FIRST INTERNATIONAL SCHOOL OF HADRON FEMTOGRAPHY

Jefferson Lab | September 16-25, 2024

QCD Factorization:

Matching hadrons to quarks and gluons with controllable approximations

- The need for factorization
- Inclusive cross sections with one, two and more identified hadrons
- Factorization for exclusive scattering
- Factorization beyond the leading power
- Joint factorization of QCD and QED
- Summary

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Frontiers of QCD and Strong Interaction

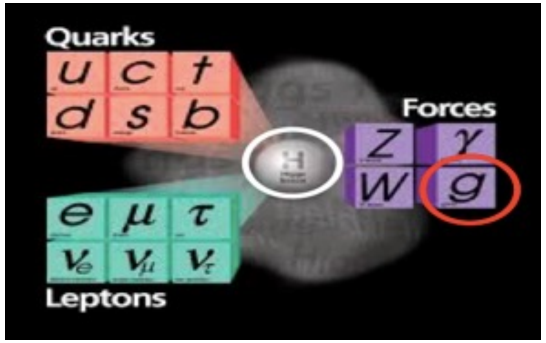
- ❑ **Understanding where did we come from?**

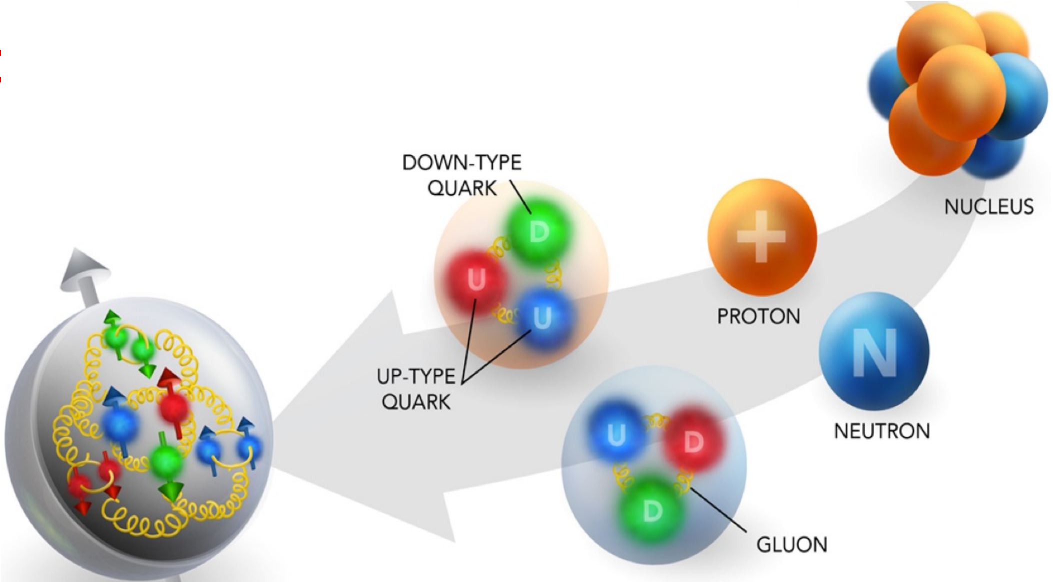
Global Time: →

QCD at high temperature, high densities, phase transition, …

Facilities – Relativistic heavy ion collisions: SPS, RHIC, the LHC

- ❑ **Understanding what are we made of?**





NO quarks and gluons can be seen in isolation!

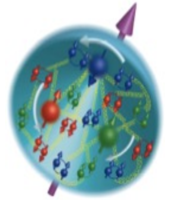
- **How to understand the emergent properties of nucleon and nuclei (elements of the periodic table) in terms of elements of the modern periodic table?**
- **How does the glue bind us all?**

Nuclear Femtography

Search for answers to these questions at a Fermi scale!

Facilities – CEBAF, EIC, EICC, LHeC, …

QCD Color is Fully Entangled



❑ QCD color confinement:

- *Do not see any quarks and gluons in isolation*
- *The structure of nucleons and nuclei – emergent properties of QCD*

Color Confinement — Asymptotic freedom — Q (GeV)

20 MeV (10 fm) — 200 MeV (1 fm) — 2 GeV (1/10 fm)

Probing scale

QCD at the Fermi Scale: Femto-science (0.1-10 fm)

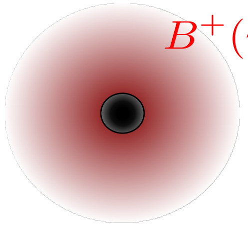
Asymptotic regime PQCD works beautifully!

- *The most interesting, rich, and complex regime of the theory!*
- *All emergent phenomena depend on the scale at which we probe them!*

❑ QCD is non-perturbative:

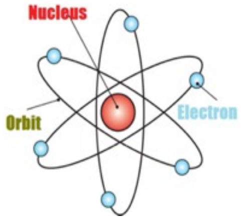
- *Any cross section/observable with identified hadron is not perturbatively calculable!*
- *Color is fully entangled!*

B-meson



Brown-Muck

Atomic structure

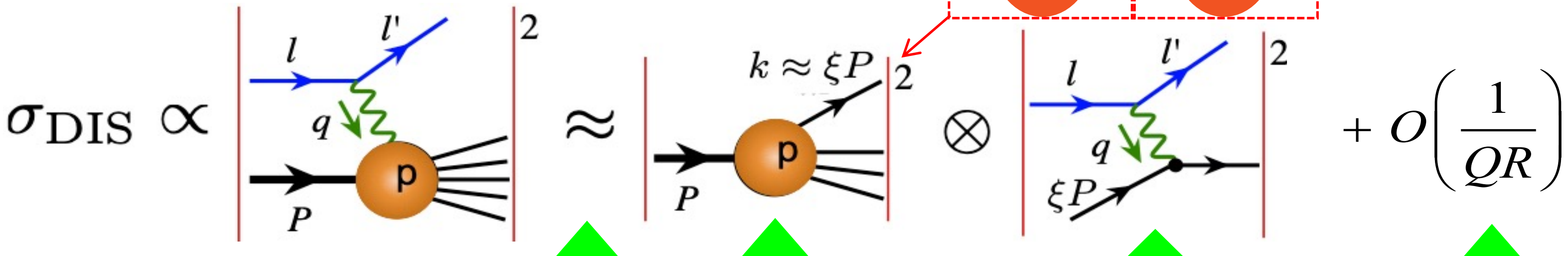


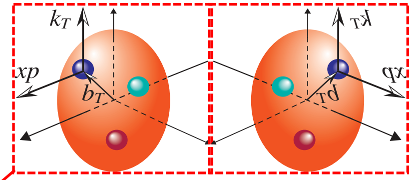
Quantum orbits

Theoretical Approaches – Approximations:

❑ Perturbative QCD Factorization:

– Approximation at Feynman diagram level





Factorization Theory	Parton-distribution "Structure"	Hard-part Probe	Power corrections Approximation

❑ Effective field theory (EFT):

– Approximation at the Lagrangian level

Soft-collinear effective theory (SCET), Non-relativistic QCD (NRQCD), Heavy quark EFT, chiral EFT(s), …

❑ Lattice QCD:

– Approximation for finite lattice spacing, finite box, lightest quark masses, … with Euclidean time formulation (removable with increased computational cost)

Hadron structure, hadron spectroscopy, nuclear structure, phase shift, …

❑ Other approaches:

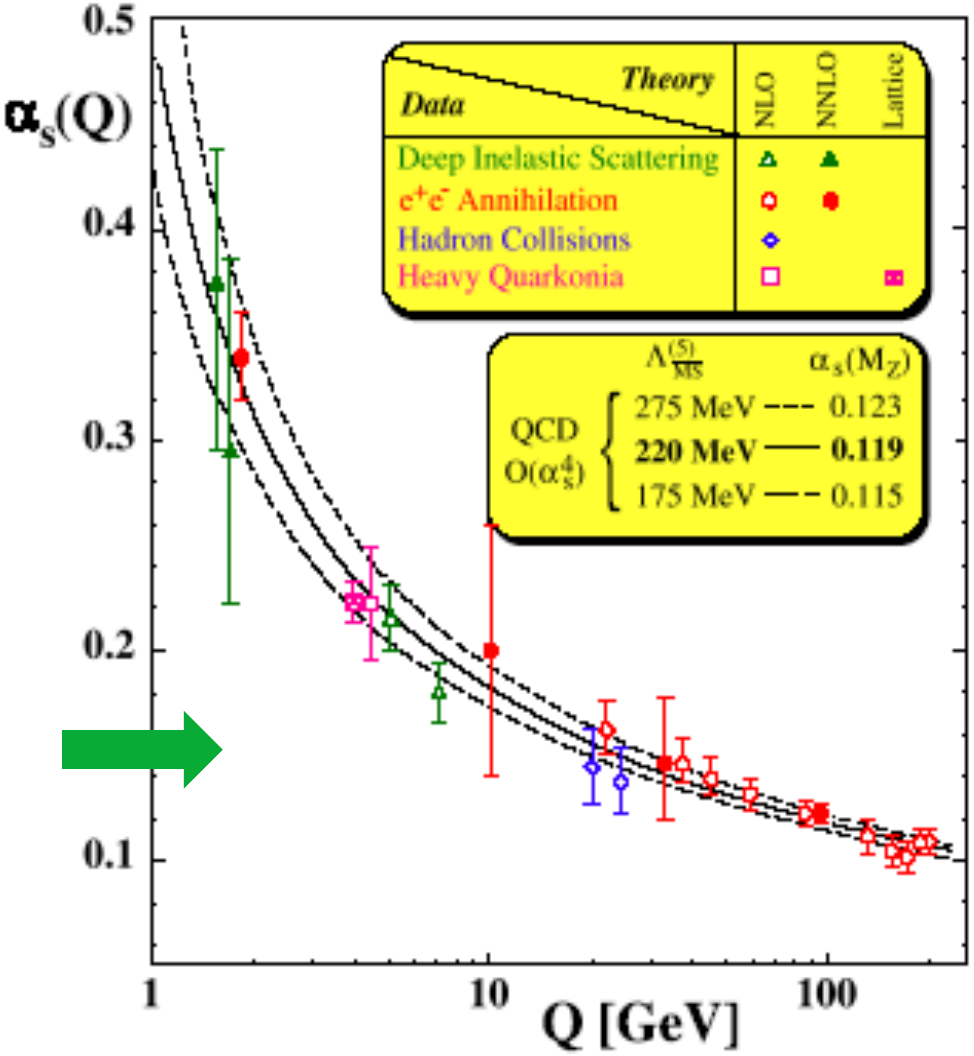
Light-cone perturbation theory, Dyson-Schwinger Equations (DSE), Constituent quark models, AdS/CFT correspondence, …

QCD Asymptotic Freedom

❑ **Interaction strength:**

$$\alpha_s(\mu_2) = \frac{\alpha_s(\mu_1)}{1 - \frac{\beta_1}{4\pi}\alpha_s(\mu_1)\ln\left(\frac{\mu_2^2}{\mu_1^2}\right)} \equiv \frac{4\pi}{-\beta_1 \ln\left(\frac{\mu_2^2}{\Lambda^2_{QCD}}\right)}$$

μ_2 and μ_1 not independent



Asymptotic Freedom ⇔ antiscreening

$$\text{QCD: } \frac{\partial \alpha_s(Q^2)}{\partial \ln Q^2} = \beta(\alpha_s) < 0$$

Compare

$$\text{QED: } \frac{\partial \alpha_{EM}(Q^2)}{\partial \ln Q^2} = \beta(\alpha_{EM}) > 0$$

D.Gross, F.Willczek, Phys.Rev.Lett 30, (1973)
H.Politzer, Phys.Rev.Lett. 30, (1973)

2004 Nobel Prize in Physics

Discovery of QCD Asymptotic Freedom

Controllable perturbative QCD calculations at HIGH ENERGY or short-distance!

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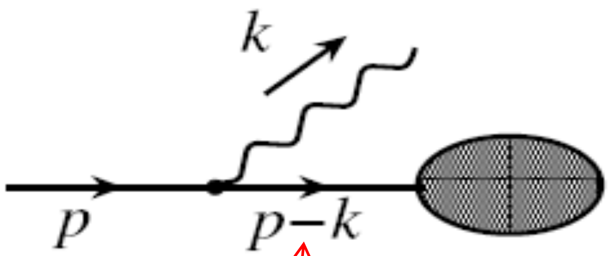
Infrared and Collinear Divergences

❑ **Consider a general diagram:**

$$p^2 = 0, \quad k^2 = 0$$ **for a massless theory**



Singularity

✧ $$k^\mu \to 0 \;\Rightarrow\; (p-k)^2 \to p^2 = 0$$

⇨ **Infrared (IR) divergence**

✧ $$k^\mu \,||\, p^\mu \;\Rightarrow\; k^\mu = \lambda p^\mu \quad \text{with} \quad 0 < \lambda < 1$$

$$\Rightarrow\; (p-k)^2 \to (1-\lambda)^2\, p^2 = 0$$

⇨ **Collinear (CO) divergence**

IR and CO divergences are generic problems of a massless perturbation theory

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Infrared Safety (IRS)

❑ Infrared safety:

$$\sigma_{\rm Phy}\left(\frac{Q^2}{\mu^2},\alpha_s(\mu^2),\frac{m^2(\mu^2)}{\mu^2}\right) \Rightarrow \hat{\sigma}\left(\frac{Q^2}{\mu^2},\alpha_s(\mu^2)\right) + \mathcal{O}\left[\left(\frac{m^2(\mu^2)}{\mu^2}\right)^{\kappa}\right]$$

Infrared safe = κ > 0

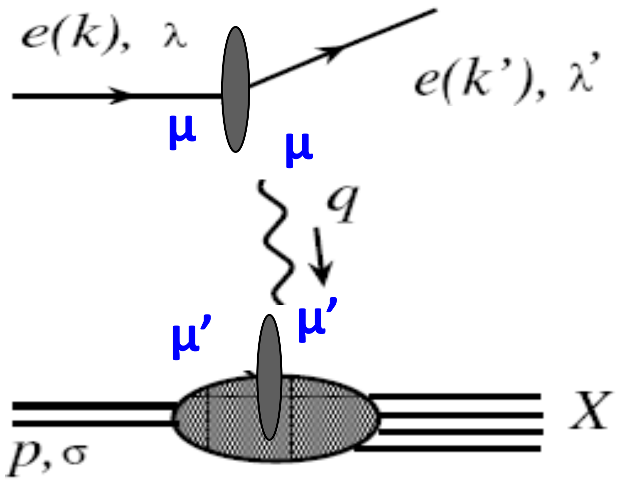
Purely perturbative calculations alone (exploiting asymptotic freedom) are only useful for quantities that are infrared safe (IRS)!

❑ Cross section with identified hadron(s):

- ***Can not be calculated perturbatively!***
- ***Solution – QCD factorization:***
  - ***to isolated what can be calculated perturbatively,***
  - ***to represent the leading non-perturbative information by universal functions***
  - ***to justify the approximation to neglect other nonperturbative information,***
    such as power corrections, …

Inclusive Lepton-Hadron DIS – One Identified Hadron

❑ **Scattering amplitude:**



$$\mathrm{M}(\lambda,\lambda';\sigma,q) = \bar{u}_{\lambda'}(k')\left[-ie\gamma_{\mu}\right]u_{\lambda}(k)$$

$$* \quad \left(\frac{i}{q^2}\right)\left(-g^{\mu\mu'}\right)$$

$$* \quad \langle X|eJ^{em}_{\mu'}(0)|p,\sigma\rangle$$

❑ **Cross section:**

$$d\sigma^{\mathrm{DIS}} = \frac{1}{2s}\left(\frac{1}{2}\right)^2 \sum_X \sum_{\lambda,\lambda',\sigma} \left|\mathrm{M}(\lambda,\lambda';\sigma,q)\right|^2 \left[\prod_{i=1}^{X}\frac{d^3 l_i}{(2\pi)^3 2E_i}\right]\frac{d^3k'}{(2\pi)^3 2E'}(2\pi)^4\delta^4\left(\sum_{i=1}^{X} l_i + k' - p - k\right)$$

$$\boxed{E'\frac{d\sigma^{\mathrm{DIS}}}{d^3k'} = \frac{1}{2s}\left(\frac{1}{Q^2}\right)^2 L^{\mu\nu}(k,k')W_{\mu\nu}(q,p)}$$

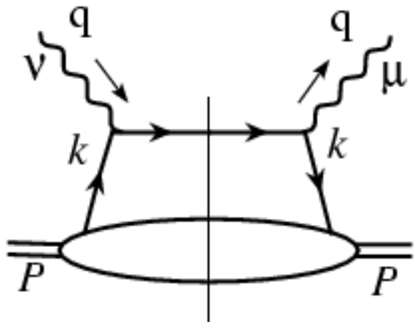
❑ **Leptonic tensor:**

– known from QED: $L^{\mu\nu}(k,k') = \frac{e^2}{2\pi^2}\left(k^{\mu}k'^{\nu} + k^{\nu}k'^{\mu} - k\cdot k' g^{\mu\nu}\right)$

DIS Structure Functions

❑ Hadronic tensor:

$$W_{\mu\nu}(q,p,S) = \frac{1}{4\pi}\int d^4z\; e^{iq\cdot z}\;\langle p,S|J_\mu^\dagger(z)J_\nu(0)|p,S\rangle$$



Cut-diagram

❑ Symmetries:

- ✧ **Parity invariance (EM current)** ⟶ $W_{\mu\nu} = W_{\nu\mu}$ sysmetric for spin avg.
- ✧ **Time-reversal invariance** ⟶ $W_{\mu\nu} = W^*_{\mu\nu}$ real
- ✧ **Current conservation** ⟶ $q^\mu W_{\mu\nu} = q^\nu W_{\mu\nu} = 0$

$$W_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)F_1\left(x_B,Q^2\right) + \frac{1}{p\cdot q}\left(p_\mu - q_\mu\frac{p\cdot q}{q^2}\right)\left(p_\nu - q_\nu\frac{p\cdot q}{q^2}\right)F_2\left(x_B,Q^2\right)$$

$$+iM_p\varepsilon^{\mu\nu\rho\sigma}q_\rho\left[\frac{S_\sigma}{p.q}g_1\left(x_B,Q^2\right) + \frac{\left(p.q\right)S_\sigma - \left(S.q\right)p_\sigma}{\left(p.q\right)^2}g_2\left(x_B,Q^2\right)\right]$$

❑ Structure functions – infrared sensitive:

$$F_1\left(x_B,Q^2\right), F_2\left(x_B,Q^2\right), g_1\left(x_B,Q^2\right), g_2\left(x_B,Q^2\right)$$

No QCD parton dynamics used in above derivation!

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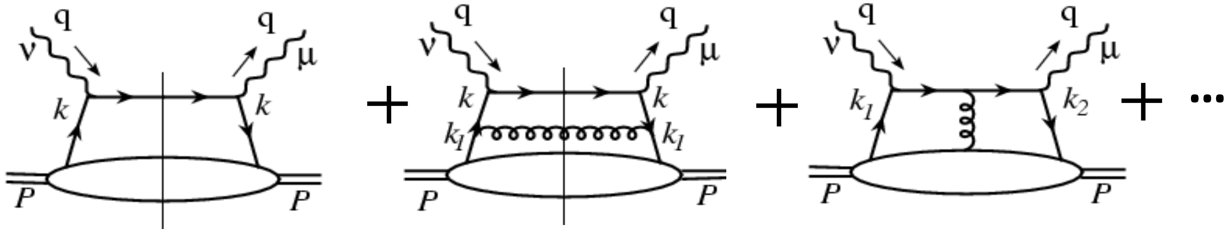
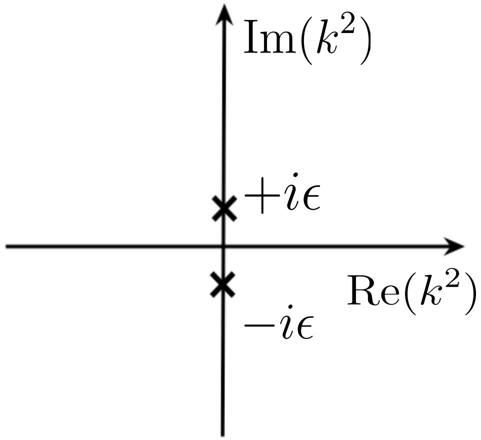
Long-Lived Parton States – Necessary for Separation of Scales

❑ Feynman diagram representation of the hadronic tensor:

$W^{\mu\nu} \propto$





❑ Perturbative pinched poles:

$$\int d^4k\ \mathrm{H}(Q,k)\left(\frac{1}{k^2+i\varepsilon}\right)\left(\frac{1}{k^2-i\varepsilon}\right)\mathrm{T}(k,\frac{1}{r_0}) \Rightarrow \infty \text{ perturbatively}$$

❑ Perturbative factorization:

$$k^\mu = xp^\mu + \frac{k^2+k_T^2}{2xp\cdot n}n^\mu + k_T^\mu$$

Light-cone coordinate:

$$v^\mu = (v^+, v^-, v^\perp), \qquad v^\pm = \frac{1}{\sqrt{2}}(v^0 \pm v^3)$$

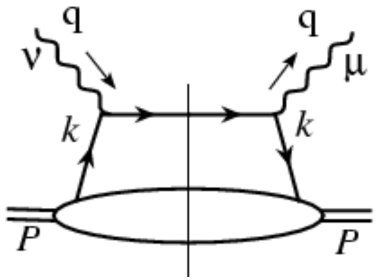
$$\int \frac{dx}{x} d^2k_T\ \mathrm{H}(Q,k^2=0)\int dk^2\left(\frac{1}{k^2+i\varepsilon}\right)\left(\frac{1}{k^2-i\varepsilon}\right)\mathrm{T}(k,\frac{1}{r_0}) + \mathcal{O}\left(\frac{\langle k^2\rangle}{Q^2}\right)$$

Short-distance

Nonperturbative matrix element

Collinear Factorization – Further Approximation

- **Collinear approximation, if** $\boxed{Q \sim xp\cdot n \gg k_T,\ \sqrt{k^2}}$ **– Lowest order:**



$$W^{\mu\nu}_{\gamma^* p} = \sum_f \int \frac{d^4k}{(2\pi)^4} \sum_{ij} \underbrace{(\gamma^\mu \gamma\cdot(k+q)\gamma^\nu)_{ij}\,(2\pi)\delta((k+q)^2)} \int d^4y\, e^{iky}\langle p|\overline{\psi}_j(0)\psi_i(y)|p\rangle + \ldots$$

$$\equiv \sum_f \int \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\left[\mathcal{H}^{\mu\nu}_{\gamma^* f}(Q,k)\mathcal{F}_{f/p}(k,p)\right] + \ldots$$

$$\boxed{\delta\big((k+q)^2\big) = \tfrac{1}{2P\cdot q}\delta(x-\xi) = \tfrac{1}{2P\cdot q}\delta\left(x - \tfrac{k^+}{P^+}\right)}$$

$$\approx \sum_f \int dx\, \mathrm{Tr}\left[\mathcal{H}^{\mu\nu}_{\gamma^* f}(Q, k\approx xp) \int \frac{d^4k}{(2\pi)^4}\delta\Big(x - \frac{k\cdot n}{p\cdot n}\Big)\mathcal{F}_{f/p}(k,p)\right] + \mathcal{O}\Big(\frac{\langle k^2\rangle}{Q^2}, \frac{\langle k_T^2\rangle}{Q^2}\Big) + \ldots$$

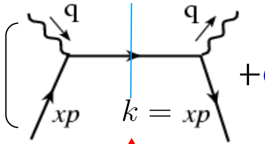
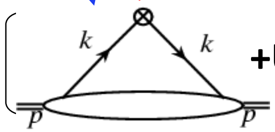
– Collinear Approx.

$$\approx \sum_f \int \frac{dx}{x}\, \mathrm{Tr}\left[\mathcal{H}^{\mu\nu}_{\gamma^* f}(Q, xp)\frac{1}{2}\gamma\cdot(xp)\right] \int \frac{d^4k}{(2\pi)^4}\delta\Big(x - \frac{k\cdot n}{p\cdot n}\Big)\mathrm{Tr}\left[\frac{\gamma\cdot n}{2p\cdot n}\mathcal{F}_{f/p}(k,p)\right] + \ldots$$

– Spin decomposition

$$\approx \sum_f \int \frac{dx}{x}\, \widehat{W}^{\mu\nu}_{\gamma^* f}(x, Q^2/\mu^2)\ \phi_{f/p}(x,\mu^2) + \ldots$$

$$\int \frac{dx}{x} \qquad \frac{\gamma\cdot n}{2p\cdot n}\,\delta\left(x - \frac{k\cdot n}{p\cdot n}\right)\frac{d^4k}{(2\pi)^4}$$

$$\approx \Bigg[$$  $$+\,O\left(\frac{k_T^2}{Q^2}\right)\Bigg] \otimes \Bigg[$$  **+UVCT(μ)** $$\Bigg]$$

$$\frac{1}{2}\gamma\cdot(xp) \qquad \widehat{W}^{\mu\nu}_{\gamma^* f}(x, Q^2/\mu^2) = \mathrm{Tr}\left[\mathcal{H}^{\mu\nu}_{\gamma^* f}(Q, xp)\frac{1}{2}\gamma\cdot(xp)\right]$$

$\int d^4k$ **in line 1 is limited, no UV**

But, factorization allows $\int d^4k$

to generate UV – Need UVCT(μ) to define parton distribution!

