Exclusive processes and generalized parton distributions

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Jefferson Lab

I) Exclusive processes in lepton-hadron scattering

Kinematic variables
Collinear frames and light-cone components
L/T currents
Cross section
Meson production and Virtual Compton Scattering VCS

"kinematics"

II) Asymptotic behavior and factorization

Bjorken regime
QCD factorization
Quantum correlation functions
DVCS factorization from collinear expansion
Generalized parton distributions

"dynamics" ←

III) Asymptotic behavior and factorization II

Meson production factorization and GPDs
GPD evolution
Characteristics of large and small x
Heavy quarkonium production near threshold
TBD

Notice

Slides are work in progress
References to literature still missing; will be added

High energy and momentum transfer regime

$W^2, Q^2 \gg \mu_{\text{had}}^2$ Energy / momentum transfer $\gg$ hadronic scale

$\mu_{\text{had}}^2 \sim M_\rho^2 \approx 0.6\ \text{GeV}^2$ reasonable reference - meson mass, nucleon size

- Can we predict/compute the asymptotic behavior of the amplitudes of exclusive processes?

- Can we describe exclusive processes in terms of QCD degrees of freedom and interactions?
  Asymptotic freedom: Effective coupling decreases at short distances
  Non-perturbative effects suppressed at short distances

- Can we use exclusive processes to quantify and measure the quark/gluon structure of hadrons?

Asymptotics enables systematic theory: Parametric expansion, ordering principle
Practical applicability to be demonstrated

Asymptotic regimes

$Q^2 \sim W^2 \to \infty$

Deep-inelastic limit (Bjorken limit)

$x_B \sim Q^2/W^2$ fixed ← considered here

DVCS, meson production

$W^2 \to \infty, \; Q^2$ fixed

High-energy limit (Gribov-Regge limit)

$x_B \to 0$

High-energy vector meson production

$Q^2 \to \infty, \; W^2$ fixed

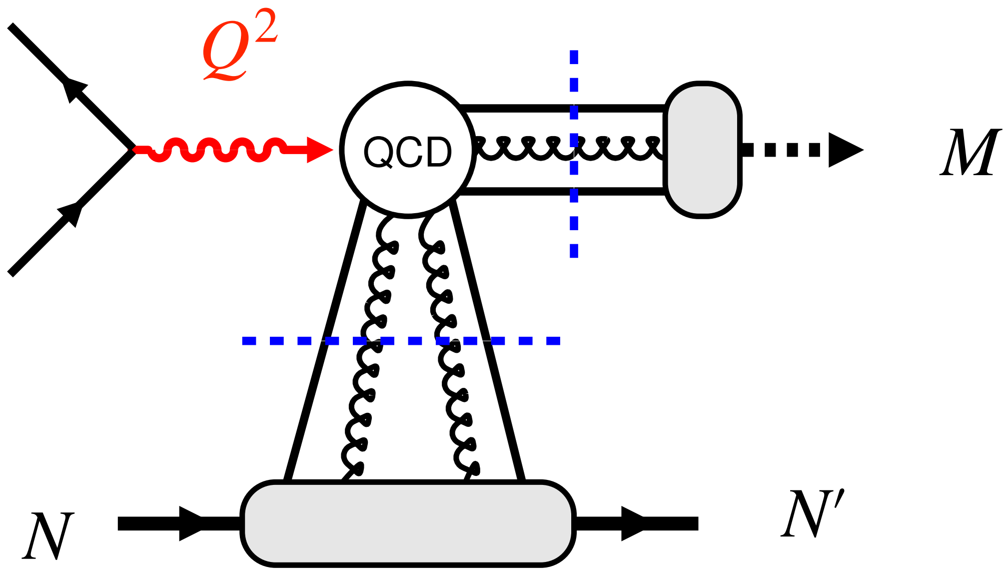
High-momentum elastic or transition form factors

$x_B \to 1$

Near-threshold production $\gamma^* N \; \to \; \pi N, \; \bar{Q}QN$

In processes involving production of heavy quarkonia $Q\bar{Q}$, the heavy quark mass m_Q can serve as the scale for the asymptotic expansion

Factorization



Amp $(\gamma^* N \to M N')$

$= \langle M \,|\, \Phi \ldots \Phi \,|\, 0 \rangle$

$\times$ Amp(γ^* – quark/gluon)

