Exclusive processes and generalized parton distributions

C. Weiss (JLab), Lectures at International School of Hadron Femtography,
JLab, 16-25 Sep 2024 [Webpage]

Jefferson Lab

I) Exclusive processes in lepton-hadron scattering

Kinematic variables
Collinear frames and light-cone components
L/T currents
Cross section
Meson production and Virtual Compton Scattering VCS

"kinematics"

II) Asymptotic behavior and factorization

Bjorken regime
QCD factorization
Quantum correlation functions
DVCS factorization from collinear expansion
Generalized parton distributions

"dynamics"

III) Meson production and phenomenology

Meson production factorization and GPDs

Characteristics of large and small x ←

Large x: DVCS, pseudoscalar mesons, JLab results

Small x: Diffractive processes, space-time picture,
vector meson production, DVCS

Notice

Slides are work in progress
Several references to literature still missing; will be added

Recap

Factorization of DVCS at tree level $Q^2 \gg \mu_{\text{had}}^2$

Computed “handbag” amplitude containing leading asymptotic contribution

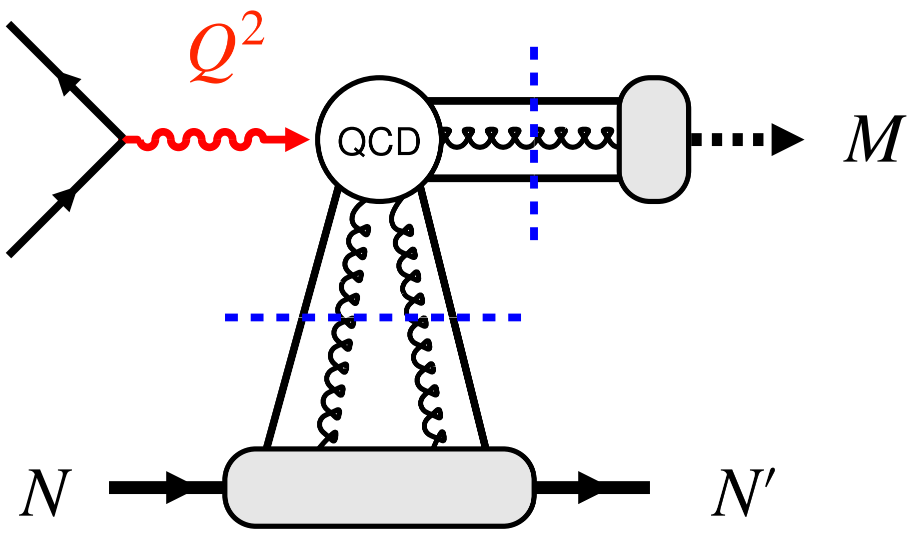
Considered process in collinear frame of external momenta $P = [P^+, P^-, \mathbf{0}], \quad q = [q^+, q^-, \mathbf{0}]$

Performed collinear expansion of loop integral, obtained light-cone correlation function of quark fields

Obtained DVCS amplitude in “convolution” form

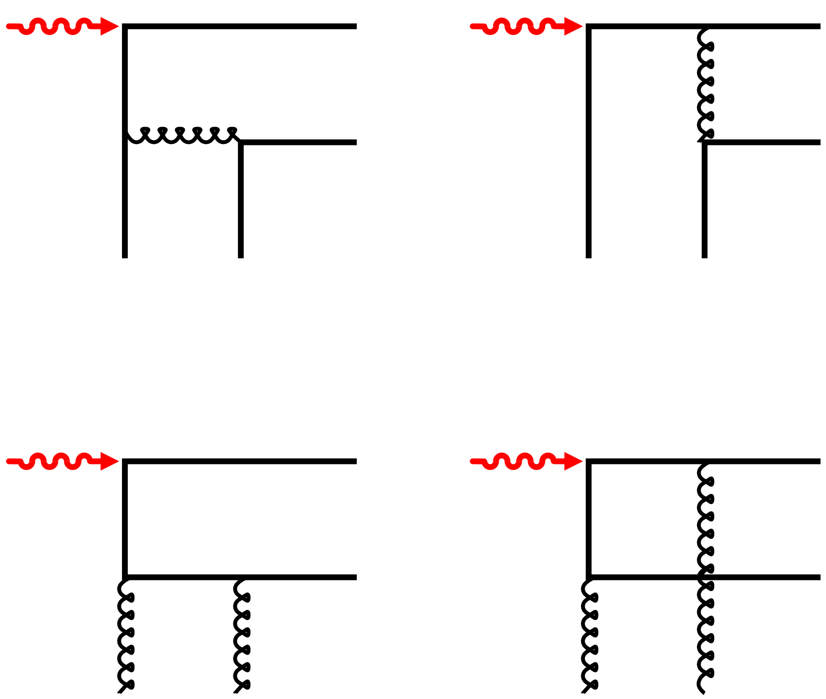
$$\text{Amp} = \int_0^1 dx \, \frac{1}{x \pm \xi \pm i0} \, \text{GPD}(x, \xi, t)$$

DVMP: Factorization



QCD subprocess requires perturbative interaction to produce collinear $q\bar{q}$ pair, has amplitude $\mathcal{O}(g^2)$

Amplitude involves meson distribution amplitude $\langle M \,|\, \bar{\psi}\psi \,|\, 0 \rangle$