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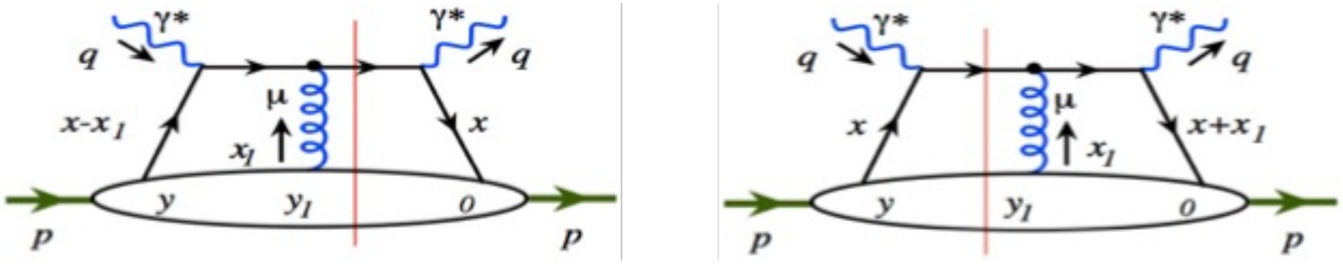
Gauge Link – 1st order in coupling "g"

- **Longitudinal gluon:**



- **Left diagram:**

$$\int dx_1 \left[\int \frac{p^+ dy_1^-}{2\pi}\, e^{ix_1 p^+(y_1^- - y^-)}\, n\cdot A^a(y_1^-)\right] \mathcal{M}(-igt^a)\frac{\gamma\cdot p}{p^+}\frac{i((x-x_1-x_B)\gamma\cdot p + (Q^2/2x_B p^+)\gamma\cdot n)}{(x-x_1-x_B)Q^2/x_B + i\epsilon}$$

$$= g\int \frac{dy_1^-}{2\pi}\, n\cdot A^a(y_1^-) t^a \left[\int dx_1\, \frac{1}{-x_1+i\epsilon}\, e^{ix_1 p^+(y_1^- - y^-)}\right] \mathcal{M} = \boxed{-ig\int_{y^-}^{\infty} dy_1^-\, n\cdot A(y_1^-)\mathcal{M}}$$

- **Right diagram:**

$$\int dx_1 \left[\int \frac{p^+ dy_1^-}{2\pi}\, e^{ix_1 p^+ y_1^-}\, n\cdot A^a(y_1^-)\right] \frac{-i((x+x_1-x_B)\gamma\cdot p + (Q^2/2x_B p^+)\gamma\cdot n)}{(x+x_1-x_B)Q^2/x_B - i\epsilon}(+igt^a)\,\frac{\gamma\cdot p}{p^+}\,\mathcal{M}$$

$$= g\int \frac{dy_1^-}{2\pi}\, n\cdot A^a(y_1^-) t^a \left[\int dx_1\, \frac{1}{x_1-i\epsilon}\, e^{ix_1 p^+ y_1^-}\right] \mathcal{M} = \boxed{ig\int_{0}^{\infty} dy_1^-\, n\cdot A(y_1^-)\,\mathcal{M}}$$

- **Total contribution:**

$$-ig\left[\int_0^{\infty} - \int_{y^-}^{\infty}\right] dy_1^-\, n\cdot A(y_1^-)\,\mathcal{M}_{\mathrm{LO}}$$

O(g)-term of the gauge link!

An Instructive Exercise for Factorization beyond the Leading Order

- **Consider a cross section:**

$$\sigma(Q^2,m^2)=\sigma_0\left[1+\alpha_s I(Q^2,m^2)+\mathcal{O}(\alpha_s^2)\right]$$

- **Leading order quantum correction:**

$$I(Q^2,m^2)=\int_0^\infty dk^2\,\frac{1}{k^2+m^2}\,\frac{Q^2}{Q^2+k^2}$$

- **Leading power contribution in O(m²/Q²):**

$$I(Q^2,m^2)=\int_{k^2\ll Q^2} dk^2\,\frac{1}{k^2+m^2}+\int_{k^2\sim Q^2} dk^2\,\frac{1}{k^2}\,\frac{Q^2}{Q^2+k^2}+\mathcal{O}\left(\frac{m^2}{Q^2}\right)$$

- **Leading power contribution to the cross section:**

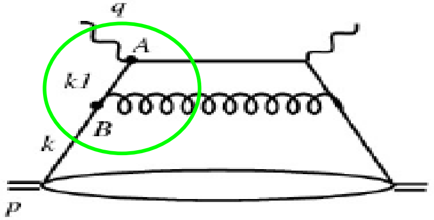
$$\sigma(Q^2,m^2)=\left[1+\alpha_s\int_{k^2\ll Q^2} dk^2\,\frac{1}{k^2+m^2}\right]\left[1+\alpha_s\int_{k^2\sim Q^2} dk^2\,\frac{1}{k^2}\,\frac{Q^2}{Q^2+k^2}\right]$$
$$+\mathcal{O}(\alpha_s^2)+\mathcal{O}\left(\frac{m^2}{Q^2}\right)$$
$$\equiv \phi\otimes\hat{\sigma}+\mathcal{O}(\alpha_s^2)+\mathcal{O}\left(\frac{m^2}{Q^2}\right)$$

Long-distance distribution

Short-distance hard part

QCD Corrections at the Next-to-Leading Order

- **NLO partonic diagram to structure functions:**

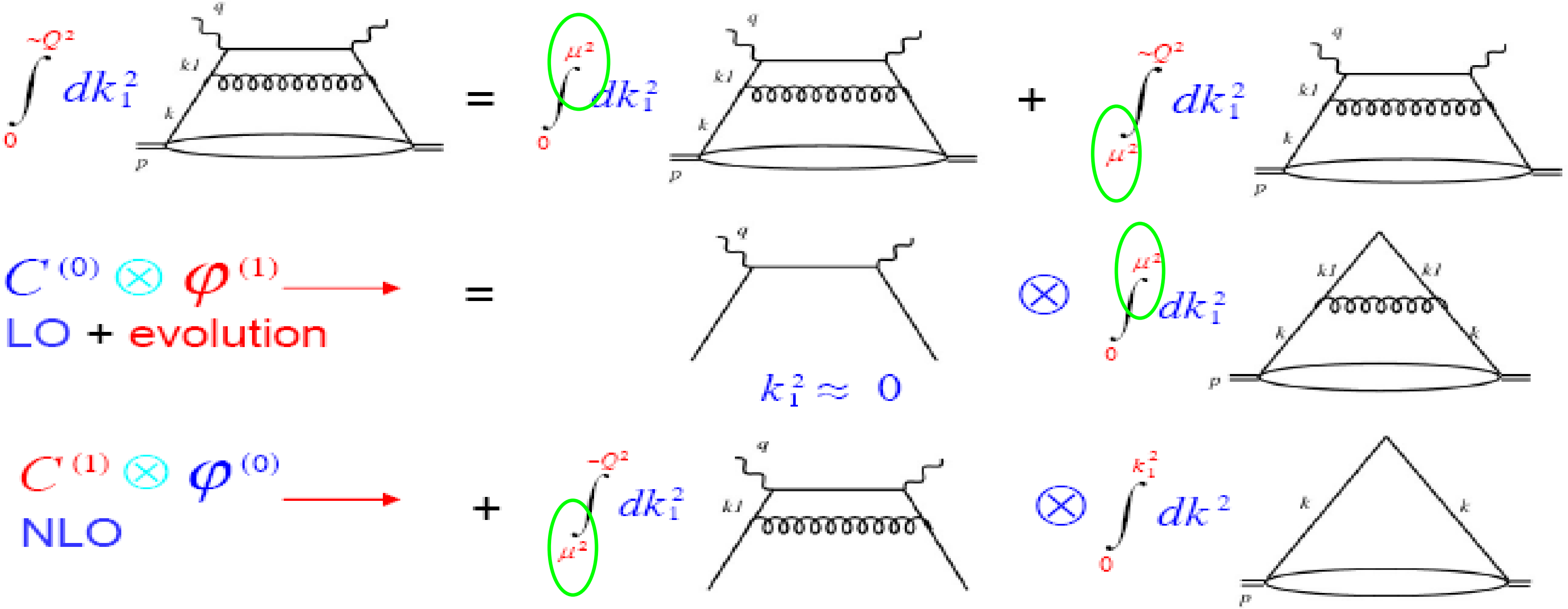


$$\propto \int_0^{-Q^2} \frac{dk_1^2}{k_1^2}$$

Dominated by $\begin{cases} k_1^2 \sim 0 \\ t_{AB} \rightarrow \infty \end{cases}$

Diagram has both long- and short-distance physics

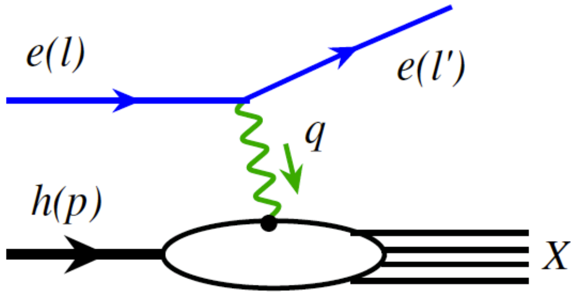
- **Factorization, separation of short- from long-distance:**



Same idea as the Instructive Exercise for Factorization

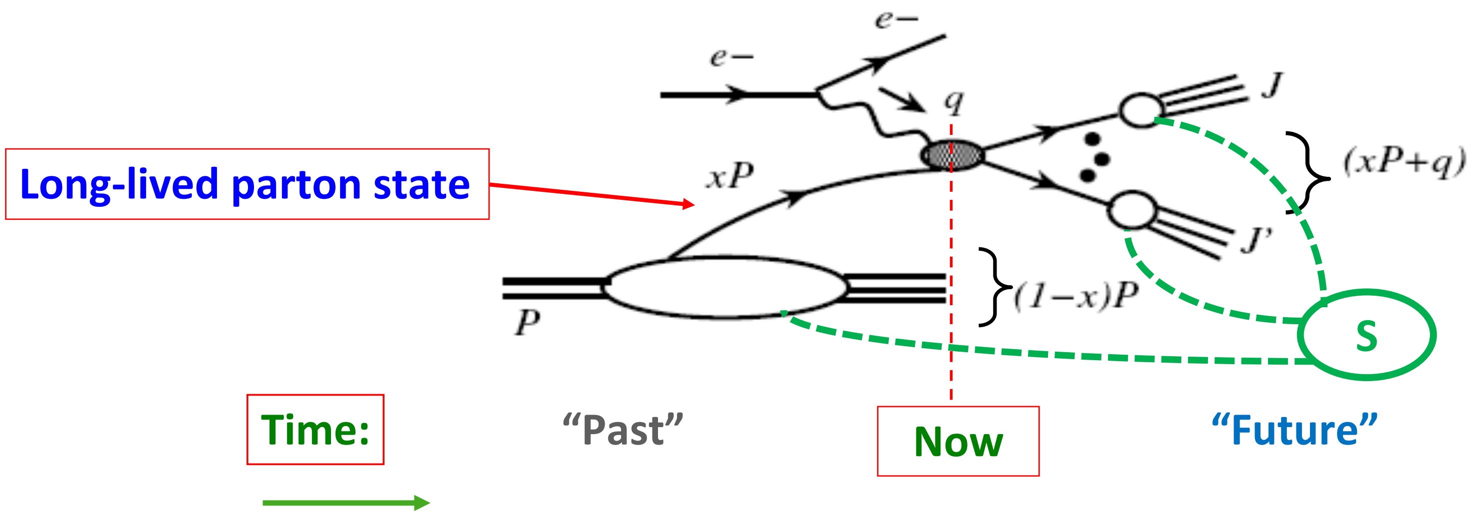
Factorization to All Order – One Identified Hadron

❑ Inclusive lepton-hadron DIS:



$$e(l) + h(p) \to e(l') + X$$

$\Rightarrow$



❑ Soft interaction – trouble maker:

Jet J direction: $\bar{n}_J = (1, 0, 0_\perp)$

Loop l in Jet J: $l \sim (1, \lambda^2, \lambda)\, E_J$

Soft gluons: $k = (\lambda^2, \lambda^2, \lambda^2)\, Q$

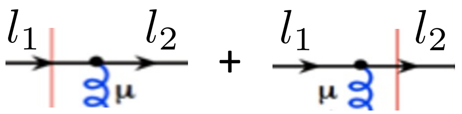
Hard scale: $Q \sim E_J \sim \sqrt{S}$

Propagator in J:

$$(r_J \pm k_i)^2 + i\epsilon \approx r_J^2 \pm 2 r_J^+ k_i^- + \mathcal{O}(\lambda^3) + i\epsilon$$

Soft gluon pol: $J^{\mu_i}(\ldots k_i \ldots) \propto \bar{n}_J^{\mu_i}$

Unitarity:

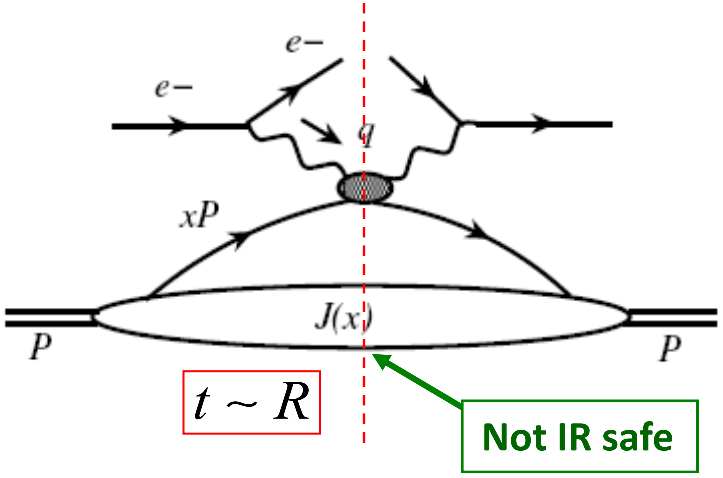


$$\Rightarrow\ \delta(l_1^2)\,\frac{1}{l_2^2 - i\epsilon} + \frac{1}{l_1^2 + i\epsilon}\,\delta(l_2^2)$$

$$\Rightarrow\ \delta(l_1^2)\,\delta(l_2^2) - \delta(l_1^2)\,\delta(l_2^2)$$

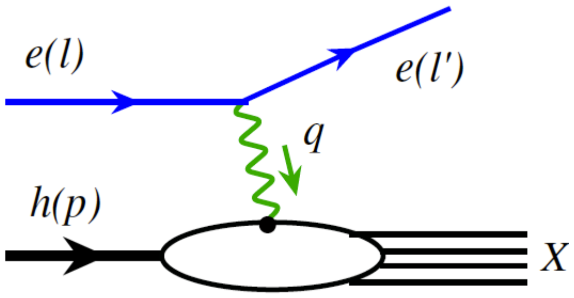
$$\Rightarrow\ 0$$

$\Rightarrow$

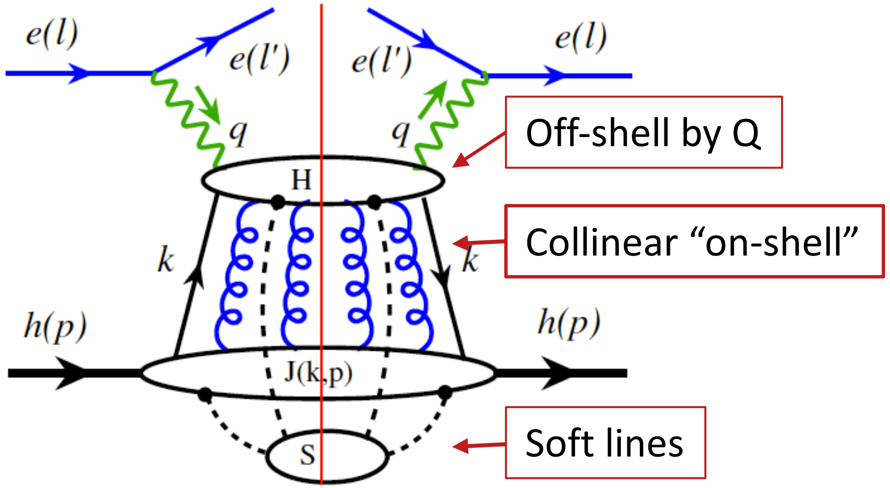


Factorization at the Leading Power – One Identified Hadron

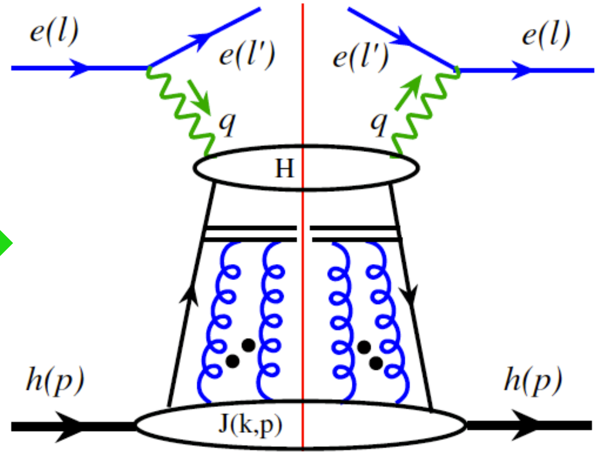
❑ Leading pinch surface – Inclusive DIS:



$$e(l) + h(p) \rightarrow e(l') + X$$



"Leading pinch surface"
Reduced diagrams
Soft lines to "H" power suppressed



Apply Ward identity
Longitudinally polarized gluons
decoupled from "H" to gauge link

- **Factorization formalism – leading power:**

$$E'\frac{d\sigma^{\rm DIS}_{eh\to eX}}{d^3l'}(l,p;l') = \sum_{f=q,\bar{q},g}\int dx\ \phi_{f/h}(x,\mu^2)\, E'\frac{d\hat{\sigma}_{ef\to eX}}{d^3l'}(l,\hat{k};l',\mu^2) + \mathcal{O}\left[\frac{\Lambda^2_{\rm QCD}}{Q^2}\right]$$

$$\boxed{\hat{k}\equiv xp^+}$$

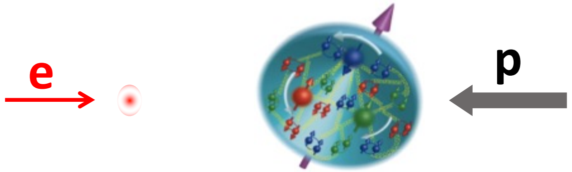
Hard part: $E'\frac{d\hat{\sigma}^{(\rm n)}_{ef\to eX}}{d^3l'} = E'\frac{d\sigma^{\rm DIS(n)}_{ef\to eX}}{d^3l'}(l,p;l') - \sum_{\rm m=0}^{\rm n-1}\left[\sum_{f'=q,\bar{q},g} E'\frac{d\hat{\sigma}^{(\rm m)}_{ef'\to eX}}{d^3l'} \otimes \phi^{(\rm n-m)}_{f'/f}(x,\mu^2)\right]$

- **Renormalization improvement:**

$$d\sigma_{eh\to eX}/d\log\mu^2 = 0 \quad \Rightarrow \quad \frac{d\phi_{f/h}(x,\mu^2)}{d\log\mu^2} = \sum_{f'}\int_x^1 \frac{dx'}{x'} P_{f/f'}\left(\frac{x}{x'},\alpha_s(\mu^2)\right)\phi_{f'/h}(x',\mu^2)$$

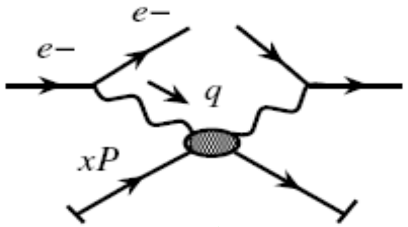
DGLAP equation

From One Hadron to Two Hadrons

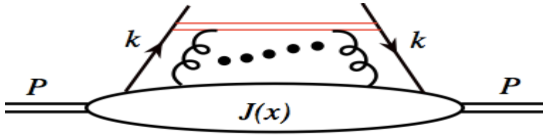


- **One hadron:**

$\sigma^{\mathrm{DIS}}_{\mathrm{tot}} \sim$



$\otimes$

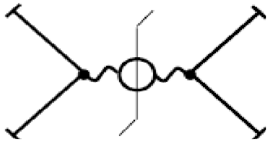


$+ O\left(\frac{1}{QR}\right)$

Hard-part Probe	Parton-distribution Structure	Power corrections Approximation

- **Two hadrons:**

$\sigma^{\mathrm{DY}}_{\mathrm{tot}} \sim$



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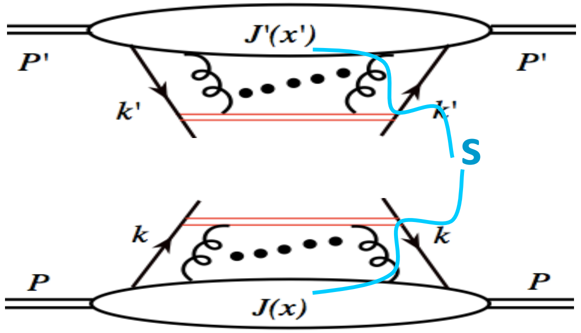
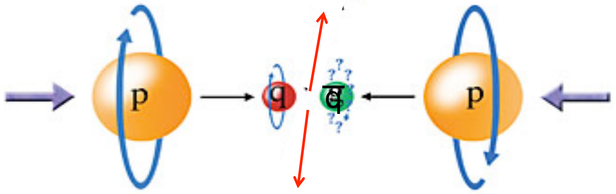


$+ O\left(\frac{1}{QR}\right)$



Predictive power: Universal Parton Distributions
Calculable coefficient functions

Drell-Yan Process – Two Identified Hadrons

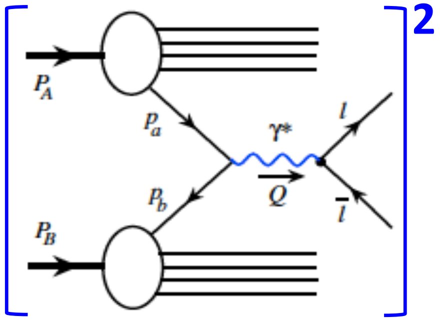
- **Drell-Yan mechanism:**

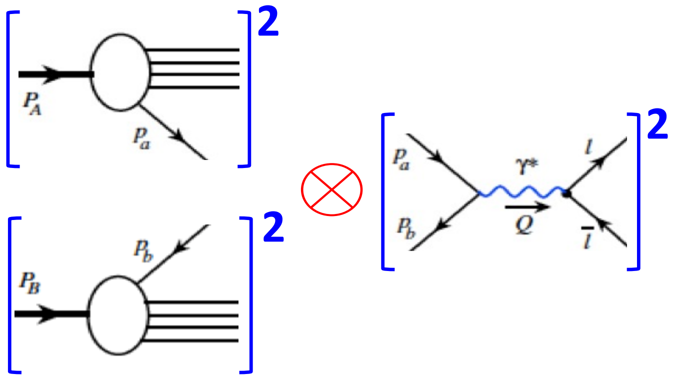
S.D. Drell and T.-M. Yan
Phys. Rev. Lett. 25, 316 (1970)
Before QCD

$$A(P_A) + B(P_B) \to \gamma^*(q)[\to l\bar{l}(q)] + X \quad \textbf{with} \quad q^2 \equiv Q^2 \gg \Lambda^2_{\rm QCD} \sim 1/\text{fm}^2$$

Lepton pair – from decay of a virtual photon, or in general,
a massive boson, e.g., W, Z, H^0, … (called Drell-Yan like processes)

- **Original Drell-Yan formula:**





$$\frac{d\sigma_{A+B\to l\bar{l}+X}}{dQ^2 dy} = \frac{4\pi\alpha^2_{em}}{3Q^4} \sum_{p,\bar{p}} x_A \phi_{p/A}(x_A)\, x_B \phi_{\bar{p}/B}(x_B) \qquad x_A = \frac{Q}{\sqrt{S}} e^{y} \quad x_B = \frac{Q}{\sqrt{S}} e^{-y}$$

No color yet!