$\times \langle N' \,|\, \Phi \ldots \Phi \,|\, N \rangle$

Separation of scales $Q^2 \longleftrightarrow \mu^2_{\text{had}}$

Quark-gluon scattering process involving momenta $\sim Q$ (meaning invariant scalars $\sim Q^2$)
mediated by perturbative QCD interactions

Correlation functions of QCD fields in hadron states $(\Phi = \psi, \bar{\psi}, F)$
describing distributions of quarks/gluons at scale μ_{had}

$\langle N' \,|\, \Phi(x') \ldots \Phi(x) \,|\, N \rangle$ "generalized parton distributions"

$\langle M \,|\, \Phi(y') \ldots \Phi(y) \,|\, 0 \rangle$ "distribution amplitude"

Exact definition and properties
to be elaborated in following

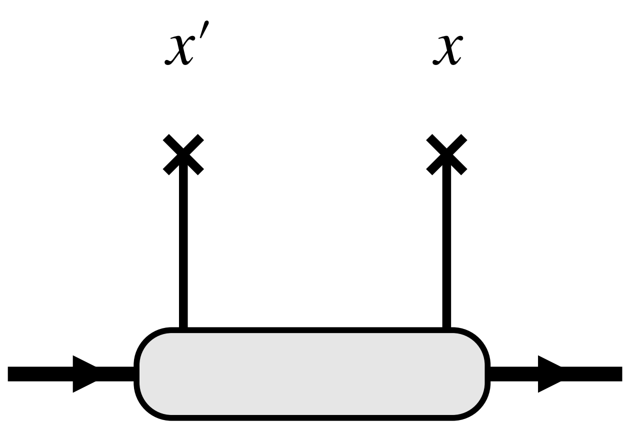
Factorization

Asymptotic behavior given by

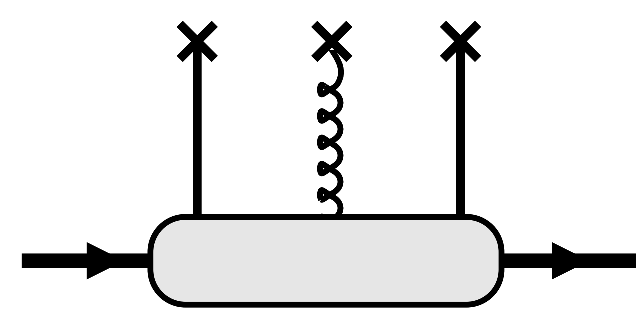
Light-like separation of fields in correlation functions: $(x' - x)^2 = 0$

Minimal number of physical fields (up to gauge invariance)

Major simplifications, make program practical!
Simple renormalization properties of operators in correlation function
Small number of structures in matrix element

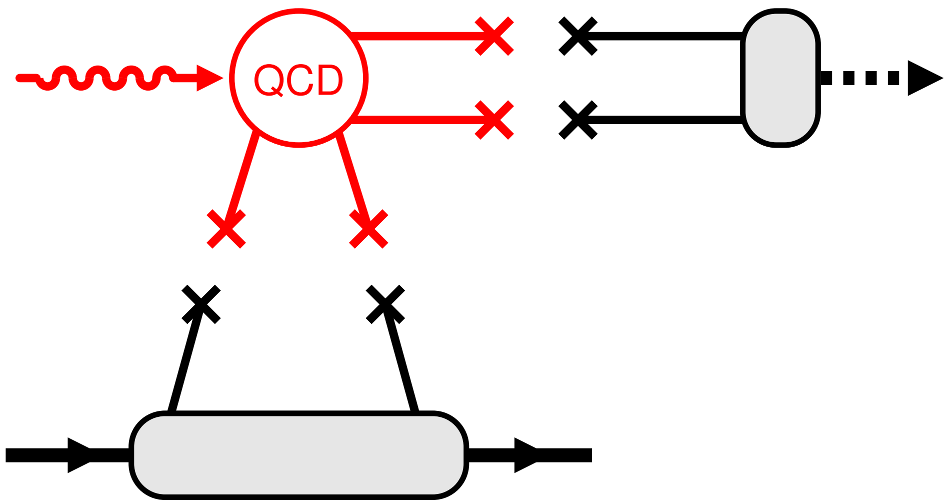


$(x' - x)^2 = 0$



$$\langle N' \,|\, \Phi(x') \ldots \Phi(x) \,|\, N \rangle$$

Factorization



Alt. view: Operator expansion

QCD process as transition operator

Expansion in "basis operators"