$M =$ any

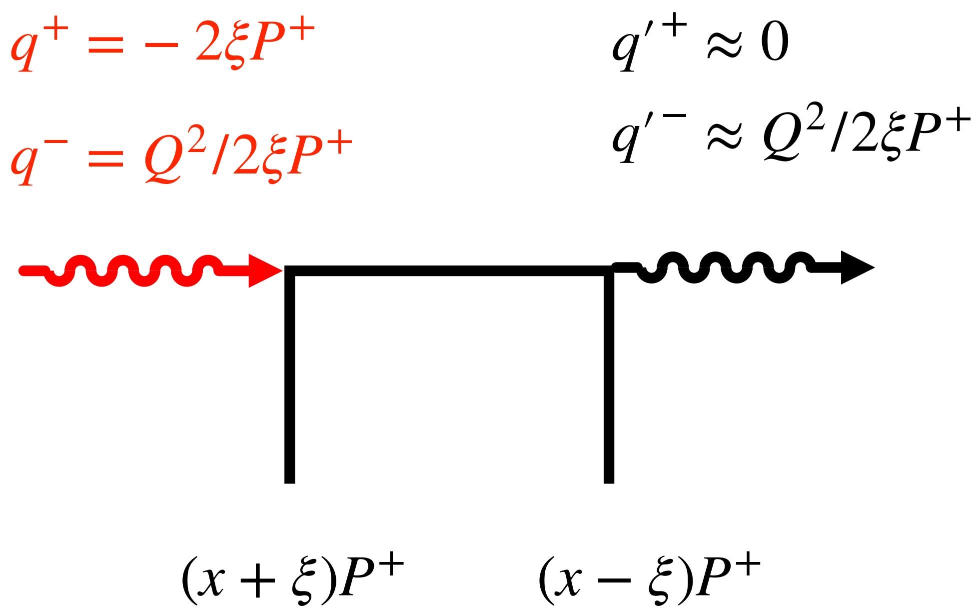
$M = \rho^0, \omega, \phi, J/\psi$

Here: Explain DVMP factorization by analogy with DVCS

Go to collinear frame

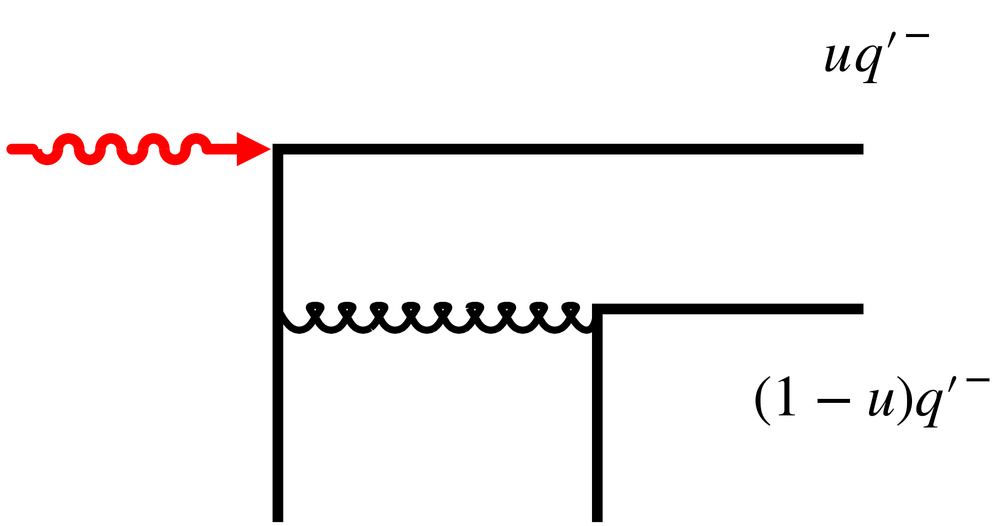
Inspect +, - momentum flow

DVMP: Factorization



Virtual photon injects large “minus” momentum $q^- \sim Q^2$

DVCS: q^- carried away entirely by outgoing real photon $q'^- \approx q^-$



DVMP: q^- split between quark and antiquark of outgoing meson

Virtuality in handbag graph reduced by factors $(1-u)$ or u compared to DVCS

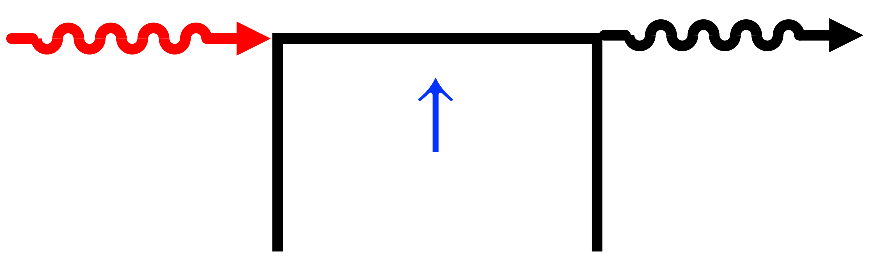
DVMP: Factorization

$q^+ = -2\xi P^+$

$q^- = Q^2/2\xi P^+$

$q'^+ \approx 0$

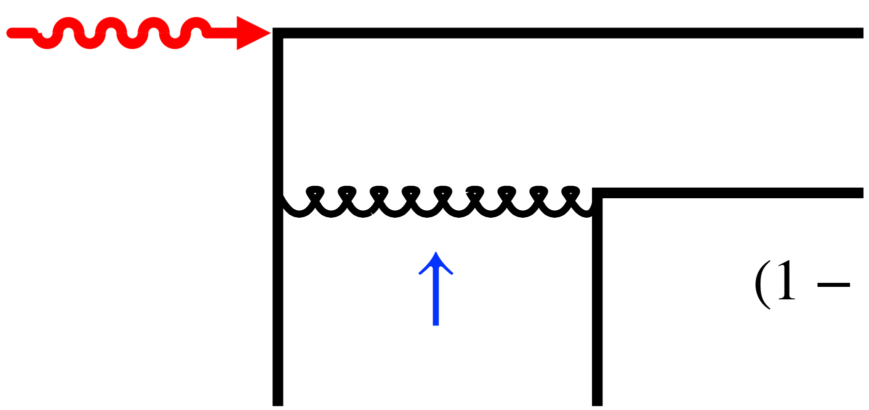
$q'^- \approx Q^2/2\xi P^+$



$(x+\xi)P^+$ $(x-\xi)P^+$

$$\frac{1}{(x-\xi+i0)\,\dfrac{Q^2}{2\xi} + \text{[transverse]}}$$

uq'^-



$\mathbf{p}_T$

$(1-u)q'^-$

$$\frac{1}{(1-u)(x-\xi+i0)\,\dfrac{Q^2}{2\xi} + \text{[transverse]}}$$

or u

$$\int_0^1 du \int d^2p_T\,[\ldots]$$

$u, p_T =$ internal longitudinal and transverse momentum in $q\bar{q}$ pair

DVMP: End-point singularities, L vs T

Problem: Collinear factorization of DVMP amplitude leads to integrals of type

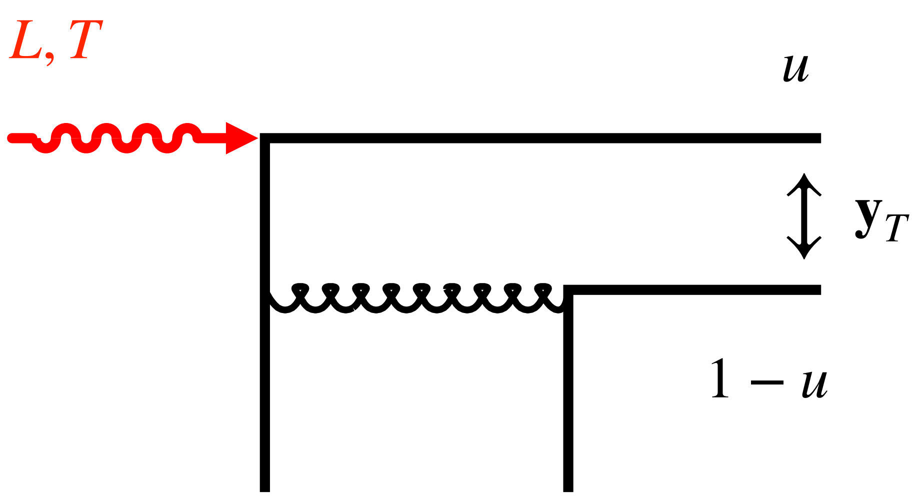
$$\int_0^1 du \int d^2 p_T \, \frac{1}{(1-u)\,Q^2 + p_T^2} \quad \text{or} \quad \frac{1}{uQ^2 + p_T^2}$$

$u, p_T =$ internal longitudinal and transverse momentum in $q\bar{q}$ pair

End points $u \to 1{,}0$: Factors $(1-u)$, u "neutralize" hard scale Q^2
Integral can become IR divergent ("end-point singularities")

L virtual photon	**T virtual photon**
Regions $u \to 1{,}0$ suppressed by numerators	Regions $u \to 1{,}0$ not suppressed
p_T integral not IR sensitive	p_T integral IR divergent
factorization possible	factorization not possible (at least not without further approximations)

DVMP: Factorization



Transverse coordinate representation

$\mathbf{y}_T$ = transverse separation of $q\bar{q}$ pair

End-point singularities = contributions of large-size pairs $|\mathbf{y}_T| \gg 1/Q$