Rapidity: $y = \frac{1}{2} \ln(x_A/x_B)$

Right shape – But – not normalization

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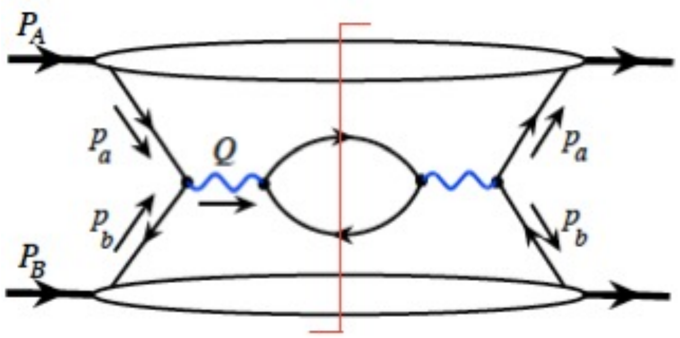
Drell-Yan Process in QCD – Factorization

❑ Factorization – approximation:

✧ Require the suppression of quantum interference between short-distance (1/Q) and long-distance (fm ~ $1/\Lambda_{QCD}$) physics

➡ Need "long-lived" active parton states linking the two hadrons



$$\int d^4 p_a \frac{1}{p_a^2 + i\varepsilon} \frac{1}{p_a^2 - i\varepsilon} \to \infty$$

Perturbatively pinched at $p_a^2 = 0$

➡ Active parton is effectively on-shell for the hard collision

$$p_a^\mu = (p_a^+, p_a^-, p_{a\perp}) \sim Q(1, \lambda^2, \lambda) \quad \text{with} \quad \lambda \sim M/Q$$

➡ $p_a^2 \sim M^2 \ll Q^2$

✧ Maintain the universality of PDFs:

Long-range soft gluon interaction has to be power suppressed

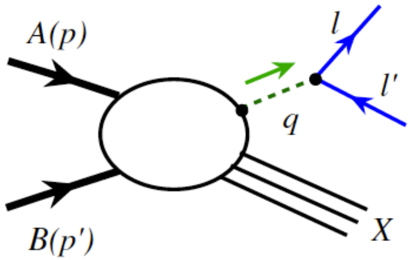
✧ Infrared safe of partonic parts:

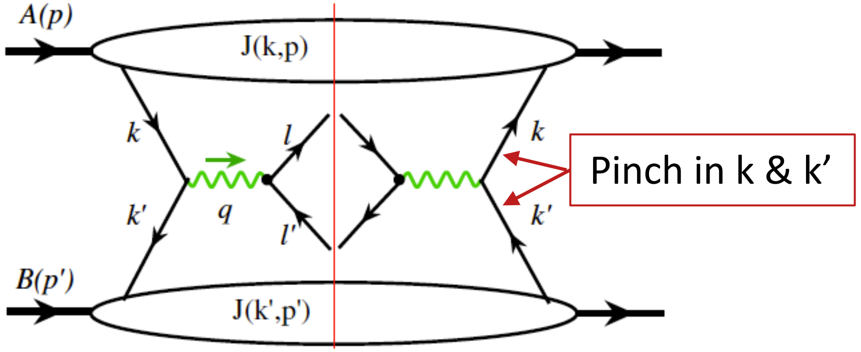
Cancelation of IR behavior
Absorb all CO divergences into PDFs

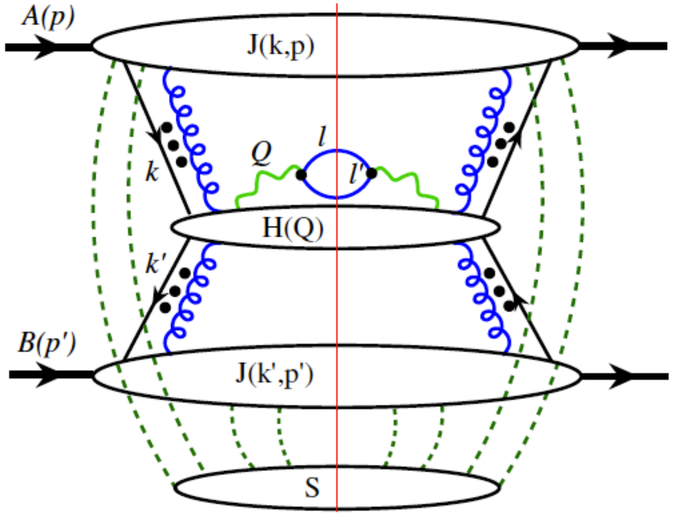
on-shell:	$p_a^2,\ p_b^2 \ll Q^2$;
collinear:	$p_{aT}^2,\ p_{bT}^2 \ll Q^2$;
higher-power:	$p_a^- \ll q^-$; and $p_b^+ \ll q^+$

Factorization at the Leading Power – Two Identified Hadrons

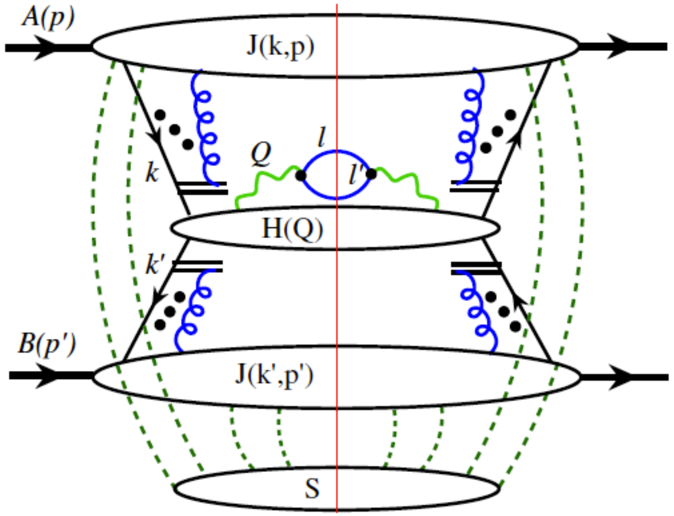
- QCD factorization with Two identified hadrons – Drell-Yan type:



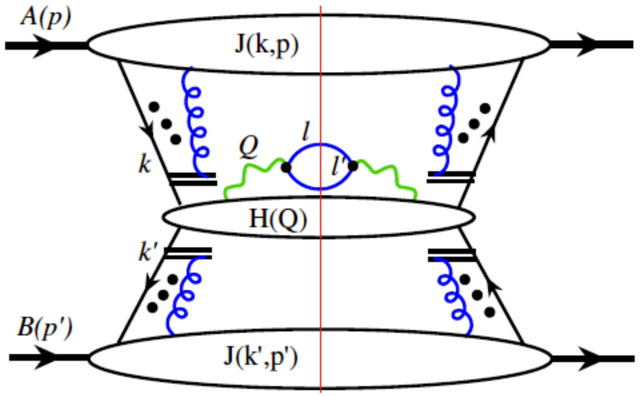




"Leading pinch surface"
n "beam jets"



Apply Ward identity
to decouple CO gluons from "H"



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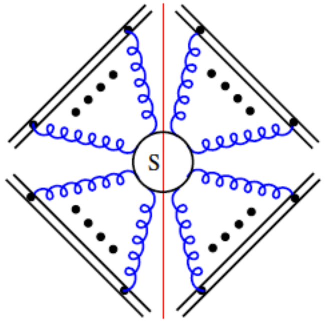

Single soft component to the beam jet
Apply Ward identity to decouple soft gluons into soft factor(s)
Soft factor = 1 for CO factorization!

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Factorization at the Leading Power – Two Identified Hadrons

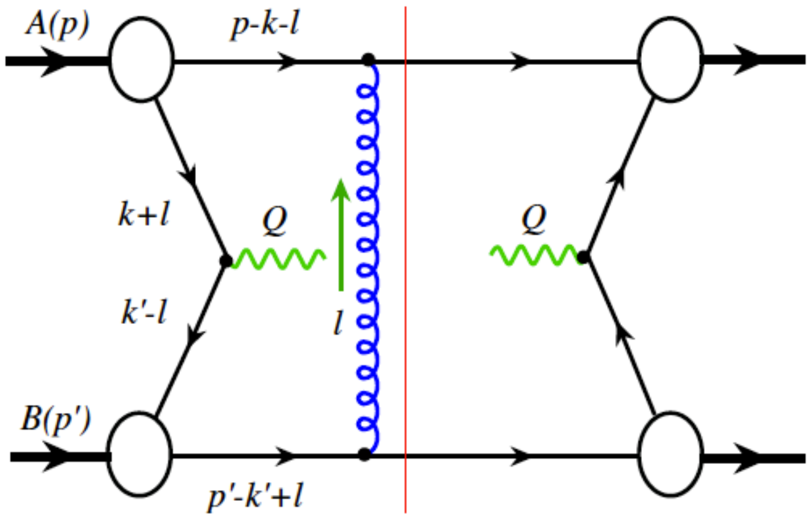
- QCD factorization with Two identified hadrons – Drell-Yan type:

$$\frac{d\sigma^{(\mathrm{DY})}_{A+B\to ll'+X}}{dQ^2dy} = \sum_{ff'}\int dx\, dx'\, \phi_{f/A}(x,\mu)\,\phi_{f'/B}(x',\mu)\,\frac{d\hat{\sigma}_{f+f'\to ll'+X}(x,x',\mu,\alpha_s)}{dQ^2dy} + \mathcal{O}\left[\frac{\Lambda^2_{\mathrm{QCD}}}{Q^2}\right]$$

Same as that in DIS
"Universality"

But, this factorization can fail if the soft gluon momenta are trapped in the Glauber region: $k_i^{\pm} \ll k_i^{\perp}$

Soft spectator interaction is responsible for this – the most challenge part of the factorization proof:



$$\boxed{\frac{1}{(p-k-l)^2+i\varepsilon}\,\frac{1}{(k+l)^2++i\varepsilon} \longrightarrow \frac{1}{-l^-+i\varepsilon}\,\frac{1}{l^-+i\varepsilon}}$$

Solution: (1) sum over all cuts, unitarity cancels all poles
in upper half plane for l^-, and
in lower half plane for l^+
(2) deform the other component out of Glauber region

$$\boxed{\frac{1}{(p'-k'+l)^2+i\varepsilon}\,\frac{1}{(k'-l)^2++i\varepsilon} \longrightarrow \frac{1}{l^++i\varepsilon}\,\frac{1}{-l^++i\varepsilon}}$$

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Factorized Drell-Yan Cross Section

- Collinear factorization – single hard scale ($q_\perp \sim Q$):

$$\frac{d\sigma_{AB}}{d^4q} = \int dx_a\, f_{a/A}(x_a,\mu) \int dx_b\, f_{b/B}(x_b,\mu)\, \frac{d\hat{\sigma}_{ab}}{d^4q}(x_a, x_b, \alpha_s(\mu), \mu) + \mathcal{O}(1/Q)$$

for $q_\perp \sim Q$ or $q_\perp$ integrated Drell-Yan cross sections: $d^4q = dQ^2\, dy\, d^2q_T$

- TMD factorization ($q_\perp \ll Q$) – active parton is still pinched to be on-shell:

$$\frac{d\sigma_{AB}}{d^4q} = \sigma_0 \int d^2k_{a\perp} d^2k_{b\perp} d^2k_{s\perp} \delta^2(q_\perp - k_{a\perp} - k_{b\perp} - k_{s\perp}) \mathcal{F}_{a/A}(x_A, k_{a\perp}) \mathcal{F}_{b/B}(x_B, k_{b\perp}) \mathcal{S}(k_{s\perp})$$
$$+ \mathcal{O}(q_\perp/Q) \qquad x_A = \frac{Q}{\sqrt{s}} e^{y} \qquad x_B = \frac{Q}{\sqrt{s}} e^{-y}$$

The soft factor, $\mathcal{S}$, is universal, could be absorbed into the definition of TMD parton distribution

- Spin dependence:

The factorization arguments at the leading power are independent of the spin states of the colliding hadrons

→ Same formula with polarized PDFs for γ*,W/Z, H^0…

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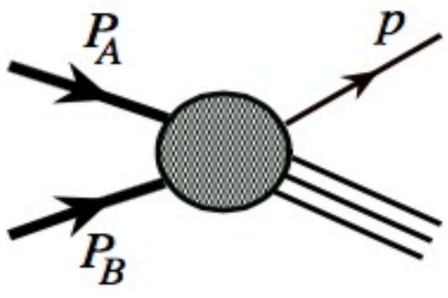
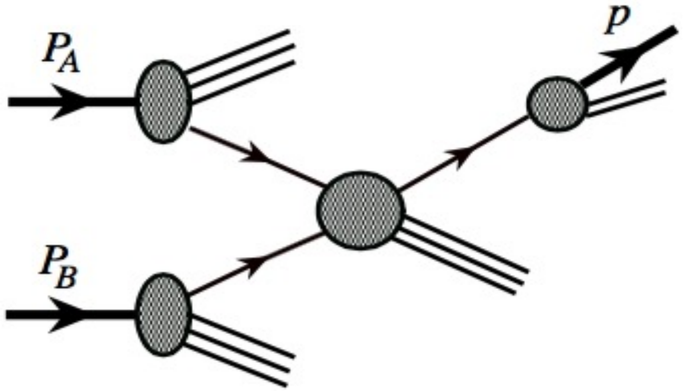
Factorization for More Than Two Hadrons

- **Factorization for high p_T single hadron:**

Nayak, Qiu, Sterman, 2005





$\gamma, W/Z, \ell(s), \mathrm{jet(s)}$
$B, D, \Upsilon, J/\psi, \pi, \ldots$

+ O (1/P_T^2)

$p_T \gg m \gtrsim \Lambda_{\mathrm{QCD}}$

$$\frac{d\sigma_{AB\to C+X}(p_A, p_B, p)}{dy\,dp_T^2} = \sum_{a,b,c} \phi_{A\to a}(x,\mu_F^2) \otimes \phi_{B\to b}(x',\mu_F^2)$$

$$\otimes \frac{d\hat{\sigma}_{ab\to c+X}(x,x',z,y,p_T^2\mu_F^2)}{dy\,dp_T^2} \otimes D_{c\to C}(z,\mu_F^2)$$

Same arguments work for more final-state hadrons if every pair of hadrons have an invariant mass >> Λ_{QCD}

- **Fragmentation function:** $D_{c\to C}(z,\mu_F^2)$

- **Choice of the scales:** $\mu_{\mathrm{Fac}}^2 \approx \mu_{\mathrm{ren}}^2 \approx p_T^2$

To minimize the size of logs in the coefficient functions

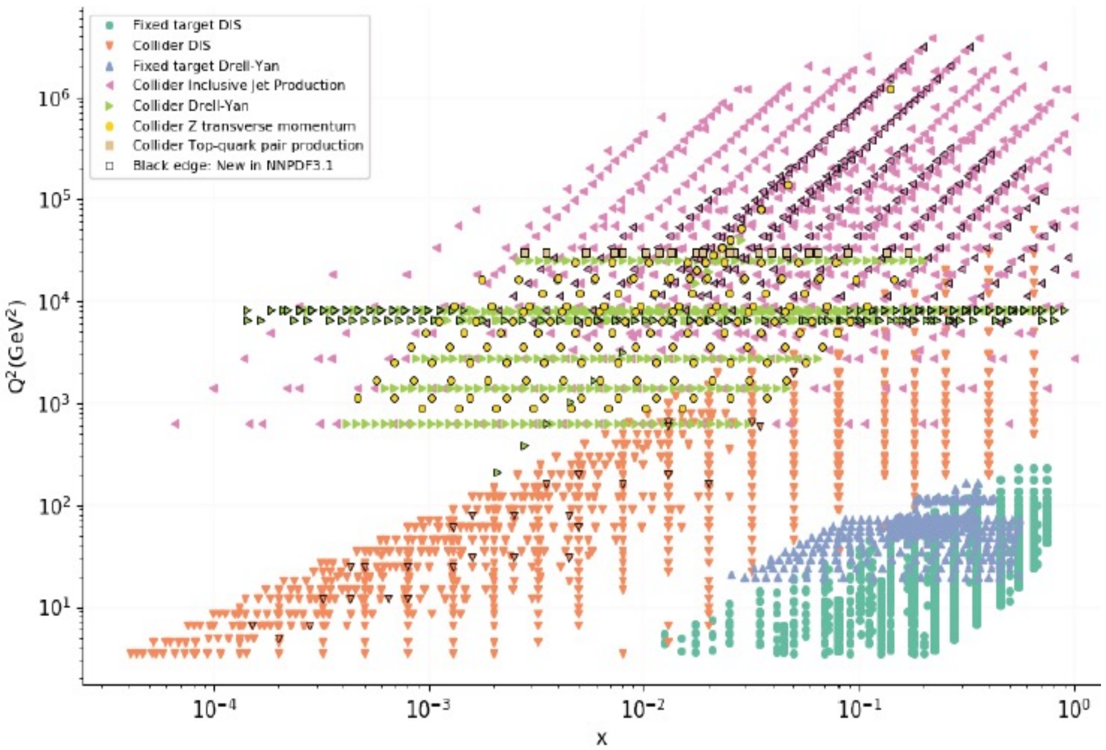
QCD Factorization Works to the Precision

❑ Data sets for Global Fits:

	Process	Subprocess	Partons	x range
Fixed Target	$\ell^\pm \{p,n\} \to \ell^\pm + X$	$\gamma^* q \to q$	$q, \bar{q}, g$	$x \gtrsim 0.01$
	$\ell^\pm n/p \to \ell^\pm + X$	$\gamma^* d/u \to d/u$	d/u	$x \gtrsim 0.01$
	$pp \to \mu^+\mu^- + X$	$u\bar{u}, d\bar{d} \to \gamma^*$	$\bar{q}$	$0.015 \lesssim x \lesssim 0.35$
	$pn/pp \to \mu^+\mu^- + X$	$(u\bar{d})/(u\bar{u}) \to \gamma^*$	$\bar{d}/\bar{u}$	$0.015 \lesssim x \lesssim 0.35$
	$\nu(\bar{\nu}) N \to \mu^-(\mu^+) + X$	$W^* q \to q'$	$q, \bar{q}$	$0.01 \lesssim x \lesssim 0.5$
	$\nu N \to \mu^-\mu^+ + X$	$W^* s \to c$	s	$0.01 \lesssim x \lesssim 0.2$
	$\bar{\nu} N \to \mu^+\mu^- + X$	$W^* \bar{s} \to \bar{c}$	$\bar{s}$	$0.01 \lesssim x \lesssim 0.2$
Collider DIS	$e^\pm p \to e^\pm + X$	$\gamma^* q \to q$	$g, q, \bar{q}$	$0.0001 \lesssim x \lesssim 0.1$
	$e^+ p \to \bar{\nu} + X$	$W^+ \{d,s\} \to \{u,c\}$	d, s	$x \gtrsim 0.01$
	$e^\pm p \to e^\pm c\bar{c} + X$	$\gamma^* c \to c, \gamma^* g \to c\bar{c}$	c, g	$10^{-4} \lesssim x \lesssim 0.01$
	$e^\pm p \to e^\pm b\bar{b} + X$	$\gamma^* b \to b, \gamma^* g \to b\bar{b}$	b, g	$10^{-4} \lesssim x \lesssim 0.01$
	$e^\pm p \to \text{jet} + X$	$\gamma^* g \to q\bar{q}$	g	$0.01 \lesssim x \lesssim 0.1$
Tevatron	$p\bar{p} \to \text{jet} + X$	$gg, qg, qq \to 2j$	g, q	$0.01 \lesssim x \lesssim 0.5$
	$p\bar{p} \to (W^\pm \to \ell^\pm \nu) + X$	$ud \to W^+, \bar{u}\bar{d} \to W^-$	$u, d, \bar{u}, \bar{d}$	$x \gtrsim 0.05$
	$p\bar{p} \to (Z \to \ell^+\ell^-) + X$	$uu, dd \to Z$	u, d	$x \gtrsim 0.05$
	$p\bar{p} \to t\bar{t} + X$	$qq \to t\bar{t}$	q	$x \gtrsim 0.1$
LHC	$pp \to \text{jet} + X$	$gg, qg, q\bar{q} \to 2j$	g, q	$0.001 \lesssim x \lesssim 0.5$
	$pp \to (W^\pm \to \ell^\pm \nu) + X$	$u\bar{d} \to W^+, d\bar{u} \to W^-$	$u, d, \bar{u}, \bar{d}, g$	$x \gtrsim 10^{-3}$
	$pp \to (Z \to \ell^+\ell^-) + X$	$q\bar{q} \to Z$	$q, \bar{q}, g$	$x \gtrsim 10^{-3}$
	$pp \to (Z \to \ell^+\ell^-) + X, p_\perp$	$gq(\bar{q}) \to Zq(\bar{q})$	$g, q, \bar{q}$	$x \gtrsim 0.01$
	$pp \to (\gamma^* \to \ell^+\ell^-) + X$, Low mass	$q\bar{q} \to \gamma^*$	$q, \bar{q}, g$	$x \gtrsim 10^{-4}$
	$pp \to (\gamma^* \to \ell^+\ell^-) + X$, High mass	$q\bar{q} \to \gamma^*$	$\bar{q}$	$x \gtrsim 0.1$
	$pp \to W^+\bar{c}, W^-c$	$sg \to W^+c, \bar{s}g \to W^-\bar{c}$	$s, \bar{s}$	$x \sim 0.01$
	$pp \to t\bar{t} + X$	$gg \to t\bar{t}$	g	$x \gtrsim 0.01$
	$pp \to D, B + X$	$gg \to c\bar{c}, b\bar{b}$	g	$x \gtrsim 10^{-6}, 10^{-5}$
	$pp \to J/\psi, \Upsilon + pp$	$\gamma^*(gg) \to c\bar{c}, b\bar{b}$	g	$x \gtrsim 10^{-6}, 10^{-5}$
	$pp \to \gamma + X$	$gq(\bar{q}) \to \gamma q(\bar{q})$	g	$x \gtrsim 0.005$

❑ Kinematic Coverage:

NNPDF3.1

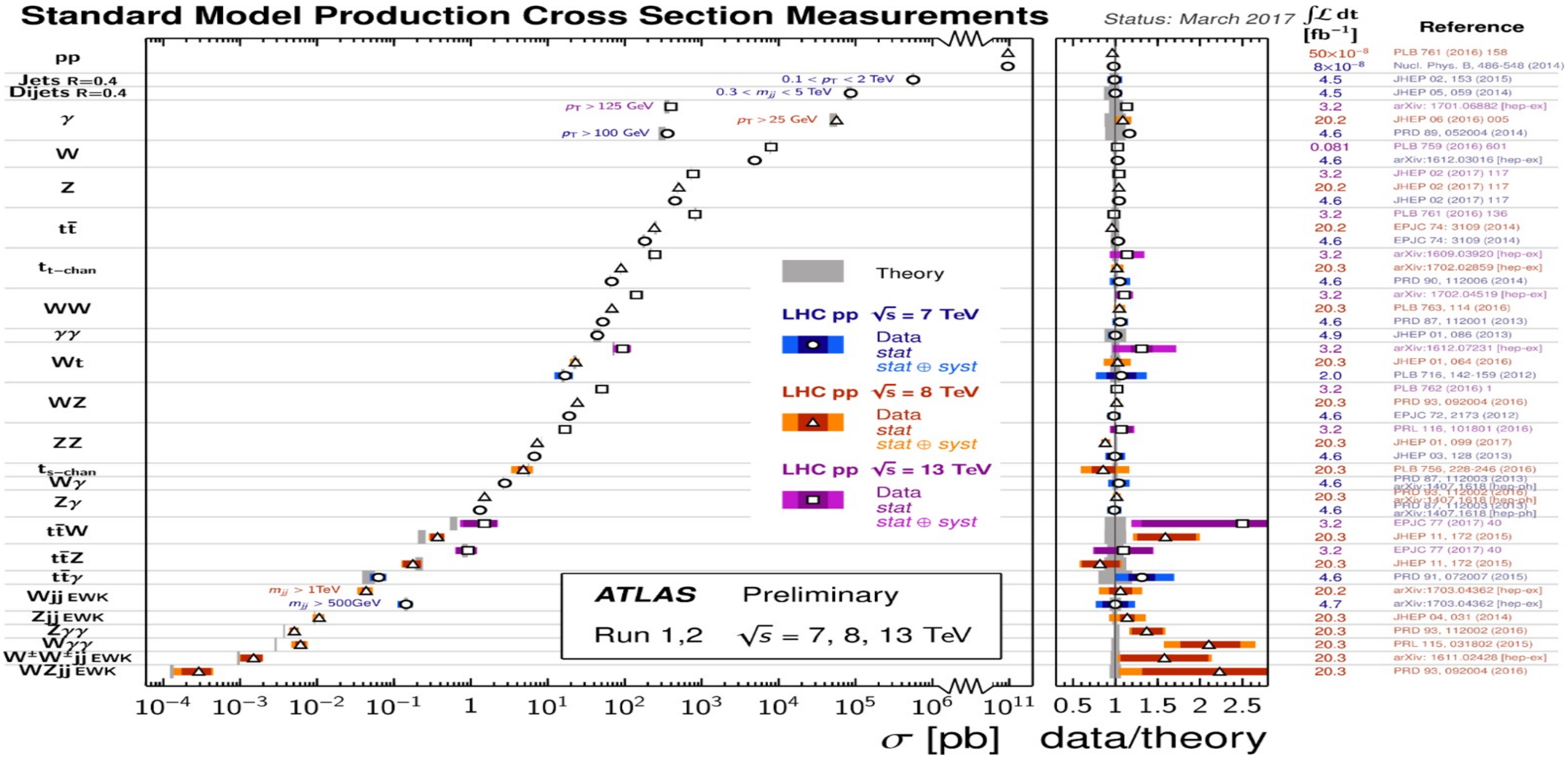


❑ Fit Quality:

$\chi^2/\text{dof} \sim 1 \Rightarrow$ Non-trivial check of QCD

All data sets	3706 / 2763	3267 / 2996	2717 / 2663
	LO	NLO	NNLO

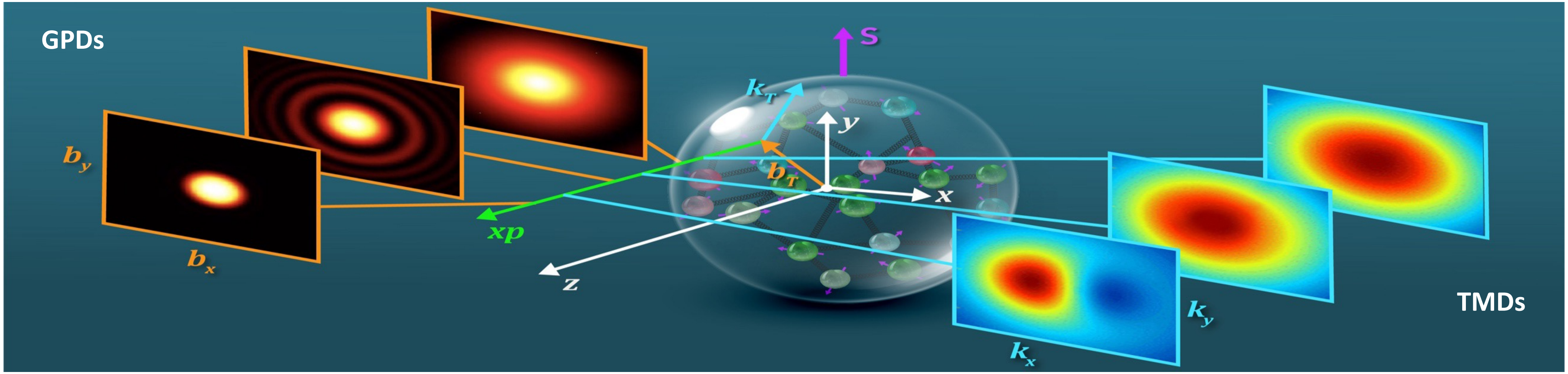
Unprecedent Success of QCD and Standard Model



SM: Electroweak processes + QCD perturbation theory + PDFs works!