$$\bar{\psi}(x')\,[x',x]\,\Gamma\,\psi(x)\;\Big|_{(x'-x)^2=0}$$

$$[x',x] \;=\; \mathrm{P}\,\exp\left[i\int_x^{x'} dy^\alpha A_\alpha(y)\right]$$

y along path between x and x'

$y = \lambda x' + (1-\lambda)x, \;\; 0 < \lambda < 1$

$A_\alpha \equiv A_\alpha^a \, \frac{1}{2}\lambda^a$ QCD gauge potential

Similarly: Gluon light-ray operators $F(x')\ldots F(x)$

Light-ray operators

Nonlocal composite operators

Gauge-invariant, contain gauge link (connection)

Can be represented as series of local tensor operators classified by twist = dimension - spin: Twist-2, 3, 4 etc.

Renormalization → scale dependence

Factorization: Collinear frame

Factorization performed in frame where external momenta q, P are collinear along z-axis

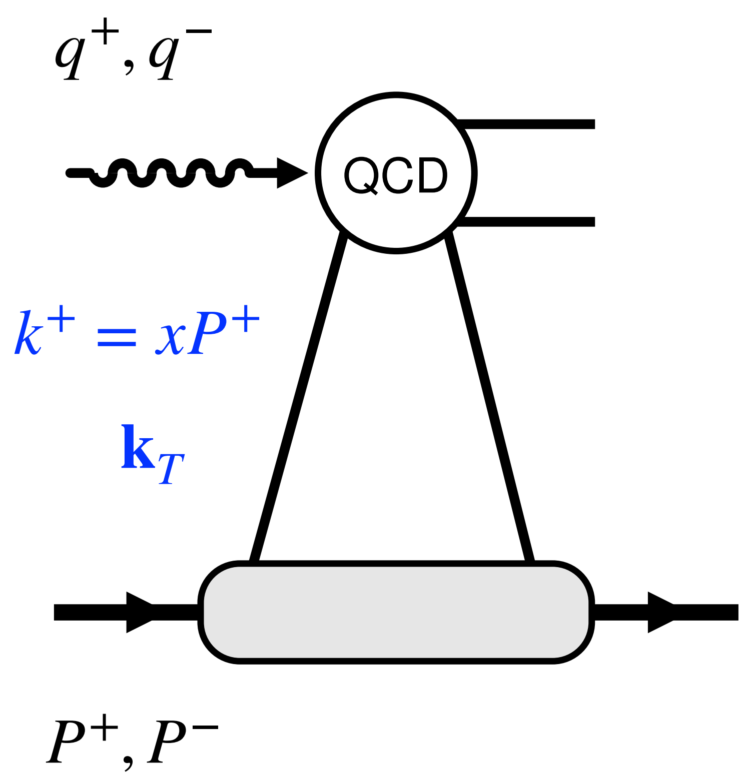
$$\mathbf{q}_T, \mathbf{P}_T = 0 \qquad q^\pm, P^\pm \neq 0 \qquad P^+ q^- \sim Q^2$$

Light-like separation of fields in correlation function along collinear direction

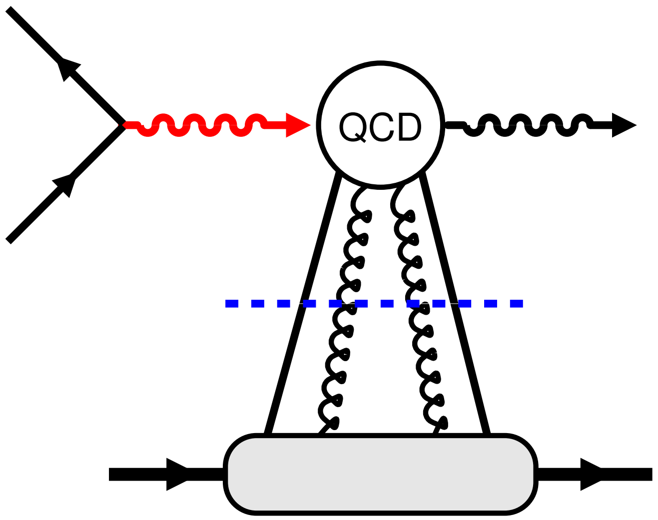
Momenta of quarks/gluons entering in QCD process