L virtual photon

DVMP amplitude dominated by small-size pairs $|\mathbf{y}_T| \sim 1/Q$

QCD subprocess can be limited to production of bare $q\bar{q}$ pair. Radiation suppressed by small size (color dipole)

Collinear expansion approximation is self-consistent, leads to factorization

T virtual photon

DVMP amplitude receives contributions from large-size pairs $|\mathbf{y}_T| \sim 1/\mu_{\text{had}} \gg 1/Q$

QCD radiation not suppressed. Large-size $q\bar{q}$ pair dressed up with gluons.
Standard collinear expansion is not self-consistent, does not permit factorization

Factorization can be achieved only with separate treatment of large-size configurations:
QCD Sudakov form factor, light-cone sum rules, matching with hadronic amplitudes…

DVMP: Amplitude

$l + N \to l' + M + N$ $\quad$ $M =$ longitudinal vector meson, pseudoscalar meson

$$e_L^\mu \,\langle MN' | J_\mu | N\rangle \;=\; \text{factor} \times \frac{\alpha_s}{Q^2} \times f_M \int du \left(\frac{1}{u} \pm \frac{1}{1-u}\right) \Phi_M(u)$$

$$\times \int dx \left(\frac{1}{x - \xi + i0} \pm \frac{1}{x + \xi - i0}\right) \text{GPD}(x, \xi; t)$$

$$f_M\,\Phi_M(u) \;=\; \int \frac{dy^-}{2\pi}\, e^{i(u-\frac{1}{2})\frac{p^+y^-}{2}} \langle M(p) | \bar\psi(y/2)\,\Gamma^+\,\psi(-y/2)\,|0\rangle_{y^+, y_T = 0}$$

$$\int_0^1 du\;\Phi_V(u) = 1$$

meson distribution amplitude / decay constant

$M =$ longitudinal vector meson: $\quad \Gamma^+ = \gamma^+ \quad$ GPD $= H, E$

$M =$ pseudoscalar meson: $\quad \Gamma^+ = \gamma^+\gamma_5 \quad$ GPD $= \tilde{H}, \tilde{E}$

Isospin/flavor structure specified later

DVMP: Amplitude

- DVMP amplitude is $\mathcal{O}(\alpha_s)$ because of QCD interaction in subprocess
- DVMP amplitude is $\propto 1/Q^2$. Factor results from "additional" hard quark/gluon propagator in QCD subprocess compared to DVCS
- Integrals over momentum fractions x (parton in nucleon) and u (quark/antiquark in meson) are independent; see discussion of end-point singularities
- Transverse momenta of parton in nucleon and quark/antiquark in meson are integrated over; integrals are independent; contained in definition of GPD and DA
- Convolution integral of GPDs arising from DVMP factorization has the same form as the Compton form factors arising from DVCS factorization

DVMP: Channels

$$\mathscr{H}(\xi,t) = \frac{4\pi\alpha_s}{27} f_M \left[\int_0^1 du\, \frac{1}{u(1-u)}\, \Phi_M(u) \int_{-1}^{1} dx\, \frac{2H(x,\xi,t)+H(-x,\xi,t)}{\xi - x - i0} \right.$$
$$\left. - \int_0^1 du\, \frac{2u-1}{u(1-u)}\, \Phi_M(u) \int_{-1}^{1} dx\, \frac{2H(x,\xi,t)-H(-x,\xi,t)}{\xi - x - i0} \right]$$

+ same expressions
$\mathscr{E} \to E,\ \tilde{\mathscr{H}} \to \tilde{H}$

$\rho^+ n$	$2[H^u - H^d] - [H^{\bar u} - H^{\bar d}]$
$\rho^0 p$	$\frac{1}{\sqrt 2}\Big([2H^u + H^d] + [2H^{\bar u} + H^{\bar d}] + \frac{9}{4}\, x^{-1} H^g \Big)$
ωp	$\frac{1}{\sqrt 2}\Big([2H^u - H^d] + [2H^{\bar u} - H^{\bar d}] + \frac{3}{4}\, x^{-1} H^g \Big)$
$K^{*+}\Lambda$	$-\frac{1}{\sqrt 6}\Big(2\,[2H^u - H^d - H^s] - [2H^{\bar u} - H^{\bar d} - H^{\bar s}] \Big)$
$K^{*+}\Sigma^0$	$-\frac{1}{\sqrt 2}\Big(2\,[H^d - H^s] - [H^{\bar d} - H^{\bar s}] \Big)$
$K^{*0}\Sigma^+$	$[H^d - H^s] + [H^{\bar d} - H^{\bar s}]$
ϕp	$-\Big([H^s + H^{\bar s}] + \frac{3}{4}\, x^{-1} H^g \Big)$
$\pi^+ n$	$2[\tilde H^u - \tilde H^d] + [\tilde H^{\bar u} - \tilde H^{\bar d}]$
$\pi^0 p$	$\frac{1}{\sqrt 2}\Big([2\tilde H^u + \tilde H^d] - [2\tilde H^{\bar u} + \tilde H^{\bar d}] \Big)$
$K^+\Lambda$	$-\frac{1}{\sqrt 6}\Big(2\,[2\tilde H^u - \tilde H^d - \tilde H^s] + [2\tilde H^{\bar u} - \tilde H^{\bar d} - \tilde H^{\bar s}] \Big)$
$K^+\Sigma^0$	$-\frac{1}{\sqrt 2}\Big(2\,[\tilde H^d - \tilde H^s] + [\tilde H^{\bar d} - \tilde H^{\bar s}] \Big)$
$K^0\Sigma^+$	$[\tilde H^d - \tilde H^s] - [\tilde H^{\bar d} - \tilde H^{\bar s}]$

Compact form obtained combining $\pm u, \pm x$
[M. Diehl 2003]

GPD spin/flavor components sampled in $\gamma^* p \to MB$ given in table

$$H^{\bar q}(x,\xi,t) \equiv -H^q(-x,\xi,t)$$
$$\tilde H^{\bar q}(x,\xi,t) \equiv \tilde H^q(-x,\xi,t)$$

DVMP: Channels

- Quantum numbers of produced meson (parity, C-parity, isospin) select spin/flavor components of GPDs in amplitude
- DVMP can be used to separate the spin-flavor components of the GPDs
- Gluon GPD contributes to amplitude of neutral vector meson production ($\rho^0, \omega, \phi, J/\psi$) at LO. These vector mesons have the same C and P quantum numbers as the virtual photon.