Nuclear Femtography

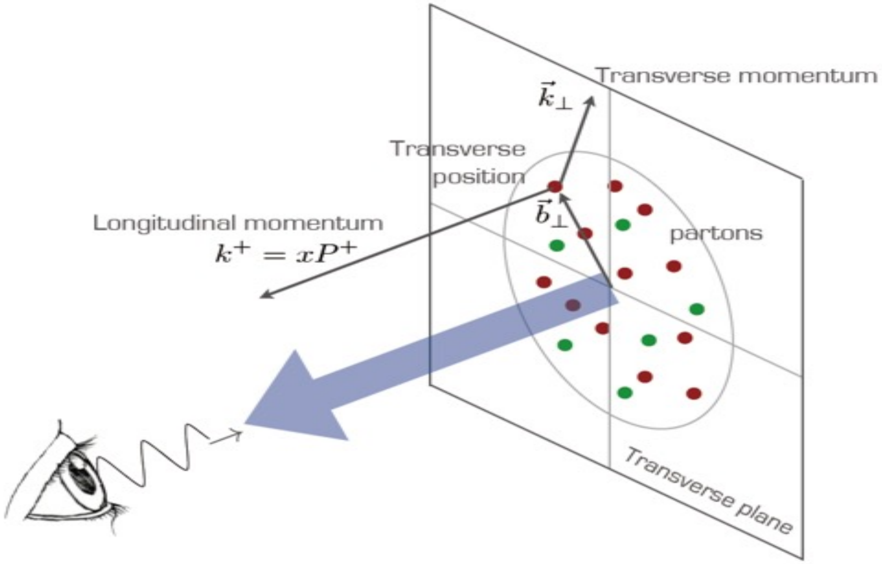
- **3D hadron structure:**



- **Need new observables with two distinctive scales:**

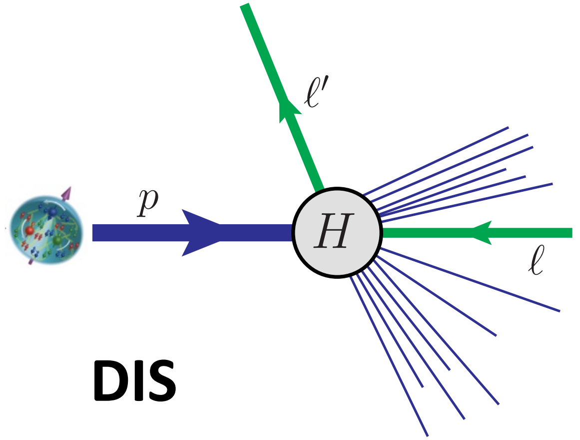
$$Q_1 \gg Q_2 \sim 1/R \sim \Lambda_{\mathrm{QCD}}$$

  - **Hard scale:** Q_1 **to localize the probe to see the particle nature of quarks/gluons**
  - **"Soft" scale:** Q_2 **to be more sensitive to the emergent regime of hadron structure ~ 1/fm**

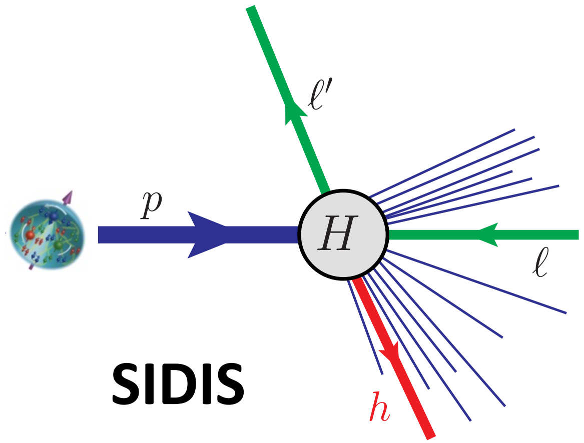


Inclusive vs. Exclusive – Partonic Structure without Breaking the Hadron!

Inclusive scattering

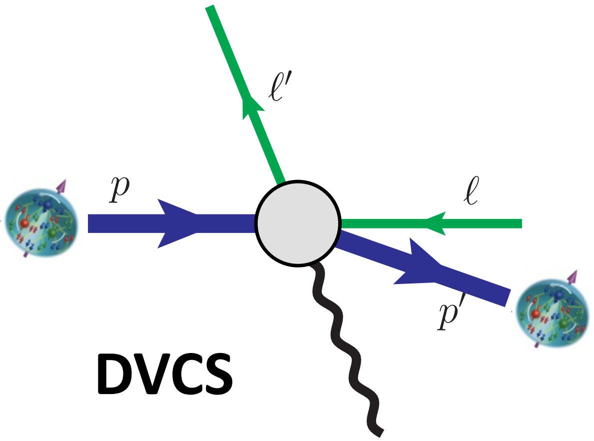


DIS



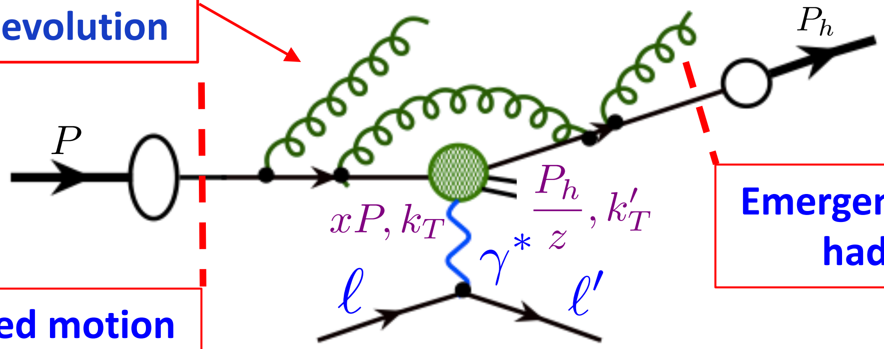
SIDIS

Exclusive diffraction



DVCS

Gluon shower – QCD evolution



Confined motion

Emergence of a hadron hadronization

$$e + P \to e + h + X$$

See Z. Yu's talk on Tuesday

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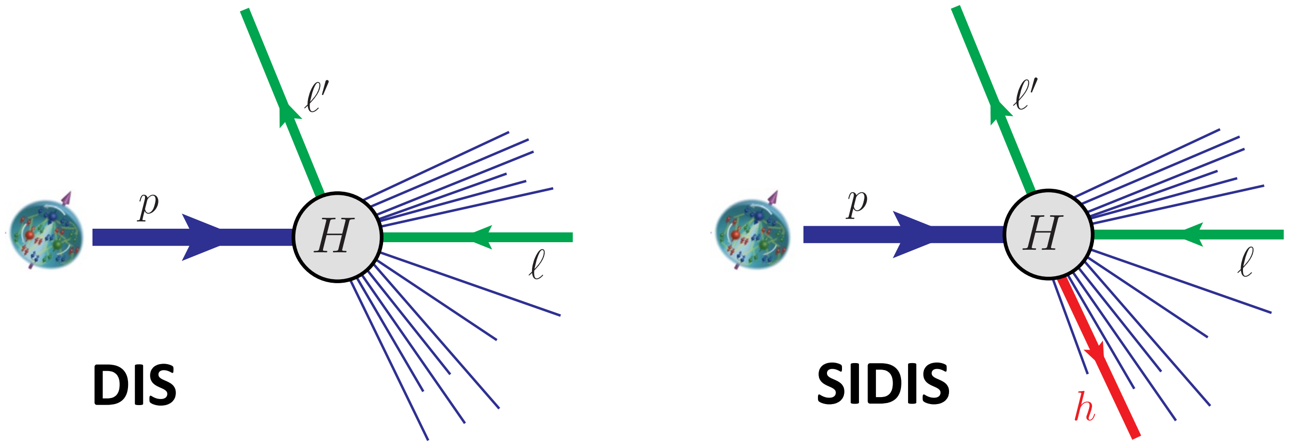
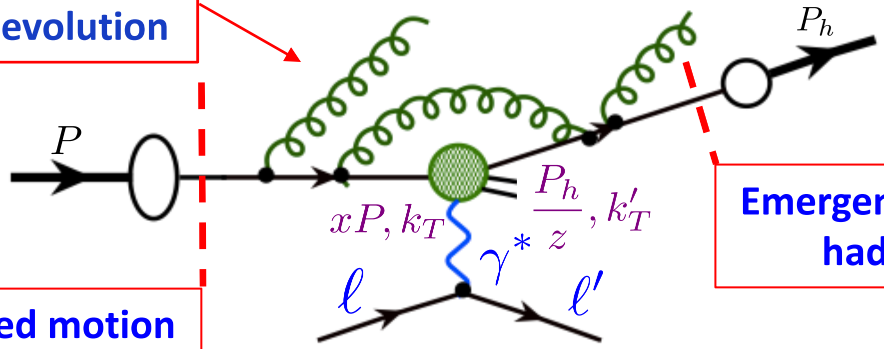
Inclusive vs. Exclusive – Partonic Structure without Breaking the Hadron!

Inclusive scattering



DIS **SIDIS**

Gluon shower – QCD evolution

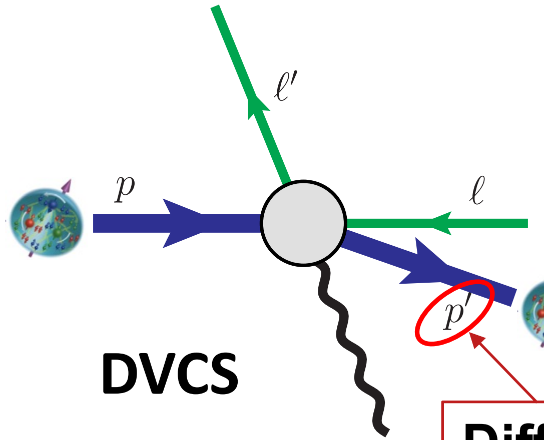


Confined motion

Emergence of a hadron hadronization

$$e + P \to e + h + X$$

Exclusive diffraction



DVCS

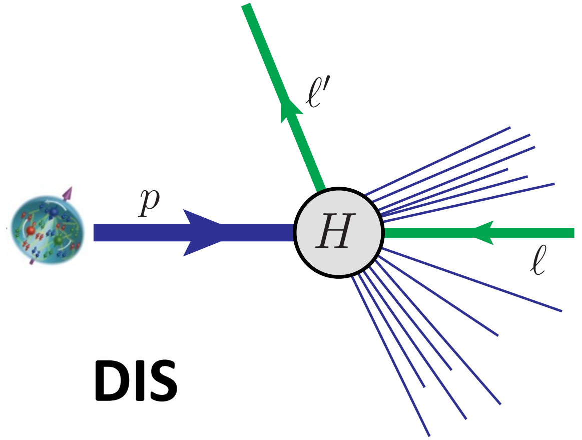
Diffraction

$$Q^2 = -(\ell - \ell')^2$$
$$\gg -(p - p')^2 = -t$$

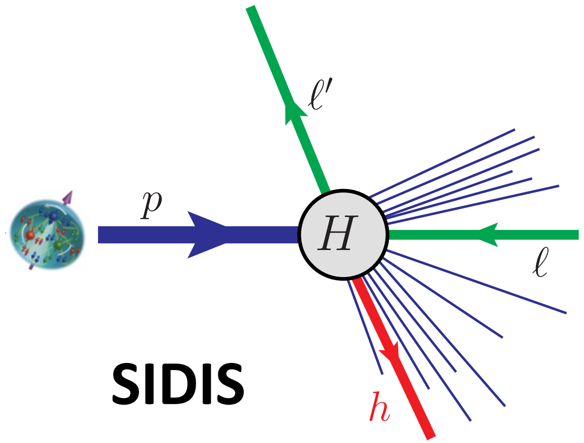
See Z. Yu's talk on Tuesday

Inclusive vs. Exclusive – Partonic Structure without Breaking the Hadron!

Inclusive scattering

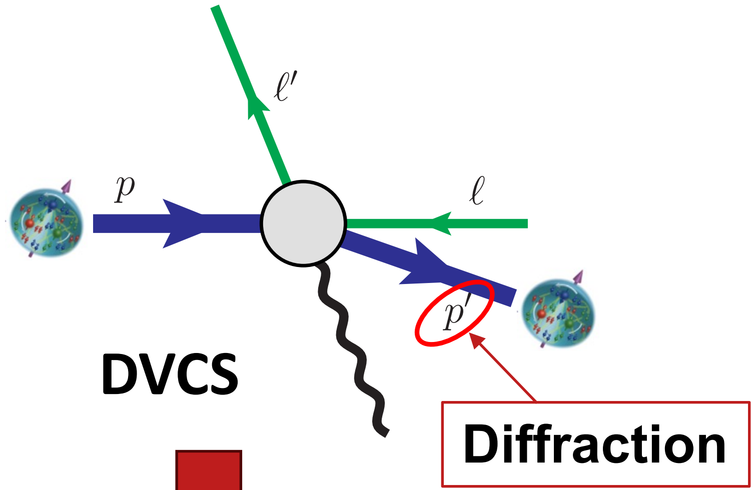


DIS



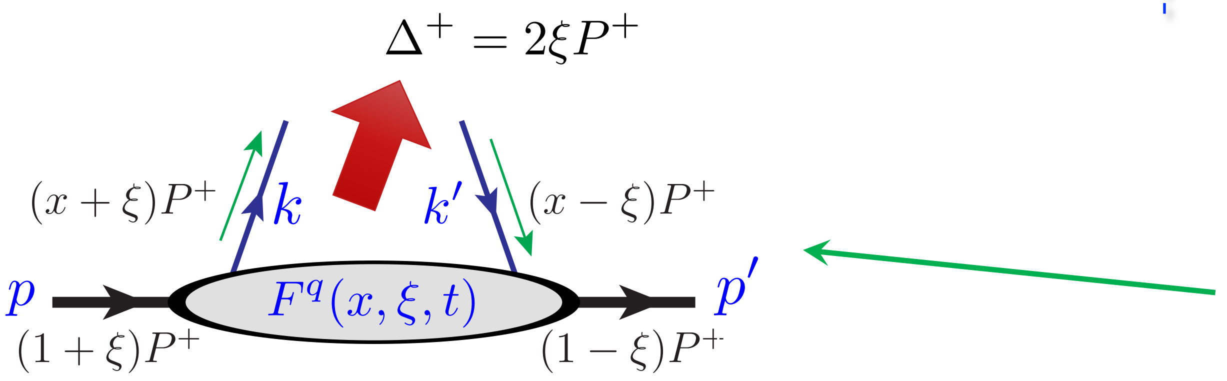
SIDIS

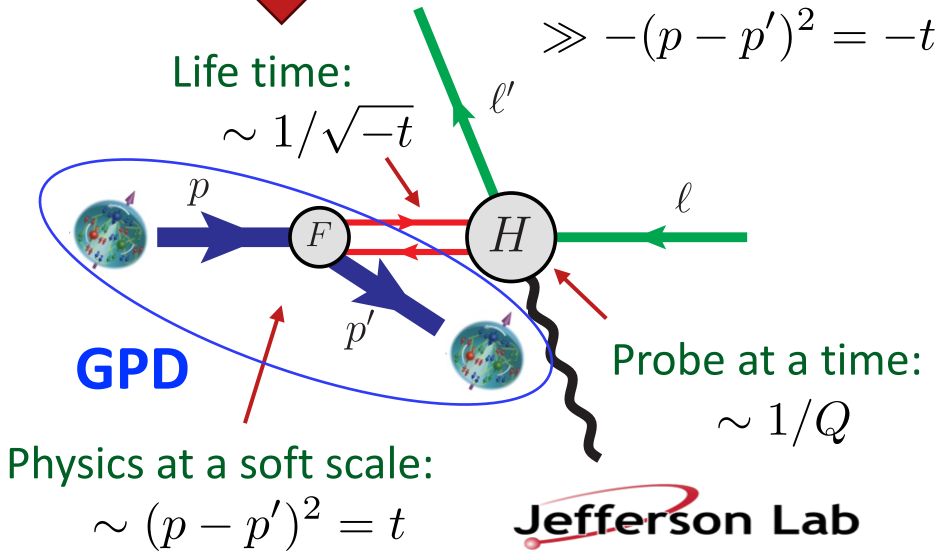
Exclusive diffraction



DVCS

$$Q^2 = -(\ell - \ell')^2 \gg -(p-p')^2 = -t$$





$$F^q(x,\xi,t) = \int \frac{\mathrm{d}z^-}{4\pi} e^{-ixP^+z^-} \langle p'|\bar{q}(z^-/2)\gamma^+ q(-z^-/2)|p\rangle$$

$$\widetilde{F}^q(x,\xi,t) = \int \frac{\mathrm{d}z^-}{4\pi} e^{-ixP^+z^-} \langle p'|\bar{q}(z^-/2)\gamma^+\gamma_5 q(-z^-/2)|p\rangle$$

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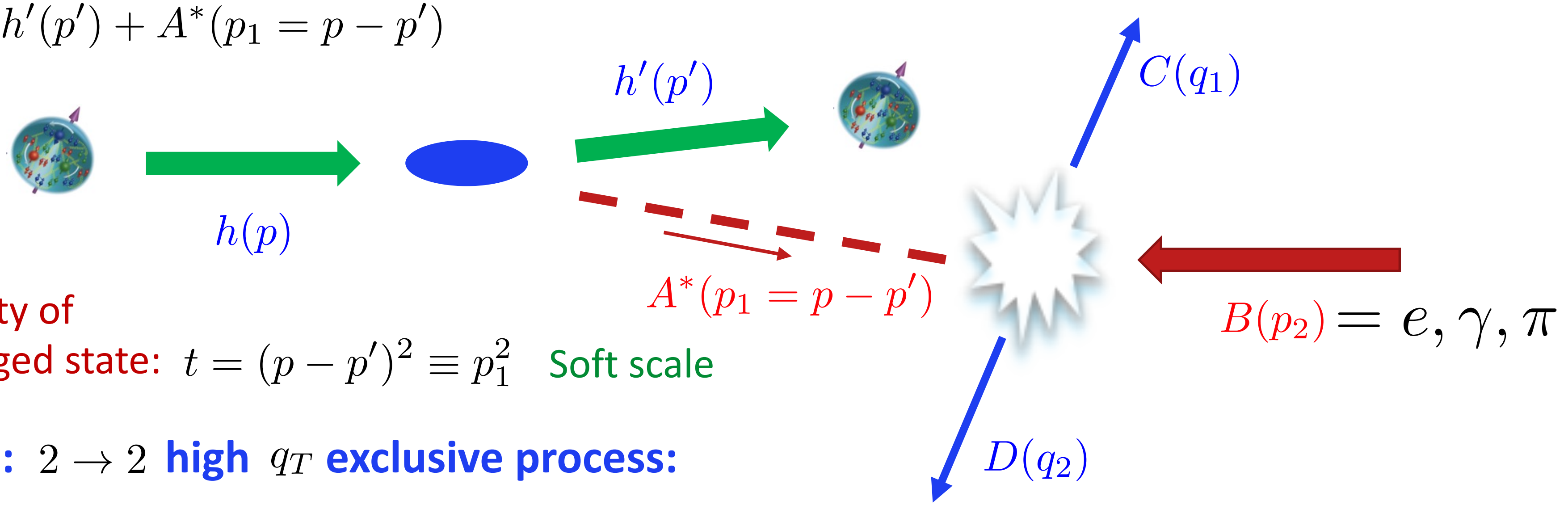
Single-Diffractive Hard Exclusive Processes (SDHEP) for Extracting GPDs

Qiu & Yu, JHEP 08 (2022) 103
PRD 107 (2023) 014007
PRL 131 (2023) 161902

❑ Separation of physics taken place at soft (t) and hard (Q) scales:

- **Single diffractive – keep the hadron intact:**

$$h(p) \to h'(p') + A^*(p_1 = p - p')$$



Virtuality of exchanged state: $t = (p - p')^2 \equiv p_1^2$ Soft scale

- **Hard probe: $2 \to 2$ high q_T exclusive process:**

$$A^*(p_1) + B(p_2) \to C(q_1) + D(q_2)$$

Probing time: $\sim 1/|q_{1T}| \approx 1/|q_{2T}|$

- **Necessary condition for QCD factorization:**

Lifetime of $A^*(p_1)$ is much longer than collision time of the probe! ➡ $|q_{1_T}| = |q_{2_T}| \gg \sqrt{-t}$

Not necessarily sufficient!

Two- stage $2 \to 3$ single diffractive exclusive hard processes (SDHEP):

$$h(p) + B(p_2) \to h'(p') + C(q_1) + D(q_2)$$

A 2-scale 2-stage observable!

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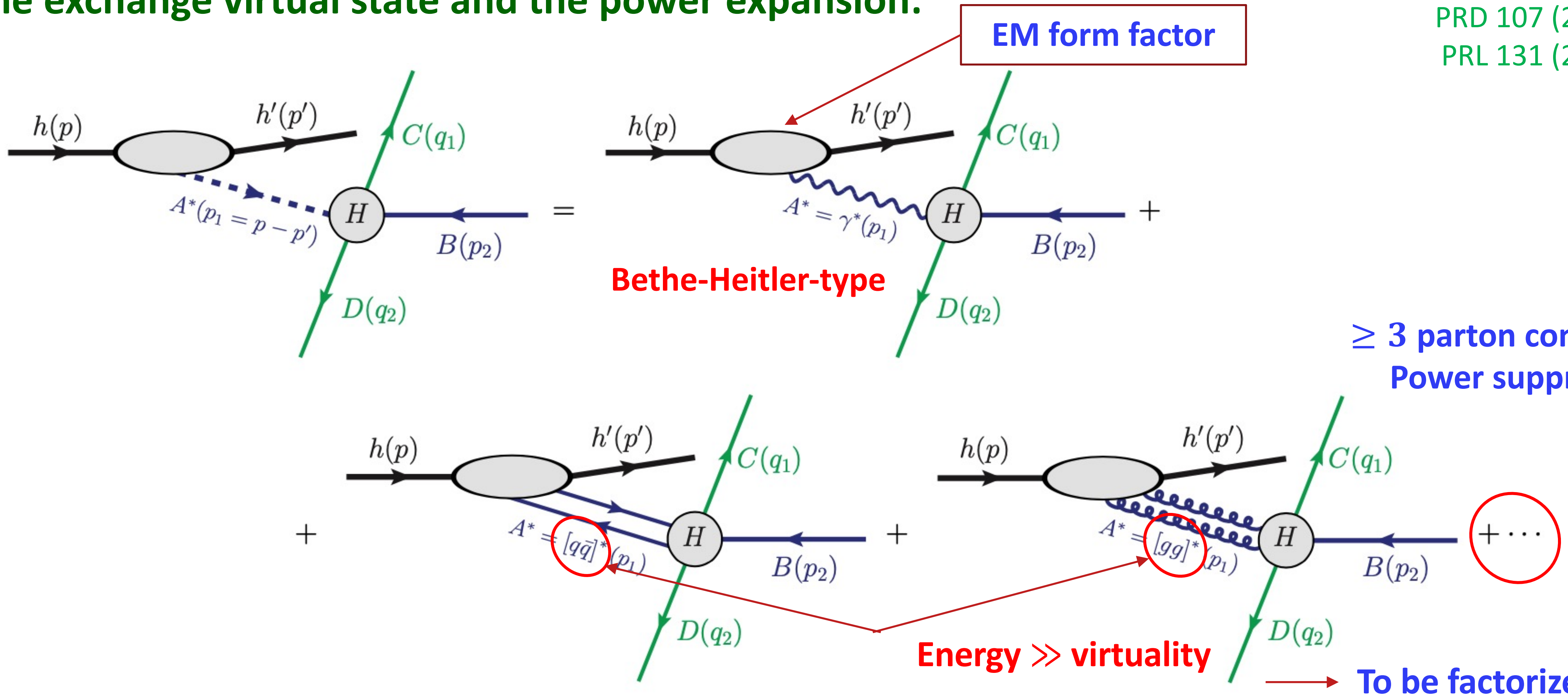
Single-Diffractive Hard Exclusive Processes (SDHEP) for Extracting GPDs

Qiu & Yu, JHEP 08 (2022) 103
PRD 107 (2023) 014007
PRL 131 (2023) 161902

❑ The exchange virtual state and the power expansion:



To be factorized into GPD

The exchanged state $A^*(p-p')$ is a sum of all possible partonic states, n=1,2, … , allowed by

- Quantum numbers of *h(p) – h'(p')*
- Symmetry of producing non-vanishing *H*

Need to separate different contributions!

Proper angular modulations!

