in the hadron,

$$M(k)^{\alpha\beta} = \int d^4\xi e^{i\xi\cdot k}\langle P|T\overline{\psi}^\beta_{\mathrm{low}}(0)\psi^\alpha_{\mathrm{low}}(\xi)|P\rangle \qquad (4)$$

where $|P\rangle$ is the hadron state.

Bjorken limit in the rest frame

- Choose photon momentum along z
- -q is the photon z-momentum, ω is photon energy

$$Q^2 = q^2 - \omega^2 = (q - \omega)(q + \omega)$$

On the other hand, $\nu = P \cdot \frac{q}{M} = \omega$

$$x = \frac{Q^2}{2M\nu} = \frac{(q-\omega)(q+\omega)}{2M\omega} = finite$$

$$q = \omega + 2Mx$$

- Virtual photon momentum $q^\mu = (\omega, -(\omega + 2xM))$, a quasi-light-like vector as $\omega \to \infty, q^+ \to \infty, q^- \sim 2xM$

Performing k integral

- $S(q+k) \sim \frac{1}{(q+k)^2} \sim \frac{1}{-Q^2+2kq+i\epsilon}$

  $\sim \frac{\gamma^+ q^-}{-Q^2+2k^+q^-+i\epsilon} \sim \gamma^+/(k^+ - x_B P^+ + i\epsilon)$

  here $P^+ = M/\sqrt{2}$ nucleon + momentum in rest frame.
- The 4-momentum integral is only non-trivial for k+,

$$d^4k = dk^+ dk^- d^2\vec{k}_\perp$$

$$\int dk^- d^2k_\perp M(k)^{\alpha\beta} = (2\pi)^3 \int d\xi^- e^{ik^+\xi^-} \left\langle P \middle| \bar{\psi}^\beta(0)\psi^\alpha(\xi^-) \middle| P \right\rangle$$

$\sim f(x)$ with $x = k^+/P^+$

Hand-Bag Result

- $W^{\mu\nu} = -\frac{1}{2} g^{\mu\nu} \sum_i e_i^2 \left(f_i(x_B) + f_i(-x_B)\right)$

  standard parton model result.

- More gluon exchanges add a gauge link between 0 and ξ^- and make PDF gauge invariant.
- PDF is a light-cone correlation function!
- In light-front quantization, this is a "equal-time" correlator, which becomes a number operator, which

  counts the number of quarks with momentum $k^+ = xP^+$,

and transverse momentum integrated over.

Trouble in computing light-cone correlators

- Time-dependent correlation function.
- It has a sign problem, cannot be done with classical lattice simulations
- Quantum computer? You might need to wait for a long time.

Factorizations in longitudinal momentum distributions (LMDF or quasi-PDF)

An assessment of partons

- Great language for high-energy scattering
- But
  - The concept of partons is an idealized one:
    Partons do not exist in the real world
  - We don't know how to calculate!

- Are partons in literal sense dispensable?

  yes!