DVMP: Cross section

$$F_L(x_B, Q^2, t) \;\equiv\; \frac{d\sigma_L}{dt}(\gamma^* p \to MB)$$

$$= \frac{\alpha_{\text{em}}}{Q^6}\frac{x_B^2}{1 - x_B}\left[(1-\xi^2)\mathcal{H}^2 - \left(\xi^2 + \frac{t}{4M^2}\right)\mathcal{H}^2 - 2\xi^2 \text{Re}(\mathcal{E}^* \mathcal{H})\right]$$

Expression for vector meson, for pseudoscalar $\mathcal{H}, \mathcal{E} \to \tilde{\mathcal{H}}, \tilde{\mathcal{E}}$

Expression neglects baryon mass differences $M_B = M_p \equiv M$

Exclusive cross sections decrease strongly with increasing Q^2 (here x_B fixed)

DVMP: $\frac{d\sigma_L}{dt}(\gamma^* p \to MB) \sim Q^{-6}$ $\qquad \frac{d\sigma}{dx_B dQ^2 dt}(lp \to l' MB) \sim Q^{-8}$

DVCS: $\frac{d\sigma_L}{dt}(\gamma^* p \to \gamma p) \sim Q^{-2}$ $\qquad \frac{d\sigma}{dx_B dQ^2 dt}(lp \to l' \gamma p) \sim Q^{-4}$

Low rates in asymptotic regime. Exclusive cross section is small fraction of total (inclusive) cross section

Applications

Apply asymptotic expressions to real processes at finite energy/momentum transfer

Proceed in two steps

I) Test approach to asymptotic regime, applicability of expressions, size of corrections $\leftarrow$

II) Extract information on GPDs / DAs from experimental data

Step I: Focus on model-independent qualitative features of asymptotic regime:
Q^2 scaling, L/T ratio, relative kinematic dependences, comparison of channels

It is difficult to test the approach to the asymptotic regime using absolute cross sections depending on subprocess amplitudes and GPD/DA models. Large variations in predictions.
Cannot separate: Shape of GPD/DA $\leftrightarrow$ higher-order perturbative corrections $\leftrightarrow$ power corrections

Onset of asymptotic regime depends on channel and kinematics.
Assessment needs to be made case-by-case

Large vs. small x_B

$x_B \gtrsim 0.1$ “large”

Valence quarks, large-x gluons

Spin-flavor quantum numbers

Small phase space for radiation,
evolution minor

Non-perturbative dynamics:
Vacuum fields, chiral symmetry
breaking, confinement

$x_B \ll 0.1$ “small”

Gluons and sea quarks

Singlet dominance

Large phase space for QCD radiation,
evolution essential

Non-perturbative dynamics:
Regge-type dynamics hadronic/partonic

Space-time picture: Dipole model,
aligned-jet model

- Fundamental distinction in analysis of exclusive processes in lepton-hadron scattering
- Recruit knowledge/experience about specifics of dynamics at large and small x_B
  gained from theory and “other” experiments (inclusive, diffractive, photon/hadron-hadron)

Large x_B: General features

- Spin-flavor dependent structures generally $\mathcal{O}(1)$

  e.g. $|E| > |H|$ because of nucleon anomalous magnetic moment

  $|E^{u-d}| \gg |E^{u+d}|$ because of isovector anomalous magnetic moment

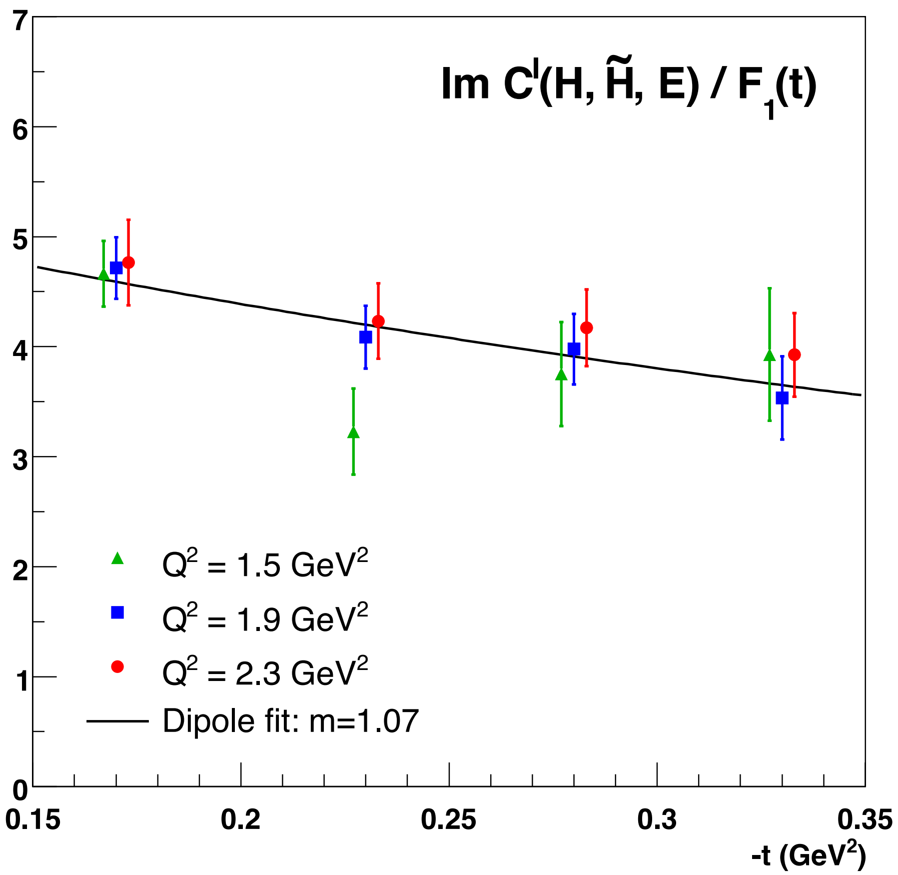
- Skewness effects large $\xi \sim x_B/2$. GPDs far from forward PDFs, behavior unknown

- DVCS and DVMP behave very differently re applicability of asymptotic approximations

  DVCS: Quark subprocess = handbag graph $\mathcal{O}(g^0)$. Quality of asymptotic approximation expected to be similar to inclusive DIS

  DVMP: Quark subprocess = perturbative one-gluon exchange $\mathcal{O}(g^2)$.
  Not relevant at $Q^2 \sim$ few GeV^2, overwhelmed by strong nonperturbative interactions.
  Application of asymptotic approximation questionable.