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Generalized Parton Distributions (GPDs)

❑ Definition:

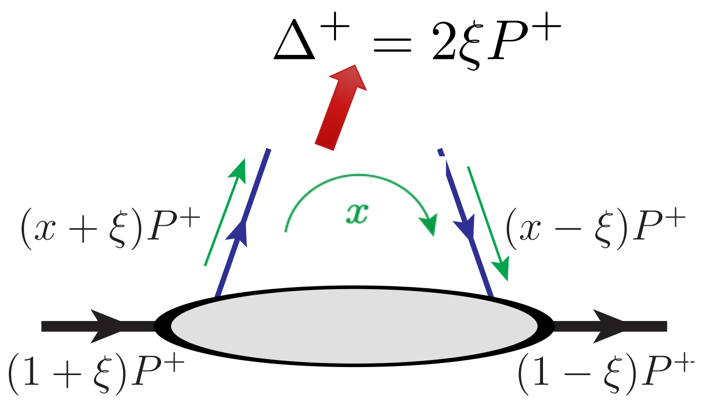
D. Müller, D. Robaschik, B. Geyer, F.-M. Dittes, J. Hořejši, *Fortsch. Phys.* 42 (1994) 101

$$F^q(x,\xi,t) = \int \frac{dz^-}{4\pi} e^{-ixP^+z^-} \langle p' | \bar{q}(z^-/2)\gamma^+ q(-z^-/2) | p \rangle$$

$$= \frac{1}{2P^+}\left[H^q(x,\xi,t)\, \bar{u}(p')\,\gamma^+ u(p) - E^q(x,\xi,t)\,\bar{u}(p')\,\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m} u(p)\right],$$

$$\widetilde{F}^q(x,\xi,t) = \int \frac{dz^-}{4\pi} e^{-ixP^+z^-} \langle p' | \bar{q}(z^-/2)\gamma^+\gamma_5 q(-z^-/2) | p \rangle$$

$$= \frac{1}{2P^+}\left[\widetilde{H}^q(x,\xi,t)\, \bar{u}(p')\,\gamma^+\gamma_5 u(p) - \widetilde{E}^q(x,\xi,t)\,\bar{u}(p')\,\frac{\gamma_5\Delta^+}{2m} u(p)\right].$$



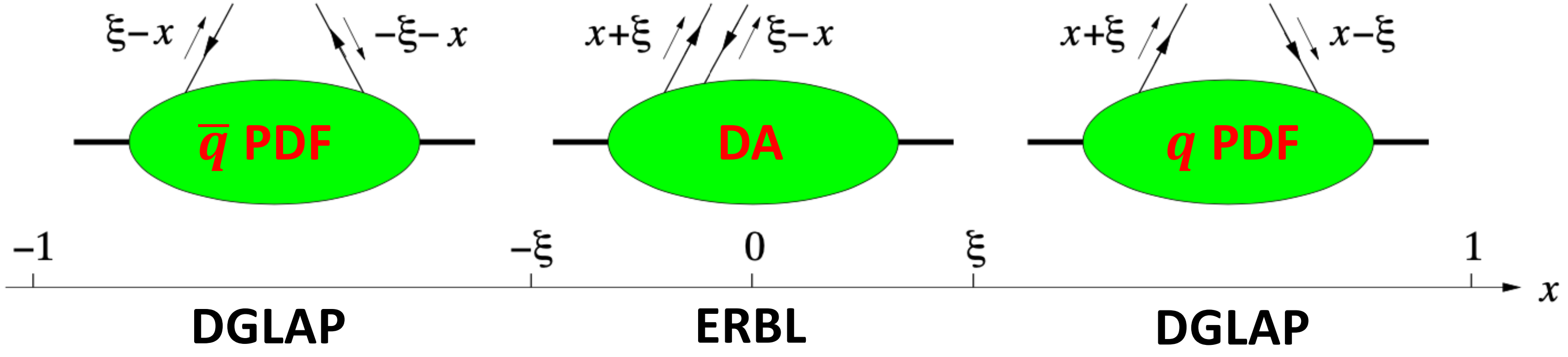
$$P^+ = \frac{p^+ + p'^+}{2}$$

$$\Delta = p - p' \qquad t = \Delta^2$$

Similar definition for gluon GPDs

❑ Combine *PDF* and *Distribution Amplitude (DA):*

Forward limit $\xi = t = 0$ **:** $\quad H^q(x,0,0) = q(x), \quad \tilde{H}^q(x,0,0) = \Delta q(x)$



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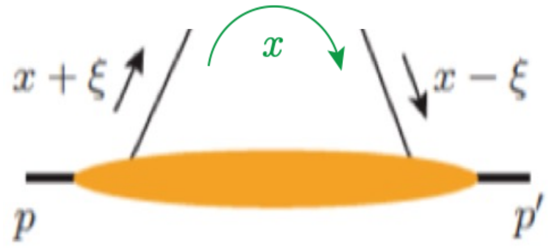
Properties of GPDs – Partonic

- **Impact parameter dependent parton density distribution:**

$$q(x, b_\perp, Q) = \int d^2\Delta_\perp e^{-i\Delta_\perp \cdot b_\perp} H_q(x, \xi = 0, t = -\Delta_\perp^2, Q)$$



Measurement of p' fixes (t, ξ)

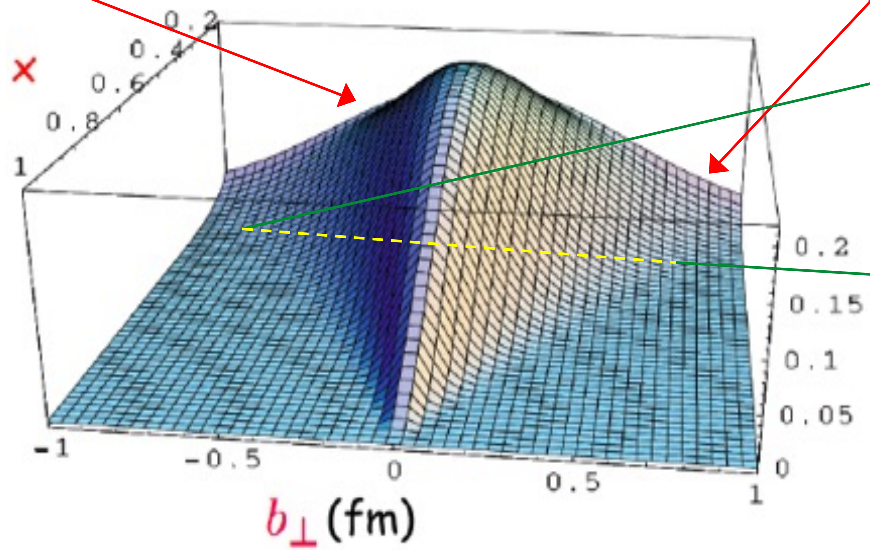
x = momentum flow between the pair

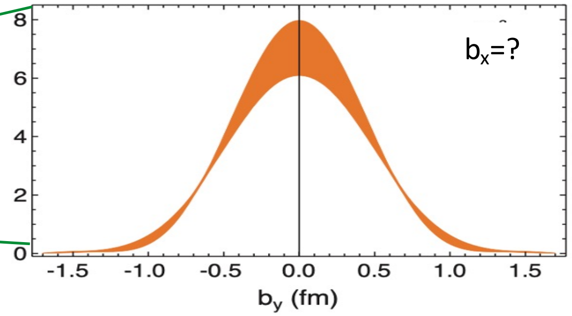
➡ **Quark density in** $\mathrm{d}x\,\mathrm{d}^2\boldsymbol{b}_T$

Tomographic image of hadron in slice of x

How fast does glue density fall?

How far does glue density spread?





Slice in (x,Q)

Modeled by M. Burkdart, PRD 2000

$$\langle q_\perp^N \rangle \equiv \int db_\perp b_\perp^N\, q(x, b_\perp, Q)$$

➡ **Proton radii from quark and gluon spatial density distribution,** $r_q(x)$ **&** $r_g(x)$

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Properties of GPDs – Hadronic = Moments of GPDs

"Mass" – QCD energy-momentum tensor:

Ji, PRL78, 1997

$$T^{\mu\nu} = \sum_{i=q,g} T_i^{\mu\nu} \quad \textbf{with} \quad T_q^{\mu\nu} = \bar{\psi}_q \, i\gamma^{(\mu} \overleftrightarrow{D}^{\nu)} \, \psi_q - g^{\mu\nu} \bar{\psi}_q \left(i\gamma \cdot \overleftrightarrow{D} - m_q \right) \psi_q \quad \textbf{and} \quad T_g^{\mu\nu} = F^{a,\mu\eta} F^{a,}{}_{\eta}{}^{\nu} + \frac{1}{4} g^{\mu\nu} \left(F^a_{\rho\eta} \right)^2$$

Gravitational form factors:

$$\langle p' \, | T_i^{\mu\nu} | \, p \rangle = \bar{u}(p') \left[A_i(t) \frac{P^\mu P^\nu}{m} + J_i(t) \frac{i P^{(\mu} \sigma^{\nu)\Delta}}{2m} + D_i(t) \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2}{4m} + m \, \bar{c}_i(t) \, g^{\mu\nu} \right] u(p)$$

$$C_i(t) \leftrightarrow D_i(t)/4$$

Connection to GPD moments:

$$\int_{-1}^{1} dx \, x \, F_i(x,\xi,t) \propto \langle p' | T_i^{++} | p \rangle \quad \propto \quad \bar{u}(p') \left[\underbrace{\left(A_i + \xi^2 D_i \right)}_{\int_{-1}^{1} dx \, x \, H_i(x,\xi,t)} \gamma^+ + \underbrace{\left(B_i - \xi^2 D_i \right)}_{\int_{-1}^{1} dx \, x \, E_i(x,\xi,t)} \frac{i\sigma^{+\Delta}}{2m} \right] u(p)$$

Related to pressure & stress force inside h

Polyakov, schweitzer, Inntt. J. Mod. Phys. A33, 1830025 (2018)
Burkert, Elouadrhiri , Girod Nature 557, 396 (2018)

"Spin" – Angular momentum sum rule:

$$J_i = \lim_{t \to 0} \int_{-1}^{1} dx \, x \left[H_i(x,\xi,t) + E_i(x,\xi,t) \right]$$

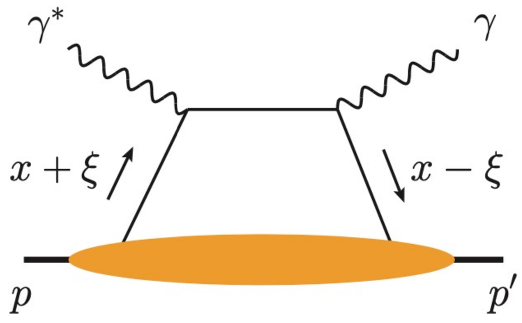
$$i = q, g$$

➡ **3D tomography**
Relation to GFF
Angular Momentum ➡ ***x*-dependence of GPDs!**

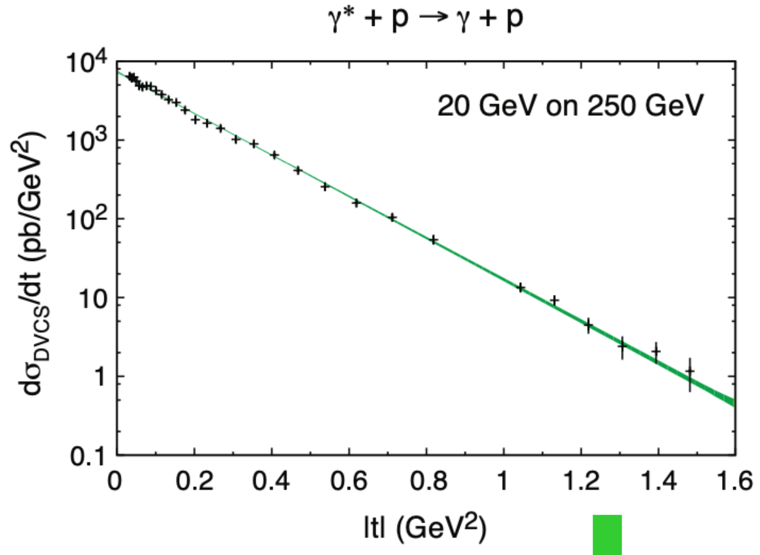
Need to know the x-dependence of GPDs to construct the proper moments!

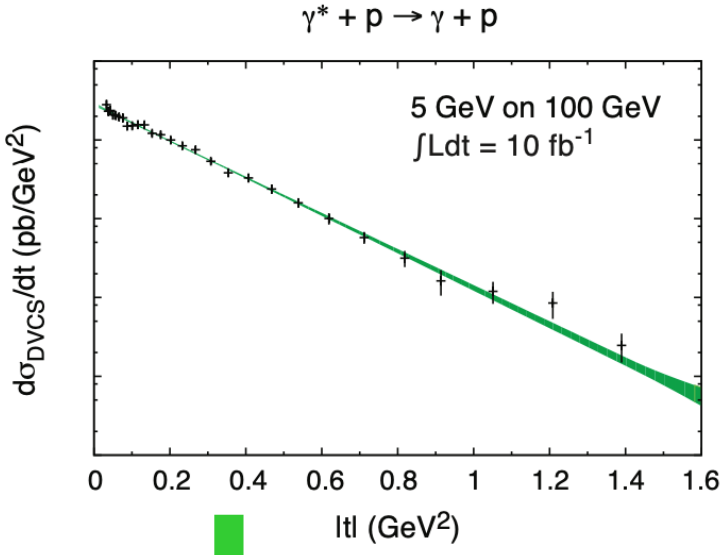
DVCS at a Future EIC (White Paper)

Cross Sections:

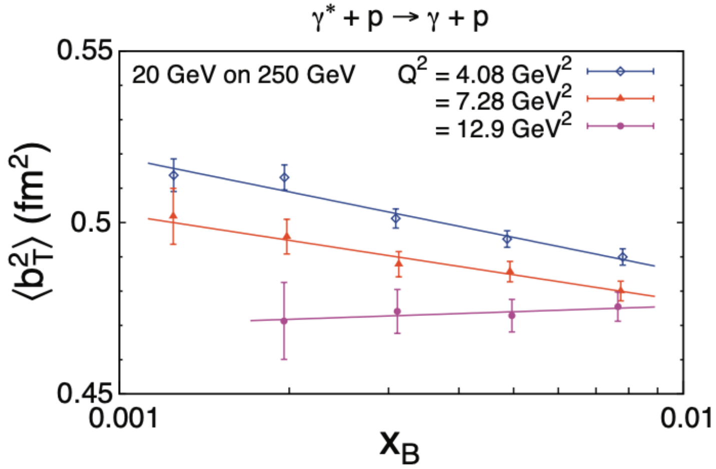


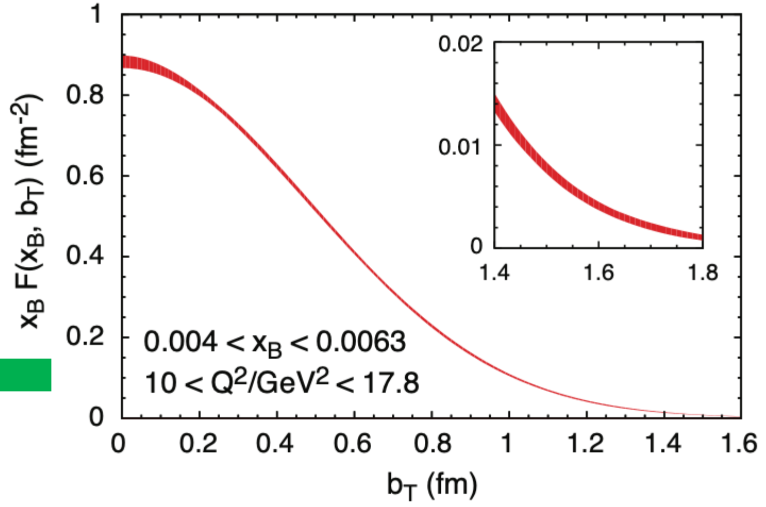
$$t = (p - p')^2$$

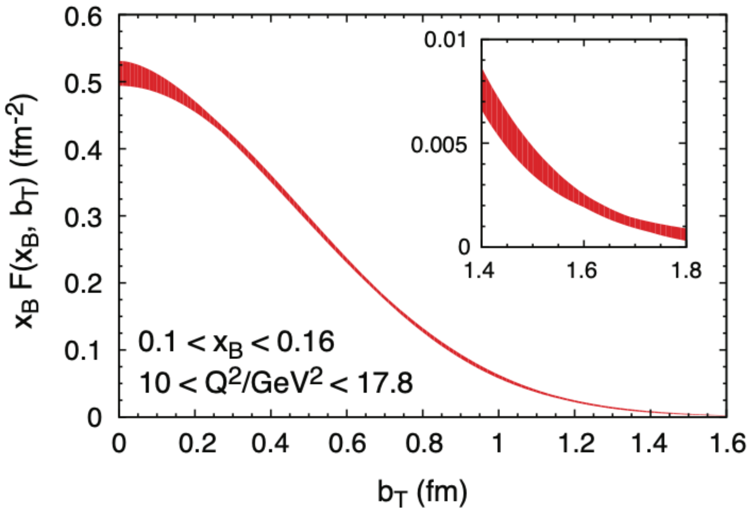




Spatial distributions:







Effective "proton radius" in terms of quarks as a function of x_B

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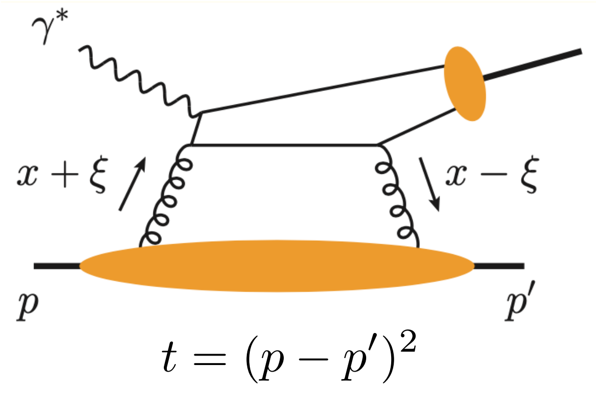
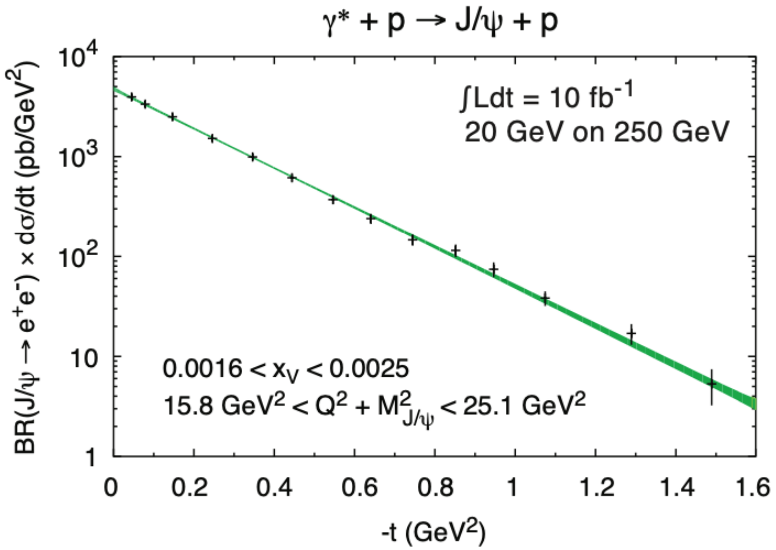
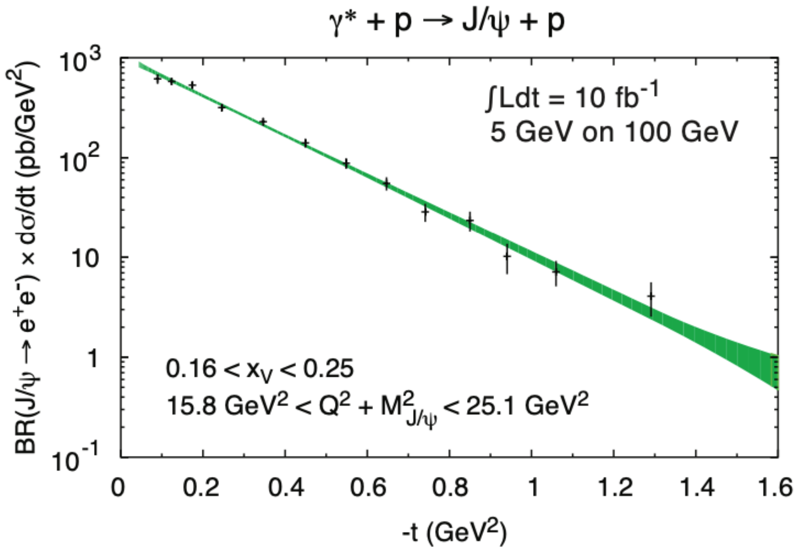
Imaging the Gluon at the EIC (White Paper)

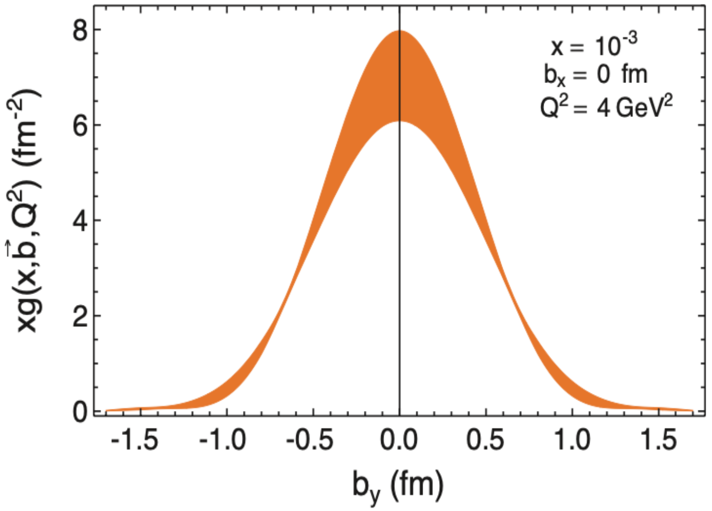
❑ Exclusive vector meson production:

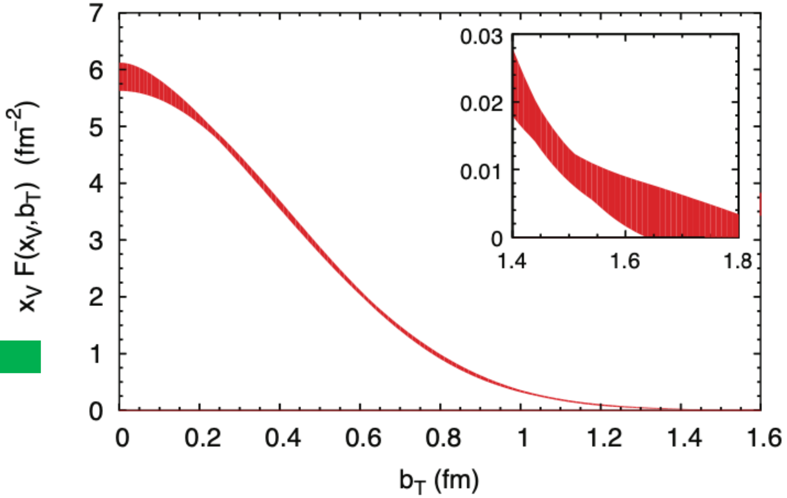


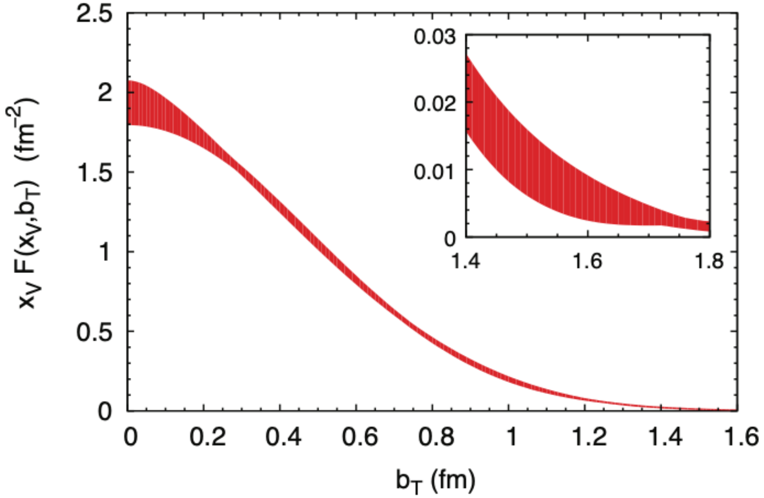




❑ Spatial distributions:



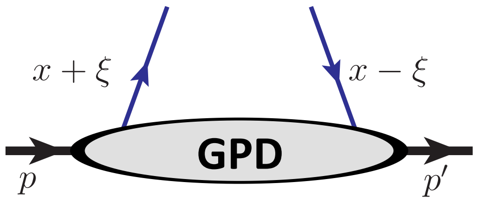




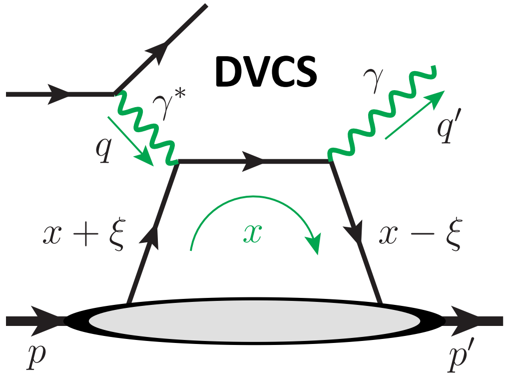
The b_T space density for gluons **Effective "proton radius" in terms of gluons**

Why is the GPD's *x*-Dependence so *difficult* to Measure?

❑ Amplitude nature: exclusive processes



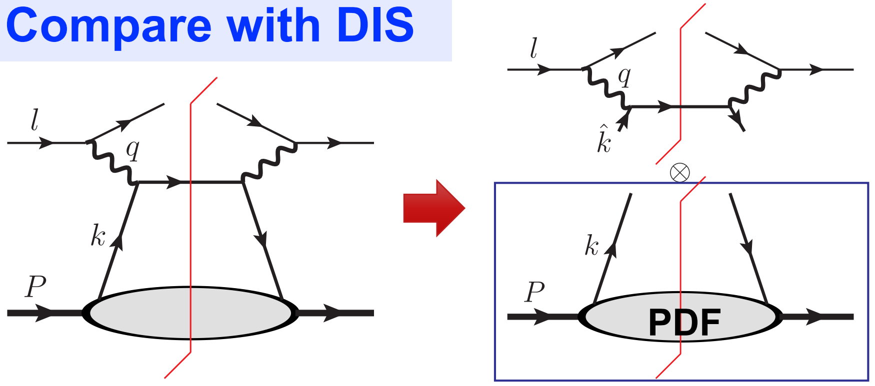
$x \sim$ loop momentum



$$i\mathcal{M} \sim \int_{-1}^{1} \mathrm{d}x\, F(x,\xi,t) \cdot C(x,\xi;Q/\mu)$$

Full range of x, including $x = 0; \quad x = \pm\xi$

Compare with DIS



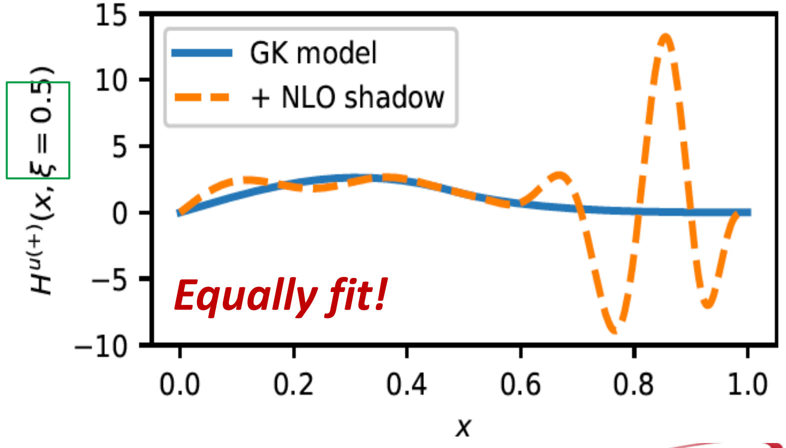
cross section: **cut diagram**

$$\sigma_{\rm DIS} \simeq \int_{x_B}^{1} \mathrm{d}x\, f(x)\, \hat{\sigma}(x/x_B)$$

❑ Sensitivity to *x*: comes from $C(x,\xi;Q/\mu)$

$$C(x,\xi;Q/\mu) = T(Q/\mu)\cdot G(x,\xi) \propto \frac{1}{x-\xi+i\epsilon}\cdots$$

$$\Rightarrow\ i\mathcal{M} \propto \int_{-1}^{1} \mathrm{d}x\, \frac{F(x,\xi,t)}{x-\xi+i\varepsilon} \equiv \text{“}F_0(\xi,t)\text{”}$$ **“moment”**



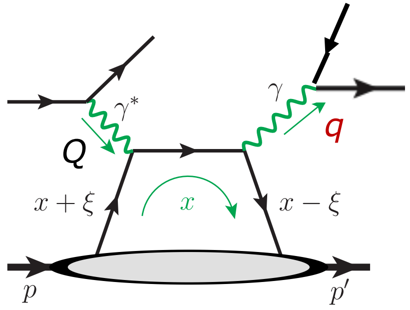
[Bertone et al. PRD `21]

What Kind of Process Could be Sensitive to the *x*-Dependence?