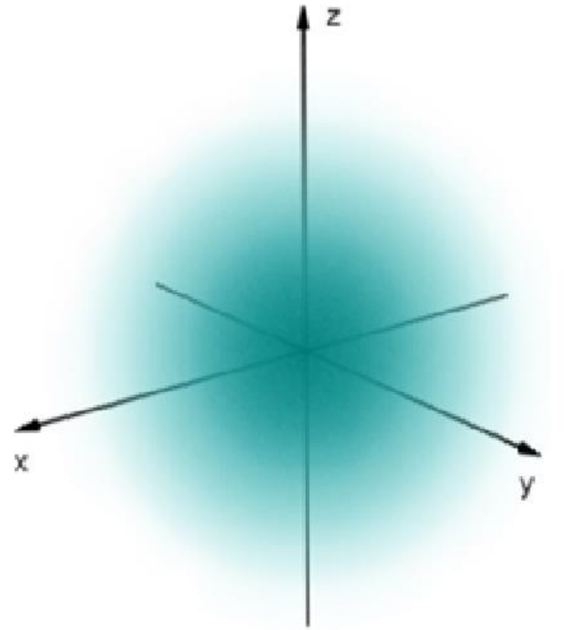
Momentum distributions

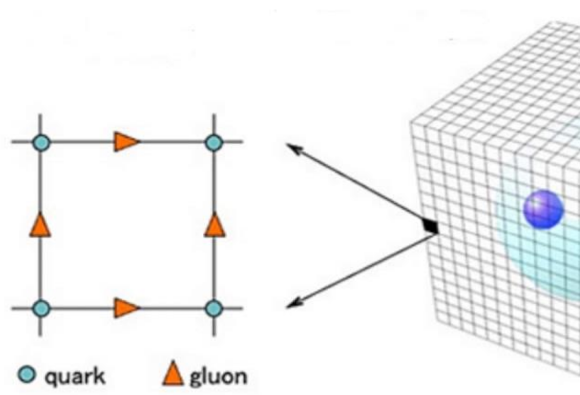
- A well-know physical observable which can defined in a moving hadron is momentum distribution $n(\vec{k}, P^z)$

  fundamental property of a quantum (many-body) system,

$$n(\vec{k}\,) = \left|\psi(\vec{k})\right|^2 \sim \int \psi^*(\vec{r})\psi(0)e^{i\vec{k}\vec{r}}d^3\vec{r}$$

- They are static correlation functions can be calculated using Monte Carlo method such as lattice QCD.





Longitudinal momentum distribution

- Longitudinal momentum distribution is

$$n(k^z, P^z) = \int d^2\vec{k}_\perp \, n(k^z, \vec{k}_\perp, P^z)$$

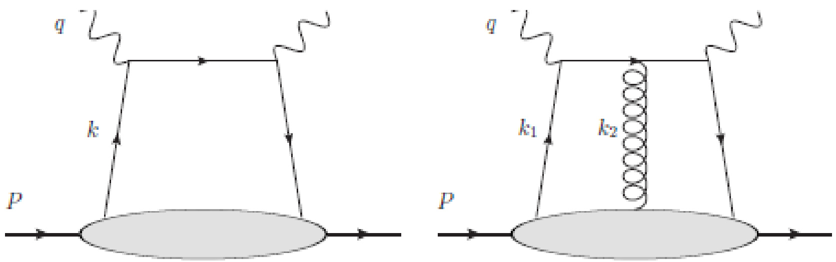
it depends on the momentum scale Pz

- For large P^z, asymptotic freedom allows one to calculate the P^z dependence through a RG equation

$$\frac{dn(k^z, P^z)}{\mathrm{dln\ P^z}} = \gamma \times n(k^z, P^z)$$

- All high-energy scattering can be factorized in term of LMD (quasi-PDF)

DIS Factorization in LMD



IG. 1: Tree-level diagrams for DIS process.

$$q^\mu = (0,0,0,-Q) \ ,$$

$$P^\mu = M\gamma v^\mu \ , \qquad \gamma = \sqrt{1+\frac{Q^2}{4x_B M^2}} \ ,$$

respectively, where $v^\mu = (1,0,0,v)$ and $v^\mu v_\mu = 1/\gamma^2$. In the Bjorken limit, $0 < x_B < 1$, $\gamma \sim Q \to \infty$ and $v \to 1$.

Fig. 1, in which the hadron tensor is,

$$W^{\mu\nu}(x_B, Q^2) = \frac{1}{2\pi}\mathrm{Im}\int i\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left[\gamma^\mu S(k+q)\gamma^\nu M(k)\right] + \text{ crossing} \tag{3}$$

where $S(k)$ is the single quark propagator of four-momentum k^μ, and $M(k)$ is the single quark Green's function in the hadron,

$$M(k)^{\alpha\beta} = \int d^4\xi e^{i\xi\cdot k}\langle P|T\overline{\psi}^\beta_{\mathrm{low}}(0)\psi^\alpha_{\mathrm{low}}(\xi)|P\rangle \tag{4}$$

where $|P\rangle$ is the hadron state.