Large x_B: DVCS



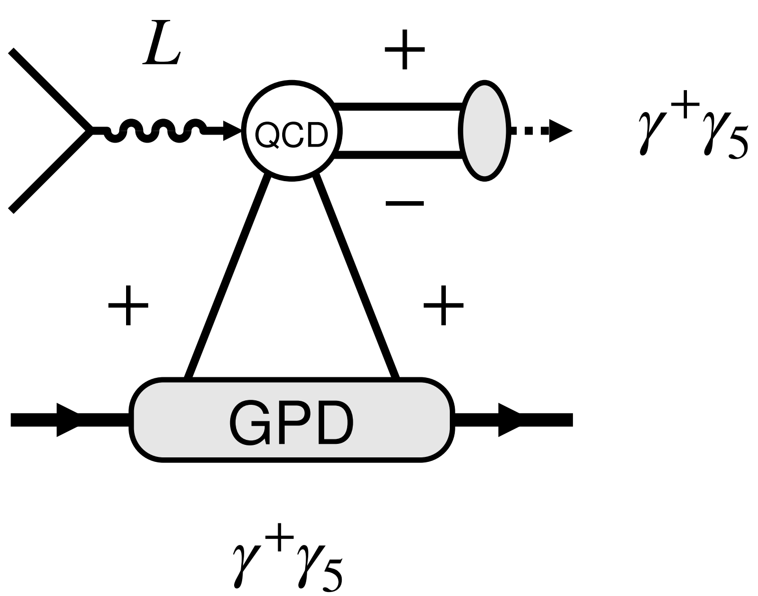
JLab 6 GeV Hall A DVCS measurement 2006

DVCS amplitudes show Q^2 scaling consistent with approach to asymptotic regime

Expected size of power corrections $\sim \dfrac{M_\rho^2}{Q^2}, \dfrac{t}{Q^2}$

Further experimental results : Lecture Charles Hyde

Large x_B: Pseudoscalar mesons



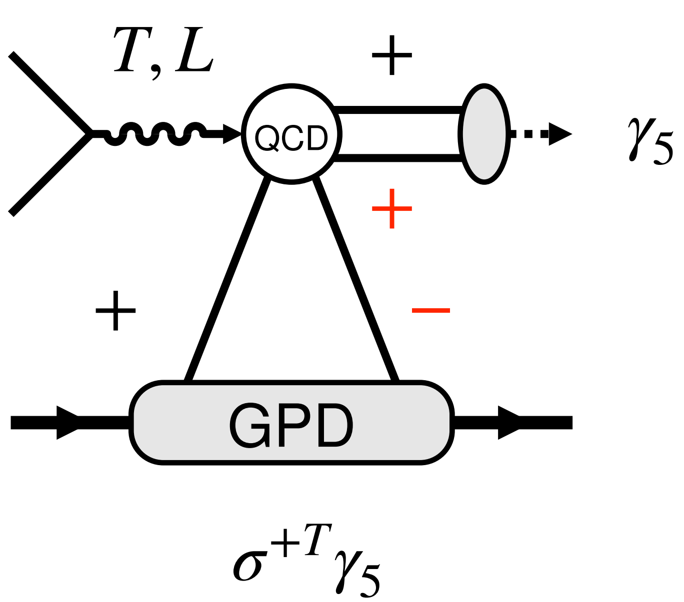
Asymptotically leading amplitude

Chiral-even pion DA $\gamma^+\gamma_5$

Chiral-even nucleon GPD $\gamma^+\gamma_5$ = quark helicity conserving

L current only

Frankfurt, Pobylitsa, Polyakov, Strikman 1998



Subleading amplitude

Chiral-odd pion DA γ_5

Chiral-odd nucleon GPD $\sigma^{+T}\gamma_5$ = quark helicity flipping

T and L currents

Chiral-odd pion DA is numerically large because of chiral symmetry breaking - nonperturbative dynamics

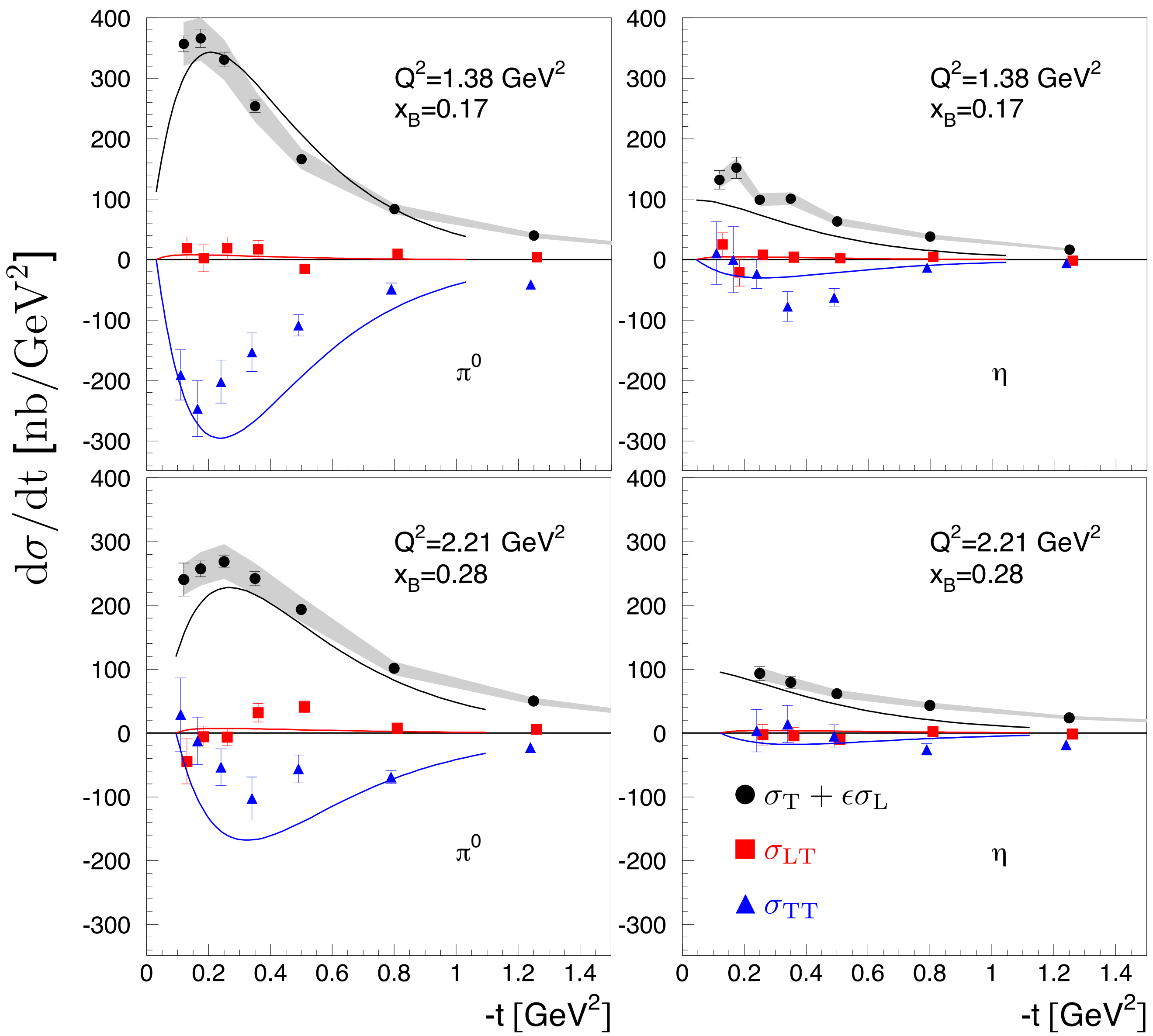
Subleading amplitude is numerically dominant at momentum transfers $Q^2 \sim$ few GeV^2

Goldstein, Liuti et al. 08+; Goloskokov, Kroll 11+

$l + p \rightarrow l' + M + p$

$M = \pi^0, \eta$ pseudoscalar

Large x_B: Pseudoscalar mesons



Practical implementation

Invoke QCD Sudakov form factor to suppress large-size $q\bar{q}$ pairs

Include finite-size corrections from intrinsic p_T of $q\bar{q}$ pair

Approximations beyond strict asymptotics

Goloskokov, Kroll 11+

Comparison with data

Predictions of chiral-odd mechanism consistent with JLab 6 GeV π^0, η data

Possible to sample chiral-odd GPDs in pseudo scalar DVMP at JLab

Properties of chiral-odd GPDs: Active area of theoretical research

$1/N_c$ expansion: Schweitzer, Weiss 2016

Will chiral-even mechanism become relevant at higher W^2 and/or Q^2?

COMPASS exclusive $\pi 0$ data. EIC

Large x_B: Vector mesons

Description of vector meson production at large x_B in context of GPDs and factorization still not well understood

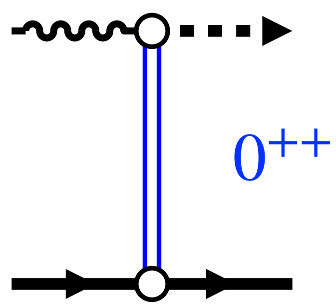
Asymptotically leading expressions not adequate, qualitative discrepancies

Large subleading contributions: Chiral-odd structures? Nonperturbative interactions?