Longitudinal: Determine kinematics of QCD process, produce scalars $\sim Q^2$

Transverse: Cannot produce scalars $\sim Q^2$, determine phase space of QCD process, integrated over in correlation function



DVCS: Factorization



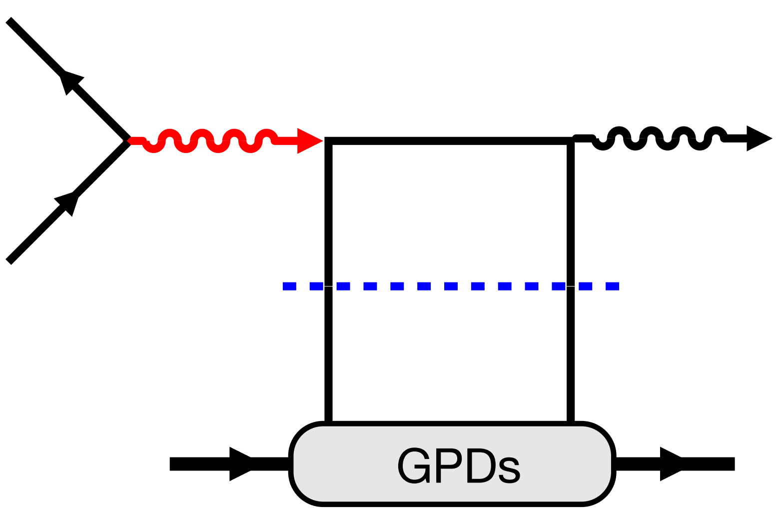
Apply factorization to deeply-virtual Compton scattering

$$l + N \to l' + \gamma + N' \qquad Q^2, W^2 \gg \mu^2_{\text{had}}$$

Simplest exclusive process

Leading asymptotic contribution to amplitude contained in virtual Compton scattering on free quark $\gamma^* q \to \gamma q'$ ("handbag graph")

No pQCD interaction required at LO, amplitude $\mathcal{O}(g^0)$



Outgoing real photon couples to quarks through pointlike QED vertex; no distribution amplitude involved at leading power accuracy in $1/Q^2$

Involves twist-2 generalized parton distributions

Factorization of DVCS amplitude formally similar to that of DIS cross section

DVCS: Calculation

1. Compute Compton amplitude from “handbag graph” with unspecified quark correlation function

2. Parametrize external and internal momenta in collinear frame

3. Perform collinear expansion of loop integral, neglecting terms $k_T^2/Q^2, k^2/Q^2$, reducing the quark correlation function to light-cone distances

4. Parametrize light-cone quark correlation function in terms of GPDs, including spin-flavor structure

DVCS: Compton amplitude from handbag graph

$$T^{\mu\nu} \equiv i \int d^4 z \, e^{i(q'+q)z/2} \, \langle N' | \, T \, J^\mu(-z/2) \, J^\nu(z/2) \, | N \rangle$$

Compton tensor, general

$$J^\mu(x) = \bar{\psi}(x) \, \gamma^\mu \, \psi(x)$$

EM current of quark field
(one flavor, unit charge)

$$T \, J^\mu(-z/2) \, J^\nu(z/2) \; = \; \underset{2}{\bar{\psi}}(-z/2) \, \gamma^\mu \, \underset{1}{\psi}(-z/2) \, \underset{1}{\bar{\psi}}(z/2) \, \gamma^\mu \, \underset{2}{\psi}(z/2)$$

Apply Wick's theorem.
Two contractions 1, 2

$$T \psi(x) \, \bar{\psi}(y) \; = \; iG(x-y)$$

Quark propagator in coordinate representation

$$T \, J^\mu(-z/2) \, J^\nu(z/2) \; = - \, \bar{\psi}(-z/2) \, \gamma^\mu \, G(-z) \, \gamma^\nu \psi(z/2) \; + \; (\mu \leftrightarrow \nu, z \rightarrow -z)$$