- **Create an entanglement between the internal *x* and an externally measured variable?**

$$i\mathcal{M} \propto \int_{-1}^{1} \mathrm{d}x \frac{F(x,\xi,t)}{x - x_p(\xi,q) + i\varepsilon}$$

Change external *q* to sample different part of x.



- **Double DVCS (two scales):**

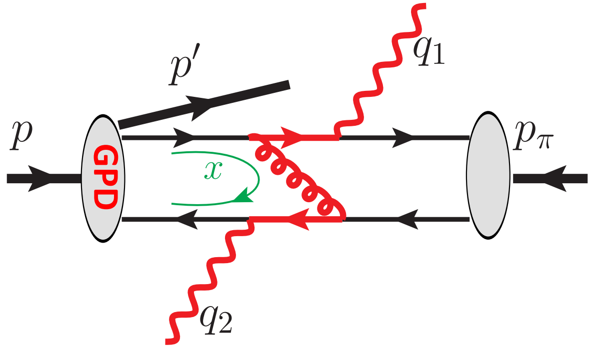
$$x_p(\xi,q) = \xi\left(\frac{1-q^2/Q^2}{1+q^2/Q^2}\right) \to \xi \quad \text{same as} \quad \text{DVCS} \quad \text{if } q \to 0$$

- **Production of two back-to-back high pT particles (say, two photons):**

$$\pi^-(p_\pi) + P(p) \to \gamma(q_1) + \gamma(q_2) + N(p')$$

Qiu & Yu
JHEP 08 (2022) 103

Hard scale: $q_T \gg \Lambda_{\text{QCD}}$ Soft scale: $t \sim \Lambda_{\text{QCD}}^2$



- **Factorization:**

[*suppressing pion DA factor*]

$$\mathcal{M}(t,\xi,q_T) = \int_{-1}^{1} \mathrm{d}x\, F(x,\xi,t;\mu)\cdot C(x,\xi;q_T/\mu) + \mathcal{O}(\Lambda_{\text{QCD}}/q_T)$$

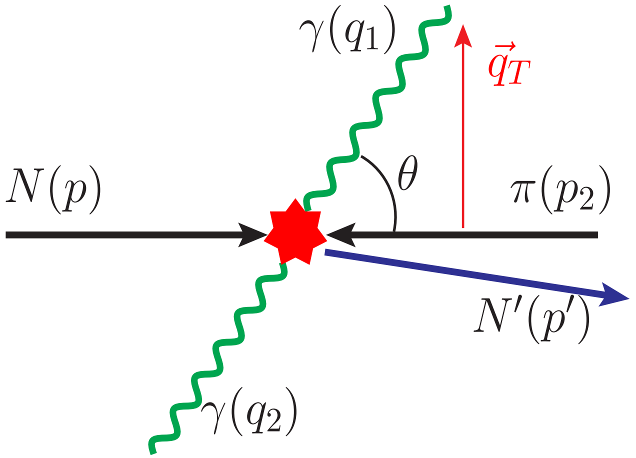
$$\frac{\mathrm{d}\sigma}{\mathrm{d}t\;\mathrm{d}\xi\;\mathrm{d}q_T} \sim |\mathcal{M}(t,\xi,q_T)|^2$$

q_T distribution is "conjugate" to x distribution

$$x \leftrightarrow q_T$$

Enhanced x-Sensitivity: (1) Diphoton Meso-production

Qiu & Yu, PRD 109 (2024) 074023



In addition to

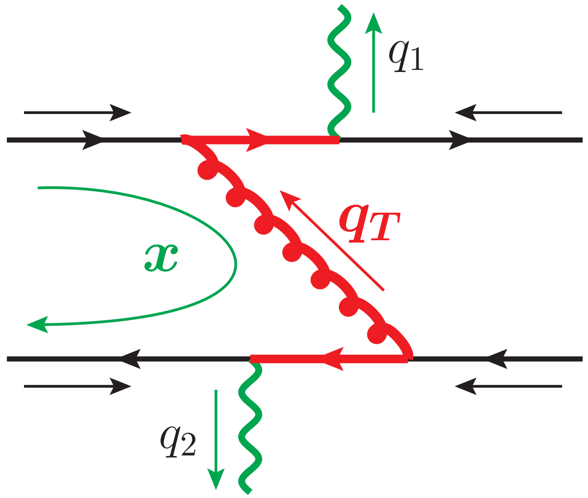
$$F_0(\xi,t) = \int_{-1}^{1} \frac{dx\, F(x,\xi,t)}{x-\xi+i\epsilon}$$

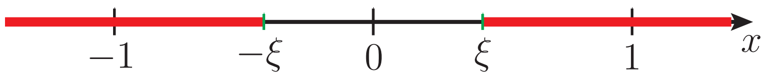
When two photons are radiated from the same charged line

$i\mathcal{M}$ **also contains**

$$I(t,\xi;z,\theta) = \int_{-1}^{1} \frac{dx\, F(x,\xi,t)}{x-\rho(z;\theta)+i\epsilon\,\mathrm{sgn}\left[\cos^2(\theta/2)-z\right]}$$

$$\rho(z;\theta) = \xi \cdot \left[\frac{1-z+\tan^2(\theta/2)\,z}{1-z-\tan^2(\theta/2)\,z}\right] \in (-\infty,-\xi] \cup [\xi,\infty)$$





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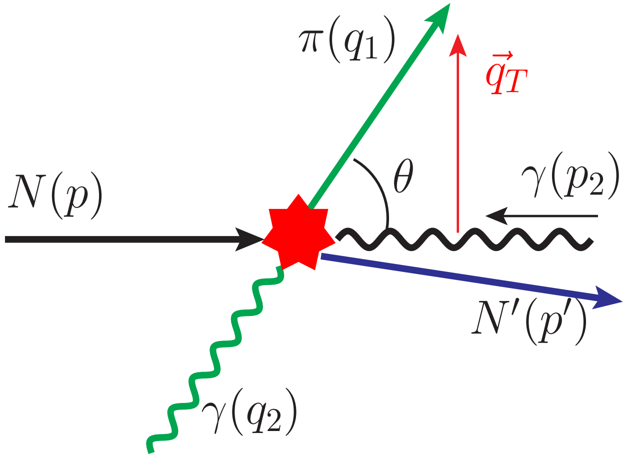
Enhanced x-Sensitivity: (2) γ-π Pair Photoproduction



G. Duplancic et al., JHEP 11 (2018) 179
G. Duplancic et al., JHEP 03 (2023) 241
G. Duplancic et al., PRD 107 (2023), 094023
Qiu & Yu, PRL 131 (2023), 161902

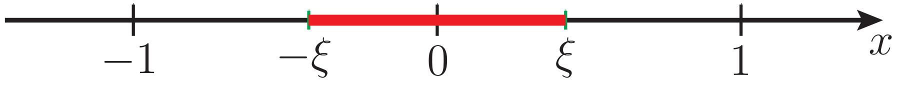
$i\mathcal{M}$ **also contains the special integral:**

$$I'(t,\xi;z,\theta) = \int_{-1}^{1} \frac{dx\, F(x,\xi,t)}{x - \rho'(z;\theta) + i\epsilon}$$

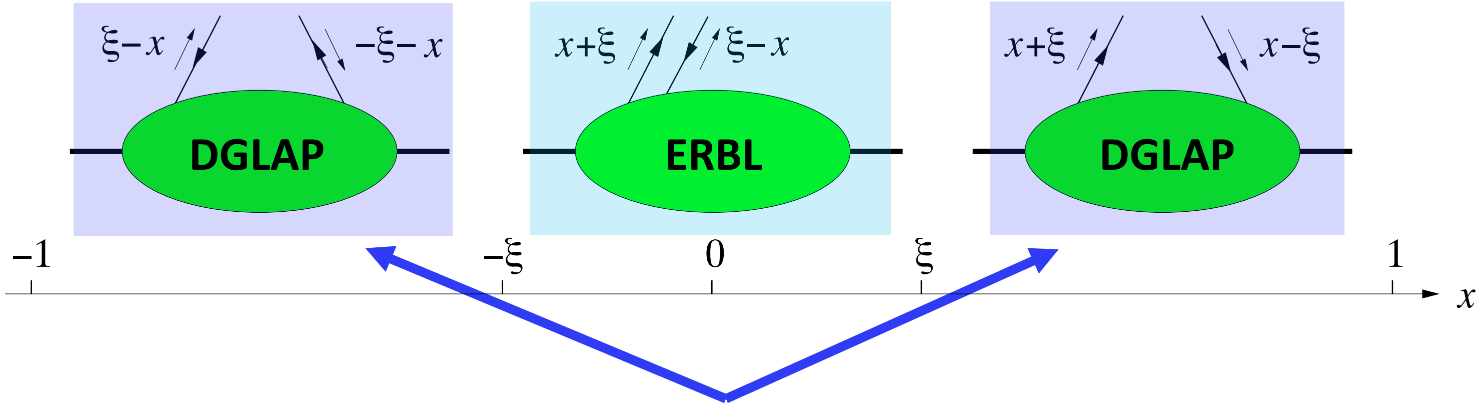
$$\rho'(z;\theta) = \xi \cdot \left[\frac{\cos^2(\theta/2)\,(1-z) - z}{\cos^2(\theta/2)\,(1-z) + z}\right] \in [-\xi, \xi]$$

For DVCS/DVMP

$$\rho'(z,\theta) \to \xi$$



Complementary sensitivity:



$N\,\pi \to N'\,\gamma\,\gamma$

Exclusive Massive Photon-Pair Production in Meson-Hadron Collision

❑ Challenge for QCD factorization: $\pi^-(p_\pi) + P(p) \to \gamma(q_1) + \gamma(q_2) + N(p')$ $\lambda \sim m_\pi/Q,$ $Q \sim q_T$

Gluons in the Glauber region: $k_s = (\lambda^2, \lambda^2, \lambda)\, Q$

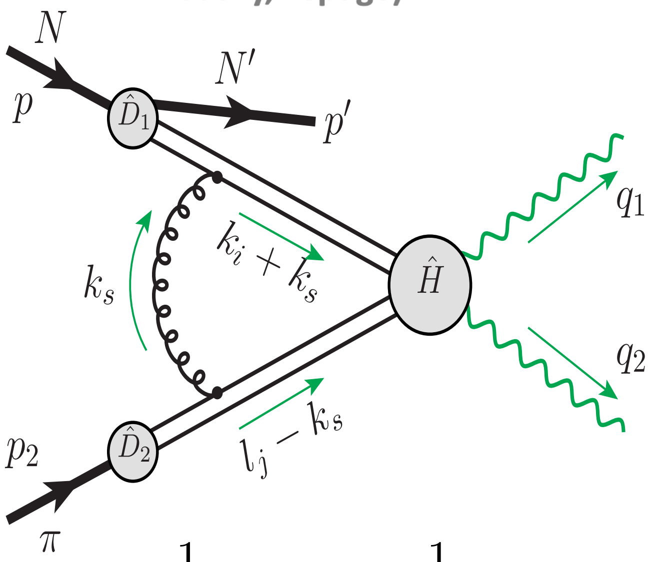
Transverse component contribute to the leading region!

$$k_i = (1, \lambda^2, \lambda)\, Q$$

$$l_j = (\lambda^2, 1, \lambda)\, Q$$

ERBL region

(Efremov, Radyushkin, Brodsky, Lepage)



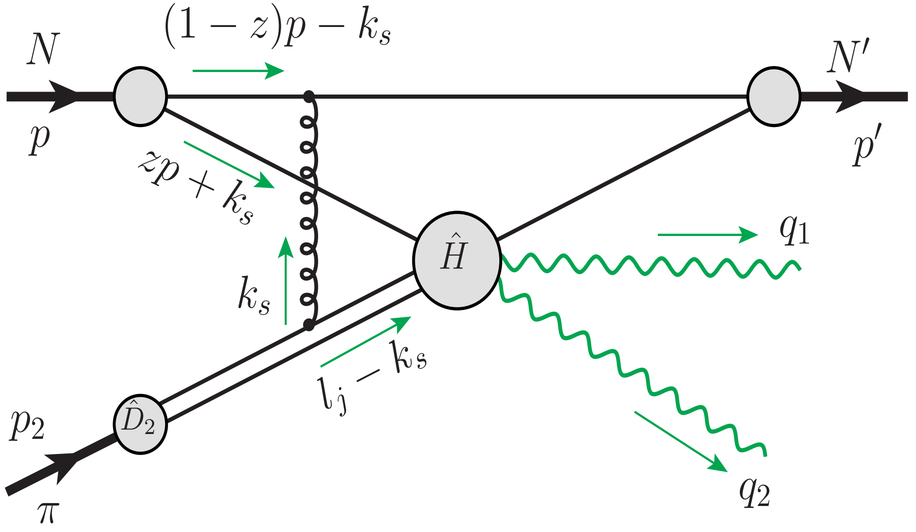
$$\frac{1}{k_s^2 + i\varepsilon} \to \frac{1}{-\mathbf{k}_s^2 + i\varepsilon}$$

$$\frac{1}{(k_i + k_s)^2 + i\varepsilon} \to \frac{1}{k_s^- + i\varepsilon}$$

$$\frac{1}{(l_j - k_s)^2 + i\varepsilon} \to \frac{1}{-k_s^+ + i\varepsilon}$$

No pinch!

DGLAP region



$$\frac{1}{((1-z)p - k_s)^2 + i\varepsilon} \to \frac{1}{k_s^- - i\varepsilon}$$

$$\frac{1}{(zp + k_s)^2 + i\varepsilon} \to \frac{1}{k_s^- + i\varepsilon}$$

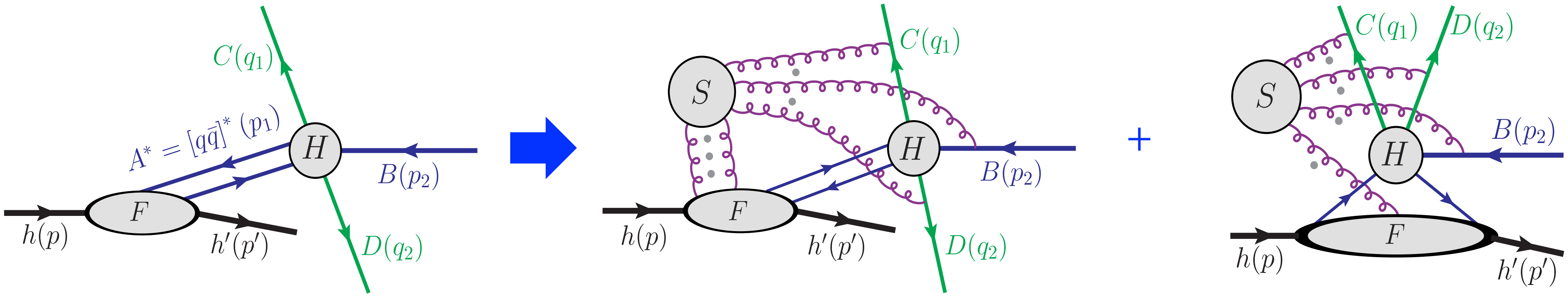
Pinched!

Same conclusion if k_s flows Through N'!

Factorization for SDHEP in the Two-Stage Paradigm

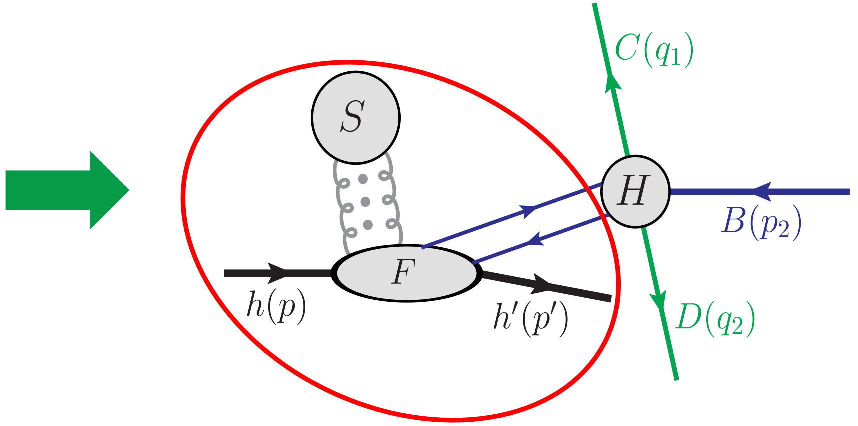
Qiu & Yu, JHEP 08 (2022) 103,
PRD 107 (2023) 1

- **Factorization for 2-parton channel factorization:**



ERBL region: $[q\bar{q}']\sim$ meson **DGLAP region: Glauber pinch**

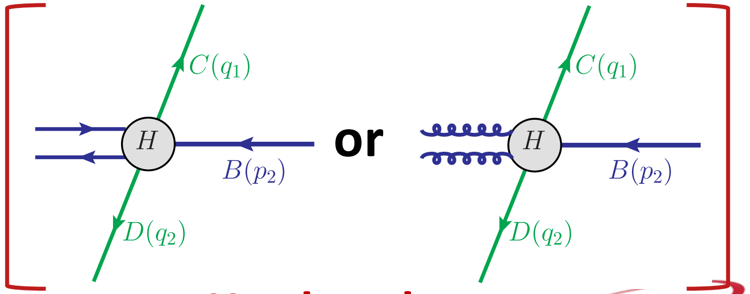
- **Soft gluons cancel when coupling to color neutral hadrons (differs from coupling to jet(s)):**



Glauber gluons of SDHEP:

$$k_s = (\lambda^2, \lambda^2, \lambda) \;\rightarrow\; (1, \lambda^2, \lambda)$$ **Collinear gluons**

$$\mathrm{GPD} \otimes$$



Hard probes

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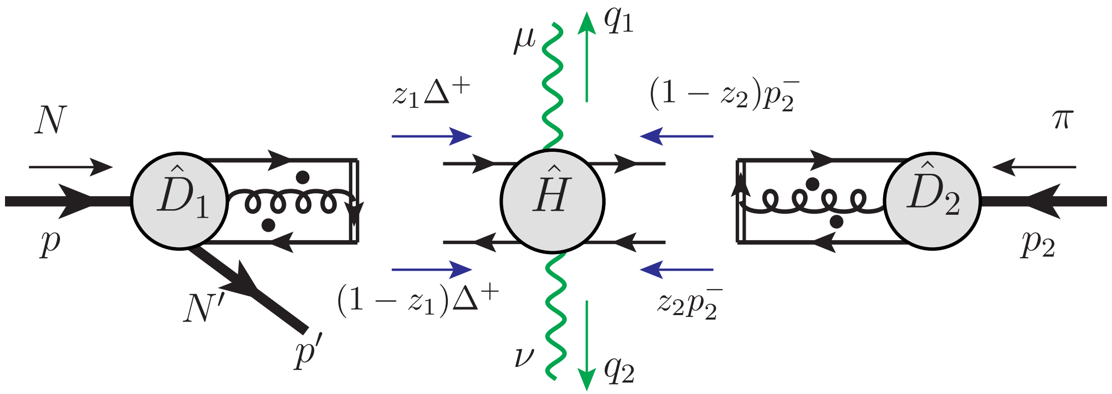
Exclusive Massive Photon-Pair Production in Meson-Hadron Collision

❑ Factorization formula:

$$\pi^-(p_\pi) + P(p) \to \gamma(q_1) + \gamma(q_2) + N(p')$$

Qiu & Yu, JHEP 08 (2022) 103

$$\mathcal{M}^{\mu\nu} = \int dz_1 dz_2 \left[\widetilde{\mathcal{F}}^{ud}_{NN'}(z_1,\xi,t) D(z_2) C^{\mu\nu}(z_1,z_2) + \mathcal{F}^{ud}_{NN'}(z_1,\xi,t) D(z_2) \widetilde{C}^{\mu\nu}(z_1,z_2) \right] + \mathcal{O}(\Lambda_{\rm QCD}/q_T)$$



Similar factorized form for SDHEP with lepton, photon beam

PRD 107 (2023) 1

$$\mathcal{F}^{ud}_{NN'}(z_1,\xi,t) = \int \frac{dy^-}{4\pi} e^{iz_1\Delta^+y^-} \langle N'(p')|\bar{d}(0)\gamma^+\Phi(0,y^-;w_2)\,u(y^-)|N(p)\rangle$$

$$= \frac{1}{2P^+}\left[H^{ud}_{NN'}(z_1,\xi,t)\,\bar{u}(p')\gamma^+u(p) - E^{ud}_{NN'}(z_1,\xi,t)\,\bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m_p}u(p)\right],$$

$$\widetilde{\mathcal{F}}^{ud}_{NN'}(z_1,\xi,t) = \int \frac{dy^-}{4\pi} e^{iz_1\Delta^+y^-} \langle N'(p')|\bar{d}(y^-)\gamma^+\gamma_5\Phi(0,y^-;w_2)\,u(0)|N(p)\rangle$$

$$= \frac{1}{2P^+}\left[\widetilde{H}^{ud}_{NN'}(z_1,\xi,t)\,\bar{u}(p')\gamma^+\gamma_5 u(p) - \widetilde{E}^{ud}_{NN'}(z_1,\xi,t)\,\bar{u}(p')\frac{i\gamma_5\sigma^{+\alpha}\Delta_\alpha}{2m_p}u(p)\right]$$

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Numerical Results

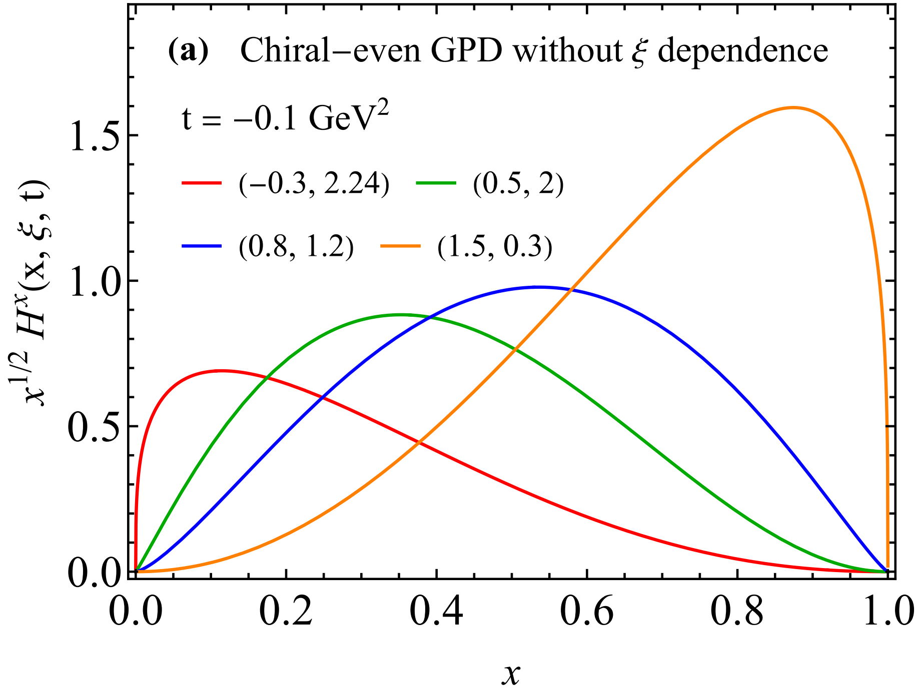
❑ GPD models – simplified GK model:

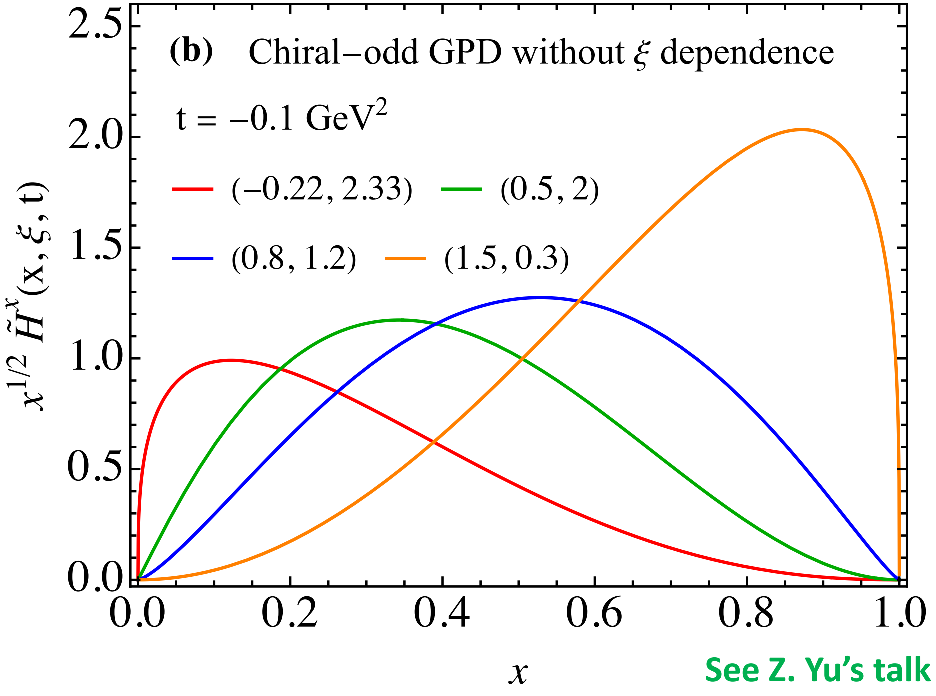
Goloskokov, Kroll
hep-ph/0501242
arXiv: 0708.3569
arXiv: 0906.0460

$$H_{pn}(x,\xi,t) = \theta(x)\, x^{-0.9\,(t/\mathrm{GeV}^2)}\, \frac{x^\rho (1-x)^\tau}{B(1+\rho, 1+\tau)}$$

$$\widetilde{H}_{pn}(x,\xi,t) = \theta(x)\, x^{-0.45\,(t/\mathrm{GeV}^2)}\, \frac{1.267\, x^\rho (1-x)^\tau}{B(1+\rho, 1+\tau)}$$