We will now restrict ψ_{low} to those collinear fields making up the hadron with velocity v,

$$\psi_{\mathrm{low}}(x) = \psi_v(x) + \dots\ , \tag{5}$$

then

$$M(k)^{\alpha\beta} = \int d^4\xi e^{i\xi\cdot k}\langle P|T\overline{\psi}^\beta_v(0)\psi^\alpha_v(\xi)|P\rangle\ . \tag{6}$$

The effective Fourier components of $\psi_v(x)$ have momentum k^μ, with the following decomposition,

$$k^\mu = \alpha v^\mu + \beta \bar{v}^\mu + k_\perp^\mu , \quad k^2 \sim \Lambda_{\rm QCD}^2 , \tag{7}$$

where $\bar{v}^\mu = (v, -1)$, $\bar{v}_\mu^2 = -1/\gamma^2$, and $v \cdot \bar{v} = 2v$; $\alpha \sim \gamma\Lambda_{\rm QCD}$ and $\beta \sim \Lambda_{\rm QCD}/\gamma$. Thus the coefficient of $\bar{v}^\mu$ are suppressed by $1/\gamma$. Moreover, ψ_v satifies,

$$\not{v}\psi_v = 0 \ , \tag{8}$$

following from the leading order equations of motion (EOM) in $1/\gamma$.

The leading contribution to the hadron tensor comes from transverse polarization of the photon and thus $i, j = \perp$. In light of the trace in Eq.(3), the quark propagator can be simplified,

$$S(k+q) = \frac{i(\not{k} + \not{q})}{(k+q)^2 + i\epsilon} = \frac{i\not{q}}{2k \cdot q - Q^2 + i\epsilon} = \frac{i\gamma^z}{2k^z - Q + i\epsilon} \ , \tag{9}$$

where in the second equality, we use Eq. (8) in the numerator to eliminate $\not{k}$ and neglected $k^2 \sim \Lambda_{\rm QCD}^2$ in the denominator. Defining $k^z = Qy/2x_B$, the integration over k^0 and $k_\perp$ in Eq. (3) can be carried out,

$$\begin{aligned} W^{\mu\nu} &= -g_\perp^{\mu\nu} {\rm Im} \int_\infty^\infty \frac{dy}{2\pi} \tilde{f}(y) \frac{1}{y/x_B - 1 + i\epsilon} + \text{crossing} \\ &= -g^{\mu\nu} \frac{1}{2} \left(\tilde{f}(x_B) + \tilde{f}(-x_B) \right) \ , \end{aligned}$$

where

$$\begin{aligned} \tilde{f}(y, P^z) &= \frac{1}{2} \int dz e^{izk^z} \langle P | \bar{\psi}_v(z) \gamma^z \psi_v(0) | P \rangle \\ &= \frac{1}{2P^z} \int d\lambda e^{iy\lambda} \langle P | \bar{\psi}_v(z) \gamma^z \psi_v(0) | P \rangle \ , \end{aligned} \tag{10}$$

with dimensionless $\lambda \equiv zP^z$. The above result is identical to the standard QCD factorization result, except that the distribution $\tilde{f}(y, P^z)$ replaces the light-cone distribution

$$f(x) = \frac{1}{2P \cdot n} \int_{-\infty}^\infty d\lambda e^{ix\lambda} \langle P | \bar{\psi}(\lambda n) \gamma^+ W(\lambda n, 0) \psi(0) | P \rangle \ . \tag{11}$$

The key of the derivation is that the k^0 components of the quark four-momentum can be eliminated through the equation of motion (EOM) of the effective field $\not{v}\psi_v$=0. And therefore, k^0 integration can be carried out in the

$$A_v^\mu = \alpha v^\mu + \beta \bar{v}^\mu + A_\perp^\mu \ , \tag{12}$$

and at leading order, only the α component dominates. For example, when there is one interaction with the gauge potential (see Fig. 1), we replace the quark propagator in Eq. (3) by

$$S(k_1+q)\gamma^\alpha S(k_1+k_2+q) \ , \tag{13}$$

and $M(k)$ by $M^\alpha(k_1,k_2)$ where an additional $A_v^\alpha(k_2)$ appears. Both S can be simplified by the EOM of the effective field,

$$\begin{aligned} S(k_1+q) &\sim \frac{i\gamma^z}{2k_1^z - Q + i\epsilon}, \\ S(k_1+k_2+q) &\sim \frac{i\gamma^z}{2k_1^z + 2k_2^z - Q + i\epsilon} \ , \end{aligned} \tag{14}$$

For $\not{A}_v$, the situation is a bit involved, since $A_v^z \sim A_v^0$ as $v \to c$, we have the leading contributon,

$$\not{A}_v = A_v^0\gamma^0 - A_v^z\gamma^z \tag{15}$$

However, after commutation with γ^z, we have

$$\gamma^z \not{A}_v = -(A_v^0\gamma^0 + A_v^z\gamma^z)\gamma^z \sim 2A_v^z \tag{16}$$

where again we have used quark's EOM and $A_v^z \sim A_v^0$. Therefore, effectively all $\not{A}_v = -2A_v^z\gamma^z$, which allows one to calculate diagrams with an arbitrary number of A_v^μ interactions.