Contractions 1 + 2

$$= \; \mathrm{tr} \left[\gamma^\mu \, G(-z) \, \gamma^\nu \psi(z/2) \, \bar{\psi}(-z/2) \; + \; (\mu \leftrightarrow \nu, z \rightarrow -z) \right]$$

Anticommuted fields to form
density $\psi \bar{\psi}$, trace in spinor ind.

$$T^{\mu\nu} = i \int d^4 z \, e^{i(q+q')z/2} \, \mathrm{tr} \Big[\gamma^\mu \, G(-z) \, \gamma^\nu \, \langle N' | \, \psi(z/2) \, \bar{\psi}(-z/2) \, | N \rangle$$
$$+ \, \gamma^\nu \, G(z) \, \gamma^\mu \, \langle N' | \, \psi(-z/2) \, \bar{\psi}(z/2) \, | N \rangle \Big]$$

Nucleon matrix element

DVCS: Compton amplitude from handbag graph

Switch to momentum representation of quark subprocess

$$G(z) = \int \frac{d^4k}{(2\pi)^4} e^{-ikz} \frac{1}{k\gamma + i0}$$

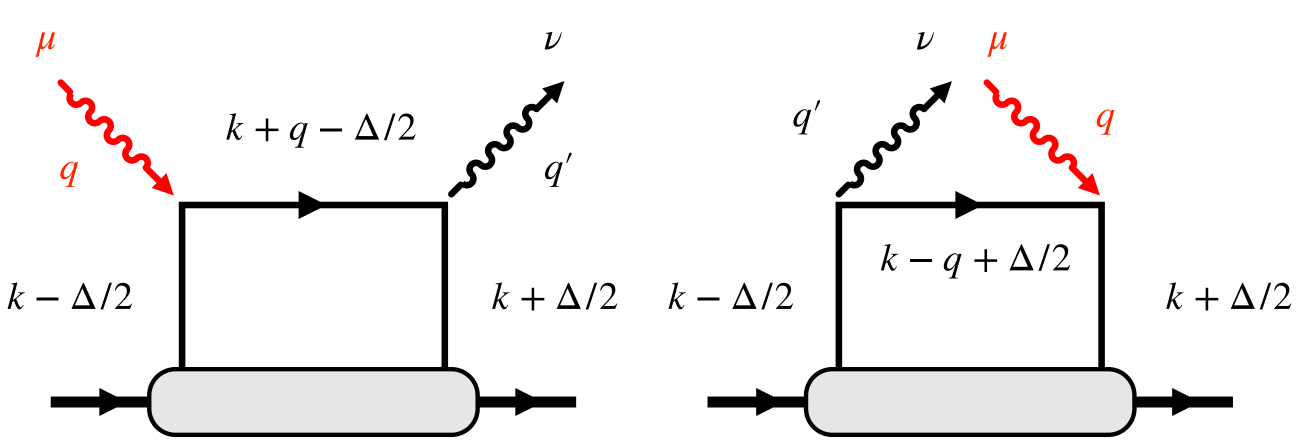
Quark propagator in momentum representation
Loop momentum k integration variable, can be shifted

$$\frac{q+q'}{2} = q + \frac{\Delta}{2} \qquad \Delta \equiv q - q' = \text{momentum transfer}$$

Momenta in Compton amplitude Fourier integral

Shift loop momentum in contractions 1 and 2 such that terms can be combined.
Change $z \to -z$ in contraction 2

$$T^{\mu\nu} = i \int \frac{d^4k}{(2\pi)^4} \int d^4z \, e^{ikz} \, \mathrm{tr} \left[\left(\gamma^\mu \frac{1}{(k-q+\Delta/2)\gamma + i0} \gamma^\nu + \gamma^\nu \frac{1}{(k+q-\Delta/2)\gamma + i0} \gamma^\mu \right) \times \langle N' | \psi(z/2) \, \bar{\psi}(-z/2) \, | N \rangle \right]$$



Agrees with result obtained by applying Feynman rules in momentum space

DVCS: Compton amplitude from handbag graph

- The expression derived from the handbag graph is an intermediate result, to be simplified further by the collinear expansion. The handbag graph contains the leading asymptotic contribution in $1/Q^2$. Its features beyond that, e.g. higher power corrections in $1/Q^2$ should not be taken seriously.