Goloskokov, Kroll 2008+

Small x_B: General features

- Processes without quantum number exchange between target and virtual photon (diffractive processes) dominate in limit $W \to \infty$ at fixed Q^2



$\gamma^* \to \rho^0, \phi, \omega, J/\psi, \gamma$ diffractive - dominant

$\gamma^* \to \pi, \eta, K, \rho^+, K^*$ nondiffractive - suppressed

- In factorized description, diffractive processes couple to the gluon and/or singlet quark GPDs

- All diffractive processes are "similar", differ only in relative coupling to gluon and singlet quark GPDs

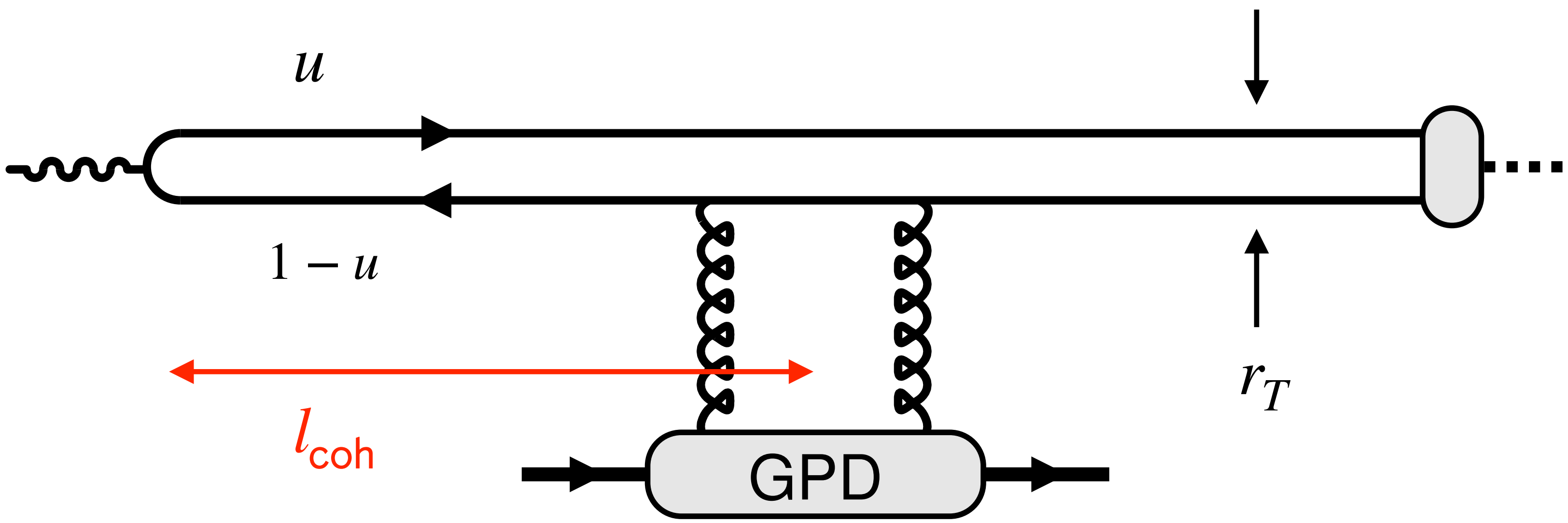
J/ψ: G only ρ^0: G and $q + \bar{q}$ γ: $q + \bar{q}$ at LO, G at NLO

- Evolution essential, determines behavior of gluon and singlet quark GPDs at $x, \xi \to 0$

- Skewness effect can be implemented theoretically. GPDs can be reconstructed from $\xi = 0$ functions in controlled approximation

Shuvaev, Golec-Biernat, Martin 1999. Polyakov, Shuvaev 2002. GUMP → Lecture Yuxun Guo

Small x_B: Space-time picture in target rest frame



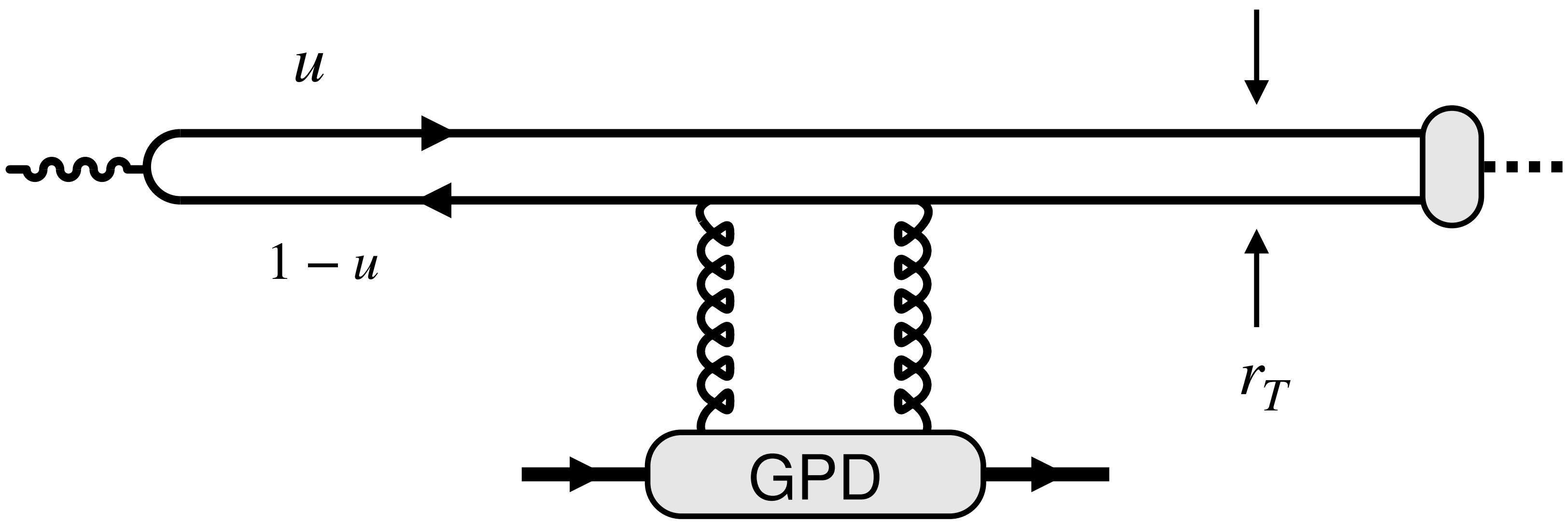
Virtual photon fluctuates into quark-antiquark pair which then interacts with target

Coherence length $\sim 1/x_B M \gg$ 1 fm for $x_B \ll 0.1$

L photon: $u, 1 - u \sim 1/2$. Quark/antiquark share momentum equally.
"Color dipole", transverse size $r_T \sim 1/Q$

T photon: $u, 1 - u \to 0, 1$. Quark/antiquark share momenta unequally, fast + slow.
"Aligned jet", transverse size $r_T \sim 1/\mu_{\text{had}}$

Small x_B: Space-time picture in target rest frame



Correspondence with GPDs and factorization

Gluon GPD determines dipole-target scattering amplitude: $A(dp \to dp) \propto r_T^2\, G(x, \xi, t), \quad x, \xi \sim x_B$