- Neglect $E, \widetilde{E}$. Neglect evolution effect.
- Tune (ρ, τ) to control x shape.
- Fix DA: $D(z) = N\, z^{0.63}(1-z)^{0.63}$





See Z. Yu's talk on Tuesday

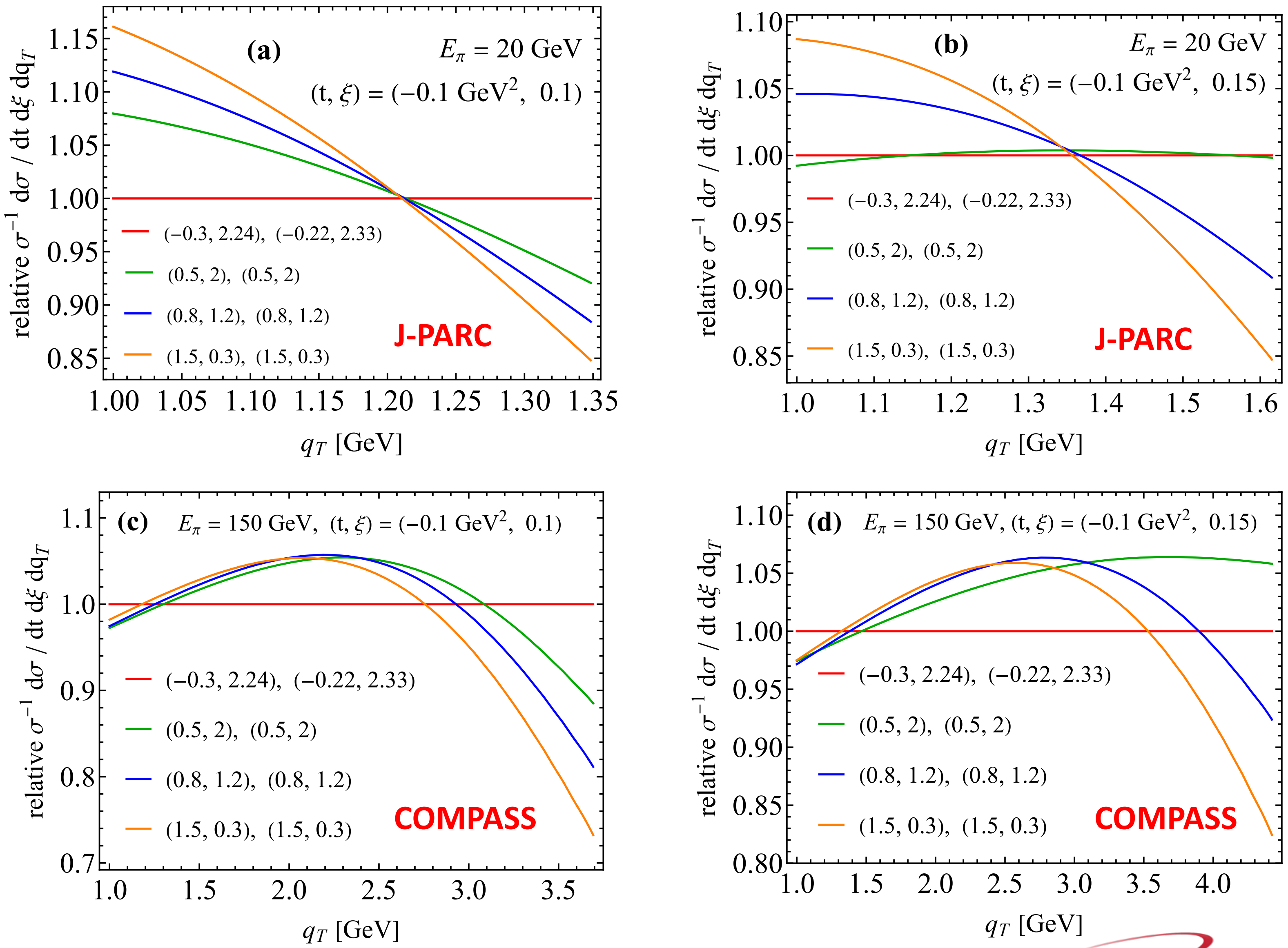
Numerical results

$$\frac{d\sigma}{dt\, d\xi\; dq_T} \sim |H(x,\xi,t)|^2$$

Relative q_T shape

$$\frac{\sigma_{\rm tot}^{-1} d\sigma/dq_T}{\text{some shape func}}$$

$$\sigma_{\rm tot} = \int_{1\ \rm GeV}^{\sqrt{\hat{s}}/2} dq_T \frac{d\sigma}{dt d\xi dq_T}$$



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Why and How QCD factorization works?

❑ Necessary conditions for QCD factorization to work:

- All process-dependent nonperturbative contributions to "good" cross sections are suppressed by powers of O(1/QR), which could be neglected if the hard scale Q is sufficiently large
- All factorizable nonperturbative contributions are process independent, representing the characteristics of identified hadron(s), and
- The process dependence of factorizable contributions is perturbatively calculable from partonic scattering at the short-distance

❑ Predictive power and the value of factorization:

- Our ability to calculate the process-dependent short distance partonic scatterings at the hard scale Q
- Prediction follows when cross sections with different hard scatterings but the same nonperturbative long-distance effect of identified hadron are compared
- Factorization supplies physical content to these universal long-distance effects of identified hadrons by matching them to hadronic matrix elements of active quark and/or gluon operators, which could be interpreted as parton distribution or correlation functions of the identified hadrons, and allows them to be measured experimentally or by numerical simulations and model calculations

Factorization beyond the Leading Power

❑ Single-Hadron – Inclusive DIS:

$$W_{\mu\nu}(q,p,S)=\frac{1}{4\pi}\int d^4z\, e^{iq\cdot z}\,\langle p,S|J_\mu^\dagger(z)J_\nu(0)|p,S\rangle$$

OPE ensures that perturbative factorization is valid to all powers in 1/Q expansion

❑ Single-Hadron – the Role of LQCD:

$$\langle h(P)|\mathcal{O}^{\mathrm{R}}_{q,g}(z,\mu_R)|h(P)\rangle=\sum_f\int_{-1}^{1}\frac{dx}{x}\,\widehat{K}_f(xP\cdot z,z^2,\mu_R,\mu_F)\,\phi_{f/h}(x,\mu_F)+\mathcal{O}(z^2)^n$$

LQCD matrix elements need renormalization!

Lattice “cross section”

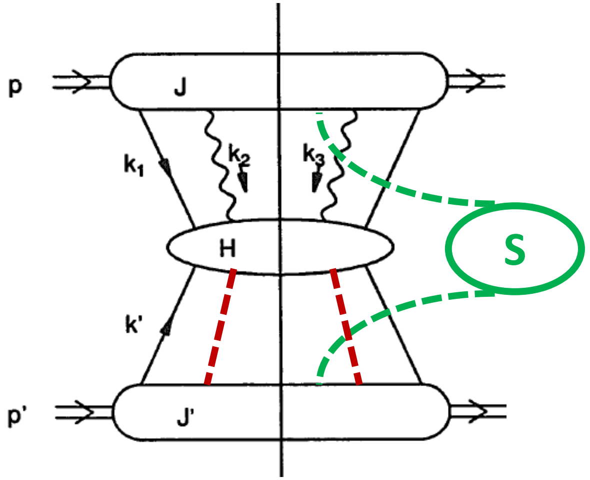
Calculable partonic “Probe”

Matching Parton to Hadron

Approximation “controllable?”

❑ Two-Hadron – Drell-Yan and beyond:

Qiu & Sterman, 1991



Only the first subleading power can be factorized!

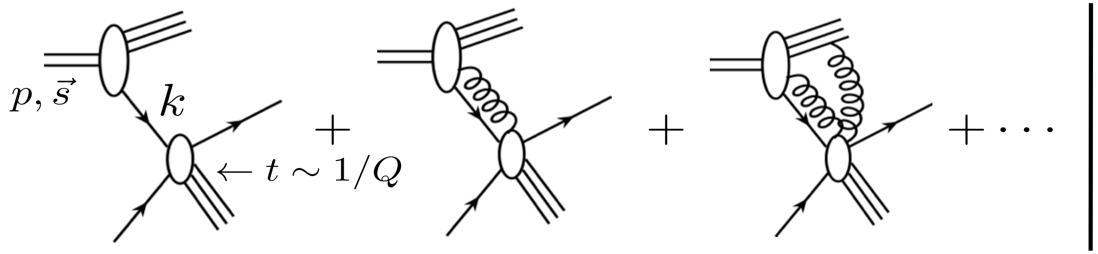
Single scale transverse single-spin asymmetry
(vanishes at the leading power)

Heavy quarkonium production at high pT
(necessary to produce a pair of heavy Q)

…

Factorization at Twist-3 – Transverse Single-Spin Asymmetry

❑ Collinear factorization beyond leading power:

$\sigma(Q,\vec{s}) \propto \Big|$  $\Big|^2$

$\left(\frac{\langle k_\perp \rangle}{Q}\right)^n$ **– Expansion**

$$\sigma(Q, s_T) = H_0 \otimes f_2 \otimes f_2 + (1/Q)\, H_1 \otimes f_2 \otimes f_3 + \mathcal{O}(1/Q^2)$$

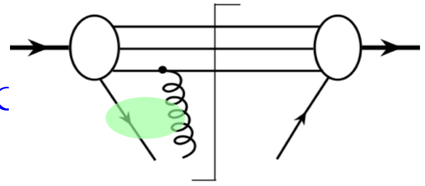
Too large to compete!

Three-parton correlation

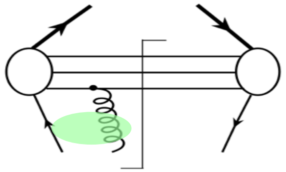
❑ Single transverse spin asymmetry:

$$\Delta\sigma(s_T) \propto T^{(3)}(x,x) \otimes \hat{\sigma}_T \otimes D(z) + \delta q(x) \otimes \hat{\sigma}_D \otimes D^{(3)}(z,z) + \ldots$$

Efremov, Teryaev, 82;
Qiu, Sterman, 91, etc.

$T^{(3)}(x,x) \propto$ 

Qiu, Sterman, 1991, …

$D^{(3)}(z,z) \propto$ 

Kang, Yuan, Zhou, 2010

Integrated information on parton's transverse motion!

Needed Phase: **Integration of "dx" using unpinched poles**

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Twist-3 Distributions Relevant to A_N

- **Twist-2 distributions:**
  - **Unpolarized PDFs:**
  - **Polarized PDFs:**

$$q(x) \propto \langle P|\overline{\psi}_q(0)\frac{\gamma^+}{2}\psi_q(y)|P\rangle$$
$$G(x) \propto \langle P|F^{+\mu}(0)F^{+\nu}(y)|P\rangle(-g_{\mu\nu})$$
$$\Delta q(x) \propto \langle P,S_{\|}|\overline{\psi}_q(0)\frac{\gamma^+\gamma^5}{2}\psi_q(y)|P,S_{\|}\rangle$$
$$\Delta G(x) \propto \langle P,S_{\|}|F^{+\mu}(0)F^{+\nu}(y)|P,S_{\|}\rangle(i\epsilon_{\perp\mu\nu})$$

- **Two-sets Twist-3 correlation functions:** ***No probability interpretation!***

Kang, Qiu, 2009

$$\widetilde{T}_{q,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\langle P,s_T|\overline{\psi}_q(0)\,\frac{\gamma^+}{2}\left[\epsilon^{s_T\sigma n\bar{n}}\,F_\sigma^{\ +}(y_2^-)\right]\psi_q(y_1^-)|P,s_T\rangle$$

$$\widetilde{T}_{G,F}^{(f,d)} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\,\frac{1}{P^+}\langle P,s_T|F^{+\rho}(0)\left[\epsilon^{s_T\sigma n\bar{n}}\,F_\sigma^{\ +}(y_2^-)\right]F^{+\lambda}(y_1^-)|P,s_T\rangle(-g_{\rho\lambda})$$

$$\widetilde{T}_{\Delta q,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\langle P,s_T|\overline{\psi}_q(0)\,\frac{\gamma^+\gamma^5}{2}\left[i\,s_T^\sigma\,F_\sigma^{\ +}(y_2^-)\right]\psi_q(y_1^-)|P,s_T\rangle$$

$$\widetilde{T}_{\Delta G,F}^{(f,d)} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2}\, e^{ixP^+y_1^-}\, e^{ix_2P^+y_2^-}\,\frac{1}{P^+}\langle P,s_T|F^{+\rho}(0)\left[i\,s_T^\sigma\,F_\sigma^{\ +}(y_2^-)\right]F^{+\lambda}(y_1^-)|P,s_T\rangle\,(i\epsilon_{\perp\rho\lambda})$$

Role of color magnetic force!

- **Twist-3 fragmentation functions:**

See Kang, Yuan, Zhou, 2010, Kang 2010

Test QCD at Twist-3 Level

- Scaling violation – "DGLAP" evolution:

Kang, Qiu, 2009

$$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \underbrace{\begin{bmatrix} \widetilde{\mathcal{T}}_{q,F} \\ \widetilde{\mathcal{T}}_{\Delta q,F} \\ \widetilde{\mathcal{T}}^{(f)}_{G,F} \\ \widetilde{\mathcal{T}}^{(d)}_{G,F} \\ \widetilde{\mathcal{T}}^{(f)}_{\Delta G,F} \\ \widetilde{\mathcal{T}}^{(d)}_{\Delta G,F} \end{bmatrix}}_{(x,\, x+x_2,\, \mu,\, s_T)} = \underbrace{\begin{pmatrix} K_{qq} & K_{q\Delta q} & K^{(f)}_{qG} & K^{(d)}_{qG} & K^{(f)}_{q\Delta G} & K^{(d)}_{q\Delta G} \\ K_{\Delta qq} & K_{\Delta q\Delta q} & K^{(f)}_{\Delta qG} & K^{(d)}_{\Delta qG} & K^{(f)}_{\Delta q\Delta G} & K^{(d)}_{\Delta q\Delta G} \\ K^{(f)}_{Gq} & K^{(f)}_{G\Delta q} & K^{(ff)}_{GG} & K^{(fd)}_{GG} & K^{(ff)}_{G\Delta G} & K^{(fd)}_{G\Delta G} \\ K^{(d)}_{Gq} & K^{(d)}_{G\Delta q} & K^{(df)}_{GG} & K^{(dd)}_{GG} & K^{(df)}_{G\Delta G} & K^{(dd)}_{G\Delta G} \\ K^{(f)}_{\Delta Gq} & K^{(f)}_{\Delta G\Delta q} & K^{(ff)}_{\Delta GG} & K^{(fd)}_{\Delta GG} & K^{(ff)}_{\Delta G\Delta G} & K^{(fd)}_{\Delta G\Delta G} \\ K^{(d)}_{\Delta Gq} & K^{(d)}_{\Delta G\Delta q} & K^{(df)}_{\Delta GG} & K^{(dd)}_{\Delta GG} & K^{(df)}_{\Delta G\Delta G} & K^{(dd)}_{\Delta G\Delta G} \end{pmatrix}}_{(\xi,\, \xi+\xi_2;\, x,\, x+x_2,\, \alpha_s)} \underbrace{\otimes}_{\int d\xi \int d\xi_2} \begin{bmatrix} \widetilde{\mathcal{T}}_{q,F} \\ \widetilde{\mathcal{T}}_{\Delta q,F} \\ \widetilde{\mathcal{T}}^{(f)}_{G,F} \\ \widetilde{\mathcal{T}}^{(d)}_{G,F} \\ \widetilde{\mathcal{T}}^{(f)}_{\Delta G,F} \\ \widetilde{\mathcal{T}}^{(d)}_{\Delta G,F} \end{bmatrix}$$

- Evolution equation – consequence of factorization:

Factorization: $\Delta\sigma(Q, s_T) = (1/Q) H_1(Q/\mu_F, \alpha_s) \otimes f_2(\mu_F) \otimes f_3(\mu_F)$

DGLAP for f_2: $\dfrac{\partial}{\partial \ln(\mu_F)} f_2(\mu_F) = P_2 \otimes f_2(\mu_F)$

Evolution for f_3: $\dfrac{\partial}{\partial \ln(\mu_F)} f_3 = \left(\dfrac{\partial}{\partial \ln(\mu_F)} H_1^{(1)} - P_2^{(1)} \right) \otimes f_3$

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Factorization at Twist-4 – Heavy Quarkonium Production

Lee, Qiu, Sterman, Watanabe, 2022

❑ Heavy quarkonium production at high P_T:

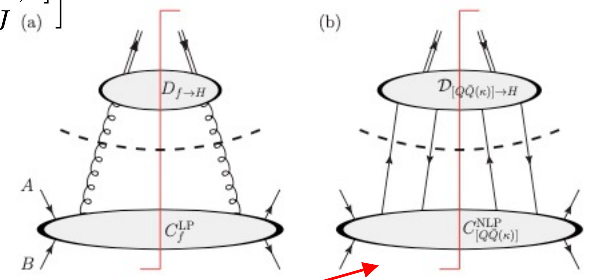
$$E\frac{d\sigma_{hh'\to J/\psi(P)X}}{d^3P} = \sum_{c\bar{c}[n]} F_{c\bar{c}[n]\to J/\psi} \otimes \sum_{a,b} \int dx_a\, f_{a/h}(x_a,\mu_f^2) \int dx_b\, f_{b/h'}(x_b,\mu_f^2) \times E\frac{d\tilde{\sigma}^{\text{Resum}}_{ab\to c\bar{c}[n](P)X}}{d^3P}$$

NRQCD: $F_{c\bar{c}[n]\to J/\psi} = \langle O^{J/\psi}_{c\bar{c}[n]}(0)\rangle$ with $c\bar{c}[n] = c\bar{c}[{}^{2S+1}L_J^{[1,8]}]$

- **Factorized partonic scattering:**

$$E\frac{d\tilde{\sigma}^{\text{Resum}}_{ab\to c\bar{c}[n](P)X}}{d^3P} \approx \sum_f \int \frac{dz}{z^2} D_{f\to c\bar{c}[n]}(z,\mu_f^2) E_f \frac{d\hat{\sigma}_{ab\to f(p_f)X}}{d^3p_f}(z, p_f = P/z, \mu_f^2)$$

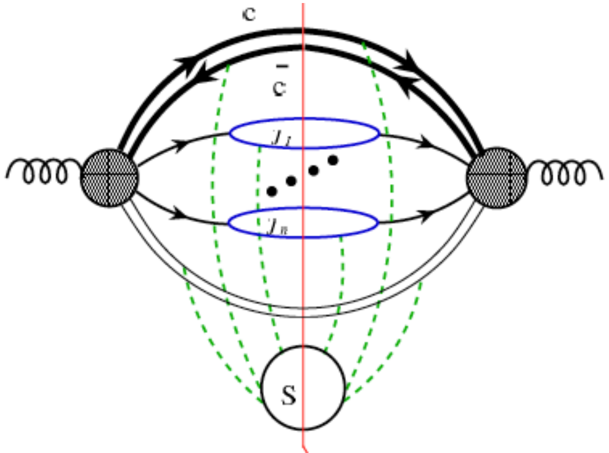
$$+ \sum_{[c\bar{c}(\kappa)]} \int \frac{dz}{z^2} D_{[c\bar{c}(\kappa)]\to c\bar{c}[n]}(z,\mu_f^2) E_c \frac{d\hat{\sigma}_{ab\to[c\bar{c}(\kappa)](p_c)X}}{d^3p_c}(z, p_c = P/z, \mu_f^2)$$

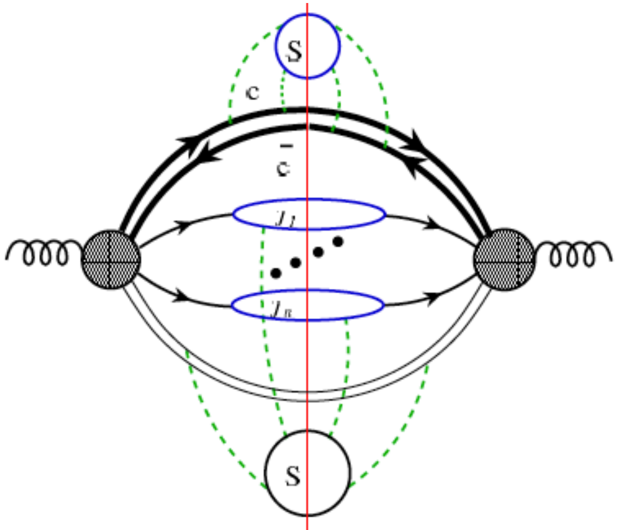


NRQCD factorization for Fragmentation functions

$$\kappa = (v,a,t)^{[1,8]}$$
$$= (\gamma^+, \gamma^+\gamma_5, \gamma^+\gamma_\perp^i)^{[1,8]}$$

Kang, Ma, Qiu, Sterman, 2014





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Renormalization group improvement

Kang, Ma, Qiu, Sterman, PRD 90, 034006 (2014)

❑ Renormalization group:

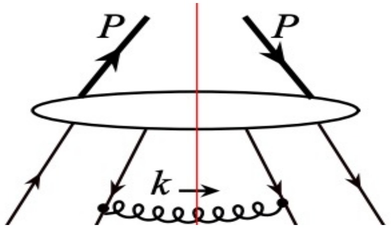
$$\frac{d}{d\ln\mu_f^2}\left[E\frac{d\tilde{\sigma}^{\mathrm{Resum}}_{ab\to c\bar{c}[n](P)X}}{d^3P}\right]=0$$

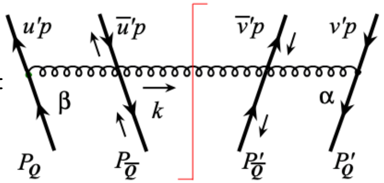
To be accurate up to the 1st power correction

❑ Modified evolution equations:

NRQCD: $H = c\bar{c}[^{2S+1}L_J^{[1,8]}]$

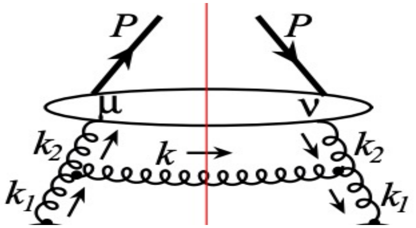
$$\frac{\partial\mathcal{D}_{[Q\bar{Q}(n)]\to H}}{\partial\ln\mu_f^2}=\Gamma_{[Q\bar{Q}(n)]\to[Q\bar{Q}(\kappa)]}\otimes\mathcal{D}_{[Q\bar{Q}(\kappa)]\to H}$$



$\Gamma =$ 

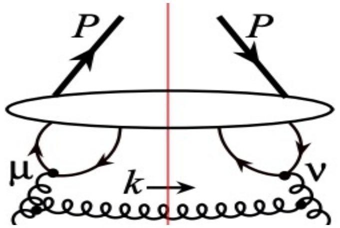
DGLAP-type: **Heavy quark pair produced at the hard scale**

$$\frac{\partial D_{[f]\to H}}{\partial\ln\mu_f^2}=\gamma_{[f]\to[f']}\otimes D_{[f']\to H}$$



Heavy quark pair produced at the input scale

$$+\frac{1}{\mu_f^2}\,\overline{\gamma}_{[f]\to[Q\bar{Q}(\kappa)]}\otimes\mathcal{D}_{[Q\bar{Q}(\kappa)]\to H}$$



$\longleftarrow\quad\overline{\gamma}_{g\to[Q\bar{Q}]}$

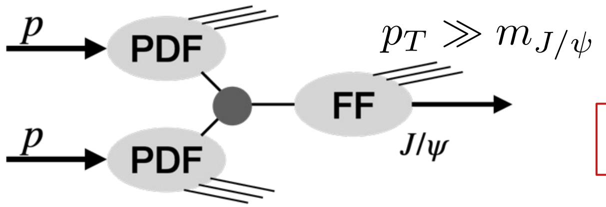
Heavy quark pair produced between the hard scale and the input scale

Modified DGLAP – inhomogeneous evolution

Single inclusive high P_T J/ψ-production in hadronic collisions

❑ Test the consistency:

$$p + p \to J/\psi + X$$



NLO

LO x K_{NLP}

$$\frac{d\sigma_{p+p\to J/\psi+X}}{dp_T} \approx f_{i/p} \otimes f_{j/p} \otimes \left[D_k^{J/\psi} \otimes C_{ij\to k} + D_{c\bar{c}}^{J/\psi} \otimes C_{ij\to c\bar{c}} \right]$$

❑ Input FFs from NRQCD:

Ma, Qiu, Zhang, PRD89 (2014) 094029; ibid. 94030

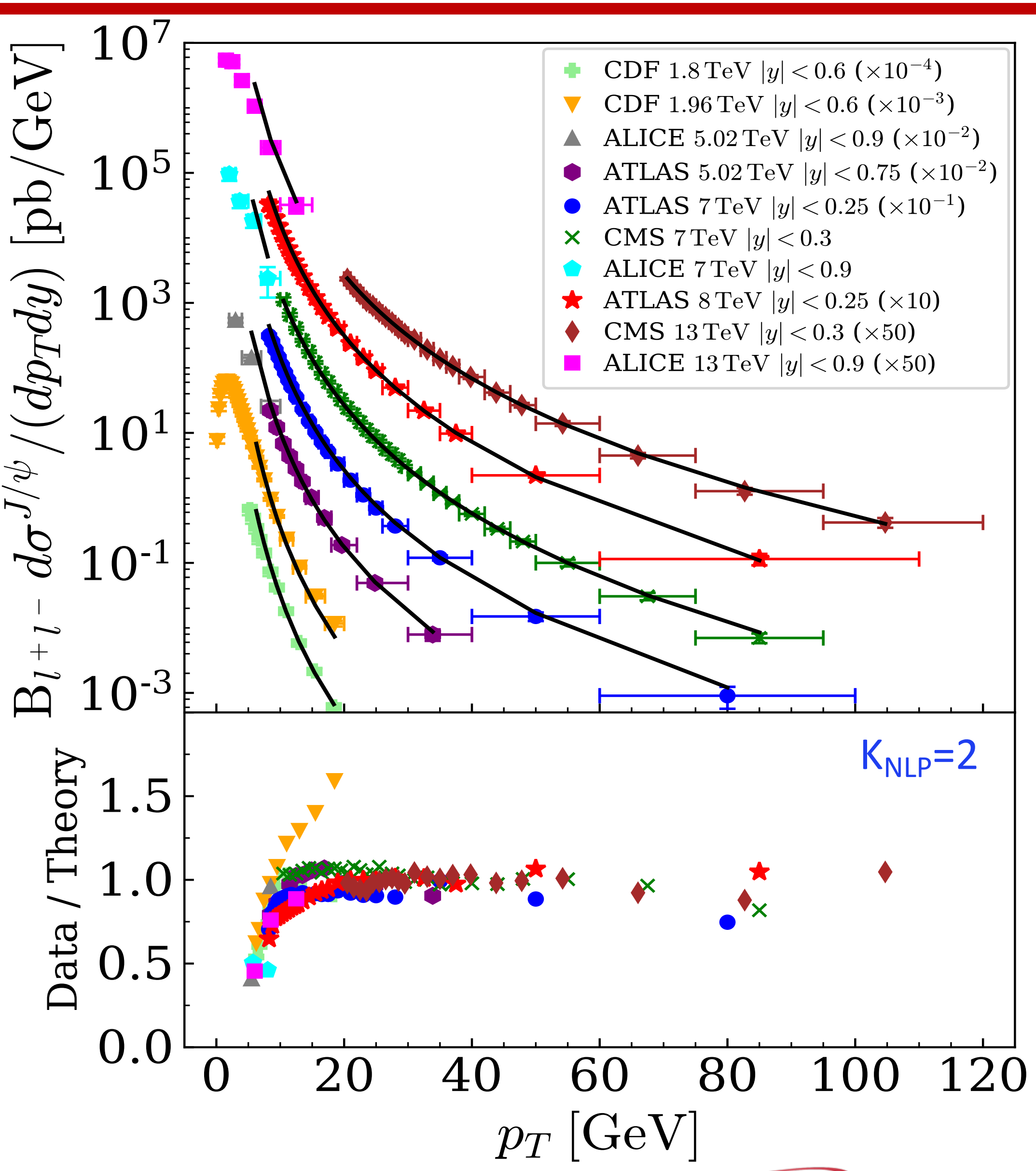
$$D_{f\to H}(z;m,\mu_0) = \sum_{[Q\bar{Q}(n)]} \pi\alpha_s \left\{ \hat{d}^{(1)}_{f\to[Q\bar{Q}(n)]}(z;m,\mu_0,\mu_\Lambda) + \frac{\alpha_s}{\pi}\hat{d}^{(2)}_{f\to[Q\bar{Q}(n)]}(z;m,\mu_0,\mu_\Lambda) + \mathcal{O}(\alpha_s^2) \right\} \frac{\langle \mathcal{O}^H_{[Q\bar{Q}(n)]}(\mu_\Lambda)\rangle}{m^{2L+3}}$$

$$\boxed{\kappa = v^{[c]}, a^{[c]}, t^{[c]}, \quad n = {}^{2S+1}L_J^{[c]}}$$

$$D_{[Q\bar{Q}(\kappa)]\to H}(z;m,\mu_0) = \sum_{[Q\bar{Q}(n)]} \left\{ \hat{d}^{(0)}_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(n)]}(z;m,\mu_0,\mu_\Lambda) + \frac{\alpha_s}{\pi}\hat{d}^{(1)}_{[Q\bar{Q}(\kappa)]\to[Q\bar{Q}(n)]}(z;m,\mu_0,\mu_\Lambda) + \mathcal{O}(\alpha_s^2) \right\} \frac{\langle \mathcal{O}^H_{[Q\bar{Q}(n)]}(\mu_\Lambda)\rangle}{m^{2L+1}}$$

$\mu_0 = \mathcal{O}(2m)$: input scale, $\mu_\Lambda = \mathcal{O}(m)$: NRQCD factorization scale

➡ $$D_{f\to H}(z) = N_f \frac{z^{\alpha_f}(1-z)^{\beta_f}}{B(1+\alpha_f, 1+\beta_f)}$$

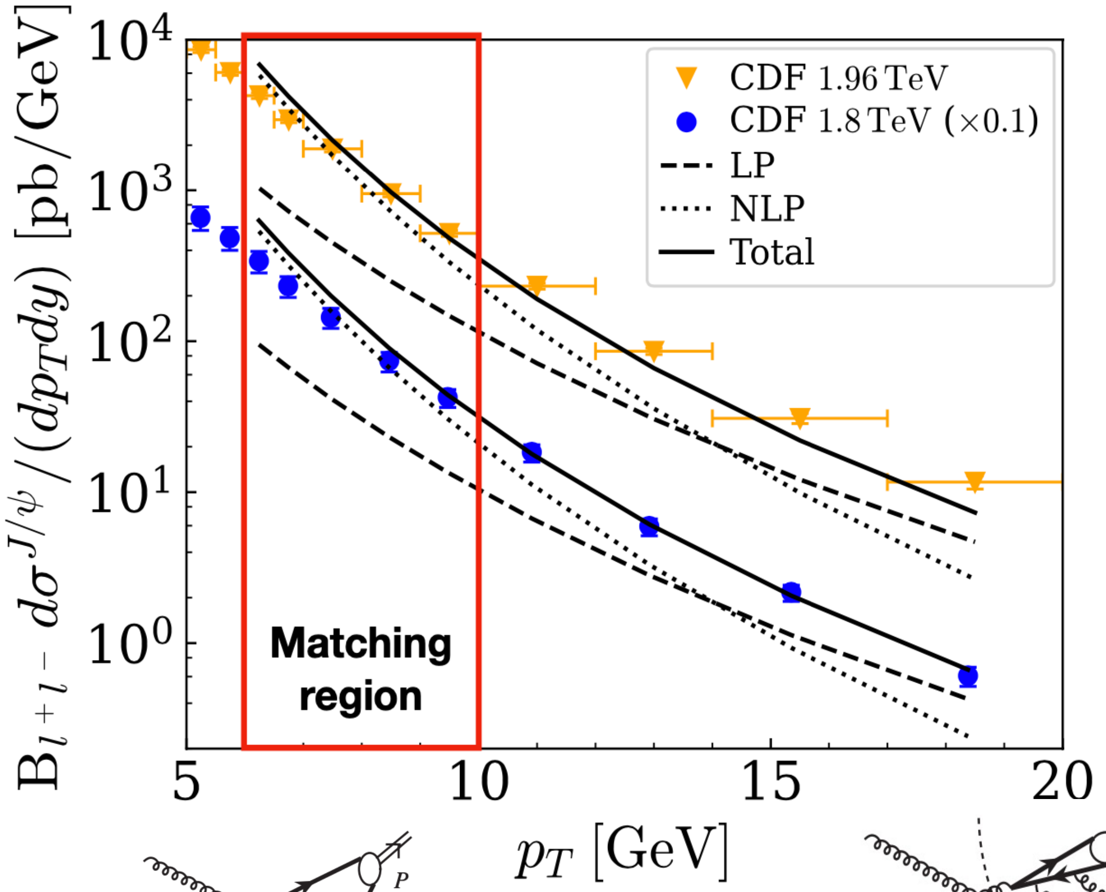


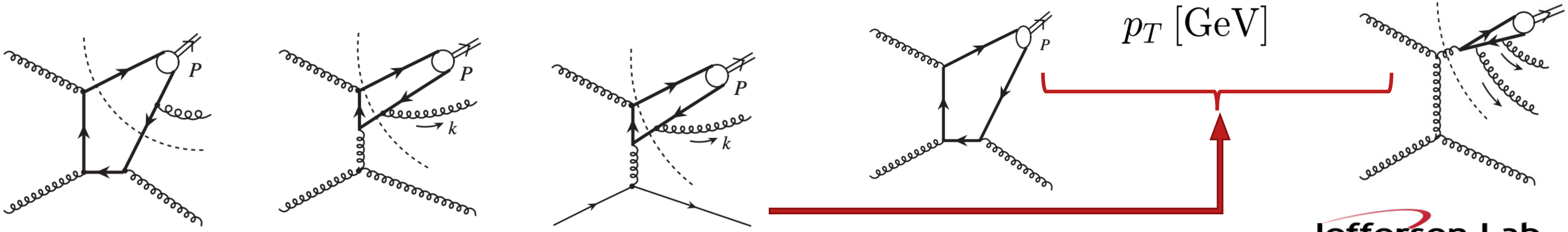
Lee, Qiu, Sterman, Watanabe, 2108.00305, 2211.12648

Matching to fixed-order PQCD calculation

- **Leading power logarithmically enhanced contributions start to dominate when**
  $P_T \gtrsim 5(2m_c) \sim 15$ GeV
- **Next-to-leading power is important for**
  $5(2m_c) \gtrsim P_T \gtrsim (2m_c)$
- **Matching to fixed-order NRQCD calculation**
  $P_T \sim (2m_c)$
  NLP term is necessary for the matching
- **Further improvement by exploring the FFs**
  Use the medium as a filter?

Lee, Qiu, Sterman, Watanabe, 2022





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Summary and Outlook

- Reliable factorization is necessary for probing QCD dynamics with identified hadrons(s)
  - Need for exploring QCD dynamics
  - Need for probing hadron's internal structure
- QCD factorization beyond the leading power is important and necessary
  - It is necessary for heavy quarkonium production where a heavy quark-pair is required
  - It is also necessary for better understanding of QCD contribution to transverse single-spin asymmetries
  - New form of evolution equations and modified scale dependence
- Joint factorization for both QCD and QED is critical for lepton-hadron collisions (not discussed in this talk)
  - QED radiation is a part of production cross sections, treated in the same way as QCD radiation from quarks and gluons
  - No artificial and/or process dependent scale(s) introduced for treating QED radiation, other than the standard factorization scale, universal lepton distribution and fragmentation functions
  - All perturbatively calculable hard parts are IR safe for both QCD and QED
  - All lepton mass or resolution sensitivity are included into "Universal" lepton distribution and fragmentation functions (or jet functions)

Liu, Melnitchouk, Qiu, Sato,
Phys.Rev.D 104 (2021) 094033
JHEP 11 (2021) 157

Thank you!

Backups

How to Calculate the Perturbative Parts?

- **Use DIS structure function F_2 as an example:**

$$F_{2h}(x_B,Q^2)=\sum_{q,f} C_{q/f}\left(\frac{x_B}{x},\frac{Q^2}{\mu^2},\alpha_s\right)\otimes\varphi_{f/h}\left(x,\mu^2\right)+O\left(\frac{\Lambda^2_{\text{QCD}}}{Q^2}\right)$$

  - **Apply the factorized formula to a parton state:** $h \to q$

Feynman diagrams $\longrightarrow$ $F_{2q}(x_B,Q^2)=\sum_{q,f} C_{q/f}\left(\frac{x_B}{x},\frac{Q^2}{\mu^2},\alpha_s\right)\otimes\varphi_{f/q}\left(x,\mu^2\right)$ $\longleftarrow$ **Feynman diagrams**

  - **Express both SFs and PDFs in terms of powers of α_s:**

0th order:

$$F^{(0)}_{2q}(x_B,Q^2)=C^{(0)}_q(x_B/x,Q^2/\mu^2)\otimes\varphi^{(0)}_{q/q}\left(x,\mu^2\right)$$

$$\Longrightarrow \quad \boxed{C^{(0)}_q(x)=F^{(0)}_{2q}(x)} \qquad \varphi^{(0)}_{q/q}\left(x\right)=\delta_{qq}\delta\left(1-x\right)$$

1th order:

$$F^{(1)}_{2q}(x_B,Q^2)=C^{(1)}_q(x_B/x,Q^2/\mu^2)\otimes\varphi^{(0)}_{q/q}\left(x,\mu^2\right)$$
$$+C^{(0)}_q(x_B/x,Q^2/\mu^2)\otimes\varphi^{(1)}_{q/q}\left(x,\mu^2\right)$$

$$\Longrightarrow \quad \boxed{C^{(1)}_q(x,Q^2/\mu^2)=F^{(1)}_{2q}(x,Q^2)-F^{(0)}_{2q}(x,Q^2)\otimes\varphi^{(1)}_{q/q}\left(x,\mu^2\right)}$$

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PDFs of a Parton

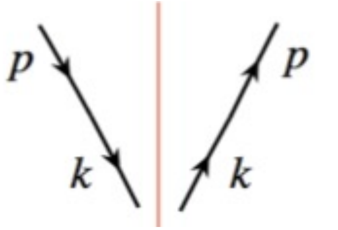
❑ Change the state without changing the operator:

$$\phi_{q/h}(x,\mu^2) = \int \frac{dy^-}{2\pi}\, e^{ixp^+y^-}\, \langle h(p)|\overline{\psi}_q(0)\frac{\gamma^+}{2}\, U^n_{[0,y^-]}\,\psi_2(y^-)|h(p)\rangle$$

$|h(p)\rangle \;\Rightarrow\; |\text{parton}(p)\rangle$ ➡ $\phi_{f/q}(x,\mu^2)$ **– given by Feynman diagrams**

❑ Lowest order quark distribution:

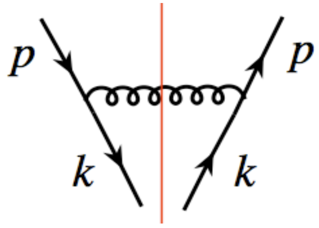
✧ From the operator definition:



$$\begin{aligned}\phi^{(0)}_{q'/q}(x) &= \delta_{qq'}\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}\left[\left(\frac{1}{2}\gamma\cdot p\right)\left(\frac{\gamma^+}{2p^+}\right)\right]\delta\left(x-\frac{k^+}{p^+}\right)(2\pi)^4\delta^4(p-k)\\ &= \delta_{qq'}\;\delta(1-x)\end{aligned}$$

❑ Leading order in α_s quark distribution:

✧ Expand to $(g_s)^2$ – logarithmic divergent:



$+\ldots$

$$\phi^{(1)}_{q/q}(x) = C_F\frac{\alpha_s}{2\pi}\int\frac{dk_T^2}{k_T^2}\left[\frac{1+x^2}{(1-x)_+}+\frac{3}{2}\,\delta(1-x)\right] + \text{UVCT}$$

UV and CO divergence

Choice of regularization

Partonic Cross Sections

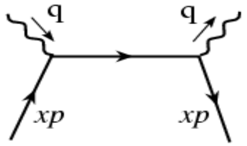
❑ Projection operators for SFs:

$$W_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) F_1\left(x, Q^2\right) + \frac{1}{p\cdot q}\left(p_\mu - q_\mu \frac{p\cdot q}{q^2}\right)\left(p_\nu - q_\nu \frac{p\cdot q}{q^2}\right) F_2\left(x, Q^2\right)$$

$$\boxed{\begin{aligned} F_1(x,Q^2) &= \frac{1}{2}\left(-g^{\mu\nu} + \frac{4x^2}{Q^2} p^\mu p^\nu\right) W_{\mu\nu}(x,Q^2) \\ F_2(x,Q^2) &= x\left(-g^{\mu\nu} + \frac{12x^2}{Q^2} p^\mu p^\nu\right) W_{\mu\nu}(x,Q^2) \end{aligned}}$$

❑ 0th order:

$$F_{2q}^{(0)}(x) = x g^{\mu\nu} W_{\mu\nu,q}^{(0)} = x g^{\mu\nu} \left[\frac{1}{4\pi} \right.$$



$$\left. \right]$$

$$= \left(x g^{\mu\nu}\right) \frac{e_q^2}{4\pi} \mathrm{Tr}\left[\frac{1}{2} \gamma\cdot p \gamma_\mu \gamma\cdot\left(p+q\right)\gamma_\nu\right] 2\pi\delta\left((p+q)^2\right)$$

$$= e_q^2 x \delta(1-x)$$

$$\boxed{C_q^{(0)}(x) = e_q^2 x\delta(1-x)}$$

NLO Coefficient Function – a Complete Example

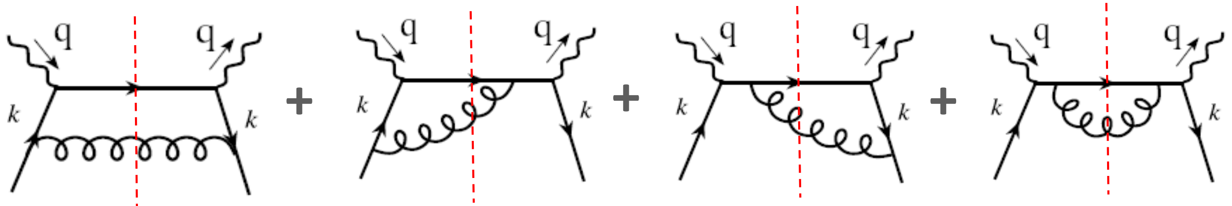
$$\boxed{C_q^{(1)}(x, Q^2/\mu^2) = F_{2q}^{(1)}(x, Q^2) - F_{2q}^{(0)}(x, Q^2) \otimes \varphi_{q/q}^{(1)}(x, \mu^2)}$$

- **Projection operators in n-dimension:** $g_{\mu\nu} g^{\mu\nu} = n \equiv 4 - 2\varepsilon$

$$\boxed{(1-\varepsilon) F_2 = x\left(-g^{\mu\nu} + (3-2\varepsilon)\frac{4x^2}{Q^2} p^\mu p^\nu \right) W_{\mu\nu}}$$

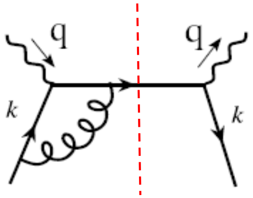
- **Feynman diagrams:**

$W_{\mu\nu,q}^{(1)}$

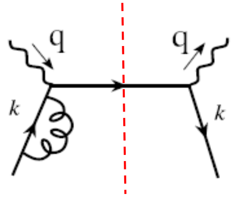


Real

Virtual { +



+ c.c. [+



+ c.c. + UV CT]

- **Calculation:**

$$-g^{\mu\nu} W_{\mu\nu,q}^{(1)} \quad \text{and} \quad p^\mu p^\nu W_{\mu\nu,q}^{(1)}$$

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Contribution from the Trace of $W_{\mu\nu}$

- **Lowest order in n-dimension:**

$$-g^{\mu\nu}W^{(0)}_{\mu\nu,q} = e_q^2(1-\varepsilon)\delta(1-x)$$

- **NLO virtual contribution:**

$$-g^{\mu\nu}W^{(1)V}_{\mu\nu,q} = e_q^2(1-\varepsilon)\delta(1-x)$$

$$*\left(-\frac{\alpha_s}{\pi}\right)C_F\left[\frac{4\pi\mu^2}{Q^2}\right]^{\varepsilon}\frac{\Gamma(1+\varepsilon)\Gamma^2(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\left[\frac{1}{\varepsilon^2}+\frac{3}{2}\frac{1}{\varepsilon}+4\right]$$

- **NLO real contribution:**

$$-g^{\mu\nu}W^{(1)R}_{\mu\nu,q} = e_q^2(1-\varepsilon)C_F\left(-\frac{\alpha_s}{2\pi}\right)\left[\frac{4\pi\mu^2}{Q^2}\right]^{\varepsilon}\frac{\Gamma(1+\varepsilon)}{\Gamma(1-2\varepsilon)}$$

$$*\left\{-\frac{1-\varepsilon}{\varepsilon}\left[1-x+\left(\frac{2x}{1-x}\right)\left(\frac{1}{1-2\varepsilon}\right)\right]+\frac{1-\varepsilon}{2(1-2\varepsilon)(1-x)}+\frac{2\varepsilon}{1-2\varepsilon}\right\}$$

Jefferson Lab

Contribution from the trace of $W_{\mu\nu}$

❑ The "+" distribution:

$$\left(\frac{1}{1-x}\right)^{1+\varepsilon} = -\frac{1}{\varepsilon}\delta(1-x) + \frac{1}{(1-x)_+} + \varepsilon\left(\frac{\ell n(1-x)}{1-x}\right)_+ + O\left(\varepsilon^2\right)$$

$$\int_z^1 dx \frac{f(x)}{(1-x)_+} \equiv \int_z^1 dx \frac{f(x)-f(1)}{1-x} + \ell n(1-z) f(1)$$

❑ One loop contribution to the trace of $W_{\mu\nu}$:

$$-g^{\mu\nu} W^{(1)}_{\mu\nu,q} = e_q^2(1-\varepsilon)\left(\frac{\alpha_s}{2\pi}\right)\left\{-\frac{1}{\varepsilon} P_{qq}(x) + P_{qq}(x)\ell n\left(\frac{Q^2}{\mu^2(4\pi e^{-\gamma_E})}\right)\right.$$
$$+C_F\left[\left(1+x^2\right)\left(\frac{\ell n(1-x)}{1-x}\right)_+ - \frac{3}{2}\left(\frac{1}{1-x}\right)_+ - \frac{1+x^2}{1-x}\ell n(x)\right.$$
$$\left.\left. +3 - x - \left(\frac{9}{2}+\frac{\pi^2}{3}\right)\delta(1-x)\right]\right\}$$

❑ Splitting function:

$$P_{qq}(x) = C_F\left[\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\delta(1-x)\right]$$

One-Loop Contribution to Partonic F2 and Quark-PDF:

❑ One loop contribution to $p^\mu p^\nu W_{\mu\nu}$:

$$\left(1-\varepsilon\right)F_2 = x\left(-g^{\mu\nu}+(3-2\varepsilon)\frac{4x^2}{Q^2}p^\mu p^\nu\right)W_{\mu\nu}$$

$$p^\mu p^\nu W^{(1)V}_{\mu\nu,q} = 0 \qquad p^\mu p^\nu W^{(1)R}_{\mu\nu,q} = e_q^2 C_F \frac{\alpha_s}{2\pi}\frac{Q^2}{4x}$$

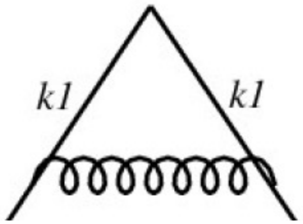
❑ One loop contribution to F_2 of a quark:

$$F^{(1)}_{2q}(x,Q^2) = e_q^2 x \frac{\alpha_s}{2\pi}\left\{\left(-\frac{1}{\varepsilon}\right)_{\mathrm{CO}} P_{qq}(x)\left(1+\varepsilon\,\ell n(4\pi e^{-\gamma_E})\right) + P_{qq}(x)\,\ell n\left(\frac{Q^2}{\mu^2}\right)\right.$$
$$\left. + C_F\left[(1+x^2)\left(\frac{\ell n(1-x)}{1-x}\right)_+ - \frac{3}{2}\left(\frac{1}{1-x}\right)_+ - \frac{1+x^2}{1-x}\ell n(x) + 3 + 2x - \left(\frac{9}{2}+\frac{\pi^2}{3}\right)\delta(1-x)\right]\right\}$$

$$\Rightarrow \quad \infty \quad \text{as} \;\; \varepsilon \to 0$$

❑ One loop contribution to quark PDF of a quark:

$$\varphi^{(1)}_{q/q}(x,\mu^2) = \left(\frac{\alpha_s}{2\pi}\right)P_{qq}(x)\left\{\left(\frac{1}{\varepsilon}\right)_{\mathrm{UV}} + \left(-\frac{1}{\varepsilon}\right)_{\mathrm{CO}}\right\} + \text{UV-CT}$$



– in the dimensional regularization

Different UV-CT = different factorization scheme!

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NLO Coefficient Function for Inclusive DIS (at EIC):

❑ Common UV-CT terms:

✧ **MS scheme:**

$$\text{UV-CT}\big|_{\text{MS}} = -\frac{\alpha_s}{2\pi} P_{qq}(x)\left(\frac{1}{\varepsilon}\right)_{\text{UV}}$$

✧ **$\overline{\text{MS}}$ scheme:**

$$\text{UV-CT}\big|_{\overline{\text{MS}}} = -\frac{\alpha_s}{2\pi} P_{qq}(x)\left(\frac{1}{\varepsilon}\right)_{\text{UV}} \left(1+\varepsilon \ell n(4\pi e^{-\gamma_E})\right)$$

✧ **DIS scheme: choose a UV-CT, such that**

$$C_g^{(1)}(x, Q^2/\mu^2)\big|_{\text{DIS}} = 0$$

❑ One loop coefficient function:

$$\boxed{C_q^{(1)}(x,Q^2/\mu^2) = F_{2q}^{(1)}(x,Q^2) - F_{2q}^{(0)}(x,Q^2) \otimes \varphi_{q/q}^{(1)}\left(x,\mu^2\right)}$$

$$C_q^{(1)}(x,Q^2/\mu^2) = e_q^2 x \frac{\alpha_s}{2\pi}\left\{ P_{qq}(x)\ell n\left(\frac{Q^2}{\mu_{\overline{\text{MS}}}^2}\right) \right.$$

$$\left. + C_F\left[(1+x^2)\left(\frac{\ell n(1-x)}{1-x}\right)_+ - \frac{3}{2}\left(\frac{1}{1-x}\right)_+ - \frac{1+x^2}{1-x}\ell n(x) + 3 + 2x - \left(\frac{9}{2}+\frac{\pi^2}{3}\right)\delta(1-x)\right]\right\}$$

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