- The nonlocal quark density in the handbag graph, as it stands, is not a gauge-invariant QCD operator. Gauge invariance will be restored in the asymptotic contribution obtained from collinear expansion.

- The handbag amplitude, as it stands, is not electromagnetically gauge invariant. EM gauge invariance will be restored in the asymptotic contribution

DVCS: Collinear frame

$\Delta \equiv q - q' = P' - P \qquad t \equiv \Delta^2 < 0$ — Momentum transfer to nucleon

$\bar{P} \equiv (P' + P)/2 \qquad \bar{M}^2 \equiv \bar{P}^2 = m^2 - t/4 > 0$ — Average of initial and final nucleon momenta

Here: Use frame where q and $\bar{P}$ collinear (convenient, other choices possible)

External momenta:

$$\bar{P} = \left[\bar{P}^+, \frac{\bar{M}^2}{\bar{P}^+}, \mathbf{0}_T\right] \qquad q = \left[-2\xi\bar{P}^+, \frac{Q^2}{2\xi\bar{P}^+}, \mathbf{0}_T\right]$$

Light-cone components $[+, -, T]$
Satisfy $\bar{P}^2 = \bar{M}^2,\ q^2 = -Q^2$

$$\Delta = \left[-2\eta\bar{P}^+,\ -\frac{t + |\mathbf{\Delta}_T|^2}{2\eta\bar{P}^+}, \mathbf{\Delta}_T\right]$$

Satisfies $\Delta^2 = t$

Parameters ξ and η determined by the conditions: $Pq = \left(\bar{P} - \frac{\Delta}{2}\right) q = \frac{Q^2}{2x_B} \qquad \bar{P}\Delta = 0$

$$\xi = \frac{x_B}{2}\left(1 + \text{terms } \frac{\{x_B^2 m^2, t\}}{Q^2}\right) \qquad \eta = \xi\left(1 + \text{terms } \frac{\{x_B^2 m^2, t\}}{Q^2}\right)$$

DVCS: Collinear expansion of Compton amplitude

$$T^{\mu\nu} = i \int \frac{d^4 k}{(2\pi)^4} \int d^4 z \, e^{ikz} \, \mathrm{tr} \left[\left(\frac{\gamma^\mu (k - q + \Delta/2)\cdot\gamma \, \gamma^\nu}{(k - q + \Delta/2)^2 + i0} + \frac{\gamma^\nu (k + q - \Delta/2)\cdot\gamma \, \gamma^\mu}{(k + q - \Delta/2)^2 + i0} \right) \times \langle N' | \psi(z/2) \, \bar\psi(-z/2) | N \rangle \right]$$

Compton amplitude as loop integral

$$k = \left[x\bar{P}^+, \frac{k^2 + |\mathbf{k}_T|^2}{x\bar{P}^+}, \mathbf{k}_T \right]$$

Loop momentum in collinear frame
k^- expressed through $k^2, |\mathbf{k}_T|^2$

$$\int d^4 k = \frac{1}{2} \int dk^+ dk^- d^2 k_T = \frac{\bar{P}^+}{2} \int dx \; dk^- d^2 k_T$$

Integration measure in light-cone components

Expand quark propagators in integral to capture leading asymptotic contribution

$Q^2 \quad \longleftrightarrow \quad k^2, k_T^2 \sim \mu^2_{\mathrm{had}} \quad \Delta^2, \mathbf{\Delta}_T^2, m^2 \sim \mu^2_{\mathrm{had}}$ "hard" $\longleftrightarrow$ "soft"

Expansion performed under integral, approximates integrand in dominant region
("Method of regions")