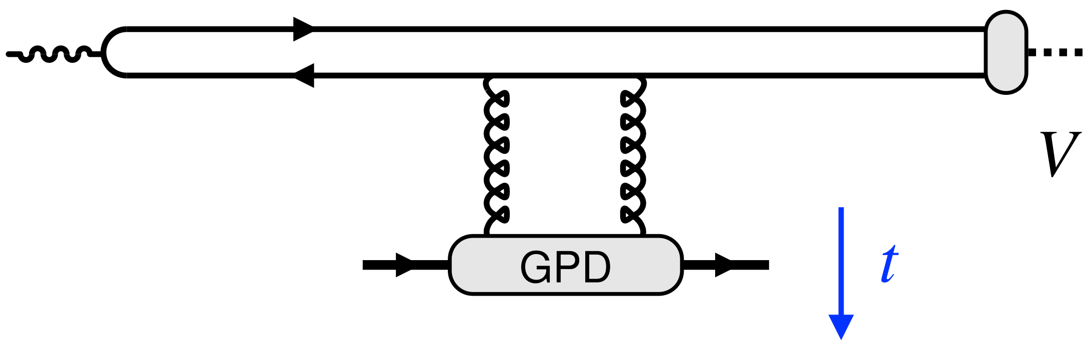
Scale set by inverse dipole size $Q^2_{\text{eff}} = \text{const} \times r_T^{-2}$

Collinear factorization can be discussed/understood in space-time picture:
L vs. T, end-point singularities, size of power corrections, …

Very useful representation~

Baym, Blaettel, Frankfurt, Strikman 1993. Brodsky, Frankfurt, Union, Strikman 1994. Frankfurt, Strikman, Köpf 1996+.
Frankfurt, Radyushkin, Strikman 1998.

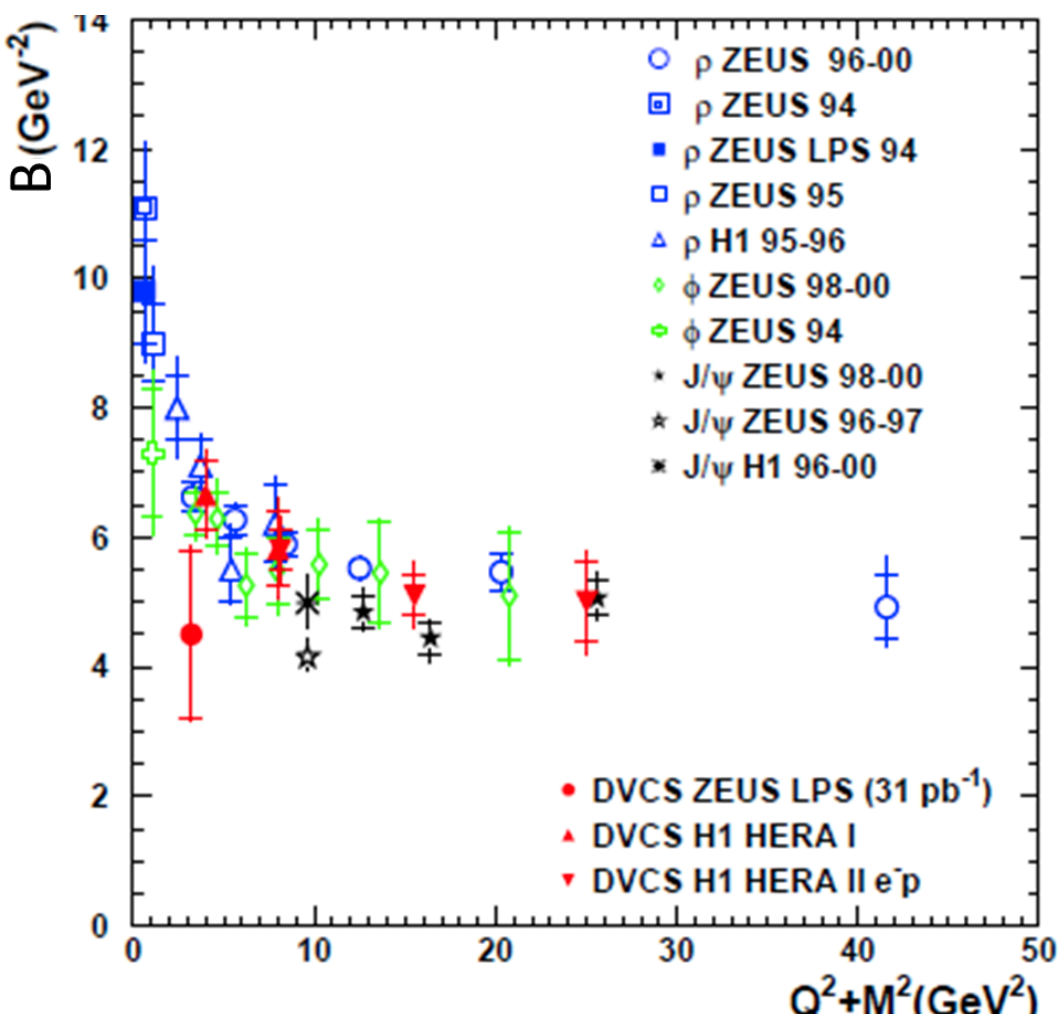
Small x_B: Vector meson production



Test approach to asymptotic regime, applicability of factorization?