Adding all the quark eikonal interactions, one has

$$\tilde{f}(y,P^z) = \frac{1}{2P^z}\int_{-\infty}^{\infty} d\lambda e^{iy\lambda}\langle P|\bar{\psi}_v(z)W(z,0)\gamma^z\psi_v(0)|P\rangle$$

LMDF factorization

- LMDF factorization can be carried out for more complicated Feynman diagrams and for other processes like Drell-Yan.
- Global analysis can be carried out for LMDF (qPDF)
- LMDF can be calculated directly on lattice QCD, there is no sign problem, no inverse problem!

- So we don’t need partons at all!

What is a parton?

parton as EFT and EFT for parton

What is a parton?

- Partons are obtained in the limit of $P^z \to \infty$, it does not exist in reality.
- In field theories, there is also a UV cut-off Λ
- Light-front quantization/Weinberg rules/light-cone correlations correspond to an unphysical limit
  - $P^z \gg \Lambda \to \infty$

    This is not even legitimate in theory.
  - A legitimate field theory requires any physical scale

    $\mathrm{P} \ll \Lambda \to \infty$

Legitimacy of partons

- Depends on the UV properties of a theory
  1. If the UV limit is simple, $P^z \gg \Lambda \to \infty$ is the same as $\Lambda \gg P^z \to \infty$, so partons correspond the infinite momentum limit of LMDF.

  or $n(k^z, P^z)$ is analytic at $P^z = \infty$

$$n(k^z, P^z) = f(x) + f_4(x)(M/P^z)^2 + ...$$

t' Hooft model (most 2D QFT)

- 1+1 D QCD with $N_c = \infty$

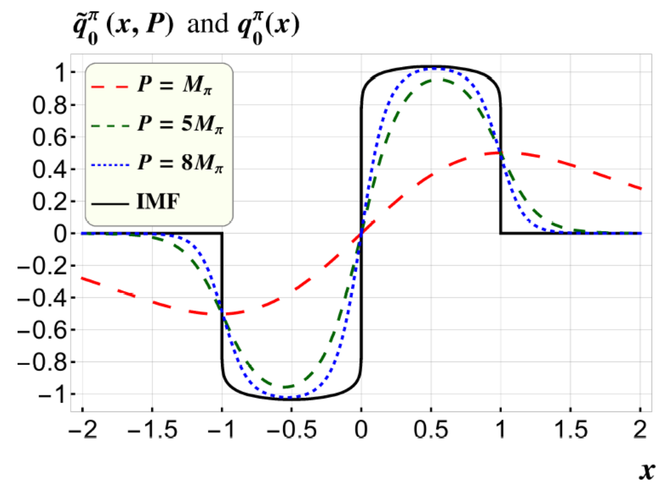
  Can be solved exactly at any finite Pz.

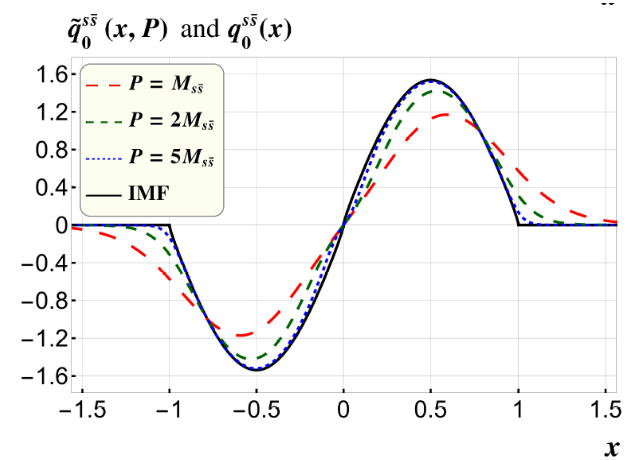
- Mom dis. Calculated at various mom:

  $p_\pi^z = m_\pi, 5m_\pi, 8m_\pi \ldots$

  $p_\phi^z = m_\phi, 2m_\phi, 5m_\phi \ldots$

- PDF obtained from the smooth limit of $p^z \to \infty$





Legitimacy of partons

- Depends on the UV properties of a theory
  1. If the UV limit is simple, $P^z \gg \Lambda \to \infty$ is the same as $\Lambda \gg P^z \to \infty$, so partons correspond to the infinite momentum limit of LMDF.
  2. If a theory is asymptotically free, and limits $P^z \gg \Lambda \to \infty$ and $\Lambda \gg P^z \to \infty$ differ in perturbation theory, and partons can be obtained from LMDF through a perturbative matching.