DVCS: Collinear expansion of Compton amplitude

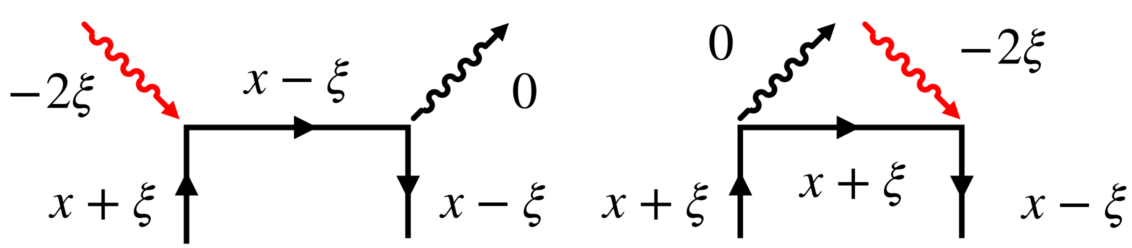
Denominators of quark propagators:

$$
\begin{aligned}
(k-\Delta/2+q)^2+i0 \;&=\; (\ldots)^+(\ldots)^- - (\ldots)_T^2 \\
&=\; (x+\eta-2\xi)\left(\frac{k^2+|\mathbf{k}|_T^2}{x}+\frac{\Delta^2+|\boldsymbol{\Delta}|_T^2}{\eta}+\frac{Q^2}{2\xi}\right)-(\mathbf{k}_T-\boldsymbol{\Delta}_T/2)^2\;+i0 \\
&=\; (x-\xi)\,\frac{Q^2}{2\xi}+\text{terms}\,\mu_{\text{had}}^2\;+\;i0 \\
&=\; (x-\xi+i0)\,\frac{Q^2}{2\xi}\left(1+\frac{\mu_{\text{had}}^2}{Q^2}\right)
\end{aligned}
$$

[Terms $\sim\mu_{\text{had}}^2$ marked in blue]

$$
(k+\Delta/2-q)^2+i0 \;=-\,(x+\xi-i0)\,\frac{Q^2}{2\xi}\left(1+\frac{\mu_{\text{had}}^2}{Q^2}\right)
$$

Denominators given by “effective plus momentum” of quark propagator times hard scale



Plus momentum flow in hard process

DVCS: Collinear expansion of Compton amplitude

Numerators of quark propagators:

$$(k \mp \Delta/2 \pm q) \cdot \gamma \;=\; \frac{1}{2}(\ldots)^{+}\gamma^{-} \;+\; \frac{1}{2}(\ldots)^{-}\gamma^{+} \;-\; (\ldots)_T \cdot \gamma_T$$

$$= \pm \frac{1}{2} q^{-}\gamma^{+} + \text{ terms } \mu_{\text{had}} \qquad = \pm \frac{Q^2}{4\xi \bar{P}^{+}}\gamma^{+} + \text{ terms } \mu_{\text{had}}$$

$$\gamma^{\mu}\gamma^{+}\gamma^{\nu},\; \gamma^{\nu}\gamma^{+}\gamma^{\mu} \;=\; g_T^{\mu\nu}\gamma^{+} \;\pm\; i\epsilon_T^{\mu\nu}\gamma^{+}\gamma_5 \;+\ldots$$

From gamma matrix algebra

$$\mu, \nu = i, j: \quad g_T^{ij} = \delta^{ij},\; \epsilon_T^{ij} = \epsilon^{ij}$$

Unit tensors in transverse space, symmetric/antisymmetric

In leading-order expansion

Compton tensor $T^{\mu\nu} \rightarrow$ transverse tensors

Spinor projection of quark correlation function $\rightarrow \gamma^{+}$

DVCS: Collinear expansion of Compton amplitude

Final result for Compton tensor:

$$T^{\mu\nu} = g_T^{\mu\nu} \int_{-1}^{1} dx \left(\frac{1}{x - \xi + i0} + \frac{1}{x + \xi - i0} \right) \times \int \frac{dz^-}{2\pi} e^{ix\bar{P}^+ z^-} \langle P' | \bar{\psi}(-z/2)\, \gamma^+ \psi(z/2) | P \rangle_{z^+, z_T = 0}$$

$$+ \epsilon_T^{\mu\nu} \int_{-1}^{1} dx \left(\frac{1}{x - \xi + i0} - \frac{1}{x + \xi - i0} \right) \times \int \frac{dz^-}{2\pi} e^{ix\bar{P}^+ z^-} \langle P' | \bar{\psi}(-z/2)\, \gamma^+ \gamma_5\, \psi(z/2) | P \rangle_{z^+, z_T = 0}$$