$$\frac{d\sigma}{dt} \propto \exp Bt$$

t-dependence of differential cross section



A. Levy, summary of HERA results

Slope B measures transverse radius of dipole-proton interaction

≈ proton's gluonic size + dipole size

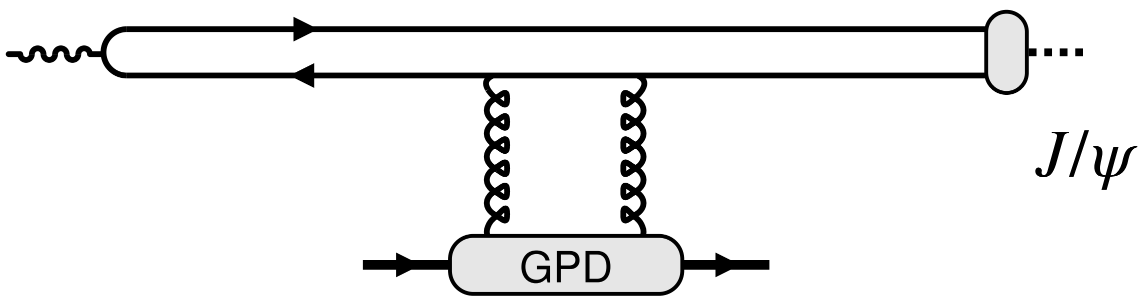
HERA vector meson data

Dipole size decreases with increasing Q^2, becomes pointlike

All diffractive channels show same $t-$slope = t-slope of gluon GPD

Confirms approach to asymptotic regime

Small x_B: Vector meson production

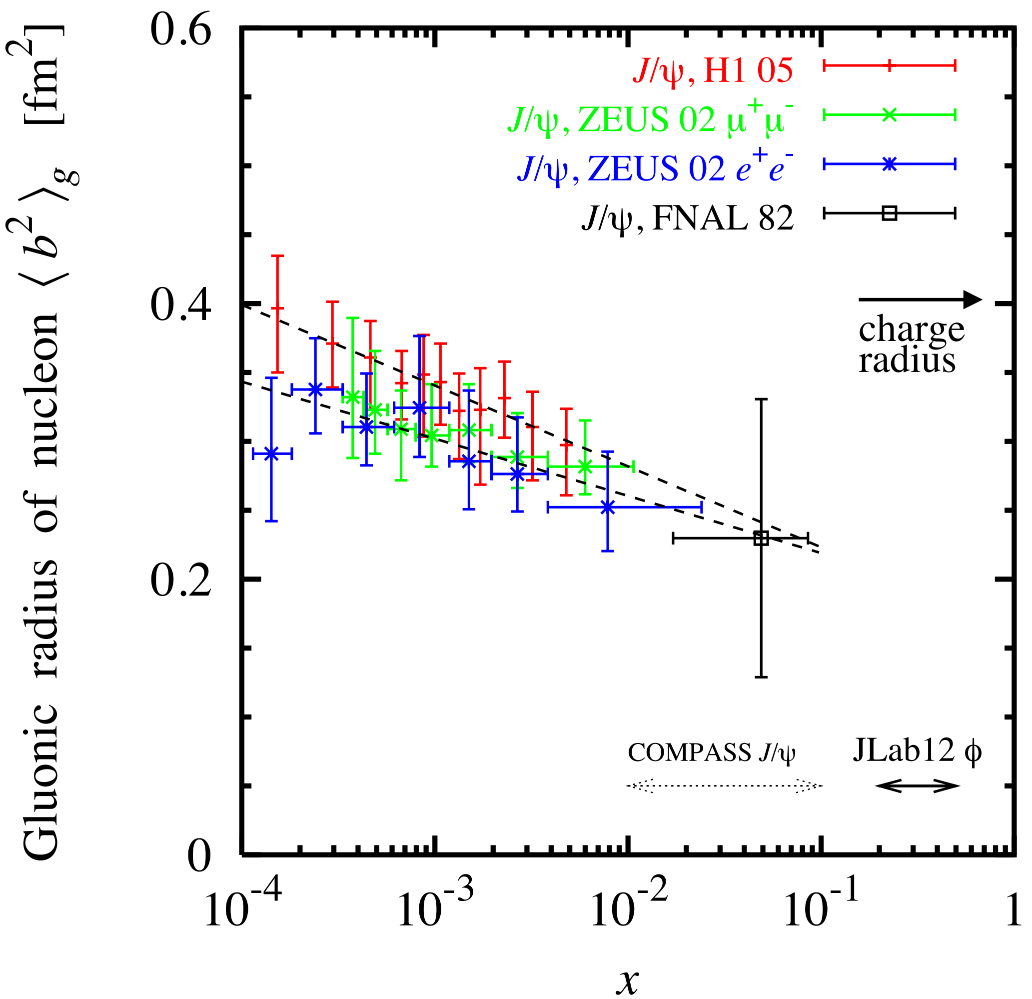


Transverse distribution of gluons in proton extracted from J/ψ production data

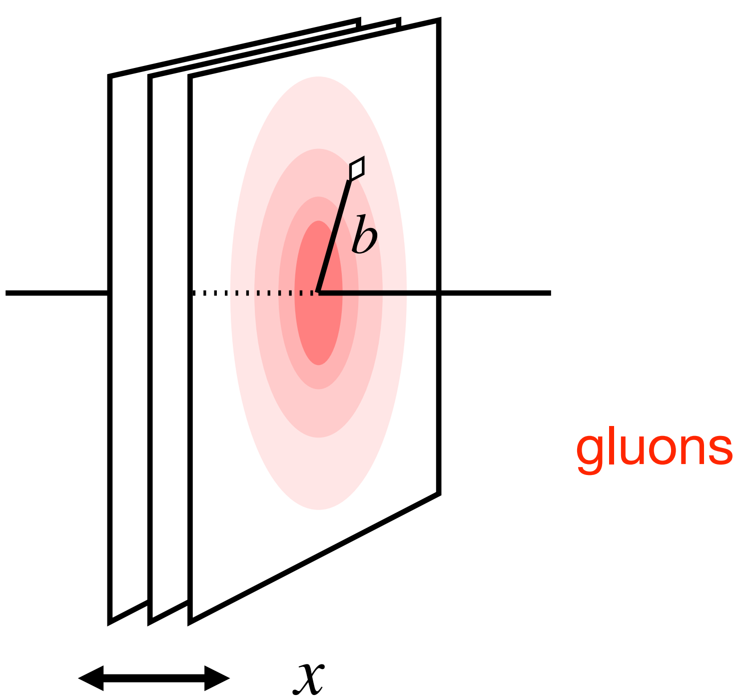
Transverse size grows with decreasing x

Dynamical mechanisms: Bound-state dynamics, parton diffusion at small x

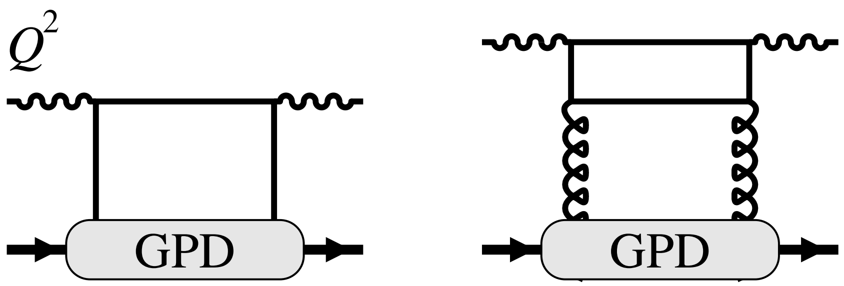
Gluon imaging of nucleon, quantitative

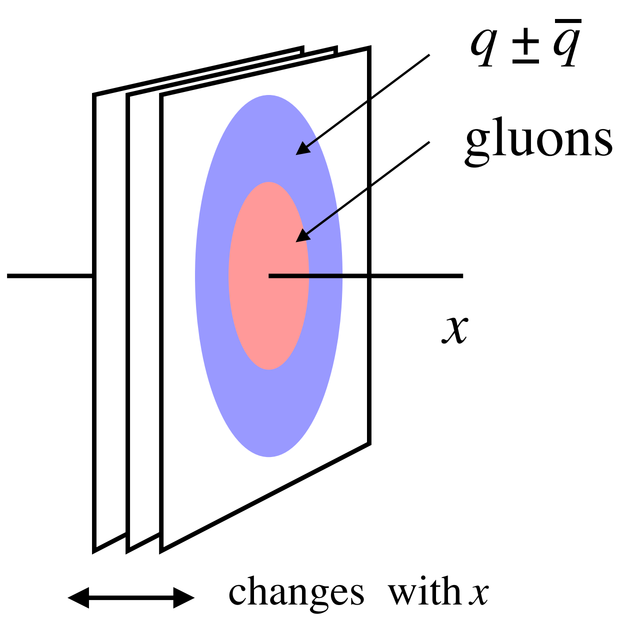


Frankfurt, Strikman, Weiss 2011



Small x_B: DVCS





DVCS analysis at small x_B well established

→ Lecture Marija Cuic

Gluon GPD enters at NLO: Large effect, cancellations between singlet quark and gluon GPD amplitudes

Need model of singlet quark and gluon GPDs at input scale based on nonperturbative dynamics

Aligned-jet model based on “soft” hadronic high-energy scattering dynamics
Frankfurt, Strikman, Guzey 1997. Freund, Strikman 2003

String-based “holographic” model
Mamo, Zahed 2024

Analysis can determine transverse distributions of singlet quarks and gluons in nucleon