Matching condition for PDFs in QCD

The relation between mom dis. in full QCD and PDFs (Ji, 2013)

$$\tilde{f}(y, P^z) = \int Z(y/x, xP^z/\mu) f(x, \mu) dx + \mathcal{O}\Big(\frac{\Lambda^2_{\rm QCD}}{y^2(P^z)^2}, \frac{\Lambda^2_{\rm QCD}}{(1-y)^2(P^z)^2}\Big),$$

Partons are now an effective degrees of freedom (like infinite heavy quarks, HQET) which simplifies the theoretical description.

However, since partons cannot be calculated directly, we need to find an EFT for partons.

EFT for partons

- High-energy physical observables can be factorized in terms of PDFs
- The same can be done in terms of qPDFs
- Therefore, PDFs can be entirely expressed in terms of qPDFs
- This is the large-momentum effective theory for partons: LaMET

Leading LaMET lagrangian

- LaMET starts with hadrons with large momentum P or velocity v,

of velocity v. The leading effective lagrangian for the quark collinear modes can be written as,

$$\mathcal{L}^{(0)}_{q,v} = \overline{\psi}_v \left[iv \cdot D + \frac{i\bar{v} \cdot D}{2\gamma^2} + (iD_\perp)\frac{1}{2i\bar{v} \cdot D}(iD_\perp) \right] \not{\bar{v}} \psi_v \tag{19}$$

where $\bar{v} = (v, 0, 0, -1)/2v$ and $v_\mu \bar{v}^\mu = 1$. One can also add the leading-order lagrangian for the gluon collinear modes. This effective theory formally converges to SCET or light-front quantization in the $v \to c$ limit. However,

Features of LaMET

- There is no light-cone singularities

  no extra renormalization/ zero mode problem
- It is a Euclidean theory, can be calculated on lattice or instanton liquid.
- P serves as a rapidity regulator, and evolution equation can be derived
  - For PDF, it is DGLAP
  - For TMDs, it is the Collins-Soper evolution.

There is no inverse problem

- Take parton observables as physical, one can an EFT expansion

$$f(x,\mu) = \int_{-\infty}^{\infty} \frac{dy}{y} C_2\left(\frac{y}{x}, \frac{P^z}{\mu}\right) \tilde{f}\left(y, \frac{P^z}{\mu}\right)$$
$$+ \left(\frac{\Lambda_{\rm QCD}}{P^z}\right)^2 \sum_i \int_{-\infty}^{\infty} \frac{dy_1}{y_1}\frac{dy_2}{y_2}\frac{dy_3}{y_3} C_{4i}\left(\frac{y_1}{x}, \frac{y_2}{x}, \frac{y_3}{x}, \frac{P^z}{\mu}\right)$$
$$\times \tilde{f}_i\left(y_1, y_2, y_3, \frac{P^z}{\mu}\right) + \ldots \qquad (22)$$

- So long as the expansion converges, PDF at any x can be computed with controlled errors.

x-dependence

- LaMET calcaluates/predicts the x-dependence without model-dependent fits (inverse problem)
- No other methods can do this.

Legitimacy of partons

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  2. If a theory is asymptotically free, and limits $P^z \gg \Lambda \to \infty$ and $\Lambda \gg P^z \to \infty$ differ in perturbation theory, and partons can be obtained from LMDF through a perturbative matching.
  3. If a theory is not asymptotically free, or if it is, but two limits differ non-perturbatively, the concept of partons is useless in the sense that no theory can bridge it with experimental data.

Summary

- Since Feynman and Bjorken introduced parton 55 years ago, we have learned a lot about the nature of partons.
- Partons will become the most useful concept about the 3D structure of the nucleon.
- The best approach to partons is through physical, finite large momentum hadron computable in lattice QCD.