Quark subprocess amplitudes
in collinear approximation

Correlation functions of quark fields
with light-like separation

- Include quark flavor: $\sum_{f=u,d} e_f^2\, \bar{\psi}_f \ldots \psi_f$ $\quad e_f$ = quark charge

- Leading asymptotic behavior of DVCS amplitude in limit $Q^2 \gg \mu^2_{\text{had}}$

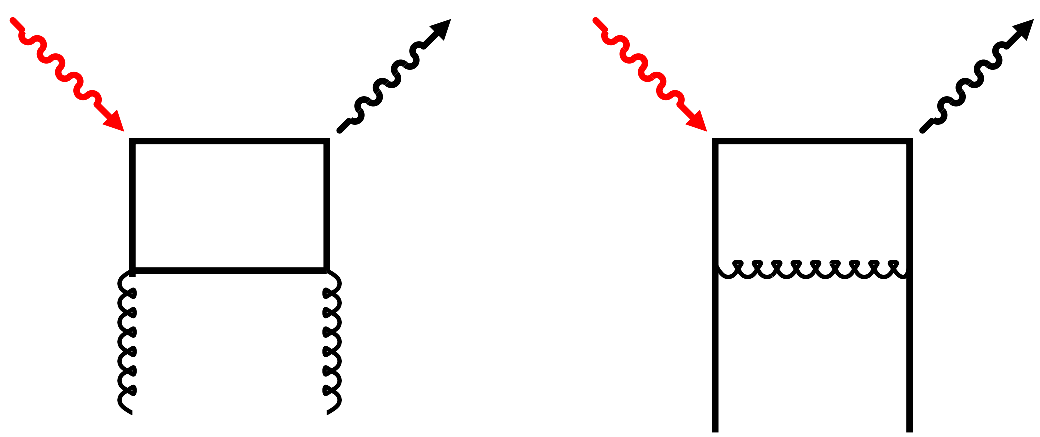
- Amplitude independent of Q^2 in asymptotic regime (Bjorken scaling).
  Cross section depends on Q^2 through kinematic factors

- QCD gauge invariance can be restored by including gauge link along light-like path $[-z/2, z/2]$.
  Results agrees with result of operator methods where gauge invariance maintained throughout.

DVCS: Collinear expansion of Compton amplitude

- Asymptotic result derived assuming quarks in hadron have $k^2, \mathbf{k}_T^2 \sim \mu^2_{\text{had}}$.
  Valid at tree level, but needs to be modified when quantum corrections are included.
  → QCD evolution, logarithmic scale dependence

- Factorization can be extended to higher order in perturbative coupling:
  QCD subprocess amplitudes $\mathcal{O}(g^2)$, contributions from quark and gluon correlators



- Power corrections $\sim \mu^2_{\text{had}}/Q^2$ can arise from several sources, e.g. t/Q^2 terms, m^2/Q^2 terms, other spin projections in field correlators, higher field correlators. They can be classified within the collinear expansion but are difficult to evaluate. They may not be factorizable.

DVCS: Generalized parton distributions

Parametrization of correlation function

$$\int \frac{dz^-}{2\pi} e^{ix\bar{P}^+ z^-} \langle P' | \bar{\psi}(-z/2)\, \gamma^+\, \psi(z/2)\, | P \rangle_{z^+, z_T = 0} \quad = \quad \bar{U}(P', \sigma') \left[\gamma^+ H(x, \xi, t) \; + \; \frac{i\sigma^{+\nu}\Delta_\nu}{2m} E(x, \xi, t) \right] U(P, \sigma)$$

$$\int \frac{dz^-}{2\pi} e^{ix\bar{P}^+ z^-} \langle P' | \bar{\psi}(-z/2)\, \gamma^+ \gamma_5\, \psi(z/2)\, | P \rangle_{z^+, z_T = 0} \quad = \quad \bar{U}(P', \sigma') \left[\gamma^+ \gamma_5\, \tilde{H}(x, \xi, t) \; + \; \frac{\gamma_5\, \Delta^+}{2m} \tilde{E}(x, \xi, t) \right] U(P, \sigma)$$

$U(P, \sigma), \bar{U}(P', \sigma')$ Bispinors of initial/final nucleon (spin wave functions)

$H, E, \tilde{H}, \tilde{E} =$ functions(x, ξ, t) Generalized parton distributions = form factors of light-ray operator